Quantum Mechanics

Syllabus

- Introduction to Quantum Mechanics
- Group velocity and phase velocity (No relation)
- de-Broglie waves
- Uncertainty principle (statement only)
- Wave function and its significance
- Normalised wave function
- Time Independent Schrodinger wave equations
- Time dependent Schrodinger wave equations
- Particle in a one dimensional box.

Quantum Physics?

At the end of 19th century, most of the physical problems were explained on the basis of classical physics, i.e. Classical Mechanics, Maxwell's theory of Electromagnetism and Thermodynamics. These three branches of physics are collectively known as classical physics.

- Classical mechanics (which is based on Newton's laws of motion) was used to predict the dynamics of material bodies
- Maxwell's theory of electromagnetism provide the proper framework to study electromagnetic radiation &
- Thermodynamics was used to explain the interaction of radiation with matter

The overwhelming success of classical physics is that it made people believe that the ultimate description of nature had been achieved. It seemed that all known physical phenomena could be explained on the basis of classical physics (general theories of matter and radiation).

Challenges for Classical Physics

At the turn of 20th century, classical physics was seriously challenged on two major fronts.

Relativistic domain: In 1905, Einstein's theory of relativity showed that the validity of classical physics, i.e. Newtonian Physics ceases at very high speeds (i.e. at speeds comparable to that of light).

Microscopic domain: As soon as the new experimental techniques were developed to the point of probing atomic and subatomic structures, it turned out that classical physics fails miserably in providing the proper explanation for several newly discovered phenomena.

Thus, it became evident that the validity of classical physics ceases at microscopic level and new concepts had to be invoked to describe, for instance, the structure of atoms and molecules and how light interacts with them.

The failure of classical physics to explain several microscopic phenomena-such as

- **Stability of atoms**
- **Spectral distribution of black body radiation (BBR)**
- **Photo Electric Effect**
- **Compton Effect**
- **Atomic spectroscopy (line spectra of atoms)**

– had cleared the way for seeking new ideas outside its purview and then comes Modern Physics/Quantum Physics.

So, Modern Physics is a branch of Physics which deals with the Post Newtonian concepts in the world of physics. It is based on the major breakthroughs of the 20th century: Relativity and Quantum Mechanics. Modern Physics is an advanced description of nature through new theories which were different from the classical description.

De Broglie Waves, Phase Velocity & Group Velocity

- In 1924, de Broglie proposed that a moving material particle constitutes a wave associated with its motion. In other words, a subatomic material particle behave like a wave. This wave associated with the motion of the particle is known as de Broglie wave and the wavelength of this wave associated with a particle of mass m moving with velocity v is given by

$$\lambda = h/mv$$

where, h = Planck's Constant,

- The velocity of de Broglie wave is known as Phase velocity or Wave velocity.
- In 1926, Schrodinger came up with an idea that a moving material particle constitute a group of waves instead of a single wave train associated with its motion. The group of waves is known as wave packet. The velocity with which this group of waves moves in a medium is called the group velocity.

Heisenberg's Uncertainty Principle

- According to Newtonian physics, it is possible to determine exactly the position and momentum of a particle simultaneously. But, the concept of dual nature of matter presents difficulty in locating the exact position and momentum at the same time.
- A moving particle, according to quantum mechanics, may be considered as a group of waves (wave-packet) and the particle may be anywhere within the wave packet. Thus, the position of the particle is uncertain within the limit of the wave packet.
- Also, since the wave packet has a velocity spread, there is uncertainty about the velocity or momentum of the particle. Hence, it is impossible to know exactly where within the wave packet the particle is and what is its exact momentum.

Heisenberg, in 1927, proposed the uncertainty principle which states that "the simultaneous determination of the exact position and momentum of a moving particle is impossible."

If Δx denotes the error in determining its position and Δp the error in determining its momentum, then according to this principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi} \quad (1)$$

The product of the two errors is, thus, approximately of the order of Planck's constant h.

i.e.

$$\Delta x \cdot \Delta p \approx h \quad (2)$$

Thus, more accurately we measure the momentum of a particle, the less defined becomes its position and vice-versa.

Wave Function & its Significance

- **The concept of wave packet demanded the existence of a certain guiding wave.**
- The equation of this guiding wave was derived by Schrodinger and is of immense use in the problem concerned with almost all the domains of sciences & Engineering.
- The Schrodinger's wave equation is given by

$$\nabla^2\Psi + \frac{2m}{\hbar^2}(E - V)\Psi = 0$$

- The quantity, Ψ , in this equation is known as wave function which is of immense importance in quantum mechanics.
- The physical significance of the wave function Ψ is that **$\Psi^*\Psi dV$ represents the probability of finding the particle in the volume element dV,** where dV = dx dy dz for a 3 –D space.
- Moreover, the value $\Psi^*\Psi$ is called the probability density of the particle.

Normalized Wave Function

- The wave function Ψ is said to be normalized wave function if

$$\int_{-\infty}^{\infty} \boldsymbol{\Psi} * \boldsymbol{\Psi} dV = 1$$

It means that the probability of finding the particle over the whole space is unity.

A wave function can be made normalized by dividing it by a factor known as normalization factor.