Computation. Antecedent / Input → (Computing $y = f(x)$) → Consequent / output

Control action

$f \Rightarrow$ Mapping function or ~~Band~~ Basically an algo. used to convert input x to output y.

Characteristics are :- a) Should provide precise Solution b) Unambiguous & accurate

c) Easy to model mathematically.

Hard Computing ⇒ Precise result guaranteed
Unambiguous
Formally defined

eg roots of poly., integration, Searching & Sorting etc.

Soft Computing ⇒ Basically Collection of methodologies aim to exploit tolerance for imprecision & Uncertainty. Its principal Constituents are fuzzy logic, neuro-Computing & probabilistic reasoning.

eg.

A, A, A, A → Soft Computing → A

Hand written Character Recognition

(ANN) Artifical Neural Network

Hard Computing	Soft Computing
* Requires a precisely stated analytical model	* It is tolerant of imprecison, Uncertainity
* Based on binary logic / Crisp System	* Based on fuzzy logic / Neural Sets
* It has characteristics of Precision	* Char. of approximation

---

* Fuzzy Logic :- It is a kind of mathematical language like relational algebra, boolean algebra

↳ It works with fuzzy Set.

Crisp Logic ⇒ Answer basically is Yes or No / True or False

[• Milk • Water] → (A liquid) → Crisp → Yes / No

Fuzzy Logic :- Answer is not fixed

(Man) → Fuzzy ← (Is it honest?)

Fuzzy →
- Extremely 99
- Very 75
- Honest at times 55
- Dishonest 35

Crisp Set :- All the Sets of finite no. of individuals. Such a Set is Called Crisp Set

Fuzzy Set

eg. X = All students in NPTEL

S = All good Students

$S = \{ (s, g(s)) \mid s \in X \}$ where $g(s)$ is a measurement of goodness of students

eg $S = \{ (\text{Rajat}, 0.8), (\text{Kabita}, 0.7), (\text{Salman}, 0.1), (\text{Ankit}, 0.9) \}$ etc.

A Crisp Set is a fuzzy Set but a fuzzy Set is not necessarily a Crisp set

Degree of Membership

Let take eg of cities how the Comfort level of each City is Calculated

eg. City	a	b	c	d	e	f
	0.95	0.90	0.80	0.01	0.65	0.75

Lies in Range $[0, 1]$

---

* Membership function :- If X is Universe of discourse & $x \in X$ then fuzzy Set A in X is defined as a set of ordered pairs

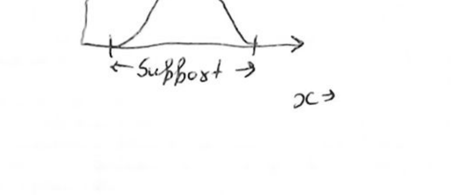
$A = \{ (x, \mu_A(x)) \mid x \in X \}$ where $\mu_A(x)$ is the membership function for Set A.

Values lies b/w $0 \to 1$

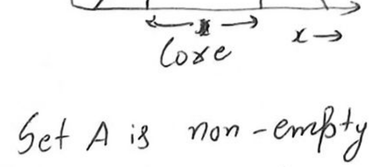
Membership Values are may be discrete or Continuous.

Support :- Set of all points on Set A where $\mu_A(x) > 0$

Support $(A) = \{ x \mid \mu_A(x) > 0 \}$



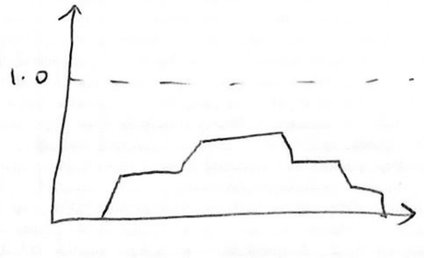
Core :- Set of all points on Set A where $\mu_A(x) = 1$



Core $(A) = \{ x \mid \mu_A(x) = 1 \}$

Normality :- Set A is non-empty. In other words, we Can always find a point $x \in X$ s.t $\mu_A(x) = 1$

Normality (A) = FALSE

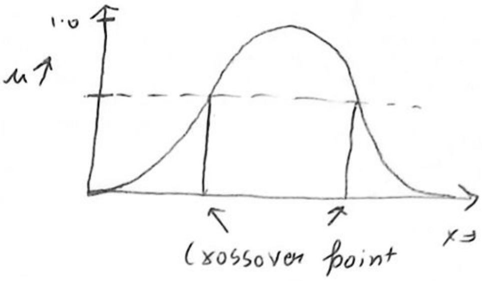


3

Crossover points :- Set of point on A in $x \in X$ where

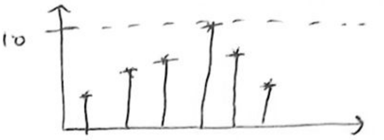
$\mu_A(x) = 0.5$

Crossover (A): $\{x \mid \mu_A(x) = 0.5\}$



Fuzzy Singleton :- Single point x with $\mu_A(x) = 1$

$|A| = \{x \mid \mu_A(x) = 1\}$



α-Cut & strong α-Cut

α-Cut of fuzzy set A is a crisp set $A_\alpha = \{x \mid \mu_A(x) >= \alpha\}$

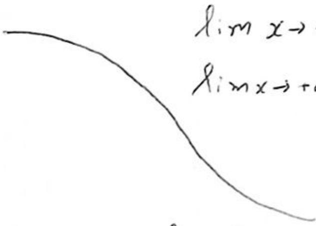
Strong α-Cut is defined $A'_\alpha = \{x \mid \mu_A(x) > \alpha\}$

Bandwidth :- Distance b/w two Unique Crossover points $\mu_A(x_1) = \mu_A(x_2) = 0.5$

Band width (A) = $|x_1 - x_2|$

Symmetry :- Set A is Symmetric if its memb. func. around a certain point $x = c$ $\mu_A(x+c) = \mu_A(x-c)$ for all $x \in X$

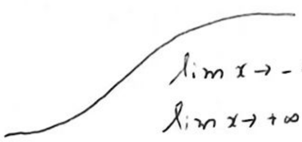
Open left



$\lim_{x \to -\infty} \mu_A(x) = 1$

$\lim_{x \to +\infty} \mu_A(x) = 0$

Open right



$\lim_{x \to -\infty} \mu_A(x) = 0$

$\lim_{x \to +\infty} \mu_A(x) = 1$

Closed

$\lim_{x \to -\infty} \mu_A(x) = 0$

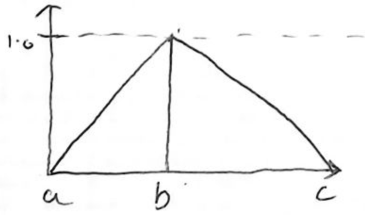
$\lim_{x \to +\infty} \mu_A(x) = 0$

A fuzzy set is characterised by its memb. func (μ)

Discrete — Continuous

↳ Universe of discourse

Triangle MF :- By three parameters {a, b, c}



$$\text{triangle}(x : a, b, c) = \begin{cases} 0 & \text{if } x <= a \\ \frac{x-a}{a-b} & \text{if } a <= x <= b \\ \frac{c-x}{c-b} & \text{if } b <= x <= c \\ 0 & \text{if } c \le x \end{cases}$$

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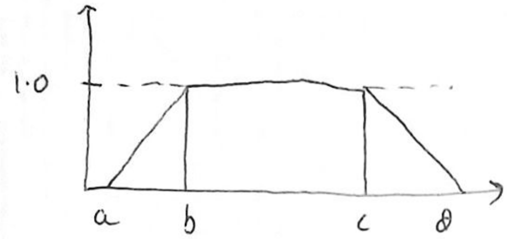
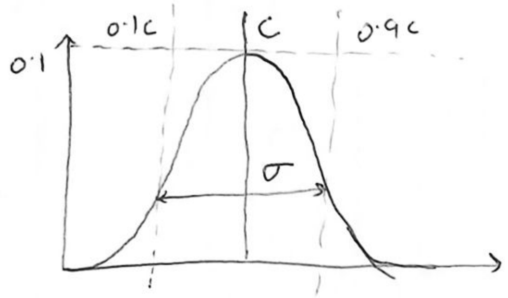
Trapezoidal :- Four parameters $\{a,b,c,d\}$



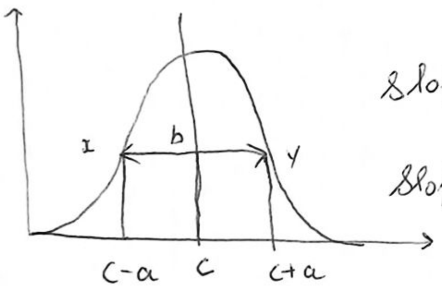
$$trapozoid(x:a,b,c,d) = \begin{cases} 0 & \text{if } x \le a \\ \frac{x-a}{b-a} & \text{if } a <= x <= b \\ 1 & \text{if } b <= x <= c \\ \frac{d-x}{d-c} & \text{if } c <= x <= d \\ 0 & \text{if } d <= x \end{cases}$$

Gaussian :- Two parameters $\{c, \sigma\}$



$$Gaussian(x:c,\sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$$

Generalized bell ⇒ Cauchy MF $\{a,b,c\}$



Slope at $x = \frac{b}{2a}$

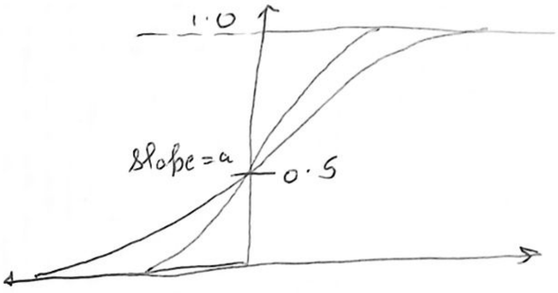
Slope at $y = -\frac{b}{2a}$

$$bell(x:a,b,c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$

In can be of diff shape by changing values of a & b

Sigmoidal MF $\{a,c\}$ ↳ crossover point

slope at c



$$Sigmoid(x;a,c) = \frac{1}{1+e^{-\left[\frac{x-c}{a}\right]}}$$

Generation of MFs :- Given a memb. func. of a fuzzy set rep. a linguistic hedge, we derive many more MF using Concentration & Dilation

Concentration : $A^k = [\mu_A(x)]^k \; ; k > 1$

Dilation $A^k = [\mu_A(x)]^k \; ; k < 1$

Use this We Can derive fuzzy Sets

eg Age: {Young, Middle, old} $\longrightarrow$ {old, very old, very very old, Extremely old}

$$\mu_{extremely\ old}(x) = (((\mu_{old}(x))^2)^2)^2$$

$$\mu_{more\ or\ less\ old}(x) = (\mu_{old}(x))^{0.5}$$

Linguistic Variables & Values

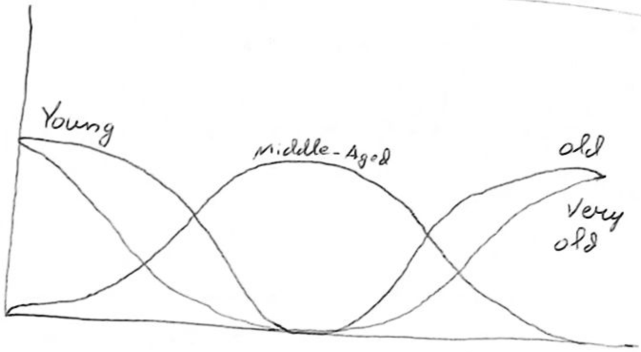
$$\mu_{young}(x) = bell(x, 20, 2, 0) = \frac{1}{1+(x/20)^4}$$

$$\mu_{old}(x) = bell(x, 30, 3, 100) = \frac{1}{1+\left(\frac{x-100}{30}\right)^6}$$

$$\mu_{middle\text{-}aged}(x) = bell(x, 30, 60, 50)$$

Not Young $= \overline{\mu_{young}(x)} = 1 - \mu_{young}(x)$

Young but not too Young $= \mu_{young}(x) \cap \overline{\mu_{young}(x)}$



Fuzzy Set Operations

1. Union $(A \cup B)$ $\quad \mu_{A\cup B}(x) = max(\mu_A(x), \mu_B(x))$
2. Intersection $(A \cap B) \Rightarrow \mu_{A\cap B}(x) = min(\mu_A(x), \mu_B(x))$
3. Complement (A^c) $\quad \mu_{A^c}(x) = 1 - \mu_A(x)$
4. Algebra product or Vector product

$$\mu_{A\cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$$

5. Scalar product $(\alpha \times A)$ $\quad \mu_{\alpha A}(x) = \alpha \times \mu_A(x)$
6. Sum $(A+B)$ $\quad \mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$
7. Difference $(A - B = A \cap B^c)$ $\quad \mu_{A-B}(x) = \mu_{A\cap B^c}(x)$
8. Disjunctive Sum $\quad A \oplus B = (A^c \cap B) \cup (A \cap B^c)$
9. Bounded Sum $\quad |A(x) \oplus B(x)| = \mu_{|A(x)\oplus B(x)|} = min\{1, \mu_A(x) + \mu_B(x)\}$
10. Bounded Diff $\quad |A(x) \ominus B(x)| = \mu_{|A(x)\ominus B(x)|} = max\{0, \mu_A(x) + \mu_B(x) - 1\}$

(11) Equality $(A = B)$ $\quad \mu_A(x) = \mu_B(x)$

(12) Power of a Set A^α $\quad \mu_{A^\alpha}(x) = [\mu_A(x)]^\alpha$

If $\alpha < 1 \Rightarrow$ Dilation

$\alpha > 1 \Rightarrow$ Concentration

(13) Cartesian Product $(A \times B)$: $\mu_{A\times B}(x,y) = \min(\mu_A(x), \mu_B(y))$

eg. $A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$$A \times B = \min(\mu_A(x), \mu_B(y)) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 \\ 0.5 & 0.5 & 0.3 \\ 0.6 & 0.6 & 0.3 \end{bmatrix} \end{array}$$

Properties of fuzzy Set

a) Commutativity $A \cap B = B \cap A$ | $A \cup B = B \cup A$

b) Associativity $A \cup (B \cup C) = (A \cup B) \cup C$ | $A \cap (B \cap C) = (A \cap B) \cap C$

c) Distributivity $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ | $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

d) Idempotence $A \cup A = A$ $\quad A \cap A = \phi$ $\quad A \cup \phi = A$ $\quad A \cap \phi = \phi$

e) Transitivity If $A \subseteq B$; $B \subseteq C$ then $A \subseteq C$

f) Involution $(A^c)^c = A$

g) De Morgan's law $(A \cap B)^c = A^c \cup B^c$ $\quad (A \cup B)^c = A^c \cap B^c$

Crisp relations $A \times B = \{(a,b) \mid a \in A \text{ \& } b \in B\}$ $\quad A \times B \neq B \times A$

$|A \times B| = |A| \times |B|$

Operations on Crisp relations $R(x,y) \cup S(x,y) = \max(R(x,y), S(x,y))$

$R(x,y) \cap S(x,y) = \min(R(x,y), S(x,y))$

Complement $\overline{R(x,y)} = 1 - R(x,y)$

eg. $R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $R \cup S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R \cap S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\bar{R} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Composition of two crisp relations — $R \circ S \to$ Relation of X & Z

$R \to$ Relation on X, Y $S \to$ Relation on Y, Z

$$R \circ S = \{ (x,z) \mid (x,y) \in R \ \& \ (y,z) \in S \ \& \ \forall y \in Y \}$$

Max- Min Composition defined as $T = R \circ S$

$$T(x,z) = \max \{ \min \{ R(x,y), S(y,z) \ \& \ \forall y \in Y \} \}$$

eg. $X = \{1,3,5\}$ $Y = \{1,3,5\}$

$R = \{ (1,3), (3,5) \}$ $S = \{ (1,3), (1,5), (3,5) \}$

$R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $R \circ S = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Methord go like Matrix Multiplication $\xrightarrow[R]{} \downarrow S$

eg $\underline{\max\{\min(0,0), \min(1,0), \min(0,0)\} = 0}$ for first cell $(0,0)$

$\max\{\min(0,1), \min(1,1), \min(0,0)\} = 1$ for third cell $(0,2)$

<u>Fuzzy relations</u> eg $X = \{a, b, c\}$ $Y = \{x, y, z\}$

$$R = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{bmatrix} 0.1 & 0.9 & 0.8 \\ 0.2 & 0.9 & 0.7 \\ 0.9 & 0.4 & 0.6 \end{bmatrix} \end{array}$$

Fuzzy Cartesian Product

$A \to \mu_A(x) \mid x \in X$

$R = A \times B \subset X \times Y$ $B \to \mu_B(y) \mid y \in Y$

$$\mu_R(x,y) = \mu_{A \times B}(x,y) = \min(\mu_A(x), \mu_B(y))$$

Operations on fuzzy set relations

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Union $\Rightarrow \mu_{R \cup S}(x,y) = \max(\mu_R(x,y), \mu_S(x,y))$

Intersection $\rightarrow \mu_{R \cap S}(x,y) = \min(\mu_R(x,y), \mu_S(x,y))$

Complement $\Rightarrow \mu_{\bar{R}}(x,y) = 1 - \mu_R(x,y)$

Composition $R \circ S \Rightarrow \mu_{R \circ S} = \max\{\min\{\mu_R(x,y), \mu_S(y,z)\}\}$

Same as Crisp

eg.

$$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.5 & 0.1 \\ x_2 & 0.2 & 0.9 \\ x_3 & 0.8 & 0.6 \end{array}_{3\times 2} \qquad S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline y_1 & 0.6 & 0.4 & 0.7 \\ y_2 & 0.5 & 0.8 & 0.9 \end{array}_{2\times 3}$$

$$R \circ S = \begin{array}{c|ccc} & y_1 & \cancel{y_2}\ y_2 & y_3 \\ \hline x_1 & 0.5 & 0.4 & 0.5 \\ x_2 & 0.5 & 0.8 & 0.9 \\ x_3 & 0.6 & 0.6 & 0.7 \end{array}$$

1st cell

$\max\{\min(0.5, 0.6), \min(0.1, 0.5)\}$

0.5

Binary fuzzy relation are fuzzy sets with 2 dimensional MFs & so on

eg. $X = R^+ = Y$ (the +ve real line) &

$R = X \times X$ $\quad$ Y is much greater than x

$$\mu_R(x,y) = \begin{cases} \frac{y-x}{4} & \text{if } y > x \\ 0 & y \le x \end{cases}$$

eg. If x is a or Y is b then Z is c

$R_1 =$ x is a then Z is c $\quad R_1 \in A \times C$

$R_2 =$ Y is b then Z is c $\quad R_2 \in B \times C$

] take Union