Assignment-1

AI1110: Probability and Random Variables
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Question:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

1) A king of red colour
2) A face card
3) A red face card
4) The jack of hearts
5) A spade
6) The queen of diamonds

Solution:

Consider 3 random variables X, Y and Z, which represent the Colour, Class and Value of each card respectively.

The pmfs of each random variable are:

$$p_X(i) = \frac{1}{2} \ \forall \ i \in [0,1] \tag{1}$$

$$p_Y(i) = \frac{1}{4} \ \forall \ i \in [1,4] \tag{2}$$

$$p_Z(i) = \frac{1}{13} \ \forall \ i \in [1,13] \tag{3}$$

The cdf of Z is defined by,

$$F_Z(z) = \Pr(Z \le z) = \sum_{i=1}^{z} \Pr(Z = i)$$
$$= z \times \Pr(Z = 1) = \frac{z}{13} \tag{4}$$

Also, the random variable pairs X,Z and Y,Z are independent.

1) Probability of drawing a King of Red colour:

$$\Pr(X = 1, Z = 3) = \Pr(X = 1) \times \Pr(Z = 3)$$
$$= \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \tag{5}$$

Event	Value of X	Value of Y	Value of Z
Draw Red King	1	N/A	3
Draw Face Card	N/A	N/A	1,2 or 3
Draw Red Face Card	1	N/A	1,2 or 3
Draw Hearts Jack	N/A	3	1
Draw Spade	N/A	4	N/A
Draw Diamonds Queen	N/A	1	2

TABLE 6
VALUES OF X,Y,Z FOR EACH EVENT

2) Probability of drawing a Face Card:

$$\Pr(1 \le Z \le 3) = F_Z(3) = \frac{3}{13}(From(4)) \tag{6}$$

3) Probability of drawing a Red Face Card:

$$\Pr(X = 1, 1 \le Z \le 3) = \Pr(X = 1) \times F_Z(3)$$
$$= \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}(From(4)) \tag{7}$$

4) Probability of drawing the Jack of Hearts:

$$\Pr(Y = 3, Z = 1) = \Pr(Y = 3) \times \Pr(Z = 1)$$
$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{8}$$

5) Probability of drawing a Spade:

$$\Pr(Y = 4) = \frac{1}{4} \tag{9}$$

6) Probability of drawing the Queen of Diamonds:

$$\Pr(Y = 1, Z = 2) = \Pr(Y = 1) \times \Pr(Z = 2)$$
$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{10}$$