

# Assignment-1

## AI1110: Probability and Random Variables

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Question:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card (iii) a red face card

(iv) the jack of hearts (v) a spade (vi) the queen of diamonds

Solution:

Consider 3 random variables  $X$ ,  $Y$  and  $Z$ , which represent the Colour, Class and Value of each card respectively.

The value of each random variable is,

$$X = \begin{cases} 0 & \text{Black} \\ 1 & \text{Red} \end{cases} \quad (1)$$

$$Y = \begin{cases} 1 & \text{Diamond} \\ 2 & \text{Heart} \\ 3 & \text{Jack} \\ 4 & \text{spade} \end{cases} \quad (2)$$

$$Z = \begin{cases} 1 & \text{Ace} \\ i & i \in [2, 10] \\ 11 & \text{Jack} \\ 12 & \text{Queen} \\ 13 & \text{King} \end{cases} \quad (3)$$

Also the pmfs of each random variable are:

$$\Pr(X = i) = \frac{1}{2} \quad \forall i \in [0, 1] \quad (4)$$

$$\Pr(Y = i) = \frac{1}{4} \quad \forall i \in [1, 4] \quad (5)$$

$$\Pr(Z = i) = \frac{1}{13} \quad \forall i \in [1, 13] \quad (6)$$

Also, the random variable pairs  $X, Z$  and  $Y, Z$  are independent.

Event	Value of X	Value of Y	Value of Z
Draw Red Card	1	N/A	13
Draw Face Card	N/A	N/A	11, 12 or 13
Draw Red Face Card	1	N/A	11, 12 or 13
Draw Jack of Hearts	N/A	3	11
Draw Spade	N/A	4	N/A
Draw Queen of Diamonds	N/A	1	12

TABLE 0  
VALUES OF  $X, Y, Z$  FOR EACH EVENT

1) Probability of drawing a King of Red colour:

$$\begin{aligned} \Pr(\text{Red King}) &= \Pr(X = 1, Z = 13) \\ &= \Pr(X = 1) \times \Pr(Z = 13) \\ &= \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \end{aligned} \quad (7)$$

2) Probability of drawing a Face Card:

$$\begin{aligned} \Pr(\text{Face Card}) &= \Pr(Z = 11, 12, 13) \\ &= \sum_{i=11}^{13} \Pr(Z = i) \\ &= 3 \times \frac{1}{13} = \frac{3}{13} \end{aligned} \quad (8)$$

3) Probability of drawing a Red Face Card:

$$\begin{aligned}
 \Pr(\text{Red Face Card}) &= \Pr(X = 1, Z = 11, 12, 13) \\
 &= \sum_{i=11}^{13} \Pr(Z = i) \times \Pr(X = 1) \\
 &= \frac{1}{2} \left( 3 \times \frac{1}{13} \right) = \frac{3}{26}
 \end{aligned}
 \tag{9}$$

4) Probability of drawing the Jack of Hearts:

$$\begin{aligned}
 \Pr(\text{Jack of Hearts}) &= \Pr(Y = 3, Z = 11) \\
 &= \Pr(Y = 3) \times \Pr(Z = 11) \\
 &= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}
 \end{aligned}
 \tag{10}$$

5) Probability of drawing a Spade:

$$\begin{aligned}
 \Pr(\text{spade}) &= \Pr(Y = 4) \\
 &= \frac{1}{4}
 \end{aligned}
 \tag{11}$$

6) Probability of drawing the Queen of Diamonds:

$$\begin{aligned}
 \Pr(\text{Queen of Diamonds}) &= \Pr(Y = 1, Z = 12) \\
 &= \Pr(Y = 1) \times \Pr(Z = 12) \\
 &= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}
 \end{aligned}
 \tag{12}$$