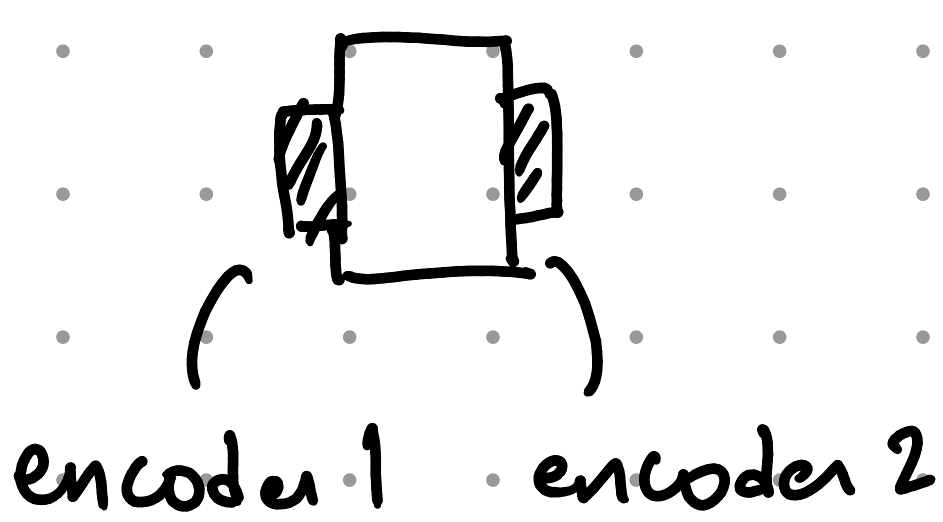
SLAM Youtube Course:

- **Step 1:**

→ Create a motion model that converts raw data from sensors into real-world co-ordinate estimates.

→ In the assignment, the sensors are incremental encoders.

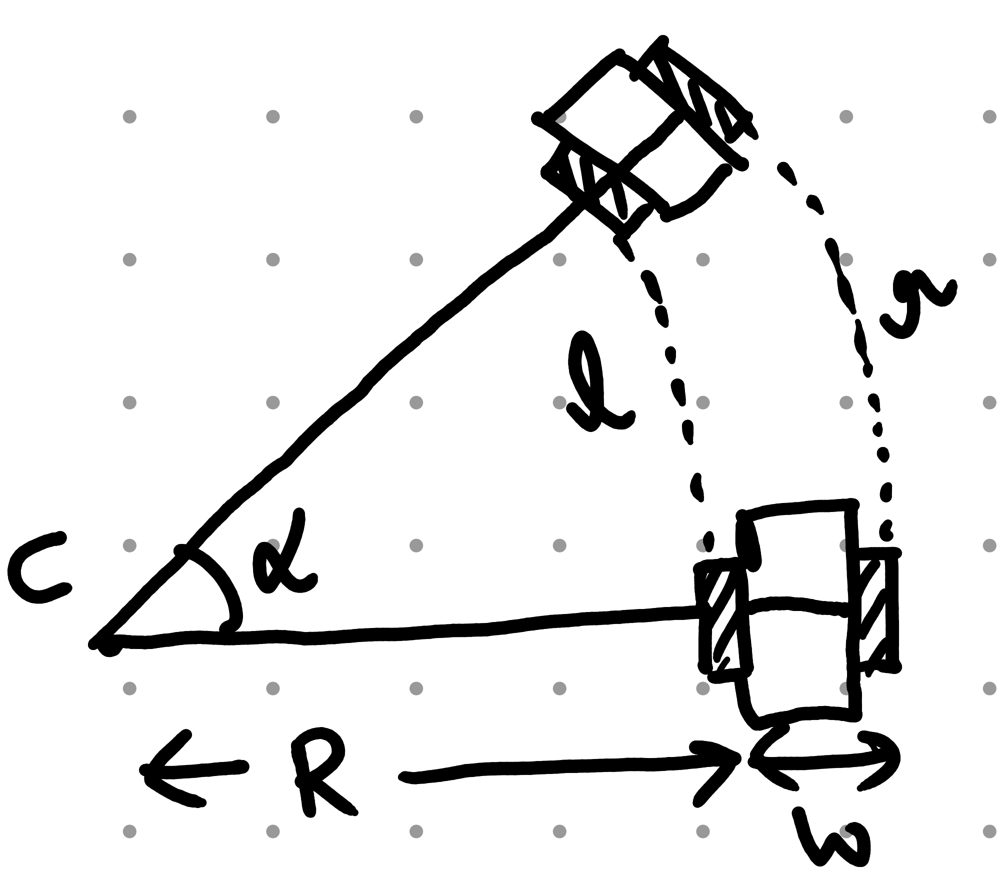


(Differential Drive robot)

It was found empirically that,

$$\boxed{1 \text{ tick} \simeq 0.349 \text{ mm}}$$

→ Now, we need to convert distance covered by right and left wheels (l and r) to rotation angle α about center of rotation C.



Total peri $= 2\pi R$

$$\frac{l}{\text{Total peri}} = \frac{\alpha}{2\pi}$$

$$\frac{l}{\cancel{2\pi} R} = \frac{\alpha}{\cancel{2\pi}}$$

$$l = \alpha R \quad (1)$$

$$r = \alpha(R+w) \quad (2)$$

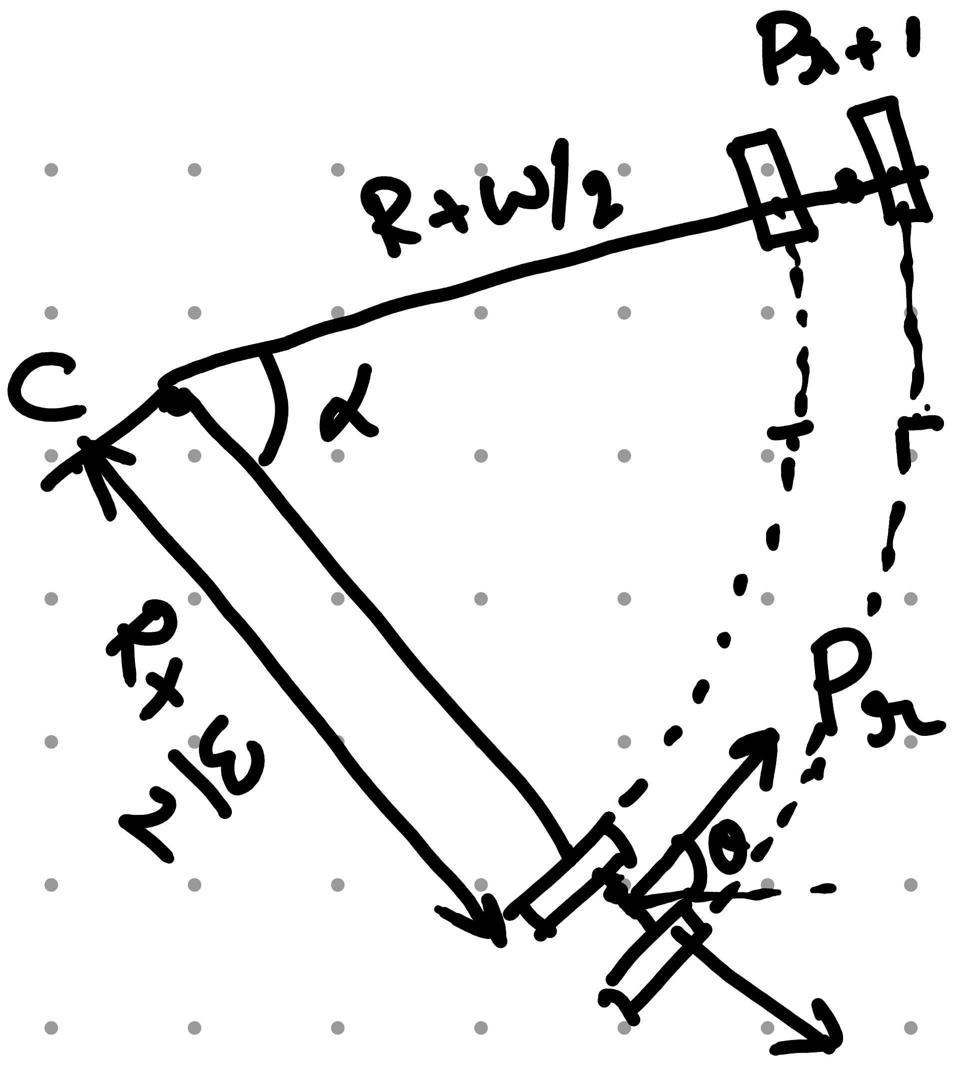
(2) − (1)

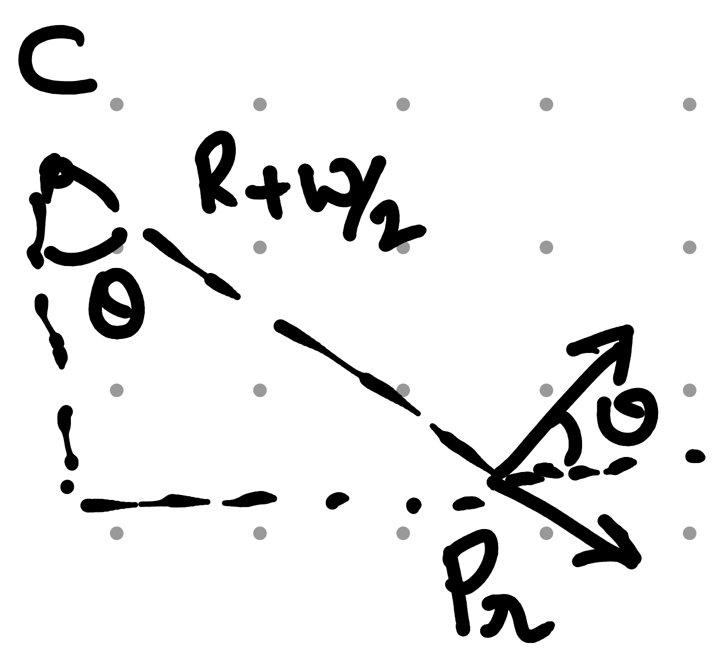
$$\Rightarrow r - l = \alpha \cdot w$$

$$\Rightarrow \boxed{\alpha = \frac{r-l}{w}}$$

$$\Rightarrow \boxed{R = \frac{l}{\alpha}}$$

where $\alpha \neq 0$





⇒ Consider a Robot frame $\{R\}$ where $P_0 = (0,0)$ and $\theta_0 = 0$ are the initial position and heading angle of the robot.

⇒ (P_r, θ_r) is the pose (position and heading) of the robot at encoder tick r

$$\boxed{\theta_{r+1} = \theta_r + \alpha}$$

⇒ We know that the position of C is constant between pose r and pose $r+1$

$$C = \begin{bmatrix} P_{rx} - \left(R+\frac{W}{2}\right)\sin\theta \\ P_{ry} + \left(R+\frac{W}{2}\right)\cos\theta \end{bmatrix} = \boxed{P_r - \left(R+\frac{W}{2}\right)\begin{bmatrix}\sin\theta \\ -\cos\theta\end{bmatrix}}$$

⇒ Now, find P_{r+1}: $C = P_{r+1} + \left(R+\frac{W}{2}\right)\begin{bmatrix}-\sin(\theta+\alpha) \\ \cos(\theta+\alpha)\end{bmatrix}$

$$\boxed{P_{r+1} = C + \left(R+\frac{W}{2}\right)\begin{bmatrix}\sin(\theta+\alpha) \\ -\cos(\theta+\alpha)\end{bmatrix}}$$

where $\theta \in [0, 2\pi]$

Therefore, when $\boxed{r \neq l}$ or $\boxed{\alpha \neq 0}$

① $\alpha = \dfrac{r - l}{w}$ and $R = \dfrac{l}{\alpha}$

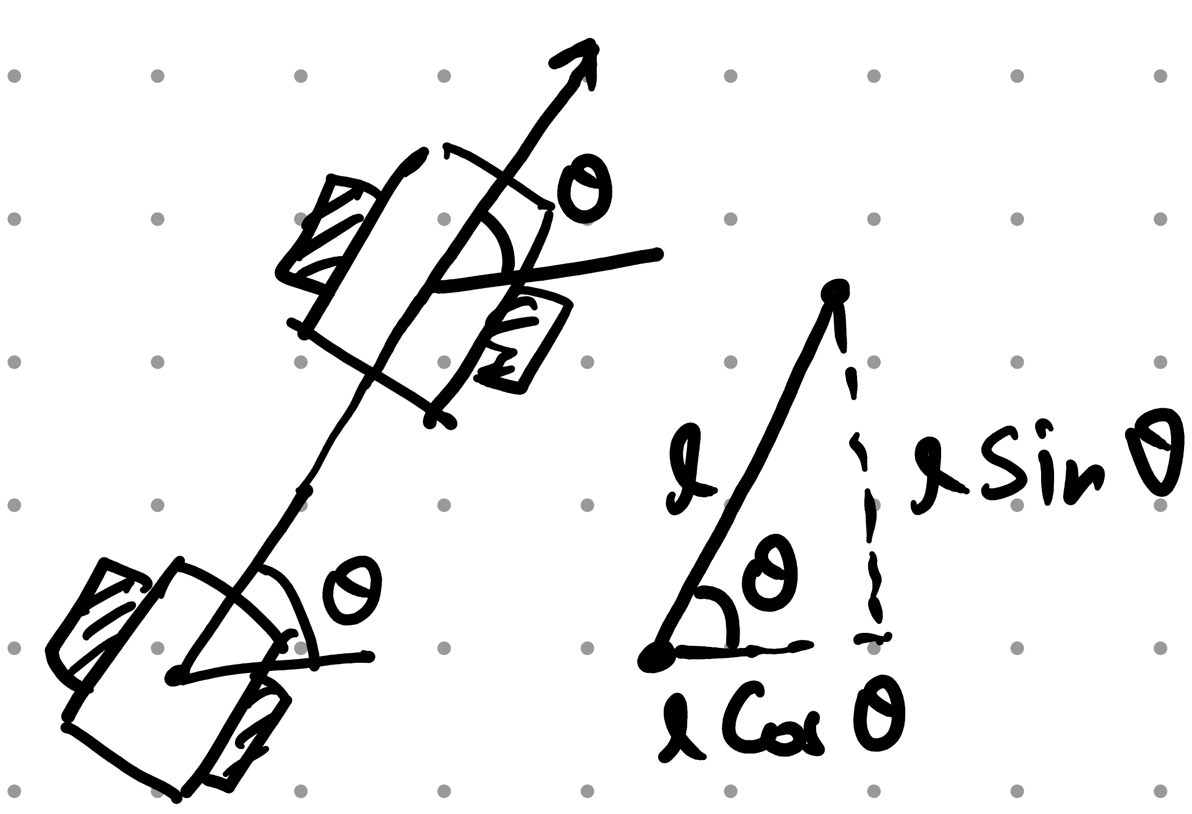
② $$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} - \left(R + \frac{w}{2}\right) \begin{bmatrix} \sin\theta \\ -\cos\theta \end{bmatrix}$$

$$\theta' = (\theta + \alpha) \bmod 2\pi$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} C_x \\ C_y \end{bmatrix} + \left(R + \frac{w}{2}\right) \cdot \begin{bmatrix} \sin\theta' \\ -\cos\theta' \end{bmatrix}$$

But, if $\boxed{r = l}$ or $\boxed{\alpha = 0}$

$$\theta' = \theta \Rightarrow \boxed{\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + l \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}}$$

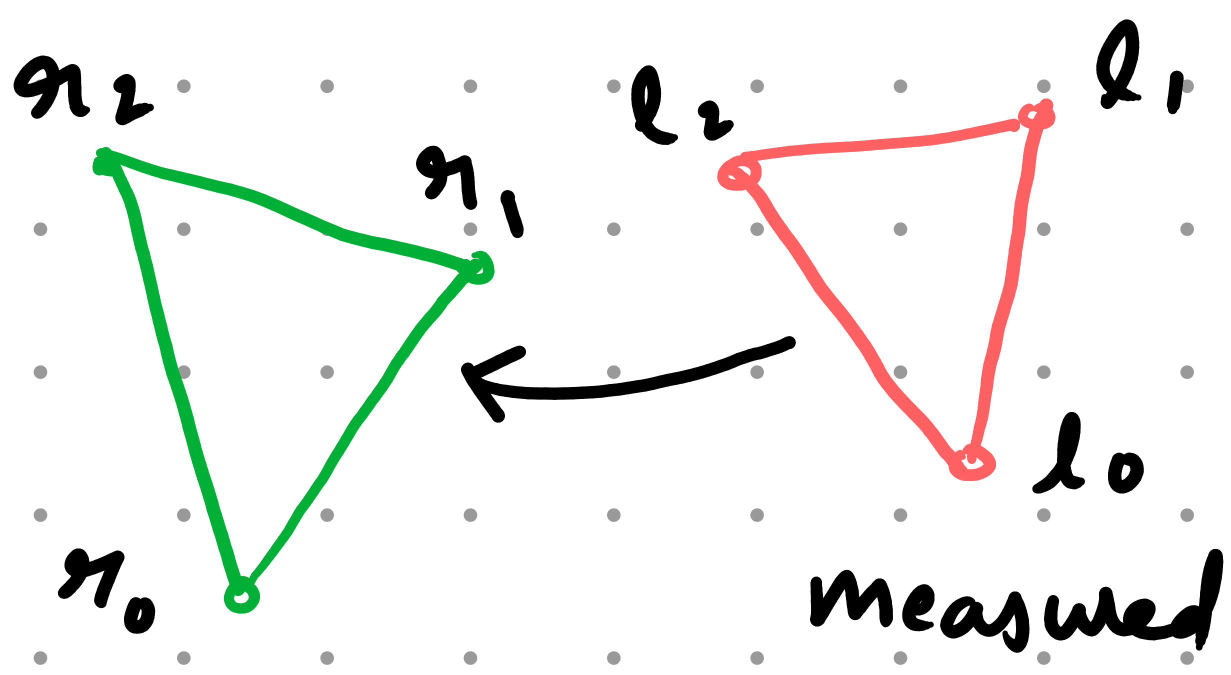


Step 2 :

→ Improve the estimated poses using known surroundings.

a) Feature based correction (Depends on features / Landmarks)

- This is possible if we know specific objects in the surroundings that can be used as landmarks.



Actual landmark locations from the robot's estimated position (r_i)

measured landmark locations (l_i)

- We need to find the transformation between the l_i and r_i to improve the robot's pose estimation.

- To do this, we get the similarity transform between l_i and r_i points

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} l_{ix} \\ l_{iy} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}$$

&

$$\boxed{\lambda R l_i + t = r_i}$$

where; λ – Scale factor $\in \mathbb{R}$
α – Angle $\in [0, 2\pi]$
t – translation vector $\in \mathbb{R}^2$

* This is only for 2D transformation

To find the best λ, α and t, we minimize the squared distance between l_i and r_i:

$$\min \sum_i \| \lambda R l_i + t - r_i \|^2$$

But, this is a non-linear problem. Because, R consists of $\cos\alpha$ and $\sin\alpha$. So, we have to linearize and iterate until we find a global minimum.

- However, we have to remember that we might not find a global minimum.

→ So, another approach is proposed:

1) Compute the center of mass

$$\bar{l} = \frac{1}{m} \sum_i l_i \qquad \bar{r} = \frac{1}{m} \sum_i r_i$$

2) Get the reduced co-ordinates

$$l_i' = l_i - \bar{l} \qquad r_i' = r_i - \bar{r} \qquad \text{so, } \sum_i l_i' = 0$$
$$\sum_i r_i' = 0$$

3) Now, replace the equation to be minimized with l_i' and r_i':

$$\min \sum_i \| \lambda R l_i - r_i + t \|^2$$
$$\min \sum_i \| \lambda R (l_i' + \bar{l}) - (r_i' + \bar{r}) + t \|^2$$
$$\min \sum_i \| \lambda R l_i' - r_i' + \underbrace{\lambda R \bar{l} - \bar{r} + t}_{t'} \|^2$$

$$\boxed{\min \sum_i \| \lambda R l_i' - r_i' + t' \|^2}$$

$$\sum_i \|\lambda R l_i' - r_i' + t'\|^2 \quad \left(\text{Use } (a+b)^2 = a^2 + 2ab + b^2\right)$$

$$= \sum_i \|\lambda R l_i' - r_i'\|^2 + 2t'^T \sum_i \lambda R l_i' - r_i' + \sum_i \|t'\|^2$$

$$= \sum_i \|\lambda R l_i' - r_i'\|^2 + 2t'^T\left(\lambda R \underbrace{\sum_i l_i'}_{0} - \underbrace{\sum_i r_i'}_{0}\right) + m\|t'\|^2$$

$$= \underbrace{\sum_i \|\lambda R l_i' - r_i'\|^2}_{\geq 0} \; (1) + \underbrace{m\|t'\|^2}_{\geq 0} \; (2)$$

→ so, now, we can minimize both of these terms individually

(2) $\Rightarrow m\|t'\|^2 = 0$

$$t' = 0$$

$$\lambda R \bar{l} - \bar{r} + t = 0$$

$$\boxed{t = \bar{r} - \lambda R \bar{l}}$$

(1) $\Rightarrow \min \sum_i \|\lambda R l_i' - r_i'\|^2$

Now, the equation is changed a bit by assuming the l_i point cloud is scaled by $\sqrt{\lambda}$ and r_i point cloud is scaled by $\frac{1}{\sqrt{\lambda}}$. Now, the new equation to minimize is

$$\min \sum_i \|\sqrt{\lambda} R l_i' - \frac{1}{\sqrt{\lambda}} r_i'\|^2$$

$$\min \lambda \underbrace{\sum \|R l_i'\|^2}_{a} - \underbrace{2\sum r_i'^T R l_i'}_{b} + \frac{1}{\lambda} \underbrace{\sum \|r_i'\|^2}_{c}$$

$$\min \| \lambda a + b + \frac{1}{\lambda} c \| \quad — (3)$$

differentiating λ

$$a + \frac{(-1)}{\lambda^2} c = 0$$

$$\frac{c}{\lambda^2} = a$$

$$\lambda^2 = \frac{c}{a}$$

$$\lambda^2 = \frac{\Sigma \| r_i' \|^2}{\Sigma \| R l_i' \|^2}$$

We know that rotation R does not change the length

$$\boxed{\lambda = \sqrt{\frac{\Sigma \| r_i' \|^2}{\Sigma \| l_i' \|^2}}}$$

λ is independent of rotation and translation.

And, since b wasn't utilized and (3) had to be minimized, we minimize b

$$\min -2 \Sigma r_i^T R l_i'$$

$$\Rightarrow \max \Sigma r_i^T R l_i'$$

$$\max \sum \begin{bmatrix} r'_x & r'_y \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} l'_x \\ l'_y \end{bmatrix}$$

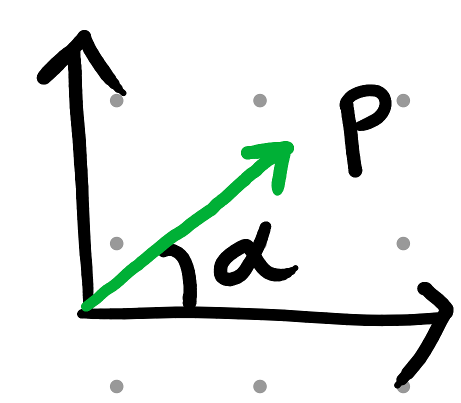
$$\max \sum \begin{bmatrix} r'_x & r'_y \end{bmatrix} \begin{bmatrix} l'_x \cos\alpha - l'_y \sin\alpha \\ l'_x \sin\alpha + l'_y \cos\alpha \end{bmatrix}$$

$$\max \sum r'_x l'_x \cos\alpha - r'_x l'_y \sin\alpha + r'_y l'_x \sin\alpha + r'_y l'_y \cos\alpha$$

$$\max \sum \cos\alpha \left(r'_x l'_x + r'_y l'_y \right) + \sin\alpha \left(-r'_x l'_y + r'_y l'_x \right)$$

$$\max \cos\alpha \sum r'_x l'_x + r'_y l'_y + \sin\alpha \sum -r'_x l'_y + r'_y l'_x$$

$$\max \begin{bmatrix} \cos\alpha & \sin\alpha \end{bmatrix} \underbrace{\begin{bmatrix} \sum r'_x l'_x + r'_y l'_y \\ \sum -r'_x l'_y + r'_y l'_x \end{bmatrix}}_{P}$$



From the equation, it can be seen that α is the angle made by the normalized vector P.

$$\boxed{\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} = \begin{bmatrix} \dfrac{P_x}{\sqrt{P_x^2 + P_y^2}} \\ \dfrac{P_y}{\sqrt{P_x^2 + P_y^2}} \end{bmatrix}}$$

* For rigid body motion, we ignore the λ from the similarity transformation. We set $\lambda = 1$.

Advantages:

- It is efficient because we only deal with limited number of features and discard all the other sensor data.

Disadvantages:

- Since we rely on features for correction, when no features are detected, the robot has more errors in its pose estimation.