# Principles of Programming Languages

## Assignment → 3

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① ⓐ

let $true : Bool \in EC$

Given $E_0 \cup \{y : Ref\ Bool\}$

$E_0 \cup \{y : Ref\ Bool\} \vdash y : Ref\ Bool$ — ①

[By Identifier Rule]

$E_0 \cup \{y : Ref\ Bool\} \vdash true : Bool$ — ②

[By Const. Rule]

$E_0 \cup \{y : Ref.\ Bool\} \vdash y := true : Command$

— ③

[By App. Rule]

i.e.

$$\dfrac{\dfrac{}{E_0 \cup \{y : Ref\ Bool\} \vdash y : Ref\ Bool}\ ① \qquad \dfrac{}{E_0 \cup \{y : Ref\ Bool\} \vdash true : Bool}\ ②}{E_0 \cup \{y : Ref\ Bool\} \vdash y := true : Command}\ ③$$

(b) Given :-

$func1 : A \to B$

$func2 : C \to B$

$\lambda(x : A).\ (func1\ x);\ \lambda(q : C).\ (func2\ q)$

The steps of derivations are :-

let $\mathcal{E}_1 = \mathcal{E}_0 \cup \{x : A\}$

$\mathcal{E}_1 \vdash func1 : A \to B$ —— (1, Const.)

$\mathcal{E}_1 \vdash x : A$ —— (2, Id.)

$\mathcal{E}_1 \vdash func1\ x : B$ —— (3, App.)

$\mathcal{E}_1 \vdash (func1\ x) : B$ —— (4, Paren)

$\mathcal{E}_0 \vdash \lambda(x : A).\ (func1\ x) : A \to B$ —— (5, Func.)

let $\mathcal{E}_2 = \mathcal{E}_0 \cup \{q : C\}$

$\mathcal{E}_2 \vdash func2 : C \to B$ —— (6, Const.)

$\mathcal{E}_2 \vdash q : C$ —— (7, Id.)

$\mathcal{E}_2 \vdash func2\ q : B$ —— (8, App.)

$\mathcal{E}_2 \vdash (func2\ q) : B$ —— (9, Paren)

$\mathcal{E}_0 \vdash \lambda(q : C).\ (func2\ q) : C \to B$ —— (10, Func.)

$\mathcal{E}_0 \vdash \lambda(x : A)\ (func1\ x);\ \lambda(q : C).\ (func2\ q) : C \to B$

—— (11, Seq. Rule)

$$\dfrac{\dfrac{\dfrac{\mathcal{E}_1 \vdash func1 : A \to B\ (1) \qquad \mathcal{E}_1 \vdash x : A\ (2)}{\mathcal{E}_1 \vdash func1\ x : B}\ (3)}{\mathcal{E}_1 \vdash (func1\ x) : B}\ (4) \qquad \dfrac{\dfrac{\mathcal{E}_2 \vdash func2 : C \to B\ (6) \qquad \mathcal{E}_2 \vdash q : C\ (7)}{\mathcal{E}_2 \vdash func2\ q : B}\ (8)}{\mathcal{E}_2 \vdash (func2\ q) : B}\ (9)}{}$$

[11, Function Rule]

2 q)

(11)

$$\frac{\quad}{\mathcal{E}_0 \vdash \lambda(x:A).(\text{func1 } x) : A \to B} \; (5) \qquad \frac{\quad}{\mathcal{E}_0 \vdash \lambda(a:C).(\text{func2 } a) : C \to B} \; (10)$$

$$\frac{\quad}{\mathcal{E}_0 \vdash \lambda(x:A)(\text{func1 } x) ; \lambda(a:C).(\text{funct2 } a) : C \to B} \; (11)$$

(c)

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$| : \text{Bool} \to \text{Bool} \to \text{Bool}$

$\text{true} : \text{Bool}$

Let $\mathcal{E}_1 = \mathcal{E}_0 \cup \{ w : \text{Bool} \to \pi \}$

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash x : \text{Bool}$ — [1, Id]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash | : \text{Bool} \to \text{Bool} \to \text{Bool}$ — [2, Const.]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash | x : \text{Bool} \to \text{Bool}$ — [3, App]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash \text{true} : \text{Bool}$ — [4, Const]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash | x \text{ true} : \text{Bool}$ — [5, App]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash w : \text{Bool} \to \pi$ — [6, Id]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash (| x \text{ true}) : \text{Bool}$ — [7, Paren]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash w(| x \text{ true}) : \pi$ — [8, App]

$\mathcal{E}_1 \cup \{x : \text{Bool}\} \vdash (w(| x \text{ true})) : \pi$ — [9, Paren]

$\mathcal{E}_1 \vdash \lambda(x : \text{Bool}).(w(| x \text{ true})) : \text{Bool} \to \pi$ [10, Function Rule]

$\therefore \mathcal{E}_0 \vdash \lambda(w : \text{Bool} \to \pi).\lambda(x : \text{Bool}).(w(| x \text{ true})) : (\text{Bool} \to \pi) \to (\text{Bool} \to \pi)$

[11, Function Rule]

(d) $+ : S \to S$

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Let $\mathcal{E}_1 = \mathcal{E}_0 \cup \{f : S \to C\}$

$\mathcal{E}_2 = \mathcal{E}_1 \cup \{x : S\}$

$\mathcal{E}_2 \vdash + : S \to S$ — [1, Const.]

$\mathcal{E}_2 \vdash x : S$ — [2, Id.]

$\mathcal{E}_2 \vdash + x : S$ — [3, App]

$\mathcal{E}_2 \vdash (+x) : S$ — [4, Paren]

$\mathcal{E}_2 \vdash f : S \to C$ — [5, Id]

$\mathcal{E}_2 \vdash f(+x) : C$ — [6, App.]

$\mathcal{E}_1 \vdash \lambda(x : S).\ f(+x) : S \to C$ — [7, Func.]

$\mathcal{E}_0 \vdash \lambda(f : S \to C).\ \lambda(x : S).\ f(+x) : (S \to C) \to (S \to C)$

— [8, Func.]

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\mathcal{E}_2 \vdash + : S \to S}\ (1) \quad \dfrac{}{\mathcal{E}_2 \vdash x : S}\ (2)}{\mathcal{E}_2 \vdash +x : S}\ (3)}{\mathcal{E}_2 \vdash (+x) : S}\ (4) \quad \dfrac{}{\mathcal{E}_2 \vdash f : S \to C}\ (5)}{\dfrac{\mathcal{E}_2 \vdash f(+x) : C}{\mathcal{E}_1 \vdash \lambda(x : S).\ f(+x) : S \to C}\ (7)}\ (6)}{\mathcal{E}_0 \vdash \lambda(f : S \to C).\ \lambda(x : S).\ f(+x) : S \to C \to S \to C}\ (8)$$

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Thu

$\mathcal{E}_3 \vdash f : float \to float$ —— [1, Id.]

$\mathcal{E}_3 \vdash y : float$ —— [2, Id.]

$\mathcal{E}_3 \vdash fy : Float$ —— [3, Appl.]

$\mathcal{E}_3 \vdash (fy) : Float$ —— [4, Paren.]

$\mathcal{E}_3 \vdash f(fy) : Float$ —— [5, Appl.]

$\mathcal{E}_3 \vdash (f(fy)) : Float$ —— [6, Paren]

$\mathcal{E}_3 \vdash p : Float \to Integer$ —— [7, Id]

$\mathcal{E}_3 \vdash p(f(fy)) : Int$ —— [8, Appl.]

$\mathcal{E}_2 \vdash \lambda(y : Float).\, p(f(fy)) : Float \to Int$

—— [9, Func.]

$\mathcal{E}_1 \vdash \lambda(f : Float \to Float).\, \lambda(y : Float).\, p(f(f\,y)) :$
$Float \to Float \to Float \to Int$

—— [10, Function Rule]

$\mathcal{E}_0 \vdash (\lambda(p : Float \to Int).\, \lambda(f : Float \to Float).$
$\lambda(y : Float).\, p(f(fy))) :$
$Float \to Int \to Float \to Float \to Float \to Int$

—— [11, Funct. Rule, Paren Rule]

$\mathcal{E}_0 \vdash \phi : Float \to Int.$ —— [12, Const.]

~~$\mathcal{E}_0 \vdash (\lambda(p : Float \to In$~~

$\therefore\ \mathcal{E}_0 \vdash (\lambda(p : Float \to Int).\, \lambda(f : Float \to Float).$
$\lambda(y : Float).\, p(f(f\,y)))\ \phi :$
$Float \to Float \to Float \to Int$

1 c

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$\mathcal{E}_0 = \{x : \text{Ref Bool},\ y : \text{Bool}\}$

$succ : Int \to Int$

$true : Bool$

$4 : Int$

Given

$\mathcal{E}_0 \vdash succ : Int \to Int$ — [1, Const.]

$\mathcal{E}_0 \vdash 4 : Int$ — [2, Const.]

$\mathcal{E}_0 \vdash succ\ 4 : Int$ — [3, App.]

$\mathcal{E}_0 \vdash x : \text{Ref. Bool}$ — [4, Id]

$\mathcal{E}_0 \vdash true : Bool$ — [5, Const.]

$\mathcal{E}_0 \vdash x := true : Command$ — [6, Assign.]

$\therefore\ \mathcal{E}_0 \vdash succ\ 4;\ x := true : Command$ — [7, sequencing rule]

Thus :-

~~2 a~~

$$\dfrac{\dfrac{\dfrac{}{\mathcal{E}_0 \vdash succ : Int \to Int}\,①\quad \dfrac{}{\mathcal{E}_0 \vdash 4 : Int}\,②}{\mathcal{E}_0 \vdash succ\ 4 : Int}\,③ \qquad \dfrac{\dfrac{}{\mathcal{E}_0 \vdash x : \text{Ref. Bool}}\,④\quad \dfrac{}{\mathcal{E}_0 \vdash true : Bool}\,⑤}{\mathcal{E}_0 \vdash x := true : Command}\,⑥}{\mathcal{E}_0 \vdash succ\ 4;\ x := true : Command}\,⑦$$

2 a

$\phi : Float \to Integer$

$\mathcal{E}_1 = \mathcal{E}_0 \cup \{p : float \to Integer\}$

$\mathcal{E}_2 = \mathcal{E}_1 \cup \{f : float \to float\}$

$\mathcal{E}_3 = \mathcal{E}_2 \cup \{y : float\}$

(iv) The derivation is :-

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$$\frac{}{\mathcal{E}_3 \vdash f : F \to F}\ (1) \qquad \frac{}{\mathcal{E}_3 \vdash y : F}\ (2)$$

$$\frac{}{\mathcal{E}_3 \vdash f\ y : F}\ (3) \qquad \frac{}{\mathcal{E}_3 \vdash f : F \to F}\ (1)$$

$$\frac{}{\mathcal{E}_3 \vdash (f(f y)) : F}\ (5,6) \qquad \frac{}{\mathcal{E}_3 \vdash p : F \to I}\ (7)$$

$$\frac{}{\mathcal{E}_3 \vdash p(f(f\ y)) : I}\ (8)$$

$$\frac{}{\mathcal{E}_2 \vdash \lambda(y:F).\ p(f(f\ y)) : F \to I}\ (9)$$

$$\frac{}{\mathcal{E}_1 \vdash \lambda(f : F \to F).\ \lambda(y:F).\ p(f(f y)) : F \to F \to F \to I}\ (10)$$

$$\frac{}{\mathcal{E}_0 \vdash (\lambda(p : F \to I).\ \lambda(f : F \to F).\ \lambda(y:F).\ p(f(f y))) : F \to I \to F \to F \to F \to I}\ (11) \qquad \frac{}{\mathcal{E}_0 \vdash \phi : F \to I}\ (13)$$

$$\frac{}{\mathcal{E}_0 \vdash (\lambda(p : F \to I).\ \lambda(f : F \to F).\ \lambda(y:F).\ p(f(f y)))\ \phi : F \to F \to F \to I}\ (14)$$

Here $F \Rightarrow$ Float
$I \Rightarrow$ Int.

(2) (b) $\left.\begin{array}{l}\phi : \text{Bool} \to \text{Bool} \to \text{Bool} \\ \text{true} : \text{Bool}\end{array}\right\} \in C, \mathcal{E}I$

Assume, $\mathcal{E}_1 = \mathcal{E}_0 \cup \{ \text{func1} : \text{Bool} \to \text{Char} \}$
$\mathcal{E}_2 = \mathcal{E}_1 \cup \{ \tau : \text{Bool} \}$

$\mathcal{E}_2 \vdash \tau : \text{Bool}$ — [1, Id.]
$\mathcal{E}_2 \vdash \phi : \text{Bool} \to \text{Bool} \to \text{Bool}$ — [2, Const.]

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$\mathcal{E}_2 \vdash \phi\, \gamma : Bool \rightarrow Bool$ — [3, Appl.]

$\mathcal{E}_2 \vdash true : Bool$ — [4, Const.]

$\mathcal{E}_2 \vdash \phi\, \gamma\ true : Bool$ — [5, Application Rule]

$\mathcal{E}_2 \vdash (\phi\, \gamma\ true) : Bool$ — [6, Paren Rule]

$\mathcal{E}_2 \vdash func1 : Bool \rightarrow Char$ — [7, Id Rule]

$\mathcal{E}_2 \vdash func1\ (\phi\, \gamma\ true) : Char$ — [8, Application]

$\mathcal{E}_1 \vdash \lambda(\gamma : Bool).\ func1(\phi\, \gamma\ true) : Bool \rightarrow Char$
[9, Func.]

$\therefore\ \mathcal{E}_0 \vdash \lambda(funct1 : Bool \rightarrow Char).\ \lambda(\gamma : Bool).\ func1(\phi\, \gamma\ true) : (Bool \rightarrow Char) \rightarrow (Bool \rightarrow Char)$
— [10, Func.]

i.e. the derivation is:-

$$\frac{}{\mathcal{E}_2 \vdash \phi : Bool \rightarrow Bool \rightarrow Bool}\ (2) \qquad \frac{}{\mathcal{E}_2 \vdash \gamma : Bool}\ (1)$$

$$\frac{\mathcal{E}_2 \vdash \phi : Bool \rightarrow Bool \rightarrow Bool \qquad \mathcal{E}_2 \vdash \gamma : Bool}{\mathcal{E}_2 \vdash \phi\, \gamma : Bool \rightarrow Bool}\ (3) \qquad \frac{}{\mathcal{E}_2 \vdash true : Bool}\ (4)$$

$$\frac{\mathcal{E}_2 \vdash \phi\, \gamma : Bool \rightarrow Bool \qquad \mathcal{E}_2 \vdash true : Bool}{\mathcal{E}_2 \vdash (\phi\, \gamma\ true) : Bool}\ (5), (6) \qquad \frac{}{\mathcal{E}_2 \vdash func1 : Bool \rightarrow Char}\ (7)$$

$$\frac{\mathcal{E}_2 \vdash (\phi\, \gamma\ true) : Bool \qquad \mathcal{E}_2 \vdash func1 : Bool \rightarrow Char}{\mathcal{E}_2 \vdash func1(\phi\, \gamma\ true) : Char}\ (8)$$

$$\frac{\mathcal{E}_2 \vdash func1(\phi\, \gamma\ true) : Char}{\mathcal{E}_1 \vdash \lambda(\gamma : Bool).\ func1\ (\phi\, \gamma\ true) : Bool \rightarrow Char}$$

$$\frac{\mathcal{E}_1 \vdash \lambda(\gamma : Bool).\ func1\ (\phi\, \gamma\ true) : Bool \rightarrow Char}{\mathcal{E}_0 \vdash \lambda(func1 : Bool \rightarrow Char).\ \lambda(\gamma : Bool).\ func1\ (\phi\, \gamma\ true) : (Bool \rightarrow Char) \rightarrow (Bool \rightarrow Char)}\ (10)$$