

# Unified Emergent Action: Gravity and Gauge Fields from Filament Topology

## Abstract

We present the unified emergent action incorporating both gravitational and gauge field dynamics derived from filament ensemble statistics and topology. The action is built without manual insertion of gravitational or gauge structures, relying solely on properties of filament tension, rigidity, and topological linking densities.

## 1 Emergent Gravity Sector

The gravitational sector arises from the collective statistical behavior of filament worldlines:

$$S_{\text{gravity}}[g] = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_{\text{induced}}} R - \Lambda_{\text{induced}} + \mathcal{O}(R^2) \right), \quad (1)$$

where:

$$G_{\text{induced}} \sim \frac{36\pi}{\log\left(\frac{T}{2A}\right)}, \quad (2)$$

$$\Lambda_{\text{induced}} \sim \frac{1}{32\pi^2 \ell_f^4}, \quad \ell_f = \left(\frac{2A}{T}\right)^{1/3}. \quad (3)$$

## 2 Emergent Gauge Sector

Gauge fields emerge from the topological linking of filaments:

$$S_{\text{gauge}}[g, \mathcal{L}_\mu] = - \sum_G \frac{1}{4g_G^2} \int d^4x \sqrt{-g} \text{Tr}_G(F_{\mu\nu} F^{\mu\nu}), \quad (4)$$

with gauge couplings determined by:

$$g_G \sim \frac{1}{\sqrt{\rho_{\text{link},G}} \ell_f}, \quad (5)$$

where  $\rho_{\text{link},G}$  is the filament linking density corresponding to gauge group  $G$  (e.g.,  $U(1)$ ,  $SU(2)$ ,  $SU(3)$ ).

## 3 Unified Emergent Action

Combining both sectors:

$$S_{\text{unified}}[g, \mathcal{L}_\mu] = \int d^4x \sqrt{-g} \left( \frac{1}{16\pi G_{\text{induced}}} R - \Lambda_{\text{induced}} - \sum_G \frac{1}{4g_G^2} \text{Tr}_G(F_{\mu\nu} F^{\mu\nu}) + \mathcal{O}(R^2) \right). \quad (6)$$

## 4 Features

- No manually inserted gravitational or gauge terms.
- Gravitational dynamics emerge from filament worldline statistics.
- Gauge field dynamics emerge from filament topological linking.
- Gauge couplings are determined by filament linking densities and fundamental scale  $\ell_f$ .

## References

Placeholder for references.