Zero as a Dimension Switch: A Conceptual Framework for Reinterpreting the Number Line

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Abstract

In the conventional real number line, zero is positioned at the center between positive and negative values. However, zero exhibits fundamentally different properties from other numbers: division by zero is undefined, and it often marks the location of singularities. This paper proposes a reinterpretation of zero as a *dimension switch* — a non-numeric boundary between two distinct spaces, thereby avoiding certain divergences and redefining the continuity around zero.

1 Introduction

In standard arithmetic, zero acts as the additive identity and lies at the center of symmetry in the real number line. Despite this central role, operations involving zero, such as division by zero, remain undefined. This conceptual framework explores treating zero not as a number but as a transition point between two dimensions.

2 Motivation

Two main issues motivate this reinterpretation:

1. **Singularities:** Functions such as $f(x) = 1/x$ diverge at $x = 0$.
2. **Physical Analogies:** In thermodynamics, absolute zero represents a one-sided boundary; temperatures do not extend below 0 K in the classical sense.

By redefining zero as a structural boundary, these difficulties may be reframed rather than patched within existing frameworks.

3 Definition of Spaces

We define:

- $\mathbb{R}^+$: the *positive space*, containing numbers such as $1, 2, \pi, \ldots$
- $\mathbb{R}^-$: the *mirror space*, containing elements $1^-, 2^-, \pi^-, \ldots$

- 0: a non-numeric *switch point* between the two spaces

The overall structure can be expressed as:

$$\mathbb{R} = \mathbb{R}^+ \ \cup \ \{0\} \ \cup \ \mathbb{R}^-$$

Here, $\mathbb{R}^-$ is not merely the set of numbers less than zero; it is an entirely separate dimension with symbolic elements.

4 Transformation Across Zero

We introduce a *space transformation*:

$$T : \mathbb{R}^+ \to \mathbb{R}^-, \quad T(a) = a^-$$

and its inverse

$$T^{-1} : \mathbb{R}^- \to \mathbb{R}^+, \quad T^{-1}(a^-) = a$$

These transformations are not arithmetic inverses; they represent a conceptual translation across the zero boundary.

5 Behavior Near Zero

In this framework, there is no requirement for continuity across zero:

$$\lim_{x \to 0^+} f(x) \ \neq \ \lim_{x \to 0^-} f(x)$$

The right-hand limit is taken in $\mathbb{R}^+$ and the left-hand limit in $\mathbb{R}^-$, and they may belong to incompatible domains.

This separation allows for reinterpretation of problematic expressions like $1/x$ at $x = 0$ as involving a change of dimension rather than a blow-up to infinity.

6 Potential Applications

Potential mathematical and physical applications include:

- Reinterpreting singularities in rational functions
- Modeling one-sided physical quantities (e.g., absolute temperature)
- Exploring connections to projective geometry and Riemann surfaces
- Developing algebraic systems with multiple interconnected spaces

7 Future Work and Open Questions

Future developments may include:

- Rigorous algebraic formalization
- Geometric representation using animation tools such as Manim
- Computational simulations to explore functional behavior under T
- Extensions to higher-dimensional spaces and non-commutative algebra

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