Chapter 2: Equations and Inequalities

2.1 Exercises

$\boxed{1}$ $-3x+4=-1 \;\Rightarrow\; -3x=-5 \;\Rightarrow\; x=\frac{5}{3}$

$\boxed{2}$ $2x-2=-9 \;\Rightarrow\; 2x=-7 \;\Rightarrow\; x=-\frac{7}{2}$

$\boxed{3}$ $4x-3=-5x+6 \;\Rightarrow\; 4x+5x=6+3 \;\Rightarrow\; 9x=9 \;\Rightarrow\; x=1$

$\boxed{4}$ $5x-4=2(x-2) \;\Rightarrow\; 5x-4=2x-4 \;\Rightarrow\; 3x=0 \;\Rightarrow\; x=0$

$\boxed{5}$ $4(2y+5)=3(5y-2) \;\Rightarrow\; 8y+20=15y-6 \;\Rightarrow\; 26=7y \;\Rightarrow\; y=\frac{26}{7}$

$\boxed{6}$ $6(2y+3)-3(y-5)=0 \;\Rightarrow\; 12y+18-3y+15=0 \;\Rightarrow\; 9y=-33 \;\Rightarrow\; y=-\frac{11}{3}$

$\boxed{7}$ $\left[\frac{1}{5}x+2=3-\frac{2}{7}x\right]\cdot 35 \;\Rightarrow\; 7x+70=105-10x \;\Rightarrow\; 17x=35 \;\Rightarrow\; x=\frac{35}{17}$

$\boxed{8}$ $\frac{5}{3}x-1=4+\frac{2}{3}x \;\Rightarrow\; \frac{5}{3}x-\frac{2}{3}x=4+1 \;\Rightarrow\; x=5$

$\boxed{9}$ $\left[0.3(3+2x)+1.2x=3.2\right]\cdot 10 \;\Rightarrow\; 9+6x+12x=32 \;\Rightarrow\; 18x=23 \;\Rightarrow\; x=\frac{23}{18}$

$\boxed{10}$ $\left[1.5x-0.7=0.4(3-5x)\right]\cdot 10 \;\Rightarrow\; 15x-7=12-20x \;\Rightarrow\; 35x=19 \;\Rightarrow\; x=\frac{19}{35}$

$\boxed{11}$ $\left[\frac{3+5x}{5}=\frac{4-x}{7}\right]\cdot 35 \;\Rightarrow\; 21+35x=20-5x \;\Rightarrow\; 40x=-1 \;\Rightarrow\; x=-\frac{1}{40}$

$\boxed{12}$ $\left[\frac{2x-9}{4}=2+\frac{x}{12}\right]\cdot 12 \;\Rightarrow\; 6x-27=24+x \;\Rightarrow\; 5x=51 \;\Rightarrow\; x=\frac{51}{5}$

$\boxed{13}$ $\left[\frac{13+2x}{4x+1}=\frac{3}{4}\right]\cdot 4(4x+1) \;\Rightarrow\; 52+8x=12x+3 \;\Rightarrow\; 49=4x \;\Rightarrow\; x=\frac{49}{4}$

$\boxed{14}$ $\left[\frac{3}{7x-2}=\frac{9}{3x+1}\right]\cdot (7x-2)(3x+1) \;\Rightarrow\; 9x+3=63x-18 \;\Rightarrow\; 21=54x \;\Rightarrow\; x=\frac{7}{18}$

$\boxed{15}$ $\left[8-\frac{5}{x}=2+\frac{3}{x}\right]\cdot x \;\Rightarrow\; 8x-5=2x+3 \;\Rightarrow\; 6x=8 \;\Rightarrow\; x=\frac{4}{3}$

$\boxed{16}$ $\left[\frac{3}{y}+\frac{6}{y}-\frac{1}{y}=11\right]\cdot y \;\Rightarrow\; 3+6-1=11y \;\Rightarrow\; y=\frac{8}{11}$

$\boxed{17}$ $(3x-2)^2=(x-5)(9x+4) \;\Rightarrow\; 9x^2-12x+4=9x^2-41x-20 \;\Rightarrow$

$$29x=-24 \;\Rightarrow\; x=-\frac{24}{29}$$

$\boxed{18}$ $(x+5)^2+3=(x-2)^2 \;\Rightarrow\; x^2+10x+25+3=x^2-4x+4 \;\Rightarrow\; 14x=-24 \;\Rightarrow$

$$x=-\frac{12}{7}$$

$\boxed{19}$ $(5x-7)(2x+1)-10x(x-4)=0 \;\Rightarrow\; 10x^2-9x-7-10x^2+40x=0 \;\Rightarrow$

$$31x=7 \;\Rightarrow\; x=\frac{7}{31}$$

$\boxed{20}$ $(2x+9)(4x-3)=8x^2-12 \;\Rightarrow\; 8x^2+30x-27=8x^2-12 \;\Rightarrow\; 30x=15 \;\Rightarrow\; x=\frac{1}{2}$

$\boxed{21}$ $\left[\frac{3x+1}{6x-2}=\frac{2x+5}{4x-13}\right]\cdot (6x-2)(4x-13) \;\Rightarrow\; 12x^2-35x-13=12x^2+26x-10 \;\Rightarrow$

$$-3=61x \;\Rightarrow\; x=-\frac{3}{61}$$

$\boxed{22}$ $\left[\frac{5x+2}{10x-3}=\frac{x-8}{2x+3}\right]\cdot (10x-3)(2x+3) \;\Rightarrow\; 10x^2+19x+6=10x^2-83x+24 \;\Rightarrow$

$$102x=18 \;\Rightarrow\; x=\frac{3}{17}$$

$\boxed{23}$ $\left[\frac{2}{5}+\frac{4}{10x+5}=\frac{7}{2x+1}\right]\cdot 5(2x+1) \;\Rightarrow\; (4x+2)+4=35 \;\Rightarrow\; 4x=29 \;\Rightarrow\; x=\frac{29}{4}$

$\boxed{24}$ $\left[\frac{-5}{3x-9}+\frac{4}{x-3}=\frac{5}{6}\right]\cdot 6(x-3) \Rightarrow -5(2)+4(6)=5(x-3) \Rightarrow 29=5x \Rightarrow x=\frac{29}{5}$

$\boxed{25}$ $\left[\frac{3}{2x-4}-\frac{5}{3x-6}=\frac{3}{5}\right]\cdot 30(x-2) \Rightarrow 3(15)-5(10)=3(6)(x-2) \Rightarrow 18x=31 \Rightarrow$
$x=\frac{31}{18}$

$\boxed{26}$ $\left[\frac{9}{2x+6}-\frac{7}{5x+15}=\frac{2}{3}\right]\cdot 30(x+3) \Rightarrow 9(15)-7(6)=2(10)(x+3) \Rightarrow$
$33=20x \Rightarrow x=\frac{33}{20}$

$\boxed{27}$ $2-\frac{5}{3x-7}=2 \Rightarrow \frac{5}{3x-7}=0 \Rightarrow$ *no solution* since the numerator is never 0.

$\boxed{28}$ $\frac{6}{2x+11}+5=5 \Rightarrow \frac{6}{2x+11}=0 \Rightarrow$ *no solution* since the numerator is never 0.

$\boxed{29}$ $\frac{1}{2x-1}=\frac{4}{8x-4} \Rightarrow \frac{1}{2x-1}=\frac{4}{4(2x-1)} \Rightarrow \frac{1}{2x-1}=\frac{1}{2x-1}$. This is an identity,

and the solutions consist of every number in the domains of the given expressions.

Thus, the solutions are all real numbers except $\frac{1}{2}$, which we denote by $\mathbb{R}-\{\frac{1}{2}\}$.

$\boxed{30}$ $\frac{4}{5x+2}-\frac{12}{15x+6}=0 \Rightarrow \frac{4}{5x+2}=\frac{12}{3(5x+2)} \Rightarrow \frac{4}{5x+2}=\frac{4}{5x+2}$, an identity.
$\mathbb{R}-\{-\frac{2}{5}\}$

$\boxed{31}$ $\left[\frac{7}{y^2-4}-\frac{4}{y+2}=\frac{5}{y-2}\right]\cdot (y+2)(y-2) \Rightarrow 7-4(y-2)=5(y+2) \Rightarrow 5=9y \Rightarrow$
$y=\frac{5}{9}$

$\boxed{32}$ $\left[\frac{4}{2u-3}+\frac{10}{4u^2-9}=\frac{1}{2u+3}\right]\cdot (2u+3)(2u-3) \Rightarrow 4(2u+3)+10=2u-3 \Rightarrow$
$6u=-25 \Rightarrow u=-\frac{25}{6}$

$\boxed{33}$ $(x+3)^3-(3x-1)^2=x^3+4 \Rightarrow (x^3+9x^2+27x+27)-(9x^2-6x+1)=$
$x^3+4 \Rightarrow 33x=-22 \Rightarrow x=-\frac{2}{3}$

$\boxed{34}$ $(x-1)^3=(x+1)^3-6x^2 \Rightarrow x^3-3x^2+3x-1=(x^3+3x^2+3x+1)-6x^2 \Rightarrow$
$-1=1$. This is a contradiction and there is *no solution.*

$\boxed{35}$ $\left[\frac{9x}{3x-1}=2+\frac{3}{3x-1}\right]\cdot (3x-1) \Rightarrow 9x=2(3x-1)+3 \Rightarrow 9x=6x+1 \Rightarrow$
$3x=1 \Rightarrow x=\frac{1}{3}$, which is not in the domain of the given expressions. No solution

$\boxed{36}$ $\left[\frac{2x}{2x+3}+\frac{6}{4x+6}=5\right]\cdot 2(2x+3) \Rightarrow 2x(2)+6=5(2)(2x+3) \Rightarrow$
$4x+6=20x+30 \Rightarrow -24=16x \Rightarrow x=-\frac{3}{2}$, which is not in the domain of the given expressions. No solution

$\boxed{37}$ $\left[\frac{1}{x+4}+\frac{3}{x-4}=\frac{3x+8}{x^2-16}\right]\cdot (x+4)(x-4) \Rightarrow x-4+3(x+4)=3x+8 \Rightarrow x=0$

$\boxed{38}$ $\left[\frac{2}{2x+3}+\frac{4}{2x-3}=\frac{5x+6}{4x^2-9}\right]\cdot (2x+3)(2x-3) \Rightarrow$
$2(2x-3)+4(2x+3)=5x+6 \Rightarrow 7x=0 \Rightarrow x=0$

$\boxed{39}$ $\left[\frac{4}{x+2}+\frac{1}{x-2}=\frac{5x-6}{x^2-4}\right]\cdot (x+2)(x-2) \Rightarrow 4(x-2)+x+2=5x-6 \Rightarrow 0=0$,
indicating an identity. $\mathbb{R}-\{\pm 2\}$

40 $\left[\dfrac{2}{2x+5}+\dfrac{3}{2x-5}=\dfrac{10x+5}{4x^2-25}\right]\cdot(2x+5)(2x-5) \;\Rightarrow$

$2(2x-5)+3(2x+5)=10x+5 \;\Rightarrow\; 10x+5=10x+5$, indicating an identity.

$\mathbb{R}-\{\pm\frac{5}{2}\}$

41 $\left[\dfrac{2}{2x+1}-\dfrac{3}{2x-1}=\dfrac{-2x+7}{4x^2-1}\right]\cdot(2x+1)(2x-1) \;\Rightarrow$

$2(2x-1)-3(2x+1)=-2x+7 \;\Rightarrow\; -2x-5=-2x+7 \;\Rightarrow\; -5=7$,

a contradiction. No solution

42 $\left[\dfrac{3}{2x+5}+\dfrac{4}{2x-5}=\dfrac{14x+3}{4x^2-25}\right]\cdot(2x+5)(2x-5) \;\Rightarrow$

$3(2x-5)+4(2x+5)=14x+3 \;\Rightarrow\; 14x+5=14x+3 \;\Rightarrow\; 2=0$,

a contradiction. No solution

43 $\left[\dfrac{5}{2x+3}+\dfrac{4}{2x-3}=\dfrac{14x+3}{4x^2-9}\right]\cdot(2x+3)(2x-3) \;\Rightarrow$

$5(2x-3)+4(2x+3)=14x+3 \;\Rightarrow\; 18x-3=14x+3 \;\Rightarrow\; 4x=6 \;\Rightarrow\; x=\frac{3}{2}$,

which is not in the domain of the given expressions. No solution

44 $\left[\dfrac{-3}{x+4}+\dfrac{7}{x-4}=\dfrac{-5x+4}{x^2-16}\right]\cdot(x+4)(x-4) \;\Rightarrow\; -3(x-4)+7(x+4)=-5x+4 \;\Rightarrow$

$4x+40=-5x+4 \;\Rightarrow\; 9x=-36 \;\Rightarrow\; x=-4$,

which is not in the domain of the given expressions. No solution

45 $(4x-3)^2-16x^2=(16x^2-24x+9)-16x^2=9-24x$

46 $(3x-4)(2x+1)+5x=6x^2-5x-4+5x=6x^2-4$

47 $\dfrac{x^2-9}{x+3}=\dfrac{(x+3)(x-3)}{x+3}=x-3$

48 $\dfrac{x^3+8}{x+2}=\dfrac{(x+2)(x^2-2x+4)}{x+2}=x^2-2x+4$

49 $\dfrac{3x^2+8}{x}=\dfrac{3x^2}{x}+\dfrac{8}{x}=\dfrac{8}{x}+3x$

50 $\dfrac{49x^2-25}{7x-5}=\dfrac{(7x+5)(7x-5)}{7x-5}=7x+5$

51 Substituting -2 for x in $4x+1+2c=5c-3x+6$ yields $-7+2c=5c+12 \;\Rightarrow$

$3c=-19 \;\Rightarrow\; c=-\frac{19}{3}$.

52 Substituting 4 for x in $3x-2+6c=2c-5x+1$ yields $10+6c=2c-19 \;\Rightarrow$

$4c=-29 \;\Rightarrow\; c=-\frac{29}{4}$.

53 (a) $\dfrac{7x}{x-5}=\dfrac{42}{x-5} \;\Rightarrow\; 7x=42 \;\Rightarrow\; x=6$,

and the two equations are equivalent since they have the same solution.

(b) No, 5 is not a solution of the first equation.

54 (a) Yes (b) No, 7 is not a solution of the first equation.

55 Substituting $\frac{5}{3}$ for x yields $\frac{5}{3}a+b=0$, or, equivalently, $b=-\frac{5}{3}a$.

Choose any a and b such that $b=-\frac{5}{3}a$. For example, let $a=3$ and $b=-5$.

56 Substituting $\frac{5}{3}$ for x yields $\frac{25}{9}a+\frac{5}{3}b=0$, or, equivalently, $b=-\frac{5}{3}a$.

Choose any a and b such that $b=-\frac{5}{3}a$. For example, let $a=3$ and $b=-5$.

[57] Division by the variable expression $x-2$ is not allowed. ★ $x+1=x+2$

[58] Division by the variable expression $x+3$ is not allowed. ★ $x+2=x+1$

[59] $EK+L=D-TK \Rightarrow EK+TK=D-L \Rightarrow K(E+T)=D-L \Rightarrow K=\dfrac{D-L}{E+T}$

[60] $CD+C=PC+N \Rightarrow CD+C-PC=N \Rightarrow C(D+1-P)=N \Rightarrow C=\dfrac{N}{D+1-P}$

[61] $M=\dfrac{Q+1}{Q} \Rightarrow MQ=Q+1 \Rightarrow MQ-Q=1 \Rightarrow Q(M-1)=1 \Rightarrow Q=\dfrac{1}{M-1}$

[62] $\beta=\dfrac{\alpha}{1-\alpha} \Rightarrow \beta(1-\alpha)=\alpha \Rightarrow \beta-\beta\alpha=\alpha \Rightarrow \beta=\alpha+\beta\alpha \Rightarrow \beta=\alpha(1+\beta) \Rightarrow \alpha=\dfrac{\beta}{1+\beta}$

[63] $I=Prt \Rightarrow P=\dfrac{I}{rt}$

[64] $C=2\pi r \Rightarrow r=\dfrac{C}{2\pi}$

[65] $A=\frac{1}{2}bh \Rightarrow 2A=bh \Rightarrow h=\dfrac{2A}{b}$

[66] $V=\frac{1}{3}\pi r^2h \Rightarrow 3V=\pi r^2h \Rightarrow h=\dfrac{3V}{\pi r^2}$

[67] $F=g\dfrac{mM}{d^2} \Rightarrow Fd^2=gmM \Rightarrow m=\dfrac{Fd^2}{gM}$

[68] $R=\dfrac{V}{I} \Rightarrow RI=V \Rightarrow I=\dfrac{V}{R}$

[69] $P=2l+2w \Rightarrow P-2l=2w \Rightarrow w=\dfrac{P-2l}{2}$

[70] $A=P+Prt \Rightarrow A-P=Prt \Rightarrow r=\dfrac{A-P}{Pt}$

[71] $A=\frac{1}{2}(b_1+b_2)h \Rightarrow \dfrac{2A}{h}=b_1+b_2 \Rightarrow b_1=\dfrac{2A}{h}-b_2$, or $b_1=\dfrac{2A-hb_2}{h}$

[72] $s=\frac{1}{2}gt^2+v_0t \Rightarrow 2s=gt^2+2v_0t \Rightarrow 2s-gt^2=2v_0t \Rightarrow v_0=\dfrac{2s-gt^2}{2t}$

[73] $S=\dfrac{p}{q+p(1-q)} \Rightarrow Sq+Sp(1-q)=p \Rightarrow Sq+Sp-Spq=p \Rightarrow$

$Sq-Spq=p-Sp \Rightarrow Sq(1-p)=p(1-S) \Rightarrow q=\dfrac{p(1-S)}{S(1-p)}$

[74] $S=2(lw+hw+hl) \Rightarrow S=2lw+2hw+2hl \Rightarrow S-2lw=2h(w+l) \Rightarrow h=\dfrac{S-2lw}{2(w+l)}$

[75] $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$ {multiply by fpq} $\Rightarrow pq=fq+fp \Rightarrow pq-fq=fp \Rightarrow$

$q(p-f)=fp \Rightarrow q=\dfrac{fp}{p-f}$

[76] $\dfrac{1}{R}=\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}$ {multiply by $RR_1R_2R_3$} $\Rightarrow$

$R_1R_2R_3=RR_2R_3+RR_1R_3+RR_1R_2 \Rightarrow$

$R_1R_2R_3-RR_2R_3-RR_1R_2=RR_1R_3 \Rightarrow R_2(R_1R_3-RR_3-RR_1)=RR_1R_3 \Rightarrow$

$$R_2=\frac{RR_1R_3}{R_1R_3-RR_3-RR_1}$$

[77] The y-value decreases 1.2 units for each 1 unit increase in the x-value. The data is best described by equation (1), $y = -1.2x + 2$.

[78] The y-values are increasing rapidly and can best be described by equation (4), $y = x^3 - x^2 + x - 10$.

2.2 Exercises

[1] Let x denote the score on the next test.

$\dfrac{75 + 82 + 71 + 84 + x}{5} = 80 \Rightarrow 312 + x = 5(80) \Rightarrow x = 400 - 312 = 88$

[2] The pre-final average is $\dfrac{72 + 80 + 65 + 78 + 60}{5} = 71$. If x denotes the score on the final exam, then $\frac{2}{3}(71) + \frac{1}{3}(x) = 76 \Rightarrow \frac{1}{3}x = \frac{86}{3} \Rightarrow x = 86$.

[3] Let x denote the gross pay. Gross pay $-$ deductions $=$ Net (take-home) pay $\Rightarrow$

$x - 0.40x = 492 \Rightarrow 0.60x = 492 \Rightarrow x = 820$.

[4] Let x denote the amount of the bill before the tax and tip are added.

Bill + Tax + Tip $= 70 \Rightarrow x + 0.06x + 0.15(x + 0.06x) = 70 \Rightarrow$

$1.06x + 0.15(1.06x) = 70 \Rightarrow 1.15(1.06x) = 70 \Rightarrow 1.219x = 70 \Rightarrow x \approx 57.42$.

[5] (a) $\text{IQ} = \dfrac{\text{mental age (MA)}}{\text{chronological age (CA)}} \times 100 = \dfrac{15}{12} \times 100 = 125$.

(b) CA $= 15$ and IQ $= 140 \Rightarrow 140 = \dfrac{\text{MA}}{15} \times 100 \Rightarrow \text{MA} = 140 \times \dfrac{15}{100} = 21$.

[6] Let S denote the surface area of the earth.

Then, $0.708S = 361 \times 10^6 \Rightarrow S = \dfrac{361 \times 10^6}{0.708} \Rightarrow S \approx 510 \times 10^6 \text{ km}^2$.

[7] Let x denote the number of months needed to recover the cost of the insulation. The savings in one month is 10% of \$60 = \$6. $6x = 1080 \Rightarrow x = 180$ months (or 15 yr).

[8] Let x denote the number of hours the workman made \$15 per hour.

$40(\$10) + x(\$15) = \$595 \Rightarrow x = 13$ hr.

[9] Let x denote the amount invested in the 8% account.

$x(0.08) + (100{,}000 - x)(0.064) = 7500 \Rightarrow 0.016x = 1100 \Rightarrow x = 68{,}750$.

Since only \$50,000 can be insured in the 8% account,

we *cannot* fully insure the money and earn annual interest of \$7,500.

[10] Let x denote the amount (in millions) invested in bonds.

$x(0.12) + (50 - x)(0.10) = 5.2 \Rightarrow 0.02x = 0.2 \Rightarrow x = 10$. The arena should be financed by selling \$10 million in bonds and borrowing \$40 million.

[11] Let x denote the number of children.

$\text{Receipts}_{\text{children}} + \text{Receipts}_{\text{adults}} = \text{Receipts}_{\text{total}} \Rightarrow x(2) + (600 - x)(5) = 2400 \Rightarrow$

$-3x = -600 \Rightarrow x = 200$ children.

$\boxed{12}$ Let x denote the engineer's hours. $\text{Bill}_{\text{engineer}} + \text{Bill}_{\text{assistant}} = \text{Bill}_{\text{total}} \Rightarrow$

$60(x) + 20(x-5) = 580 \Rightarrow 80x = 680 \Rightarrow x = 8.5.$

The engineer spent 8.5 hr on the job and the assistant spent $8.5 - 5 = 3.5$ hr.

$\boxed{13}$ Let x denote the number of ounces of glucose solution.

$x(0.30) + (7-x)(0) = 7(0.20) \Rightarrow 0.3x = 1.4 \Rightarrow x = \frac{14}{3}.$

Use $\frac{14}{3}$ oz of the 30% glucose solution and $7 - \frac{14}{3} = \frac{7}{3}$ oz of water.

$\boxed{14}$ Let x denote the number of mL of 1% solution.

$x(1) + (15-x)(10) = 15(2)$ {all in %} $\Rightarrow -9x = -120 \Rightarrow x = \frac{40}{3}.$

Use $\frac{40}{3}$ mL of the 1% solution and $15 - \frac{40}{3} = \frac{5}{3}$ mL of the 10% solution.

$\boxed{15}$ Let x denote the number of grams of British sterling silver.

$(0.075)x + 1(200-x) = (0.10)(200) \Rightarrow 180 = 0.925x \Rightarrow x = 194.6.$

Use 194.6 g of British sterling silver and 5.4 g of copper.

$\boxed{16}$ Let x denote the number of mL of the elixir. $x(5) + (100-x)(0) = 100(2) \Rightarrow$

$x = 40$. Use 40 mL of the elixir and $100 - 40 = 60$ mL of the cherry-flavored syrup.

$\boxed{17}$ (a) Let t denote the desired number of seconds. $1.5t + 2t = 224 \Rightarrow t = 64$ sec

(b) $64(1.5) = 96$ m and $64(2) = 128$ m, respectively

$\boxed{18}$ Let t denote the number of seconds that the second runner has been running.

The distance of the first runner at time t is $6t + 6(\frac{5}{60})$. The distance of the second runner is $8t$. Equating yields $6t + \frac{1}{2} = 8t \Rightarrow t = \frac{1}{4}$ hr, or 15 min.

$\boxed{19}$ Let r denote the rate of the snowplow. At 8:30 A.M., the car has traveled 15 miles and the snowplow has been traveling for $2\frac{1}{2}$ hours. Thus, $\frac{5}{2}r = 15 \Rightarrow r = 6$ mi/hr.

$\boxed{20}$ Let t denote the time in hours after 1:15 P.M.

The first child's distance is $1 + 4t$ miles and the second child's distance is $6t$.

$(1 + 4t) + 6t = 2 \Rightarrow t = \frac{1}{10}$ hr, or 6 min. After 1:21 P.M.

$\boxed{21}$ (a) Let r denote the rate of the river's current.

$\text{Distance}_{\text{upstream}} = \text{Distance}_{\text{downstream}} \Rightarrow (5-r)\frac{15}{60} = (5+r)\frac{12}{60} \Rightarrow$

$5(5-r) = 4(5+r) \Rightarrow 25 - 5r = 20 + 4r \Rightarrow 5 = 9r \Rightarrow r = \frac{5}{9}$ mi/hr.

(b) The distance upstream is $(5 - \frac{5}{9})\frac{1}{4} = \frac{10}{9}$. The total distance is $2 \cdot \frac{10}{9} = \frac{20}{9}$, or $2\frac{2}{9}$ mi.

$\boxed{22}$ Let x denote the number of gallons used in the city.

$\text{Miles}_{\text{city}} + \text{Miles}_{\text{highway}} = \text{Miles}_{\text{total}} \Rightarrow x(25) + (51-x)(40) = 1800 \Rightarrow$

$240 = 15x \Rightarrow x = 16$. The number of miles driven in the city is $16 \cdot 25 = 400$ mi.

$\boxed{23}$ Let x denote the distance to the target.

$\text{Time}_{\text{to target}} + \text{Time}_{\text{from target}} = \text{Time}_{\text{total}} \Rightarrow \frac{x}{3300} + \frac{x}{1100} = 1.5 \Rightarrow$

$x + 3x = 1.5(3300) \Rightarrow 4x = 4950 \Rightarrow x = 1237.5$ ft.

[24] Let x denote the miles in one direction. A 6-minute-mile pace is equivalent to a rate of $\frac{1}{6}$ mile/min. $\text{Minutes}_{\text{north}} + \text{Minutes}_{\text{south}} = \text{Minutes}_{\text{total}}$ $\Rightarrow$

$\dfrac{x}{1/6} + \dfrac{x}{1/7} = 45 \Rightarrow 6x + 7x = 45 \Rightarrow x = \frac{45}{13}$.

The total distance is $2 \cdot \frac{45}{13} = \frac{90}{13}$, or $6\frac{12}{13}$ mi.

[25] Let l denote the length of the side parallel to the river bank. $P = 2w + l$

(a) $P = 2w + 2w = 4w$; $4w = 180 \Rightarrow w = 45$ ft and $A = (45)(90) = 4050$ ft^2.

(b) $P = 2w + \frac{1}{2}w = \frac{5}{2}w$; $\frac{5}{2}w = 180 \Rightarrow w = 72$ ft and $A = (72)(36) = 2592$ ft^2.

(c) $P = 2w + w = 3w$; $3w = 180 \Rightarrow w = 60$ ft and $A = (60)(60) = 3600$ ft^2.

[26] The first story has cross-sectional area $8 \times 30 = 240$. The second story has cross-sectional area $(30 \times 3) + 2(\frac{1}{2})(15)(h - 3) = 90 + 15h - 45 = 15h + 45$.

Equating yields $15h + 45 = 240 \Rightarrow 15h = 195 \Rightarrow h = 13$ ft.

[27] $A = \frac{1}{2}\pi r^2 + lw \Rightarrow 24 = \frac{1}{2}\pi(\frac{3}{2})^2 + (h - \frac{3}{2})3 \Rightarrow h - \frac{3}{2} = 8 - \frac{3\pi}{8} \Rightarrow$

$h = \frac{19}{2} - \frac{3\pi}{8} \approx 8.32$ ft.

[28] $A = \frac{1}{2}(b_1 + b_2)h \Rightarrow 5 = \frac{1}{2}(3 + b_2)(1) \Rightarrow b_2 = 7$ ft.

[29] Let h_1 denote the height of the cylinder.

$V = \frac{2}{3}\pi r^3 + \pi r^2 h_1 = 11{,}250\pi$ and $r = 15 \Rightarrow$

$2250\pi + 225\pi h_1 = 11{,}250\pi \Rightarrow 225\pi h_1 = 9000\pi \Rightarrow h_1 = \dfrac{9000\pi}{225\pi} = 40$.

The total height is 40 ft + 15 ft = 55 ft.

[30] $V = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$, $r = 1$, and $V = 8 \Rightarrow 8 = \frac{\pi}{3}h + \frac{2\pi}{3} \Rightarrow$

$h = \frac{3}{\pi}(8 - \frac{2\pi}{3}) = \frac{24}{\pi} - 2 \approx 5.64$ in.

[31] Let x denote the desired time.

Using the rates (in minutes), $\frac{1}{90} + \frac{1}{60} = \frac{1}{x} \Rightarrow 2x + 3x = 180 \Rightarrow x = 36$ min.

[32] Let x denote the desired time. Using the hourly rates, $\frac{1}{8} + \frac{1}{5} = \frac{1}{x} \Rightarrow x = \frac{40}{13}$ hr.

[33] Let x denote the desired time.

Using the rates (in minutes), $\frac{1}{45} + \frac{1}{x} = \frac{1}{20} \Rightarrow 4x + 180 = 9x \Rightarrow x = 36$ min.

[34] The larger pump will empty $\frac{4}{5}$ of the tank in 4 hours. The smaller pump can empty the remaining $\frac{1}{5}$ tank in $\frac{1}{5}$ of 8 hours, or 1 hr 36 min. Start the smaller pump at 3:24 P.M.

[35] First, a simple example of calculating a GPA. Suppose a student gets a 3-credit A (worth 4 honor points) and a 4-credit C (worth 2 honor points). Then,

$$\text{GPA} = \frac{\text{total weighted honor points}}{\text{total credit hours}} = \frac{3(4.0) + 4(2.0)}{3 + 4} = \frac{12 + 8}{7} = \frac{20}{7} \approx 2.86.$$

(continued)

Let x denote the number of additional credit hours.

$\text{GPA} = 3.2 \Rightarrow \dfrac{48(2.75) + x(4.0)}{48 + x} = 3.2 \Rightarrow 132 + 4x = 3.2(48 + x) \Rightarrow$

$132 + 4x = 153.6 + 3.2x \Rightarrow 21.6 = 0.8x \Rightarrow x = \frac{21.6}{0.8} = 27.$

$\boxed{36}$ Let x denote the numerical amount to be *added* to V and R.

$I = \dfrac{V}{R} = \dfrac{110}{50}$. Thus, $2I = \dfrac{110 + x}{50 + x} \Rightarrow 220(50 + x) = 50(110 + x) \Rightarrow$

$170x = -5500 \Rightarrow x = -\frac{550}{17}$. Decrease both V and R by $\frac{550}{17} \approx 32.35$.

$\boxed{37}$ (a) $h = 5280$ and $T_0 = 70 \Rightarrow T = 70 - \left(\dfrac{5.5}{1000}\right)5280 = 40.96°\text{F}$.

(b) $T = 32 \Rightarrow 32 = 70 - \left(\dfrac{5.5}{1000}\right)h \Rightarrow h = (70 - 32)\left(\dfrac{1000}{5.5}\right) \approx 6909$ ft.

$\boxed{38}$ (a) $T = 70$ and $D = 55 \Rightarrow h = 227(70 - 55) = 3405$ ft.

(b) $h = 3500$ and $D = 65 \Rightarrow 3500 = 227(T - 65) \Rightarrow T = \dfrac{3500}{227} + 65 \approx 80.4°\text{F}$

$\boxed{39}$ $B = 55$ and $h = 10{,}000 - 4000 = 6000 \Rightarrow T = 55 - \left(\dfrac{3}{1000}\right)(6000) = 37°\text{F}$.

$\boxed{40}$ (a) $x = 30 \Rightarrow h = 65 + 3.14(30) = 159.2$ cm.

(b) $x = 34 \Rightarrow h = 73.6 + 3(34) = 175.6$ cm. The height of the skeleton has decreased by $175.6 - 174 = 1.6$ cm due to aging after age 30. $\dfrac{1.6}{0.06} \approx 27$ years. The male was approximately $30 + 27 = 57$ years old at death.

2.3 Exercises

$\boxed{1}$ $6x^2 + x - 12 = 0 \Rightarrow (2x + 3)(3x - 4) = 0 \Rightarrow x = -\frac{3}{2}, \frac{4}{3}$

$\boxed{2}$ $4x^2 + x - 14 = 0 \Rightarrow (x + 2)(4x - 7) = 0 \Rightarrow x = -2, \frac{7}{4}$

$\boxed{3}$ $15x^2 - 12 = -8x \Rightarrow 15x^2 + 8x - 12 = 0 \Rightarrow (5x + 6)(3x - 2) = 0 \Rightarrow x = -\frac{6}{5}, \frac{2}{3}$

$\boxed{4}$ $15x^2 - 14 = 29x \Rightarrow 15x^2 - 29x - 14 = 0 \Rightarrow (5x + 2)(3x - 7) = 0 \Rightarrow x = -\frac{2}{5}, \frac{7}{3}$

$\boxed{5}$ $2x(4x + 15) = 27 \Rightarrow 8x^2 + 30x - 27 = 0 \Rightarrow (2x + 9)(4x - 3) = 0 \Rightarrow x = -\frac{9}{2}, \frac{3}{4}$

$\boxed{6}$ $x(3x + 10) = 77 \Rightarrow 3x^2 + 10x - 77 = 0 \Rightarrow (x + 7)(3x - 11) = 0 \Rightarrow x = -7, \frac{11}{3}$

$\boxed{7}$ $75x^2 + 35x - 10 = 0 \Rightarrow 15x^2 + 7x - 2 = 0 \Rightarrow (3x + 2)(5x - 1) = 0 \Rightarrow x = -\frac{2}{3}, \frac{1}{5}$

$\boxed{8}$ $48x^2 + 12x - 90 = 0 \Rightarrow 8x^2 + 2x - 15 = 0 \Rightarrow (2x + 3)(4x - 5) = 0 \Rightarrow x = -\frac{3}{2}, \frac{5}{4}$

$\boxed{9}$ $12x^2 + 60x + 75 = 0 \Rightarrow 4x^2 + 20x + 25 = 0 \Rightarrow (2x + 5)^2 = 0 \Rightarrow x = -\frac{5}{2}$

$\boxed{10}$ $4x^2 - 72x + 324 = 0 \Rightarrow x^2 - 18x + 81 = 0 \Rightarrow (x - 9)^2 = 0 \Rightarrow x = 9$

$\boxed{11}$ $\left[\dfrac{2x}{x + 3} + \dfrac{5}{x} - 4 = \dfrac{18}{x^2 + 3x}\right] \cdot x(x + 3) \Rightarrow 2x(x) + 5(x + 3) - 4(x^2 + 3x) = 18 \Rightarrow$

$0 = 2x^2 + 7x + 3 \Rightarrow (2x + 1)(x + 3) = 0 \Rightarrow$

$x = -\frac{1}{2}$ { -3 is not in the domain of the given expressions }

$\boxed{12}$ $\left[\dfrac{5x}{x-2}+\dfrac{3}{x}+2=\dfrac{-6}{x^2-2x}\right]\cdot x(x-2) \;\Rightarrow\; 5x(x)+3(x-2)+2(x^2-2x)=-6 \;\Rightarrow$

$7x^2-x=0 \;\Rightarrow\; x(7x-1)=0 \;\Rightarrow$

$x=\frac{1}{7}$ { 0 is not in the domain of the given expressions }

$\boxed{13}$ $\left[\dfrac{5x}{x-3}+\dfrac{4}{x+3}=\dfrac{90}{x^2-9}\right]\cdot(x+3)(x-3) \;\Rightarrow\; 5x(x+3)+4(x-3)=90 \;\Rightarrow$

$5x^2+19x-102=0 \;\Rightarrow\; (5x+34)(x-3)=0 \;\Rightarrow$

$x=-\frac{34}{5}$ { 3 is not in the domain of the given expressions }

$\boxed{14}$ $\left[\dfrac{3x}{x-2}+\dfrac{1}{x+2}=\dfrac{-4}{x^2-4}\right]\cdot(x+2)(x-2) \;\Rightarrow\; 3x(x+2)+1(x-2)=-4 \;\Rightarrow$

$3x^2+7x+2=0 \;\Rightarrow\; (3x+1)(x+2)=0 \;\Rightarrow$

$x=-\frac{1}{3}$ { -2 is not in the domain of the given expressions }

$\boxed{15}$ (a) $x^2=16$ has solutions $x=\pm 4$.

The equations are not equivalent since -4 is not a solution of $x=4$.

(b) $x=\sqrt{9}=3$.

The equations are equivalent since they have exactly the same solutions.

$\boxed{16}$ (a) No, -5 is not a solution of $x=5$. (b) Yes

$\boxed{17}$ Using the special quadratic equation in this section,

$x^2=169 \;\Rightarrow\; x=\pm\sqrt{169}=\pm 13$.

$\boxed{18}$ $x^2=361 \;\Rightarrow\; x=\pm\sqrt{361}=\pm 19$

$\boxed{19}$ $25x^2=9 \;\Rightarrow\; x^2=\frac{9}{25} \;\Rightarrow\; x=\pm\sqrt{\frac{9}{25}}=\pm\frac{3}{5}$

$\boxed{20}$ $16x^2=49 \;\Rightarrow\; x^2=\frac{49}{16} \;\Rightarrow\; x=\pm\sqrt{\frac{49}{16}}=\pm\frac{7}{4}$

$\boxed{21}$ $(x-3)^2=17 \;\Rightarrow\; x-3=\pm\sqrt{17} \;\Rightarrow\; x=3\pm\sqrt{17}$

$\boxed{22}$ $(x+4)^2=31 \;\Rightarrow\; x+4=\pm\sqrt{31} \;\Rightarrow\; x=-4\pm\sqrt{31}$

$\boxed{23}$ $4(x+2)^2=11 \;\Rightarrow\; (x+2)^2=\frac{11}{4} \;\Rightarrow\; x+2=\pm\sqrt{\frac{11}{4}} \;\Rightarrow\; x=-2\pm\frac{1}{2}\sqrt{11}$

$\boxed{24}$ $9(x-1)^2=7 \;\Rightarrow\; (x-1)^2=\frac{7}{9} \;\Rightarrow\; x-1=\pm\sqrt{\frac{7}{9}} \;\Rightarrow\; x=1\pm\frac{1}{3}\sqrt{7}$

$\boxed{25}$ (a) In general, $d=(\frac{1}{2}b)^2$. In this case, $d=\left[\frac{1}{2}(9)\right]^2=\frac{81}{4}$.

(b) As in part (a), $d=(\frac{1}{2}b)^2=\left[\frac{1}{2}(-8)\right]^2=16$. *Note:* It is appropriate to use 8 or -8.

(c) In general, $d=2(\pm\sqrt{c})$ for $c>0$.

In this case, $c=36 \;\Rightarrow\; \sqrt{c}=6$, and $d=2(\pm 6)=\pm 12$.

(d) $c=\frac{49}{4} \;\Rightarrow\; \sqrt{c}=\frac{7}{2}$, and $d=2(\pm\frac{7}{2})=\pm 7$.

$\boxed{26}$ (a) $d=(\frac{1}{2}b)^2=\left[\frac{1}{2}(13)\right]^2=\frac{169}{4}$.

(b) $d=(\frac{1}{2}b)^2=\left[\frac{1}{2}(-6)\right]^2=9$. *Note:* It is appropriate to use 6 or -6.

(c) $c=25 \;\Rightarrow\; \sqrt{c}=5$, and $d=2(\pm 5)=\pm 10$.

(d) $c=\frac{81}{4} \;\Rightarrow\; \sqrt{c}=\frac{9}{2}$, and $d=2(\pm\frac{9}{2})=\pm 9$.

[27] $x^2+6x+7=0 \Rightarrow x^2+6x+\underline{\;9\;}=-7+\underline{\;9\;} \Rightarrow (x+3)^2=2 \Rightarrow$
$x+3=\pm\sqrt{2} \Rightarrow x=-3\pm\sqrt{2}$

[28] $x^2-8x+11=0 \Rightarrow x^2-8x+\underline{\;16\;}=-11+\underline{\;16\;} \Rightarrow (x-4)^2=5 \Rightarrow$
$x-4=\pm\sqrt{5} \Rightarrow x=4\pm\sqrt{5}$

[29] $4x^2-12x-11=0 \Rightarrow x^2-3x+\frac{9}{4}=\frac{11}{4}+\frac{9}{4} \Rightarrow (x-\frac{3}{2})^2=5 \Rightarrow$
$x-\frac{3}{2}=\pm\sqrt{5} \Rightarrow x=\frac{3}{2}\pm\sqrt{5}$

[30] $4x^2+20x+13=0 \Rightarrow x^2+5x+\frac{25}{4}=-\frac{13}{4}+\frac{25}{4} \Rightarrow (x+\frac{5}{2})^2=3 \Rightarrow$
$x+\frac{5}{2}=\pm\sqrt{3} \Rightarrow x=-\frac{5}{2}\pm\sqrt{3}$

[31] $6x^2-x=2 \Rightarrow 6x^2-x-2=0 \Rightarrow x=\dfrac{1\pm\sqrt{1+48}}{12}=\dfrac{1\pm7}{12}=-\frac{1}{2}, \frac{2}{3}$

[32] $5x^2+13x=6 \Rightarrow 5x^2+13x-6=0 \Rightarrow$
$x=\dfrac{-13\pm\sqrt{169+120}}{10}=\dfrac{-13\pm17}{10}=-3, \frac{2}{5}$

[33] $x^2+4x+2=0 \Rightarrow x=\dfrac{-4\pm\sqrt{16-8}}{2}=\dfrac{-4\pm2\sqrt{2}}{2}=-2\pm\sqrt{2}$

[34] $x^2-6x-3=0 \Rightarrow x=\dfrac{6\pm\sqrt{36+12}}{2}=\dfrac{6\pm4\sqrt{3}}{2}=3\pm2\sqrt{3}$

[35] $2x^2-3x-4=0 \Rightarrow x=\dfrac{3\pm\sqrt{9+32}}{4}=\frac{3}{4}\pm\frac{1}{4}\sqrt{41}$

[36] $3x^2+5x+1=0 \Rightarrow x=\dfrac{-5\pm\sqrt{25-12}}{6}=-\frac{5}{6}\pm\frac{1}{6}\sqrt{13}$

[37] $\frac{3}{2}z^2-4z-1=0 \Rightarrow z=\dfrac{4\pm\sqrt{16+6}}{3}=\frac{4}{3}\pm\frac{1}{3}\sqrt{22}$

[38] $\frac{5}{3}s^2+3s+1=0 \Rightarrow 5s^2+9s+3=0 \Rightarrow s=\dfrac{-9\pm\sqrt{81-60}}{10}=-\frac{9}{10}\pm\frac{1}{10}\sqrt{21}$

[39] $\left[\dfrac{5}{w^2}-\dfrac{10}{w}+2=0\right]\cdot w^2 \Rightarrow 5-10w+2w^2=0 \Rightarrow$
$w=\dfrac{10\pm\sqrt{100-40}}{4}=\dfrac{10\pm2\sqrt{15}}{4}=\frac{5}{2}\pm\frac{1}{2}\sqrt{15}$

[40] $\left[\dfrac{x+1}{3x+2}=\dfrac{x-2}{2x-3}\right]\cdot(3x+2)(2x-3) \Rightarrow (x+1)(2x-3)=(x-2)(3x+2) \Rightarrow$
$2x^2-x-3=3x^2-4x-4 \Rightarrow 0=x^2-3x-1 \Rightarrow x=\dfrac{3\pm\sqrt{9+4}}{2}=\frac{3}{2}\pm\frac{1}{2}\sqrt{13}$

[41] $4x^2+81=36x \Rightarrow 4x^2-36x+81=0 \Rightarrow x=\dfrac{36\pm\sqrt{1296-1296}}{8}=\dfrac{36}{8}=\dfrac{9}{2}$

[42] $24x+9=-16x^2 \Rightarrow 16x^2+24x+9=0 \Rightarrow x=\dfrac{-24\pm\sqrt{576-576}}{32}=-\dfrac{24}{32}=-\dfrac{3}{4}$

[43] $\dfrac{5x}{x^2+9}=-1 \Rightarrow 5x=-x^2-9 \Rightarrow x^2+5x+9=0 \Rightarrow x=\dfrac{-5\pm\sqrt{25-36}}{2}.$
Since the discriminant is negative, there are no real solutions.

[44] $\frac{1}{7}x^2+1=\frac{4}{7}x \Rightarrow x^2+7=4x \Rightarrow x^2-4x+7=0 \Rightarrow x=\dfrac{4\pm\sqrt{16-28}}{2} \Rightarrow$
no real solutions

$\boxed{45}$ The *expression* is $x^2 + x - 30$. The associated quadratic *equation* is $x^2 + x - 30 = 0$.

Using the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve for x with $a = 1$, $b = 1$,

and $c = -30$ gives us: $\quad x = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(-30)}}{2(1)} = \dfrac{-1 \pm \sqrt{1 + 120}}{2}$

$$= \frac{-1 \pm \sqrt{121}}{2} = \frac{-1 \pm 11}{2} = \frac{10}{2}, \frac{-12}{2} = 5, -6$$

Write the equation as a product of linear factors: $[x - (5)]\,[x - (-6)] = 0$

Now simplify: $(x - 5)(x + 6) = 0$

So the final factored form of $x^2 + x - 30$ is $(x - 5)(x + 6)$.

$\boxed{46}$ $x^2 + 7x = 0$ $\{a = 1,\ b = 7,\ c = 0\}$, so $x = \dfrac{-7 \pm \sqrt{49 - 0}}{2} = \dfrac{-7 \pm 7}{2} = 0, -7$.

Thus, $x^2 + 7x = [x - (0)]\,[x - (-7)] = x(x + 7)$.

$\boxed{47}$ $12x^2 - 16x - 3 = 0$ $\{a = 12,\ b = -16,\ c = -3\}$,

so $x = \dfrac{16 \pm \sqrt{256 + 144}}{24} = \dfrac{16 \pm 20}{24} = \dfrac{3}{2}, -\dfrac{1}{6}$.

Write the equation as a product of linear factors: $\left[x - \frac{3}{2}\right]\left[x - \left(-\frac{1}{6}\right)\right] = 0$

Now multiply the first factor by 2 and the second factor by 6. $(2x - 3)(6x + 1) = 0$

So the final factored form of $12x^2 - 16x - 3$ is $(2x - 3)(6x + 1)$.

$\boxed{48}$ $15x^2 + 34x - 16 = 0$ $\{a = 15,\ b = 34,\ c = -16\}$,

so $x = \dfrac{-34 \pm \sqrt{1156 + 960}}{30} = \dfrac{-34 \pm 46}{30} = \dfrac{2}{5}, -\dfrac{8}{3}$.

Thus, $15x^2 + 34x - 16 = 5\left[x - \frac{2}{5}\right] \cdot 3\left[x - \left(-\frac{8}{3}\right)\right] = (5x - 2)(3x + 8)$.

$\boxed{49}$ (a) $4x^2 - 4xy + 1 - y^2 = 0 \ \Rightarrow\ (4)x^2 + (-4y)x + (1 - y^2) = 0 \ \Rightarrow$

$$x = \frac{4y \pm \sqrt{16y^2 - 16(1 - y^2)}}{8} = \frac{4y \pm 4\sqrt{2y^2 - 1}}{8} = \frac{y \pm \sqrt{2y^2 - 1}}{2}$$

(b) $4x^2 - 4xy + 1 - y^2 = 0 \ \Rightarrow\ (-1)y^2 + (-4x)y + (4x^2 + 1) = 0 \ \Rightarrow$

$$y = \frac{4x \pm \sqrt{16x^2 + 4(4x^2 + 1)}}{-2} = \frac{4x \pm 2\sqrt{8x^2 + 1}}{-2} = -2x \pm \sqrt{8x^2 + 1}$$

$\boxed{50}$ (a) $2x^2 - xy = 3y^2 + 1 \ \Rightarrow\ (2)x^2 + (-y)x + (-3y^2 - 1) = 0 \ \Rightarrow$

$$x = \frac{y \pm \sqrt{y^2 - 8(-3y^2 - 1)}}{4} = \frac{y \pm \sqrt{25y^2 + 8}}{4}$$

(b) $2x^2 - xy = 3y^2 + 1 \ \Rightarrow\ (-3)y^2 + (-x)y + (2x^2 - 1) = 0 \ \Rightarrow$

$$y = \frac{x \pm \sqrt{x^2 + 12(2x^2 - 1)}}{-6} = \frac{x \pm \sqrt{25x^2 - 12}}{-6}$$

$\boxed{51}$ $K = \frac{1}{2}mv^2 \;\Rightarrow\; v^2 = \frac{2K}{m} \;\Rightarrow\; v = \pm\sqrt{\frac{2K}{m}} \;\Rightarrow\; v = \sqrt{\frac{2K}{m}}$ since $v > 0$.

$\boxed{52}$ $F = g\frac{mM}{d^2} \;\Rightarrow\; d^2 = \frac{gmM}{F} \;\Rightarrow\; d = \pm\sqrt{\frac{gmM}{F}} \;\Rightarrow\; d = \sqrt{\frac{gmM}{F}}$ since $d > 0$.

$\boxed{53}$ $A = 2\pi r(r+h) \;\Rightarrow\; A = 2\pi r^2 + 2\pi rh \;\Rightarrow\; (2\pi)r^2 + (2\pi h)r - A = 0 \;\Rightarrow$

$r = \frac{-2\pi h \pm \sqrt{4\pi^2h^2 + 8\pi A}}{4\pi} = \frac{-\pi h \pm \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$.

Since $r > 0$, we must use the plus sign, and $r = \frac{-\pi h + \sqrt{\pi^2h^2 + 2\pi A}}{2\pi}$.

$\boxed{54}$ $s = \frac{1}{2}gt^2 + v_0t \;\Rightarrow\; (\frac{1}{2}g)t^2 + (v_0)t - s = 0 \;\Rightarrow\; t = \frac{-v_0 \pm \sqrt{v_0^2 + 2gs}}{g}$.

Since $t > 0$, we must use the plus sign, and $t = \frac{-v_0 + \sqrt{v_0^2 + 2gs}}{g}$.

$\boxed{55}$ $V = V_{\max}\left[1 - \left(\frac{r}{r_0}\right)^2\right] \;\Rightarrow\; \frac{V}{V_{\max}} = 1 - \left(\frac{r}{r_0}\right)^2 \;\Rightarrow\; \left(\frac{r}{r_0}\right)^2 = 1 - (V/V_{\max}) \;\Rightarrow$

$r^2 = r_0^2[1 - (V/V_{\max})]\ \{r > 0\} \;\Rightarrow\; r = r_0\sqrt{1 - (V/V_{\max})}$

$\boxed{56}$ $D = 0.74 \;\Rightarrow\; 0.74 = 1.225 - (1.12 \times 10^{-4})h + (3.24 \times 10^{-9})h^2 \;\Rightarrow$

$(3.24 \times 10^{-9})h^2 - (1.12 \times 10^{-4})h + 0.485 = 0 \;\Rightarrow\; h \approx 5076$ and 29,492.

Since the formula is valid only for $0 \le h \le 10{,}000$, $h \approx 5076$ m.

$\boxed{57}$ Using $V = \pi r^2h$ with $V = 3000$ and $h = 20$ gives us:

$3000 = \pi r^2(20) \;\Rightarrow\; r^2 = 150/\pi \;\Rightarrow\; r = \sqrt{150/\pi} \approx 6.9$ cm

$\boxed{58}$ Let x denote the original width, $2x$ the length. $V = lwh \;\Rightarrow$

$60 = (2x-6)(x-6)(3) \;\Rightarrow\; 10 = (x-3)(x-6) \;\Rightarrow\; x^2 - 9x + 8 = 0 \;\Rightarrow$

$(x-1)(x-8) = 0 \;\Rightarrow\; x = 8$ for $x > 6$. The sheet should be 8 in. by 16 in.

$\boxed{59}$ (a) $s = 48 \;\Rightarrow\; -16t^2 + 64t = 48 \;\Rightarrow\; t^2 - 4t + 3 = 0 \;\Rightarrow\; (t-1)(t-3) = 0 \;\Rightarrow$

$t = 1, 3$. After 1 sec and after 3 sec

(b) It will hit the ground when $s = 0$.

$s = 0 \;\Rightarrow\; -16t^2 + 64t = 0 \;\Rightarrow\; t(t-4) = 0 \;\Rightarrow\; t = 0, 4$. After 4 seconds.

$\boxed{60}$ (a) $v = 55 \;\Rightarrow\; d = v + (v^2/20) = 55 + (55^2/20) = 206.25$ ft

(b) $d = 120 \;\Rightarrow\; 120 = v + (v^2/20) \;\Rightarrow\; 2400 = 20v + v^2 \;\Rightarrow$

$(v+60)(v-40) = 0 \;\Rightarrow\; v = 40$ mi/hr

$\boxed{61}$ (a) $T = 98 \;\Rightarrow\; h = 1000(100-T) + 580(100-T)^2 = 1000(2) + 580(2)^2 = 4320$ m.

(b) If $x = 100 - T$ and $h = 8840$, then $8840 = 1000x + 580x^2 \;\Rightarrow$

$29x^2 + 50x - 442 = 0 \;\Rightarrow\; x = \frac{-25 \pm \sqrt{13{,}443}}{29} \approx -4.86, 3.14$.

$T = 100 - x \;\Rightarrow\; x = 3.14$ and $T = 96.86\,^{\circ}\mathrm{C}$ for $95 \le T \le 100$.

$\boxed{62}$ $F = 0 \Rightarrow \dfrac{2k}{(2-x)^2} = \dfrac{k}{(x+2)^2} \Rightarrow 2k(x+2)^2 = k(2-x)^2 \Rightarrow$

$2x^2 + 8x + 8 = x^2 - 4x + 4 \Rightarrow x^2 + 12x + 4 = 0 \Rightarrow$

$x = -6 + 4\sqrt{2} \approx -0.34$ for $-2 \le x \le 2$.

$\boxed{63}$ Let x denote the width of the walk. $\text{Area}_{\text{plot}} + \text{Area}_{\text{walk}} = \text{Area}_{\text{total}} \Rightarrow$

$26 \cdot 30 + 240 = (26 + 2x)(30 + 2x) \Rightarrow 240 = 4x^2 + 112x \Rightarrow$

$x^2 + 28x - 60 = 0 \Rightarrow (x+30)(x-2) = 0 \Rightarrow x = 2$ ft.

$\boxed{64}$ Let x denote the width of the side or top margin, $2x$ the bottom.

Printed area $= lw \Rightarrow 661.5 = (24 - 2x)(36 - 3x) \Rightarrow$

$6x^2 - 144x + 202.5 = 0 \Rightarrow x = 12 \pm \frac{21}{2} = 1.5$.

The margins are 1.5 in. for the sides and the top, and 3 in. for the bottom.

$\boxed{65}$ Let x denote the length of one side. $\text{Cost}_{\text{preparation}} + \text{Cost}_{\text{fence}} = \text{Cost}_{\text{total}} \Rightarrow$

$x^2(\$0.50) + 4x(\$1) = \$120 \Rightarrow x^2 + 8x - 240 = 0 \Rightarrow (x+20)(x-12) = 0 \Rightarrow$

$x = 12$. The size of the garden should be 12 ft by 12 ft.

$\boxed{66}$ Let x denote the length of an adjacent side, $2x$ the parallel side.

$A = lw \Rightarrow 128 = (2x)(x) \Rightarrow 2x^2 = 128 \Rightarrow x = 8$.

The farmer should purchase $8 + 8 + 16 = 32$ ft of fencing.

$\boxed{67}$ Let $d(A, P) = x$ and $d(P, B) = 6 - x$.

$x^2 + (6-x)^2 = 5^2 \Rightarrow 2x^2 - 12x + 11 = 0 \Rightarrow x = 3 \pm \frac{1}{2}\sqrt{14} \approx 4.9,\ 1.1$ mi.

There are 4 possible roads since P could be on either side of segment AB.

$\boxed{68}$ Let r denote the city's original radius. $\text{Area}_{\text{original}} + \text{Area}_{\text{growth}} = \text{Area}_{\text{current}} \Rightarrow$

$\pi r^2 + 16\pi = \pi(5)^2 \Rightarrow r^2 + 16 = 25 \Rightarrow r^2 = 9 \Rightarrow r = 3$, and $5 - r = 2$ miles.

$\boxed{69}$ (a) The distances of the northbound and eastbound planes are $100 + 200t$ and $400t$, respectively. Using the Pythagorean theorem,

$$d = \sqrt{(100 + 200t)^2 + (400t)^2} = \sqrt{100^2(1+2t)^2 + 100^2(4t)^2} = 100\sqrt{20t^2 + 4t + 1}.$$

(b) $d = 500 \Rightarrow 500 = 100\sqrt{20t^2 + 4t + 1} \Rightarrow 5^2 = 20t^2 + 4t + 1 \Rightarrow$

$5t^2 + t - 6 = 0 \Rightarrow (5t + 6)(t - 1) = 0 \Rightarrow$

$t = 1$ hour after 2:30 P.M., or 3:30 P.M.

$\boxed{70}$ Let t denote the desired time. Using the Pythagorean theorem,

$(4t)^2 + (3t)^2 = 2^2 \Rightarrow 25t^2 = 4 \Rightarrow t = \frac{2}{5}$ hr, or 24 min.

They will be in range until 9:24 A.M.

$\boxed{71}$ Let x denote the outer width of the box, $x - 2$ the inner width.

Since the base is square, $(x-2)^2 = 144 \Rightarrow x = 14$.

The length is $3(1) + 2(x-2) = 2x - 1$. Thus, the size is 14 in. by 27 in.

[72] Let x denote the length of one side of the larger frame. $4x$ and $(100-4x)$ are the perimeters. Larger area $= 2 \times$ (smaller area) $\Rightarrow$ $\left(\frac{4x}{4}\right)^2 = 2\left(\frac{100-4x}{4}\right)^2$ $\Rightarrow$

$8x^2 = 10{,}000 - 800x + 16x^2 \Rightarrow x^2 - 100x + 1250 = 0 \Rightarrow$

$x = 50 - 25\sqrt{2} \approx 14.64$ in. for $x < 25$.

The length of a side for the smaller frame is $25 - x = 25\sqrt{2} - 25 \approx 10.36$ in.

[73] Let x denote the rate of the canoeist in still water. Then $x-5$ is the rate upstream and $x+5$ is the rate downstream.

$\text{Time}_{\text{up}} = \text{Time}_{\text{down}} + \frac{1}{2} \Rightarrow \left\{t = \frac{d}{r}\right\} \frac{1.2}{x-5} = \frac{1.2}{x+5} + \frac{1}{2} \Rightarrow$

$2.4(x+5) = 2.4(x-5) + x^2 - 25 \Rightarrow x^2 = 49 \Rightarrow x = 7$ mi/hr.

[74] Let t denote the number of seconds the rock falls.

$\text{Distance}_{\text{down}} = \text{Distance}_{\text{up}} \Rightarrow 16t^2 = 1100(4-t)\ \{d = rt\} \Rightarrow$

$4t^2 + 275t - 1100 = 0 \Rightarrow t = \dfrac{-275 + 5\sqrt{3729}}{8} \approx 3.79.$

The height is $16t^2 \approx 229.94$, or 230 ft.

[75] Let x denote the number of pairs ordered. Cost = (# of pairs)(cost per pair) $\Rightarrow$

$8400 = x(40 - 0.04x) \Rightarrow \frac{1}{25}x^2 - 40x + 8400 = 0 \Rightarrow$

$x^2 - 1000x + 210{,}000 = 0 \Rightarrow (x-300)(x-700) = 0 \Rightarrow x = 300$ for $0 \le x \le 600$.

[76] Let x denote the number of \$10 reductions in price.

Revenue = (unit price) $\times$ (# of units) $\Rightarrow$ $7000 = (300 - 10x)(15 + 2x) \Rightarrow$

$700 = -2x^2 + 45x + 450 \Rightarrow 2x^2 - 45x + 250 = 0 \Rightarrow (2x - 25)(x - 10) = 0 \Rightarrow$

$x = 10$ or 12.5. The selling price is \$300 − \$10(10) = \$200,

or \$300 − \$10(12.5) = \$175.

[77] The total surface area is the sum of the surface area of the cylinder and that of the top and bottom. $S = 2\pi rh + 2\pi r^2 \Rightarrow 10\pi = 8\pi r + 2\pi r^2 \Rightarrow$

$r^2 + 4r - 5 = 0 \Rightarrow (r+5)(r-1) = 0 \Rightarrow r = 1$, and the diameter is 2 ft.

[78] (a) $\text{Area}_{\text{capsule}} = \text{Area}_{\text{sphere}}$ {the two ends are hemispheres} $+ \text{Area}_{\text{cylinder}} =$

$4\pi r^2 + 2\pi rh = 4\pi(\frac{1}{4})^2 + 2\pi(\frac{1}{4})(2 - \frac{1}{2}) = \frac{\pi}{4} + \frac{3\pi}{4} = \pi \text{ cm}^2.$

$\text{Area}_{\text{tablet}} = \text{Area}_{\text{top and bottom}} + \text{Area}_{\text{cylinder}} = 2\pi r^2 + 2\pi r(\frac{1}{2}) = 2\pi r^2 + \pi r.$

Equating the two surface areas yields $2\pi r^2 + \pi r = \pi \Rightarrow$

$2r^2 + r - 1 = 0 \Rightarrow (2r-1)(r+1) = 0 \Rightarrow r = \frac{1}{2}$, and the diameter is 1 cm.

(b) $\text{Volume}_{\text{capsule}} = \text{Volume}_{\text{sphere}} + \text{Volume}_{\text{cylinder}} =$

$\frac{4}{3}\pi r^3 + \pi r^2 h = \frac{4}{3}\pi(\frac{1}{4})^3 + \pi(\frac{1}{4})^2\frac{3}{2} = \frac{\pi}{48} + \frac{3\pi}{32} = \frac{11\pi}{96} \approx 0.360 \text{ cm}^3.$

$\text{Volume}_{\text{tablet}} = \text{Volume}_{\text{cylinder}} = \pi r^2 h = \pi(\frac{1}{2})^2\frac{1}{2} = \frac{\pi}{8} \approx 0.393 \text{ cm}^3.$

[79] V is 95% of V_0 $\Rightarrow$ $V = 0.95V_0$ $\Rightarrow$ $\dfrac{V}{V_0} = 0.95$.

$0.95 = 0.8197 + 0.007752t + 0.0000281t^2 \Rightarrow 0.281t^2 + 77.52t - 1303 = 0 \Rightarrow$

$t \approx -291.76,\ 15.89$. Thus, the volume of the fireball will be 95% of the maximum volume approximately 15.89 seconds after the explosion.

[80] $0.95 = 0.831 + 0.00598t + 0.0000919t^2 \Rightarrow 0.919t^2 + 59.8t - 1190 = 0 \Rightarrow$

$t \approx -81.05,\ 15.98$. Approximately 15.98 seconds after the explosion.

[81] (a) $x = \dfrac{-4{,}500{,}000 \pm \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}}{2} \approx 0$ and $-4{,}500{,}000$

(b) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \cdot \dfrac{-b \mp \sqrt{b^2 - 4ac}}{-b \mp \sqrt{b^2 - 4ac}} = \dfrac{b^2 - (b^2 - 4ac)}{2a(-b \mp \sqrt{b^2 - 4ac})} =$

$\dfrac{4ac}{2a(-b \mp \sqrt{b^2 - 4ac})} = \dfrac{2c}{-b \mp \sqrt{b^2 - 4ac}}$. The root near zero was obtained in part (a) using the plus sign. In the second formula, it corresponds to the minus sign.

$$x = \frac{2(-0.96)}{-4{,}500{,}000 - \sqrt{4{,}500{,}000^2 - 4(1)(-0.96)}} \approx 2.13 \times 10^{-7}$$

[82] (a) $x = \dfrac{73{,}000{,}000 \pm \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}}{2} \approx 73{,}000{,}000$ and 0

(b) The root near zero was obtained in part (a) using the minus sign,

In the second formula, it corresponds to the plus sign.

$$x = \frac{2(2.01)}{73{,}000{,}000 + \sqrt{(-73{,}000{,}000)^2 - 4(1)(2.01)}} \approx 2.75 \times 10^{-8}$$

[83] (a) Let $Y_1 = T_1 = -1.09L + 96.01$ and $Y_2 = T_2 = -0.011L^2 - 0.126L + 81.45$.

Table each equation and compare them to the actual temperatures.

x (L)	Y_1	Y_2	S. Hem.
85	3.36	−8.74	−5
75	14.26	10.13	10
65	25.16	26.79	27
55	36.06	41.25	42
45	46.96	53.51	53
35	57.86	63.57	65
25	68.76	71.43	75
15	79.66	77.09	78
5	90.56	80.55	79

(continued)

Comparing Y_1 (T_1) with Y_2 (T_2), we can see that the linear equation T_1 is not as accurate as the quadratic equation T_2.

(b) $L = 50 \Rightarrow T_2 = -0.011(50)^2 - 0.126(50) + 81.45 = 47.65°F$.

2.4 Exercises

$\boxed{1}$ $(5-2i)+(-3+6i) = [5+(-3)]+(-2+6)i = 2+4i$

$\boxed{2}$ $(-5+7i)+(4+9i) = (-5+4)+(7+9)i = -1+16i$

$\boxed{3}$ $(7-6i)-(-11-3i) = (7+11)+(-6+3)i = 18-3i$

$\boxed{4}$ $(-3+8i)-(2+3i) = (-3-2)+(8-3)i = -5+5i$

$\boxed{5}$ $(3+5i)(2-7i) = (6-35i^2)+(10-21)i = (6+35)-11i = 41-11i$

$\boxed{6}$ $(-2+6i)(8-i) = (-16-6i^2)+(2+48)i = (-16+6)+50i = -10+50i$

$\boxed{7}$ $(1-3i)(2+5i) = (2-15i^2)+(5-6)i = (2+15)-i = 17-i$

$\boxed{8}$ $(8+2i)(7-3i) = (56-6i^2)+(-24+14)i = (56+6)-10i = 62-10i$

$\boxed{9}$ $(5-2i)^2 = 5^2-2(5)(2i)+(2i)^2 = (25-4)-20i = 21-20i$

$\boxed{10}$ $(6+7i)^2 = 6^2+2(6)(7i)+(7i)^2 = (36-49)+84i = -13+84i$

$\boxed{11}$ $i(3+4i)^2 = i\big[(9-16)+2(3)(4i)\big] = i(-7+24i) = -24-7i$

$\boxed{12}$ $i(2-7i)^2 = i\big[(4-49)-2(2)(7i)\big] = i(-45-28i) = 28-45i$

$\boxed{13}$ $(3+4i)(3-4i) = 3^2-(4i)^2 = 9-(-16) = 9+16 = 25$

$\boxed{14}$ $(4+9i)(4-9i) = 4^2-(9i)^2 = 16-(-81) = 16+81 = 97$

$\boxed{15}$ $i^{43} = i^{40}i^3 = (i^4)^{10}(-i) = 1^{10}(-i) = -i$ $\boxed{16}$ $i^{92} = (i^4)^{23} = 1^{23} = 1$

$\boxed{17}$ $i^{73} = i^{72}i = (i^4)^{18}i = 1^{18}i = i$ $\boxed{18}$ $i^{66} = i^{64}i^2 = (i^4)^{16}(-1) = 1^{16}(-1) = -1$

$\boxed{19}$ $\dfrac{3}{2+4i}\cdot\dfrac{2-4i}{2-4i} = \dfrac{6-12i}{4-(-16)} = \dfrac{6-12i}{20} = \dfrac{3}{10}-\dfrac{3}{5}i$

$\boxed{20}$ $\dfrac{5}{2-7i}\cdot\dfrac{2+7i}{2+7i} = \dfrac{10+35i}{4-(-49)} = \dfrac{10+35i}{53} = \dfrac{10}{53}+\dfrac{35}{53}i$

$\boxed{21}$ $\dfrac{1-7i}{6-2i}\cdot\dfrac{6+2i}{6+2i} = \dfrac{(6+14)+(2-42)i}{36-(-4)} = \dfrac{20-40i}{40} = \dfrac{1}{2}-i$

$\boxed{22}$ $\dfrac{2+9i}{-3-i}\cdot\dfrac{-3+i}{-3+i} = \dfrac{(-6-9)+(2-27)i}{9-(-1)} = \dfrac{-15-25i}{10} = -\dfrac{3}{2}-\dfrac{5}{2}i$

$\boxed{23}$ $\dfrac{-4+6i}{2+7i}\cdot\dfrac{2-7i}{2-7i} = \dfrac{(-8+42)+(28+12)i}{4-(-49)} = \dfrac{34+40i}{53} = \dfrac{34}{53}+\dfrac{40}{53}i$

$\boxed{24}$ $\dfrac{-3-2i}{5+2i}\cdot\dfrac{5-2i}{5-2i} = \dfrac{(-15-4)+(6-10)i}{25-(-4)} = \dfrac{-19-4i}{29} = -\dfrac{19}{29}-\dfrac{4}{29}i$

$\boxed{25}$ $\dfrac{4-2i}{-5i} = \dfrac{4-2i}{-5i}\cdot\dfrac{i}{i} = \dfrac{4i-2i^2}{-5i^2} = \dfrac{2+4i}{5} = \dfrac{2}{5}+\dfrac{4}{5}i$

$\boxed{26}$ $\dfrac{-2+6i}{3i} = \dfrac{-2+6i}{3i}\cdot\dfrac{-i}{-i} = \dfrac{2i-6i^2}{-3i^2} = \dfrac{6+2i}{3} = 2+\dfrac{2}{3}i$

$\boxed{27}$ $(2+5i)^3 = (2)^3 + 3(2)^2(5i) + 3(2)(5i)^2 + (5i)^3 = (8+150i^2) + (60i + 125i^3) =$
$(8-150) + (60-125)i = -142 - 65i$

$\boxed{28}$ $(3-2i)^3 = (3)^3 + 3(3)^2(-2i) + 3(3)(-2i)^2 + (-2i)^3 = (27+36i^2) + (-54i - 8i^3) =$
$(27-36) + (-54+8)i = -9 - 46i$

$\boxed{29}$ $(2-\sqrt{-4})(3-\sqrt{-16}) = (2-2i)(3-4i) = -2-14i$

$\boxed{30}$ $(-3+\sqrt{-25})(8-\sqrt{-36}) = (-3+5i)(8-6i) = 6+58i$

$\boxed{31}$ $\dfrac{4+\sqrt{-81}}{7-\sqrt{-64}} = \dfrac{4+9i}{7-8i} \cdot \dfrac{7+8i}{7+8i} = \dfrac{(28-72)+(32+63)i}{49-(-64)} = \dfrac{-44+95i}{113} = -\dfrac{44}{113} + \dfrac{95}{113}i$

$\boxed{32}$ $\dfrac{5-\sqrt{-121}}{1+\sqrt{-25}} = \dfrac{5-11i}{1+5i} \cdot \dfrac{1-5i}{1-5i} = \dfrac{(5-55)+(-25-11)i}{1-(-25)} = \dfrac{-50-36i}{26} = -\dfrac{25}{13} - \dfrac{18}{13}i$

$\boxed{33}$ $\dfrac{\sqrt{-36}\sqrt{-49}}{\sqrt{-16}} = \dfrac{(6i)(7i)}{4i} \cdot \dfrac{-i}{-i} = \dfrac{(-42)(-i)}{-4i^2} = \dfrac{42i}{4} = \dfrac{21}{2}i$

$\boxed{34}$ $\dfrac{\sqrt{-25}}{\sqrt{-16}\sqrt{-81}} = \dfrac{5i}{(4i)(9i)} = \dfrac{5i}{36i^2} = \dfrac{5i}{-36} = -\dfrac{5}{36}i$

$\boxed{35}$ $4 + (x+2y)i = x + 2i \;\Rightarrow\; 4 = x$ and $x + 2y = 2 \;\Rightarrow$
$x = 4$ and $4 + 2y = 2 \;\Rightarrow\; 2y = -2 \;\Rightarrow\; y = -1$, so $x = 4$ and $y = -1$.

$\boxed{36}$ $(x-y) + 3i = 7 + yi \;\Rightarrow\; 3 = y$ and $x - y = 7 \;\Rightarrow\; x = 10,\ y = 3$

$\boxed{37}$ $(2x-y) - 16i = 10 + 4yi \;\Rightarrow\; 2x - y = 10$ and $-16 = 4y \;\Rightarrow$
$y = -4$ and $2x - (-4) = 10 \;\Rightarrow\; 2x + 4 = 10 \;\Rightarrow\; 2x = 6 \;\Rightarrow\; x = 3$,
so $x = 3$ and $y = -4$.

$\boxed{38}$ $8 + (3x+y)i = 2x - 4i \;\Rightarrow\; 2x = 8$ and $3x + y = -4 \;\Rightarrow\; x = 4,\ y = -16$

$\boxed{39}$ $x^2 - 6x + 13 = 0 \;\Rightarrow\; x = \dfrac{6 \pm \sqrt{36-52}}{2} = \dfrac{6 \pm 4i}{2} = 3 \pm 2i$

$\boxed{40}$ $x^2 - 2x + 26 = 0 \;\Rightarrow\; x = \dfrac{2 \pm \sqrt{4-104}}{2} = \dfrac{2 \pm 10i}{2} = 1 \pm 5i$

$\boxed{41}$ $x^2 + 4x + 13 = 0 \;\Rightarrow\; x = \dfrac{-4 \pm \sqrt{16-52}}{2} = \dfrac{-4 \pm 6i}{2} = -2 \pm 3i$

$\boxed{42}$ $x^2 + 8x + 17 = 0 \;\Rightarrow\; x = \dfrac{-8 \pm \sqrt{64-68}}{2} = \dfrac{-8 \pm 2i}{2} = -4 \pm i$

$\boxed{43}$ $x^2 - 5x + 20 = 0 \;\Rightarrow\; x = \dfrac{5 \pm \sqrt{25-80}}{2} = \dfrac{5}{2} \pm \dfrac{1}{2}\sqrt{55}\,i$

$\boxed{44}$ $x^2 + 3x + 6 = 0 \;\Rightarrow\; x = \dfrac{-3 \pm \sqrt{9-24}}{2} = -\dfrac{3}{2} \pm \dfrac{1}{2}\sqrt{15}\,i$

$\boxed{45}$ $4x^2 + x + 3 = 0 \;\Rightarrow\; x = \dfrac{-1 \pm \sqrt{1-48}}{8} = -\dfrac{1}{8} \pm \dfrac{1}{8}\sqrt{47}\,i$

$\boxed{46}$ $-3x^2 + x - 5 = 0 \;\Rightarrow\; x = \dfrac{-1 \pm \sqrt{1-60}}{-6} = \dfrac{1}{6} \pm \dfrac{1}{6}\sqrt{59}\,i$

$\boxed{47}$ $x^3+125=0 \Rightarrow (x+5)(x^2-5x+25)=0 \Rightarrow$

$$x=-5 \text{ or } x=\frac{5\pm\sqrt{25-100}}{2}=\frac{5\pm5\sqrt{3}\,i}{2}.$$ The three solutions are $-5, \frac{5}{2}\pm\frac{5}{2}\sqrt{3}\,i$.

$\boxed{48}$ $x^3-27=0 \Rightarrow (x-3)(x^2+3x+9)=0 \Rightarrow$

$$x=3 \text{ or } x=\frac{-3\pm\sqrt{9-36}}{2}=\frac{-3\pm3\sqrt{3}\,i}{2}.$$ The three solutions are $3, -\frac{3}{2}\pm\frac{3}{2}\sqrt{3}\,i$.

$\boxed{49}$ $x^4=256 \Rightarrow x^4-256=0 \Rightarrow (x^2-16)(x^2+16)=0 \Rightarrow x=\pm 4, \pm 4i$

$\boxed{50}$ $x^4=81 \Rightarrow x^4-81=0 \Rightarrow (x^2-9)(x^2+9)=0 \Rightarrow x=\pm 3, \pm 3i$

$\boxed{51}$ $4x^4+25x^2+36=0 \Rightarrow (x^2+4)(4x^2+9)=0 \Rightarrow x=\pm 2i, \pm\frac{3}{2}i$

$\boxed{52}$ $27x^4+21x^2+4=0 \Rightarrow (9x^2+4)(3x^2+1)=0 \Rightarrow x=\pm\frac{2}{3}i, \pm\frac{1}{3}\sqrt{3}\,i$

$\boxed{53}$ $x^3+3x^2+4x=0 \Rightarrow x(x^2+3x+4)=0 \Rightarrow x=0, -\frac{3}{2}\pm\frac{1}{2}\sqrt{7}\,i$

$\boxed{54}$ $8x^3-12x^2+2x-3=0 \Rightarrow 4x^2(2x-3)+1(2x-3)=0 \Rightarrow$

$$(4x^2+1)(2x-3)=0 \Rightarrow x=\tfrac{3}{2}, \pm\tfrac{1}{2}i$$

Note: Exer. 55–60: Let $z=a+bi$ and $w=c+di$.

$\boxed{55}$ $\overline{z+w}=\overline{(a+bi)+(c+di)}$

$$=\overline{(a+c)+(b+d)i}=(a+c)-(b+d)i=(a-bi)+(c-di)=\overline{z}+\overline{w}.$$

$\boxed{56}$ $\overline{z-w}=\overline{(a+bi)-(c+di)}$

$$=\overline{(a-c)+(b-d)i}=(a-c)-(b-d)i=(a-bi)-(c-di)=\overline{z}-\overline{w}.$$

$\boxed{57}$ $\overline{z\cdot w}=\overline{(a+bi)\cdot(c+di)}=\overline{(ac-bd)+(ad+bc)i}=$

$$(ac-bd)-(ad+bc)i=ac-adi-bd-bci=a(c-di)-bi(c-di)=$$

$$(a-bi)\cdot(c-di)=\overline{z}\cdot\overline{w}$$

$\boxed{58}$ $$\overline{\left(\frac{z}{w}\right)}=\overline{\left(\frac{a+bi}{c+di}\right)}=\overline{\left(\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}\right)}=\overline{\left(\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}\right)}=$$

$$\overline{\left(\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i\right)}=\frac{ac+bd}{c^2+d^2}-\frac{bc-ad}{c^2+d^2}i=\frac{(ac+bd)+(ad-bc)i}{c^2+d^2}=$$

$$\frac{a-bi}{c-di}\cdot\frac{c+di}{c+di}=\frac{a-bi}{c-di}=\frac{\overline{(a+bi)}}{\overline{(c+di)}}=\frac{\overline{z}}{\overline{w}}$$

$\boxed{59}$ If $\overline{z}=z$, then $a-bi=a+bi$ and hence $-bi=bi$, or $2bi=0$.

Thus, $b=0$ and $z=a$ is real. Conversely, if z is real, then $b=0$ and hence

$$\overline{z}=\overline{a+0i}=a-0i=a+0i=z.$$

$\boxed{60}$ $\overline{z^2}=\overline{(a+bi)^2}=\overline{a^2+2abi-b^2}=\overline{(a^2-b^2)+2abi}=$

$$(a^2-b^2)-2abi=a^2-2abi-b^2=(a-bi)^2=(\overline{z})^2$$

2.5 Exercises

$\boxed{1}$ $|x+4|=11 \Rightarrow x+4=11 \text{ or } x+4=-11 \Rightarrow x=7 \text{ or } x=-15$

$\boxed{2}$ $|x-5|=2 \Rightarrow x-5=2 \text{ or } x-5=-2 \Rightarrow x=7 \text{ or } x=3$

$\boxed{3}$ $|3x-2|+3=7 \Rightarrow |3x-2|=4 \Rightarrow 3x-2=4$ or $3x-2=-4 \Rightarrow$
$3x=6$ or $3x=-2 \Rightarrow x=2$ or $x=-\frac{2}{3}$

$\boxed{4}$ $2|5x+2|-1=5 \Rightarrow 2|5x+2|=6 \Rightarrow |5x+2|=3 \Rightarrow$
$5x+2=3$ or $5x+2=-3 \Rightarrow 5x=1$ or $5x=-5 \Rightarrow x=\frac{1}{5}$ or $x=-1$

$\boxed{5}$ $3|x+1|-2=-11 \Rightarrow 3|x+1|=-9 \Rightarrow |x+1|=-3$.
Since the absolute value of an expression is nonnegative, $|x+1|=-3$ has no solution.

$\boxed{6}$ $|x-2|+5=5 \Rightarrow |x-2|=0$. Since the absolute value of an expression can only
equal 0 if the expression itself is 0, $|x-2|=0 \Rightarrow x-2=0 \Rightarrow x=2$.

$\boxed{7}$ $9x^3-18x^2-4x+8=0 \Rightarrow 9x^2(x-2)-4(x-2)=0 \Rightarrow$
$(9x^2-4)(x-2)=0 \Rightarrow x=\pm\frac{2}{3}, 2$

$\boxed{8}$ $3x^3-4x^2-27x+36=0 \Rightarrow x^2(3x-4)-9(3x-4)=0 \Rightarrow$
$(x^2-9)(3x-4)=0 \Rightarrow x=\pm 3, \frac{4}{3}$

$\boxed{9}$ $4x^4+10x^3=6x^2+15x \Rightarrow x(4x^3+10x^2-6x-15)=0 \Rightarrow$
$x\left[2x^2(2x+5)-3(2x+5)\right]=0 \Rightarrow x(2x^2-3)(2x+5)=0 \Rightarrow x=0, \pm\frac{1}{2}\sqrt{6}, -\frac{5}{2}$

$\boxed{10}$ $15x^5-20x^4=6x^3-8x^2 \Rightarrow x^2(15x^3-20x^2-6x+8)=0 \Rightarrow$
$x^2\left[5x^2(3x-4)-2(3x-4)\right]=0 \Rightarrow x^2(5x^2-2)(3x-4)=0 \Rightarrow x=0, \pm\frac{1}{5}\sqrt{10}, \frac{4}{3}$

$\boxed{11}$ $y^{3/2}=5y \Rightarrow y^{3/2}-5y=0 \Rightarrow y(y^{1/2}-5)=0 \Rightarrow y=0$ or $y^{1/2}=5$.
$y^{1/2}=5 \Rightarrow (y^{1/2})^2=5^2 \Rightarrow y=25$. $y=0, 25$

$\boxed{12}$ $y^{4/3}=-3y \Rightarrow y^{4/3}+3y=0 \Rightarrow y(y^{1/3}+3)=0 \Rightarrow y=0$ or $y^{1/3}=-3$.
$y^{1/3}=-3 \Rightarrow (y^{1/3})^3=(-3)^3 \Rightarrow y=-27$. $y=0, -27$

$\boxed{13}$ $\sqrt{7-5x}=8 \Rightarrow (\sqrt{7-5x})^2=8^2 \Rightarrow 7-5x=64 \Rightarrow x=-\frac{57}{5}$

$\boxed{14}$ $\sqrt{2x-9}=\frac{1}{3} \Rightarrow (\sqrt{2x-9})^2=(\frac{1}{3})^2 \Rightarrow 2x-9=\frac{1}{9} \Rightarrow x=\frac{41}{9}$

$\boxed{15}$ $2+\sqrt[3]{1-5t}=0 \Rightarrow (\sqrt[3]{1-5t})^3=(-2)^3 \Rightarrow 1-5t=-8 \Rightarrow t=\frac{9}{5}$

$\boxed{16}$ $\sqrt[3]{6-s^2}+5=0 \Rightarrow (\sqrt[3]{6-s^2})^3=(-5)^3 \Rightarrow 6-s^2=-125 \Rightarrow s=\pm\sqrt{131}$

$\boxed{17}$ $\sqrt[5]{2x^2+1}-2=0 \Rightarrow (\sqrt[5]{2x^2+1})^5=2^5 \Rightarrow 2x^2+1=32 \Rightarrow x^2=\frac{31}{2} \Rightarrow$
$x=\pm\frac{1}{2}\sqrt{62}$

$\boxed{18}$ $\sqrt[4]{2x^2-1}=x \Rightarrow (\sqrt[4]{2x^2-1})^4=x^4 \Rightarrow 2x^2-1=x^4 \Rightarrow (x^2-1)^2=0 \Rightarrow$
$\left[(x+1)(x-1)\right]^2 \Rightarrow x=1$ and -1 is an extraneous solution.

$\boxed{19}$ $\sqrt{7-x}=x-5 \Rightarrow 7-x=x^2-10x+25 \Rightarrow x^2-9x+18=0 \Rightarrow$
$(x-3)(x-6)=0 \Rightarrow x=6$ and 3 is an extraneous solution.

$\boxed{20}$ $\sqrt{3-x}-x=3 \Rightarrow (\sqrt{3-x})^2=(x+3)^2 \Rightarrow 3-x=x^2+6x+9 \Rightarrow$
$x^2+7x+6=0 \Rightarrow (x+1)(x+6)=0 \Rightarrow$
$x=-1$ and -6 is an extraneous solution.

$\boxed{21}$ $3\sqrt{2x-3}+2\sqrt{7-x}=11 \;\Rightarrow\; 3\sqrt{2x-3}=11-2\sqrt{7-x} \;\Rightarrow$
$9(2x-3)=121-44\sqrt{7-x}+4(7-x) \;\Rightarrow\; 44\sqrt{7-x}=-22x+176 \;\Rightarrow$
$2\sqrt{7-x}=8-x \;\Rightarrow\; 4(7-x)=64-16x+x^2 \;\Rightarrow\; x^2-12x+36=0 \;\Rightarrow$
$(x-6)^2=0 \;\Rightarrow\; x=6$

$\boxed{22}$ $\sqrt{2x+15}-2=\sqrt{6x+1} \;\Rightarrow\; 2x+15-4\sqrt{2x+15}+4=6x+1 \;\Rightarrow$
$4\sqrt{2x+15}=-4x+18 \;\Rightarrow\; 2\sqrt{2x+15}=-2x+9 \;\Rightarrow$
$4(2x+15)=4x^2-36x+81 \;\Rightarrow\; 4x^2-44x+21=0 \;\Rightarrow$
$(2x-1)(2x-21)=0 \;\Rightarrow\; x=\frac{1}{2}$ and $\frac{21}{2}$ is an extraneous solution.

$\boxed{23}$ $x=4+\sqrt{4x-19} \;\Rightarrow\; x-4=\sqrt{4x-19} \;\Rightarrow\; x^2-8x+16=4x-19 \;\Rightarrow$
$x^2-12x+35=0 \;\Rightarrow\; (x-5)(x-7)=0 \;\Rightarrow\; x=5,\ 7$

$\boxed{24}$ $x=3+\sqrt{5x-9} \;\Rightarrow\; x-3=\sqrt{5x-9} \;\Rightarrow\; x^2-6x+9=5x-9 \;\Rightarrow$
$x^2-11x+18=0 \;\Rightarrow\; (x-2)(x-9)=0 \;\Rightarrow\; x=9$ and 2 is an extraneous solution.

$\boxed{25}$ $x+\sqrt{5x+19}=-1 \;\Rightarrow\; \sqrt{5x+19}=-x-1 \;\Rightarrow\; 5x+19=x^2+2x+1 \;\Rightarrow$
$x^2-3x-18=0 \;\Rightarrow\; (x-6)(x+3)=0 \;\Rightarrow\; x=-3$ and 6 is an extraneous solution.

$\boxed{26}$ $x-\sqrt{-7x-24}=-2 \;\Rightarrow\; x+2=\sqrt{-7x-24} \;\Rightarrow\; x^2+4x+4=-7x-24 \;\Rightarrow$
$x^2+11x+28=0 \;\Rightarrow\; (x+4)(x+7)=0 \;\Rightarrow$
there is no solution since -4 and -7 are extraneous.

$\boxed{27}$ $\sqrt{7-2x}-\sqrt{5+x}=\sqrt{4+3x} \;\Rightarrow$
$(7-2x)-2\sqrt{(7-2x)(5+x)}+(5+x)=4+3x \;\Rightarrow$
$-4x+8=2\sqrt{-2x^2-3x+35} \;\Rightarrow\; -2x+4=\sqrt{-2x^2-3x+35} \;\Rightarrow$
$4x^2-16x+16=-2x^2-3x+35 \;\Rightarrow\; 6x^2-13x-19=0 \;\Rightarrow$
$(x+1)(6x-19)=0 \;\Rightarrow\; x=-1$ and $\frac{19}{6}$ is an extraneous solution.

$\boxed{28}$ $4\sqrt{1+3x}+\sqrt{6x+3}=\sqrt{-6x-1} \;\Rightarrow$
$16(1+3x)+8\sqrt{(3x+1)(6x+3)}+(6x+3)=-6x-1 \;\Rightarrow$
$8\sqrt{18x^2+15x+3}=-60x-20 \;\Rightarrow\; 2\sqrt{18x^2+15x+3}=-15x-5 \;\Rightarrow$
$4(18x^2+15x+3)=225x^2+150x+25 \;\Rightarrow\; 0=153x^2+90x+13 \;\Rightarrow$
$(3x+1)(51x+13)=0 \;\Rightarrow\; x=-\frac{1}{3}$ and $-\frac{13}{51}$ is an extraneous solution.

$\boxed{29}$ $\sqrt{11+8x}+1=\sqrt{9+4x} \;\Rightarrow\; (11+8x)+2\sqrt{11+8x}+1=9+4x \;\Rightarrow$
$2\sqrt{8x+11}=-4x-3 \;\Rightarrow\; 4(8x+11)=16x^2+24x+9 \;\Rightarrow$
$16x^2-8x-35=0 \;\Rightarrow\; (4x-7)(4x+5)=0 \;\Rightarrow$
$x=-\frac{5}{4}$ and $\frac{7}{4}$ is an extraneous solution.

$\boxed{30}$ $2\sqrt{x}-\sqrt{x-3}=\sqrt{5+x} \;\Rightarrow\; 4x-4\sqrt{x(x-3)}+(x-3)=5+x \;\Rightarrow$
$4x-8=4\sqrt{x^2-3x} \;\Rightarrow\; x-2=\sqrt{x^2-3x} \;\Rightarrow\; x^2-4x+4=x^2-3x \;\Rightarrow\; x=4$

[31] $\sqrt{2\sqrt{x+1}} = \sqrt{3x-5} \Rightarrow 2\sqrt{x+1} = 3x-5 \Rightarrow 4(x+1) = 9x^2 - 30x + 25 \Rightarrow$
$9x^2 - 34x + 21 = 0 \Rightarrow (x-3)(9x-7) = 0 \Rightarrow$
$x = 3$ and $\frac{7}{9}$ is an extraneous solution.

[32] $\sqrt{5\sqrt{x}} = \sqrt{2x-3} \Rightarrow 5\sqrt{x} = 2x-3 \Rightarrow 25x = 4x^2 - 12x + 9 \Rightarrow$
$4x^2 - 37x + 9 = 0 \Rightarrow (4x-1)(x-9) = 0 \Rightarrow$
$x = 9$ and $\frac{1}{4}$ is an extraneous solution.

[33] $\sqrt{1+4\sqrt{x}} = \sqrt{x}+1 \Rightarrow 1+4\sqrt{x} = x+2\sqrt{x}+1 \Rightarrow 2\sqrt{x} = x \Rightarrow$
$4x = x^2 \Rightarrow x(4-x) = 0 \Rightarrow x = 0,\ 4$

[34] $\sqrt{x+1} = \sqrt{x-1} \Rightarrow x+1 = x-1 \Rightarrow 1 = -1 \Rightarrow$ No solution

[35] $x^4 - 25x^2 + 144 = 0 \Rightarrow (x^2-9)(x^2-16) = 0 \Rightarrow x = \pm 3,\ \pm 4$

[36] $2x^4 - 10x^2 + 8 = 0 \Rightarrow 2(x^2-1)(x^2-4) = 0 \Rightarrow x = \pm 1,\ \pm 2$

Note: Substitution could be used instead of factoring for the following exercises.

[37] $5y^4 - 7y^2 + 1 = 0 \Rightarrow y^2 = \frac{7 \pm \sqrt{29}}{10} \cdot \frac{10}{10} = \frac{70 \pm 10\sqrt{29}}{100} \Rightarrow y = \pm\frac{1}{10}\sqrt{70 \pm 10\sqrt{29}}$

[38] $3y^4 - 5y^2 + 1 = 0 \Rightarrow y^2 = \frac{5 \pm \sqrt{13}}{6} \cdot \frac{6}{6} = \frac{30 \pm 6\sqrt{13}}{36} \Rightarrow y = \pm\frac{1}{6}\sqrt{30 \pm 6\sqrt{13}}$

[39] $36x^{-4} - 13x^{-2} + 1 = 0 \Rightarrow (4x^{-2}-1)(9x^{-2}-1) = 0 \Rightarrow x^{-2} = \frac{1}{4}, \frac{1}{9} \Rightarrow$
$x^2 = 4,\ 9 \Rightarrow x = \pm 2,\ \pm 3$

[40] $x^{-2} - 2x^{-1} - 35 = 0 \Rightarrow (x^{-1}-7)(x^{-1}+5) = 0 \Rightarrow x^{-1} = 7,\ -5 \Rightarrow x = \frac{1}{7}, -\frac{1}{5}$

[41] $3x^{2/3} + 4x^{1/3} - 4 = 0 \Rightarrow (3x^{1/3}-2)(x^{1/3}+2) = 0 \Rightarrow \sqrt[3]{x} = \frac{2}{3}, -2 \Rightarrow$
$x = \frac{8}{27}, -8$

[42] $2y^{1/3} - 3y^{1/6} + 1 = 0 \Rightarrow (2y^{1/6}-1)(y^{1/6}-1) = 0 \Rightarrow \sqrt[6]{y} = \frac{1}{2}, 1 \Rightarrow y = \frac{1}{64}, 1$

[43] $6w - 23w^{1/2} + 20 = 0 \Rightarrow (2w^{1/2}-5)(3w^{1/2}-4) = 0 \Rightarrow \sqrt{w} = \frac{5}{2}, \frac{4}{3} \Rightarrow$
$w = \frac{25}{4}, \frac{16}{9}$

[44] $2x^{-2/3} - 7x^{-1/3} - 15 = 0 \Rightarrow (2x^{-1/3}+3)(x^{-1/3}-5) = 0 \Rightarrow x^{-1/3} = -\frac{3}{2}, 5 \Rightarrow$
$\sqrt[3]{x} = -\frac{2}{3}, \frac{1}{5} \Rightarrow x = -\frac{8}{27}, \frac{1}{125}$

[45] $\left(\frac{t}{t+1}\right)^2 - \frac{2t}{t+1} - 8 = 0 \Rightarrow \left(\frac{t}{t+1} - 4\right)\left(\frac{t}{t+1} + 2\right) = 0 \Rightarrow \frac{t}{t+1} = 4,\ -2 \Rightarrow$
$t = 4t+4,\ -2t-2 \Rightarrow t = -\frac{4}{3}, -\frac{2}{3}$

[46] $6u^{-1/2} - 13u^{-1/4} + 6 = 0 \Rightarrow (2u^{-1/4}-3)(3u^{-1/4}-2) \Rightarrow u^{-1/4} = \frac{3}{2}, \frac{2}{3} \Rightarrow$
$\sqrt[4]{u} = \frac{2}{3}, \frac{3}{2} \Rightarrow u = \frac{16}{81}, \frac{81}{16}$

[47] $27x^3 = (x+5)^3 \Rightarrow \left(\frac{x+5}{x}\right)^3 = 27 \Rightarrow \frac{x+5}{x} = 3 \Rightarrow x+5 = 3x \Rightarrow x = \frac{5}{2}$

[48] $16x^4 = (x-4)^4 \Rightarrow \left(\frac{x-4}{x}\right)^4 = 16 \Rightarrow \frac{x-4}{x} = \pm 2 \Rightarrow$
$x-4 = 2x$ or $x-4 = -2x \Rightarrow x = -4$ or $\frac{4}{3}$

[49] $\sqrt[3]{x} = 2\sqrt[4]{x} \Rightarrow (\sqrt[3]{x})^{12} = (2\sqrt[4]{x})^{12} \Rightarrow x^4 = 2^{12}x^3 \Rightarrow x^4 - 4096x^3 = 0 \Rightarrow$
$x^3(x-4096) = 0 \Rightarrow x = 0,\ 4096$

[50] $\sqrt{x+3} = \sqrt[4]{2x+6} \Rightarrow (\sqrt{x+3})^4 = (\sqrt[4]{2x+6})^4 \Rightarrow (x+3)^2 = 2x+6 \Rightarrow$
$x^2+6x+9 = 2x+6 \Rightarrow x^2+4x+3=0 \Rightarrow (x+1)(x+3)=0 \Rightarrow x=-1, -3$

[51] (a) $x^{5/3} = 32 \Rightarrow (x^{5/3})^{3/5} = (32)^{3/5} \Rightarrow x = (\sqrt[5]{32})^3 = 2^3 = 8$

(b) $x^{4/3} = 16 \Rightarrow (x^{4/3})^{3/4} = \pm(16)^{3/4} \Rightarrow x = \pm(\sqrt[4]{16})^3 = \pm 2^3 = \pm 8$

(c) $x^{2/3} = -36 \Rightarrow (x^{2/3})^{3/2} = \pm(-36)^{3/2} \Rightarrow x = \pm(\sqrt{-36})^3$,

which are not real numbers. No real solutions

(d) $x^{3/4} = 125 \Rightarrow (x^{3/4})^{4/3} = (125)^{4/3} \Rightarrow x = (\sqrt[3]{125})^4 = 5^4 = 625$

(e) $x^{3/2} = -27 \Rightarrow (x^{3/2})^{2/3} = (-27)^{2/3} \Rightarrow x = (\sqrt[3]{-27})^2 = (-3)^2 = 9$,

which is an extraneous solution. No real solutions

[52] (a) $x^{3/5} = -27 \Rightarrow (x^{3/5})^{5/3} = (-27)^{5/3} \Rightarrow x = (\sqrt[3]{-27})^5 = (-3)^5 = -243$

(b) $x^{2/3} = 25 \Rightarrow (x^{2/3})^{3/2} = \pm(25)^{3/2} \Rightarrow x = \pm(\sqrt{25})^3 = \pm 5^3 = \pm 125$

(c) $x^{4/3} = -49 \Rightarrow (x^{4/3})^{3/4} = \pm(-49)^{3/4} \Rightarrow x = \pm(\sqrt[4]{-49})^3$,

which are not real numbers. No real solutions

(d) $x^{3/2} = 27 \Rightarrow (x^{3/2})^{2/3} = (27)^{2/3} \Rightarrow x = (\sqrt[3]{27})^2 = 3^2 = 9$

(e) $x^{3/4} = -8 \Rightarrow (x^{3/4})^{4/3} = (-8)^{4/3} \Rightarrow x = (\sqrt[3]{-8})^4 = (-2)^4 = 16$,

which is an extraneous solution. No real solutions

[53] $T = 2\pi\sqrt{\dfrac{l}{g}} \Rightarrow \dfrac{T}{2\pi} = \sqrt{\dfrac{l}{g}} \Rightarrow \dfrac{T^2}{4\pi^2} = \dfrac{l}{g} \Rightarrow l = \dfrac{gT^2}{4\pi^2}$

[54] $d = \frac{1}{2}\sqrt{4R^2-C^2} \Rightarrow 2d = \sqrt{4R^2-C^2} \Rightarrow 4d^2 = 4R^2 - C^2 \Rightarrow$
$C^2 = 4(R^2-d^2) \Rightarrow C = \pm 2\sqrt{R^2-d^2} \Rightarrow C = 2\sqrt{R^2-d^2}$ since $C > 0$

[55] $S = \pi r\sqrt{r^2+h^2} \Rightarrow \dfrac{S}{\pi r} = \sqrt{r^2+h^2} \Rightarrow \dfrac{S^2}{\pi^2r^2} = h^2+r^2 \Rightarrow \dfrac{S^2}{\pi^2r^2} - r^2 = h^2 \Rightarrow$
$h^2 = \dfrac{1}{\pi^2r^2}(S^2-\pi^2r^4) \Rightarrow h = \pm\frac{1}{\pi r}\sqrt{S^2-\pi^2r^4} \Rightarrow h = \frac{1}{\pi r}\sqrt{S^2-\pi^2r^4}$ since $h > 0$

[56] $\omega = \dfrac{1}{\sqrt{LC}} \Rightarrow \omega^2 = \dfrac{1}{LC} \Rightarrow C\omega^2 = \dfrac{1}{L} \Rightarrow C = \dfrac{1}{L\omega^2}$

[57] From the Pythagorean theorem, $d^2+h^2 = L^2$. Since d is to be 25% of L, we have $d = \frac{1}{4}L$, so $(\frac{1}{4}L)^2 + h^2 = L^2 \Rightarrow h^2 = L^2 - (\frac{1}{4}L)^2 \Rightarrow h^2 = 1L^2 - \frac{1}{16}L^2 \Rightarrow$
$h^2 = \frac{15}{16}L^2 \Rightarrow h = \sqrt{\frac{15}{16}L^2}$ (since $h > 0$) $= \dfrac{\sqrt{15}}{4}L \approx 0.97L$. Thus, $h \approx 97\%L$.

[58] $A = k\sqrt{\dfrac{t}{T}} \Rightarrow \dfrac{A}{k} = \sqrt{\dfrac{t}{T}} \Rightarrow \dfrac{A^2}{k^2} = \dfrac{t}{T} \Rightarrow t = \dfrac{TA^2}{k^2}$

[59] $P = 0.31ED^2V^3 \Rightarrow V = \left(\dfrac{P}{0.31ED^2}\right)^{1/3} = \left(\dfrac{10{,}000}{(0.31)(0.42)10^2}\right)^{1/3} \approx 9.16$ ft/sec.

Multiplying by $\frac{60}{88}$ (or $\frac{15}{22}$) to convert to mi/hr gives us approximately 6.24 mi/hr.

[60] $P = 15{,}700S^{5/2}RD \Rightarrow S = \left(\dfrac{P}{15{,}700RD}\right)^{2/5} = \left(\dfrac{380}{(15{,}700)(0.113/2)(2)}\right)^{2/5} \approx 0.54$

$\boxed{61}$ $Q = kP^{-c} = 10^5 P^{-1/2} \Rightarrow$

$$\sqrt{P} = \frac{10^5}{Q} \Rightarrow P = \left(\frac{10^5}{Q}\right)^2 = \left(\frac{100{,}000}{5000}\right)^2 = (20)^2 = 400 \text{ cents, or, \$4.00.}$$

$\boxed{62}$ $T = 0.25P^{1/4}/\sqrt{v} \Rightarrow P^{1/4} = 4T\sqrt{v} \Rightarrow P = (4T)^4v^2 = 4^4 3^4 5^2 = 518{,}400$

$\boxed{63}$ $V = \frac{1}{3}\pi r^2 h \Rightarrow 144 = \frac{1}{3}\pi r^3$ {since $r = h$} $\Rightarrow r^3 = 432/\pi \Rightarrow$

$r = \sqrt[3]{432/\pi}$, and the diameter is $2\sqrt[3]{432/\pi} \approx 10.3$ cm.

$\boxed{64}$ Original: $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{32}{3} = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{8}{\pi} \Rightarrow r = \frac{2}{\sqrt[3]{\pi}}$ and $d = \frac{4}{\sqrt[3]{\pi}}$

Inflated: $V = 25\frac{1}{3} + 10\frac{2}{3} \Rightarrow \frac{4}{3}\pi r^3 = 36 \Rightarrow r^3 = \frac{27}{\pi} \Rightarrow r = \frac{3}{\sqrt[3]{\pi}}$ and $d = \frac{6}{\sqrt[3]{\pi}}$

The change in the diameter is $\frac{6}{\sqrt[3]{\pi}} - \frac{4}{\sqrt[3]{\pi}} = \frac{2}{\sqrt[3]{\pi}} \approx 1.37$ ft.

$\boxed{65}$ $y = 60\% \Rightarrow \frac{x^3}{x^3 + (1-x)^3} = \frac{3}{5} \Rightarrow 5x^3 = 3x^3 + 3(1-x)^3 \Rightarrow$

$2x^3 = 3(1-x)^3 \Rightarrow \left(\frac{x}{1-x}\right)^3 = \frac{3}{2} \Rightarrow \frac{x}{1-x} = \sqrt[3]{1.5} \Rightarrow x = \sqrt[3]{1.5} - \sqrt[3]{1.5}\,x \Rightarrow$

$x + \sqrt[3]{1.5}\,x = \sqrt[3]{1.5} \Rightarrow (1 + \sqrt[3]{1.5})x = \sqrt[3]{1.5} \Rightarrow x = \frac{\sqrt[3]{1.5}}{1 + \sqrt[3]{1.5}} \approx 0.534$, or 53.4%

$\boxed{66}$ $S = \pi r\sqrt{r^2 + h^2}$ with $S = 6\pi$ in.^{2} and $h = 3$ in. $\Rightarrow 6\pi = \pi r\sqrt{r^2 + 9} \Rightarrow$

$36 = r^2(r^2 + 9) \Rightarrow r^4 + 9r^2 - 36 = 0 \Rightarrow (r^2 + 12)(r^2 - 3) = 0 \Rightarrow r = \sqrt{3}$ in.

$\boxed{67}$ $\text{Cost}_{\text{underwater}} + \text{Cost}_{\text{overland}} = \text{Cost}_{\text{total}} \Rightarrow$

$7500\sqrt{x^2 + 1} + 6000(5 - x) = 35{,}000 \Rightarrow 15\sqrt{x^2 + 1} = 12x + 10 \Rightarrow$

$225(x^2 + 1) = 144x^2 + 240x + 100 \Rightarrow 81x^2 - 240x + 125 = 0 \Rightarrow$

$x = \frac{240 \pm \sqrt{17{,}100}}{162} = \frac{40 \pm 5\sqrt{19}}{27} \approx 2.2887,\ 0.6743$ mi. There are two possible routes.

$\boxed{68}$ (a) $h + \frac{h}{1.684 - h} = 0.545t + \frac{0.5 + t}{1 + \frac{4}{3}t}$ with $t = 12 \Rightarrow$

$\frac{1.684h - h^2 + h}{1.684 - h} = 6.54 + \frac{12.5}{17} \Rightarrow \frac{h^2 - 2.684h}{h - 1.684} = \frac{123.68}{17} \Rightarrow$

$17h^2 - 45.628h = 123.68h - 208.27712 \Rightarrow$

$17h^2 - 169.308h + 208.27712 = 0 \Rightarrow$

$h \approx 8.5216,\ 1.4377$; only 1.4377 meters ($\approx 56.60$ inches) makes sense.

(b) Let $h = \frac{1}{2}h_{\text{M}} = \frac{1}{2}(1.684) = 0.842$. $0.842 + \frac{0.842}{1.684 - 0.842} = 0.545t + \frac{0.5 + t}{1 + \frac{4}{3}t} \Rightarrow$

$(1.842 - 0.545t)\left(\frac{3 + 4t}{3}\right) = \frac{2t + 1}{2} \Rightarrow 4360t^2 - 5466t - 8052 = 0 \Rightarrow$

$t \approx -0.8697,\ 2.1234$; only 2.1234 years ($\approx 25.5$ months) makes sense.

69 (a) Let $Y_1 = D_1 = 6.096L + 685.7$ and

$Y_2 = D_2 = 0.00178L^3 - 0.072L^2 + 4.37L + 719.$

Table each equation and compare them to the actual values.

x (L)	Y_1	Y_2	Summer
0	686	719	720
10	747	757	755
20	808	792	792
30	869	833	836
40	930	893	892
50	991	980	978
60	1051	1106	1107

Comparing Y_1 (D_1) with Y_2 (D_2) we can see that the linear equation D_1 is not as accurate as the cubic equation D_2.

(b) $L = 35 \Rightarrow D_2 = 0.00178(35)^3 - 0.072(35)^2 + 4.37(35) + 719 \approx 860$ min.

70 (a) The volume of the box is given by $V = x(24 - 2x)(36 - 2x)$.

(b) Let $Y_1 = x(24 - 2x)(36 - 2x)$.

The maximum V is $1825.292 \approx 1825.3$ in.^{2} when $x = 4.7$ in.

x	V	x	V
4.5	1822.5	4.8	1824.8
4.6	1824.5	4.9	1823.0
4.7	1825.3	5.0	1820.0

71 The volume of the box is $V = hw^2 = 25$, where h is the height and w is the length of a side of the square base. The amount of cardboard will be minimized when the surface area of the box is a minimum. The surface area is given by $S = w^2 + 4wh$. Since $h = 25/w^2$, we have $S = w^2 + 100/w$. Form a table for w and S.

w	S	w	S
3.4	40.972	3.7	40.717
3.5	40.821	3.8	40.756
3.6	40.738	3.9	40.851

The minimum surface area is $S \approx 40.717$ when $w \approx 3.7$ and $h = 25/w^2 \approx 1.8$.

2.6 Exercises

1 (a) 5 is added to both sides: $-7+5 < -3+5 \;\Rightarrow\; -2 < 2$

(b) 4 is subtracted from both sides: $-7-4 < -3-4 \;\Rightarrow\; -11 < -7$

(c) both sides are multiplied by $\frac{1}{3}$: $-7 \cdot \frac{1}{3} < -3 \cdot \frac{1}{3} \;\Rightarrow\; -\frac{7}{3} < -1$

(d) both sides are multiplied by $-\frac{1}{3}$: $-7 \cdot -\frac{1}{3} > -3 \cdot -\frac{1}{3} \;\Rightarrow\; \frac{7}{3} > 1 \;\Rightarrow\; 1 < \frac{7}{3}$

2 (a) 7 is added to both sides: $4+7 > -5+7 \;\Rightarrow\; 11 > 2$

(b) -5 is subtracted from both sides: $4-(-5) > -5-(-5) \;\Rightarrow\; 9 > 0$

(c) both sides are divided by 6: $4/6 > -5/6 \;\Rightarrow\; \frac{2}{3} > -\frac{5}{6}$

(d) both sides are divided by -6: $4/(-6) < -5/(-6) \;\Rightarrow\; -\frac{2}{3} < \frac{5}{6}$

3 $x < -2 \;\Leftrightarrow\; (-\infty, -2)$

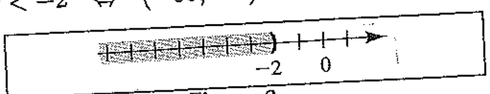

Figure 3

4 $x \leq 5 \;\Leftrightarrow\; (-\infty, 5]$

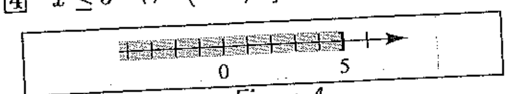

Figure 4

5 $x \geq 4 \;\Leftrightarrow\; [4, \infty)$

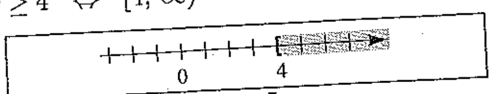

Figure 5

6 $x > -3 \;\Leftrightarrow\; (-3, \infty)$

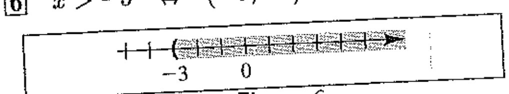

Figure 6

7 $-2 < x \leq 4 \;\Leftrightarrow\; (-2, 4]$

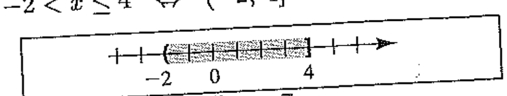

Figure 7

8 $-3 \leq x < 5 \;\Leftrightarrow\; [-3, 5)$

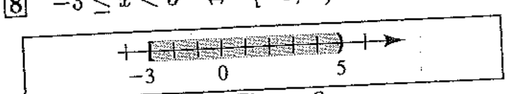

Figure 8

9 $3 \leq x \leq 7 \;\Leftrightarrow\; [3, 7]$

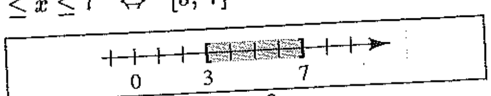

Figure 9

10 $-3 < x < -1 \;\Leftrightarrow\; (-3, -1)$

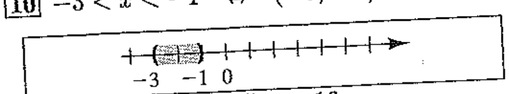

Figure 10

11 $5 > x \geq -2 \;\Rightarrow\; -2 \leq x < 5 \;\Leftrightarrow\; [-2, 5)$

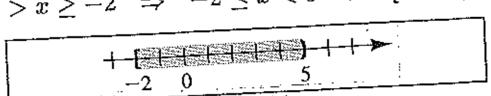

Figure 11

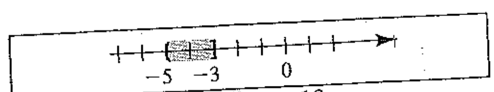

Figure 12

12 $-3 \geq x > -5 \;\Rightarrow\; -5 < x \leq -3 \;\Leftrightarrow\; (-5, -3]$

13 $(-5, 8] \;\Leftrightarrow\; -5 < x \leq 8$

14 $[0, 4) \;\Leftrightarrow\; 0 \leq x < 4$

15 $[-4, -1] \;\Leftrightarrow\; -4 \leq x \leq -1$

16 $(3, 7) \;\Leftrightarrow\; 3 < x < 7$

17 $[4, \infty) \;\Leftrightarrow\; x \geq 4$

18 $(-3, \infty) \;\Leftrightarrow\; x > -3$

19 $(-\infty, -5) \;\Leftrightarrow\; x < -5$

20 $(-\infty, 2] \;\Leftrightarrow\; x \leq 2$

21 $3x - 2 > 14 \;\Rightarrow\; 3x > 16 \;\Rightarrow\; x > \frac{16}{3} \;\Leftrightarrow\; (\frac{16}{3}, \infty)$

22 $2x + 5 \leq 7 \;\Rightarrow\; 2x \leq 2 \;\Rightarrow\; x \leq 1 \;\Leftrightarrow\; (-\infty, 1]$

23 $-2 - 3x \geq 2 \;\Rightarrow\; -3x \geq 4 \;\Rightarrow\; x \leq -\frac{4}{3} \;\Leftrightarrow\; (-\infty, -\frac{4}{3}]$

[24] $3 - 5x < 11 \Rightarrow -5x < 8 \Rightarrow x > -\frac{8}{5} \Leftrightarrow (-\frac{8}{5}, \infty)$

[25] $2x + 5 < 3x - 7 \Rightarrow -x < -12 \Rightarrow x > 12 \Leftrightarrow (12, \infty)$

[26] $x - 8 > 5x + 3 \Rightarrow -4x > 11 \Rightarrow x < -\frac{11}{4} \Leftrightarrow (-\infty, -\frac{11}{4})$

[27] $\left[9 + \frac{1}{3}x \geq 4 - \frac{1}{2}x\right] \cdot 6 \Rightarrow 54 + 2x \geq 24 - 3x \Rightarrow 5x \geq -30 \Rightarrow x \geq -6 \Leftrightarrow [-6, \infty)$

[28] $\left[\frac{1}{4}x + 7 \leq \frac{1}{3}x - 2\right] \cdot 12 \Rightarrow 3x + 84 \leq 4x - 24 \Rightarrow -x \leq -108 \Rightarrow$
$x \geq 108 \Leftrightarrow [108, \infty)$

[29] $-3 < 2x - 5 < 7 \Rightarrow 2 < 2x < 12 \Rightarrow 1 < x < 6 \Leftrightarrow (1, 6)$

[30] $4 \geq 3x + 5 > -1 \Rightarrow -1 < 3x + 5 \leq 4 \Rightarrow -6 < 3x \leq -1 \Rightarrow$
$-2 < x \leq -\frac{1}{3} \Leftrightarrow (-2, -\frac{1}{3}]$

[31] $\left[3 \leq \frac{2x - 3}{5} < 7\right] \cdot 5 \Rightarrow 15 \leq 2x - 3 < 35 \Rightarrow 18 \leq 2x < 38 \Rightarrow$
$9 \leq x < 19 \Leftrightarrow [9, 19)$

[32] $\left[-2 < \frac{4x + 1}{3} \leq 0\right] \cdot 3 \Rightarrow -6 < 4x + 1 \leq 0 \Rightarrow -7 < 4x \leq -1 \Rightarrow$
$-\frac{7}{4} < x \leq -\frac{1}{4} \Leftrightarrow (-\frac{7}{4}, -\frac{1}{4}]$

[33] $4 > \frac{2 - 3x}{7} \geq -2 \Rightarrow 28 > 2 - 3x \geq -14 \Rightarrow 26 > -3x \geq -16 \Rightarrow$
$-\frac{26}{3} < x \leq \frac{16}{3} \Leftrightarrow (-\frac{26}{3}, \frac{16}{3}]$

[34] $5 \geq \frac{6 - 5x}{3} > 2 \Rightarrow 15 \geq 6 - 5x > 6 \Rightarrow 9 \geq -5x > 0 \Rightarrow -\frac{9}{5} \leq x < 0 \Leftrightarrow [-\frac{9}{5}, 0)$

[35] $0 \leq 4 - \frac{1}{3}x < 2 \Rightarrow -4 \leq -\frac{1}{3}x < -2 \Rightarrow 12 \geq x > 6 \Rightarrow 6 < x \leq 12 \Leftrightarrow (6, 12]$

[36] $-2 < 3 + \frac{1}{4}x \leq 5 \Rightarrow -5 < \frac{1}{4}x \leq 2 \Rightarrow -20 < x \leq 8 \Leftrightarrow (-20, 8]$

[37] $(2x - 3)(4x + 5) \leq (8x + 1)(x - 7) \Rightarrow 8x^2 - 2x - 15 \leq 8x^2 - 55x - 7 \Rightarrow$
$53x \leq 8 \Rightarrow x \leq \frac{8}{53} \Leftrightarrow (-\infty, \frac{8}{53}]$

[38] $(x - 3)(x + 3) \geq (x + 5)^2 \Rightarrow x^2 - 9 \geq x^2 + 10x + 25 \Rightarrow -34 \geq 10x \Rightarrow$
$10x \leq -34 \Rightarrow x \leq -\frac{17}{5} \Leftrightarrow (-\infty, -\frac{17}{5}]$

[39] $(x - 4)^2 > x(x + 12) \Rightarrow x^2 - 8x + 16 > x^2 + 12x \Rightarrow -20x > -16 \Rightarrow$
$x < \frac{4}{5} \Leftrightarrow (-\infty, \frac{4}{5})$

[40] $2x(6x + 5) < (3x - 2)(4x + 1) \Rightarrow 12x^2 + 10x < 12x^2 - 5x - 2 \Rightarrow 15x < -2 \Rightarrow$
$x < -\frac{2}{15} \Leftrightarrow (-\infty, -\frac{2}{15})$

[41] By the law of signs, a quotient is positive if the sign of the numerator and the sign of the denominator are the same. Since the numerator is positive, $\frac{4}{3x + 2} > 0 \Rightarrow$ $3x + 2 > 0 \Rightarrow x > -\frac{2}{3} \Leftrightarrow (-\frac{2}{3}, \infty)$. The expression is never equal to 0 since the numerator is never 0. Thus, the solution of $\frac{4}{3x + 2} \geq 0$ is $(-\frac{2}{3}, \infty)$.

[42] $\frac{3}{2x + 5} \leq 0 \Rightarrow 2x + 5 < 0 \Rightarrow x < -\frac{5}{2} \Leftrightarrow (-\infty, -\frac{5}{2})$

[43] $\frac{-2}{4 - 3x} > 0 \Rightarrow 4 - 3x < 0$ {denominator must also be negative} $\Rightarrow$
$x > \frac{4}{3} \Leftrightarrow (\frac{4}{3}, \infty)$

$\boxed{44}$ $\dfrac{-3}{2-x} < 0 \;\Rightarrow\; 2-x>0 \;\Rightarrow\; x<2 \;\Leftrightarrow\; (-\infty,\, 2)$

$\boxed{45}$ $(1-x)^2 > 0\ \forall x$ except 1. Thus, $\dfrac{2}{(1-x)^2} > 0$ has solution $\mathbb{R} - \{1\}$.

$\boxed{46}$ $x^2+4>0\ \forall x$. Hence, $\dfrac{4}{x^2+4} > 0\ \forall x$, and $\dfrac{4}{x^2+4} < 0$ has no solution.

$\boxed{47}$ $|x| < 3 \;\Rightarrow\; -3 < x < 3 \;\Leftrightarrow\; (-3,\, 3)$

$\boxed{48}$ $|x| \leq 7 \;\Rightarrow\; -7 \leq x \leq 7 \;\Leftrightarrow\; [-7,\, 7]$

$\boxed{49}$ $|x| \geq 5 \;\Rightarrow\; x \geq 5$ or $x \leq -5 \;\Leftrightarrow\; (-\infty,\, -5] \cup [5,\, \infty)$

$\boxed{50}$ $|-x| > 2 \;\Rightarrow\; -x > 2$ or $-x < -2$ { or else first use $|-x| = |x|$ } $\Rightarrow$

$x < -2$ or $x > 2 \;\Leftrightarrow\; (-\infty,\, -2) \cup (2,\, \infty)$

$\boxed{51}$ $|x+3| < 0.01 \;\Rightarrow\; -0.01 < x+3 < 0.01 \;\Rightarrow$

$-3.01 < x < -2.99 \;\Leftrightarrow\; (-3.01,\, -2.99)$

$\boxed{52}$ $|x-4| \leq 0.03 \;\Rightarrow\; -0.03 \leq x-4 \leq 0.03 \;\Rightarrow\; 3.97 \leq x \leq 4.03 \;\Leftrightarrow\; [3.97,\, 4.03]$

$\boxed{53}$ $|x+2| + 0.1 \geq 0.2 \;\Rightarrow\; |x+2| \geq 0.1 \;\Rightarrow\; x+2 \geq 0.1$ or $x+2 \leq -0.1 \;\Rightarrow$

$x \geq -1.9$ or $x \leq -2.1 \;\Leftrightarrow\; (-\infty,\, -2.1] \cup [-1.9,\, \infty)$

$\boxed{54}$ $|x-3| - 0.3 > 0.1 \;\Rightarrow\; |x-3| > 0.4 \;\Rightarrow\; x-3 > 0.4$ or $x-3 < -0.4 \;\Rightarrow$

$x > 3.4$ or $x < 2.6 \;\Leftrightarrow\; (-\infty,\, 2.6) \cup (3.4,\, \infty)$

$\boxed{55}$ $|2x+5| < 4 \;\Rightarrow\; -4 < 2x+5 < 4 \;\Rightarrow\; -9 < 2x < -1 \;\Rightarrow$

$-\frac{9}{2} < x < -\frac{1}{2} \;\Leftrightarrow\; (-\frac{9}{2},\, -\frac{1}{2})$

$\boxed{56}$ $|3x-7| \geq 5 \;\Rightarrow\; 3x-7 \geq 5$ or $3x-7 \leq -5 \;\Rightarrow$

$x \geq 4$ or $x \leq \frac{2}{3} \;\Leftrightarrow\; (-\infty,\, \frac{2}{3}] \cup [4,\, \infty)$

$\boxed{57}$ $-\frac{1}{3}|6-5x| + 2 \geq 1 \;\Rightarrow\; -\frac{1}{3}|6-5x| \geq -1 \;\Rightarrow\; |6-5x| \leq 3 \;\Rightarrow$

$-3 \leq 6-5x \leq 3 \;\Rightarrow\; -9 \leq -5x \leq -3 \;\Rightarrow\; \frac{9}{5} \geq x \geq \frac{3}{5} \;\Leftrightarrow\; [\frac{3}{5},\, \frac{9}{5}]$

$\boxed{58}$ $2|-11-7x| - 2 > 10 \;\Rightarrow\; 2|-11-7x| > 12 \;\Rightarrow\; |-11-7x| > 6 \;\Rightarrow$

$-11-7x > 6$ or $-11-7x < -6 \;\Rightarrow\; -7x > 17$ or $-7x < 5 \;\Rightarrow$

$x < -\frac{17}{7}$ or $x > -\frac{5}{7} \;\Leftrightarrow\; (-\infty,\, -\frac{17}{7}) \cup (-\frac{5}{7},\, \infty)$

$\boxed{59}$ Since $|7x+2| \geq 0\ \forall x$, $|7x+2| > -2$ has solution $(-\infty,\, \infty)$.

$\boxed{60}$ Since $|6x-5| \geq 0\ \forall x$, $|6x-5| \leq -2$ has no solution.

$\boxed{61}$ $|3x-9| > 0\ \forall x$ except when $3x-9=0$, or $x=3$. The solution is $(-\infty,\, 3) \cup (3,\, \infty)$.

$\boxed{62}$ $|5x+2| = 0$ if $x = -\frac{2}{5}$, but is never less than 0.

Thus, $|5x+2| \leq 0$ has solution $x = -\frac{2}{5}$.

$\boxed{63}$ $\left|\dfrac{2-3x}{5}\right| \geq 2 \;\Rightarrow\; \dfrac{|2-3x|}{|5|} \geq 2 \;\Rightarrow\; |2-3x| \geq 10 \;\Rightarrow$

$2-3x \geq 10$ or $2-3x \leq -10 \;\Rightarrow\; -3x \geq 8$ or $-3x \leq -12 \;\Rightarrow$

$x \leq -\frac{8}{3}$ or $x \geq 4 \;\Leftrightarrow\; (-\infty,\, -\frac{8}{3}] \cup [4,\, \infty)$

64 $\left|\dfrac{2x+5}{3}\right|<1 \;\Rightarrow\; \dfrac{|2x+5|}{|3|}<1 \;\Rightarrow\; |2x+5|<3 \;\Rightarrow\; -3<2x+5<3 \;\Rightarrow$
$-8<2x<-2 \;\Rightarrow\; -4<x<-1 \;\Leftrightarrow\; (-4,\,-1)$

65 $\dfrac{3}{|5-2x|}<2 \;\Rightarrow\; |5-2x|>\frac{3}{2} \;\Rightarrow\; 5-2x>\frac{3}{2}$ or $5-2x<-\frac{3}{2} \;\Rightarrow$
$-2x>-\frac{7}{2}$ or $-2x<-\frac{13}{2} \;\Rightarrow\; x<\frac{7}{4}$ or $x>\frac{13}{4}\ \{x\neq\frac{5}{2}\} \;\Leftrightarrow\; (-\infty,\,\frac{7}{4})\cup(\frac{13}{4},\,\infty)$

66 $\dfrac{2}{|2x+3|}\geq 5 \;\Rightarrow\; |2x+3|\leq\frac{2}{5} \;\Rightarrow\; -\frac{2}{5}\leq 2x+3\leq\frac{2}{5} \;\Rightarrow\; -\frac{17}{5}\leq 2x\leq-\frac{13}{5} \;\Rightarrow$
$-\frac{17}{10}\leq x\leq-\frac{13}{10}\ \{x\neq-\frac{3}{2}\} \;\Leftrightarrow\; [-\frac{17}{10},\,-\frac{3}{2})\cup(-\frac{3}{2},\,-\frac{13}{10}]$

67 $-2<|x|<4 \;\Rightarrow\; |x|>-2$ *and* $|x|<4 \;\Rightarrow$
$|x|<4$ {since $|x|$ is always greater than -2} $\Rightarrow\; -4<x<4 \;\Leftrightarrow\; (-4,\,4)$

68 $1<|x|<5 \;\Rightarrow\; 1<x<5$ *or* $1<-x<5 \;\Rightarrow\; 1<x<5$ or $-1>x>-5 \;\Rightarrow$
$1<x<5$ or $-5<x<-1 \;\Leftrightarrow\; (-5,\,-1)\cup(1,\,5)$

69 From the definition of absolute value, $|x-2|$ equals either $x-2$ or $-(x-2)$.
Thus, $1<|x-2|<4 \;\Rightarrow\; 1<x-2<4$ or $1<-(x-2)<4 \;\Rightarrow$
$1<x-2<4$ or $-1>x-2>-4 \;\Rightarrow\; 3<x<6$ or $1>x>-2 \;\Leftrightarrow\; (-2,\,1)\cup(3,\,6)$.
An alternative method is to rewrite the inequality as $|x-2|>1$ *and* $|x-2|<4$.
Solving independently gives us
$x-2>1$ or $x-2<-1 \;\Rightarrow\; x>3$ or $x<1$ *and* $-4<x-2<4 \;\Rightarrow\; -2<x<6$.
Taking the *intersection* of these intervals gives $(-2,\,1)\cup(3,\,6)$.

70 $2<|2x-1|<3 \;\Rightarrow\; 2<2x-1<3$ or $2<-(2x-1)<3 \;\Rightarrow$
$2<2x-1<3$ or $-2>2x-1>-3 \;\Rightarrow\; 3<2x<4$ or $-1>2x>-2 \;\Rightarrow$
$\frac{3}{2}<x<2$ or $-\frac{1}{2}>x>-1 \;\Leftrightarrow\; (-1,\,-\frac{1}{2})\cup(\frac{3}{2},\,2)$

71 (a) $|x+5|=3 \;\Rightarrow\; x+5=3$ or $x+5=-3 \;\Rightarrow\; x=-2$ or $x=-8$.
(b) $|x+5|<3$ has solutions between the values found in part (a), that is, $(-8,\,-2)$.
(c) The solutions of $|x+5|>3$ are the portions of the real line that are not in parts (a) and (b), that is, $(-\infty,\,-8)\cup(-2,\,\infty)$.

72 (a) $|x-3|<2 \;\Rightarrow\; -2<x-3<2 \;\Rightarrow\; 1<x<5 \;\Leftrightarrow\; (1,\,5)$.
(b) $|x-3|=2$ has solutions at the endpoints of the interval in part (a); that is, at $x=1$ and $x=5$.
(c) As in Exercise 71(c), $|x-3|>2$ has solutions in $(-\infty,\,1)\cup(5,\,\infty)$.

73 $|w-148|\leq 2$

74 $|r-1|\leq 0.01$

75 $5<|T_1-T_2|<10$

76 5 minutes is $\frac{1}{12}$ of an hour. $|t-4|\geq\frac{1}{12}$

77 $30\leq C\leq 40 \;\Rightarrow\; 30\leq\frac{5}{9}(F-32)\leq 40 \;\Rightarrow\; 54\leq F-32\leq 72 \;\Rightarrow\; 86\leq F\leq 104$

78 $10\leq F\leq 18 \;\Rightarrow\; 10\leq(4.5)x\leq 18 \;\Rightarrow\; 10\leq\frac{9}{2}x\leq 18 \;\Rightarrow\; \frac{20}{9}\leq x\leq 4$

79 $R=\dfrac{110}{I} \;\Rightarrow\; I=\dfrac{110}{R}.\ I\leq 10 \;\Rightarrow\; \dfrac{110}{R}\leq 10 \;\Rightarrow\; 110\leq 10R\ \{R>0\} \;\Rightarrow\; R\geq 11$

$\boxed{80}$ $\frac{1}{R}=\frac{1}{10}+\frac{1}{R_2} \Rightarrow \frac{1}{R}=\frac{R_2+10}{10R_2} \Rightarrow R=\frac{10R_2}{R_2+10}$. $R<5 \Rightarrow \frac{10R_2}{R_2+10}<5 \Rightarrow$

$10R_2<5R_2+50\ \{R_2+10>0\} \Rightarrow 5R_2<50 \Rightarrow R_2<10 \Rightarrow$

$0<R_2<10\ \{R_2>0\}$

$\boxed{81}$ $M\geq 3 \Rightarrow \frac{6}{6-p}\geq 3 \Rightarrow 6\geq 18-3p\ \{6-p>0\} \Rightarrow$

$p\geq 4$, but $p<6$ since $p<f$. Thus, $4\leq p<6$.

$\boxed{82}$ $c>1.5 \Rightarrow \frac{3.5t}{t+1}>1.5 \Rightarrow \{t+1>0\}\ 3.5t>1.5t+1.5 \Rightarrow t>\frac{3}{4}$ hr

$\boxed{83}$ $\text{Cost}_A<\text{Cost}_B \Rightarrow 50{,}000+4000x<40{,}000+5500x \Rightarrow 10{,}000<1500x \Rightarrow$

$x>\frac{20}{3}$, or $6\frac{2}{3}$ yr

$\boxed{84}$ Let t denote the time in years from the present. $\text{Cost}_B<\text{Cost}_A \Rightarrow$

$\text{Purchase}_B+\text{Insurance}_B+\text{Gas}_B<\text{Purchase}_A+\text{Insurance}_A+\text{Gas}_A \Rightarrow$

$12{,}000+600t+\frac{15{,}000}{50}\cdot 1.25t<10{,}000+550t+\frac{15{,}000}{30}\cdot 1.25t \Rightarrow$

$12{,}000+975t<10{,}000+1175t \Rightarrow 2000<200t \Rightarrow t>10$ yr.

$\boxed{85}$ (a) 5 ft 9 in $=69$ in. In a 40 year period, a person's height will decrease by $40\times 0.024=0.96$ in ≈ 1 in. The person will be approximately one inch shorter, or 5 ft 8 in. at age 70.

(b) 5 ft 6 in $=66$ in. In 20 years, a person's height $(h=66)$ will change by $0.024\times 20=0.48$ in. Thus, $66-0.48\leq h\leq 66+0.48 \Rightarrow 65.52\leq h\leq 66.48$.

2.7 Exercises

$\boxed{1}$ $(3x+1)(5-10x)>0$ has solutions in the interval $(-\frac{1}{3},\frac{1}{2})$. See *Diagram 1* for details concerning the signs of the individual factors and the resulting sign.

Resulting sign:	⊖	⊕	⊖
Sign of $5-10x$:	+	+	−
Sign of $3x+1$:	−	+	+
x values:	$-1/3$	$1/2$	

Diagram 1

Resulting sign:	⊖	⊕	⊖
Sign of $4x-7$:	−	−	+
Sign of $2-3x$:	+	−	−
x values:	$2/3$	$7/4$	

Diagram 2

$\boxed{2}$ $(2-3x)(4x-7)\geq 0$; $[\frac{2}{3},\frac{7}{4}]$

$\boxed{3}$ $(x+2)(x-1)(4-x)\leq 0$; $[-2,1]\cup[4,\infty)$

Resulting sign:	⊕	⊖	⊕	⊖
Sign of $4-x$:	+	+	+	−
Sign of $x-1$:	−	−	+	+
Sign of $x+2$:	−	+	+	+
x values:	-2	1	4	

Diagram 3

Resulting sign:	⊕	⊖	⊕	⊖
Sign of $-2-x$:	+	+	−	−
Sign of $x-5$:	−	−	−	+
Sign of $x+3$:	−	+	+	+
x values:	-3	-2	5	

Diagram 4

$\boxed{4}$ $(x-5)(x+3)(-2-x)<0$; $(-3,-2)\cup(5,\infty)$

[5] $x^2 - x - 6 < 0 \;\Rightarrow\; (x-3)(x+2) < 0$; $(-2, 3)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-3$:	$-$	$-$	$+$
Sign of $x+2$:	$-$	$+$	$+$
x values:	-2	3	

Diagram 5

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x+1$:	$-$	$-$	$+$
Sign of $x+3$:	$-$	$+$	$+$
x values:	-3	-1	

Diagram 6

[6] $x^2 + 4x + 3 \geq 0 \;\Rightarrow\; (x+1)(x+3) \geq 0$; $(-\infty, -3] \cup [-1, \infty)$

[7] $x^2 - 2x - 5 > 3 \;\Rightarrow\; x^2 - 2x - 8 > 0 \;\Rightarrow\; (x-4)(x+2) > 0$; $(-\infty, -2) \cup (4, \infty)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-4$:	$-$	$-$	$+$
Sign of $x+2$:	$-$	$+$	$+$
x values:	-2	4	

Diagram 7

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-7$:	$-$	$-$	$+$
Sign of $x+3$:	$-$	$+$	$+$
x values:	-3	7	

Diagram 8

[8] $x^2 - 4x - 17 \leq 4 \;\Rightarrow\; x^2 - 4x - 21 \leq 0 \;\Rightarrow\; (x-7)(x+3) \leq 0$; $[-3, 7]$

[9] $x(2x+3) \geq 5 \;\Rightarrow\; 2x^2 + 3x - 5 \geq 0 \;\Rightarrow\; (2x+5)(x-1) \geq 0$; $(-\infty, -\frac{5}{2}] \cup [1, \infty)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-1$:	$-$	$-$	$+$
Sign of $2x+5$:	$-$	$+$	$+$
x values:	$-5/2$	1	

Diagram 9

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $3x-4$:	$-$	$-$	$+$
Sign of $x+1$:	$-$	$+$	$+$
x values:	-1	$4/3$	

Diagram 10

[10] $x(3x-1) \leq 4 \;\Rightarrow\; 3x^2 - x - 4 \leq 0 \;\Rightarrow\; (3x-4)(x+1) \leq 0$; $[-1, \frac{4}{3}]$

[11] $6x - 8 > x^2 \;\Rightarrow\; x^2 - 6x + 8 < 0 \;\Rightarrow\; (x-2)(x-4) < 0$; $(2, 4)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-4$:	$-$	$-$	$+$
Sign of $x-2$:	$-$	$+$	$+$
x values:	2	4	

Diagram 11

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-4$:	$-$	$-$	$+$
Sign of $x+3$:	$-$	$+$	$+$
x values:	-3	4	

Diagram 12

[12] $x + 12 \leq x^2 \;\Rightarrow\; x^2 - x - 12 \geq 0 \;\Rightarrow\; (x-4)(x+3) \geq 0$; $(-\infty, -3] \cup [4, \infty)$

Note: Solving $x^2 <$ (or $>$) a^2 for $a > 0$ may be solved using factoring, that is, $x^2 - a^2 < 0 \;\Rightarrow\; (x+a)(x-a) < 0 \;\Rightarrow\; -a < x < a$; or by taking the square root of each side, that is, $\sqrt{x^2} < \sqrt{a^2} \;\Rightarrow\; |x| < a \;\Rightarrow\; -a < x < a$.

[13] $x^2 < 16 \;\Rightarrow\; |x| < 4 \;\Rightarrow\; -4 < x < 4 \;\Leftrightarrow\; (-4, 4)$

[14] $x^2 > 9 \;\Rightarrow\; |x| > 3 \;\Rightarrow\; x > 3$ or $x < -3 \;\Leftrightarrow\; (-\infty, -3) \cup (3, \infty)$

[15] $25x^2 - 9 < 0 \;\Rightarrow\; x^2 < \frac{9}{25} \;\Rightarrow\; |x| < \frac{3}{5} \;\Rightarrow\; -\frac{3}{5} < x < \frac{3}{5} \;\Leftrightarrow\; (-\frac{3}{5}, \frac{3}{5})$

[16] $25x^2 - 9x < 0 \;\Rightarrow\; x(25x-9) < 0$; $(0, \frac{9}{25})$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $25x-9$:	$-$	$-$	$+$
Sign of x:	$-$	$+$	$+$
x values:	0	$9/25$	

Diagram 16

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $16x-9$:	$-$	$-$	$+$
Sign of x:	$-$	$+$	$+$
x values:	0	$9/16$	

Diagram 17

[17] $16x^2 \geq 9x \;\Rightarrow\; x(16x-9) \geq 0$; $(-\infty, 0] \cup [\frac{9}{16}, \infty)$

[18] $16x^2 > 9 \;\Rightarrow\; x^2 > \frac{9}{16} \;\Rightarrow\; |\,x\,| > \frac{3}{4} \;\Rightarrow\; x > \frac{3}{4}$ or $x < -\frac{3}{4} \;\Leftrightarrow\; (-\infty, -\frac{3}{4}) \cup (\frac{3}{4}, \infty)$

[19] $x^4 + 5x^2 \geq 36 \;\Rightarrow\; x^4 + 5x^2 - 36 \geq 0 \;\Rightarrow\; (x^2+9)(x^2-4) \geq 0 \;\Rightarrow$
$x^2 - 4 \geq 0 \; \{\, x^2+9 > 0 \,\} \;\Rightarrow\; x^2 \geq 4 \;\Rightarrow\; |\,x\,| \geq 2 \;\Rightarrow$
$x \geq 2$ or $x \leq -2 \;\Leftrightarrow\; (-\infty, -2] \cup [2, \infty)$

[20] $x^4 + 15x^2 < 16 \;\Rightarrow\; x^4 + 15x^2 - 16 < 0 \;\Rightarrow\; (x^2+16)(x^2-1) < 0 \;\Rightarrow$
$x^2 - 1 < 0 \; \{\, x^2 + 16 > 0 \,\} \;\Rightarrow\; x^2 < 1 \;\Rightarrow\; |\,x\,| < 1 \;\Rightarrow\; -1 < x < 1 \;\Leftrightarrow\; (-1, 1)$

[21] $x^3 + 2x^2 - 4x - 8 \geq 0 \;\Rightarrow\; x^2(x+2) - 4(x+2) \geq 0 \;\Rightarrow\; (x^2-4)(x+2) \geq 0 \;\Rightarrow$
$(x-2)(x+2)^2 \geq 0 \;\Rightarrow\; x \geq 2$ or $x = -2 \;\Leftrightarrow\; \{-2\} \cup [2, \infty)$

[22] $2x^3 - 3x^2 - 2x + 3 \leq 0 \;\Rightarrow$

$x^2(2x-3) - 1(2x-3) \leq 0 \;\Rightarrow$

$(x^2-1)(2x-3) \leq 0$; $(-\infty, -1] \cup [1, \frac{3}{2}]$

Resulting sign:	⊖	⊕	⊖	⊕
Sign of $2x-3$:	−	−	−	+
Sign of $x-1$:	−	−	+	+
Sign of $x+1$:	−	+	+	+
x values:		-1	1	$3/2$

Diagram 22

[23] $\dfrac{x^2(x+2)}{(x+2)(x+1)} \leq 0 \;\Rightarrow\; \dfrac{x^2}{x+1} \leq 0 \; \{\, \text{exclude } -2 \,\} \;\Rightarrow\; \dfrac{1}{x+1} \leq 0 \; \{\, \text{include } 0 \,\} \;\Rightarrow$
$x + 1 < 0 \;\Rightarrow\; x < -1$; $(-\infty, -2) \cup (-2, -1) \cup \{\, 0 \,\}$

[24] $\dfrac{(x^2+1)(x-3)}{x^2-9} \geq 0 \;\Rightarrow\; \dfrac{x-3}{(x+3)(x-3)} \geq 0 \; \{\, x^2+1 > 0 \,\} \;\Rightarrow$
$\dfrac{1}{x+3} \geq 0 \; \{\, \text{exclude } 3 \,\} \;\Rightarrow\; x + 3 > 0 \; \{\, \text{exclude } -3 \,\} \;\Rightarrow\; x > -3$; $(-3, 3) \cup (3, \infty)$

[25] $\dfrac{x^2 - x}{x^2 + 2x} \leq 0 \;\Rightarrow\; \dfrac{x(x-1)}{x(x+2)} \leq 0 \;\Rightarrow\; \dfrac{x-1}{x+2} \leq 0 \; \{\, \text{exclude } 0 \,\}$; $(-2, 0) \cup (0, 1]$

Resulting sign:	⊕	⊖	⊕
Sign of $x-1$:	−	−	+
Sign of $x+2$:	−	+	+
x values:		-2	1

Diagram 25

Resulting sign:	⊕	⊖	⊕
Sign of $x+2$:	−	−	+
Sign of $x+4$:	−	+	+
x values:		-4	-2

Diagram 26

[26] $\dfrac{(x+3)^2(2-x)}{(x+4)(x^2-4)} \leq 0 \;\Rightarrow\; \dfrac{2-x}{(x+4)(x+2)(x-2)} \leq 0 \; \{\, \text{include } -3 \,\} \;\Rightarrow$

$\dfrac{1}{(x+4)(x+2)} \geq 0 \; \{\, \text{cancel, change inequality, exclude } 2 \,\}$;
$(-\infty, -4) \cup \{\, -3 \,\} \cup (-2, 2) \cup (2, \infty)$

[27] $\dfrac{x-2}{x^2 - 3x - 10} \geq 0 \;\Rightarrow\; \dfrac{x-2}{(x-5)(x+2)} \geq 0$; $(-2, 2] \cup (5, \infty)$

Resulting sign:	⊖	⊕	⊖	⊕
Sign of $x-5$:	−	−	−	+
Sign of $x-2$:	−	−	+	+
Sign of $x+2$:	−	+	+	+
x values:		-2	2	5

Diagram 27

Resulting sign:	⊖	⊕	⊖	⊕
Sign of $x-4$:	−	−	−	+
Sign of $x-3$:	−	−	+	+
Sign of $x+5$:	−	+	+	+
x values:		-5	3	4

Diagram 28

[28] $\dfrac{x+5}{x^2 - 7x + 12} \leq 0 \;\Rightarrow\; \dfrac{x+5}{(x-3)(x-4)} \leq 0$; $(-\infty, -5] \cup (3, 4)$

29 $\dfrac{-3x}{x^2-9} > 0 \;\Rightarrow\; \dfrac{x}{(x+3)(x-3)} < 0$ {divide by -3}; $(-\infty, -3) \cup (0, 3)$

Resulting sign:	$\ominus$	$\oplus$	$\ominus$	$\oplus$
Sign of $x-3$:	$-$	$-$	$-$	$+$
Sign of x:	$-$	$-$	$+$	$+$
Sign of $x+3$:	$-$	$+$	$+$	$+$
x values:	-3	0	3	

Diagram 29

Resulting sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$
Sign of $4-x$:	$+$	$+$	$+$	$-$
Sign of x:	$-$	$-$	$+$	$+$
Sign of $4+x$:	$-$	$+$	$+$	$+$
x values:	-4	0	4	

Diagram 30

30 $\dfrac{2x}{16-x^2} < 0 \;\Rightarrow\; \dfrac{x}{(4+x)(4-x)} < 0$ {divide by 2}; $(-4, 0) \cup (4, \infty)$

31 $\dfrac{x+1}{2x-3} > 2 \;\Rightarrow\; \dfrac{x+1-2(2x-3)}{2x-3} > 0 \;\Rightarrow\; \dfrac{-3x+7}{2x-3} > 0$; $(\frac{3}{2}, \frac{7}{3})$

Resulting sign:	$\ominus$	$\oplus$	$\ominus$
Sign of $-3x+7$:	$+$	$+$	$-$
Sign of $2x-3$:	$-$	$+$	$+$
x values:	$3/2$	$7/3$	

Diagram 31

Resulting sign:	$\ominus$	$\oplus$	$\ominus$
Sign of $3x+5$:	$-$	$-$	$+$
Sign of $-11x-22$:	$+$	$-$	$-$
x values:	-2	$-5/3$	

Diagram 32

32 $\dfrac{x-2}{3x+5} \le 4 \;\Rightarrow\; \dfrac{x-2-4(3x+5)}{3x+5} \le 0 \;\Rightarrow\; \dfrac{-11x-22}{3x+5} \le 0$; $(-\infty, -2] \cup (-\frac{5}{3}, \infty)$

33 $\dfrac{1}{x-2} \ge \dfrac{3}{x+1} \;\Rightarrow\; \dfrac{1(x+1)-3(x-2)}{(x-2)(x+1)} \ge 0 \;\Rightarrow\; \dfrac{-2x+7}{(x-2)(x+1)} \ge 0$; $(-\infty, -1) \cup (2, \frac{7}{2}]$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$
Sign of $-2x+7$:	$+$	$+$	$+$	$-$
Sign of $x-2$:	$-$	$-$	$+$	$+$
Sign of $x+1$:	$-$	$+$	$+$	$+$
x values:	-1	2	$7/2$	

Diagram 33

Resulting sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$
$x-5$:	$-$	$-$	$-$	$+$
$2x+3$:	$-$	$-$	$+$	$+$
$-2x-16$:	$+$	$-$	$-$	$-$
x values:	-8	$-3/2$	5	

Diagram 34

34 $\dfrac{2}{2x+3} \le \dfrac{2}{x-5} \;\Rightarrow\; \dfrac{2(x-5)-2(2x+3)}{(2x+3)(x-5)} \le 0 \;\Rightarrow\; \dfrac{-2x-16}{(2x+3)(x-5)} \le 0$;

$[-8, -\frac{3}{2}) \cup (5, \infty)$

35 $\dfrac{4}{3x-2} \le \dfrac{2}{x+1} \;\Rightarrow\; \dfrac{4(x+1)-2(3x-2)}{(3x-2)(x+1)} \le 0 \;\Rightarrow\; \dfrac{-2x+8}{(3x-2)(x+1)} \le 0$; $(-1, \frac{2}{3}) \cup [4, \infty)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$
Sign of $-2x+8$:	$+$	$+$	$+$	$-$
Sign of $3x-2$:	$-$	$-$	$+$	$+$
Sign of $x+1$:	$-$	$+$	$+$	$+$
x values:	-1	$2/3$	4	

Diagram 35

Resulting sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$
$x-3$:	$-$	$-$	$-$	$+$
$5x+1$:	$-$	$-$	$+$	$+$
$-2x-10$:	$+$	$-$	$-$	$-$
x values:	-5	$-1/5$	3	

Diagram 36

36 $\dfrac{3}{5x+1} \ge \dfrac{1}{x-3} \;\Rightarrow\; \dfrac{3(x-3)-1(5x+1)}{(5x+1)(x-3)} \ge 0 \;\Rightarrow\; \dfrac{-2x-10}{(5x+1)(x-3)} \ge 0$;

$(-\infty, -5] \cup (-\frac{1}{5}, 3)$

37 $\dfrac{x}{3x-5} \le \dfrac{2}{x-1} \;\Rightarrow\; \dfrac{x(x-1)-2(3x-5)}{(3x-5)(x-1)} \le 0 \;\Rightarrow\; \dfrac{(x-2)(x-5)}{(3x-5)(x-1)} \le 0$; $(1, \frac{5}{3}) \cup [2, 5]$

Res. sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$	$\oplus$
$x-5$:	$-$	$-$	$-$	$-$	$+$
$x-2$:	$-$	$-$	$-$	$+$	$+$
$3x-5$:	$-$	$-$	$+$	$+$	$+$
$x-1$:	$-$	$+$	$+$	$+$	$+$
x values:	1	5/3	2	5	

Diagram 37

Res. sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$	$\oplus$
$x-3$:	$-$	$-$	$-$	$-$	$+$
$x-1$:	$-$	$-$	$-$	$+$	$+$
$2x-1$:	$-$	$-$	$+$	$+$	$+$
$x+2$:	$-$	$+$	$+$	$+$	$+$
x values:	-2	1/2	1	3	

Diagram 38

38 $\dfrac{x}{2x-1} \ge \dfrac{3}{x+2} \;\Rightarrow\; \dfrac{x(x+2)-3(2x-1)}{(2x-1)(x+2)} \ge 0 \;\Rightarrow\; \dfrac{(x-1)(x-3)}{(2x-1)(x+2)} \ge 0$;

$(-\infty, -2) \cup (\frac{1}{2}, 1] \cup [3, \infty)$

39 $x^3 > x \;\Rightarrow\; x^3 - x > 0 \;\Rightarrow\; x(x^2-1) > 0 \;\Rightarrow\; x(x+1)(x-1) > 0$; $(-1, 0) \cup (1, \infty)$

Resulting sign:	$\ominus$	$\oplus$	$\ominus$	$\oplus$
Sign of $x-1$:	$-$	$-$	$-$	$+$
Sign of x:	$-$	$-$	$+$	$+$
Sign of $x+1$:	$-$	$+$	$+$	$+$
x values:	-1	0	1	

Diagram 39

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-1$:	$-$	$-$	$+$
Sign of $x+1$:	$-$	$+$	$+$
x values:	-1	1	

Diagram 40

40 $x^4 \ge x^2 \;\Rightarrow\; x^4 - x^2 \ge 0 \;\Rightarrow\; x^2(x+1)(x-1) \ge 0$.

Since $x^2 \ge 0$, x^2 does not need to be included in the sign diagram, but 0 must be included in the answer because of the equality. ★ $(-\infty, -1] \cup \{0\} \cup [1, \infty)$

41 $v \ge k \;\Rightarrow\; t^3 - 3t^2 - 4t + 20 \ge 8 \;\Rightarrow\; t^3 - 3t^2 - 4t + 12 \ge 0 \;\Rightarrow$

$t^2(t-3) - 4(t-3) \ge 0 \;\Rightarrow\; (t^2-4)(t-3) \ge 0 \;\Rightarrow\; (t+2)(t-2)(t-3) \ge 0$.

See *Diagram 41*. For $[0, 5]$, we have $[0, 2] \cup [3, 5]$.

Resulting sign:	$\ominus$	$\oplus$	$\ominus$	$\oplus$
Sign of $t-3$:	$-$	$-$	$-$	$+$
Sign of $t-2$:	$-$	$-$	$+$	$+$
Sign of $t+2$:	$-$	$+$	$+$	$+$
t values:	-2	2	3	

Diagram 41

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $t-2$:	$-$	$-$	$+$
Sign of $t+2$:	$-$	$+$	$+$
t values:	-2	2	

Diagram 42

42 $v \ge k \;\Rightarrow\; t^4 - 4t^2 + 10 \ge 10 \;\Rightarrow\; t^4 - 4t^2 \ge 0 \;\Rightarrow\; t^2(t^2-4) \ge 0 \;\Rightarrow$

$t^2(t+2)(t-2) \ge 0$. See *Diagram 42*. For $[1, 6]$, we have $[2, 6]$.

43 $s > 9 \;\Rightarrow\; -16t^2 + 24t + 1 > 9 \;\Rightarrow\; -16t^2 + 24t - 8 > 0 \;\Rightarrow$

$2t^2 - 3t + 1 < 0$ {divide by -8} $\;\Rightarrow\; (2t-1)(t-1) < 0 \;\Rightarrow\; \frac{1}{2} < t < 1$.

The dog is more than 9 ft off the ground for $1 - \frac{1}{2} = \frac{1}{2}$ sec.

44 $s \ge 1536 \;\Rightarrow\; -16t^2 + 320t \ge 1536 \;\Rightarrow\; -16t^2 + 320t - 1536 \ge 0 \;\Rightarrow$

$t^2 - 20t + 96 \le 0 \;\Rightarrow\; (t-8)(t-12) \le 0 \;\Leftrightarrow\; 8 \le t \le 12$

45 $d < 75 \;\Rightarrow\; v + \frac{1}{20}v^2 < 75 \;\Rightarrow\; v^2 + 20v - 1500 < 0 \;\Rightarrow\; (v+50)(v-30) < 0 \;\Rightarrow$

$-50 < v < 30 \;\Rightarrow\; 0 \le v < 30$ $\{v \ge 0\}$

[46] $M \geq 45 \;\Rightarrow\; -\frac{1}{30}v^2 + \frac{5}{2}v \geq 45 \;\Rightarrow\; -\frac{1}{30}v^2 + \frac{5}{2}v - 45 \geq 0 \;\Rightarrow$

$v^2 - 75v + 1350 \leq 0 \;\Rightarrow\; (v-30)(v-45) \leq 0 \;\Leftrightarrow\; 30 \leq v \leq 45$

[47] $R > S \;\Rightarrow\; \dfrac{4500\,S}{S+500} > S \;\Rightarrow\; \dfrac{S(S-4000)}{S+500} < 0 \;\{S > 0\} \;\Rightarrow\; 0 < S < 4000$

[48] $D > 400 \;\Rightarrow\; \dfrac{5000x}{x^2+36} > 400 \;\Rightarrow$

$25x > 2(x^2+36) \left\{\text{multiply by } \dfrac{x^2+36}{200}, \text{ which is positive}\right\} \;\Rightarrow$

$2x^2 - 25x + 72 < 0 \;\Rightarrow\; (2x-9)(x-8) < 0 \;\Rightarrow\; 4.5 < x < 8$

[49] $W < 5 \;\Rightarrow\; 125\left(\dfrac{6400}{6400+x}\right)^2 < 5 \;\Rightarrow\; \left(\dfrac{6400}{6400+x}\right)^2 < \left(\dfrac{1}{5}\right)^2 \;\Rightarrow$

$\dfrac{6400}{6400+x} < \dfrac{1}{5} \left\{\text{since } \dfrac{6400}{6400+x} > 0\right\} \;\Rightarrow\; 32{,}000 < x + 6400 \;\Rightarrow\; x > 25{,}600 \text{ km.}$

[50] $L < \frac{1}{2}L_0 \;\Rightarrow\; L^2 < \frac{1}{4}L_0^2 \;\Rightarrow\; L_0^2\left(1 - \dfrac{v^2}{c^2}\right) < \frac{1}{4}L_0^2 \;\Rightarrow\; 1 - \dfrac{v^2}{c^2} < \dfrac{1}{4} \;\Rightarrow\; \dfrac{3}{4} < \dfrac{v^2}{c^2} \;\Rightarrow$

$v^2 > \frac{3}{4}c^2 \;\Rightarrow\; v > \frac{1}{2}\sqrt{3}\,c$ since $v > 0$ and $c > 0$

[51] $7500 \leq 0.00334V^2S \leq 10{,}000 \;\Rightarrow\; 7500 \leq 0.00334(210)V^2 \leq 10{,}000 \;\Rightarrow$

$\dfrac{7500}{0.7014} \leq V^2 \leq \dfrac{10{,}000}{0.7014} \;\Rightarrow\; \sqrt{\dfrac{7500}{0.7014}} \leq V \leq \sqrt{\dfrac{10{,}000}{0.7014}} \;\Rightarrow$

$103.4 \leq V \leq 119.4$ ft/sec $\;\Rightarrow$

{multiply by $\frac{60}{88} = \frac{15}{22}$ to convert} $70.5 \leq V \leq 81.4$ mi/hr.

[52] The numerator is equal to zero when $x = 2,\ 3$ and the denominator is equal to zero when $x = \pm 1$. From the table, the expression $Y_1 = \dfrac{(2-x)(3x-9)}{(1-x)(x+1)}$ is positive when $x \in [-2, -1) \cup (1, 2) \cup (3, 3.5]$.

x	Y_1	x	Y_1
−2.0	20	1.0	ERROR
−1.5	37.8	1.5	1.8
−1.0	ERROR	2.0	0
−0.5	−35	2.5	−0.1429
0.0	−18	3.0	0
0.5	−15	3.5	0.2

[53] By using a table it can be shown that the expression is equal to zero when $x = -3$, -2, 2, 4. The expression $Y_1 = x^4 - x^3 - 16x^2 + 4x + 48$ is negative when $x \in (-3, -2) \cup (2, 4)$.

x	Y_1	x	Y_1
−3.5	30.938	1.0	36
−3.0	0	1.5	19.688
−2.5	−7.313	2.0	0
−2.0	0	2.5	−18.56
−1.5	14.438	3.0	−30
−1.0	30	3.5	−26.81
−0.5	42.188	4.0	0
0.0	48	4.5	60.938
0.5	45.938	5.0	168

Chapter 2 Review Exercises

[1] $\left[\frac{3x+1}{5x+7} = \frac{6x+11}{10x-3}\right] \cdot (5x+7)(10x-3) \Rightarrow (3x+1)(10x-3) =$

$(6x+11)(5x+7) \Rightarrow 30x^2 + x - 3 = 30x^2 + 97x + 77 \Rightarrow 96x = -80 \Rightarrow x = -\frac{5}{6}$

[2] $\left[2 - \frac{1}{x} = 1 + \frac{4}{x}\right] \cdot x \Rightarrow 2x - 1 = x + 4 \Rightarrow x = 5$

[3] $\left[\frac{2}{x+5} - \frac{3}{2x+1} = \frac{5}{6x+3}\right] \cdot 3(x+5)(2x+1) \Rightarrow 6(2x+1) - 9(x+5) = 5(x+5) \Rightarrow$

$3x - 39 = 5x + 25 \Rightarrow -2x = 64 \Rightarrow x = -32$

[4] $\left[\frac{7}{x-2} - \frac{6}{x^2-4} = \frac{3}{2x+4}\right] \cdot 2(x+2)(x-2) \Rightarrow 14(x+2) - 12 = 3(x-2) \Rightarrow$

$11x = -22 \Rightarrow x = -2$, which is not in the domain of the given expressions.

No solution

[5] $\text{LS} = \frac{1}{\sqrt{x}} - 2 = \frac{1 - 2\sqrt{x}}{\sqrt{x}} = \text{RS}$, an identity.

The given equation is true for every $x > 0$.

[6] $2x^2 + 5x - 12 = 0 \Rightarrow (x+4)(2x-3) = 0 \Rightarrow x = -4, \frac{3}{2}$

[7] $x(3x+4) = 5 \Rightarrow 3x^2 + 4x - 5 = 0 \Rightarrow$

$$x = \frac{-4 \pm \sqrt{16+60}}{6} = \frac{-4 \pm 2\sqrt{19}}{6} = -\frac{2}{3} \pm \frac{1}{3}\sqrt{19}$$

[8] $\left[\frac{x}{3x+1} = \frac{x-1}{2x+3}\right] \cdot (3x+1)(2x+3) \Rightarrow x(2x+3) = (x-1)(3x+1) \Rightarrow$

$2x^2 + 3x = 3x^2 - 2x - 1 \Rightarrow x^2 - 5x - 1 = 0 \Rightarrow x = \frac{5 \pm \sqrt{25+4}}{2} = \frac{5}{2} \pm \frac{1}{2}\sqrt{29}$

$\boxed{9}$ $(x-2)(x+1)=3 \Rightarrow x^2-x-5=0 \Rightarrow x=\dfrac{1\pm\sqrt{1+20}}{2}=\frac{1}{2}\pm\frac{1}{2}\sqrt{21}$

$\boxed{10}$ $4x^4-33x^2+50=0 \Rightarrow (4x^2-25)(x^2-2) \Rightarrow x^2=\frac{25}{4}, 2 \Rightarrow x=\pm\frac{5}{2}, \pm\sqrt{2}$

$\boxed{11}$ $x^{2/3}-2x^{1/3}-15=0 \Rightarrow (x^{1/3}+3)(x^{1/3}-5)=0 \Rightarrow \sqrt[3]{x}=-3, 5 \Rightarrow$
$x=-27, 125$

$\boxed{12}$ $20x^3+8x^2-35x-14=0 \Rightarrow 4x^2(5x+2)-7(5x+2)=0 \Rightarrow$
$(4x^2-7)(5x+2)=0 \Rightarrow x=\pm\frac{1}{2}\sqrt{7}, -\frac{2}{5}$

$\boxed{13}$ $5x^2=2x-3 \Rightarrow 5x^2-2x+3=0 \Rightarrow x=\dfrac{2\pm\sqrt{4-60}}{10}=\dfrac{2\pm2\sqrt{14}i}{10}=\dfrac{1}{5}\pm\dfrac{1}{5}\sqrt{14}i$

$\boxed{14}$ $x^2+\frac{1}{3}x+2=0 \Rightarrow 3x^2+x+6=0 \Rightarrow x=\dfrac{-1\pm\sqrt{1-72}}{6}=-\dfrac{1}{6}\pm\dfrac{1}{6}\sqrt{71}i$

$\boxed{15}$ $6x^4+29x^2+28=0 \Rightarrow (2x^2+7)(3x^2+4)=0 \Rightarrow x^2=-\frac{7}{2}, -\frac{4}{3} \Rightarrow$
$x=\pm\frac{1}{2}\sqrt{14}i, \pm\frac{2}{3}\sqrt{3}i$

$\boxed{16}$ $x^4-3x^2+1=0 \Rightarrow x^2=\dfrac{3\pm\sqrt{5}}{2}\cdot\dfrac{2}{2}=\dfrac{6\pm2\sqrt{5}}{4} \Rightarrow$
$x=\pm\frac{1}{2}\sqrt{6\pm2\sqrt{5}}\approx\pm1.62, \pm0.62$

$\boxed{17}$ $|4x-1|=7 \Rightarrow 4x-1=7$ or $4x-1=-7 \Rightarrow$
$4x=8$ or $4x=-6 \Rightarrow x=2$ or $x=-\frac{3}{2}$

$\boxed{18}$ $2|2x+1|+1=19 \Rightarrow 2|2x+1|=18 \Rightarrow |2x+1|=9 \Rightarrow$
$2x+1=9$ or $2x+1=-9 \Rightarrow 2x=8$ or $2x=-10 \Rightarrow x=4$ or $x=-5$

$\boxed{19}$ $\left[\frac{1}{x}+6=\dfrac{5}{\sqrt{x}}\right]\cdot x \Rightarrow 1+6x=5\sqrt{x} \Rightarrow 6x-5\sqrt{x}+1=0 \Rightarrow$
$(2\sqrt{x}-1)(3\sqrt{x}-1)=0 \Rightarrow \sqrt{x}=\frac{1}{2}, \frac{1}{3} \Rightarrow x=\frac{1}{4}, \frac{1}{9}$

$\boxed{20}$ $\sqrt[3]{4x-5}-2=0 \Rightarrow (\sqrt[3]{4x-5})^3=2^3 \Rightarrow 4x-5=8 \Rightarrow x=\frac{13}{4}$

$\boxed{21}$ $\sqrt{7x+2}+x=6 \Rightarrow (\sqrt{7x+2})^2=(6-x)^2 \Rightarrow 7x+2=36-12x+x^2 \Rightarrow$
$x^2-19x+34=0 \Rightarrow (x-2)(x-17)=0 \Rightarrow$
$x=2$ and 17 is an extraneous solution.

$\boxed{22}$ $\sqrt{x+4}=\sqrt[4]{6x+19} \Rightarrow (\sqrt{x+4})^4=(\sqrt[4]{6x+19})^4 \Rightarrow (x+4)^2=6x+19 \Rightarrow$
$x^2+8x+16=6x+19 \Rightarrow x^2+2x-3=0 \Rightarrow (x+3)(x-1)=0 \Rightarrow x=-3, 1$

$\boxed{23}$ $\sqrt{3x+1}-\sqrt{x+4}=1 \Rightarrow 3x+1=1+2\sqrt{x+4}+x+4 \Rightarrow$
$2\sqrt{x+4}=2x-4 \Rightarrow (\sqrt{x+4})^2=(x-2)^2 \Rightarrow x+4=x^2-4x+4 \Rightarrow$
$x^2-5x=0 \Rightarrow x(x-5)=0 \Rightarrow x=5$ and 0 is an extraneous solution.

$\boxed{24}$ $x^{4/3}=16 \Rightarrow (x^{4/3})^{3/4}=\pm16^{3/4} \Rightarrow x=\pm(\sqrt[4]{16})^3=\pm2^3=\pm8$

$\boxed{25}$ $3x^2-12x+3=0 \Rightarrow x^2-4x+1=0 \Rightarrow x^2-4x+4=-1+4 \Rightarrow$
$(x-2)^2=3 \Rightarrow x-2=\pm\sqrt{3} \Rightarrow x=2\pm\sqrt{3}$

$\boxed{26}$ $x^2+10x+38=0 \Rightarrow x^2+10x+25=-38+25 \Rightarrow (x+5)^2=-13 \Rightarrow$
$x+5=\pm\sqrt{-13} \Rightarrow x=-5\pm\sqrt{13}i$

[27] $(x-3)^2$ is never less than 0. It is equal to 0 when $x=3$.

[28] $10-7x<4+2x \Rightarrow -9x<-6 \Rightarrow x>\frac{2}{3} \Leftrightarrow (\frac{2}{3}, \infty)$

[29] $\left[-\frac{1}{2}<\frac{2x+3}{5}<\frac{3}{2}\right]\cdot 10 \Rightarrow -5<4x+6<15 \Rightarrow -11<4x<9 \Rightarrow$
$-\frac{11}{4}<x<\frac{9}{4} \Leftrightarrow (-\frac{11}{4}, \frac{9}{4})$

[30] $(3x-1)(10x+4)\geq(6x-5)(5x-7) \Rightarrow 30x^2+2x-4\geq 30x^2-67x+35 \Rightarrow$
$69x\geq 39 \Rightarrow x\geq\frac{13}{23} \Leftrightarrow [\frac{13}{23}, \infty)$

[31] $\frac{6}{10x+3}<0 \Rightarrow 10x+3<0$ {since $6>0$} $\Rightarrow x<-\frac{3}{10} \Leftrightarrow (-\infty, -\frac{3}{10})$

[32] $|4x+7|<21 \Rightarrow -21<4x+7<21 \Rightarrow -28<4x<14 \Rightarrow$
$-7<x<\frac{7}{2} \Leftrightarrow (-7, \frac{7}{2})$

[33] $2\,|3-x|+1>5 \Rightarrow 2\,|3-x|>4 \Rightarrow |3-x|>2 \Rightarrow$
$3-x>2$ or $3-x<-2 \Rightarrow 1>x$ or $5<x \Rightarrow$
$x<1$ or $x>5 \Leftrightarrow (-\infty, 1)\cup(5, \infty)$

[34] $-2\,|x-3|+1\geq -5 \Rightarrow -2\,|x-3|\geq -6 \Rightarrow |x-3|\leq 3 \Rightarrow$
$-3\leq x-3\leq 3 \Rightarrow 0\leq x\leq 6 \Leftrightarrow [0, 6]$

[35] $|16-3x|\geq 5 \Rightarrow 16-3x\geq 5$ or $16-3x\leq -5 \Rightarrow$
$-3x\geq -11$ or $-3x\leq -21 \Rightarrow x\leq\frac{11}{3}$ or $x\geq 7 \Leftrightarrow (-\infty, \frac{11}{3}]\cup[7, \infty)$

[36] $2<|x-6|<4 \Rightarrow 2<x-6<4$ or $2<-(x-6)<4 \Rightarrow$
$8<x<10$ or $-2>x-6>-4 \Rightarrow 8<x<10$ or $4>x>2 \Leftrightarrow (2, 4)\cup(8, 10)$

[37] $10x^2+11x>6 \Rightarrow 10x^2+11x-6>0 \Rightarrow (2x+3)(5x-2)>0;\ (-\infty, -\frac{3}{2})\cup(\frac{2}{5}, \infty)$

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $5x-2$:	$-$	$-$	$+$
Sign of $2x+3$:	$-$	$+$	$+$
x values:	$-3/2$	$2/5$	

Diagram 37

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-5$:	$-$	$-$	$+$
Sign of $x+2$:	$-$	$+$	$+$
x values:	-2	5	

Diagram 38

[38] $x(x-3)\leq 10 \Rightarrow x^2-3x-10\leq 0 \Rightarrow (x-5)(x+2)\leq 0;\ [-2, 5]$

[39] $\frac{x^2(3-x)}{x+2}\leq 0 \Rightarrow \frac{3-x}{x+2}\leq 0$ {include 0}; $(-\infty, -2)\cup\{0\}\cup[3, \infty)$

Resulting sign:	$\ominus$	$\oplus$	$\ominus$
Sign of $3-x$:	$+$	$+$	$-$
Sign of $x+2$:	$-$	$+$	$+$
x values:	-2	3	

Diagram 39

Resulting sign:	$\oplus$	$\ominus$	$\oplus$
Sign of $x-2$:	$-$	$-$	$+$
Sign of $x+3$:	$-$	$+$	$+$
x values:	-3	2	

Diagram 40

[40] $\frac{x^2-x-2}{x^2+4x+3}\leq 0 \Rightarrow \frac{(x-2)(x+1)}{(x+1)(x+3)}\leq 0 \Rightarrow \frac{x-2}{x+3}\leq 0$ {exclude -1};
$(-3, -1)\cup(-1, 2]$

$\boxed{41}$ $\dfrac{3}{2x+3} < \dfrac{1}{x-2} \Rightarrow \dfrac{3(x-2)-1(2x+3)}{(2x+3)(x-2)} < 0 \Rightarrow \dfrac{x-9}{(2x+3)(x-2)} < 0;$

$(-\infty, -\frac{3}{2}) \cup (2, 9)$

Resulting sign:	$\ominus$	$\oplus$	$\ominus$	$\oplus$
Sign of $x-9$:	$-$	$-$	$-$	$+$
Sign of $x-2$:	$-$	$-$	$+$	$+$
Sign of $2x+3$:	$-$	$+$	$+$	$+$
x values:		$-3/2$	2	9

Diagram 41

Resulting sign:	$\ominus$	$\oplus$	$\ominus$	$\oplus$
Sign of $x-5$:	$-$	$-$	$-$	$+$
Sign of $x+1$:	$-$	$-$	$+$	$+$
Sign of $x+5$:	$-$	$+$	$+$	$+$
x values:		-5	-1	5

Diagram 42

$\boxed{42}$ $\dfrac{x+1}{x^2-25} \le 0 \Rightarrow \dfrac{x+1}{(x+5)(x-5)} \le 0;\ (-\infty, -5) \cup [-1, 5)$

$\boxed{43}$ $x^3 > x^2 \Rightarrow x^2(x-1) > 0 \Rightarrow x-1 > 0 \Rightarrow x > 1 \Leftrightarrow (1, \infty)$

$\boxed{44}$ $(x^2-x)(x^2-5x+6) < 0 \Rightarrow$

$x(x-1)(x-2)(x-3) < 0;\ (0, 1) \cup (2, 3)$

Res. sign:	$\oplus$	$\ominus$	$\oplus$	$\ominus$	$\oplus$
$x-3$:	$-$	$-$	$-$	$-$	$+$
$x-2$:	$-$	$-$	$-$	$+$	$+$
$x-1$:	$-$	$-$	$+$	$+$	$+$
x:	$-$	$+$	$+$	$+$	$+$
x values:		0	1	2	3

Diagram 44

$\boxed{45}$ $P+N = \dfrac{C+2}{C} \Rightarrow C(P+N) = C+2 \Rightarrow CP+CN = C+2 \Rightarrow$

$$CP+CN-C = 2 \Rightarrow C(P+N-1) = 2 \Rightarrow C = \frac{2}{P+N-1}$$

$\boxed{46}$ $A = B\sqrt[3]{\dfrac{C}{D}} - E \Rightarrow A+E = B\sqrt[3]{\dfrac{C}{D}} \Rightarrow \dfrac{A+E}{B} = \sqrt[3]{\dfrac{C}{D}} \Rightarrow \left(\dfrac{A+E}{B}\right)^3 = \dfrac{C}{D} \Rightarrow$

$$D \cdot \frac{(A+E)^3}{B^3} = C \Rightarrow D(A+E)^3 = C \cdot B^3 \Rightarrow D = \frac{CB^3}{(A+E)^3}$$

$\boxed{47}$ $V = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \dfrac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\dfrac{3V}{4\pi}}$

$\boxed{48}$ $F = \dfrac{\pi P R^4}{8VL} \Rightarrow R^4 = \dfrac{8FVL}{\pi P} \Rightarrow R = \pm\sqrt[4]{\dfrac{8FVL}{\pi P}} \Rightarrow R = \sqrt[4]{\dfrac{8FVL}{\pi P}}$ since $R > 0$

$\boxed{49}$ $c = \sqrt{4h(2R-h)} \Rightarrow c^2 = 8Rh - 4h^2 \Rightarrow 4h^2 - 8Rh + c^2 = 0 \Rightarrow$

$$h = \frac{8R \pm \sqrt{64R^2 - 16c^2}}{8} = \frac{8R \pm 4\sqrt{4R^2 - c^2}}{8} = R \pm \tfrac{1}{2}\sqrt{4R^2 - c^2}$$

$\boxed{50}$ $V = \frac{1}{3}\pi h(r^2 + R^2 + rR) \Rightarrow r^2 + Rr + R^2 - \dfrac{3V}{\pi h} = 0 \Rightarrow$

$(\pi h)r^2 + (\pi hR)r + (\pi hR^2 - 3V) = 0 \Rightarrow$

$$r = \frac{-\pi hR \pm \sqrt{\pi^2 h^2 R^2 - 4\pi h(\pi hR^2 - 3V)}}{2\pi h} = \frac{-\pi hR \pm \sqrt{12\pi hV - 3\pi^2 h^2 R^2}}{2\pi h}.$$

Since $r > 0$, we must use the plus sign, and $r = \dfrac{-\pi hR + \sqrt{12\pi hV - 3\pi^2 h^2 R^2}}{2\pi h}$.

$\boxed{51}$ $(7+5i) - (-8+3i) = (7+8) + (5-3)i = 15 + 2i$

$\boxed{52}$ $(4+2i)(-5+4i) = (-20-8) + (16-10)i = -28 + 6i$

[53] $(3+8i)^2 = 3^2 + 2(3)(8i) + (8i)^2 = (9-64) + 48i = -55 + 48i$

[54] $\dfrac{1}{9-\sqrt{-4}} = \dfrac{1}{9-2i} = \dfrac{1}{9-2i} \cdot \dfrac{9+2i}{9+2i} = \dfrac{9+2i}{81+4} = \dfrac{9}{85} + \dfrac{2}{85}i$

[55] $\dfrac{6-3i}{2+7i} = \dfrac{6-3i}{2+7i} \cdot \dfrac{2-7i}{2-7i} = \dfrac{(12-21)+(-42-6)i}{53} = -\dfrac{9}{53} - \dfrac{48}{53}i$

[56] $\dfrac{20-8i}{4i} = \dfrac{4(5-2i)}{4i} = \dfrac{5-2i}{i} \cdot \dfrac{-i}{-i} = \dfrac{-5i+2i^2}{-i^2} = \dfrac{-2-5i}{1} = -2-5i$

[57] Let x denote the number of years from now to retirement eligibility.

Age + service $\geq 90 \Rightarrow (37+x)+(15+x) \geq 90 \Rightarrow 2x+52 \geq 90 \Rightarrow$

$2x \geq 38 \Rightarrow x \geq 19$. The teacher will be eligible to retire at age $37+19=56$.

[58] $R=2$ and $R_1=5 \Rightarrow \left[\frac{1}{2} = \frac{1}{5} + \frac{1}{R_2}\right] \cdot 10R_2 \Rightarrow 5R_2 = 2R_2 + 10 \Rightarrow R_2 = \frac{10}{3}$ ohms

[59] Let P denote the principal that will be invested, and r the yield rate of the stock fund. $\text{Income}_{\text{stocks}} - 28\%$ federal tax $- 7\%$ state tax $= \text{Income}_{\text{bonds}} \Rightarrow$

$(Pr) - 0.28(Pr) - 0.07(Pr) = 0.07186P \Rightarrow 1r - 0.28r - 0.07r = 0.07186 \Rightarrow$

$0.65r = 0.07186 \Rightarrow r \approx 0.11055$, or, 11.055%.

[60] Let x denote the number of cm^3 of gold. $\text{Grams}_{\text{gold}} + \text{Grams}_{\text{silver}} = \text{Grams}_{\text{total}} \Rightarrow$

$x(19.3) + (5-x)(10.5) = 80 \Rightarrow 8.8x = 27.5 \Rightarrow x = 3.125$.

The number of grams of gold is $19.3x = 60.3125 \approx 60.3$.

[61] Let x denote the number of ounces of the vegetable portion, $10-x$ the number of ounces of meat. $\text{Protein}_{\text{vegetable}} + \text{Protein}_{\text{meat}} = \text{Protein}_{\text{total}} \Rightarrow$

$\frac{1}{2}(x) + 1(10-x) = 7 \Rightarrow -\frac{1}{2}x = -3 \Rightarrow x = 6$.

Use 6 oz of vegetables and 4 oz of meat.

[62] Let x denote the number of grams of 95% ethyl alcohol solution used, $400-x$ the number of grams of water. $95(x) + 0(400-x) = 75(400)$ {all in %} $\Rightarrow$

$95x = 75(400) \Rightarrow x = \frac{6000}{19} \approx 315.8$.

Use 315.8 g of ethyl alcohol and 84.2 g of water.

[63] Let x denote the number of gallons of 20% solution, $120-x$ the number of gallons of 50% solution. $20(x) + 50(120-x) = 30(120)$ {all in %} $\Rightarrow 20 \cdot 120 = 30x \Rightarrow$

$x = 80$. Use 80 gal of the 20% solution and 40 gal of the 50% solution.

[64] Let x denote the distance upstream. 10 gallons of gas @ 16 mi/gal = 160 miles.

At 20 mi/hr, there is enough fuel for 8 hours of travel.

The rate of the boat upstream is 15 mi/hr and the rate downstream is 25 mi/hr.

$\text{Time}_{\text{up}} + \text{Time}_{\text{down}} = \text{Time}_{\text{total}} \Rightarrow \left[\frac{x}{15} + \frac{x}{25} = 8\right] \cdot 75 \Rightarrow$

$5x + 3x = 600 \Rightarrow 8x = 600 \Rightarrow x = 75$ mi.

$\boxed{65}$ Let x denote the number of hours spent traveling in the smaller cities, $5\frac{1}{2} - x$ the number of hours in the country. $\text{Distance}_{\text{country}} + \text{Distance}_{\text{cities}} = \text{Distance}_{\text{total}} \Rightarrow$

$$100(5\tfrac{1}{2} - x) + 25(x) = 400 \Rightarrow 150 = 75x \Rightarrow x = 2 \text{ hr.}$$

$\boxed{66}$ Let x denote the speed of the wind. $\text{Distance}_{\text{with wind}} = \text{Distance}_{\text{against wind}} \Rightarrow$

$(320 + x)\frac{1}{2} = (320 - x)\frac{3}{4}$ $\{d = rt\}$ $\Rightarrow$ $640 + 2x = 960 - 3x \Rightarrow 5x = 320 \Rightarrow$

$$x = 64 \text{ mi/hr}$$

$\boxed{67}$ Let $50 + r$ denote the rate the automobile, that is, r is the rate over 50 mi/hr. The automobile must travel $40 + 20 = 60$ ft more than the truck (traveling at 50 mi/hr) in 5 seconds. Since 1 mi/hr $= \frac{5280}{3600} = \frac{22}{15}$ ft/sec, the automobile's rate in *excess* of 50 mi/hr is $\frac{22}{15}r$. Thus, $d = rt \Rightarrow 60 = (\frac{22}{15}r)(5) \Rightarrow r = \frac{90}{11}$.

The rate is $50 + \frac{90}{11} = \frac{640}{11} \approx 58.2$ mi/hr.

$\boxed{68}$ Let x denote the number of hours needed to fill an empty bin.

Using the hourly rates, $\left[\frac{1}{2} - \frac{1}{5} = \frac{1}{x}\right] \cdot 10x \Rightarrow 5x - 2x = 10 \Rightarrow 3x = 10 \Rightarrow$

$x = \frac{10}{3}$ hr. Since the bin was half-full at the start, $\frac{1}{2}x = \frac{1}{2} \cdot \frac{10}{3} = \frac{5}{3}$ hr, or, 1 hr 40 min.

$\boxed{69}$ Let x denote the number of gallons used in the city, $24 - x$ the number on the highway. $\text{Distance}_{\text{city}} + \text{Distance}_{\text{highway}} = \text{Distance}_{\text{total}} \Rightarrow$

$22x + 28(24 - x) = 627 \Rightarrow 45 = 6x \Rightarrow x = \frac{15}{2}$.

The number of miles in the city is $22x = 22(\frac{15}{2}) = 165$.

$\boxed{70}$ Let d denote the distance from the center of the city to a corner and $2x$ denote the length of one side of the city. $x^2 + x^2 = d^2 \Rightarrow d = \sqrt{2}x$.

A = area of the city $= (2x)^2 = 4x^2$, or $2d^2$. Currently: $d = 10 \Rightarrow A = 200$. One decade ago: $A = 150 \Rightarrow d = \sqrt{75} = 5\sqrt{3}$. The change in d is $10 - 5\sqrt{3} \approx 1.34$ mi.

$\boxed{71}$ Let x denote the change in the radius.

New surface area = 125% of original surface area $\Rightarrow$

$4\pi(6 + x)^2 = 1.25\,[4\pi(6)^2] \Rightarrow (x + 6)^2 = 45 \Rightarrow x + 6 = \sqrt{45} \Rightarrow$

$$x = 3\sqrt{5} - 6 \approx 0.71 \text{ micron.}$$

$\boxed{72}$ (a) The eastbound car has distance $20t$ and the southbound car has distance

$$(-2 + 50t).\ d^2 = (20t)^2 + (-2 + 50t)^2 \Rightarrow d = \sqrt{2900t^2 - 200t + 4}$$

(b) $104 = \sqrt{2900t^2 - 200t + 4} \Rightarrow 2900t^2 - 200t - 10{,}812 = 0 \Rightarrow$

$725t^2 - 50t - 2703 = 0 \Rightarrow$

$$t = \frac{50 \pm \sqrt{7{,}841{,}200}}{1450}\ \{t > 0\} = \frac{5 + 2\sqrt{19{,}603}}{145} \approx 1.97,$$

or approximately 11:58 A.M.

$\boxed{73}$ Let l and w denote the length and width, respectively. $3l + 6w = 270 \Rightarrow$
$w = 45 - \frac{1}{2}l$. The total area is to be $10 \cdot 100 = 1000$ ft^2.
Area $= lw \Rightarrow 1000 = l(45 - \frac{1}{2}l) \Rightarrow 2000 = 90l - l^2 \Rightarrow$
$(l - 40)(l - 50) = 0 \Rightarrow l = 40, 50$ and $w = 25, 20$.
There are two arrangements: 40 ft $\times$ 25 ft and 50 ft $\times$ 20 ft.

$\boxed{74}$ Let x denote the length of one side of an end.

(a) $V = lwh \Rightarrow 48 = 6 \cdot x \cdot x \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$ ft

(b) $S = lw + 2wh + 2lh \Rightarrow 44 = 6x + 2(x^2) + 2(6x) \Rightarrow 44 = 2x^2 + 18x \Rightarrow$
$x^2 + 9x - 22 = 0 \Rightarrow (x + 11)(x - 2) = 0 \Rightarrow x = 2$ ft

$\boxed{75}$ Let x and $4x$ denote the width and length of the pool, respectively.
$A = lw \Rightarrow 1440 = (x + 12)(4x + 12) \Rightarrow x^2 + 15x + 36 = 360 \Rightarrow$
$x^2 + 15x - 324 = 0 \Rightarrow (x + 27)(x - 12) = 0 \Rightarrow x = 12$.
The dimensions of the pool are 12 ft by 48 ft.

$\boxed{76}$ Let x denote the width of the tiled area, $2x$ the length.
The bathing area has measurements $x - 2$ and $2x - 2$.
$(x - 2)(2x - 2) = 40 \Rightarrow x^2 - 3x + 2 = 20 \Rightarrow (x - 6)(x + 3) = 0 \Rightarrow x = 6$.
The tiled area is 12 ft by 6 ft and the bathing area is 10 ft by 4 ft.

$\boxed{77}$ $P = 20 \Rightarrow 15 + \sqrt{3t + 2} = 20 \Rightarrow 3t + 2 = 5^2 \Rightarrow t = \frac{23}{3}$, or after $7\frac{2}{3}$ yr.

$\boxed{78}$ $pv = 200 \Rightarrow v = \frac{200}{p}$. $25 \le v \le 50 \Rightarrow 25 \le \frac{200}{p} \le 50 \Rightarrow \frac{1}{25} \ge \frac{p}{200} \ge \frac{1}{50} \Rightarrow$
$8 \ge p \ge 4 \Rightarrow 4 \le p \le 8$

$\boxed{79}$ Let x denote the amount of yearly business. $\text{Pay}_B > \text{Pay}_A \Rightarrow$
$\$20{,}000 + 0.10x > \$25{,}000 + 0.05x \Rightarrow 0.05x > \$5000 \Rightarrow x > \$100{,}000$

$\boxed{80}$ $v > 1100 \Rightarrow 1087\sqrt{\frac{T}{273}} > 1100 \Rightarrow$
$$\sqrt{\frac{T}{273}} > \frac{1100}{1087} \Rightarrow \frac{T}{273} > \frac{(1100)^2}{(1087)^2} \Rightarrow T > \frac{273(1100)^2}{(1087)^2} \Rightarrow T > 279.57 \text{ K}$$

$\boxed{81}$ $T = 2\pi\sqrt{\frac{l}{980}} \Rightarrow l = \frac{980T^2}{4\pi^2}$. $98 \le l \le 100 \Rightarrow 98 \le \frac{980T^2}{4\pi^2} \le 100 \Rightarrow$
$\frac{2\pi^2}{5} \le T^2 \le \frac{20\pi^2}{49} \Rightarrow \frac{10\pi^2}{25} \le T^2 \le \frac{20\pi^2}{49} \Rightarrow \frac{\pi}{5}\sqrt{10} \le T \le \frac{2\pi}{7}\sqrt{5}\ \{T \ge 0\}$,
or, approximately, $1.987 \le T \le 2.007$ sec.

$\boxed{82}$ $v = \frac{626.4}{\sqrt{h + 6372}} \Rightarrow h = \frac{(626.4)^2}{v^2} - 6372$ and $h > 100 \Rightarrow$
$$\frac{(626.4)^2}{v^2} - 6372 > 100 \Rightarrow \frac{(626.4)^2}{v^2} > 6472 \Rightarrow v^2 < \frac{(626.4)^2}{6472}\ \{v > 0\} \Rightarrow$$
$$0 < v < \frac{626.4}{\sqrt{6472}} \approx 7.786 \text{ km/sec}$$

[83] $P = 2l + 2w \Rightarrow 100 = 2l + 2w \Rightarrow l = 50 - w$. $A \geq 600 \Rightarrow lw \geq 600 \Rightarrow$ $(50 - w)w \geq 600 \Rightarrow -w^2 + 50w - 600 \geq 0 \Rightarrow w^2 - 50w + 600 \leq 0 \Rightarrow$ $(w - 20)(w - 30) \leq 0 \Rightarrow 20 \leq w \leq 30$. If w is greater than 25, it would be the length. Hence, the desired values of w are between 20 and 25.

[84] Let x denote the number of trees over 24. Then $24 + x$ represents the total number of trees planted per acre, and $600 - 12x$ represents the number of apples per tree.

Total apples = (# of trees)(# of apples per tree)

$= (24 + x)(600 - 12x) = -12x^2 + 312x + 14{,}400$.

Apples $\geq 16{,}416 \Rightarrow -12x^2 + 312x + 14{,}400 \geq 16{,}416 \Rightarrow$ $-12x^2 + 312x - 2016 \geq 0 \Rightarrow x^2 - 26x + 168 \leq 0 \Rightarrow (x - 12)(x - 14) \leq 0 \Rightarrow$ $12 \leq x \leq 14$. Hence, 36 to 38 trees per acre should be planted.

[85] Let x denote the number of \$10 increases in rent. Then the number of occupied apartments is $180 - 5x$ and the rent per apartment is $300 + 10x$.

Total income = (# of occupied apartments)(rent per apartment)

$= (180 - 5x)(300 + 10x) = -50x^2 + 300x + 54{,}000$.

Income $\geq 54{,}400 \Rightarrow -50x^2 + 300x + 54{,}000 \geq 54{,}400 \Rightarrow$ $-50x^2 + 300x - 400 \geq 0 \Rightarrow x^2 - 6x + 8 \leq 0 \Rightarrow (x - 2)(x - 4) \leq 0 \Rightarrow 2 \leq x \leq 4$. Hence, the rent charged should be \$320 to \$340.

[86] The y-values are increasing slowly and can best be described by equation (3), $y = 3\sqrt{x - 0.5}$.

Chapter 2 Discussion Exercises

[1] Solve the equation $x^2 - xy + y^2 = 0$ for x.

$$x = \frac{y \pm \sqrt{y^2 - 4y^2}}{2} = \frac{y \pm \sqrt{-3y^2}}{2} = \frac{y \pm |y|\sqrt{3}\,i}{2}.$$

Since this equation has imaginary solutions, $x^2 - xy + y^2$ is not factorable over the reals. A similar argument holds for $x^2 + xy + y^2$.

[2] The solutions are $x_1 = (-b + \sqrt{b^2 - 4ac})/(2a)$ and $x_2 = (-b - \sqrt{b^2 - 4ac})/(2a)$. The average is $(x_1 + x_2)/2 = \left(\frac{-2b}{2a}\right)/2 = -b/(2a)$. Suppose you solve the equation $-x^2 + 4x + 7 = 0$ and obtain the solutions $x_1 \approx -1.32$ and $x_2 \approx 5.32$. Averaging these numbers gives us the value 2, which we can easily see is equal to $-b/(2a)$

[3] (a) $\dfrac{1}{\dfrac{a+bi}{c+di}} = \dfrac{c+di}{a+bi} \cdot \dfrac{a-bi}{a-bi} = \dfrac{ac+bd+(ad-bc)i}{a^2+b^2} = \dfrac{ac+bd}{a^2+b^2} + \dfrac{ad-bc}{a^2+b^2}i$

(b) Yes, try an example such as 3/4.

(c) a and b cannot both be 0 because then the denominator would be 0.

[4] Since we don't know the value of x, we don't know the sign of $x-2$, and hence we are unsure of whether or not to reverse the direction of the inequality sign.

[5] (1) $a > 0$, $D \leq 0$: solution is $x \in \mathbb{R}$

(2) $a > 0$, $D > 0$: let $x_1 = (-b - \sqrt{D})/(2a)$ and $x_2 = (-b + \sqrt{D})/(2a)$ $\Rightarrow$ solution is $(-\infty, x_1] \cup [x_2, \infty)$

(3) $a < 0$, $D < 0$: solution is $\{\ \}$

(4) $a < 0$, $D = 0$: solution is $x = -b/(2a)$

(5) $a < 0$, $D > 0$: solution is $[x_1, x_2]$

[6] (a) This problem is solved in three steps.

(i) First, we must determine the height of the cloud base using the formula in Exercise 38, $h = 227(T - D) = 227(80 - 68) = 2724$ ft.

(ii) Next, we must determine the temperature T at the cloud base. From (i), the height of the cloud base is $h = 2724$ and

$$T = T_0 - \left(\frac{5.5}{1000}\right)h = 80 - \left(\frac{5.5}{1000}\right)2724 = 65.018^\circ\text{F}.$$

(iii) Finally, we must solve the equation

$T = B - \left(\frac{3}{1000}\right)h$ for h, when $T = 32^\circ$F and $B = 65.018^\circ$F.

$$32 = 65.018 - \left(\frac{3}{1000}\right)h \quad\Rightarrow\quad h = (65.018 - 32)\left(\frac{1000}{3}\right) = 11{,}006 \text{ ft.}$$

(b) Following the procedure in part (a) and using $\frac{11}{2000}$ for $\frac{5.5}{1000}$, we obtain

$$h = \tfrac{1}{6}(2497D - 497G - 64{,}000).$$

[7] The first equation, $\sqrt{2x-3} + \sqrt{x+5} = 0$, is a sum of square roots that is equal to 0. The only way this could be true is if both radicals are actually equal to 0. It is easy to see that $\sqrt{x+5}$ is equal to 0 only if $x = -5$, but -5 will not make $\sqrt{2x-3}$ equal to 0, so there is no reason to try to solve the first equation.

On the other hand, the second equation, $\sqrt[3]{2x-3} + \sqrt[3]{x+5} = 0$, can be written as $\sqrt[3]{2x-3} = -\sqrt[3]{x+5}$. This just says that one cube root is equal to the negative of another cube root, which could happen since a cube root can be negative. Solving this equation gives us $2x - 3 = -(x+5) \Rightarrow 3x = -2 \Rightarrow x = -\frac{2}{3}$.

$\boxed{8}$ $\sqrt{x} = cx - 2/c \;\Rightarrow\; c\sqrt{x} = c^2x - 2 \;\Rightarrow\; c^2x = c^4x^2 - 4c^2x + 4 \;\Rightarrow$
$0 = c^4x^2 - 5c^2x + 4 \;\Rightarrow\; 0 = (c^2x - 1)(c^2x - 4) \;\Rightarrow\; x_{1,2} = \frac{1}{c^2}, \frac{4}{c^2}.$

Check x_1 $= \frac{1}{c^2} = \frac{1}{(2\times 10^{500})^2} = \frac{1}{4\times 10^{1000}}.$

$$\text{LS} = \sqrt{x_1} = \frac{1}{2\times 10^{500}}$$

$$\text{RS} = cx_1 - \frac{2}{c} = \frac{2\times 10^{500}}{4\times 10^{1000}} - \frac{2}{2\times 10^{500}} = \frac{1}{2\times 10^{500}} - \frac{2}{2\times 10^{500}} = -\frac{1}{2\times 10^{500}}$$

Check x_2 $= \frac{4}{c^2} = \frac{4}{(2\times 10^{500})^2} = \frac{4}{4\times 10^{1000}} = \frac{1}{10^{1000}}.$

$$\text{LS} = \sqrt{x_2} = \frac{1}{10^{500}}$$

$$\text{RS} = cx_2 - \frac{2}{c} = \frac{2\times 10^{500}}{10^{1000}} - \frac{2}{2\times 10^{500}} = \frac{2}{10^{500}} - \frac{1}{10^{500}} = \frac{1}{10^{500}}$$

So x_2 is a valid solution. The right side of the original equation, $cx - 2/c$, must be nonnegative since it is equal to a square root. Note that the right side equals a negative number when $x = x_1$.