Assignment-1
AI1110: Probability and Random Variables
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EE22BTECH11212

Question:

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(i) a king of red colour (ii) a face card (iii) a red face card

(iv) the jack of hearts (v) a spade (vi) the queen of diamonds

Solution:

Consider 3 random variables X, Y and Z, which represent the Colour, Class and Value of each card respectively.

The value of each random variable is,

$$X = \begin{cases} 0 & Black \\ 1 & Red \end{cases} \qquad (1)$$

$$Y = \begin{cases} 1 & Diamond \\ 2 & Heart \\ 3 & Jack \\ 4 & spade \end{cases} \qquad (2)$$

$$Z = \begin{cases} 1 & Ace \\ i & i \in [2, 10] \\ 11 & Jack \\ 12 & Queen \\ 13 & King \end{cases} \qquad (3)$$

Also the pmfs of each random variable are:

$$\Pr(X = i) = \frac{1}{2} \ \forall \ i \in [0, 1] \qquad (4)$$

$$\Pr(Y = i) = \frac{1}{4} \ \forall \ i \in [1, 4] \qquad (5)$$

$$\Pr(Z = i) = \frac{1}{13} \ \forall \ i \in [1, 13] \qquad (6)$$

Also, the random variable pairs X,Z and Y,Z are independent.

Event	Value of X	Value of Y	Value of Z
Draw Red King	1	N/A	13
Draw Face Card	N/A	N/A	11,12 or 13
Draw Red Face Card	1	N/A	11,12 or 13
Draw Hearts Jack	N/A	3	11
Draw Spade	N/A	4	N/A
Draw Diamonds Queen	N/A	1	12

TABLE 0

VALUES OF X,Y,Z FOR EACH EVENT

1) Probability of drawing a King of Red colour:

$$\begin{aligned} \Pr(Red\ King) &= \Pr(X = 1, Z = 13) \\ &= \Pr(X = 1) \times \Pr(Z = 13) \\ &= \frac{1}{2} \times \frac{1}{13} = \frac{1}{26} \qquad (7) \end{aligned}$$

2) Probability of drawing a Face Card:

$$\begin{aligned} \Pr(Face\ Card) &= \Pr(Z = 11, 12, 13) \\ &= \sum_{i=11}^{13} \Pr(Z = i) \\ &= 3 \times \frac{1}{13} = \frac{3}{13} \qquad (8) \end{aligned}$$

3) Probability of drawing a Red Face Card:

$$\Pr(Red\ Face\ Card) = \Pr(X = 1, Z = 11, 12, 13)$$

$$= \sum_{i=11}^{13} \Pr(Z = i) \times \Pr(X = 1)$$

$$= \frac{1}{2}(3 \times \frac{1}{13}) = \frac{3}{26}$$

(9)

4) Probability of drawing the Jack of Hearts:

$$\Pr(Jack\ of\ Hearts) = \Pr(Y = 3, Z = 11)$$

$$= \Pr(Y = 3) \times \Pr(Z = 11)$$

$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

(10)

5) Probability of drawing a Spade:

$$\Pr(spade) = \Pr(Y = 4)$$

$$= \frac{1}{4}$$

(11)

6) Probability of drawing the Queen of Diamonds:

$$\Pr(Queen\ of\ Diamonds) = \Pr(Y = 1, Z = 12)$$

$$= \Pr(Y = 1) \times \Pr(Z = 12)$$

$$= \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

(12)