Emergent Gauge Couplings from Filament Topology

Abstract

We derive explicit expressions for the gauge coupling constants of emergent gauge fields based on filament topological structures. Linking densities and filament scales determine the strength of the couplings without manual insertion. The resulting expressions are testable and provide clear falsifiability conditions.

1 Emergent Gauge Coupling Constants

In standard gauge theories, the gauge field kinetic term is:

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4g^2}\text{Tr}(F_{\mu\nu}F^{\mu\nu}). \tag{1}$$

In the filament framework, gauge potentials $\mathcal{L}_\mu$ and field strengths $\mathcal{F}_{\mu\nu}$ emerge from local filament linking and twisting densities.

2 Dimensional Analysis

Gauge coupling g must be dimensionless in 4D. A natural relation is:

$$g^{-2} \sim \rho_{\text{link}}\, \ell_f^2, \tag{2}$$

where:

- ρ_{link} = filament linking density (number of links per unit volume),
- ℓ_f = filament transverse scale.

Thus:

$$g \sim \frac{1}{\sqrt{\rho_{\text{link}}\, \ell_f^2}}. \tag{3}$$

3 Specific Gauge Groups

- U(1): $g_{\text{U(1)}} \sim \frac{1}{\sqrt{\rho_{\text{winding}}\, \ell_f^2}}$.
- SU(2): $g_{\text{SU(2)}} \sim \frac{1}{\sqrt{\rho_{\text{link}}\, \ell_f^2}}$.
- SU(3): $g_{\text{SU(3)}} \sim \frac{1}{\sqrt{\rho_{\text{triple link}}\, \ell_f^2}}$.

4 Filament Parameters

Recall the filament transverse scale:

$$\ell_f = \left(\frac{2A}{T}\right)^{1/3}, \tag{4}$$

where T is filament tension and A is bending rigidity. Thus:

$$g \sim \frac{1}{\sqrt{\rho_{\text{link}}}} \left(\frac{T}{2A}\right)^{1/3}. \tag{5}$$

5 Predictions and Falsifiability

The ratios of gauge couplings are predicted:

$$\frac{g_{\text{SU(2)}}}{g_{\text{U(1)}}} \sim \sqrt{\frac{\rho_{\text{winding}}}{\rho_{\text{link}}}}, \tag{6}$$

$$\frac{g_{\text{SU(3)}}}{g_{\text{SU(2)}}} \sim \sqrt{\frac{\rho_{\text{link}}}{\rho_{\text{triple link}}}}. \tag{7}$$

- **Falsifiability**: Discrepancy between these predictions and observed SM gauge couplings implies falsification.

References

Placeholder for references.