$1\,m = 10^{-3}\,Km$

Prove the relation b/w newton and dyne

$1\,N = 10^{5}$ dyne

1 dyne $= 10^{-5}\,N$

$1\,N = 1\,kg \times 1\,m/s^2$

~~$1000g \times 100cm/s^2$~~

$1\,N = 1000 \times 100 \times 1g \times cm/s^2$

$1\,N = 10^{5}\,g\,cm/s^2$

Relation b/w Tesla and Gauss

$$1T = \frac{1N}{Am} = \frac{10^{5}\ \text{dyne}}{10^{-1}\ Bi \times 100cm} = 10^{4}\ G$$

Relation b/w Neper and decibel

1 neper = 8.68 dB

Practical file format

L.H.S (white page)	R.H.S (Pen side)
Aim	Aim
Appratus	Apparatus
Formula Used	Theory / Principle
Diagram	Procedure
Observation	Result
Calculations	Precautions
Result	Learning outcomes

Group B

Exp = AP = attenuation & propagation losses in optical fibre

$1n = 8.68$

$V_1 = 31/2 \times 0$
$V_2 = 17/2 \times 0$
$l_1 = l_2 = 1m$

$\frac{31}{17} = e$

$1.82 = e^{-2\alpha}$

$$\frac{V_1}{V_2} = e^{-\alpha(l_1+l_2)}$$

$0.26 = -2\alpha$
$\alpha = -0.13$

Ph121 Modern & Computational Physics
Book = Engineering Physics by H.K Malik

Unit-1 ⇒ Electrodynamics
- → Physical Qty
- → Curl, Gradient, Divergence

Unit-2 ⇒ Lasers
- → What is a laser
- → Laser principle
- → Types of lasers

Unit → 3 ⇒ Optical fibres
Unit - 4 ⇒ Magnetic Materials
Unit - 5 ⇒ Superconductivity
Unit - 6 ⇒ Quantum Mechanics

Any quantity which we can measure is a physical quantity, Eg → temp, length

Physical Quantity
- Scalar
- vector

23/11/21

ELECTRODYNAMICS

Physical Quantity: Any quantity which we can measure is a physical quantity.
Eg → temp, length

Physical Quantity

Scalar — only magnitude — Eg- mass, dist

Vector — magnitude & direction — Eg: vel, accel

Field: If a physical quantity varies from point to point, it becomes a field

Field

Scalar

Vector

Dell operator: It is a vector differential operator

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

It is also known as nabla

* $\vec{\nabla}\phi$ = grad ϕ, where ϕ is a scalar function/field

$$\text{grad}\cdot\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} = \vec{\nabla}\phi$$

When del operator acts upon a function it becomes gradient. Gradient of ϕ is a vector quantity

Ques: If $\phi(x,y,z) = 3x^2y - y^3z^2$, calculate the gradient of ϕ at the point $(1,-2,-1)$

Sol: $\vec{\nabla}\phi = \text{grad}\,\phi = (6xy)\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - (2y^3z)\hat{k}$ using partial diff

at point $(1,-2,-1)$

$$\vec{\nabla}\phi = -12\hat{i} - 9\hat{j} - 16\hat{k}$$

Gradient of Scalar Fields

The gradient of a scalar field is a vector field, which points in the direction of the greatest rate of increase of the scalar field, and whose magnitude is the greatest rate of change

$$|\vec{\nabla}\phi| = \sqrt{(-12)^2 + (-9)^2 + (-16)^2}$$

Ques: $\phi = x^3 + y^3 + z^3 - 3xyz$ at $(1,-1,2)$, find $\vec{\nabla}\phi$ & $|\vec{\nabla}\phi|$

Sol: $\vec{\nabla}\phi = (3x^2 - 3yz)\hat{i} + (3y^2 - 3xz)\hat{j} + (3z^2 - 3xy)\hat{k}$

at point (1, -1, 2)

$$\vec{\nabla}\phi = (3+6)\hat{i} + (3-6)\hat{j} + (12+3)\hat{k}$$

$$\vec{\nabla}\phi = 9\hat{i} - 3\hat{j} + 15\hat{k}$$

$$|\vec{\nabla}\phi| = \cancel{\sqrt{3(9+1+25)}} \quad 3\sqrt{35}$$

Divergence

It is the scalar product of a vector field with ~~gradient~~ dell operator

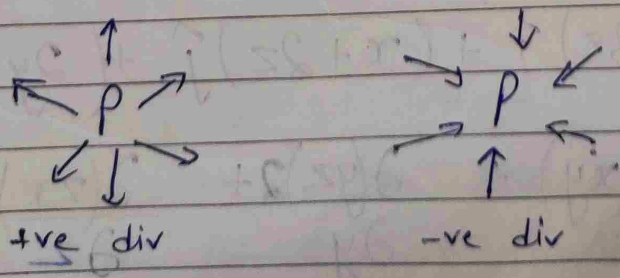
$$\text{div } \vec{F} = \vec{\nabla} \cdot \vec{F}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(F_1\hat{i} + F_2\hat{j} + F_3\hat{k}\right)$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

geometrical interpretation: It is a measure of how much a vector F spreads out (diverges) from the point in question

Eg:



P = electric field

Note: If Divergence of vector field is zero then it is also termed as Solenoidal Field.

Divergence is an operator that measures the magnitude of a vector field's source or sink at a given point.

The divergence of a vector field is a scalar and at a point it is defined as the amount of flux diverging from a unit volume element per second around that point.

Ques: Calculate divergence & dell operator

(a) $\vec{V_1} = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$

$\vec{\nabla_1} = (2x + 3z^2 - 2z)\hat{i} + (6x - 2x)\hat{k}$

$\text{div } \vec{V_1} = \cancel{2x^3 + 3x^2z^2 - 2x^2z + 6x^3 - 2x^3}$ $\quad \frac{\partial(x^2)}{dx} + \left(\frac{\partial\, 3xz^2}{\partial y}\right)$

$= \cancel{6x^3 + 3x^2z^2 - 2x^2z}$ $= 2x + 0 - 2x$ $\quad + \frac{\partial}{\partial z}(-2xz)$

$= 0$

(b) $\vec{V_2} = xy\hat{i} + 2yz\hat{j} + 3zx\hat{k}$

$\vec{\nabla_2} = (y + 3z)\hat{i} + (x + 2z)\hat{j} + (2y + 3x)\hat{k}$

$\text{div } \vec{V_2} = \frac{\partial}{\partial x}(xy) + \frac{\partial (yz)}{\partial y}2 + \frac{\partial (zx)}{\partial z}$

$= y + 2z + x$

Ques: $\phi = 2x^3 y z^2$ find div(grad ϕ)

Sol: grad $\phi = (6x^2 yz^2)\hat{i} + (2x^3z^2)\hat{j} + (4x^3yz)\hat{k}$

$= 6x^2yz^2\,\hat{i} + 2x^3z^2\,\hat{j} + 4x^3yz\,\hat{k}$

div(grad ϕ) $= 12xyz^2 + 4x^3y$

Curl Of Vector Field

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

$$\text{Curl } \vec{A} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) - \hat{j}\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

Curl measures the tendency to rotate around a point. It shows a vector field's rate of rotation i.e. the direction of axis of rotation & the magnitude of rotation

<u>Note:</u> grad ϕ = vector

div $\vec{A}$ = scalar

curl $\vec{A}$ = vector

When curl = 0 then that field is called Irrotational field or curl-free, or non-curl or conservative field

<u>Ques.</u> $\vec{F} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$. Prove F is solenoidal

Sol: $\text{div } \vec{F} = \frac{\partial (x+3y)}{\partial x} + \frac{\partial (y-3z)}{\partial y} + \frac{\partial (x-2z)}{\partial z}$

$= 1 + \cancel{3y} - \cancel{3z} + 1 + \cancel{x} - 2 = 0$

Thus F is solenoidal

Ques: Prove $\vec{F} = (x^2+xy^2)\hat{i} + (y^2+x^2y)\hat{j}$ is irrotational

Sol: $\text{Curl} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2+xy^2 & y^2+x^2y & 0 \end{vmatrix}$

$= \hat{i}\left(-\frac{\partial (y^2+x^2y)}{\partial z}\right) - \hat{j}\left(-\frac{\partial}{\partial z}(x^2+xy^2)\right) + \hat{k}\left(\frac{\partial (y^2+x^2y)}{\partial x} - \frac{\partial (x^2+xy^2)}{\partial y}\right)$

$= 0 + 0 + (2xy - 2xy)$

$= 0$ Thus F is irrotational

Ques: $\vec{A} = x^2 y\,\hat{i} - 2xz\,\hat{j} + 2yz\,\hat{k}$

Find curl curl $\vec{A}$

Sol: curl $\vec{A} = \hat{i}(-2x - \cancel{2y}\,2z) - \hat{j}(0-0) + \hat{k}(x^2 - 2z)$

$= (-2x - \cancel{2y}\,2z)\,\hat{i} + (x^2 - 2z)\,\hat{k}$

curl (curl $\vec{A}$) $= \hat{i}(0)\ \cancel{-\hat{j}(0) + \hat{k}(}$

$-\hat{j}((-2) - (2x - 0)) + \hat{k}(0)$

$= \cancel{(2x-4)}\,\hat{j}\ \ (2x+2)\,\hat{j}$

~~Maxwell's Equation~~

Gauss Divergence Theorem

The integral of a derivative (divergence) over a region (volume) is equal to the value of the function at the boundaries (surface)

$$\boxed{\int_V (\vec{\nabla}\cdot\vec{F})\, dV = \oint_S \vec{F}\cdot d\vec{S}}$$

3D to 2D

It is also known as fundamental theorem for divergence

It is used to convert volume integral into surface integral

Date
Page

Fundamental Theorem for Curl
OR
Stokes Theorem

The integral of a derivative (curl) over a region (a patch of surface) is equal to the value of the function at the boundary (perimeter of the patch)

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{S} = \oint \vec{F} \cdot d\vec{l}$$

2D to 2D

Equation of Continuity
OR
Conservation of Charge

The total current flowing out of some volume must be equal to the rate of decrease of the charge within that volume, if the charge is neither being created nor lost.

$$I = \frac{dq}{dt} = \frac{d}{dt}\int \rho \, dV \quad \text{---(i)}$$

(ρ = charge density)

$$I = \int_S \vec{J} \cdot d\vec{s} = \frac{dq}{dt} \quad \text{---(ii)}$$

(J = current density)

from (i) & (ii)

$$\oint_S \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\int_V \rho\, dV = -\frac{dq}{dt} \quad \text{---(iii)}$$

Note: −ve sign indicates the decrease of charge in volume V

From gauss divergence theorem (apply on (iii))

$$\oint_S \vec{J}\cdot d\vec{s} = \int_V (\vec{\nabla}\cdot\vec{J})\, dV \quad \text{--- (iv)}$$

from (iii) & (iv)

$$\int_V (\vec{\nabla}\cdot\vec{J})\, dV = -\frac{d}{dt}\int_V \rho\, dV = -\int_V \frac{d\rho}{dt}\, dV \quad \text{---(v)}$$

$$\boxed{\text{div}\cdot\vec{J} = \vec{\nabla}\cdot\vec{J} = -\frac{d\rho}{dt}}$$ → equation of continuity

Also $$\text{div}\,\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

At steady state

$$\frac{\partial\rho}{\partial t} = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{J} = 0}$$

as charge density remain constant

Maxwells Equation

I Gauss's Law for Electricity

It states that the total electric flux through the closed surface is equal to $(1/\varepsilon_0)$ times the total charge enclosed by the surface.

$$\phi = \oint \vec{E} \cdot \vec{dS} = \frac{1}{\varepsilon_0} \Sigma q = \frac{1}{\varepsilon_0} \cdot Q$$

Suppose that charge q is distributed over a volume V. Let ρ be the volume charge density

Total charge $q = \int \rho \, dV$

Gauss law

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{1}{\varepsilon_0} \int \rho \, dV$$

Acc. to gauss div theorem

$$\int \vec{E} \cdot \vec{dS} = \int (\vec{\nabla} \vec{E}) \, dV$$

$$\int (\vec{\nabla} \vec{E}) \, dV = \frac{1}{\varepsilon_0} \int \rho \, dV$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$ ... differential form of Gauss's Law for electricity.

Also displacement vector $\vec{D} = \varepsilon_0 \vec{E}$

$$\vec{\nabla} \cdot \vec{D} = \rho$$

Significance (i) It is time independent equation or steady state equation

(ii) It gives the relationship b/w electric flux & charge density

(iii) It shows charge acts as source or sink for electric lines of force.

II Maxwell's Second Equation

Total magnetic flux in closed surface is zero

$$\oint \vec{B}\cdot\vec{ds} = 0$$

Acc. to Gauss div

$$\oint \vec{B}\cdot\vec{ds} = \int \vec{\nabla}\cdot\vec{B}\, dV = 0$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

This is analogue of Gauss' Law in magnetism

Significance : (i) It shows that magnetic monopole does not exist

(ii) There is no source or sink for the magnetic lines of force

(iii) It is time independent differential equation.

Date
Page

III **Maxwell's Third Equation OR Faraday's Law of Electromagnetic Induction**

It states that EMF is induced in a circuit, if there is a change in the magnetic flux linked through the circuit & if the circuit is closed, an induced current begins to flow.

$$\varepsilon_{emf} = \frac{-\partial \phi}{\partial t}$$

Acc. to Faraday's Law $EMF = -\frac{d\phi}{dt}$

$$\phi_{(t)} = \int_S \vec{B}\, \vec{ds} \quad \text{.... magnetic flux}$$

$$EMF_{total} = \oint_L E \cdot dL \quad \text{....}$$

$$\oint_L E \cdot dL = \int_S \vec{\nabla} \times \vec{E} \cdot dS \quad \text{(stokes theorem)}$$

$$\int_S \vec{\nabla} \times \vec{E} \cdot dS = \frac{-d}{dt} \int_S B \cdot dS = \int_S \frac{-dB}{dt} \cdot dS$$

$$\boxed{\vec{\nabla} \times \vec{E} = \frac{-dB(t)}{dt}}$$ diff of maxwell 3rd eq

Significance: (i) It relates the space variation of electric field with time variation of magnetic field

(ii) It is time dependent differential equation

(iii) It proves that the electric field can be generated by change in magnetic field

IV Ampere's Circuital Law

The line integral of magnetic field B around any closed loop is equal to μ_0 times the net current I flowing through the area enclosed by the loop

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

where μ_0 = permeability of free space

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

From Ampere's Law

$$\oint_l \vec{H} \cdot \vec{dl} = I$$

and also,

$$\oint_l \vec{H} \cdot \vec{dl} = \oint_s (\vec{\nabla} \times \vec{H}) \cdot ds \quad \text{(from Stokes theorem)}$$

and

$$I = \oint_s \vec{J} \cdot \vec{ds} \quad , \quad q = \int_v \rho \, dv$$

$$\oint_s (\vec{\nabla} \times \vec{H}) ds = \oint_s \vec{J} \cdot \vec{ds}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times H = \frac{\partial D}{\partial t} + J$$

$$\vec{\nabla} \cdot (\vec{\nabla} \times H) = \vec{\nabla} \cdot \vec{J} \quad [\vec{\nabla} \cdot \vec{J} = 0 \text{ div of } J \text{ is always zero}]$$

$$\vec{\nabla} \cdot \vec{J} = 0$$

But from eq of continuity $\vec{\nabla} \cdot \vec{J} = \frac{-\partial \rho}{\partial t}$

So maxwell realised that the definition of total current density is incomplete & suggested to add another term

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}'$$

taking div both sides

$$\nabla \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot (J + J')$$

As $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$

$$\nabla \cdot J + \nabla + J' = 0$$

$$\nabla \cdot J' = -\nabla \cdot J = \frac{\partial \rho}{\partial t}$$

$$\nabla \cdot J' = \frac{\partial \rho}{\partial t}$$

Acc to gauss Law of electrostat

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\nabla \cdot J' = \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{D} = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\Rightarrow J^d = \frac{\partial D}{\partial t} = \varepsilon_0 \frac{\partial E}{\partial t}$$

Modified form of Ampere's Circuital law

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Significance: (i) time dependent

(ii) Relates the space variation of magnetic field with time variation of electric field

(iii) Proves that magnetic field can be generated by changing electric field.

integral form: $\oint_l \vec{H} \cdot \vec{dl} = \int_s \left[J + \frac{\partial D}{\partial t} \right] \cdot \partial \vec{s}$

Propagation of Electromagnetic Wave in free Space

Here $\rho = 0$ & $J = 0$

$\vec{\nabla} \cdot \vec{E} = 0$ —(i)

$\vec{\nabla} \cdot \vec{H} = 0$ —(ii)

$$\vec{\nabla} \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} = \frac{-\partial \vec{B}}{\partial t} \quad \text{—(iii)}$$

$\vec{B} = \mu_0 H$

$$\vec{\nabla} \times \vec{H} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = \frac{\partial \vec{D}}{\partial t} \quad \text{—(iv)}$$

taking curl of eq 3 on both sides

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

$$\left[LHS = \vec{A} \times (\vec{B} \times \vec{C}) = A(\vec{B} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B}) \right]$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\cancelto{0}{\vec{\nabla} \cdot \vec{E}}) - \vec{E}(\vec{\nabla} \cdot \vec{\nabla})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\vec{\nabla}^2 \cdot \vec{E}$$

$$\vec{\nabla}^2 \cdot \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

from eq (4)

$$\vec{\nabla}^2 \cdot \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left(\frac{\partial E}{\partial t} \right)$$

$$\vec{\nabla}^2 \cdot \vec{E} = -\mu_0 \varepsilon_0 \left(\frac{\partial^2 \vec{E}}{\partial t^2} \right)$$

$$\vec{\nabla}^2 \cdot \vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\vec{\nabla}^2 \cdot \vec{E} + \frac{1}{v^2} \frac{\partial^2 E}{\partial t^2} = 0 \quad \left[v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}, \ \frac{1}{v^2} = \mu_0 \varepsilon_0 \right]$$

$$(\mu = 4\pi \times 10^{-7} \quad \varepsilon_0 = 8.85 \times 10^{-12})$$

$$\Rightarrow \vec{\nabla}^2 \cdot \vec{E} + \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

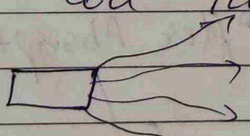
LASERS

LASER stands for Light Amplification by Stimulated Emission of Radiation

Laser is a very intense, concentrated, highly parallel and monochromatic beam of light

Coherence ~~a~~ is very important property of Laser

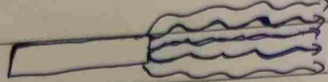
<u>Incoherent Light</u>: Incoherent light is due to spontaneous and random emission of photons which are not in phase with each other.


Incoherent Light


Directional

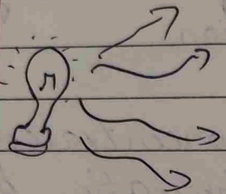
<u>Coherent Light</u>: Coherent light is uniform in frequency, amplitude, continuity and constant initial phase difference

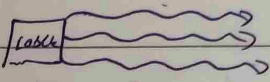
It is obtained due to stimulated emission of photons from the atoms jumping from meta stable state to lower energy state



Coherent light

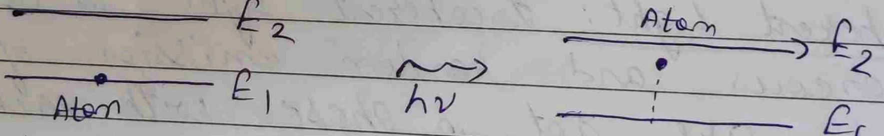
Incandescent	Laser Light
Many Wavelengths	Monochromatic
Multidirectional	Directional
Incoherent	Coherent





Various Atomic Interactions Related to LASER

a) Induced Absorption / Absorption of Radiation



Before Absorption   After Absorption

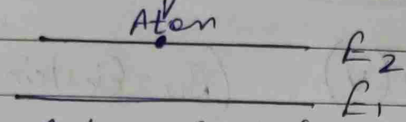
$$\boxed{\nu = \frac{E_2 - E_1}{h}}$$

(h = Planck's constant)
($h = 6.626 \times 10^{-34}$ Js)

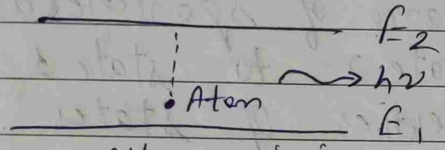
b) Spontaneous Emission

An excited atom can stay in the higher energy state only for the time of 10^{-8} sec. After this time, it returns back to lower energy state by emitting a photon of energy $h\nu = E_2 - E_1$

During spontaneous emission photons are randomly emitted and hence they will not be in phase with each other. Therefore light is incoherent



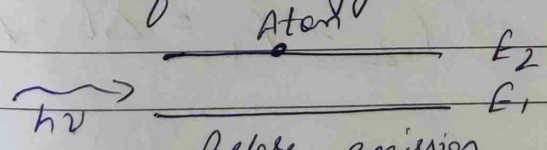
Before Emission



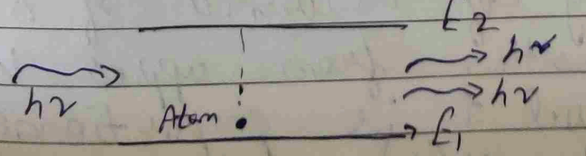
After emission

c) Stimulated Emission / Induced Emission

When photon of suitable size (energy) is showered on an excited atom in the higher energy state, the atom falls back to the ground state by emitting a photon of energy $h\nu = E_2 - E_1$



Before emission



After emission

It results in appearence of one additional photon. Light is coherent as photons emitted are in phase

Absorption of Radiation

The prob. of occurance of this absorption from state 1 to state 2 is $\propto$ to the energy density $u(\nu)$ of the radiation

$$P_{12} = B_{12}\, u(\nu) \quad (B_{12} = \text{Einstein Coefficient})$$

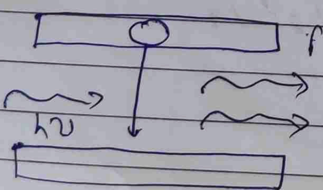
Spontaneous Emission

The prob. of occurence of spontaneous emission transition from state 2 to state 1 depends only on the properties of states 2 & 1 is given by

$$P'_{21} = A_{21}$$

A_{21} = Proportionality constant
= Einstein coefficient of spontaneous emission of radiation

Stimulated Emission



$$E_2 - E_1 = \Delta E = h\nu$$

The probability of occurence of stimulated emission transition from upper level 2 to lower level 1 is proportional to the energy density $u(\nu)$ of the radiation & is expressed as

$$P''_{21} = B_{21}\, u(\nu)$$

where B_{21} = Einstein coefficient of stimulated emission of radiation

Relation b/w Einstein's Coefficients

Let N_1 & N_2 = no. of atoms at any instant in state 1 & 2. The probability of absorption for no. of atoms from state 1 to 2 per unit time is

$$N_1 P_{12} = N_1 B_{12}\, u(\nu)$$

Total prob. of transition for no. of atoms from state 2 to 1, either by spontaneous or by stimulated emission per unit time is

$$= N_2 \left(P'_{21} + P''_{21}\right] = N_2 A_{21} + N_2 B_{21}\, u(\nu)$$

$$= N_2 P_{21} = N_2 \left[A_{21} + B_{21}\, u(\nu)\right]$$

In thermal equilibrium at temp T, the absorption probabilities are equal

$$N_1 P_{12} = N_2 P_{21}$$

$$N_1 B_{12}\, u(\nu) = N_2 A_{21} + N_2 B_{21}\, u(\nu)$$

$$N_1 B_{12}\, u(\nu) - N_2 B_{21}\, u(\nu) = N_2 A_{21}$$

$$u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}}$$

$$u(\nu) = \frac{A_{21}}{B_{21}\left(\frac{N_1}{N_2}\frac{B_{12}}{B_{21}} - 1\right)}$$

classmate
Date
Page

But acc to Einstein

$$B_{12} = B_{21}$$

$$\boxed{u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{N_1}{N_2} - 1\right)}}$$

Acc. to Boltzmann Law, the distribution of atoms among the energy states E_1 & E_2 at the thermal equilibrium at temp T is

$$\frac{N_1}{N_2} = \frac{e^{-E_1/KT}}{e^{-E_2/KT}} = e^{(E_2-E_1)/KT}$$

$$\frac{N_1}{N_2} = e^{(h\nu)/KT}$$

(where K = Boltzmann constant)

$$\boxed{u(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\left(e^{h\nu/KT} - 1\right)}}$$

Plank's radiation formula yields energy density of radiation

$$\boxed{u(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{\left(e^{h\nu/KT} - 1\right)}}$$

Relation b/w Einstein coefficients A & B

$$\boxed{\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}}$$

$$\boxed{\frac{B_{21}}{A_{21}} = \left(\frac{c^3}{8\pi h}\right)\frac{1}{\nu^3}}$$

That is B_{21}/A_{21} is inversely $\propto$ to freq of the resonant radiation. Therefore, the higher the freq smaller the value of B_{21}. That is, it is comparitively difficult to obtain the stimulated emission at higher frequencies.

Ques: Calculate the rellative population of Na atoms in sodium lamp in first & ground state at temp 350 °C. λ = 590 nm

Sol:

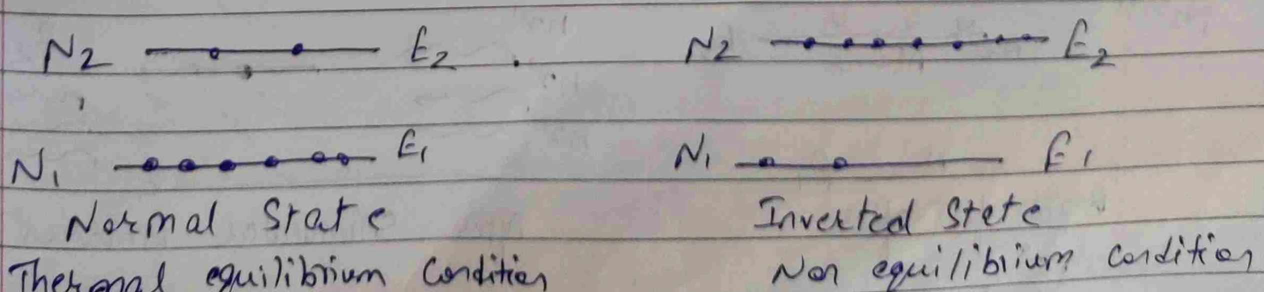
$$\frac{N_1}{N_2} = e^{(h\nu)/KT} = e^{\frac{hc}{\lambda KT}} = e^{\frac{6.626\times10^{-34}\times3\times10^{8}}{1.35\times10^{-23}\times590\times10^{-10}\times623}}$$

Population Inversion & Pumping

Acc. to Boltzmann the ratio of atoms in energy states 2 & 1 at temp T is given by

$$\frac{N_2}{N_1} = \frac{e^{-E_2/KT}}{e^{-E_1/KT}} = e^{-(E_2-E_1)/KT}$$

for population inversion: $N_2 >> N_1$ i.e.



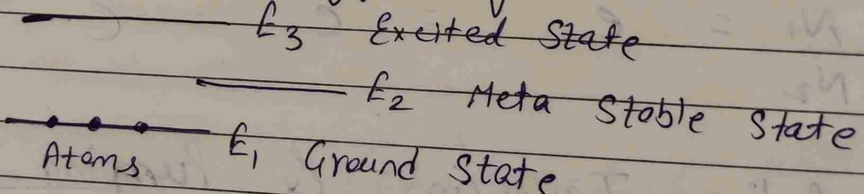
Normal State
Thermal equilibrium Condition
$N_1 >> N_2$

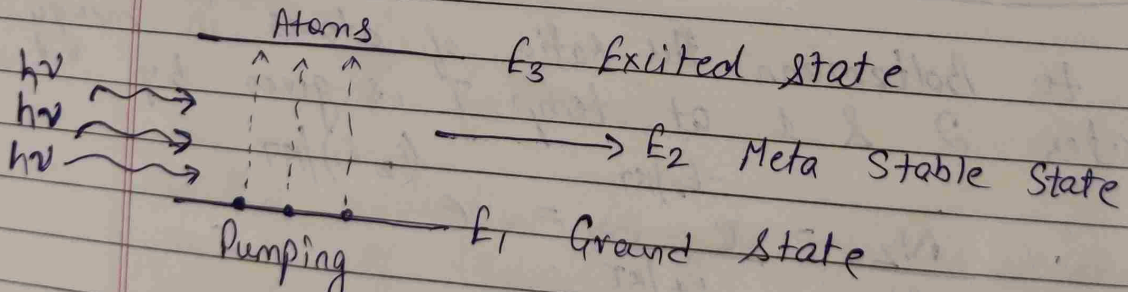
Inverted State
Non equilibrium Condition
$N_2 >> N_1$

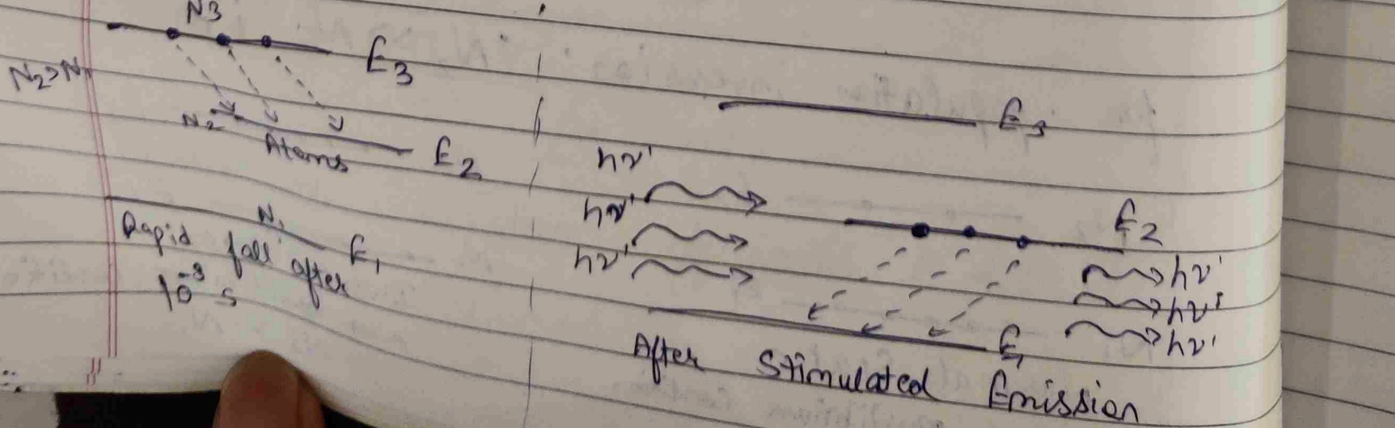
The process of making population of atoms in the higher energy state more than that in the lower energy state is known as 'population inversion'.

The method by which a population inversion is achieved is called 'pumping'. In this process atom are raised to an excited state by injecting into system photon of frequency different from the stimulating frequency.

Population inversion can be explained by three level energy diagram.







After Stimulated Emission

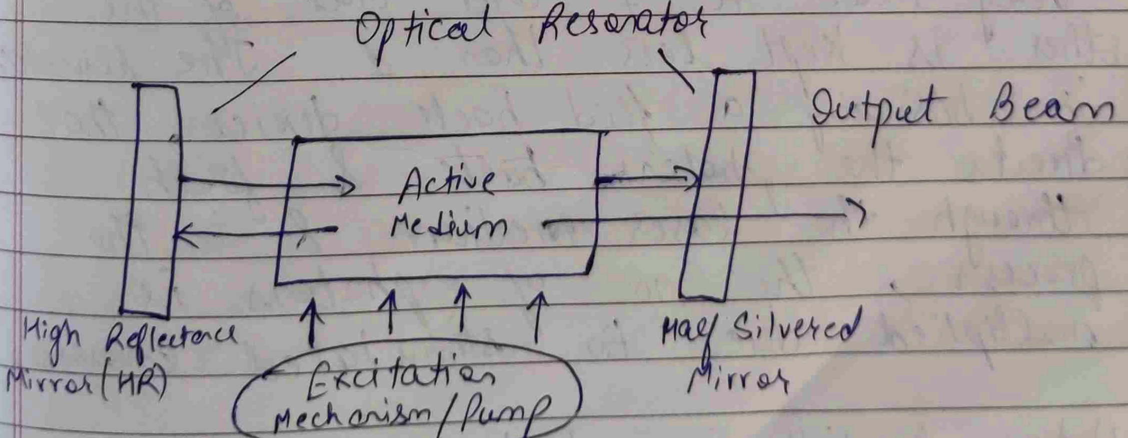
Types of Pumping

1) Electrical Pumping : Atoms can be excited by ē impact in sufficiently intense gaseous discharge. It is suited to gas & semiconductor lasers.

2) Optical Pumping: Atoms are excited by powerful lamp or a light source whose light populates excited states by photon absorption. Suitable to solid state or liquid lasers

3) Chemical Pumping: Here inversion is achieved from an exothermic chemical reaction. It usually applies to material in gas phase & it generally requires highly reactive & often explosive gas mixtures

Laser Components



1) Pump: (i) It is an external source which supplies energy to obtain population inversion. The pump can be optical, electrical or thermal. In Ruby laser we use optical pumping & in He-Ne laser electric discharge pumping

(ii) The lifetime of metastable energy state ~~is~~ ~~to~~ must be very large as compared to normal lifetime of the excited atom in any other energy State

2) <u>The Laser Medium:</u> It is the material in which laser action is made to take place. It may be solid, liquid or gas. The very imp characteristic requirement for the medium is that inversion should be possible in it.

Note: Ruby Laser output is at 694.3 nm
He-Ne Laser = 632.8 nm

3 <u>The Resonator:</u> It consists of a pair of plane or spherical mirrors having common principal axis. The reflection coefficient of one of the mirrors is very near to 1 and that of the other is kept less than 1. The resonator is basically a feed-back device, that directs the photons back & forth through the laser medium & in the process, the no. of photons is multiplied due to stimulated emission.

<u>Note:</u> <u>Amplification of light:</u> In pumping the atoms are made to fall from meta stable state to lower energy & photons are emitted by stimulated emission. The photons are reflected back & forth in the active medium to excite the other atoms.

Thus a large no. of photons are emitted simultaneously which possess the same energy, phase & direction. This process is called 'amplification of light'.

Note: To produce laser beam, the foll two conditions must be fulfilled:

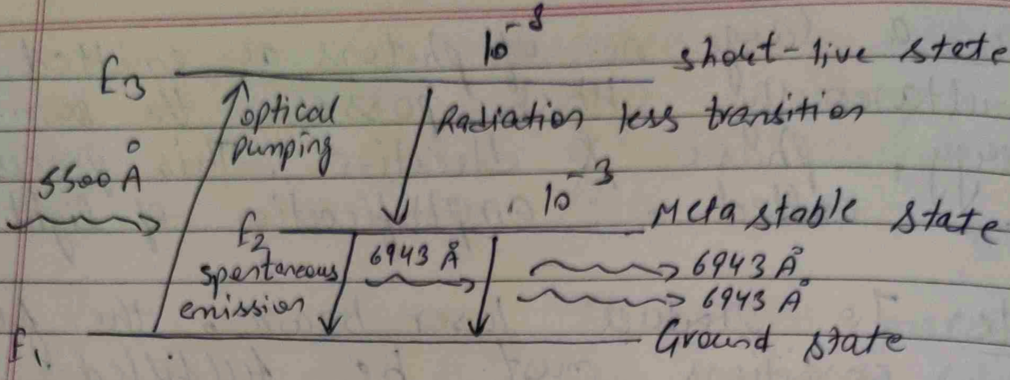
1) The meta-stable state should all the time have larger no. of atoms than the no. of atoms in lower energy state.

2) The photons emitted due to stimulated emission should stimulate other atoms to multiply the photons in the active medium.

Note: Four-level system lasers are best.

Ruby Laser

The first laser was created in 1961 by T. Maiman. It is solid state laser. He used a rod of ruby as active (lasing) medium. It is a three level laser. It has chromium ions. Ruby rod is taken in form of cylindrical rod of about 4cm in length & 1cm in diameter. The end faces of ruby rod are silvered so that they form the optical resonator. Chromium atoms play the active role for laser action. Ruby laser uses optical pumping. The wavelength of laser is 6943 Å

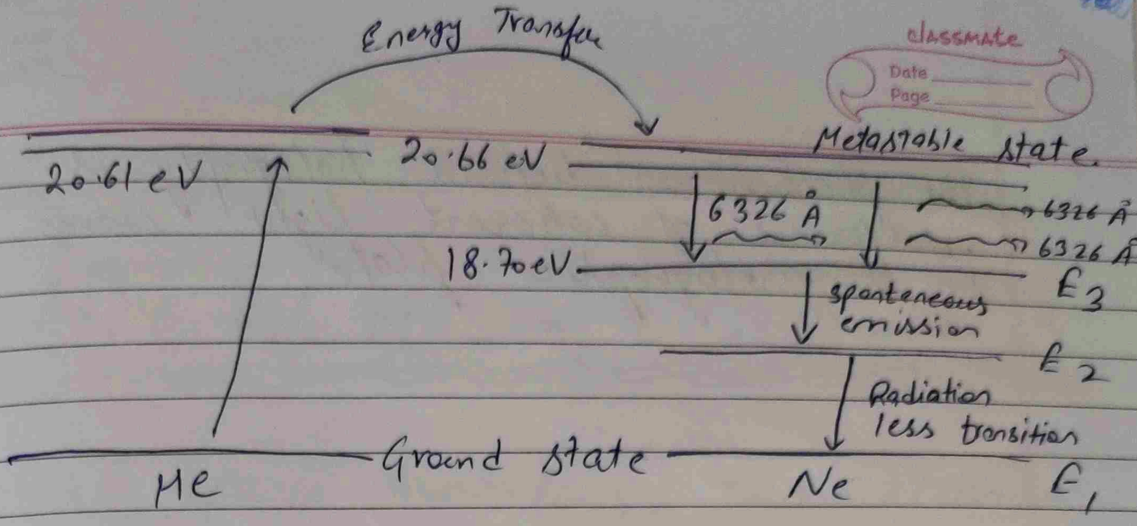


Features: Its output lies in visible spectrum. Light from Xenon flash lamp is pumping agent, Poor efficiency and it operates in pulsed mode.

Drawbacks 1) Pulsed or spiked output
2) Large fraction of pumped energy is lost as heat
3) Requires bulky cooling system.

He-Ne Laser

- It was built in 1961 by Ali Javan
- A He-Ne laser is a type of small gas laser
- Its a four-level system laser
- The most common method of exciting gas medium is by passing an electric discharge through the gases
- Laser medium is mixture of Helium & Neon gases in ratio 10:1
- Ne atom play active role here and the purpose of He atoms is to provide energy to Ne atoms through collision b/w He and Ne atoms.



Features:
- It lies in visible spectrum
- Low efficiency & low power output
- Operates in continuous mode

Applications of Lasers

- In Communication
- Industrial Application
- Medical Application
- Military Application
- In Computers
- In thermonuclear fission
- Entertainment
- Holography - The method of producing 3-dimensional image of an object. In holography both the phase and amplitude of the light waves are recorded in the film whereas in photography only amplitude is recorded.

The recorded hologram has no resemblance to the original object. It has it in a coded form of information of the object. The image is reproduced by a process called reconstruction

The phenomenon behind holography is interference of coherent light waves on a photographic plate