MIP for cost effective 3-D packing problem

Abdelrehman Abdelkarim, Aniket Satpute,
Miaad Hajighasemi, Sarthak Kinger, Tanmay Sanjeev

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1 MIP

This Mixed Integer Program solves the problem of clustering the total number of items according to the container they will be packed, in a cost effective manner. Each container has a pivot weight, which changes the cost function for that particular container if the total weight of the items in that container exceeds the pivot weight.

1.1 Objective Function

$$Y(i,j) = sum_j(b_j * (\sum_i x_{ij} w_i) * (1 - z_j) + (b_j * p_j + a_j * ((\sum_i x_{ij} * w_i) - p_j)) * z_j)$$

1.2 Constraints

$$\sum_{j=1}^{m} x_{ij} = 1 \qquad .....\forall i \in (1,2,3,....,n) \quad (1)$$

Every item to be packed only once

$$\sum_{i=1}^{n} x_{ij} w_i \leq W_j \qquad .....\forall j \in (1,2,3,....,m) \quad (2)$$

Weight capacity constraint

$$\sum_{i=1}^{n} x_{ij} v_i \leq 0.9 * V_j \qquad .....\forall j \in (1,2,3,....,m)$$

(3)

Volume capacity constraint

$$y_j \leq p_j + M z_j \qquad .....\forall j \in (1,2,3,....,m) \quad (4)$$

Check if weight of items in container j exceeds pivot weight

$x_{ij} \in \{0,1\}$ — *Item i goes in container j or not*

$z_j \in \{0,1\}$ — *Container j exceeds the pivot weight or not*

$y_j \in \mathbb{R}^+$ — *Total weight of the items in container j*

1.3 Variables

$x_{ij} \rightarrow$ Binary decision variable to decide if item i goes in container j or not
$y_j \rightarrow$Total weight of all the items stored in container j
$z_j \rightarrow$Binary Decision variable, 1 if container j has exceeded pivot weight, 0 otherwise
$w_i \rightarrow$Weight of the item i
$W_j \rightarrow$Weight capacity of container j
$v_i \rightarrow$Volume of the item i
$V_j \rightarrow$Volumetric capacity of container j
$a_j \rightarrow$Cost function above pivot weight
$b_j \rightarrow$Cost function below pivot weight
$p_j \rightarrow$Pivot weight for container j