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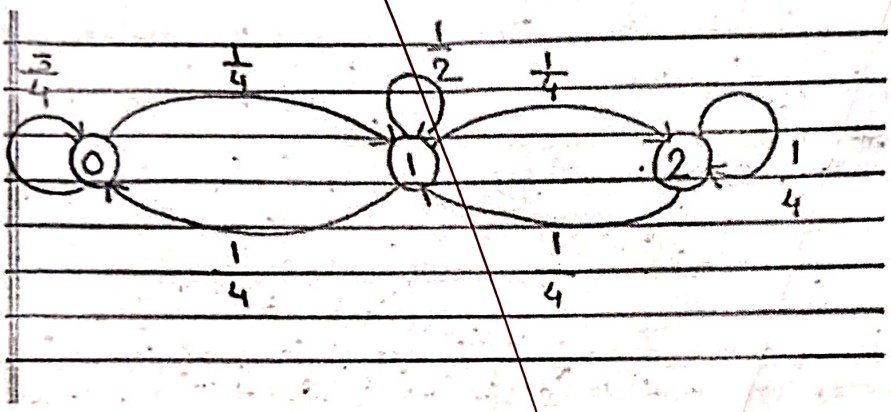
Department of Statistics

Practical: ST – 35

Practical No. 1 Realization of Markov Chain

Q.1 Write the transition matrix for the following graph.



If the initial distribution is [1/3, 1/3, 1/3] then find

1) $P(X_2 = 2 \mid X_0 = 1)$
2) $P(X_3 = 1 \mid X_1 = 2)$
3) $P(X_2 = 2)$
4) $P(X_5 = 1 \mid X_2 = 2)$
5) $P(X_4 = 2 \mid X_3 = 0)$

Q.2 Suppose probability that dry day following rainy day is 1/3 and probability of rainy day following dry day is 1/2.If 1st may is dry day .

Find probability that,

1) 3rd May is dry day
2) 5th May is rainy day
3) 4th May is dry day

Q.3 For the given matrix

$$P = \begin{bmatrix} 1/4 & 2/4 & 1/4 \\ 1/4 & 1/4 & 2/4 \\ 3/4 & 1/4 & 0 \end{bmatrix}$$

Find the distribution of X_3.

Q.4 Let $\{Xn, n \geq 0\}$ is a Markov chain with state space $s = \{0, 1, 2\}$ and initial probability distribution$\{2/6, 3/6, 1/6\}$.If the one step t.p.m. is

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Find

1) $P(X_2 = 1, X_1 = 1, X_0 = 0)$
2) $P(X_2 = 2, X_1 = 1, X_0 = 1)$

Q.5 Given TPM of a Markov chain,

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

Find:

1) $P(X_2 = 3)$
2) $P(X_3 = 2, X_2 = 1, X_1 = 2, X_0 = 1)$
3) $P(X_3 = 0, X_2 = 1, X_1 = 1, X_0 = 0)$

Q.6 Given the TPM,

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1/3 & 0 & 2/3 & 0 \\ 0 & 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2/5 & 0 & 3/5 \end{bmatrix}$$

1) Find the absorbing state.
2) Check whether state 3 is recurrent or transient.
3) Check whether state 2 is recurrent or transient.

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Practical Sheet

Class: M. Sc. II **Subject: STS-604-MJP**

Practical No: 2 Simulation / Realization of branching process and estimating its mean and variance

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1. In a population, each individual can have a random number of children. Specially, each individual has the following probabilities:
   Has o children with probability $P_0 = 0.2$
   Has 1 children with probability $P_1 = 0.5$
   Has 2 children with probability $P_2 = 0.3$
   You start with single individual and want to simulate the growth of a family tree over a specified number of generations.
   i) Simulate the growth of the family tree starting with one individuals over 6 generations.
   ii) Repeat the simulation for multiple trials.
   iii) Estimate the mean and variance of the total number of individuals in the family tree after the specified number of generations.

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Practical Sheet

Class: M. Sc. II **Subject: STS-604-MJP**

Practical No 3: Simulation of Poisson and related process.

1. Let $\{N(t), t \geq 0\}$ be the passion process with rate $(\lambda = x)$. Simulate the interarrival time and hence waiting times up $t = y$. Depict it graphically for $0 \leq t \leq y$.
   i) $x = 2$ and $y = 5$ ii) $x = 4$ and $y = 9.5$ iii) $x = 9$ and $y = 7$.

2. If $\{N(t), t \geq 0\}$ is the poisson Process and $N(x) = y$. Simulate poisson process for $0 \leq t \leq y$.
   i) $x = 6$ and $y = 5$ ii) $x = 7$ and $y = 10$ iii) $x = 9$ and $y = 7$.

3. Simulate the poisson process at time points t with rate λ without simulating its interarrival time.
   i) $t = 1.5, 2.2, 3.8, 7.5, 8.8, and\ \lambda = 1$
   ii) $t = 1.23, 2.21, 2.83, 6.05, 7.08, 17.8, and\ \lambda = 1.5$

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Practical: ST – 35

Practical No. 3 Realization of Birth and Death process

Q.1 Considering the finite set of time over a time interval [0,25]. The parameters are birth rate 0.3, death rate 0.3. Using the above information simulate the data from birth and death process (X_t) taking initial point $X_0 = 17$.

Q.2 Simulate the data from birth and death process (X_t) considering its initial and end points are 10 and 20 respectively over the interval [0,15], given the birth rate = 2 , death rate = 3.

Q.3 Simulate the data from birth and death process (X_t) given the initial point $X_0 = 9$ over the interval [0,20]. The parameters are given as birth rate = 0.5, death rate = 0.3 and immigration rate = 0.2.

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Practical: ST – 35

Practical No. 4 Realization of Brownian motion process

1) Simulate the Brownian motion process in the interval (0,1) for n=100 and n=1000.

2) Generate 1000 points of Geometric Brownian motion for the interval (0,1) taking its starting value 1 and $\mu = 0.05$ and $\sigma = 0.15$

3) Simulate Brownian motion in the interval of time [0,10] and plot the simulated returns.

4) Simulate Geometric Brownian motion process on [0,1].
Take, $X_o = 10, \mu = 0.5$ and $\sigma = 0.1$. Also plot the simulated output.

5) For n=1000 generate Brownian motion in two dimension in the interval (0,1).

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Practical Sheet

Class: M. Sc. II **Subject: STS-604-MJP**

Practical No: 6 Realization Gambler Ruin Problem

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1. Simulate a gambler starting with a given amount of money $X_0 = 10$, where the gambler wins or loses \$1 with equal probability (50%). The game ends when the gambler either reaches \$0\$(ruin) or reaches a target amount of \$20\$. Run the simulation 1000 times and determine:
   a) The probability of ruin (gambler ends with \$0\$).
   b) The average number of steps the gambler plays before the game ends.

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Class: M.Sc. II **Subject: STS-604-MJP**

Practical No: 8 Two way Classification with one observation per cell (with interaction)

1. Four experiments determine the moisture content of samples of a powder, each man taking a sample from each of six consignments. The assessments are;

Observer	Consignment					
	1	2	3	4	5	6
1	9	10	9	10	11	11
2	12	11	9	11	10	10
3	11	10	10	12	11	10
4	12	13	11	14	12	10

Carry out ANOVA and interpret results.

2. The data in Table show the birth-weights of babies born, classified according to the age of mother and order of gravida there being three observations per cell:

Order of gravida	Age-group of mother				
	15-20	20-25	25-30	30-35	35 and over
1	5.1,5.0,4.8	5.0,5.1,5.3	5.1,5.1,4.9	4.9,4.9,5.0	5.0,5.0,5.0
2	5.2,5.2,5.4	5.3,5.3,5.5	5.3,5.2,5.2	5.2,5.0,5.5	5.1,5.3,5.9
3	5.8,5.7,5.9	6.0,5.9,6.2	5.8,5.9,5.9	5.8,5.5,5.5	5.9,5.4,5.5
4	6.0,6.0,5.9	6.2,6.5,6.0	6.0,6.1,6.0	6.0,5.8,5.5	5.8,5.6,5.5
5 and over	6.0,6.0,6.0	6.0,6.1,6.3	5.9,6.0,5.8	5.9,6.0,5.5	5.5,6.0,6.2

Test whether the age of mother and order of gravida significantly affect the birth- weight.

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Class: M.Sc. II **Subject: STS-604-MJP**

Practical No: 9 Analysis of LSD and BIBD

1. An experiment was carried out to determine the effect of claying the ground on the field of barley grains; amount of clay used were as follows:

A: No Clay
B: Clay at 100 per acre
C: Clay at 200 per acre
D: Clay at 300 per acre

Column→ Row ↓	I	II	III	IV
I	D 29.1	B 18.9	C 29.4	A 5.7
II	C 16.4	A 10.2	D 21.2	B 19.1
III	A 5.4	D 38.8	B 24.0	C 37.0
IV	B 24.9	C 41.7	A 9.5	D 28.9

2. Suppose that the chemical engineer thinks that the time of reaction for a chemical process is a function of type of catalyst used. Four catalysts are investigated. The randomized incomplete block design is used. The observations recorded are shown in the following table. Analyze the data and draw the conclusion.

Catalyst	Blocks			
	1	2	3	4
1	73	74	-	71
2	-	75	67	72
3	73	75	68	-
4	75	-	72	75

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Practical Sheet

Class: M.Sc. II **Subject: STS-604-MJP**

Practical No: 10 Analysis of covariance in one way and two way model.

1. A soft drink distributor is studying the effectiveness of delivery methods. Three different types of hand trucks have been developed, and an experienced is performed in the company's methods in engineering laboratory. The variable of interest is the delivery time in minutes(y); however, delivery time is also strongly related to the case volume delivered (x). Each hand truck is used four times and data that follows are obtained.

Machines					
I		II		III	
Y	X	Y	X	Y	X
27	24	25	26	40	38
44	40	35	32	22	26
33	35	46	42	42	50
11	40	26	25	25	26

a) Analyze these data and draw appropriate conclusions. Use $\propto = 0.05$

b) Obtain an unbiased estimate of error variance.

2. An engineer is studying the effect of cutting speed on the rate of metal removable in a machine operation. However the rate of metal removable is also related to the hardness of the test specimen. There are five blocks. The amount of metal removed (y) and the hardness of the specimen (x) are shown in following table. Analyze the data. Use $\propto = 0.05$

Blocks	Cutting Speed					
	1000		1200		1400	
	Y	X	Y	X	Y	X
1	68	120	112	165	118	175
2	90	140	94	140	82	132
3	98	150	65	120	73	124
4	77	125	74	125	92	141
5	88	136	88	133	80	130

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - 2^k factorial experiment, Analysis of single replicate of 2^k.

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1. A chemical product is produced in a pressure vessel. A factorial experiment is carried out in the pilot plant to study the factors though to influence the filtration rate of this product. The four factors are temperature (A), pressure (B), concentration of formaldehyde (C), and stirring rate (D). Each factor is present at two levels. The design matrix and the response data obtained from a single replicate of the 2^k experiment are shown in table. Analyze the data.

Run No.	Factor				Run label	Response
	A	B	C	D		
1	-1	-1	-1	-1	(1)	44
2	1	-1	-1	-1	a	70
3	-1	1	-1	-1	b	49
4	1	1	-1	-1	ab	66
5	-1	-1	1	-1	c	68
6	1	-1	1	-1	ac	60
7	-1	1	1	-1	bc	80
8	1	1	1	-1	abc	65
9	-1	-1	-1	1	d	42
10	1	-1	-1	1	ad	100
11	-1	1	-1	1	bd	45
12	1	1	-1	1	abd	102
13	-1	-1	1	1	cd	77
14	1	-1	1	1	acd	85
15	-1	1	1	1	bcd	72
16	1	1	1	1	abcd	94

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Total and partial confounding in 2^k factorial experiments.

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1. Consider a 2^3 factorial experiment confounded in four replicates as given below. Identify the confounded effect and analyze the data.

Replicate 1		Replicate 2		Replicate 3		Replicate 4	
Block 1	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**
ab=3	b=1	'1'=4	a=3	'1'=6	a=3	b=3	a=4
'1' =2	a=4	c=1	ac=0	a=3	c=4	'1'=4	c=3
ac=0	abc=0	ab=3	b=0	bc=2	ab=5	ac=3	ab=5
bc=4	c=5	abc=2	bc=2	abc=1	ac=2	abc=4	bc=0

2. Analyze the following 2^3 completely confounded factorial design:

Replicate 1		Replicate 2		Replicate 3		Replicate 4	
Block 1	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**	**Block 1**	**Block 2**
'1'=101	nkp=450	'1'=106	nkp=449	'1'=87	nkp=471	'1'=131	nkp=437
nk=291	n=106	nk=306	n= 89	nk=334	n=128	nk=272	n=103
np=373	k=265	np=338	k=272	np=324	k=279	np=361	k=302
kp=391	p=312	kp=407	p= 324	kp=423	p=323	kp=445	p=324

(N = Nitrogen; P = Phosphate; K = Potash)

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Class: - M.Sc. (II) **Class Code: - STS-604-MJP**

Title: - Random effect and mixed models.

1. A manufacturer suspects that the batch4s of raw material furnished by her supplier differ significantly in calcium contents. There are a large number of batches currently in the warehouse. Five of these are randomly selected for study. A chemist makes five determinations on each batch and obtains the following data.

Batch I	Batch II	Batch III	Batch IV	Batch V
23.46	23.59	23.51	23.28	23.29
23.48	23.46	23.64	23.4	23.46
23.58	23.42	23.48	23.37	23.37
23.39	23.49	23.52	23.46	23.32
23.4	23.5	23.49	23.39	23.38

Is there any significance variation in calcium content from batch to batch?

2. In a textile company, the process engineer fix four looms and makes the random observations on strength of fabric manufactured on each loom. Following are the recorded observations.

Looms	Observations			
	1	2	3	4
1	98	97	99	96
2	91	90	93	92
3	96	95	99	95
4	95	96	97	98

Analyze the residual from the experiment. Are the analysis of variance assumptions satisfied?

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Analysis of first and second order response surface model.

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1. A researcher is studying the effect of two factors, X1 (Temperature) and X2(Time), on the yield of a chemical reaction. The researcher conducts a central composite design (CCD) with the following coded levels:
   Low level of X1 = -1 (100°C)
   Medium level of X1 = 0 (150°C)
   High level of X1 = +1 (200°C)
   Low level of X2 = -1 (30 min)
   Medium level of X2 = 0 (45 min)
   High level of X2 = +1 (60 min)
   The experimental results (in yield percentage) from the CCD are as follow

Run	X1	X2	Yield (%)
1	-1	-1	70
2	-1	1	80
3	1	-1	85
4	1	1	90
5	0	0	88
6	0	-1	82
7	0	1	87
8	-1	0	75
9	1	0	92
10	0	0	89

   a) Calculate the coefficients for first and second-order polynomial.
   b) Estimate the optimal conditions for maximizing the yield.

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Central composite design. Contour and surface plots.

1. The following data was collected by a chemical engineer. The response y is filtration time, x1 is temperature and x2 is pressure.

y	54	45	32	47	50	53	47	51	41	39	44	42	40
x1	-1	-1	1	1	-1.414	1.414	0	0	0	0	0	0	0
x2	-1	1	-1	1	0	0	-1.414	1.414	0	0	0	0	0

a) Generate the CCD and analyse the data. What operating conditions would you recommend if the objective is to minimize the filtration time.

b) Obtain the response surface plot for the fitted model. Also obtain the contour plot.

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Taguchi Methods

1. A company is optimizing the production of a plastic part to maximize tensile strength. The following factors affect tensile strength:
   Temperature (A) = 180°C (1), 200°C (2), 220°C (3)
   Cooling Time (B): 10 min (1), 20 min (2), 30 min (3)
   Pressure (C): 5 bar (1), 10 bar (2), 15bar (3)
   The tensile strength results (in MPa) from their experiments are:

Experiment	Temp(A)	Cooling Time(B)	Pressure(C)	Tensile Strength
1	1	1	1	40
2	1	2	2	45
3	1	3	3	50
4	2	1	2	55
5	2	2	3	60
6	2	3	1	52
7	3	1	3	62
8	3	2	1	58
9	3	3	2	61

   a) Calculate the average tensile strength for each level of factors A, B, and C.
   b) Determine the optimal level for each factor.
   c) What combination of factors yields the highest tensile strength?

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Application of central limit theorem and Weak law of large number

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Q.1 Let $X \rightarrow B(n, \theta)$ use the central limit theorem to find n such that,

Prob $[X > n/2] \geq 1 - \alpha$

If $\alpha = 0.1$ and $\theta = 0.45$ calculate n

Satisfied prob $[X > n/2] \geq 0.9$

Q.2 Let $X_1, X_2 \ldots\ldots X_{100}$ be a random sample from Poisson (λ) where $\lambda = 3$.

$S = \sum_{i=1}^{100} x_i \; i = 1,2,\ldots\ldots\ldots\ldots 100$. Find prob($S \geq 300$) using central limit theorem and compare with exact probability.

Q3. Let $X_1, X_2 \ldots\ldots X_{50}$ be a random sample from Bernoulli($\theta = 0.45$).

$S = \sum_{i=1}^{100} x_i \; i = 1,2,\ldots\ldots\ldots\ldots 50$.

(i) find prob($S \geq 30$) using central limit theorem and compare with exact probability.

(ii) find prob($S < 10$) using central limit theorem and compare with exact probability.

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Verification of weak law of large number.

1. A fair coin is toss 1000 times with probability of getting head is 1 and probability of getting tail is 0. Does the proportion of heads converge to the theoretical value of 0.5 as the number of tosses increases?($\varepsilon = 0.05$)
2. A store records the number of customers arriving each minute. If the expected average is 5 customers per minute, Verify that the sample mean of customer arrivals converges to the expected mean of 5 customers per minute.
3. A researcher measures the heights of students one by one. Use the WLLN to show that as the sample size increases, the average sample height converges to the true mean height of 170 cm. (Assume s.d = 10)
4. A factory produces components with a 2% defect rate and inspect 5,000 components
   a) Calculate the cumulative defect rate over time.
   b) Plot the cumulative defect rate and compare it with the 2% true rate.
   c) Use the Weak Law of Large Numbers to explain the convergence behaviour.

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Class: - M.Sc. (II) **Class code: - STS-604-MJP**

Title: - Modes of Convergence

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1. Given the sequence of functions $fn(x) = \frac{x}{n}$ for n=1:5 and x=0,1,2, answer the following:
   a) Calculate $fn(x)$ for the specified values of n and x.
   b) Does the sequence of functions $fn(x) \to 0$ as $n \to \infty$ for each value of x? If yes, conclude whether the convergence is pointwise.
2. Given the sequence of functions $gn(x) = \frac{x^2}{n}$ where $x \in [0,1]$
   a) Compute the values of $gn(x)$ for n=1, 2, 3 at x=0. 5
   b) Based on your calculations, does the sequence of functions converge to 0 uniformly over [0, 1]. Justify your answer.
3. Consider a sequence of random variables Xn

   $Xn = 1$ With probability $P = \frac{1}{n}$

   $Xn = 0$ With probability $P = 1 - \frac{1}{n}$

   Calculate probabilities, discuss whether Xn converge to 0 almost surely as n approaches infinity.