# Assignment-1

## AI1110: Probability and Random Variables
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Question:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

1) A king of red colour
2) A face card
3) A red face card
4) The jack of hearts
5) A spade
6) The queen of diamonds

Solution:

Consider 3 random variables $X$, $Y$ and $Z$, which represent the Colour, Class and Value of each card respectively.

The pmfs of each random variable are:

$$\Pr(X = i) = \frac{1}{2} \;\forall\; i \in [0, 1] \tag{1}$$

$$\Pr(Y = i) = \frac{1}{4} \;\forall\; i \in [1, 4] \tag{2}$$

$$\Pr(Z = i) = \frac{1}{13} \;\forall\; i \in [1, 13] \tag{3}$$

Also, the random variable pairs X,Z and Y,Z are independent.

| Event | Value of X | Value of Y | Value of Z |
|---|---|---|---|
| Draw Red King | 1 | N/A | 13 |
| Draw Face Card | N/A | N/A | 11,12 or 13 |
| Draw Red Face Card | 1 | N/A | 11,12 or 13 |
| Draw Hearts Jack | N/A | 3 | 11 |
| Draw Spade | N/A | 4 | N/A |
| Draw Diamonds Queen | N/A | 1 | 12 |

TABLE 6
VALUES OF X,Y,Z FOR EACH EVENT

1) Probability of drawing a King of Red colour:

$$\Pr(X = 1, Z = 13) = \Pr(X = 1) \times \Pr(Z = 13) = \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \tag{4}$$

2) Probability of drawing a Face Card:

$$\Pr(Z = 11, 12, 13) = \sum_{i=11}^{13} \Pr(Z = i) = 3 \times \frac{1}{13} = \frac{3}{13} \tag{5}$$

3) Probability of drawing a Red Face Card:

$$\Pr(X = 1, Z = 11, 12, 13) = \sum_{i=11}^{13} \Pr(Z = i) \times \Pr(X = 1) = \frac{1}{2}\left(3 \times \frac{1}{13}\right) = \frac{3}{26} \tag{6}$$

4) Probability of drawing the Jack of Hearts:

$$\Pr(Y = 3, Z = 11) = \Pr(Y = 3) \times \Pr(Z = 11) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{7}$$

5) Probability of drawing a Spade:

$$\Pr(Y = 4) = \frac{1}{4} \tag{8}$$

6) Probability of drawing the Queen of Diamonds:

$$\Pr(Y = 1, Z = 12) = \Pr(Y = 1) \times \Pr(Z = 12) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \tag{9}$$