Linear Algebra

Contents

Introduction

Linear Algebra

Linear algebra is the study of the lines and planes, vector spaces, and mappings needed for linear transforms. It involves the study of systems of linear equations, matrices, vectors, and linear transformations.

- Systems of Linear Equations
- Matrices and Matrix Operations
- Vectors and Vector Spaces
- Determinants
- Eigenvalues and Eigenvectors
- Linear Transformations

Introduction

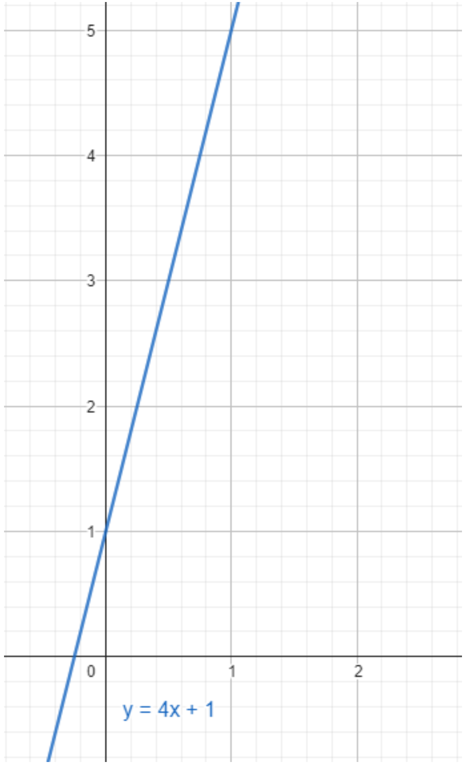
Linear Algebra

$$y = 4x + 1$$

is an example of a linear equation.

Equations such as this one are considered linear because they describe a line on a graph with only two dimensions.

The line is the result of trying a variety of different values for the variable x in order to determine how the equation or model affects the value of the variable y.



Introduction

Linear Transformation

-> translation rotati

A transformation is another name for a function. A function changes a variable from an element of one set to an element of another set (possibly the same set) through some rule.

A linear transformation is a function between two vector spaces, it takes vectors from one vector space and "transforms" them into vectors in another vector space, while preserving the essential algebraic structure of the vectors.

Note: Vector spaces are sets, the elements of which are vectors.

Introduction

Vectors and Matrices

Linear algebra basically deals with vectors and matrices (different shapes of arrays) and operations on these arrays. Vectors are basically a 1-D array of numbers but geometrically, they have both magnitude and direction.

Example:

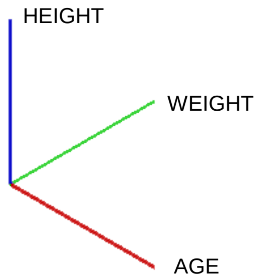
Vector for Data Representation

One row in this data is represented by a feature vector which has 3 elements or components representing 3 different dimensions.

N-entries in a vector makes it n-dimensional vector space and in this case, we can see 3-dimensions.

Height	Age	Weight
170	24	70
165	45	65
190	28	102
180	34	83
182	30	79
178	67	85

$$\begin{bmatrix} 170 \\ 24 \\ 70 \end{bmatrix}$$



Why Study Linear Algebra?

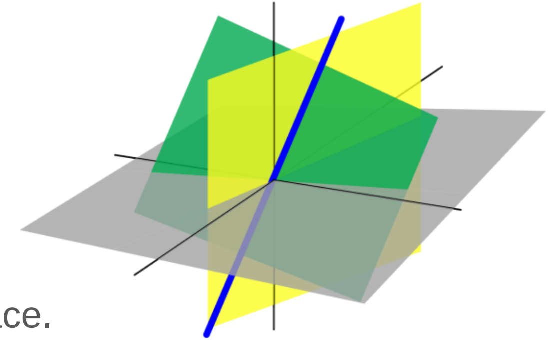
Application Areas

- Computer Graphics
- Data Science
- Machine Learning
- Linear Programming
- Cryptography
- Games

Why Study Linear Algebra?

Applications

- Used for space studies



The use of geometry as the foundation for modelling ambient space.

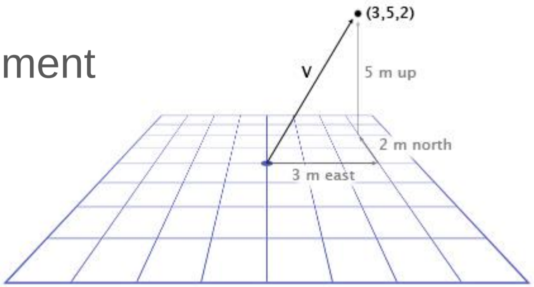
- Mechanics and robotics - to describe the dynamics of rigid bodies
- Computer vision, and computer graphics - to describe the relationship between a scene and its plane representation.

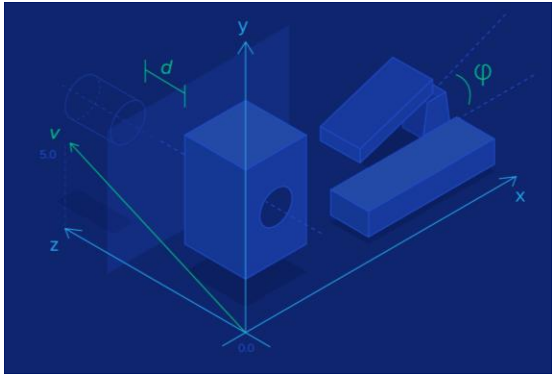
In these applications, the geometry, matrices, the computation of coordinates, vector spaces, etc. is frequently utilized.

Why Study Linear Algebra?

Applications

- Used for Game development





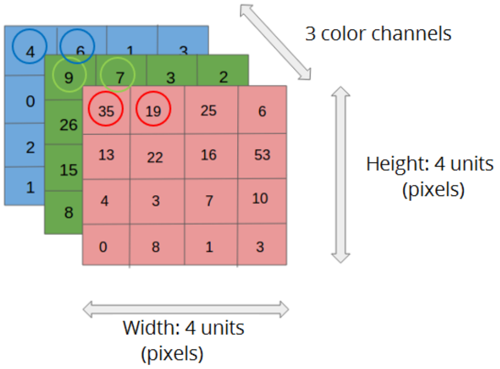
Matrices are used to represent many different types of data. Games that involve 2D or 3D graphics rely on some matrix operations to display the game environment and characters in game.

It involves modeling of a surface of an object into a representation of a collection of points in 3D space. These points are vectors in Linear Algebra, on which the processes of Linear Algebra such as transformation, rotation and scaling can be applied.

Why Study Linear Algebra?

Applications

- Used for digital images



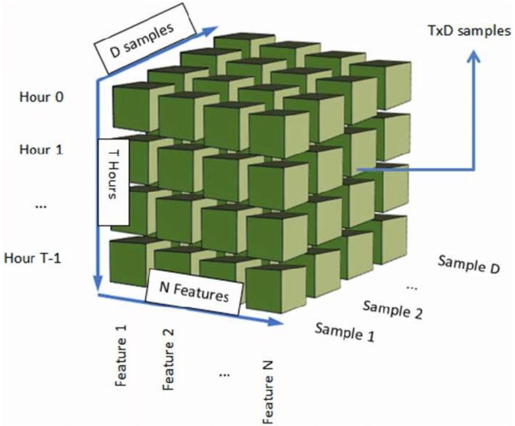
A digital image is made up of a bunch of pixels each of a single color. A pixel is a small element of a picture. Each pixel represents a color which is a combination of three-color channels: red, green and blue. With a million combination of these colors, we can represent any color and any image.

In computers, images are commonly stored as matrices. To represent a single channel intensity, we will use values from 0-255. So, for one color pixel we need three times 8 bits for a representation.

Why Study Linear Algebra?

Applications

- Used for Data Science & Machine Learning



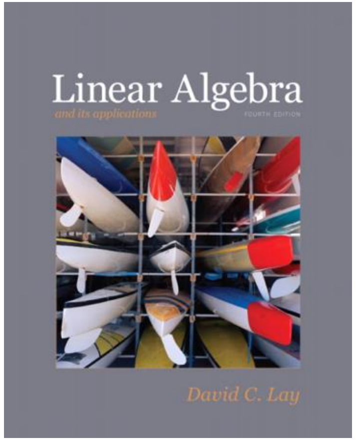
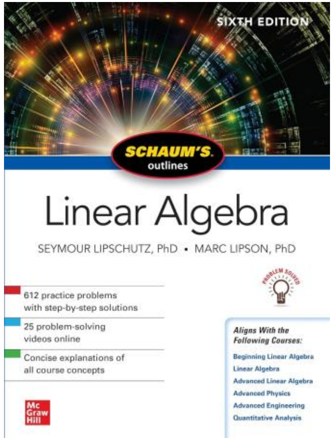
The fuel of ML models, that is data, needs to be converted into arrays before you can feed it into your models. Thus, both data and machine learning models can be expressed in matrix form. Linear algebra is used in data preprocessing, data transformation, and model evaluation.

The computations performed on these arrays include operations like matrix multiplication (dot product). This further returns the output that is also represented as a transformed matrix of numbers.

Course Objectives

- To understand the concept of a system of linear equations, and to understand the number and geometry of the solutions to these equations.
- To be able to formulate a variety of problems in terms of finding the solution to a system of linear equations.
- To understand the concept of a vector space and be able to perform basic operations with vectors and matrices.
- To be able to apply the concept of vectors and matrices to solve problems related to system of equations and linear transformations.
- To be able to apply software tools for solving problems related to linear algebra.

Syllabus

Syllabus	Textbooks	
1. Systems of Linear Equations	 **LINEAR ALGEBRA AND ITS APPLICATIONS** David C Lay Addison-Wesley (2012)	 **SCHAUM'S OUTLINE OF LINEAR ALGEBRA** Seymour Lipschutz, Marc Lipson Mc Graw Hill (2017)
2. Matrices and Matrix Operations		
3. Determinants		
4. Vectors and Vector Spaces		
5. Eigenvalues and Eigenvectors		
6. Linear Transformations		

Software Tools

Use Case	Software Tools	
Solving Equations Plotting Functions Matrix Operations Matrix Manipulations Vector Analysis etc.	**Maxima** Software Maxima is a computer algebra system based on a 1982 version of Macsyma and is still in development today. It is written in Common Lisp and runs on all POSIX platforms such as macOS, Unix, BSD, and Linux, as well as under Microsoft Windows and Android. Wikipedia	**MATLAB** Programming language MATLAB is a proprietary multi-paradigm programming language and numeric computing environment developed by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages. Wikipedia
	MAXIMA Computer Algebra System CAS Software [Free]	**MATLAB** Computing Platform [INHA License]

Lesson Plan – 1/4

Week	Lecture Topic
1	Introduction: Basic Concepts; Course Details
	Introduction: Linear Algebra - Vectors, Matrices and Determinants
2	Systems of Linear Equations: Graphical Solutions
	Systems of Linear Equations: Algebraic Solution - Substitution & Elimination
3	Matrices: Matrix Notation, Matrix Equation, Augmented Matrix
	Matrices: Matrix Operations - Multiplication, Transpose
4	Matrices: Matrix Operations - Trace and Inverse of a Matrix
	Matrices: Echelon Matrix, Row Reduction Method

Lesson Plan – 2/4

Week	Lecture Topic
5	Matrices: Matrix Solution to System of Linear Equations
	Vectors: Basic Properties, Vector Operations, Theorem
6	Vectors: Vector Equations - Vectors in $\mathbb{R}^2$, Vectors in $\mathbb{R}^3$, Vectors in $\mathbb{R}^n$
	Vectors: Linear Combinations of Vectors
7	Vector Spaces: Coordinate systems
	Vector Spaces: Vector Spaces and Subspaces
8	**Mid-term exam**
	Solution and discussion

Lesson Plan – 3/4

Week	Lecture Topic
9	Linear Transformations: Introduction
	Linear Transformations: Matrix of Linear Transformations
10	MATLAB Basics: Installation and quick tutorial
	Vector Analysis and Basic Operations with Matrices in MATLAB
11	Application of matrices to Computer Graphics
	Determinants: Introduction, Properties
12	Determinants: Cramer’s Rule for solving System of Linear Equations
	Eigen Values and Eigen Vectors: Introduction

Lesson Plan – 4/4

Week	Lecture Topic
13	Eigen Values and Eigen Vectors: Characteristic Equation
	Eigen Values and Eigen Vectors: Diagonalization
14	Eigenvectors and Linear Transformation, Application to differential Equation
15	**Final Examination**
16	Check and Update
	Result Announcement

Evaluation Criteria

Mid-term Exam	Final exam	Attendance	Assignment	Total
20%	40%	20%	20%	100%

THANK YOU