Lecture 1: Course introduction and review

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Materials in this lecture notes partially come from two lecture notes written by Jean Kyung Kim (Inha Univ.) and Pratheepa Jeganathan (Stanford Univ.).

Course introduction and review

Outline

- What is a regression model?
- Descriptive statistics – numerical
- Descriptive statistics – graphical
- Inference about a population mean
- Difference between two population means

What is course about?

- It is a course on Regression analysis.
- In class, we learn about basic concepts and theories in regression analysis.
- Taking Regression analysis Lab. is recommended.
  (In practic, we need to use a statistical software to handle data sets and fit the regression models.)
- Course notes will be R markdown.
- We will start out with a review of introductory statistics to see R in action.

What is a regression model?

A regression model is a model of the relationships between some *covariates (predictors)* and an *outcome*.

Specifically, regression is a model of the *average* outcome *given or having fixed* the covariates.

Example (Heights of mothers and daughters)

- We will consider the heights of mothers and daughters collected by Karl Pearson in the late 19th century in R package `alr4`.

```
install.packages("alr4")
```

```
library(alr4)
head(Heights)
```

```
##   mheight dheight
## 1    59.7    55.1
## 2    58.2    56.5
## 3    60.6    56.0
## 4    60.7    56.8
## 5    61.8    56.0
## 6    55.5    57.9
```

- One of our goals is to understand height of the daughter, D, knowing the height of the mother, M.
- A mathematical model might look like

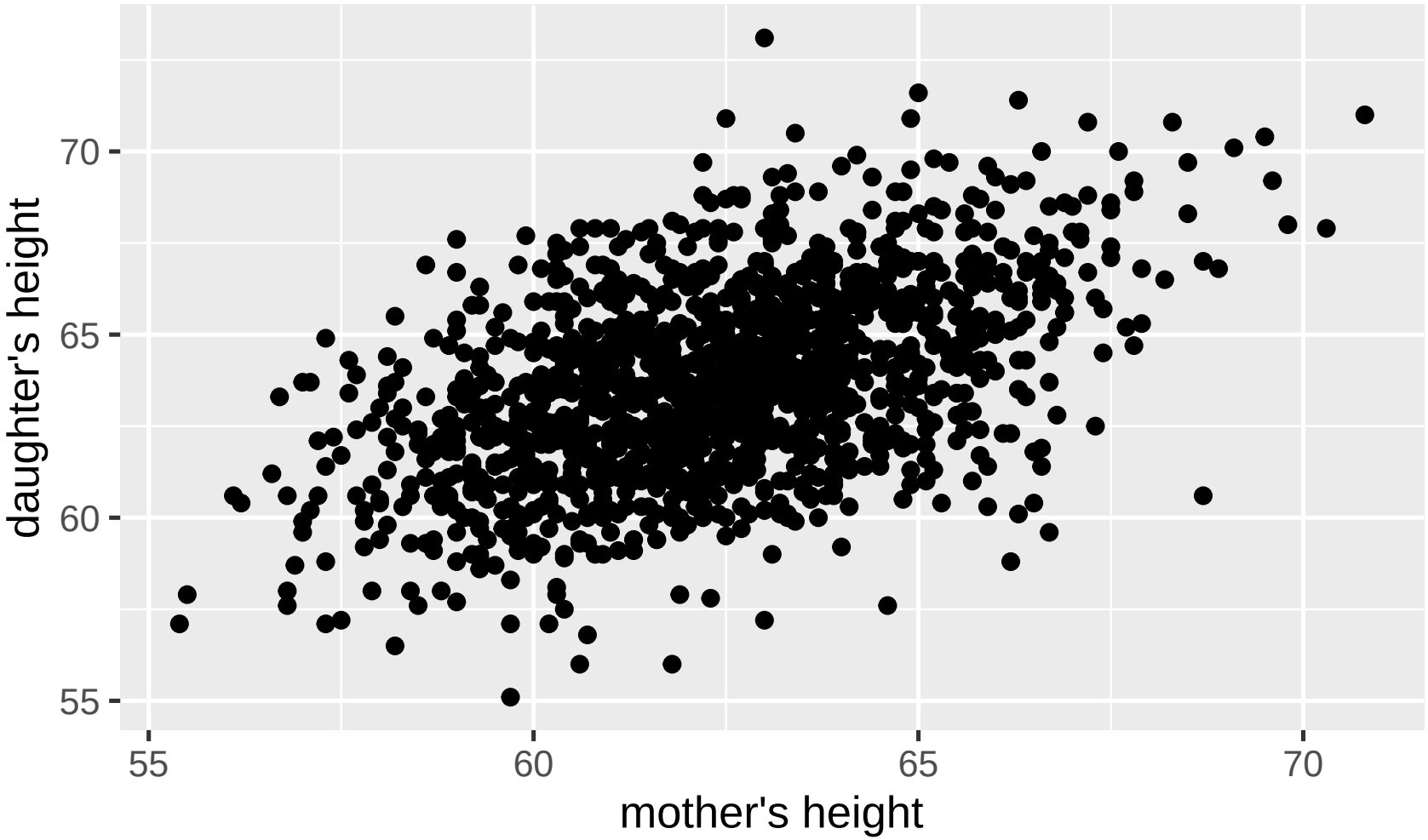
$$D = f(M) + \varepsilon,$$

  where f gives the average height of the daughter of a mother of height M and ε is *error*: not *every* daughter has the same height.
- A statistical question: is there *any* relationship between covariates and outcomes – is f just a constant?

- Let's create a plot of the heights of the mother/daughter pairs.

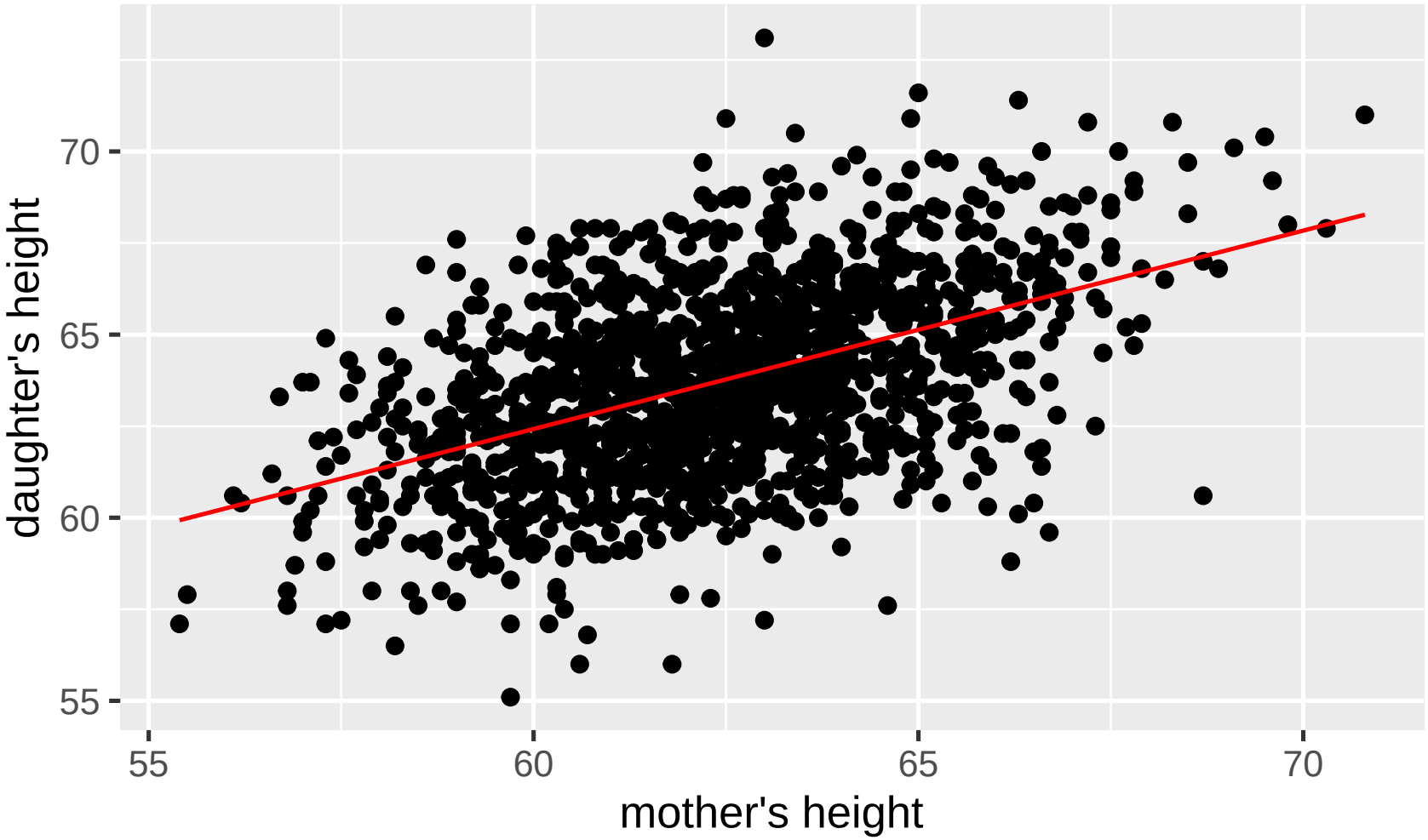
```
install.packages("ggplot2")
```

```
library(ggplot2)
p = ggplot(data = Heights) +
  geom_point(aes(x = mheight, y = dheight)) +
  xlab("mother's height") +
  ylab("daughter's height")
```


daughter's height
70
65
60
55
55
60
65
70
mother's height

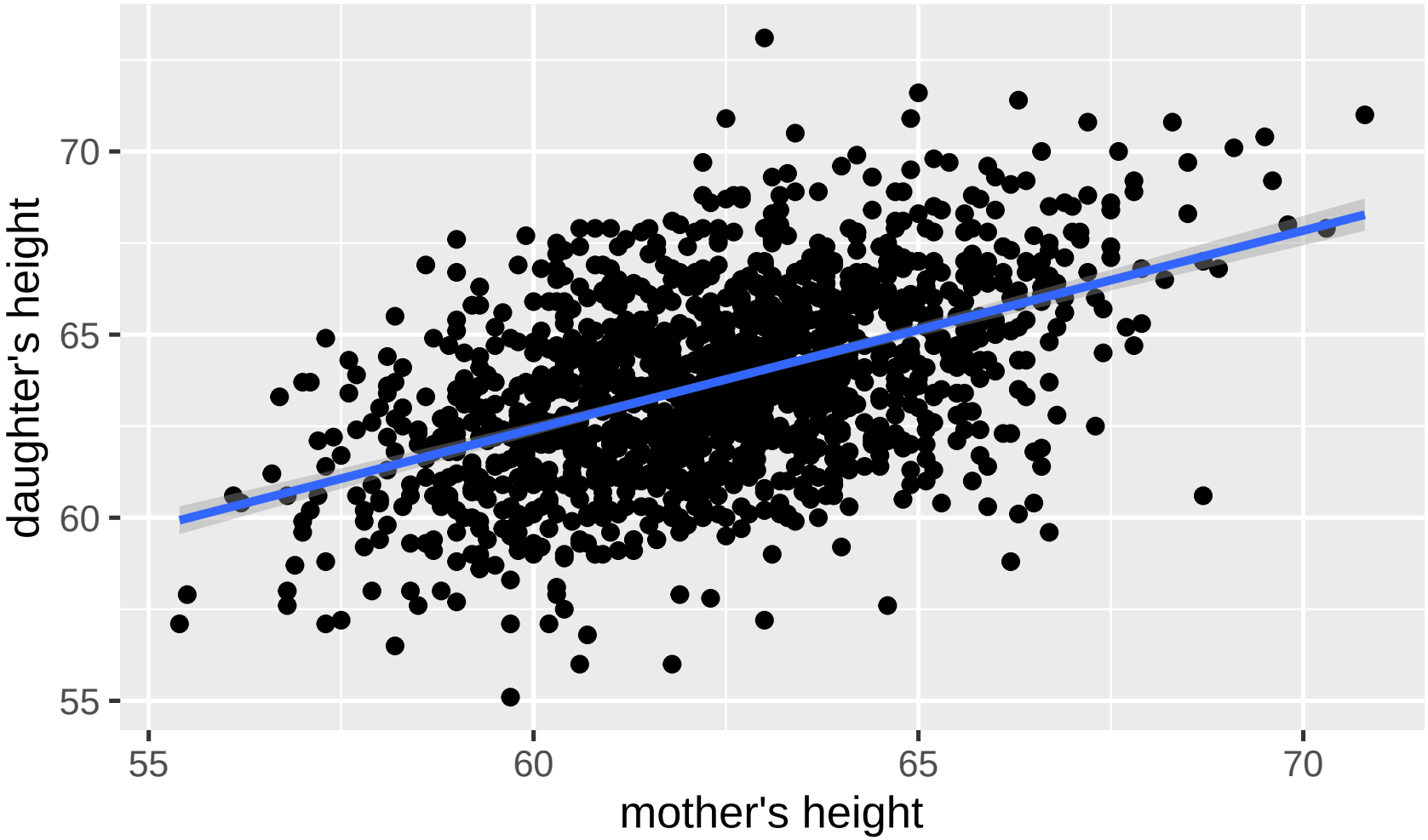
- In the first part of this course we'll talk about fitting a line to this data. Let's do that and remake the plot, including this "best fitting line".

```
fit.lm  = lm(dheight ~ mheight, data = Heights)
df = data.frame(mheight = Heights$mheight,
  dheight.fit = fitted(fit.lm))
p2 = ggplot(data = Heights) +
  geom_point(aes(x = mheight,
    y = dheight)) +
  xlab("mother's height") +
  ylab("daughter's height") +
  geom_line(data = df, aes(x = mheight,
    y = dheight.fit), color = "red")
```


daughter's height
70
65
60
55
55
60
65
70
mother's height

- We can directly call `lm` as another layer.

```
p3 = ggplot(data = Heights, aes(x = mheight,
    y = dheight)) +
  geom_point() +
  xlab("mother's height") +
  ylab("daughter's height") +
  geom_smooth(method='lm', formula = y~x)
```


daughter's height
70
65
60
55
55
60
65
70
mother's height

Linear regression model

- How do we find this line? With a model.
- We might model the data as

$$D = \beta_0 + \beta_1 M + \varepsilon.$$

- This model is *linear* in (β_0, β_1), the intercept and the coefficient of M (the mother's height), it is a *simple linear regression model*.
- Another model:

$$D = \beta_0 + \beta_1 M + \beta_2 M^2 + \beta_3 F + \varepsilon,$$

  where F is the height of the daughter's father.
- Also linear in $(\beta_0, \beta_1, \beta_2, \beta_3)$, the coefficients of $1, M, M^2, F$.
- Which model is better? We will need a tool to compare models... more to come later.

A more complex model

- Our example here was rather simple: we only had one predictor variable.
- predictor variables are sometimes called *features* or *covariates* or *independent variables*.
- In practice, we often have many more than one predictor.

Terminologies

- Regression variables
  Y : response, dependent
  X or $\mathbf{X}$: explanatory, predictor, independent, covariate, feature, regressor, factor, carrier and etc.
- Regression model

$$Y = f(X_1, X_2, \ldots, X_p) + \varepsilon,$$

where ε is called error or discrepancy in the approximate and p is a number of regressors.

  - Special case: linear regression model

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_p X_p + \varepsilon$$

Applications of Regression analysis

- Agricultural Sciences
  - DHI(Diary Herd Improvement) cooperative: produce milk
  - Interest: How to develop a suitable model to predict current milk production
  - Data: samples are taken once a month
  - Variable: Table 1.1 on p.4
- Industrial and Labor Relation
  - Wagner Act and Taft-Hartley Amendments (1947)
  - Interest: Effects of laws
  - Variable: Table 1.2 on p.5
- Government
  - Interest: Building a prediction model for domestic immigration
  - Variable: Table 1.3 on p.6

- History
  - Interest: How to estimate the age of historical objects based on some age-related characteristics
  - Variable: Table 1.4 on p.6
- Environmental Science
  - Interest: How land use around a river basin contributes to the water pollution
  - Data: Table 1.5 on p.7
- Industrial Production
  - Interest: Estimating the polishing time for new products
  - Variables: Bowlm Casserolem Dish, Tray, Plate Diam, Time, Price

Steps in regression analysis

- Statement of the Problem
- Selection of Potentially relevant variables
- Data collection
- Model specification
- Choice of fitting method
- Model fitting
- Model validation and criticism

References for this lecture

- Based on the lecture notes of Pratheepa Jeganathan
- Based on the lecture notes of Jonathan Taylor .