Emergent Gauge Algebras from Filament Topological Invariants

Abstract

We derive a mapping between topological invariants of filament configurations and Lie algebras associated with gauge groups. Winding numbers, linking numbers, and higher linking structures generate $\mathfrak{u}(1)$, $\mathfrak{su}(2)$, and $\mathfrak{su}(3)$ algebras, respectively. This construction provides a formal pathway from filament topology to gauge symmetries without manual insertion of gauge structures.

1 Filament Topological Structures

- **Loop**: Map $S^1 \to \Sigma_\phi$, classified by $\pi_1(\Sigma_\phi)$.
- **Link**: Pair of loops characterized by the linking number $\mathrm{Lk}(\gamma_1, \gamma_2) \in \mathbb{Z}$.
- **Triple Link (Borromean)**: Triple of loops associated with triple linking invariants.

2 Lie Algebra Construction

Define a bracket operation on filament configurations:

$$[\gamma_i, \gamma_j] = \mathrm{Lk}(\gamma_i, \gamma_j). \tag{1}$$

Properties:

- Bilinearity.
- Antisymmetry: $[\gamma_i, \gamma_j] = -[\gamma_j, \gamma_i]$.

This structure mirrors the Lie bracket.

For higher linking, Massey products serve as higher-order generalizations corresponding to higher Lie algebra structures.

3 Mappings to Lie Algebras

- **Winding number (loops)** $\to \mathfrak{u}(1)$: Abelian, bracket is zero.
- **Linking number (pairs)** $\to \mathfrak{su}(2)$: Non-Abelian; linking numbers correspond to structure constants.
- **Triple linking (Borromean rings)** $\to \mathfrak{su}(3)$: Higher linking invariants map to triple commutators/Massey products.

4 Higher Topological Structures and Cohomology

Massey triple products capture triple linking phenomena and are associated with 3-cohomology classes, corresponding to triple commutators in $\mathfrak{su}(3)$.

5 Emergent Gauge Symmetry

Local deformations of filament configurations correspond to gauge transformations. The structure group formed by allowable deformations defines a principal fiber bundle over the spacetime manifold with the associated gauge Lie algebra.

6 Falsifiability Criteria

- Failure of topological linking structures to reproduce Lie algebra commutation relations implies falsification.
- Jacobi identity and structure constant verification are required for validation.

References

Placeholder for references.