Linear Algebra

Linear Equations

Example:

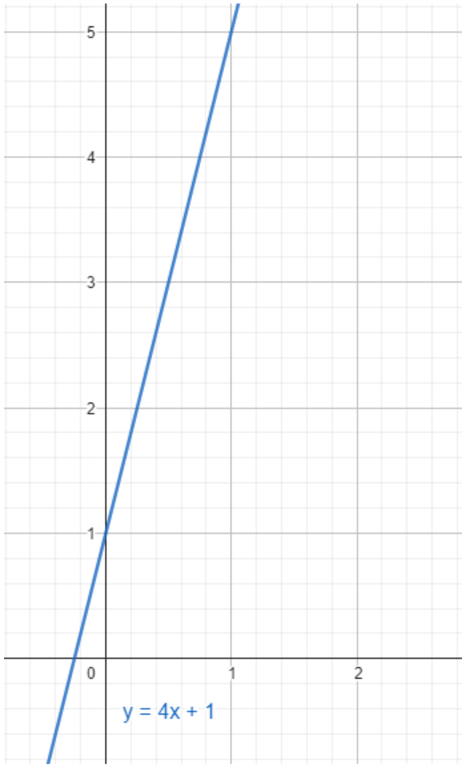
$$y = 4x + 1$$

-> value of x changes y

is an example of a linear equation.

Equations such as this one are considered linear because they describe a line on a graph with only two dimensions.

The line is the result of trying a variety of different values for the variable x in order to determine how the equation or model affects the value of the variable y.



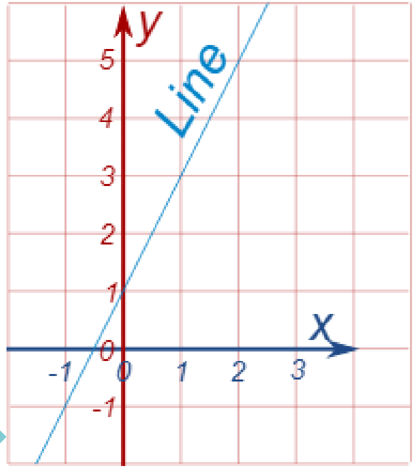
Linear Equations

Examples:

- $y = 2x + 1$
- $5y = 6 + 3y$
- $y/2 = 3 - x$

are examples of a linear equation.

Considering equation: $y = 2x + 1$



Example values:

x	y = 2x + 1
-1	y = 2 × (-1) + 1 = -1
0	y = 2 × 0 + 1 = 1
1	y = 2 × 1 + 1 = 3
2	y = 2 × 2 + 1 = 5

- When x increases, y increases twice as fast, so we need $2x$.
- When x is 0, y is already 1. So, +1 is also needed.

Linear Equations

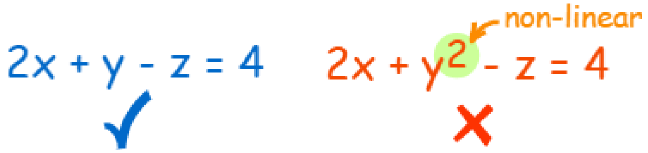
0,1 -> linear ; 2 -> non-linear

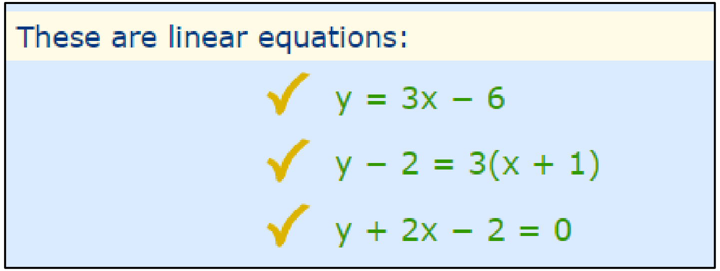
Linear vs Non-linear Equations:

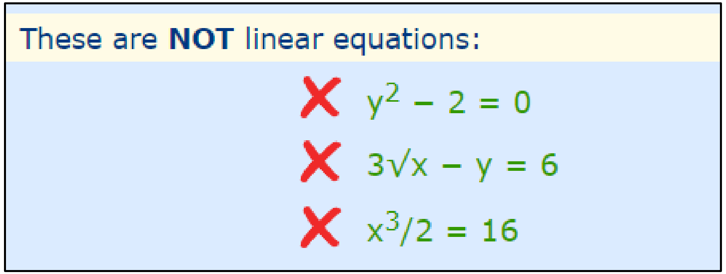
There are many ways of writing linear equations, but they usually have constants and must have simple variables (like "x" or "y").

But the variables in Linear Equations do NOT have:

- Exponents (x^2, x^3, etc.)
- Roots ($\sqrt{x}$, $\sqrt[3]{x}$, etc.)







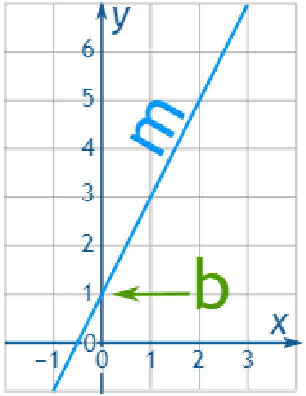
Linear Equations

Equation of Straight Line:

The equation of a straight line is usually as:

$$y = \mathrm{m}\,x + \mathrm{b}$$

y = how far up
x = how far along
m = slope or gradient (how steep the line is)
b = y intercept (value of y when x = 0)



-> slope/tangent -> distance

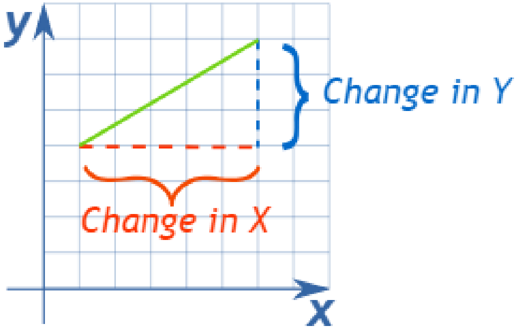
$$y = mx + b$$

Slope or Gradient

y value when **x = 0** *(see Y Intercept)*

Calculating Slope:

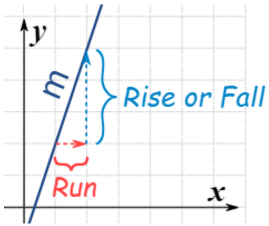
$$m = \frac{Change\ in\ Y}{Change\ in\ X}$$



Linear Equations

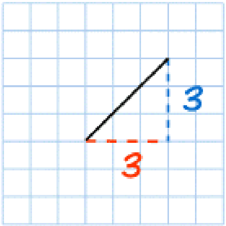
Gradient

Examples:



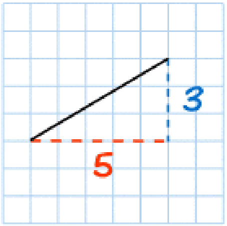
$$m = \frac{rise}{run}$$

- Rise is how far up
- Run is how far along



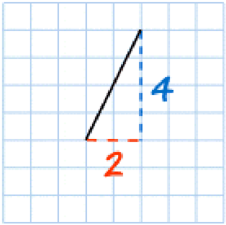
The Gradient = $\frac{3}{3}$ = 1

So the Gradient is equal to 1



The Gradient = $\frac{3}{5}$ = 0.6

The line is less steep, and so the Gradient is smaller.



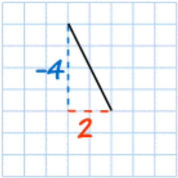
The Gradient = $\frac{4}{2}$ = 2

The line is steeper, and so the Gradient is larger.

Linear Equations

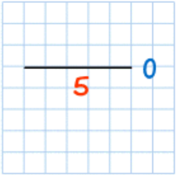
Gradient (Continued):

Examples:



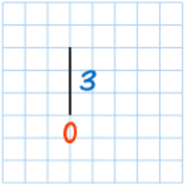
Gradient $= \frac{-4}{2} = -2$

That line goes **down** as you move along, so it has a negative Gradient.



Gradient $= \frac{0}{5} = 0$

A line that goes straight across (Horizontal) has a Gradient of zero.



Gradient $= \frac{3}{0} =$ undefined

That last one is a bit tricky ... you can't divide by zero,
so a "straight up and down" (vertical) line's Gradient is "undefined".

Linear Equations

General form of a Straight Line:

The equation of a straight line can also be represented in a general form as below:

$$Ax + By + C = 0$$

(A and B cannot both be 0)

Example: $3x + 2y - 4 = 0$

It is in the form $\mathbf{Ax + By + C = 0}$ where:

- $A = 3$
- $B = 2$
- $C = -4$

Linear Equations

Linear equations as functions:

Sometimes a linear equation is written as a function with f(x) instead of y:

For example :

$y = 2x - 3$
$f(x) = 2x - 3$
These are the same!

The functions are not always written using f(x):

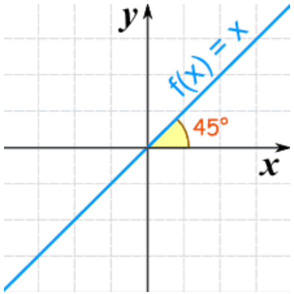
$y = 2x - 3$
$w(u) = 2u - 3$
$h(z) = 2z - 3$
These are also the same!

Linear Equations

Linear equations as functions

There are special linear functions:

<u>Identity Function</u>: f(x) = x



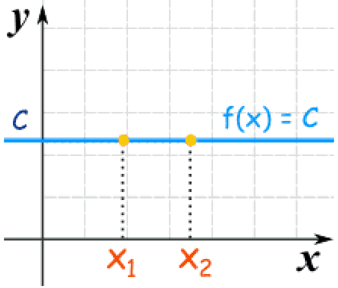
Example Values:

In	Out
0	0
5	5
−2	−2
...etc	...etc

It makes a 45° (its slope is 1).

It is called "Identity" because what comes out is identical to what goes in:

<u>Constant Function</u>: f(x) = C



No matter what value of "x", f(x) is always equal to some constant value.

System of Linear Equations

A System of Linear Equations is when we have two or more linear equations working together.

Example: Here are two linear equations.

- $2x + y = 5$
- $-x + y = 2$

Together they are a system of linear equations. We can solve the above equations to find the values of x and y.

System of Linear Equations

<u>Example</u>:

Let's say you and your dog are racing. While you can run 0.2 km every minute, your dog can run 0.5 km. If you are given a head start of 6 minutes, how far can you get before your dog catches up with you?

We can make two equations

(d = distance in km, t = time in minutes)

- You run 0.2 km every minute:

$$d = 0.2\,t$$

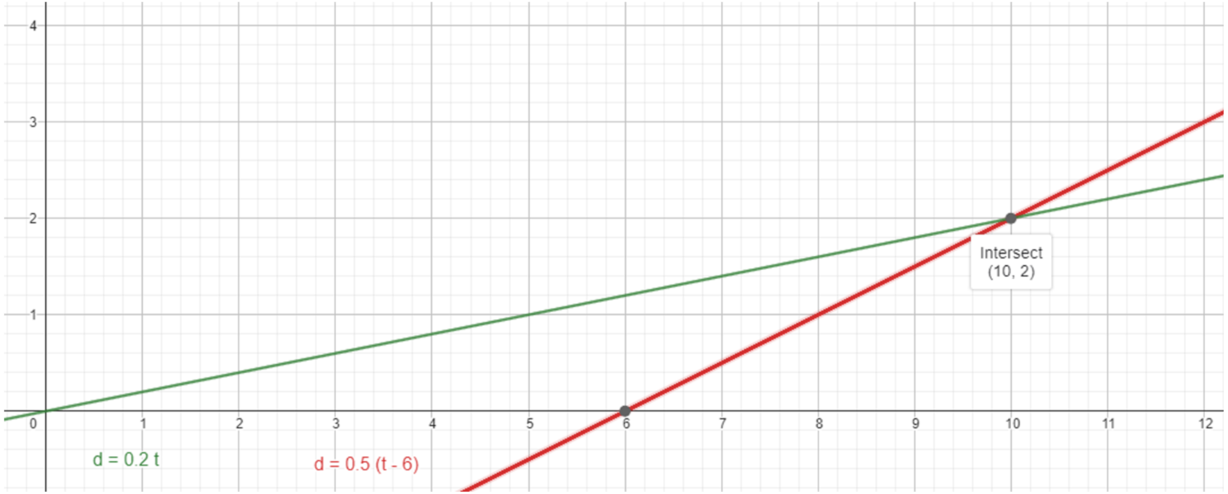
- The dog runs at 0.5 km per minute, but we take 6 off its time:

$$d = 0.5\,(t - 6)$$

System of Linear Equations

<u>Equations</u>: $d = 0.2\,t$ and $d = 0.5\,(t - 6)$

We can solve it on a graph:

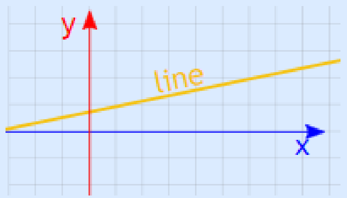
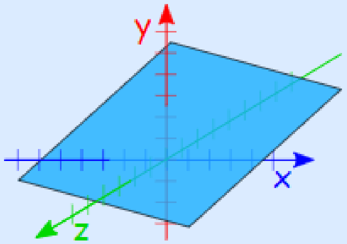


Notice that the dog starts at 6 minutes, but then runs faster.

And you get caught after 10 minutes while you only managed to get 2 km away.

System of Linear Equations

Dimensions

A **Linear Equation** can be in 2 dimensions ... (such as **x** and **y**)	
... or in 3 dimensions ... (it makes a plane)	
... or 4 dimensions ...	
... or more!	

System of Linear Equations

So a System of Equations could have **many** equations and **many** variables.

Example: 3 equations in 3 variables

$$
\begin{aligned}
2x + y - 2z &= 3 \\
x - y - z &= 0 \\
x + y + 3z &= 12
\end{aligned}
$$

There can be any combination:

- 2 equations in 3 variables,
- 6 equations in 4 variables,
- 9,000 equations in 567 variables,

System of Linear Equations

Solutions

When the number of equations is the same as the number of variables there is likely to be a solution.

There are only three possible cases:

- No solution
- One solution
- Infinitely many solutions

Here is a diagram for **2 equations** in **2 variables**: