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Research Article

Multibox Three-Dimensional Packing Problems for Heterogeneous Extrudable Items

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Aiming at the multibox 3D packing problem of heterogeneous extrudable items in e-commerce retail department stores, the optimization goal is to select appropriate packaging materials for packing all items in the order. On this basis, the number of packaging materials used is the least and the space utilization rate is high. A heuristic algorithm is designed. Firstly, the items are sorted in descending order according to volume, and then the items are boxed according to the rule of "layer generation." In order to verify the feasibility and effectiveness of the algorithm, the data provided by a large domestic e-commerce platform are used to analyse and compare the space utilization of packaging materials under the conditions of no extrusion operation and extrusion operation. The feasibility and effectiveness of the algorithm in the packaging and delivery link in the storage field of e-commerce retail department stores are proved.

1. Introduction

According to the relevant data of the China Post Bureau, by October 31, 2021, the number of courier packages in China in 2021 will have reached 86.72 billion. On November 11 alone, 696 million packages were processed, a record high. On one hand, there is the rapid development of the e-commerce logistics industry, and on the other hand, there is the huge waste of express packaging. According to incomplete statistics, the average space utilization rate of packaging is only about 50% in the packaging links of major e-commerce warehouses, and the cost waste caused by this is incalculable. The Management Measures for Express Mail Packaging issued by China's Ministry of Communications in 2021 clearly stated that green express packaging should be promoted to save resources and protect the environment. As a result, whether from an economic or social standpoint, it is critical to investigate the warehousing and packing problems of e-commerce in real life.

The packing problem refers to the loading of a certain number of items into packaging, requiring that under the premise of ensuring that each item can be loaded, the fewer the packaging items used, and the higher the total space utilization rate of the packaging items, the better, which is a typical combinatorial optimization problem [1]. The three-dimensional packing problem is a generalization of the one-dimensional and two-dimensional packing problems [2].

The packing problem can be divided into single-box packing problems and multibox packing problems according to the different types and quantities of packaging. At present, most of the research focuses on the single box packing problem, and the research on the multibox packing problem is relatively rare.

For the fixed-size single-box 3D packing problem, Huang et al. [3] proposed a simple and effective loading arrangement heuristic algorithm for solving the large-scale problem. Su et al. [4] incorporated a chemical reaction optimization algorithm into the greedy algorithm to enable faster convergence of the algorithm for solving the cradle problem. Instead of relying on genetic algorithms to generate initial solutions, Correcher et al. [5] used the stochastic strategy GRASP to generate multiple solutions and improve the algorithm to obtain high-quality solutions in a short time. Zhao et al. [6] proposed an online analysis of packing stability with a novel stacking tree to optimize packing policies. De Carvalho et al. [7] proposed a data-driven solution approach that can produce high-quality solutions in a short time.

In addition to the algorithmic improvements, some scholars have optimized the packing problem from other perspectives. Liu et al. [8] designed a "slice-strip-layer" packing rule by improving the packing order, and Gzara et al. [9] used a layer-based column generation method to improve the loading rate of the single-box 3D packing problem by representing the idle space through the very large space and designing a layer construction strategy to crate to fill the very large space, forming a crating algorithm with static stability constraints. Wang et al. [10] considered crating priorities in a single box loading problem, where all items with high priority must be loaded into the package before those with low priority and improved the average utilization of crating by a multiround local beam search method. Tresca et al. [11] minimized the number of layers by minimizing the unused space in each layer when items are stacked.

For the fixed-size multicarton 3D crating problem, Mauro et al. [12] proposed a holistic algorithm with a mixture of methods as a way to find the crating solution with the least amount of packaging used in crating. Ren et al. [13] designed a priority-conscious heuristic algorithm to prioritize the crating of bulky items by giving them a certain priority and obtained a better crating solution. Toffolo et al. [14] performed a stage decomposition of the heuristic algorithm to quickly generate feasible solutions and improved and optimized the cradle solution by local search. Nguyen et al. [15] presented a novel heuristic algorithm to solve the problem of automatically packing 3D boxes into containers. Agarwal et al. [16] presented a new bin packing system, Jampacker, to achieve faster completion of the packing process. Li et al. [17] proposed a recurrent conditional query learning (RCQL) method to solve 3D packing problems.

In the study of 3D packing with practical application scenarios, Silva et al. [18] investigated the enforcement of nonoverlapping constraints in the 3D packing problem. Iwasawa et al. [19] and Ramos et al. [20] studied the container loading problem for logistics platforms. The common container loading problem in air [21], ship, vehicle, or rail transportation [22] can also be treated as a fixed-size 3D packing problem [23, 24]. Container loading problems can be subdivided into multicontainer loading [25] and LTL loading [26]. The multicontainer loading problem has also become a research concern [5, 27].

The 3D box packing problem is usually solved by exact algorithms for small scales, such as column generation algorithms [2], dynamic programming, and branch-and-bound algorithms [28] for the solution of the 3D box packing problems. Mauro et al. [12] used a branch-and-price algorithm to solve a fixed-size boxing problem with a time window. On this basis, Liu et al. [29] investigated the variable size boxing problem with time windows using primal heuristics, iterative local search, and column generation.

In addition to the crating of regular items, the crating of irregular items is also a hot topic of research [30–32]. For example, heuristics algorithm, genetic algorithm, grey wolf optimisation [33], stochastic local search [34], and LS algorithm [35] have been used to investigate the problem of boxing irregular items. The search speed is improved by hot-start and iterative double search strategies [34].

In addition to existing research methods, the logistics sector is also being integrated with digital twins as they are used in various industry sectors [36, 37]. Marmolejo-Saucedo [36] uses digital twin technology to investigate the packaging problem in a large-scale optimization problem for supply chains. Leng et al. [38] used the digital twin as a driver for joint optimization of packaging and storage allocation in the warehouse. As one of the key concepts for accelerating and optimising management and control, the digital twin opens up new ideas for optimising the quantity and space utilization of the packaging materials used in crates.

The above studies on the three-dimensional packing problem have idealized the problem, such as

(1) Boxing only considers the items to be rigid and ignores the possible existence of some items that can be slightly extruded;
(2) The size of the package is fixed. However, in the e-commerce retail department store type storage field, packaging may not only be boxes but also bags. The size of the bags will change with the size specifications of the items loaded into them when they are boxed;
(3) Due to the numerous box models and item types in the field of e-commerce warehousing, the existing research is far from a single digit. The main optimization goal of the existing research is the space utilization rate of the box, and there is not very demanding requirement on the calculation time of the boxing scheme, which does not meet the time requirement of the boxing practice session in the field of e-commerce storage.

The soft and hard constraints of the items and packaging limit the boxing collocation, the choice of the boxing order and boxing direction of the items, as well as the choice of loading points. This leads to the study of the multibox packing problem being bound to face the problem of too many feasible boxing solutions so that it is difficult to find the optimal solution to the boxing solution.

Considering the huge amount of data in the field of e-commerce retail department stores, the calculation speed of the crating solution must be fast enough to avoid the situation of "people waiting for the crating solution." This paper will be based on the objective reality of these problems and the use of heuristic algorithms to solve the three-dimensional crating problem of heterogeneous squeezable items with multiple boxes.

Therefore, the main contributions of this paper are as follows:

(1) In this paper, we consider the problem of crating when the item has the squeezable property
(2) In this paper, we add the study of packing bags to the packing problem to solve it

(3) Based on the specific situation of the crating practice in the field of e-commerce warehousing, this paper takes the improvement of the average loading rate of orders as the main optimization objective and optimizes the crating scheme of items under the realistic condition of taking into account the computational time consumption

The focus of this paper's research is on the packing out process in the e-commerce retail department store category of warehousing. Given the wide variety of items in this industry, some item properties are explained and illustrated.

Heterogeneous items are items with different sizes, densities, weights, and other properties. The squeezable property refers to some items within a certain limit being able to change the length, width, and height of the items without affecting the value of the items themselves. There are soft constraints on the boxing operation; while some items being squeezed will damage their own value, they cannot be squeezed. The packaging refers to the packaging boxes and bags used by e-commerce retail department store enterprises in the packing and warehousing process. The packaging boxes cannot change their own specifications, and there are hard constraints on the boxing operation; while the bags can dynamically change their own length, width, and height within a certain range according to the boxed items, there are soft constraints on the boxing operation. Single box type and multibox type: the single box type refers to the order boxing's being able to choose to use the same size of the packaging specifications, that is, only one type; the multibox type refers to the order boxing's being able to use the size of the packaging specifications that are different; that is, there are a variety of models.

The remainder of the paper is organized as follows: in the next section, we describe and construct a corresponding model for the crating problem in the e-commerce retail department store class storage domain. In Section 3, we design the corresponding heuristic coding rules. The algorithm is validated and analysed in Section 4 by applying a relevant dataset provided by a large e-commerce company. We conclude and discuss future research directions in Section 5.

2. Problem Description and Model Construction

2.1. Problem Explanation. In the current e-commerce retail department store industry, there are heterogeneous items such as chips, tissues, mirrors, scissors, and so on that differ in size, density, and weight. Among them, items such as chips and paper towels have squeezable properties, while items such as mirrors and scissors cannot be squeezed. In order to meet the actual demand, the e-commerce retail department store industry has two kinds of packaging items: boxes and bags, and in order to meet the different needs of customers, the e-commerce retail department store industry has packaging items of different sizes so as to meet the needs of customers while minimizing the cost and maximizing the benefits of the enterprise. Based on this, the following problem description is given:

It is known that there is a set of items with different sizes, some of which have the property of being squeezable; there is a set of packages of different sizes, the packages include bags and boxes, where there is a soft constraint on the packing operation when using bags, and there is a hard constraint on the boxing operation; there is a set of order data, and each order contains different numbers of different kinds of items.

Based on the consideration of the economic and social-environmental benefits of the enterprise, the enterprise designs the packaging with full consideration of the practicality and economy of the packaging. By analyzing the data set provided by an e-commerce platform, it is found that most orders can be packed into the same packaging. Therefore, the following settings are made: all items in each given order are loaded into the appropriate packaging; the result of packing cannot be lost orders or items lost, overlapping, etc.; the size constraints of the packaging cannot be exceeded when loading items. The goal is to fill all orders after the use of the minimum total number of packages, and the boxing solution with the highest space utilization is better.

Based on the consideration of soft and hard constraints on items and packaging and other restrictions, the following assumptions and explanations are made for the three-dimensional packing problem of heterogeneous squeezable items based on the enterprise packing problem judging criteria of an e-commerce platform.

(1) The criteria for determining whether an item will fit into a bag are
   (a) Bag length + bag height ≥ item length + item height
   (b) Bag width + bag height ≥ item width + item height
(2) Items in different orders cannot be mixed in the same package
(3) Disregarding the item's own weight and the load-bearing capacity of the package
(4) Do not consider whether the item itself has requirements for loading direction
(5) The rule for determining the length, width, and height of the article and the package is length ≥ width ≥ height
(6) Assume that each article has at least one package that can fit the article
(7) No consideration is given to the support problem that exists when the articles are packed
(8) Assume that the items are all rectangular with uniform density, the extrusion is for a certain face of the item, the force is uniform, and the extrusion rate cannot exceed 10%, otherwise it will have an impact on the value of the item, and it is considered that the boxing scheme is not feasible

2.2. Judging Criteria. Based on the premise of the huge order parcel base, the judging principle of the packing problem for the packing out link in the field of e-commerce retail department stores is "fewer and smaller." "Fewer" means that, on the basis of ensuring that each item in the order can be packed, the smaller the number of packages used, the better, for the first optimization objective of the three-dimensional boxing problem with soft and hard constraints for heterogeneous squeezable items with multiple box types, which has a higher priority; "smaller" means that when the order uses the same number of packaging, the smaller the volume of the packaging the better, that is, the higher the space utilization of the packaging the better, for the three-dimensional packing problem of heterogeneous squeezable items with soft and hard constraints under the constraints of the secondary optimization goal, the priority level is lower than "smaller."

2.3. Model Construction

2.3.1. Variable Settings. The description of symbols is shown in Table 1.

2.3.2. Objective Functions and Constraints. Based on the "fewer and smaller" criteria and evaluation indexes given by a large e-commerce company, the objective function of this research problem is expressed as follows:

$$\max F = f - \sum_{o \in O} B_o - \alpha \cdot Z_\theta. \tag{1}$$

s.t.

$$\frac{\sum_{o \in O} \sum_{i \in B_o} \left(\sum_{j \in S_{io}} v_j / V_i \right) / m_o}{N} = f, \tag{2}$$

$$L_i + W_i + H_i = L'_{ij} + W'_{ij} + H'_{ij}, \tag{3}$$

$$V_i = \begin{cases} L'_{\max i} \cdot W'_{\max i} \cdot H'_{\max i}, & i = 1, 2, \ldots, x, \\ L_i \cdot W_i \cdot H_i, & i = x, x+1, \ldots, x+y, \end{cases} \tag{4}$$

$$l_j \cdot w_j \cdot h_j = v_j, \tag{5}$$

$$l'_j \cdot w'_j \cdot h'_j = v'_j, \tag{6}$$

$$\sum_{j \in S_{io}} v_j \leq V_i, \tag{7}$$

$$\min\left(\frac{l'_j}{l_j}, \frac{w'_j}{w_j}, \frac{h'_j}{h_j} \right) = \theta_{oj}, \tag{8}$$

$$\frac{\sum_{j \in S_{io}} \left(\sum_{j \in S_{io}} \theta_{oj} / \sum_{j \in S_{io}} j \right)}{B_o} = \theta_o, \tag{9}$$

$$\frac{\sum \theta_o}{\sum_{o=1}^{n} m_o} = Z_\theta. \tag{10}$$

In the above model, (2) indicates the average space utilization of the package after all orders are cratered; (3) indicates the calculation rule when the package is a bag and the length-width-height dynamically changes; (4) indicates the volume when the package is a bag and a box, respectively; (5) and (6) indicate the volume after the items are not crushed and are crushed, respectively; (7) indicates the volume of all the items in an order. The total volume of all the items in an order is less than the volume of the packaging; that is, all the items can be packed into the packaging; (8) indicates that the extrusion rate of the items is calculated by the minimum value of the ratio between the length, width, and height after extrusion and before extrusion; (9) indicates that the order extrusion rate is equal to the average value of the extrusion rate of each item in each package; (10) indicates the overall extrusion rate of all orders.

3. Heuristic Packing Rule Design

In this paper, both the package and the item are cubed and placed into a right-angle coordinate system, where the x-axis is the length of the package or the item, the y-axis is the width of the package or the item, and the z-axis is the height of the package or the item. As shown in Figure 1, the position of the package or item can be expressed as $\begin{bmatrix} 0 & 0 & 0 \\ L(l) & W(w) & H(h) \end{bmatrix}$. Point a coordinate is (0, 0, 0), indicating the package or item's loading point coordinate; point b coordinate is $(L(l), W(w), H(h))$, indicating the package or item's size.

3.1. The Dynamic Bag Size Change Rule Design. When the bag is in the initial state, its height is nearly negligible, as shown in Figure 2(a). At this time, the size information of the bag can be expressed as $\begin{bmatrix} 0 & 0 & 0 \\ L & W & 1 \end{bmatrix}$. When filled with the first item $(l_1 \;\; w_1 \;\; h_1)$, its height dynamics change to the height of the item h_1 and length and width according to the change in height to reduce the corresponding $(h_1/2)$. The change process is shown in Figure 2(b). At this time, the size information of the bag is dynamically updated as $\begin{bmatrix} 0 & 0 & 0 \\ L - (h_1/2) & W - (h_1/2) & h_1 \end{bmatrix}$.

Then, we find the right place to put the next item. We repeat the above steps when ready to load the nth item $(l_n \;\; w_n \;\; h_n)$. At this time, the bag size information is $\begin{bmatrix} 0 & 0 & 0 \\ L - \sum_{k=1}^{n-1} (h_k/2) & W - \sum_{k=1}^{n-1} (h_k/2) & \sum_{k=1}^{n-1} h_k \end{bmatrix}$, where the location of a loading point information is $\begin{bmatrix} \sum_{k=1}^{n-1} (h_k/2) & \sum_{k=1}^{n-1} (h_k/2) & \sum_{k=1}^{n-2} h_k \\ L - \sum_{k=1}^{n-1} (h_k/2) & W - \sum_{k=1}^{n-1} (h_k/2) & \sum_{k=1}^{n-1} h_k \end{bmatrix}$.

If $\Big\{ \max\Big\{ L - 2 \times \sum_{k=1}^{n-1} (h_k/2) + \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k,\; W - 2 \times \sum_{k=1}^{n-1} (h_k/2) + \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k, \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k, \Big\} \geq l_n \min\Big\{ L - 2 \times \sum_{k=1}^{n-1} (h_k/2) + \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k, W - 2 \times \sum_{k=1}^{n-1} (h_k/2) + \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k, \sum_{k=1}^{n-1} h_k - \sum_{k=1}^{n-2} h_k \Big\} \geq h_n$, then the loading point is considered to be filled with the item; otherwise, the loading point is considered not to be filled with the item, and the next loading point is selected for

TABLE 1: Description of symbols.

Symbols	Definitions
B_o	The set of packages used for oth order, where $o \in O$.
S_{io}	The set of items packed in the ith package used for the oth order, where $o \in O$, $i \in B$.
m_o	The number of packages used for the oth order, $m_o = \|B_o\|$.
v_j	The volume of the jth item, where $j \in S$.
l_j, w_j, h_j	The length, width, and height of the jth item, respectively.
l_j', w_j', h_j'	The new length, width, and height of the jth item after extrusion and deformation, respectively.
V_i	The maximum volume of the ith package.
L_i, W_i, H_i	The length, width, and height of the ith package, respectively.
$L_{ij}', W_{ij}', H_{ij}'$	The new length, width, and height of the ith bag after the jth item has been filled, respectively.
$L_{\max i}', W_{\max i}', H_{\max i}'$	The maximum length, width, and height of the ith bag, respectively.
θ_{oj}	The squeeze rate of the oth order of the jth squeezable item after packing, $0 \le \theta_{oj} \le 10\%$.
θ_o	The overall squeeze rate of the oth order after packing.
Z_θ	The overall squeeze rate of all orders.
α	The squeezability factor of the items in the order, $0 \le \alpha < 1$.
f	The average space utilization per order after all orders are packed.
N	The total number of orders.
F	Be used to evaluate the goodness of a crating scheme. The higher the value, the better the crating scheme.

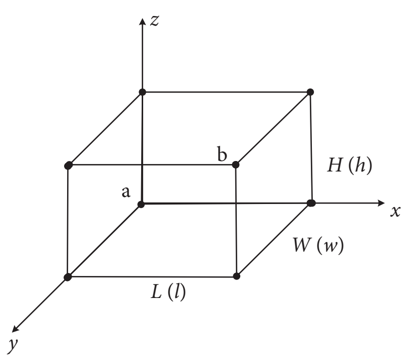

FIGURE 1: Schematic diagram of the coordinates of the package and the location of the item.

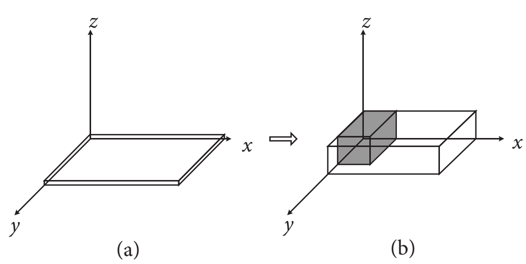

FIGURE 2: Diagram of the dynamic change in size after the bag has been filled with items.

judgment. If all the loading points are traversed and the item still cannot be filled, the bag is considered full.

3.2. Packaging Selection Rule Design. Sort the packages in descending order by the largest volume. When packing, we select a package with a volume slightly larger than the total volume of unboxed items in the current order for packing. If the items in the current order can no longer fit into the current package, we open a new package. We repeat until all items in the current order are packed.

3.3. The Design of Item Packing Orders. Because some items have deformable soft constraint limiting properties, the extrudable items are compressed first, and then the items contained in each order are sorted in descending order by volume, and the packing operation is performed starting from the item with the largest volume.

3.4. The Item Packing Direction Design. There are six states of the item in the package, as shown in Figure 3. Their position information is $\begin{bmatrix} 0 & 0 & 0 \\ l & w & h \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ l & h & w \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ w & l & h \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ h & l & w \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 0 \\ h & w & l \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 & 0 \\ w & h & l \end{bmatrix}$, respectively. When the item is boxed, we try to box the items in order according to the order in the figure, and we stop trying when the item can be put into the package. If the current package cannot be successfully put into the item, we open a new package. We repeat the above operation until the item is put into the package.

3.5. The Design of Loading Space Division Rules. When the item is loaded into the package, the package is divided into several spaces, and the small spaces are stitched together to create three large loadable spaces, as shown in Figure 4. The locations are $\begin{bmatrix} 0 & w & 0 \\ L & W-w & H \end{bmatrix}$, $\begin{bmatrix} l & 0 & 0 \\ L-l & W & H \end{bmatrix}$, and $\begin{bmatrix} 0 & 0 & h \\ L & W & H-h \end{bmatrix}$ from left to right, and they are stored in a dynamic array. The loading space is updated each time an additional item is placed.

3.6. The Design of Loading Space Division Rules. Before the item is packed, all the loading points of the current item to be loaded are judged in turn. If $\min(L-x, W-y, H-z) \ge \min(l, w, h)$, the loading point

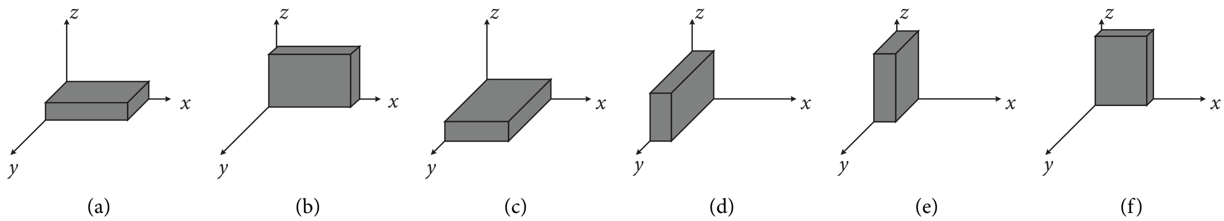

FIGURE 3: Object placement direction schematic diagram.

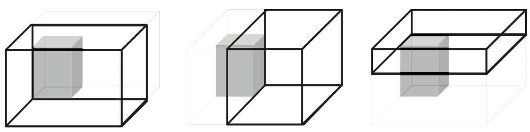

FIGURE 4: A diagram of possible loading space.

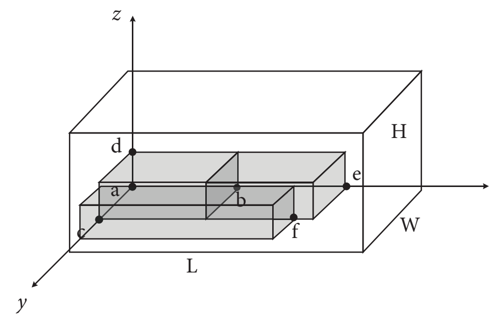

FIGURE 5: A diagram of the order of packing items.

information in the current set of loading points of the item to be loaded is retained; otherwise, it is deleted from the current set of loading points.

We compare the size of the loadable space $\min(L-x, W-y, H-z)$ of each load point in the current set of filtered load points with the current items to be loaded $\min(l, w, h)$, and we sort the order of the load points in the current set of load points, the closer the result is, the higher the order of the load points in the current set of load points.

Based on the current set of loading points of the items to be loaded, if the results are the same, boxing will be carried out according to the rule of "left and then down." As shown in Figure 5, in the packing box $\begin{bmatrix} 0 & 0 & 0 \\ L & W & H \end{bmatrix}$, the packing point a is selected first, then point b is selected, and then point e is selected. We determine whether the current item to be loaded $(l_n \ \ w_n \ \ h_n)$ can be put into the loading point $e\begin{bmatrix} \sum_{k=1}^{n-1} l_k & 0 & 0 \\ L & W & H \end{bmatrix}$. If $\begin{cases} \max\left\{L-\sum_{k=1}^{n-1} l_k, W, H\right\} \geq l_n \\ \min\left\{L-\sum_{k=1}^{n-1} l_k, W, H\right\} \geq h_n \end{cases}$, it is considered that the current item can be loaded at point e, and the loading point information is updated; otherwise, it is decided that loading point e cannot be loaded with the current item to be loaded, and point c is selected. When reloading a new item, priority is given to loading point e. If it is still impossible to place the current item, the loading point f is judged. When all the loading points at the bottom of the package cannot fit the current item, we select point d to place the item according to the "layer generation" rule. We repeat the above steps until all the items in the current order are loaded into the packaging or the packaging can no longer fit the rest of the items.

3.7. Rules for Determining whether Items Overlap. We suppose that the jth item is placed at $\begin{bmatrix} x_j & y_j & z_j \\ l_j & w_j & h_j \end{bmatrix}$ and the $j+1$th item is placed at $\begin{bmatrix} x_{j+1} & y_{j+1} & z_{j+1} \\ l_{j+1} & w_{j+1} & h_{j+1} \end{bmatrix}$. The rule to determine whether two items have overlap is that if any set of data intersection is empty, it is determined that two items do not overlap, i.e., if there is any $\{x_j, x_j+l_j\} \cap \{x_{j+1}, x_{j+1}+l_{j+1}\} = \varnothing$ or $\{y_j, y_j+w_j\} \cap \{y_{j+1}, y_{j+1}+w_{j+1}\} = \varnothing$ or $\{z_j, z_j+h_j\} \cap \{z_{j+1}, z_{j+1}+h_{j+1}\} = \varnothing$, it is considered that there is no overlap between the two items.

3.8. Rules for Determining whether an Item is Out of Bounds. We assume that the position of the jth item in the oth order is $\begin{bmatrix} x_j & y_j & z_j \\ l_j & w_j & h_j \end{bmatrix}$ and the position of its packaging is $\begin{bmatrix} 0 & 0 & 0 \\ L_i & W_i & H_i \end{bmatrix}$. If $\begin{cases} x_j + l_j \leq L_i \\ y_j + w_j \leq W_i \\ z_j + h_j \leq H_i \end{cases}$, then the item is not considered to be out of bounds.

3.9. Design of Item Extrusion Operation Rules. Assuming that the extrusion operation is performed on a certain surface of the item, the volume of the item itself is physically reduced after the extrusion without considering the resulting change in the shape of the item, i.e., the extrusion operation only leads to a change in a single property of the length, width, and height of the item. Therefore, there are three possible rules for the dimensional change of the jth squeezed item:

$$\begin{cases} (1-\alpha)l_j = l_j' \\ w_j = w_j' \\ h_j = h_j' \end{cases},$$
$$\begin{cases} l_j = l_j' \\ (1-\alpha)w_j = w_j', \\ h_j = h_j' \end{cases} \quad (11)$$
$$\begin{cases} l_j = l_j' \\ w_j = w_j' \\ (1-\alpha)h_j = h_j' \end{cases}.$$

The description of the heterogeneous extrudable item multicase 3D packing algorithm is shown in Algorithm 1.

4. Algorithm Verification and Result Analysis

4.1. Experimental Environment. The experiment was conducted in a Windows 10 environment with an Intel (R) Core (TM) i5-10210U CPU @ 1.60 GHz, 2.11 GHz, 16.0 GB of RAM (15.8 GB available), and algorithms written in Python, version Python 3.9.

4.2. Experimental Data. In this paper, we use a large domestic e-commerce platform to give a packaging size dataset, an order dataset, and an item specification attribute dataset. The packaging dataset contains 3 types of bags and 7 types of boxes, for a total of 10 types of packaging items. The maximum value of packaging volume is 516600 cm^3, and the minimum value is 120781 cm^3. The maximum number of items is 187, the minimum is 40, and the average number of items per order is 113.487. The item dataset contains 30 kinds of items, and the attributes are mainly item size and whether they are deformable or not. 0 means the items are not deformable, i.e., they cannot be squeezed, and there are 18 items that cannot be squeezed. 1 means the items are deformable, i.e., they can be squeezed, and there are 12 items that can be squeezed. The length, width, and height of the items are in the range of 33 mm–99 mm, 22 mm–96 mm, and 20 mm–76 mm, respectively. The maximum value of item volume is 5848.20 cm^3, and the minimum value is 271.04 cm^3.

All experimental data used for algorithm validation are from actual experiments with a high degree of reliability and representativeness, and they satisfy the description and assumptions made in the preceding problem, allowing us to test the algorithm's feasibility and effectiveness in real-world applications. Some of the experimental data from the three datasets involved in the experiments are shown in Tables 2 to 4. This experiment will use this dataset to perform no-squeeze operation 3D packing and squeeze operation 3D packing on the items, respectively, in order to verify the feasibility and effectiveness of the algorithm in practical applications.

4.3. Experimental Results. Based on the order data, a suitable package is selected and the items are packed. The evaluation index F value is −999.446 when the item is not squeezed and −999.323 when the item is squeezed, which shows that the packing solution obtained by squeezing the item first and then packing is better, with an overall optimization of 21.982%. A comparison of the results of the packing scheme is shown in Table 5.

As shown in Table 4, according to the evaluation criteria of “fewer and smaller”:

(1) Regardless of whether or not the items are crushed during the crating operation, the crating results are 1000 orders using a total of 1000 packages, and the number of packages used for each order is 1, achieving the crating goal of using the least number of packages;

(2) Without extrusion, the maximum space utilization value of the packaging after packing 1000 orders was 87.3852%, the minimum value was 50.8678%, and the average space utilization f value was 86.346%. When extrusion operation was performed on the items, the maximum space utilization rate of the packaging after packing 1000 orders was 92.9995%, the minimum value was 53.8206%, and the average space utilization f value was 82.799%. Both achieved a higher average space utilization rate than the actual e-commerce department store retail industry and, to a certain extent, achieved the packing goal of “smaller” packaging volume;

(3) The average time required to calculate each packing solution per order is 3.5 s when no extrusion is performed on the items. When extruding the items, the average time required to calculate each packing solution per order was 3.8 s. In contrast, squeezing the items first leads to a slight increase in the calculation time of the solution, but it can effectively meet the daily crates needs of companies in practice, regardless of whether the items are squeezed first. There are no situations where an order to be boxed has arrived and the staff don't know which packaging to choose for the box and how the item should be packed. At the same time, it also avoids to a certain extent the waste that may be caused by the packing staff's experience in the packing of order items and helps the e-commerce department store retail industry save a certain amount of time, labor trial and error costs, and operation costs in the warehouse packing out of the warehouse. When an extrusion operation is carried out on the items, the effect of the partial packing scheme is shown in Figure 6.

In the process of the study, it was found that the average space utilization value of the packaging was 3.548% lower when the extrusion operation was performed first compared to the packing without the extrusion operation, i.e., the space utilization of some orders with the extrusion operation and then packing was lower than the space utilization of the direct packing. Statistics on the packing results showed that there were 880 orders with higher space utilization when packing was done directly and 120 orders with higher space utilization when crushing operations were

```
Input: packages set B, order data set O, item set S;
Output: packing solution for each order;
Start
Step 1: sort the packages in order of maximum volume from largest to smallest.
Step 2: reading order data and sorting the squeezable items by size of the items from largest to smallest after squeezing them.
For n = 0 to N (traversing the order set)
    Calculate the total volume of all unboxed items in the current order
    Selecting packages that are slightly larger than the total volume of all unboxed items in the order
    For j = 0 to total number of items in the current order (traversing items in order o)
        Update load point information set A
        For a = 1 to |A| (traversing the set of loading points)
                For b = 1 to 6 (traversing the set of boxing directions)
                    If the current item can be loaded in the current direction
                        Then load current item
                        Break
                    Else
                        b++;
                    End If
                    End For
                    If the current item has been loaded into
                        Break
                    Else
                        a++;
                    End If
                End For
                If the current item has been loaded into
                    j++;
                Else
                    Discard the current package and select a package with a slightly larger volume than the current package to start packing again.
                End If
            End For
            n++;
        End For
Step 3: output individual order packing solutions.
End
```

Algorithm 1: Heterogeneous squeezable items multibin 3D packing algorithm.

Table 2: Packing dataset (mm).

Package type	Length	Width	Height
No. 1 self-propelled waterproof bag ($D1$)	549	424	1
No. 2 self-propelled waterproof bag ($D2$)	531	509	1
No. 3 self-propelled waterproof bag ($D3$)	449	419	1
Platform no. 1 packing box ($X1$)	330	330	225
Platform no. 2 packing box ($X2$)	430	360	227
Platform no. 3 packing box ($X3$)	330	240	258
Platform no. 4 packing box ($X4$)	330	330	260
Platform no. 5 packing box ($X5$)	430	360	283
Platform no. 6 packing box ($X6$)	550	400	311
Platform no. 7 packing box ($X7$)	410	350	360

Table 3: Order dataset (part).

Order no.	Number of item types	Total number of items
0	12	133
1	10	104
2	13	160
3	8	105
4	14	185
...	...	...
995	11	147
996	10	105
997	8	119
998	9	110
999	9	110

Table 4: Object dataset (part).

Item no.	Length	Width	Height	Deformable or not
0	94	89	62	0
1	67	40	36	0
2	87	68	63	1
3	81	67	34	0
4	45	35	25	0
5	48	36	23	0
6	73	22	22	1
7	75	62	49	0
8	37	30	28	1
9	85	60	35	1
10	72	66	45	0
11	99	96	55	0
12	56	22	22	1
13	89	75	47	1
14	86	85	71	0
15	64	62	39	0
16	87	70	26	0
17	73	55	40	1
18	56	36	22	0
19	82	50	44	1
...	...	...	...	...

Table 5: A comparison table of the results of the packing scheme.

	No squeeze operation on items	Squeeze operation on items
Evaluation index F	−999.137	−999.173
Number of packages used	1000	1000
Average space utilization f	86.346%	82.799%
Average packing solution calculation time (s)	3.5	3.8

performed first and then packing was done. The crating scheme of orders with and without crushing operation was compared, and the volume difference between the order items before and after crushing was calculated, as well as the volume difference of the packaging selected before and after crushing of the order items, and some results are shown in Table 6. When the total volume difference of the order after the squeezing operation is greater than the difference in the

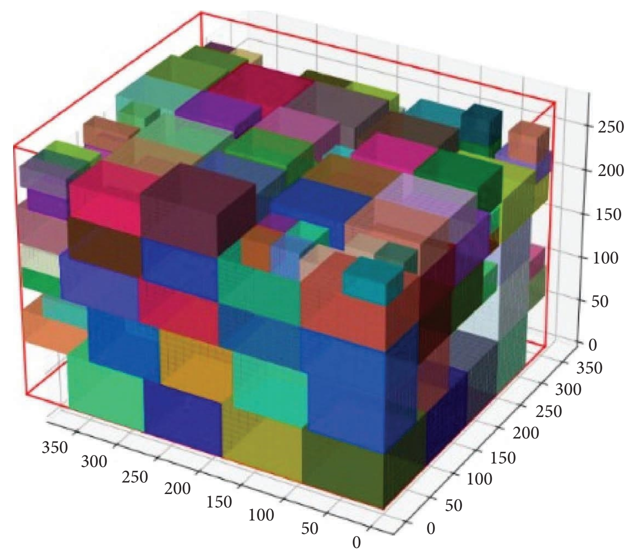

Figure 6: The result chart of item packing.

Table 6: Comparison table of the results of the packing operation (part).

Order no.	No extrusion operation				Performing extrusion operations				Order volume difference	Package volume difference
	Total order volume (cm^3)	Package	Volume (cm^3)	f(%)	Total order volume (cm^3)	Package	Volume (cm^3)	f(%)		
0	20282582	$X4$	28314000	71.634464	19461392.8	$X4$	28314000	68.734170	821189.2	0
1	17625280	$X4$	28314000	62.249347	17014602.8	$X4$	28314000	60.092544	610677.2	0
2	28438130	$X5$	43808400	64.914788	26833974.4	$X5$	43808400	61.253035	1604155.6	0
17	32455188	$X7$	51660000	62.824599	31049735.9	$X5$	43808400	70.876215	1405452.1	7851600
18	35489090	$X7$	51660000	68.697426	33465645.6	$X5$	43808400	76.390933	2023444.4	7851600
36	19893597	$X5$	68420000	57.48966	18956781.5	$X2$	35139600	54.461013	936815.5	33280400
43	26735613	$X2$	35139600	76.083999	26735613	$X2$	35139600	76.083999	0	0
51	15497871	$X1$	24502500	63.250162	14923882.5	$D2$	20809848.52	71.715479	573988.5	3692651.48
118	21344060	$X1$	35139600	60.74076	20317933.6	$X4$	28314000	71.759319	1026126.4	6825600
120	11660823	$D1$	16432968.05	70.959932	11660823	$D1$	16432968.05	70.959932	0	0
137	30737753	$X7$	51660000	59.500103	28905245	$X5$	43808400	65.981056	1832508	7851600
141	14712168	$X1$	24502500	60.043538	13852996.8	$D2$	20809848.52	66.569426	859171.2	3692651.48
233	16707377	$X1$	24502500	68.186418	15857296	$X1$	24502500	64.717053	850081.2	0
321	99391243	$X1$	24502500	45.959806	8738625	$D3$	12078072.33	72.351156	652618	8355527.67
405	18729023	$X4$	28314000	66.147570	17855141	$X1$	24502500	72.870692	873881.6	3811500
591	21276003	$X3$	20433600	60.547084	10121136	$X3$	20433600	60.547084	0	0
650	19476566	$X5$	68420000	28.466188	18069972	$X4$	28314000	63.819920	1406594	40106000
773	13630918	$X1$	24502500	55.630723	13630918	$X1$	24502500	55.630723	0	0
865	16745289	$X4$	28314000	59.141375	16160395	$X1$	24502500	65.954066	584894	3811500
989	24975915	$X5$	43808400	57.011703	23329326.3	$X2$	35139600	66.390415	1646588.7	8668800

volume change of the selected packaging, the items are first squeezed and then packed; otherwise, the items in the order are packed directly.

5. Conclusions

The three-dimensional bin packing problem is prevalent in all walks of life, such as transportation vehicle container packing links, airline container packing links, and so on. However, no scholars have paid attention to the packing problem in the warehouse packing out of the e-commerce retail industry, so it is of great practical significance to study the multicarton three-dimensional packing problem of heterogeneous squeezable items under the constraints of soft and hard constraints. In this paper, we focus on the packing and shipping process in the e-commerce department store retail industry, and we design a heuristic algorithm to solve the heterogeneous squeezable item multicarton 3D crating problem, focusing on the impact on the order crating space utilization when the item is flexible and can be squeezed to a certain extent. The feasibility and effectiveness of the algorithm are compared and verified with specific data sets, and the results prove that the algorithm can meet the packing requirements of e-commerce retail department store-type enterprises and can reduce the time cost and labor cost of enterprises to a certain extent.

The optimization of the algorithm will be carried out later to improve the speed of the algorithm and to try to use the rest of the algorithm to solve the problem. Further research will be conducted on whether and under what conditions to squeeze the items during the crate operation. At the same time, with the application of digital twin technology in the field of logistics, subsequent research will use digital twin to simulate the effect of crating in order to more intuitively find the factors affecting the effect of crating, so as to improve the crating solution, further optimize the amount of packaging materials used and improve the space utilisation of the crating solution. Based on this, the packaging size is optimized and standardised with clustering and other algorithms to improve the recycling rate of the boxes, in order to further save costs in the e-commerce retail sector, effectively reduce carbon emissions, and better respond to the national call for "green packaging."

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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