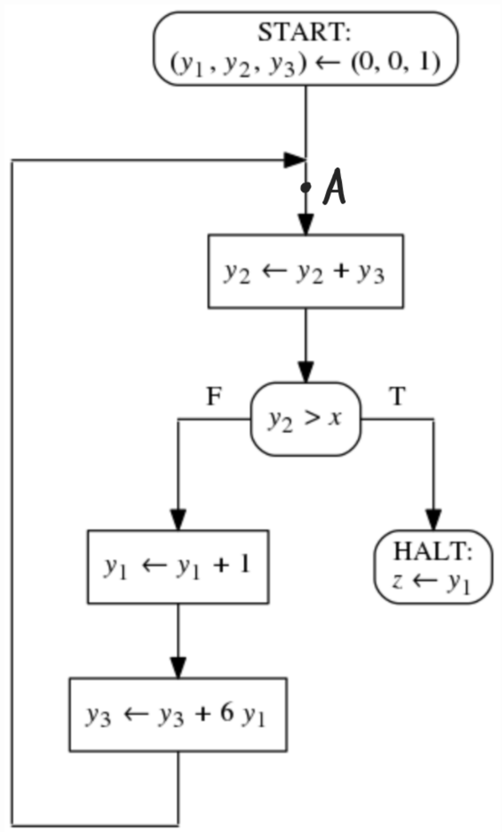
Задание 1.2

$\phi(x) \equiv x \geq 0$

$\psi(x, z) \equiv z^3 \leq x < (z + 1)^3$



$S\text{-}A: x \geqslant 0 \longrightarrow P_A(x, 0, 0, 1)$

$A\text{-}A: x \geqslant 0 \wedge y_2 + y_3 \leqslant x \wedge P_A(x, y_1, y_2, y_3) \rightarrow$

$\longrightarrow P_A(x, y_1 + 1, y_2 + y_3, y_3 + 6y_1 + 6)$

$A\text{-}H: x \geqslant 0 \wedge y_2 + y_3 > x \wedge P_A(x, y_1, y_2, y_3) \rightarrow$

$\longrightarrow y_1^3 \leq x \leq (y_1 + 1)^3$

Докажем по индукции:

$k = 0 \qquad y_1 = 0 \quad y_2 = 0 \quad y_3 = 1$

$k = n \qquad y_1 = \sum_{i=1}^{n} i = n$

$y_3 = y_{3_0} + 6 \sum_{i=1}^{n} y_{1_i} = 1 + 6 \cdot \frac{n(n+1)}{2} =$

$= 1 + 3n(n+1) = 1 + 3n + 3n^2$

$y_2 = y_{2_0} + \sum_{i=0}^{n-1} y_{3_i} = 0 + \sum_{i=0}^{n-1} (1 +$

$+ 3n + 3n^2) = \sum^{n-1} 1 + 3 \sum^{n-1} n +$

$$+3\sum_{i=0}^{n-1} n^2 = n + 3\frac{(n-1)n}{2} +$$

$$+3\,\frac{(2(n-1)+1)(n)(n-1)}{6} = (n-1)+1+$$

$$+\frac{3((n-1)+1)(n-1)}{2} + \frac{3((n-1)+1)}{6}$$

$$\frac{(n-1)(2(n-1)+1)}{6} = \{n-1=m\} =$$

$$= m+1+\frac{3(m+1)m}{2}+\frac{3(m+1)m(2m+1)}{6} =$$

$$= 1+m+\frac{3m^2+3m+2m^3+2m^2+}{2}$$

$$\frac{+m^2+m}{2} = 1+m+m^3+3m^2+2m$$

$$= 1+3m+3m^2+m^3=(m+1)^3=n^3$$

$$\Rightarrow \quad y_{1n}=n \quad y_{2n}=n^3 \quad y_{3n}=1+3n+3n^2$$

Тогда $x< y_2+y_3 = n^3+3n^2+3n+1 =$

$$= (n+1)^3 \quad \Rightarrow p(x, y_1, y_2, y_3) =$$

$$=\underline{(y_2=y_1^3)\cap(y_3=3y_1^2+3y_1+1)\cap(y_1^3\le x)}$$

$$x<(y_1+1)^3$$