Variables

A variable is a symbol, typically a letter such as x or y, for a number that can change or vary. It is called a variable because it can vary.

Why do we need a variable?
Let's take a look at the following scenario.

Hunter works as a waiter at a restaurant. He gets paid $14 an hour plus tips. How much does he make an hour?

$14 + tips = $ _____

Well, we don't know the answer because the amount of tips vary by the hour!

Let's say we use the letter t for the tips. Then the amount of money Hunter makes per hour is $14 + t$.

If he got $18 in tips in one hour, then he has made _____ + _____ = $_____

If he got $9 in tips in another hour, then he has made _____ + _____ = $_____ in that hour.

The letter t is a variable because it will vary by the hour.
A variable can be a letter like t or x and also a symbol like ∗ or ◊. A variable is a symbol for an unknown number or a number that may vary.

Use a variable to write the expression.

1. 7 times a certain number. _____

2. The number of apples in a box _____

3. 9 × □   _____

Algebraic expressions

Terms

Now that we know what a variable is, let's take a look at a term.
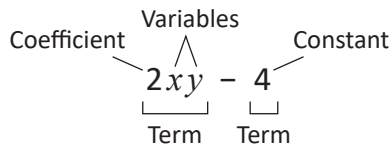
A term can be a number: 3, 7, 15, 120...

When a term does not change and is always the same, it is called a constant.

The numbers 3, 7, and 15 above are _____.

A term can be a variable: a, b, c, x, y, a^2, x^3

A term can have a number with a variable: $3x, 5y^2, 14t$...

When a term has a number with a variable, the numerical part of the term is a coefficient.

Coefficient — Variables
Constant
$2xy - 4$
Term   Term

The coefficient is a number that is multiplied by a variable.

What is the coefficient for q?

$q =$ _____ $\times q$

So, the coefficient for q is _____.

When we use variables, the multiplication symbol is often omitted to avoid confusion.

$$2 \times x \times y = 2 \cdot x \cdot y = 2xy$$

Additionally, the number 2 and the variable x are factors because a factor is any number or variable that is a part of a product.

$$2 \quad \cdot \quad x \quad = \quad 10$$
Factor   Factor   Product

> A variable with an exponent such as x^3 or m^2 is considered as one factor because it has one variable.

Terms, Expressions and Equations

A **term** is a number, a variable, or number with a variable.

Let's take a look the at the terms!

$3, \ 7, \ \frac{3}{4}, \ .7$ $x, \ y, \ t, \ a$ $3x, \ 6x^2, \ 7yz$

       Numbers   Variables   Numbers with variables

An **expression** is a group of terms with + or − symbols.

$7x + 4$ $4x^2 + 2y + 7$ $a + b + c$ $7 + x + 3$

All those above are expressions.

An algebraic equation doesn't have division!

What do we do when we need to express division? We use fraction!

$7 \div 2 = \frac{7}{2}$ $5x \div y = \frac{5x}{y}$

An **equation** is a statement with two terms or expressions that are equal.

$3x + 5 = 20$ $4x + 9 = 34 + 11$ $5y = 15x$

Expression Term   Expression   Expression   Term   Term

✏️ Write the name.

1. $3y^2$ _____

2. $9y + 3z - 14$ _____

3. $4a = \frac{1}{3}b$ _____

4. $17abc$ _____

5. $12a + 9y + 5c$ _____

6. $6x + 3y = 18$ _____

7. $7x^2 + 11y^2 + 3c = 148$ _____

8. $5t + .7m = 4x - m$ _____

✏️ Write the term for the phrase.

1. 7 times x squared times y _____

2. 5 cubed times y squared times z _____

3. 12 times m to the fourth power times n cubed _____

4. 9 times p to the fifth power times q squared _____

5. 7 divided by the product of p and q _____

✏️ Write the expression for the phrase.

1. n to the third power subtracted from 5 times m squared _____

2. 6 times a cubed added to 4 _____

3. Negative 7 times c cubed plus 5 times ab _____

4. The difference of 12 times p cubed times q squared and 12 times r squared times t. _____

✏️ Write the equation for the phrase.

1. 3 times x squared plus 4 equals the sum of 2 and y _____

2. 7 times m cubed times n squared equals 11 times a _____

3. Negative 8 times p cubed times q equals 15 times b squared _____

4. $\frac{2}{3}$ times x cubed times y added to 13 equals 7 times a to the third power subtracted from 14. _____

Algebraic expressions

Circle the statement that does not belong.

1. $b + 3$
   ① A number b plus 3
   ② The sum of b and 3
   ③ A number b and 3
   ④ b increased by 3

2. $k - 4$
   ① A number k minus 4
   ② The difference of k and 4
   ③ 4 less than k
   ④ k less than 4

3. $5a^2b$
   ① 5 times a^2b
   ② 5 times a^2 times b
   ③ 5 times a plus b
   ④ $5a^2$ times b

4. $\dfrac{q}{6}$
   ① The quotient of q and 6
   ② a number q divided by 6
   ③ 6 divided by the number q
   ④ The ratio of q to 6

5. ax^3
   ① a times x cubed
   ② a times x times 3
   ③ $a \cdot x \cdot x \cdot x$
   ④ a times x to the third power

Evaluating Algebraic Expressions

An algebraic expression is one that contains at least one variable. A numerical expression contains numbers only.

$$3x^2 + 7y + 9$$

$$5 \times (11 - 3) + 7$$

_____ expression   _____ expression

The answer to an algebraic equation will change as the value of the _____ in the expression changes.

To evaluate or to find the value of an algebraic expression, simply substitute the variable with the given value.

Let's take a look!

Sam works at a restaurant and makes $14 an hour. How much money does he make in 3, 5, and 8 hours?

The amount of money Sam makes per hour: $14h$ or $14 \times h \rightarrow h$ is the number of hours

In 3 hours, he makes $14 \times$ _____ = $ _____

In 5 hours, he makes $14 \times$ _____ = $ _____

In 8 hours, he makes $14 \times$ _____ = $ _____

Evaluate the algebraic expression when $x = \frac{1}{2}$, 3, and 5.

	$\frac{1}{2}$	3	5
1. $5x + 4$			
2. $10 - 4x$			
3. $3x - 9$			

Evaluate the algebraic expression and complete the table.

Table A	
x	$10 + x$
1	
3	
5	
8	
10	

Table B	
x	$10 - x$
1	
3	
5	
8	
10	

For table A, as the value of x increases, the value of the expression $10 + x$ _____.

For table B, as the value of x increases, the value of the expression $10 - x$ _____.

Table C	
x	$3 + \frac{1}{x}$
1	
2	
3	
4	
5	

Table D	
x	$3 - \frac{1}{x}$
1	
2	
3	
4	
5	

For table C, as the value of x increases, the value of the expression $3 + \frac{1}{x}$ _____.

For table D, as the value of x increases, the value of the expression $3 - \frac{1}{x}$ _____.

Evaluating Algebraic Expressions II

Sometimes in algebraic expression has more than one variable.

Let's take a look at the algebraic expression below.

$3x^2 + 5y + 7$ It has two variables, _____ and _____.

Now, evaluate the expression when $x = 4$ and $y = 8$

To evaluate the expression, substitute 4 for _____ and 8 for _____.

$3 \cdot 4^2 + 5 \cdot 8 + 7 =$

_____ + _____ + 7 = _____   It is actually quite simple!

Evaluate the algebraic expression.

1. $8r^2 - 5s$ when $r = 5$ and $s = 9$

    _____

2. $7a^2b + \dfrac{b^2}{3}$ when $a = 3$ and $b = 6$

    _____

3. $4x^2y - 3xy$ when $x = 4$ and $y = 7$

    _____

4. $5p^3q - 9p^2q$ when $p = 2$ and $q = 5$

    _____

5. $6r^2s - 7rs$ when $r = 3$ and $s = 4$

    _____

6. $\dfrac{12xy}{3x^2y}$ when $x = 6$ and $y = 5$

    _____

Evaluating algebraic expressions

✏️ Evaluate the algebraic expression.

1. $5xy - 3x^2$ when $x = \frac{1}{3}$ and $y = 4$

_____

2. $8m^2n + 2mn$ when $m = .7$ and $n = 6$

_____

3. $4a^2b + 6ab$ when $a = \frac{3}{5}$ and $b = 5$

_____

4. $9r^2s - 12rs$ when $r = 3$ and $s = 4$

_____

5. $5p^2q - 2pq$ when $p = \frac{7}{10}$ and $q = 4$

_____

6. $6c^2d + 4cd$ when $c = 5$ and $d = 8$

_____

7. $4x^2y - 12xy$ when $x = 3$ and $y = \frac{1}{2}$

_____

8. $24a^2 + 7ab$ when $a = .5$ and $b = 12$

_____

9. $\frac{3}{4}l^3w - l^2w$ when $l = 4$ and $w = \frac{5}{8}$

_____

Evaluating algebraic expressions

📝 Answer the questions.

d = distance	l = length
t = times	w = width
r = rate (speed)	h = height

1. Jason drove for 4 hours at the speed of 60 miles per hour.

   1A. What is the distance he drove in miles?

   1B. What is the formula for finding the distance?

2. Jason's rectangular backyard has a length of $4\frac{1}{2}$ m and a width of $7\frac{5}{12}$ m.

   2A. What is the area of his backyard?

   2B. What is the formula for finding the area?

3. Jason has a fish tank that has a length of 18 in and a width of 40 in. It's height is 23 in.

   3A. What is the volume of the fish tank?

   3B. What is the formula for finding the volume?

A formula is an algebraic equation with variables!

Evaluating algebraic expressions

Name: Date:

? Fill in the blanks with the word from the word box.

coefficient	equation	term
constant	expression	variable

1. A _____ is a symbol for a number that can change or vary.

2. A _____ is a number that does not change.

3. A number, a variable or a number with a variable is a _____ .

4. A _____ is a numerical part of a term.

5. An _____ is a statement with two terms or expressions that are equal.

6. An _____ is a group of terms with + or − symbols.

✏ Write *term*, *expression*, or *equation*.

1. $4x^2y$ _____

2. $7ab = 4x^2$ _____

3. $8a + 5b - c$ _____

4. $16a^3b^2c$ _____

5. $7k^2 + 3m + n = 141$ _____

6. $5x - 9$ _____

7. $5x^2 + 7y = 16x - 46$ _____

8. $14a$ _____

Circle the statement that does not belong.

1. $k + 5$
   - ① The sum of k and 5
   - ② The difference of k and 5
   - ③ k increased by 5
   - ④ 5 more than k

2. $t - 13$
   - ① The difference of t and 13
   - ② t subtracted from 13
   - ③ 13 less than t
   - ④ t decreased by 13

3. $3xy$
   - ① 3 times xy
   - ② 3 times x times y
   - ③ 3 times x plus y
   - ④ $3x$ times y

4. $\dfrac{y}{3x}$
   - ① the quotient of y and $3x$
   - ② y divided by 3 times x
   - ③ $3x$ divided by y
   - ④ The ratio of y to $3x$

5. x^3y^2
   - ① x cubed times y squared
   - ② x times 3 times y times 2
   - ③ $x \cdot x \cdot x \cdot y \cdot y$
   - ④ x to the third power times y squared

Algebraic expressions

Write *true* or *false*.

1. A term can be a number. _____

2. A term can be a variable. _____

3. A term can be a number with a variable. _____

4. A number that is a term is also called a coefficient because it does not change. _____

5. A coefficient is the numerical part of a term. _____

6. When a term has a number and two variables, all three are factors of the term. _____

7. The coefficient for t is 1 because $1 \times t = t$. _____

Write the coefficient, factors, and constant for each expression.

	Coefficient	Factors	Constant
1. $5x^2y + 9$	_____	_____ _____ _____	_____
2. $14x^3y + 7$	_____	_____ _____ _____	_____
3. $\frac{5}{9}m^2n + \frac{2}{3}$	_____	_____ _____ _____	_____
4. $.7ab + .53$	_____	_____ _____ _____	_____
5. $r^2s + 100$	_____	_____ _____ _____	_____
6. $21 + 8c^2d$	_____	_____ _____ _____	_____

Circle the correct statement(s) for the expression.

1. $7x^2y + \frac{3}{y} + 14$

   a. There are 2 terms in the expression.

   b. The term $7x^2y$ has 3 factors

   c. The term $\frac{3}{y}$ is a quotient.

   d. The coefficient is 14.

2. $5a^2b + \frac{a}{8} + 9^2 + 18$

   a. There are 3 terms in the expression.

   b. The term $5a^2b$ has 4 factors.

   c. The term 9^2 is a variable.

   d. The term 18 is a constant.

3. $\frac{4}{5}m^2n - mn + 15$

   a. The coefficient for the term $\frac{4}{5}m^2n$ is 4.

   b. The term mn doesn't have a coefficient.

   c. There are 2 variables in the expression.

   d. There are 3 terms in the expression.

4. $.75x^2y = \frac{1}{xy} + 6^2 + 4$

   a. There are four terms in the expression.

   b. 4 is the only constant in the expression.

   c. The coefficient for the term $.75x^2y$ is .75.

   d. The term $\frac{1}{xy}$ is a quotient.

Algebraic expressions

✏️ Write the term for the phrase

1. 9 times x cubed times y _____

2. 14 cubed times p to the 4th power _____

3. $\frac{5}{7}$ times m squared times n _____

4. Negative 6 times a to the 3rd power times b squared _____

5. .3 times d to the 4th power times t _____

6. Negative $\frac{4}{3}$ times g cubed times h squared. _____

7. q divided by the product of r and s _____

✏️ Write the algebraic expression for the phrase.

1. 7 added to 5 times x squared _____

2. The difference of n and 5 multiplied by y _____

3. q divided by the difference of r and s added to 3 _____

4. 5 multiplied by y squared subtracted from 2 times x cubed _____

5. Negative 12 times d to the fourth power added to $\frac{4}{9}$ times e cubed

   _____

6. 9.8 times m cubed times s squared subtracted from 13 times g squared

   _____

7. The difference of 8 times x cubed times y squared and $\frac{2}{3}$ times r squared times s

   _____

Write the verbal phrase for the given expression.

1. $6x - 4y$

_____

2. $\frac{1}{2}p + q$

_____

3. $4x^3y^2 - \frac{2}{3}yx$

_____

4. $3a^2b + \frac{7}{10}c$

_____

5. $16f^4g^2 - 5h$

_____

6. $\frac{3}{5}p^3 + 9rs$

_____

✏️ Evaluate the algebraic expression when $x = \frac{1}{3}$, 2, and 6.

	$\frac{1}{3}$	2	6
1. $12x - 5$	_____	_____	_____
2. $6x^2 + 8$	_____	_____	_____
3. $(9x + 4) \times 4$	_____	_____	_____
4. $12x^2 \times 6$	_____	_____	_____
5. $3 \times (4x + 7)$	_____	_____	_____
6. $10 - 5x$	_____	_____	_____

✏️ Evaluate the algebraic expression when $y = \frac{1}{4}$, $\frac{1}{2}$, and 3.

	$\frac{1}{4}$	$\frac{1}{2}$	3
1. $3y + 6$	_____	_____	_____
2. $10y^2 - 4$	_____	_____	_____
3. $5y + 3$	_____	_____	_____
4. $-4y + 7$	_____	_____	_____
5. $8y \times (y + 2y)$	_____	_____	_____
6. $y^2 - y$	_____	_____	_____

Evaluate the algebraic expression and complete the table.

Table A	
x	$100 + x$
5	
10	
20	
40	
80	
120	

Table B	
x	$100 - x$
5	
10	
20	
40	
80	
120	

For table _____, the value of the expression increases as the value of x increases.

Table C	
x	$10 + \frac{1}{x}$
1	
2	
4	
5	
10	
20	

Table D	
x	$10 - \frac{1}{x}$
1	
2	
4	
5	
10	
20	

For table D, the value of the expression _____ as the value of x increases.

✏️ Evaluate the algebraic expression.

1. $5a^2b + 2b$ when $a = 5$ and $b = 4$

_____

2. $\dfrac{4a^3b^2}{2a^2b}$ when $a = 3$ and $b = 2$

_____

3. $11ab - 7a^2b$ when $a = 6$ and $b = 4$

_____

4. $3x(4x - 5y)$ when $x = 9$ and $y = 7$

_____

5. $\dfrac{7xy + 15}{3x^2y}$ when $x = 4$ and $y = 2$

_____

6. $x(4 + 13) - 4y(7 - x)$ when $x = 6$ and $y = 4$

_____

7. $(t^3 - 5t) \times (u^2 - 9)$ when $t = 5$ and $u = 7$

_____

8. $7t^2 - 3t + 5u + 19$ when $t = 4$ and $u = 8$

_____

9. $2t^3 + 6t^2 - 8u$ when $t = 3$ and $u = 9$

_____

✏️ Evaluate the algebraic expression.

1. $6x + 2y - 7$ when $x = \frac{3}{4}$ and $y = 6$

_____

2. $3x^2 + 5y - 4$ when $x = 4$ and $y = \frac{3}{4}$

_____

3. $\frac{7xy + 36}{4xy}$ when $x = 5$ and $y = .8$

_____

4. $5c^2d - 3cd$ when $c = \frac{4}{5}$ and $d = 15$

_____

5. $\frac{1}{4c^3} - 14d$ when $c = 2$ and $d = \frac{3}{8}$

_____

6. $8c^2d + 6d$ when $c = .5$ and $d = .5$

_____

7. $\frac{3s^2t + st}{5st}$ when $s = 6$ and $t = 7$

_____

8. $12s^2 - 4st + \frac{5}{4}$ when $s = \frac{1}{2}$ and $t = \frac{7}{8}$

_____

Evaluating algebraic expressions

Write the algebraic expression and solve.

1. Jerry works at a coffee shop for x number of hours per week and makes y dollars per hour.

    1A. How much money does he make in 4 weeks?

    1B. Jerry spends of $\frac{1}{2}$ of the money he makes each week and saves the rest. How much money does he save in 4 weeks?

    1C. Jerry donates $25 to a local charity each week. How much money does he save after the donation in 4 weeks?

    1D. How much money does he actually save in 4 weeks if he makes $17 per hour and works 25 hours per week?

2. Jon works at a store for d number of days each week. He works h number of hours each day and makes m dollars per hour. He spends $75 a week for lunch and gas, and he saves $\frac{1}{3}$ of the remaining money in a bank account. How much money will he have saved in his bank account in the next 12 weeks if he already has $500 in his bank account?

Writing Your Own Formula

In the United States, as well as other countries, you pay a percentage of the income (the amount of money you make) in taxes.

The percentage of taxes varies depending on the amount of money you make.

Let's take a look at the table for taxes.

Income	Taxes
$ 0 - 29,999	12%
$ 30,000 - 59,999	16%
$ 60,000 - 99,999	18%
$ 120,000 +	23%

If you had a salary of $30,000 a year, you would pay 16% of $30,000 in taxes, which would be _____ , and receive the rest as your net salary.

1. Write the formula for the net salary for someone who makes $75,000 a year.

2. What is the net salary of that person?

3. What is the net salary of someone who has made $115,000 in a year?