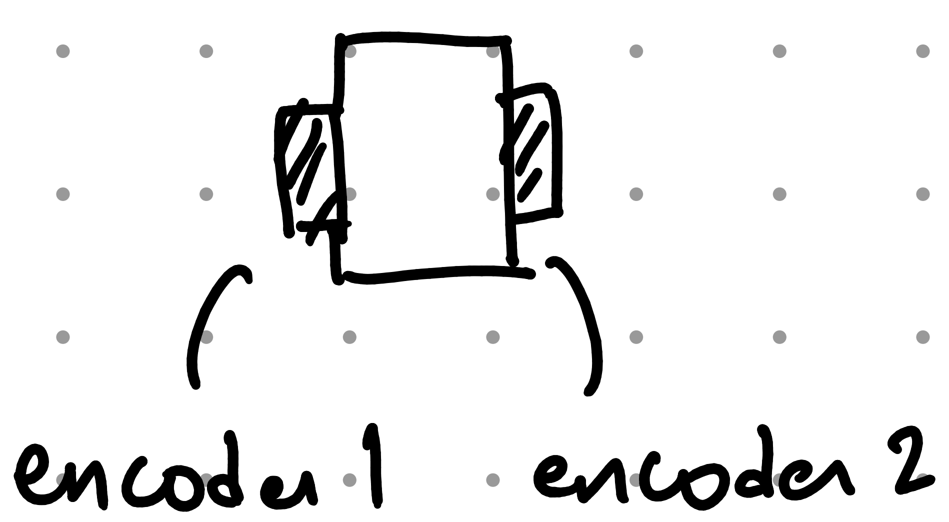
SLAM Youtube Course:

- **Step 1:**

→ Create a motion model that converts raw data from sensors into real-world co-ordinate estimates.

→ In the assignment, the sensors are incremental encoders.

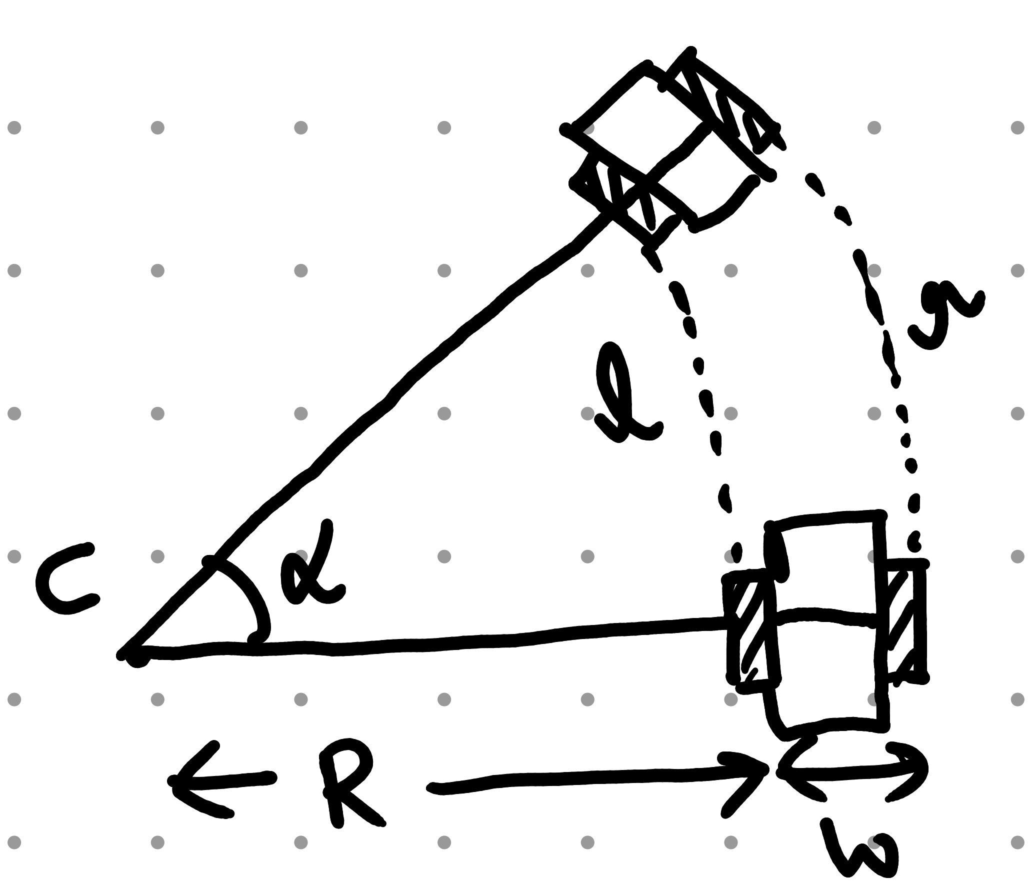


(Differential Drive robot)

It was found empirically that,

$$\boxed{1 \text{ tick} \simeq 0.349 \text{ mm}}$$

→ Now, we need to convert distance covered by right and left wheels (l and r) to rotation angle α about center of rotation C.



Total peri $= 2\pi R$

$$\frac{l}{\text{Total peri}} = \frac{\alpha}{2\pi}$$

$$\frac{l}{\cancel{2\pi} R} = \frac{\alpha}{\cancel{2\pi}}$$

$$l = \alpha R \text{ — (1)}$$

$$r = \alpha(R+w) \text{ — (2)}$$

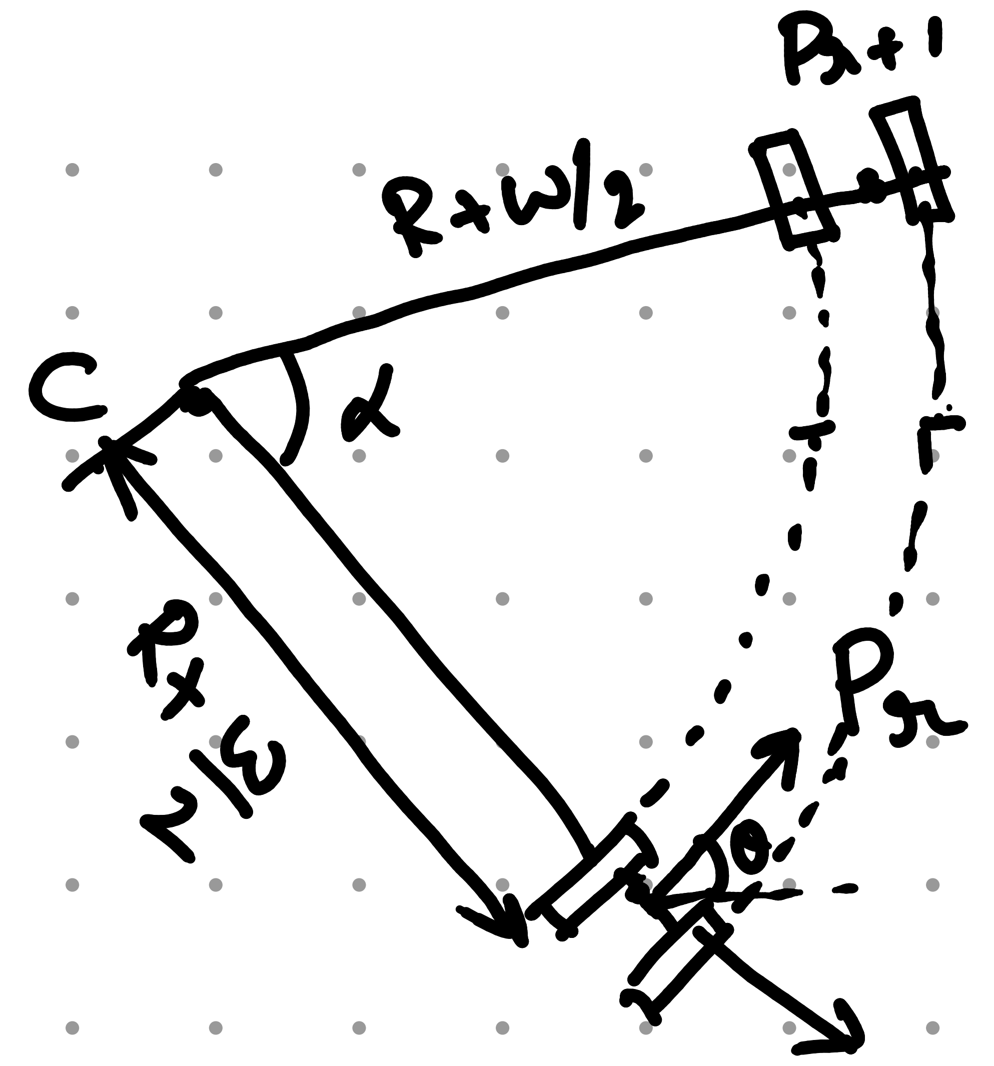
(2) − (1)

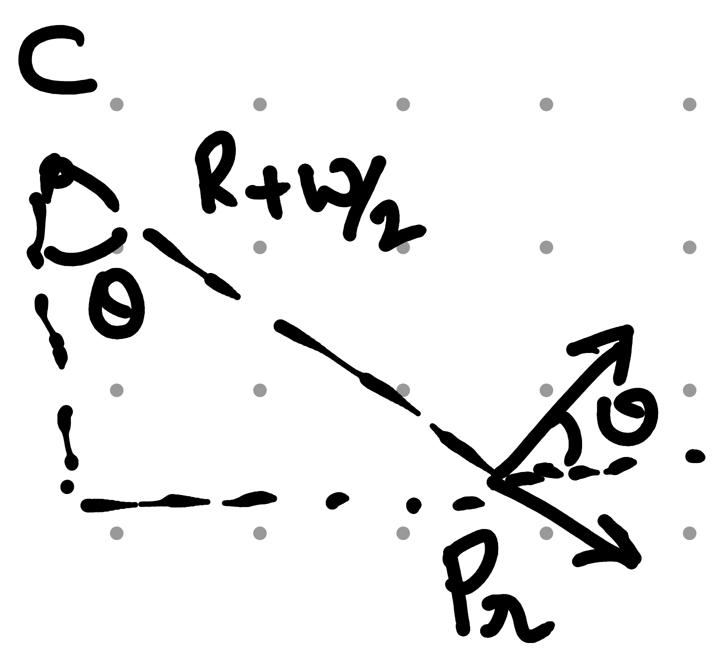
$$\Rightarrow r - l = \alpha \cdot w$$

$$\Rightarrow \boxed{\alpha = \frac{r-l}{w}}$$

$$\Rightarrow \boxed{R = \frac{l}{\alpha}}$$

where $\alpha \neq 0$





$\Rightarrow$ Consider a Robot frame $\{R\}$ where $P_0 = (0,0)$ and $\theta_0 = 0$ are the initial position and heading angle of the robot.

$\Rightarrow$ (P_r, θ_r) is the pose (position and heading) of the robot at encoder tick r

$$\boxed{\theta_{r+1} = \theta_r + \alpha}$$

$\Rightarrow$ We know that the position of C is constant between pose r and pose $r+1$

$$C = \begin{bmatrix} P_{rx} - \left(R+\frac{W}{2}\right)\sin\theta \\ P_{ry} + \left(R+\frac{W}{2}\right)\cos\theta \end{bmatrix} = \boxed{P_r - \left(R+\frac{W}{2}\right)\begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}}$$

$\Rightarrow$ Now, find P_{r+1}: $C = P_{r+1} + \left(R+\frac{W}{2}\right)\begin{bmatrix} -\sin(\theta+\alpha) \\ \cos(\theta+\alpha) \end{bmatrix}$

$$\boxed{P_{r+1} = C + \left(R+\frac{W}{2}\right)\begin{bmatrix} \sin(\theta+\alpha) \\ -\cos(\theta+\alpha) \end{bmatrix}}$$

where $\theta \in [0, 2\pi]$

Therefore, when $\boxed{r \neq l}$ or $\boxed{\alpha \neq 0}$

(1) $\alpha = \dfrac{r-l}{w}$ and $R = \dfrac{l}{\alpha}$

(2) $$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \left(R + \frac{w}{2}\right) \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}$$

$$\theta' = (\theta + \alpha) \bmod 2\pi$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} C_x \\ C_y \end{bmatrix} + \left(R + \frac{w}{2}\right) \cdot \begin{bmatrix} \sin\theta' \\ -\cos\theta' \end{bmatrix}$$

But, if $\boxed{r = l}$ or $\boxed{\alpha = 0}$

$$\theta' = \theta \Rightarrow \boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}$$