

# Assignment-1

## AI1110: Probability and Random Variables

### Indian Institute of Technology, Hyderabad

P. Satvik  
EE22BTECH11212

Question:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

- 1) A king of red colour
- 2) A face card
- 3) A red face card
- 4) The jack of hearts
- 5) A spade
- 6) The queen of diamonds

Solution:

Consider 3 random variables  $X$ ,  $Y$  and  $Z$ , which represent the Colour, Class and Value of each card respectively.

The pmfs of each random variable are:

$$P_X(i) = \frac{1}{2} \quad \forall i \in [0, 1] \quad (1)$$

$$P_Y(i) = \frac{1}{4} \quad \forall i \in [1, 4] \quad (2)$$

$$P_Z(i) = \frac{1}{13} \quad \forall i \in [1, 13] \quad (3)$$

The cdf of  $Z$  is defined by,

$$\begin{aligned} F_Z(z) &= \Pr(Z \leq z) = \sum_{i=1}^z \Pr(Z = i) \\ &= z \times \Pr(Z = 1) = \frac{z}{13} \end{aligned} \quad (4)$$

Also, the random variable pairs  $X, Z$  and  $Y, Z$  are independent.

- 1) Probability of drawing a King of Red colour:

$$\begin{aligned} \Pr(X = 1, Z = 3) &= \Pr(X = 1) \times \Pr(Z = 3) \\ &= \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \end{aligned} \quad (5)$$

Event	Value of X	Value of Y	Value of Z
Draw Red King	1	N/A	3
Draw Face Card	N/A	N/A	1, 2 or 3
Draw Red Face Card	1	N/A	1, 2 or 3
Draw Hearts Jack	N/A	3	1
Draw Spade	N/A	4	N/A
Draw Diamonds Queen	N/A	1	2

TABLE 6  
VALUES OF  $X, Y, Z$  FOR EACH EVENT

- 2) Probability of drawing a Face Card:

$$\Pr(1 \leq Z \leq 3) = F_Z(3) = \frac{3}{13} \text{ (From (4))} \quad (6)$$

- 3) Probability of drawing a Red Face Card:

$$\begin{aligned} \Pr(X = 1, 1 \leq Z \leq 3) &= \Pr(X = 1) \times F_Z(3) \\ &= \frac{1}{2} \times \frac{3}{13} = \frac{3}{26} \text{ (From (4))} \end{aligned} \quad (7)$$

- 4) Probability of drawing the Jack of Hearts:

$$\begin{aligned} \Pr(Y = 3, Z = 1) &= \Pr(Y = 3) \times \Pr(Z = 1) \\ &= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \end{aligned} \quad (8)$$

- 5) Probability of drawing a Spade:

$$\Pr(Y = 4) = \frac{1}{4} \quad (9)$$

- 6) Probability of drawing the Queen of Diamonds:

$$\begin{aligned} \Pr(Y = 1, Z = 2) &= \Pr(Y = 1) \times \Pr(Z = 2) \\ &= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \end{aligned} \quad (10)$$