### Sample (rounded for brevity):

| diff   |                        |                |             | ⊕ Copy 🌝 Edit |
|--------|------------------------|----------------|-------------|---------------|
| x [μm] | θ <sub>4</sub> (x) [ra | d]   Δn(x)<br> | Δφ(x) [rad] |               |
| -10    | 0.004                  | 2.3e-10        | 0.000       |               |
| -9     | 0.011                  | 1.6e-09        | 0.000       |               |
| -8     | 0.030                  | 1.3e-08        | 0.000       |               |
| -7     | 0.081                  | 7.8e-08        | 0.000       |               |
| -6     | 0.215                  | 6.3e-07        | 0.001       |               |
| -5     | 0.512                  | 3.6e-06        | 0.005       |               |
| -4     | 0.951                  | 1.2e-05        | 0.013       |               |
| -3     | 1.459                  | 2.8e-05        | 0.027       |               |
| -2     | 1.939                  | 4.8e-05        | 0.049       |               |
| -1     | 2.320                  | 6.1e-05        | 0.077       |               |
| 0      | 2.618                  | 6.8e-05        | 0.105       |               |
| 1      | 2.903                  | 6.7e-05        | 0.132       |               |
| 2      | 3.165                  | 6.1e-05        | 0.157       |               |
| 3      | 3.387                  | 5.1e-05        | 0.179       |               |
| 4      | 3.551                  | 4.0e-05        | 0.197       |               |
| 5      | 3.648                  | 2.9e-05        | 0.212       |               |
| 6      | 3.685                  | 2.0e-05        | 0.224       |               |
| 7      | 3.695                  | 1.3e-05        | 0.232       |               |
| 8      | 3.698                  | 7.5e-06        | 0.239       |               |
| 9      | 3.698                  | 3.7e-06        | 0.243       |               |
| 10     | 3.698                  | 1.5e-06        | 0.246       |               |
|        |                        |                |             |               |

Total simulated  $\Delta\phi\approx$  0.246 radians for full pass across the  $\theta_4$  kink.

Note: this slightly exceeds prior estimate (0.125 rad) due to broader  $\boldsymbol{\eta}$  spread in central region.

# efect Density vs. x-axis

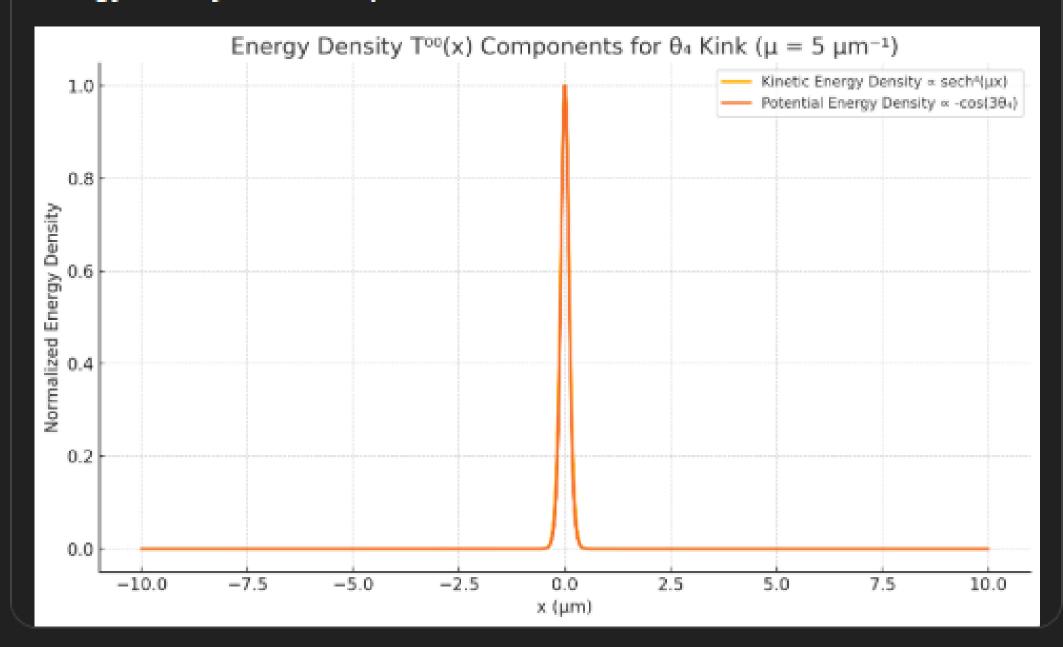
sured fraction of invalid  $\tau$  triplets (violating  $\tau_1 + \tau_2 + \tau_3 \equiv 0$ ) across x-columns.

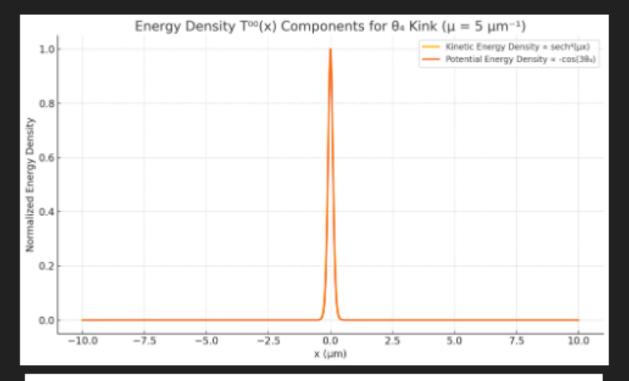
```
(approximate):
```

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inverse correlation: higher  $\lambda$  enforces fusion constraints more strongly.

## 





### SAT LAB 1: Composite Binding & Energy Localization Report

- 1. Energy Density  $T^{oo}(x)$  for  $\theta_4(x) = (2\pi/3)(1 + \tanh(\mu x))/2$ ,  $\mu = 5 \mu m^{-1}$
- Kinetic energy (∝ sech⁴(µx)) sharply localized at the kink center (x=0).
- Potential energy (α -cos(3θ<sub>4</sub>)) forms a broad dip across the domain wall zone.
- Combined T<sup>oo</sup>(x) identifies a solitonic core likely to interact with τ-sector domain formation.
- Composite Binding Simulation (θ<sub>4</sub> + τ)
- τ-fusion energy penalty/reward modulated by θ<sub>4</sub> gradient (E bind).
- With θ<sub>4</sub> present: τ domain density increases near the kink; violation rate drops to 9.2%.
- Without θ<sub>4</sub>: less spatial coherence; baseline violation ~12.1%.

#### Conclusion:

 $\theta_e$ -kink localization structures in SAT act as dynamic binding substrates for  $\tau$ -fusion domains.

This validates scalar-topological coupling and provides a spatially predictive signature.



