Chapter 6: The Trigonometric Functions

6.1 Exercises

Note: Exer. 1–4: The answers listed are the smallest (in magnitude) two positive coterminal angles and two negative coterminal angles.

$\boxed{1}$ (a) $120^\circ + 1(360^\circ) = 480^\circ$, $\quad 120^\circ + 2(360^\circ) = 840^\circ$;
$120^\circ - 1(360^\circ) = -240^\circ$, $\quad 120^\circ - 2(360^\circ) = -600^\circ$

(b) $135^\circ + 1(360^\circ) = 495^\circ$, $\quad 135^\circ + 2(360^\circ) = 855^\circ$;
$135^\circ - 1(360^\circ) = -225^\circ$, $\quad 135^\circ - 2(360^\circ) = -585^\circ$

(c) $-30^\circ + 1(360^\circ) = 330^\circ$, $\quad -30^\circ + 2(360^\circ) = 690^\circ$;
$-30^\circ - 1(360^\circ) = -390^\circ$, $\quad -30^\circ - 2(360^\circ) = -750^\circ$

$\boxed{2}$ (a) $240^\circ \rightarrow 600^\circ,\ 960^\circ,\ -120^\circ,\ -480^\circ$

(b) $315^\circ \rightarrow 675^\circ,\ 1035^\circ,\ -45^\circ,\ -405^\circ$

(c) $-150^\circ \rightarrow 210^\circ,\ 570^\circ,\ -510^\circ,\ -870^\circ$

$\boxed{3}$ (a) $620^\circ - 1(360^\circ) = 260^\circ$, $\quad 620^\circ + 1(360^\circ) = 980^\circ$;
$620^\circ - 2(360^\circ) = -100^\circ$, $\quad 620^\circ - 3(360^\circ) = -460^\circ$

(b) $\frac{5\pi}{6} + 1(2\pi) = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$, $\quad \frac{5\pi}{6} + 2(2\pi) = \frac{5\pi}{6} + \frac{24\pi}{6} = \frac{29\pi}{6}$;
$\frac{5\pi}{6} - 1(2\pi) = \frac{5\pi}{6} - \frac{12\pi}{6} = -\frac{7\pi}{6}$, $\quad \frac{5\pi}{6} - 2(2\pi) = \frac{5\pi}{6} - \frac{24\pi}{6} = -\frac{19\pi}{6}$

(c) $-\frac{\pi}{4} + 1(2\pi) = -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$, $\quad -\frac{\pi}{4} + 2(2\pi) = -\frac{\pi}{4} + \frac{16\pi}{4} = \frac{15\pi}{4}$;
$-\frac{\pi}{4} - 1(2\pi) = -\frac{\pi}{4} - \frac{8\pi}{4} = -\frac{9\pi}{4}$, $\quad -\frac{\pi}{4} - 2(2\pi) = -\frac{\pi}{4} - \frac{16\pi}{4} = -\frac{17\pi}{4}$

$\boxed{4}$ (a) $570^\circ \rightarrow 210^\circ,\ 930^\circ,\ -150^\circ,\ -510^\circ$

(b) $\frac{2\pi}{3} \rightarrow \frac{8\pi}{3},\ \frac{14\pi}{3},\ -\frac{4\pi}{3},\ -\frac{10\pi}{3}$

(c) $-\frac{5\pi}{4} \rightarrow \frac{3\pi}{4},\ \frac{11\pi}{4},\ -\frac{13\pi}{4},\ -\frac{21\pi}{4}$

$\boxed{5}$ (a) $90^\circ - 5^\circ 17' 34'' = 84^\circ 42' 26''$ $\quad$ (b) $90^\circ - 32.5^\circ = 57.5^\circ$

$\boxed{6}$ (a) $90^\circ - 63^\circ 4' 15'' = 26^\circ 55' 45''$ $\quad$ (b) $90^\circ - 82.73^\circ = 7.27^\circ$

$\boxed{7}$ (a) $180^\circ - 48^\circ 51' 37'' = 131^\circ 8' 23''$ $\quad$ (b) $180^\circ - 136.42^\circ = 43.58^\circ$

$\boxed{8}$ (a) $180^\circ - 152^\circ 12' 4'' = 27^\circ 47' 56''$ $\quad$ (b) $180^\circ - 15.9^\circ = 164.1^\circ$

Note: Multiply each degree measure by $\frac{\pi}{180}$ to obtain the listed radian measure.

$\boxed{9}$ (a) $150^\circ \cdot \frac{\pi}{180} = \frac{5 \cdot 30\pi}{6 \cdot 30} = \frac{5\pi}{6}$ $\quad$ (b) $-60^\circ \cdot \frac{\pi}{180} = -\frac{60\pi}{3 \cdot 60} = -\frac{\pi}{3}$

(c) $225^\circ \cdot \frac{\pi}{180} = \frac{5 \cdot 45\pi}{4 \cdot 45} = \frac{5\pi}{4}$

$\boxed{10}$ (a) $120^\circ \cdot \frac{\pi}{180} = \frac{2 \cdot 60\pi}{3 \cdot 60} = \frac{2\pi}{3}$ $\quad$ (b) $-135^\circ \cdot \frac{\pi}{180} = -\frac{3 \cdot 45\pi}{4 \cdot 45} = -\frac{3\pi}{4}$

(c) $210^\circ \cdot \frac{\pi}{180} = \frac{7 \cdot 30\pi}{6 \cdot 30} = \frac{7\pi}{6}$

$\boxed{11}$ (a) $450^\circ \cdot \frac{\pi}{180} = \frac{5 \cdot 90\pi}{2 \cdot 90} = \frac{5\pi}{2}$ $\quad$ (b) $72^\circ \cdot \frac{\pi}{180} = \frac{2 \cdot 36\pi}{5 \cdot 36} = \frac{2\pi}{5}$

(c) $100^\circ \cdot \frac{\pi}{180} = \frac{5 \cdot 20\pi}{9 \cdot 20} = \frac{5\pi}{9}$

[12] (a) $630^\circ \cdot \frac{\pi}{180} = \frac{7 \cdot 90\pi}{2 \cdot 90} = \frac{7\pi}{2}$ (b) $54^\circ \cdot \frac{\pi}{180} = \frac{3 \cdot 18\pi}{10 \cdot 18} = \frac{3\pi}{10}$

(c) $95^\circ \cdot \frac{\pi}{180} = \frac{19 \cdot 5\pi}{36 \cdot 5} = \frac{19\pi}{36}$

Note: Multiply each radian measure by $\frac{180}{\pi}$ to obtain the listed degree measure.

[13] (a) $\frac{2\pi}{3} \cdot (\frac{180}{\pi})^\circ = (\frac{2 \cdot 3 \cdot 60\pi}{3\pi})^\circ = 120^\circ$ (b) $\frac{11\pi}{6} \cdot (\frac{180}{\pi})^\circ = (\frac{11 \cdot 30 \cdot 6\pi}{6\pi})^\circ = 330^\circ$

(c) $\frac{3\pi}{4} \cdot (\frac{180}{\pi})^\circ = (\frac{3 \cdot 45 \cdot 4\pi}{4\pi})^\circ = 135^\circ$

[14] (a) $\frac{5\pi}{6} \cdot (\frac{180}{\pi})^\circ = (\frac{5 \cdot 30 \cdot 6\pi}{6\pi})^\circ = 150^\circ$ (b) $\frac{4\pi}{3} \cdot (\frac{180}{\pi})^\circ = (\frac{4 \cdot 60 \cdot 3\pi}{3\pi})^\circ = 240^\circ$

(c) $\frac{11\pi}{4} \cdot (\frac{180}{\pi})^\circ = (\frac{11 \cdot 45 \cdot 4\pi}{4\pi})^\circ = 495^\circ$

[15] (a) $-\frac{7\pi}{2} \cdot (\frac{180}{\pi})^\circ = -(\frac{7 \cdot 90 \cdot 2\pi}{2\pi})^\circ = -630^\circ$ (b) $7\pi \cdot (\frac{180}{\pi})^\circ = (7 \cdot 180)^\circ = 1260^\circ$

(c) $\frac{\pi}{9} \cdot (\frac{180}{\pi})^\circ = (\frac{20 \cdot 9\pi}{9\pi})^\circ = 20^\circ$

[16] (a) $-\frac{5\pi}{2} \cdot (\frac{180}{\pi})^\circ = -(\frac{5 \cdot 90 \cdot 2\pi}{2\pi})^\circ = -450^\circ$ (b) $9\pi \cdot (\frac{180}{\pi})^\circ = (9 \cdot 180)^\circ = 1620^\circ$

(c) $\frac{\pi}{16} \cdot (\frac{180}{\pi})^\circ = (\frac{45 \cdot 4\pi}{4 \cdot 4\pi})^\circ = (\frac{45}{4})^\circ = 11.25^\circ$

[17] $2 \cdot (\frac{180}{\pi})^\circ \approx 114.59156^\circ = 114^\circ + 0.59156^\circ$.

Since $60' = 1^\circ$, we have $0.59156^\circ = 0.59156\,(60') = 35.4936'$.

Since $60'' = 1'$, we have $0.4936' = 0.4936\,(60'') \approx 30''$. $\therefore$ 2 radians $\approx 114^\circ 35' 30''$

[18] $1.5 \cdot (\frac{180}{\pi})^\circ \approx 85.943669^\circ$; $0.943669\,(60') = 56.62014'$; $0.62014\,(60'') \approx 37''$.

$\therefore$ 1.5 radians $\approx 85^\circ 56' 37''$

[19] $5 \cdot (\frac{180}{\pi})^\circ \approx 286.4789^\circ$; $0.4789\,(60') = 28.734'$; $0.734\,(60'') \approx 44''$.

$\therefore$ 5 radians $\approx 286^\circ 28' 44''$

[20] $4 \cdot (\frac{180}{\pi})^\circ \approx 229.18312^\circ$; $0.18312\,(60') = 10.9872'$; $0.9872\,(60'') \approx 59''$.

$\therefore$ 4 radians $\approx 229^\circ 10' 59''$

[21] $37^\circ 41' = (37 + \frac{41}{60})^\circ \approx 37.6833^\circ$ [22] $83^\circ 17' = (83 + \frac{17}{60})^\circ \approx 83.2833^\circ$

[23] $115^\circ 26' 27'' = (115 + \frac{26}{60} + \frac{27}{3600})^\circ \approx 115.4408^\circ$

[24] $258^\circ 39' 52'' = (258 + \frac{39}{60} + \frac{52}{3600})^\circ \approx 258.6644^\circ$

[25] $0.169\,(60') = 10.14'$; $0.14\,(60'') = 8.4''$; $\therefore$ $63.169^\circ \approx 63^\circ 10' 8''$

[26] $0.864\,(60') = 51.84'$; $0.84\,(60'') = 50.4''$; $\therefore$ $12.864^\circ \approx 12^\circ 51' 50''$

[27] $0.6215\,(60') = 37.29'$; $0.29\,(60'') = 17.4''$; $\therefore$ $310.6215^\circ \approx 310^\circ 37' 17''$

[28] $0.7238\,(60') = 43.428'$; $0.428\,(60'') = 25.68''$; $\therefore$ $81.7238^\circ \approx 81^\circ 43' 26''$

[29] $s = r\theta \;\Rightarrow\; r = \frac{s}{\theta} = \frac{10}{4} = 2.5$ cm

[30] $s = r\theta \;\Rightarrow\; r = \frac{s}{\theta} = \dfrac{3}{20 \cdot \frac{\pi}{180}} = \frac{27}{\pi} \approx 8.59$ km

[31] (a) $s = r\theta = 8 \cdot (45 \cdot \frac{\pi}{180}) = 8 \cdot \frac{\pi}{4} = 2\pi \approx 6.28$ cm

(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(\frac{\pi}{4}) = 8\pi \approx 25.13\ \text{cm}^2$

[32] (a) $s = r\theta = 9 \cdot (120 \cdot \frac{\pi}{180}) = 9 \cdot \frac{2\pi}{3} = 6\pi \approx 18.85$ cm

(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(9)^2(\frac{2\pi}{3}) = 27\pi \approx 84.82\ \text{cm}^2$

[33] (a) $s = r\theta \;\Rightarrow\; \theta = \frac{s}{r} = \frac{7}{4} = 1.75$ radians; $\frac{7}{4}\cdot(\frac{180}{\pi})^\circ = (\frac{315}{\pi})^\circ \approx 100.27^\circ$
(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2(\frac{7}{4}) = 14\text{ cm}^2$

[34] (a) $\theta = \frac{s}{r} = \frac{3(12)}{20} = \frac{9}{5} = 1.8$ radians; $\frac{9}{5}\cdot(\frac{180}{\pi})^\circ = (\frac{324}{\pi})^\circ \approx 103.13^\circ$
(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(20)^2(\frac{9}{5}) = 360\text{ in}^2$

[35] (a) $s = r\theta = (\frac{1}{2}\cdot 16)(50\cdot\frac{\pi}{180}) = 8\cdot\frac{5\pi}{18} = \frac{20\pi}{9} \approx 6.98$ m
(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(8)^2(\frac{5\pi}{18}) = \frac{80\pi}{9} \approx 27.93\text{ m}^2$

[36] (a) $s = r\theta = (\frac{1}{2}\cdot 120)(2.2) = 60(2.2) = 132$ cm
(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(60)^2(2.2) = 3960\text{ cm}^2$

[37] radius $= \frac{1}{2}\cdot 8000$ miles $= 4000$ miles
(a) $s = r\theta = 4000\,(60\cdot\frac{\pi}{180}) = \frac{4000\pi}{3} \approx 4189$ miles
(b) $s = r\theta = 4000\,(45\cdot\frac{\pi}{180}) = 1000\pi \approx 3142$ miles
(c) $s = r\theta = 4000\,(30\cdot\frac{\pi}{180}) = \frac{2000\pi}{3} \approx 2094$ miles
(d) $s = r\theta = 4000\,(10\cdot\frac{\pi}{180}) = \frac{2000\pi}{9} \approx 698$ miles
(e) $s = r\theta = 4000\,(1\cdot\frac{\pi}{180}) = \frac{200\pi}{9} \approx 70$ miles

[38] $1' = (\frac{1}{60})^\circ$; $s = r\theta = 4000\,(\frac{1}{60}\cdot\frac{\pi}{180}) = \frac{10\pi}{27} \approx 1.16$ mi

[39] $\theta = \frac{s}{r} = \frac{500}{4000} = \frac{1}{8}$ radian; $(\frac{1}{8})(\frac{180}{\pi})^\circ = (\frac{45}{2\pi})^\circ \approx 7^\circ 10'$

[40] The area of equilateral $\triangle OAB$ is $\frac{1}{4}\sqrt{3}\,r^2$ (see the formula on the endpapers of the text). The area of sector OAB minus the area of $\triangle OAB$ is one-sixth of the total difference; that is, $\frac{1}{6}(24) = 4$.
Sector OAB_{area} − Triangle $OAB_{\text{area}} = 4 \;\Rightarrow$
$\frac{1}{2}r^2\cdot\frac{\pi}{3} - \frac{1}{4}\sqrt{3}\,r^2 = 4 \;\Rightarrow\; 2\pi r^2 - 3\sqrt{3}\,r^2 = 48 \;\Rightarrow$
$r^2 = \dfrac{48}{2\pi - 3\sqrt{3}} \;\Rightarrow\; r = \sqrt{\dfrac{48}{2\pi - 3\sqrt{3}}} \approx 6.645$ m.

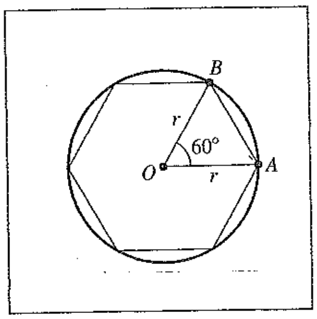

Figure 40

[41] The total rectangular area of the window is $T = 54\times 24 = 1296\text{ in.}^2$. The wiped area W is the area of a sector with radius $17+5 = 22$ inches minus the area of a sector with radius 5 inches; that is,

$$W = \tfrac{1}{2}(22)^2\cdot\tfrac{2\pi}{3} - \tfrac{1}{2}(5)^2\cdot\tfrac{2\pi}{3} = \tfrac{\pi}{3}(484-25) = \tfrac{459\pi}{3} \approx 480.66.$$

The percentage of the window's area that is wiped by the blade is $100\times\dfrac{W}{T} \approx 37.1\%$.

[42] A point on the perimeter of the core is moving 264 ft in 1 sec.
The radius of the core is 100 ft. $s = r\theta \;\Rightarrow$

$$\theta = \tfrac{s}{r} = \tfrac{264}{100} = 2.64\text{ rad/sec} = 158.4\text{ rad/min} = \tfrac{158.4}{2\pi}\text{ rev/min} \approx 25.2\text{ rev/min}.$$

$\boxed{43}$ 23 hours, 56 minutes, and 4 seconds $= 23(60)^2 + 56(60) + 4 = 86{,}164$ sec.

Since the earth turns through 2π radians in 86,164 seconds,

it rotates through $\frac{2\pi}{86{,}164} \approx 7.29 \times 10^{-5}$ radians in one second.

$\boxed{44}$ Using the result from Exercise 43, the distance s traveled in 1 second will be

$$s = r\theta \approx (3963.3)\left(\frac{2\pi}{86{,}164}\right) \approx 0.29 \text{ mi/sec, or, } 1040 \text{ mi/hr.}$$

$\boxed{45}$ (a) $\left(40\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = 80\pi\,\frac{\text{rad}}{\text{min}}$

(b) $s = r\theta = (5 \text{ in}) \cdot 80\pi = 400\pi$ in.

Linear speed $= 400\pi$ in/min $= \frac{100\pi}{3}$ ft/min ≈ 104.72 ft/min.

$\boxed{46}$ (a) $\left(2400\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = 4800\pi\,\frac{\text{rad}}{\text{min}}$

(b) $s = r\theta = (9 \text{ in}) \cdot 4800\pi = 43{,}200\pi$ in.

Linear speed $= 43{,}200\pi$ in/min $= 3600\pi$ ft/min.

$\boxed{47}$ (a) $200 \text{ rpm} = 200\,\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} = 400\pi\,\frac{\text{rad}}{\text{min}}$

(b) $s = r\theta = (5.7)(400\pi) = 2280\pi$ cm. Linear speed $= 2280\pi$ cm/min.

Divide by 60 to obtain cm/sec: $\frac{2280\pi}{60} = 38\pi \approx 119$ cm/sec

(c) Let x be the desired angular speed. The linear speed at 5.7 cm must equal the linear speed at 3 cm, so $2280\pi = 3x \Rightarrow x = 760\pi$ rad/min. Divide by 2π to obtain revolutions per minute: $\frac{760\pi}{2\pi} = 380$ rpm

(d) One approach is to simply return to part (c), replace 3 with r, and divide by 2π to create the function. $2280\pi = rx \Rightarrow x = \frac{2280\pi}{r}$, so $S(r) = \frac{2280\pi}{r(2\pi)} = \frac{1140}{r}$. As the radius increases, the drive motor speed decreases. Thus, S and r vary *inversely*.

$\boxed{48}$ $\frac{60 \text{ miles}}{\text{hour}} = \frac{60 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ hour}}{60 \text{ minutes}} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{12 \text{ inches}}{\text{foot}} = \frac{63{,}360 \text{ inches}}{\text{minute}}$.

A point of the circumference of the tire moves $2\pi r = 2\pi(\frac{1}{2} \cdot 22) = 22\pi$ inches each revolution. Thus, the number of revolutions per minute is

$$\frac{63{,}360 \text{ inches}}{\text{minute}} \cdot \frac{1 \text{ revolution}}{22\pi \text{ inches}} = \frac{31{,}680 \text{ revolutions}}{11\pi \text{ minute}} \approx 916.73 \text{ rpm.}$$

$\boxed{49}$ (a) $s = r\theta = (\frac{1}{2} \cdot 3)(\frac{7\pi}{4}) = \frac{21\pi}{8} \approx 8.25$ ft

(b) $s = r\theta \Rightarrow d = (\frac{1}{2} \cdot 3)\,\theta \Rightarrow \theta = (\frac{2}{3}d)$ radians

$\boxed{50}$ $\theta = \frac{s}{r} = \frac{6 \text{ in}}{4 \text{ ft}} = \frac{6 \text{ in}}{48 \text{ in}} = \frac{1}{8}$ radian $\approx 7.162^\circ$ or $7^\circ 10'$

[51] $\text{Area}_{\text{small}} = \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{1}{2} \cdot 18\right)^2 \cdot \left(\frac{2\pi}{6}\right) = \frac{27\pi}{2}$. $\text{Area}_{\text{large}} = \frac{1}{2}\left(\frac{1}{2} \cdot 26\right)^2 \cdot \left(\frac{2\pi}{8}\right) = \frac{169\pi}{8}$.

$\text{Cost}_{\text{small}} = \frac{27\pi}{2} \div 2 \approx 21.21 \text{ in}^2/\text{dollar}$. $\text{Cost}_{\text{large}} = \frac{169\pi}{8} \div 3 \approx 22.12 \text{ in}^2/\text{dollar}$.

The large slice provides slightly more pizza per dollar.

[52] Let s_1 and s_2 denote the lengths of the chain around the sprockets with radii r_1 and r_2, respectively. Thus, $s_1 = r_1\theta_1$ and $s_2 = r_2\theta_2$.

The lengths of chain s_1 and s_2 are equal. $s_2 = s_1 \Rightarrow r_2\theta_2 = r_1\theta_1 \Rightarrow \theta_2 = \frac{r_1\theta_1}{r_2}$.

[53] $\frac{40 \text{ miles}}{\text{hour}} = \frac{40 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ hour}}{3600 \text{ seconds}} \cdot \frac{5280 \text{ feet}}{\text{mile}} \cdot \frac{12 \text{ inches}}{\text{foot}} = \frac{704 \text{ inches}}{\text{second}}$.

The circumference of the wheel is $2\pi(14)$ inches. The back sprocket then rotates $\frac{704}{28\pi}$ revolutions per second or $\frac{704}{28\pi} \cdot 2\pi = \frac{352}{7}$ radians per second. The front sprocket's angular speed is given by $\theta_1 = \frac{r_2\theta_2}{r_1} = \frac{2 \cdot \frac{352}{7}}{5} = \frac{704}{35} \approx 20.114$ radians per second

or 3.2 revolutions per second or 192.08 revolutions per minute.

[54] $0.0017 \text{ rad/yr} = 0.0017\left(\frac{180}{\pi}\right)^\circ/\text{yr}$.

If x is the number of years required, then $5 = \frac{0.0017(180)}{\pi}x \Rightarrow x = \frac{5\pi}{0.306} \approx 51.3$.

6.2 Exercises

[1] (a–b) From the figure, α is smaller than β. Since one radian is about 57°, the only logical choices for the radian measures of α and β are 0.28 and 1.29, respectively. The answers are (a) B and (b) D.

(c–e) The sides x, y, and z must be 7, 24, and 25, respectively, since the legs x and y must be smaller than the hypotenuse z, and from the figure, x is less than y. The answers are (c) A, (d) C, and (e) E.

[2] (c–e) The sides x, y, and z must be 16, 17, and 23.35, respectively, since the legs x and y must be smaller than the hypotenuse z, and from the figure, $x < y$ (a measurement will show this). The answers are (c) B, (d) C, & (e) A.

(a–b) Since $x < y$, we have $\alpha < \beta$ since α is the angle opposite side x. The only logical choices for the radian measures of α and β are 0.76 and 0.82, respectively. The answers are (a) E and (b) D.

Note: Answers are in the order *sin, cos, tan, cot, sec, csc* for any exercises that require the values of the six trigonometric functions.

[3] $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{4}{5}$; $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{5}$; $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$;

$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$; $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}$; $\csc\theta = \frac{1}{\sin\theta} = \frac{5}{4}$

[4] $\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$; $\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$; $\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$ ★ $\frac{8}{17}, \frac{15}{17}, \frac{8}{15}, \frac{15}{8}, \frac{17}{15}, \frac{17}{8}$

[5] Using the Pythagorean theorem, $\text{adj} = \sqrt{(\text{hyp})^2 - (\text{opp})^2} = \sqrt{5^2 - 2^2} = \sqrt{21}$.

★ $\frac{2}{5}, \frac{\sqrt{21}}{5}, \frac{2}{\sqrt{21}}, \frac{\sqrt{21}}{2}, \frac{5}{\sqrt{21}}, \frac{5}{2}$

[6] Using the Pythagorean theorem, $\text{opp} = \sqrt{(\text{hyp})^2 - (\text{adj})^2} = \sqrt{3^2 - 1^2} = \sqrt{8}$.

★ $\frac{\sqrt{8}}{3}, \frac{1}{3}, \sqrt{8}, \frac{1}{\sqrt{8}}, 3, \frac{3}{\sqrt{8}}$

[7] Using the Pythagorean theorem, $\text{hyp} = \sqrt{(\text{adj})^2 + (\text{opp})^2} = \sqrt{a^2 + b^2}$.

★ $\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, \frac{a}{b}, \frac{b}{a}, \frac{\sqrt{a^2+b^2}}{b}, \frac{\sqrt{a^2+b^2}}{a}$

[8] $\text{opp} = \sqrt{c^2 - a^2}$ ★ $\frac{\sqrt{c^2-a^2}}{c}, \frac{a}{c}, \frac{\sqrt{c^2-a^2}}{a}, \frac{a}{\sqrt{c^2-a^2}}, \frac{c}{a}, \frac{c}{\sqrt{c^2-a^2}}$

[9] $\text{adj} = \sqrt{c^2 - b^2}$ ★ $\frac{b}{c}, \frac{\sqrt{c^2-b^2}}{c}, \frac{b}{\sqrt{c^2-b^2}}, \frac{\sqrt{c^2-b^2}}{b}, \frac{c}{\sqrt{c^2-b^2}}, \frac{c}{b}$

[10] $\text{hyp} = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}\,a \Rightarrow \sin\theta = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$, or, $\frac{\sqrt{2}}{2}$.

$\cos\theta = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$ and $\tan\theta = \frac{a}{a} = 1$. ★ $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1, 1, \sqrt{2}, \sqrt{2}$

[11] $\sin 30^\circ = \frac{4}{x} \Rightarrow \frac{1}{2} = \frac{4}{x} \Rightarrow x = 8$; $\tan 30^\circ = \frac{4}{y} \Rightarrow \frac{\sqrt{3}}{3} = \frac{4}{y} \Rightarrow y = 4\sqrt{3}$

[12] $\sin 60^\circ = \frac{3}{x} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3}{x} \Rightarrow x = 2\sqrt{3}$; $\tan 60^\circ = \frac{3}{y} \Rightarrow \sqrt{3} = \frac{3}{y} \Rightarrow y = \sqrt{3}$

[13] $\sin 45^\circ = \frac{7}{x} \Rightarrow \frac{\sqrt{2}}{2} = \frac{7}{x} \Rightarrow x = 7\sqrt{2}$; $\tan 45^\circ = \frac{7}{y} \Rightarrow 1 = \frac{7}{y} \Rightarrow y = 7$

[14] $\sin 30^\circ = \frac{x}{10} \Rightarrow \frac{1}{2} = \frac{x}{10} \Rightarrow x = 5$; $\cos 30^\circ = \frac{y}{10} \Rightarrow \frac{\sqrt{3}}{2} = \frac{y}{10} \Rightarrow y = 5\sqrt{3}$

[15] $\sin 60^\circ = \frac{x}{8} \Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{8} \Rightarrow x = 4\sqrt{3}$; $\cos 60^\circ = \frac{y}{8} \Rightarrow \frac{1}{2} = \frac{y}{8} \Rightarrow y = 4$

[16] $\sin 45^\circ = \frac{x}{4} \Rightarrow \frac{\sqrt{2}}{2} = \frac{x}{4} \Rightarrow x = 2\sqrt{2}$; $\cos 45^\circ = \frac{y}{4} \Rightarrow \frac{\sqrt{2}}{2} = \frac{y}{4} \Rightarrow y = 2\sqrt{2}$

Note: It may help to sketch a triangle as shown for Exercises 17 and 21.

Use the Pythagorean theorem to find the remaining side.

$\boxed{17}$ $(\text{adj})^2 + (\text{opp})^2 = (\text{hyp})^2 \Rightarrow (\text{adj})^2 + 3^2 = 5^2 \Rightarrow \text{adj} = \sqrt{25-9} = 4.$

★ $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}, \frac{4}{3}, \frac{5}{4}, \frac{5}{3}$

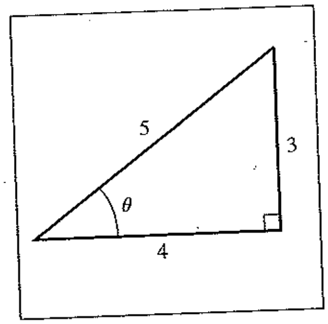

Figure 17

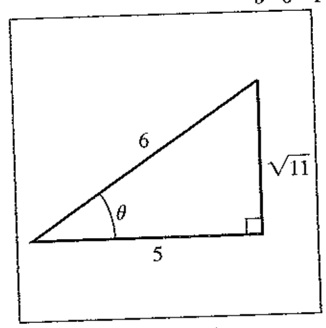

Figure 21

$\boxed{18}$ $8^2 + (\text{opp})^2 = 17^2 \Rightarrow \text{opp} = \sqrt{289-64} = 15.$ ★ $\frac{15}{17}, \frac{8}{17}, \frac{15}{8}, \frac{8}{15}, \frac{17}{8}, \frac{17}{15}$

$\boxed{19}$ $12^2 + 5^2 = (\text{hyp})^2 \Rightarrow \text{hyp} = \sqrt{144+25} = 13.$ ★ $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}, \frac{12}{5}, \frac{13}{12}, \frac{13}{5}$

$\boxed{20}$ $7^2 + 24^2 = (\text{hyp})^2 \Rightarrow \text{hyp} = \sqrt{49+576} = 25.$ ★ $\frac{24}{25}, \frac{7}{25}, \frac{24}{7}, \frac{7}{24}, \frac{25}{7}, \frac{25}{24}$

$\boxed{21}$ $5^2 + (\text{opp})^2 = 6^2 \Rightarrow \text{opp} = \sqrt{36-25} = \sqrt{11}.$ ★ $\frac{\sqrt{11}}{6}, \frac{5}{6}, \frac{\sqrt{11}}{5}, \frac{5}{\sqrt{11}}, \frac{6}{5}, \frac{6}{\sqrt{11}}$

$\boxed{22}$ $(\text{adj})^2 + 1^2 = 4^2 \Rightarrow \text{adj} = \sqrt{16-1} = \sqrt{15}.$ ★ $\frac{1}{4}, \frac{\sqrt{15}}{4}, \frac{1}{\sqrt{15}}, \sqrt{15}, \frac{4}{\sqrt{15}}, 4$

$\boxed{23}$ Let h denote the height of the tree.

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 60^\circ = \frac{h}{200} \Rightarrow h = 200\sqrt{3} \approx 346.4 \text{ ft}$$

$\boxed{24}$ Let d denote the distance to the base of the mountain.

$$\cot\theta = \frac{\text{adj}}{\text{opp}} \Rightarrow \cot 30^\circ = \frac{d}{12{,}400} \Rightarrow d = 12{,}400\sqrt{3} \approx 21{,}477.4 \text{ ft}$$

$\boxed{25}$ Let d be the distance that it was moved. $\sin 9^\circ = \frac{30}{d} \Rightarrow d = \frac{30}{\sin 9^\circ} \approx 192$ ft

$\boxed{26}$ Let h be the height of the top of the sign.

$$\tan 78.87^\circ = \frac{h}{200} \Rightarrow h = 200\tan 78.87^\circ \approx 1017 \text{ ft}$$

$\boxed{27}$ $\sin\theta = \frac{1.22\lambda}{D} \Rightarrow D = \frac{1.22\lambda}{\sin\theta} = \frac{1.22 \times 550 \times 10^{-9}}{\sin 0.00003769^\circ} \approx 1.02$ meters

$\boxed{28}$ (a) $A = \frac{1}{2}\pi R^2(1+\cos 0^\circ) = \frac{1}{2}\pi R^2(2) = \pi R^2 = \pi(1080)^2 = 1{,}166{,}400\pi \approx 3{,}664{,}354 \text{ mi}^2$

(b) $A = \frac{1}{2}\pi R^2(1+\cos 180^\circ) = \frac{1}{2}\pi R^2(0) = 0 \text{ mi}^2$

(c) $A = \frac{1}{2}\pi R^2(1+\cos 90^\circ) = \frac{1}{2}\pi R^2(1) = \frac{1}{2}\pi R^2 = \frac{\pi}{2}(1080)^2 = 583{,}200\pi \approx 1{,}832{,}177 \text{ mi}^2$

(d) $A = \frac{1}{2}\pi R^2(1+\cos 103^\circ) \approx \frac{1}{2}\pi R^2(0.77505) \approx 0.38752\pi(1080)^2 \approx 1{,}420{,}027 \text{ mi}^2$

[29] *Note:* Be sure that your calculator is in degree mode.

(a) $\sin 42^\circ \approx 0.6691$ (b) $\cos 77^\circ \approx 0.2250$

(c) $\csc 123^\circ = \dfrac{1}{\sin 123^\circ} \approx 1.1924$ (d) $\sec(-190^\circ) = \dfrac{1}{\cos(-190^\circ)} \approx -1.0154$

[30] (a) $\tan 282^\circ \approx -4.7046$ (b) $\cot(-81^\circ) = \dfrac{1}{\tan(-81^\circ)} \approx -0.1584$

(c) $\sec 202^\circ = \dfrac{1}{\cos 202^\circ} \approx -1.0785$ (d) $\sin 97^\circ \approx 0.9925$

[31] *Note:* Be sure that your calculator is in radian mode.

(a) $\cot \frac{\pi}{13} = \dfrac{1}{\tan(\pi/13)} \approx 4.0572$ (b) $\csc 1.32 = \dfrac{1}{\sin 1.32} \approx 1.0323$

(c) $\cos(-8.54) \approx -0.6335$ (d) $\tan \frac{3\pi}{7} \approx 4.3813$

[32] (a) $\sin(-0.11) \approx -0.1098$ (b) $\sec \frac{31}{27} = \dfrac{1}{\cos(31/27)} \approx 2.4380$

(c) $\tan(-\frac{3}{13}) \approx -0.2350$ (d) $\cos 2.4\pi \approx 0.3090$

[33] (a) $\sin 30^\circ = 0.5$ (b) $\sin 30 \approx -0.9880$

(c) $\cos \pi^\circ \approx 0.9985$ (d) $\cos \pi = -1$

[34] (a) $\sin 45^\circ \approx 0.7071$ (b) $\sin 45 \approx 0.8509$

(c) $\cos(3\pi/2)^\circ \approx 0.9966$ (d) $\cos(3\pi/2) = 0$

[35] (a) Since $1 + \tan^2 4\beta = \sec^2 4\beta$, $\tan^2 4\beta - \sec^2 4\beta = -1$.

(b) $4\tan^2\beta - 4\sec^2\beta = 4(\tan^2\beta - \sec^2\beta) = 4(-1) = -4$

[36] (a) Since $1 + \cot^2 3\alpha = \csc^2 3\alpha$, $\csc^2 3\alpha - \cot^2 3\alpha = 1$.

(b) $3\csc^2\alpha - 3\cot^2\alpha = 3(\csc^2\alpha - \cot^2\alpha) = 3(1) = 3$

[37] (a) $5\sin^2\theta + 5\cos^2\theta = 5(\sin^2\theta + \cos^2\theta) = 5(1) = 5$

(b) $5\sin^2(\theta/4) + 5\cos^2(\theta/4) = 5\left[\sin^2(\theta/4) + \cos^2(\theta/4)\right] = 5(1) = 5$

[38] (a) $7\sec^2\gamma - 7\tan^2\gamma = 7(\sec^2\gamma - \tan^2\gamma) = 7(1) = 7$

(b) $7\sec^2(\gamma/3) - 7\tan^2(\gamma/3) = 7\left[\sec^2(\gamma/3) - \tan^2(\gamma/3)\right] = 7(1) = 7$

[39] $$\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = \frac{(\sin\theta + \cos\theta)(\sin^2\theta - \sin\theta\,\cos\theta + \cos^2\theta)}{\sin\theta + \cos\theta} =$$
$$\sin^2\theta - \sin\theta\,\cos\theta + \cos^2\theta = (\sin^2\theta + \cos^2\theta) - \sin\theta\,\cos\theta = 1 - \sin\theta\,\cos\theta$$

[40] $$\frac{\cot^2\alpha - 4}{\cot^2\alpha - \cot\alpha - 6} = \frac{(\cot\alpha + 2)(\cot\alpha - 2)}{(\cot\alpha - 3)(\cot\alpha + 2)} = \frac{\cot\alpha - 2}{\cot\alpha - 3}$$

[41] $$\frac{2 - \tan\theta}{2\csc\theta - \sec\theta} = \frac{2 - \dfrac{\sin\theta}{\cos\theta}}{2\cdot\dfrac{1}{\sin\theta} - \dfrac{1}{\cos\theta}} = \frac{\dfrac{2\cos\theta - \sin\theta}{\cos\theta}}{\dfrac{2\cos\theta - \sin\theta}{\sin\theta\,\cos\theta}} = \frac{\dfrac{1}{1}}{\dfrac{1}{\sin\theta}} = \sin\theta$$

[42] $$\frac{\csc\theta + 1}{(1/\sin^2\theta) + \csc\theta} = \frac{\csc\theta + 1}{\csc^2\theta + \csc\theta} = \frac{\csc\theta + 1}{\csc\theta(\csc\theta + 1)} = \frac{1}{\csc\theta} = \sin\theta$$

$\boxed{43}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$ { cotangent identity } $= \dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$ { $\sin^2\theta + \cos^2\theta = 1$ }

$\boxed{44}$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ { tangent identity } $= \dfrac{\sqrt{1-\cos^2\theta}}{\cos\theta}$ { $\sin^2\theta + \cos^2\theta = 1$ }

$\boxed{45}$ $\sec\theta = \dfrac{1}{\cos\theta}$ { reciprocal identity } $= \dfrac{1}{\sqrt{1-\sin^2\theta}}$

$\boxed{46}$ $\csc\theta = \dfrac{1}{\sin\theta}$ { reciprocal identity } $= \dfrac{1}{\sqrt{1-\cos^2\theta}}$

$\boxed{47}$ One solution is $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\dfrac{1}{\sec^2\theta}} = \dfrac{\sqrt{\sec^2\theta - 1}}{\sec\theta}$.

Alternatively, $\sin\theta = \dfrac{\sin\theta/\cos\theta}{1/\cos\theta} = \dfrac{\tan\theta}{\sec\theta} = \dfrac{\sqrt{\sec^2\theta-1}}{\sec\theta}$ { $1+\tan^2\theta = \sec^2\theta$ }.

$\boxed{48}$ $\cos\theta = \dfrac{\cos\theta/\sin\theta}{1/\sin\theta} = \dfrac{\cot\theta}{\csc\theta} = \dfrac{\cot\theta}{\sqrt{1+\cot^2\theta}}$ { $1+\cot^2\theta = \csc^2\theta$ }

$\boxed{49}$ $\cos\theta\ \sec\theta = \cos\theta\,(1/\cos\theta) = 1$

$\boxed{50}$ $\tan\theta\ \cot\theta = \tan\theta\,(1/\tan\theta) = 1$

$\boxed{51}$ $\sin\theta\ \sec\theta = \sin\theta\,(1/\cos\theta) = \sin\theta/\cos\theta = \tan\theta$

$\boxed{52}$ $\sin\theta\ \cot\theta = \sin\theta\,(\cos\theta/\sin\theta) = \cos\theta$

$\boxed{53}$ $\dfrac{\csc\theta}{\sec\theta} = \dfrac{1/\sin\theta}{1/\cos\theta} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$

$\boxed{54}$ $\cot\theta\ \sec\theta = \dfrac{\cos\theta}{\sin\theta}\cdot\dfrac{1}{\cos\theta} = \dfrac{1}{\sin\theta} = \csc\theta$

$\boxed{55}$ $(1+\cos 2\theta)(1-\cos 2\theta) = 1-\cos^2 2\theta = \sin^2 2\theta$

$\boxed{56}$ $\cos^2 2\theta - \sin^2 2\theta = \cos^2 2\theta - (1-\cos^2 2\theta) = 2\cos^2 2\theta - 1$

$\boxed{57}$ $\cos^2\theta\,(\sec^2\theta - 1) = \cos^2\theta\,(\tan^2\theta) = \cos^2\theta\cdot\dfrac{\sin^2\theta}{\cos^2\theta} = \sin^2\theta$

$\boxed{58}$ $(\tan\theta + \cot\theta)\tan\theta = \tan^2\theta + \cot\theta\ \tan\theta = \tan^2\theta + 1 = \sec^2\theta$

$\boxed{59}$ $\dfrac{\sin(\theta/2)}{\csc(\theta/2)} + \dfrac{\cos(\theta/2)}{\sec(\theta/2)} = \dfrac{\sin(\theta/2)}{1/\sin(\theta/2)} + \dfrac{\cos(\theta/2)}{1/\cos(\theta/2)} = \sin^2(\theta/2) + \cos^2(\theta/2) = 1$

$\boxed{60}$ $1-2\sin^2(\theta/2) = 1-2(1-\cos^2(\theta/2)) = 1-2+2\cos^2(\theta/2) = 2\cos^2(\theta/2) - 1$

$\boxed{61}$ $(1+\sin\theta)(1-\sin\theta) = 1-\sin^2\theta = \cos^2\theta = \dfrac{1}{\sec^2\theta}$

$\boxed{62}$ $(1-\sin^2\theta)(1+\tan^2\theta) = (\cos^2\theta)(\sec^2\theta) = \cos^2\theta\,(1/\cos^2\theta) = 1$

$\boxed{63}$ $\sec\theta - \cos\theta = \dfrac{1}{\cos\theta} - \cos\theta = \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta = \tan\theta\ \sin\theta$

$\boxed{64}$ $\dfrac{\sin\theta + \cos\theta}{\cos\theta} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta} = \tan\theta + 1 = 1 + \tan\theta$

[65] $(\cot\theta + \csc\theta)(\tan\theta - \sin\theta) = \cot\theta\,\tan\theta - \cot\theta\,\sin\theta + \csc\theta\,\tan\theta - \csc\theta\,\sin\theta$

$$= \frac{1}{\tan\theta}\tan\theta - \frac{\cos\theta}{\sin\theta}\sin\theta + \frac{1}{\sin\theta}\frac{\sin\theta}{\cos\theta} - \frac{1}{\sin\theta}\sin\theta$$

$$= 1 - \cos\theta + \frac{1}{\cos\theta} - 1 = -\cos\theta + \sec\theta = \sec\theta - \cos\theta$$

[66] $\cot\theta + \tan\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta\,\cos\theta} = \dfrac{1}{\sin\theta\,\cos\theta} = \dfrac{1}{\sin\theta}\cdot\dfrac{1}{\cos\theta} = \csc\theta\,\sec\theta$

[67] $\sec^2 3\theta\,\csc^2 3\theta = (1+\tan^2 3\theta)(1+\cot^2 3\theta) = 1 + \tan^2 3\theta + \cot^2 3\theta + 1 =$
$\sec^2 3\theta + \csc^2 3\theta$

[68] $\dfrac{1+\cos^2 3\theta}{\sin^2 3\theta} = \dfrac{1}{\sin^2 3\theta} + \dfrac{\cos^2 3\theta}{\sin^2 3\theta} = \csc^2 3\theta + \cot^2 3\theta = \csc^2 3\theta + (\csc^2 3\theta - 1) =$
$2\csc^2 3\theta - 1$

[69] $\log\csc\theta = \log\left(\dfrac{1}{\sin\theta}\right) = \log 1 - \log\sin\theta = 0 - \log\sin\theta = -\log\sin\theta$

[70] $\log\tan\theta = \log\left(\dfrac{\sin\theta}{\cos\theta}\right) = \log\sin\theta - \log\cos\theta$

[71] $x = 4$ and $y = -3 \;\Rightarrow\; r = \sqrt{4^2 + (-3)^2} = 5$.
$\sin\theta = y/r = -\frac{3}{5}$, $\cos\theta = x/r = \frac{4}{5}$, and $\tan\theta = y/x = -\frac{3}{4}$. ★ $-\frac{3}{5}, \frac{4}{5}, -\frac{3}{4}, -\frac{4}{3}, \frac{5}{4}, -\frac{5}{3}$

[72] $x = -8$ and $y = -15 \;\Rightarrow\; r = \sqrt{(-8)^2 + (-15)^2} = 17$. ★ $-\frac{15}{17}, -\frac{8}{17}, \frac{15}{8}, \frac{8}{15}, -\frac{17}{8}, -\frac{17}{15}$

[73] $x = -2$ and $y = -5 \;\Rightarrow\; r = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29}$.
★ $-\dfrac{5}{\sqrt{29}}, -\dfrac{2}{\sqrt{29}}, \frac{5}{2}, \frac{2}{5}, -\dfrac{\sqrt{29}}{2}, -\dfrac{\sqrt{29}}{5}$

[74] $x = -1$ and $y = 2 \;\Rightarrow\; r = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$. ★ $\dfrac{2}{\sqrt{5}}, -\dfrac{1}{\sqrt{5}}, -2, -\frac{1}{2}, -\sqrt{5}, \dfrac{\sqrt{5}}{2}$

[75] Since the terminal side of θ is in QII, choose x to be negative.
If $x = -1$, then $y = 4$ and $(-1, 4)$ is a point on the terminal side of θ.
$x = -1$ and $y = 4 \;\Rightarrow\; r = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$. ★ $\dfrac{4}{\sqrt{17}}, -\dfrac{1}{\sqrt{17}}, -4, -\frac{1}{4}, -\sqrt{17}, \dfrac{\sqrt{17}}{4}$

[76] Since the terminal side of θ is in QIV, choose x to be positive.
If $x = 3$, then $y = -5$ and $(3, -5)$ is a point on the terminal side of θ.
$x = 3$ and $y = -5 \;\Rightarrow\; r = \sqrt{3^2 + (-5)^2} = \sqrt{34}$. ★ $-\dfrac{5}{\sqrt{34}}, \dfrac{3}{\sqrt{34}}, -\frac{5}{3}, -\frac{3}{5}, \dfrac{\sqrt{34}}{3}, -\dfrac{\sqrt{34}}{5}$

[77] An equation of the line is $y = \frac{4}{3}x$. If $x = 3$, then $y = 4$ and $(3, 4)$ is a point on the terminal side of θ. $x = 3$ and $y = 4 \;\Rightarrow\; r = \sqrt{3^2 + 4^2} = 5$. ★ $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}, \frac{3}{4}, \frac{5}{3}, \frac{5}{4}$

[78] An equation of the line bisecting the third quadrant is $y = x$.
If $x = -1$, then $y = -1$ and $(-1, -1)$ is a point on the terminal side of θ.
$x = -1$ and $y = -1 \;\Rightarrow\; r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$. ★ $-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}, 1, 1, -\sqrt{2}, -\sqrt{2}$

[79] $2y - 7x + 2 = 0 \;\Leftrightarrow\; y = \frac{7}{2}x - 1$. Thus, the slope of the given line is $\frac{7}{2}$.
An equation of the line through the origin with that slope is $y = \frac{7}{2}x$. (continued)

If $x=-2$, then $y=-7$ and $(-2,\,-7)$ is a point on the terminal side of θ.

$x=-2$ and $y=-7 \;\Rightarrow\; r=\sqrt{(-2)^2+(-7)^2}=\sqrt{53}$.

$\bigstar\; -\frac{7}{\sqrt{53}},\; -\frac{2}{\sqrt{53}},\; \frac{7}{2},\; \frac{2}{7},\; -\frac{\sqrt{53}}{2},\; -\frac{\sqrt{53}}{7}$

$\boxed{80}$ $m_{AB}=\dfrac{-2-4}{3-1}=-3$. An equation of the line through the origin with a slope of -3 is $y=-3x$. If $x=-1$, then $y=3$ and $(-1,\,3)$ is a point on the terminal side of θ.

$x=-1$ and $y=3 \;\Rightarrow\; r=\sqrt{(-1)^2+3^2}=\sqrt{10}$. $\bigstar\; \frac{3}{\sqrt{10}},\; -\frac{1}{\sqrt{10}},\; -3,\; -\frac{1}{3},\; -\sqrt{10},\; \frac{\sqrt{10}}{3}$

Note: U denotes *undefined.*

$\boxed{81}$ (a) For $\theta=90^\circ$, choose $x=0$ and $y=1$. $r=1$. $\bigstar$ 1, 0, U, 0, U, 1

(b) For $\theta=0^\circ$, choose $x=1$ and $y=0$. $r=1$. $\bigstar$ 0, 1, 0, U, 1, U

(c) For $\theta=\frac{7\pi}{2}$, choose $x=0$ and $y=-1$. $r=1$. $\bigstar$ -1, 0, U, 0, U, -1

(d) For $\theta=3\pi$, choose $x=-1$ and $y=0$. $r=1$. $\bigstar$ 0, -1, 0, U, -1, U

$\boxed{82}$ (a) For $\theta=180^\circ$, choose $x=-1$ and $y=0$. $r=1$. $\bigstar$ 0, -1, 0, U, -1, U

(b) For $\theta=-90^\circ$, choose $x=0$ and $y=-1$. $r=1$. $\bigstar$ -1, 0, U, 0, U, -1

(c) For $\theta=2\pi$, choose $x=1$ and $y=0$. $r=1$. $\bigstar$ 0, 1, 0, U, 1, U

(d) For $\theta=\frac{5\pi}{2}$, choose $x=0$ and $y=1$. $r=1$. $\bigstar$ 1, 0, U, 0, U, 1

$\boxed{83}$ (a) $\cos\theta>0 \;\Rightarrow\; \theta$ is in QI or QIV. $\sin\theta<0 \;\Rightarrow\; \theta$ is in QIII or QIV.
$\therefore\, \theta$ is in QIV.

(b) $\sin\theta<0 \;\Rightarrow\; \theta$ is in QIII or QIV. $\cot\theta>0 \;\Rightarrow\; \theta$ is in QI or QIII.
$\therefore\, \theta$ is in QIII.

(c) $\csc\theta>0 \;\Rightarrow\; \theta$ is in QI or QII. $\sec\theta<0 \;\Rightarrow\; \theta$ is in QII or QIII. $\therefore\, \theta$ is in QII.

(d) $\sec\theta<0 \;\Rightarrow\; \theta$ is in QII or QIII. $\tan\theta>0 \;\Rightarrow\; \theta$ is in QI or QIII.
$\therefore\, \theta$ is in QIII.

$\boxed{84}$ (a) $\tan\theta<0 \;\Rightarrow\; \theta$ is in QII or QIV. $\cos\theta>0 \;\Rightarrow\; \theta$ is in QI or QIV.
$\therefore\, \theta$ is in QIV.

(b) $\sec\theta>0 \;\Rightarrow\; \theta$ is QI or QIV. $\tan\theta<0 \;\Rightarrow\; \theta$ is in QII or QIV. $\therefore\, \theta$ is in QIV.

(c) $\csc\theta>0 \;\Rightarrow\; \theta$ is in QI or QII. $\cot\theta<0 \;\Rightarrow\; \theta$ is in QII or QIV. $\therefore\, \theta$ is in QII.

(d) $\cos\theta<0 \;\Rightarrow\; \theta$ is in QII or QIII. $\csc\theta<0 \;\Rightarrow\; \theta$ is in QIII or QIV.
$\therefore\, \theta$ is in QIII.

Note: Exer. 85–92: Steps to determine 2 function values using only the fundamental identities are shown. The other 3 function values are just the reciprocals of those given and are listed in the answer.

$\boxed{85}$ $\tan\theta=-\frac{3}{4}$ and $\sin\theta>0 \;\Rightarrow\; \theta$ is in QII. $\sec\theta=-\sqrt{1+\tan^2\theta}=-\sqrt{1+\frac{9}{16}}=-\frac{5}{4}$.

$\tan\theta=\dfrac{\sin\theta}{\cos\theta} \;\Rightarrow\; -\frac{3}{4}=\dfrac{\sin\theta}{-4/5} \;\Rightarrow\; \sin\theta=\frac{3}{5}$ $\bigstar\; \frac{3}{5},\; -\frac{4}{5},\; -\frac{3}{4},\; -\frac{4}{3},\; -\frac{5}{4},\; \frac{5}{3}$

$\boxed{86}$ $\cot\theta = \frac{3}{4}$ and $\cos\theta < 0 \Rightarrow \theta$ is in QIII. $\csc\theta = -\sqrt{1+\cot^2\theta} = -\sqrt{1+\frac{9}{16}} = -\frac{5}{4}$;

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} \Rightarrow \frac{3}{4} = \dfrac{\cos\theta}{-4/5} \Rightarrow \cos\theta = -\frac{3}{5}$ $\quad\bigstar\ -\frac{4}{5}, -\frac{3}{5}, \frac{4}{3}, \frac{3}{4}, -\frac{5}{3}, -\frac{5}{4}$

$\boxed{87}$ $\sin\theta = -\frac{5}{13}$ and $\sec\theta > 0 \Rightarrow \theta$ is in QIV. $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-5/13}{12/13} = -\frac{5}{12}$ $\quad\bigstar\ -\frac{5}{13}, \frac{12}{13}, -\frac{5}{12}, -\frac{12}{5}, \frac{13}{12}, -\frac{13}{5}$

$\boxed{88}$ $\cos\theta = \frac{1}{2}$ and $\sin\theta < 0 \Rightarrow \theta$ is in QIV. $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{4}} = -\frac{\sqrt{3}}{2}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\sqrt{3}/2}{1/2} = -\sqrt{3}$ $\quad\bigstar\ -\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3}, -\frac{1}{\sqrt{3}}, 2, -\frac{2}{\sqrt{3}}$

$\boxed{89}$ $\cos\theta = -\frac{1}{3}$ and $\sin\theta < 0 \Rightarrow \theta$ is in QIII. $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{9}} = -\frac{\sqrt{8}}{3}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{-\sqrt{8}/3}{-1/3} = \sqrt{8} = 2\sqrt{2}$ $\quad\bigstar\ -\frac{\sqrt{8}}{3}, -\frac{1}{3}, \sqrt{8}, \frac{1}{\sqrt{8}}, -3, -\frac{3}{\sqrt{8}}$

$\boxed{90}$ $\csc\theta = 5$ and $\cot\theta < 0 \Rightarrow \theta$ is in QII. $\cot\theta = -\sqrt{\csc^2\theta - 1} = -\sqrt{25-1} = -\sqrt{24}$.

$\cot\theta = \dfrac{\cos\theta}{\sin\theta} \Rightarrow -\sqrt{24} = \dfrac{\cos\theta}{1/5} \Rightarrow \cos\theta = -\dfrac{\sqrt{24}}{5}$

$\bigstar\ \frac{1}{5}, -\frac{\sqrt{24}}{5}, -\frac{1}{\sqrt{24}}, -\sqrt{24}, -\frac{5}{\sqrt{24}}, 5$

$\boxed{91}$ $\sec\theta = -4$ and $\csc\theta > 0 \Rightarrow \theta$ is in QII. $\tan\theta = -\sqrt{\sec^2\theta - 1} = -\sqrt{16-1} = -\sqrt{15}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \Rightarrow -\sqrt{15} = \dfrac{\sin\theta}{-1/4} \Rightarrow \sin\theta = \dfrac{\sqrt{15}}{4}$

$\bigstar\ \frac{\sqrt{15}}{4}, -\frac{1}{4}, -\sqrt{15}, -\frac{1}{\sqrt{15}}, -4, \frac{4}{\sqrt{15}}$

$\boxed{92}$ $\sin\theta = \frac{2}{5}$ and $\cos\theta < 0 \Rightarrow \theta$ is in QII. $\cos\theta = -\sqrt{1-\sin^2\theta} = -\sqrt{1-\frac{4}{25}} = -\frac{\sqrt{21}}{5}$.

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{2/5}{-\sqrt{21}/5} = -\frac{2}{\sqrt{21}}$ $\quad\bigstar\ \frac{2}{5}, -\frac{\sqrt{21}}{5}, -\frac{2}{\sqrt{21}}, -\frac{\sqrt{21}}{2}, -\frac{5}{\sqrt{21}}, \frac{5}{2}$

$\boxed{93}$ $\sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = |\tan\theta| = -\tan\theta$ since $\tan\theta < 0$ if $\pi/2 < \theta < \pi$

$\boxed{94}$ $\sqrt{1+\cot^2\theta} = \sqrt{\csc^2\theta} = |\csc\theta| = \csc\theta$ since $\csc\theta > 0$ if $0 < \theta < \pi$

$\boxed{95}$ $\sqrt{1+\tan^2\theta} = \sqrt{\sec^2\theta} = |\sec\theta| = \sec\theta$ since $\sec\theta > 0$ if $3\pi/2 < \theta < 2\pi$

$\boxed{96}$ $\sqrt{\csc^2\theta - 1} = \sqrt{\cot^2\theta} = |\cot\theta| = -\cot\theta$ since $\cot\theta < 0$ if $3\pi/2 < \theta < 2\pi$

$\boxed{97}$ $\sqrt{\sin^2(\theta/2)} = |\sin(\theta/2)| = -\sin(\theta/2)$ since

$\sin(\theta/2) < 0$ if $2\pi < \theta < 4\pi$ $\{2\pi < \theta < 4\pi \Rightarrow \pi < \theta/2 < 2\pi\}$

$\boxed{98}$ $\sqrt{\cos^2(\theta/2)} = |\cos(\theta/2)| = \cos(\theta/2)$ since

$\cos(\theta/2) > 0$ if $0 < \theta < \pi$ $\{0 < \theta/2 < \pi/2\}$

6.3 Exercises

$\boxed{1}$ $P(-\frac{15}{17}, \frac{8}{17}) \Rightarrow \sin t = y = \frac{8}{17}$, $\cos t = x = -\frac{15}{17}$, $\tan t = y/x = -\frac{8}{15}$,

$\cot t = x/y = -\frac{15}{8}$, $\sec t = 1/x = -\frac{17}{15}$, $\csc t = 1/y = \frac{17}{8}$

$\boxed{2}$ $P(\frac{4}{5}, \frac{3}{5})$ gives us $\frac{3}{5}, \frac{4}{5}, \frac{3}{4}, \frac{4}{3}, \frac{5}{4}, \frac{5}{3}$.

[3] $P(\frac{24}{25}, -\frac{7}{25})$ gives us $-\frac{7}{25}, \frac{24}{25}, -\frac{7}{24}, -\frac{24}{7}, \frac{25}{24}, -\frac{25}{7}$.

[4] $P(-\frac{5}{13}, -\frac{12}{13})$ gives us $-\frac{12}{13}, -\frac{5}{13}, \frac{12}{5}, \frac{5}{12}, -\frac{13}{5}, -\frac{13}{12}$.

[5] (a) $P(t) = (\frac{3}{5}, \frac{4}{5}) \Rightarrow P(t+\pi) = (-\frac{3}{5}, -\frac{4}{5})$ (b) $P(t-\pi) = P(t+\pi) = (-\frac{3}{5}, -\frac{4}{5})$
(c) $P(-t) = (\frac{3}{5}, -\frac{4}{5})$ (d) $P(-t-\pi) = (-\frac{3}{5}, \frac{4}{5})$

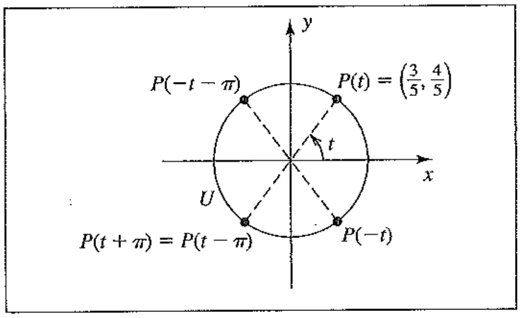

Figure 5

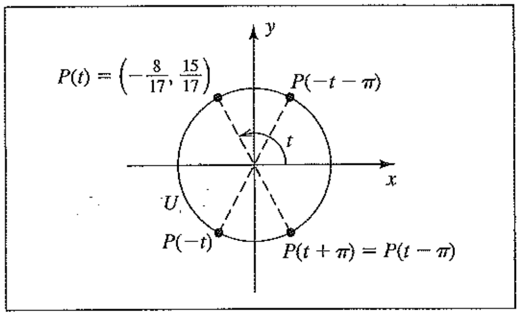

Figure 6

[6] (a) $P(t) = (-\frac{8}{17}, \frac{15}{17}) \Rightarrow P(t+\pi) = (\frac{8}{17}, -\frac{15}{17})$ (b) $P(t-\pi) = P(t+\pi) = (\frac{8}{17}, -\frac{15}{17})$
(c) $P(-t) = (-\frac{8}{17}, -\frac{15}{17})$ (d) $P(-t-\pi) = (\frac{8}{17}, \frac{15}{17})$

[7] (a) $P(t) = (-\frac{12}{13}, -\frac{5}{13}) \Rightarrow P(t+\pi) = (\frac{12}{13}, \frac{5}{13})$ (b) $P(t-\pi) = P(t+\pi) = (\frac{12}{13}, \frac{5}{13})$
(c) $P(-t) = (-\frac{12}{13}, \frac{5}{13})$ (d) $P(-t-\pi) = (\frac{12}{13}, -\frac{5}{13})$

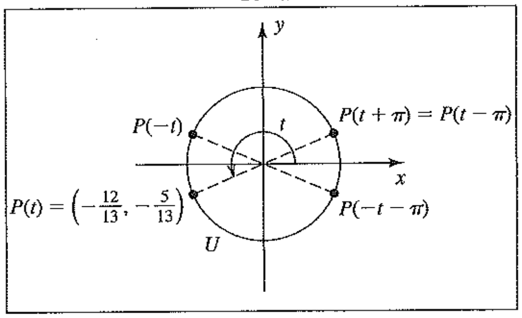

Figure 7

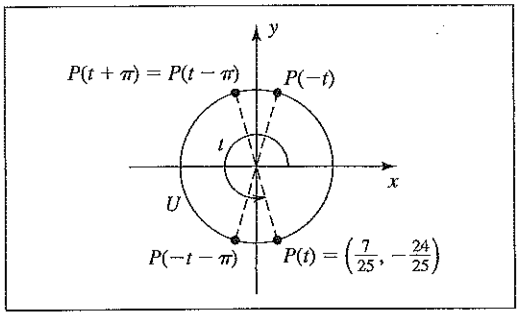

Figure 8

[8] (a) $P(t) = (\frac{7}{25}, -\frac{24}{25}) \Rightarrow P(t+\pi) = (-\frac{7}{25}, \frac{24}{25})$ (b) $P(t-\pi) = P(t+\pi) = (-\frac{7}{25}, \frac{24}{25})$
(c) $P(-t) = (\frac{7}{25}, \frac{24}{25})$ (d) $P(-t-\pi) = (-\frac{7}{25}, -\frac{24}{25})$

Note: U denotes *undefined*.

[9] (a) $t = 2\pi \Rightarrow P(x, y) = (1, 0)$.

The exact values of the trigonometric functions at t are 0, 1, 0, U, 1, U.

(b) $t = -3\pi \Rightarrow P(x, y) = (-1, 0)$. 0, −1, 0, U, −1, U

[10] (a) $t = -\pi \Rightarrow P(x, y) = (-1, 0)$. 0, −1, 0, U, −1, U

(b) $t = 6\pi \Rightarrow P(x, y) = (1, 0)$. 0, 1, 0, U, 1, U

[11] (a) $t = \frac{3\pi}{2} \Rightarrow P(x, y) = (0, -1)$. −1, 0, U, 0, U, −1

(b) $t = -\frac{7\pi}{2} \Rightarrow P(x, y) = (0, 1)$. 1, 0, U, 0, U, 1

$\boxed{12}$ (a) $t = \frac{5\pi}{2} \Rightarrow P(x, y) = (0, 1)$. 1, 0, U, 0, U, 1

(b) $t = -\frac{\pi}{2} \Rightarrow P(x, y) = (0, -1)$. -1, 0, U, 0, U, -1

$\boxed{13}$ (a) $t = \frac{9\pi}{4} \Rightarrow P(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1, 1, \sqrt{2}, \sqrt{2}$

(b) $t = -\frac{5\pi}{4} \Rightarrow P(x, y) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -1, -1, -\sqrt{2}, \sqrt{2}$

$\boxed{14}$ (a) $t = \frac{3\pi}{4} \Rightarrow P(x, y) = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. $\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -1, -1, -\sqrt{2}, \sqrt{2}$

(b) $t = -\frac{7\pi}{4} \Rightarrow P(x, y) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$. $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1, 1, \sqrt{2}, \sqrt{2}$

$\boxed{15}$ (a) $t = \frac{5\pi}{4} \Rightarrow P(x, y) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. $-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1, 1, -\sqrt{2}, -\sqrt{2}$

(b) $t = -\frac{\pi}{4} \Rightarrow P(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1, -1, \sqrt{2}, -\sqrt{2}$

$\boxed{16}$ (a) $t = \frac{7\pi}{4} \Rightarrow P(x, y) = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. $-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1, -1, \sqrt{2}, -\sqrt{2}$

(b) $t = -\frac{3\pi}{4} \Rightarrow P(x, y) = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$. $-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1, 1, -\sqrt{2}, -\sqrt{2}$

$\boxed{17}$ (a) $\sin(-90°) = -\sin 90° = -1$ (b) $\cos\left(-\frac{3\pi}{4}\right) = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$

(c) $\tan(-45°) = -\tan 45° = -1$

$\boxed{18}$ (a) $\sin\left(-\frac{3\pi}{2}\right) = -\sin\frac{3\pi}{2} = -(-1) = 1$ (b) $\cos(-225°) = \cos 225° = -\frac{\sqrt{2}}{2}$

(c) $\tan(-\pi) = -\tan\pi = 0$

$\boxed{19}$ (a) $\cot\left(-\frac{3\pi}{4}\right) = -\cot\frac{3\pi}{4} = -(-1) = 1$ (b) $\sec(-180°) = \sec 180° = -1$

(c) $\csc\left(-\frac{3\pi}{2}\right) = -\csc\frac{3\pi}{2} = -(-1) = 1$

$\boxed{20}$ (a) $\cot(-225°) = -\cot 225° = -1$ (b) $\sec\left(-\frac{\pi}{4}\right) = \sec\frac{\pi}{4} = \sqrt{2}$

(c) $\csc(-45°) = -\csc 45° = -\sqrt{2}$

$\boxed{21}$ $\sin(-x)\sec(-x) = (-\sin x)\sec x = (-\sin x)(1/\cos x) = -\tan x$

$\boxed{22}$ $\csc(-x)\cos(-x) = (-\csc x)\cos x = (-1/\sin x)(\cos x) = -\cot x$

$\boxed{23}$ $\dfrac{\cot(-x)}{\csc(-x)} = \dfrac{-\cot x}{-\csc x} = \dfrac{\cos x/\sin x}{1/\sin x} = \cos x$

$\boxed{24}$ $\dfrac{\sec(-x)}{\tan(-x)} = \dfrac{\sec x}{-\tan x} = -\dfrac{1/\cos x}{\sin x/\cos x} = -\dfrac{1}{\sin x} = -\csc x$

$\boxed{25}$ $\dfrac{1}{\cos(-x)} - \tan(-x)\sin(-x) = \frac{1}{\cos x} - (-\tan x)(-\sin x) =$

$$\frac{1}{\cos x} - \frac{\sin x}{\cos x}\sin x = \frac{1 - \sin^2 x}{\cos x} = \frac{\cos^2 x}{\cos x} = \cos x$$

$\boxed{26}$ $\cot(-x)\cos(-x) + \sin(-x) = -\cot x \cos x - \sin x =$

$$-\frac{\cos x}{\sin x}\cos x - \sin x = -\frac{\cos^2 x + \sin^2 x}{\sin x} = -\frac{1}{\sin x} = -\csc x$$

27 (a) As $x \to 0^+$, $\sin x \to \underline{0}$ (b) As $x \to -\frac{\pi}{2}^-$, $\sin x \to \underline{-1}$

28 (a) As $x \to \pi^+$, $\sin x \to \underline{0}$ (b) As $x \to \frac{\pi}{6}^-$, $\sin x \to \underline{\tfrac{1}{2}}$

29 (a) As $x \to \frac{\pi}{4}^+$, $\cos x \to \underline{\sqrt{2}/2}$ (b) As $x \to \pi^-$, $\cos x \to \underline{-1}$

30 (a) As $x \to 0^+$, $\cos x \to \underline{1}$ (b) As $x \to -\frac{\pi}{3}^-$, $\cos x \to \underline{\tfrac{1}{2}}$

31 (a) As $x \to \frac{\pi}{4}^+$, $\tan x \to \underline{1}$ (b) As $x \to \frac{\pi}{2}^+$, $\tan x \to \underline{-\infty}$

32 (a) As $x \to 0^+$, $\tan x \to \underline{0}$ (b) As $x \to -\frac{\pi}{2}^-$, $\tan x \to \underline{\infty}$

33 (a) As $x \to -\frac{\pi}{4}^-$, $\cot x \to \underline{-1}$ (b) As $x \to 0^+$, $\cot x \to \underline{\infty}$

34 (a) As $x \to \frac{\pi}{6}^+$, $\cot x \to \underline{\sqrt{3}}$ (b) As $x \to \pi^-$, $\cot x \to \underline{-\infty}$

35 (a) As $x \to \frac{\pi}{2}^-$, $\sec x \to \underline{\infty}$ (b) As $x \to \frac{\pi}{4}^+$, $\sec x \to \underline{\sqrt{2}}$

36 (a) As $x \to \frac{\pi}{2}^+$, $\sec x \to \underline{-\infty}$ (b) As $x \to 0^-$, $\sec x \to \underline{1}$

37 (a) As $x \to 0^-$, $\csc x \to \underline{-\infty}$ (b) As $x \to \frac{\pi}{2}^+$, $\csc x \to \underline{1}$

38 (a) As $x \to \pi^+$, $\csc x \to \underline{-\infty}$ (b) As $x \to \frac{\pi}{4}^-$, $\csc x \to \underline{\sqrt{2}}$

39 Refer to Figure 8 and the accompanying table. We see that $\sin\frac{3\pi}{2} = -1$.

Since the period of the sine is 2π, the second value in $[0, 4\pi]$ is $\frac{3\pi}{2} + 2\pi = \frac{7\pi}{2}$.

★ $\frac{\pi}{2}, \frac{5\pi}{2}$

40 $\sin x = 1$ •

★ $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

41 $\sin x = \frac{1}{2}$ •

★ $\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

42 $\sin x = -\sqrt{2}/2$ •

★ $0, 2\pi, 4\pi$

43 $\cos x = 1$ •

★ $\pi, 3\pi$

44 $\cos x = -1$ •

45 Refer to Figure 10 and the accompanying table. We see that $\cos\frac{\pi}{4} = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$.

Since the period of the cosine is 2π,

other values in $[0, 4\pi]$ are $\frac{\pi}{4} + 2\pi = \frac{9\pi}{4}$ and $\frac{7\pi}{4} + 2\pi = \frac{15\pi}{4}$.

★ $\frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

46 $\cos x = -\frac{1}{2}$ •

47 Refer to Figure 13. In the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, $\tan x = 1$ only if $x = \frac{\pi}{4}$. Since the period of the tangent is π, the desired value in the interval $(\frac{\pi}{2}, \frac{3\pi}{2})$ is $\frac{\pi}{4} + \pi = \frac{5\pi}{4}$.

★ $\frac{\pi}{3}, \frac{4\pi}{3}$

48 $\tan x = \sqrt{3}$ •

★ $0, \pi$

49 $\tan x = 0$ •

★ $-\frac{\pi}{6}, \frac{5\pi}{6}$

50 $\tan x = -1/\sqrt{3}$ •

51 $y = \sin x$; $[-2\pi, 2\pi]$; $a = \frac{1}{2}$ • Refer to Figure 8. $\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$.

Also, $\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$ and $\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$. $\sin x > \frac{1}{2}$ when the graph is above the horizontal line $y = \frac{1}{2}$. $\sin x < \frac{1}{2}$ when the graph is below the horizontal line $y = \frac{1}{2}$.

★ (a) $-\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$ (b) $-\frac{11\pi}{6} < x < -\frac{7\pi}{6}$ and $\frac{\pi}{6} < x < \frac{5\pi}{6}$

(c) $-2\pi \le x < -\frac{11\pi}{6}$, $-\frac{7\pi}{6} < x < \frac{\pi}{6}$, and $\frac{5\pi}{6} < x \le 2\pi$

[52] $y = \cos x$; $[0, 4\pi]$; $a = \dfrac{\sqrt{3}}{2}$ • ★ (a) $\frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

(b) $0 \le x < \frac{\pi}{6}$, $\frac{11\pi}{6} < x < \frac{13\pi}{6}$, and $\frac{23\pi}{6} < x \le 4\pi$ (c) $\frac{\pi}{6} < x < \frac{11\pi}{6}$ and $\frac{13\pi}{6} < x < \frac{23\pi}{6}$

[53] $y = \cos x$; $[-2\pi, 2\pi]$; $a = -\frac{1}{2}$ • ★ (a) $-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

(b) $-2\pi \le x < -\frac{4\pi}{3}$, $-\frac{2\pi}{3} < x < \frac{2\pi}{3}$, and $\frac{4\pi}{3} < x \le 2\pi$

(c) $-\frac{4\pi}{3} < x < -\frac{2\pi}{3}$ and $\frac{2\pi}{3} < x < \frac{4\pi}{3}$

[54] $y = \sin x$; $[0, 4\pi]$; $a = -\dfrac{\sqrt{2}}{2}$ • ★ (a) $\frac{5\pi}{4}, \frac{7\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

(b) $0 \le x < \frac{5\pi}{4}$, $\frac{7\pi}{4} < x < \frac{13\pi}{4}$, and $\frac{15\pi}{4} < x \le 4\pi$ (c) $\frac{5\pi}{4} < x < \frac{7\pi}{4}$ and $\frac{13\pi}{4} < x < \frac{15\pi}{4}$

[55] $y = 2 + \sin x$ • shift $y = \sin x$ up 2 units

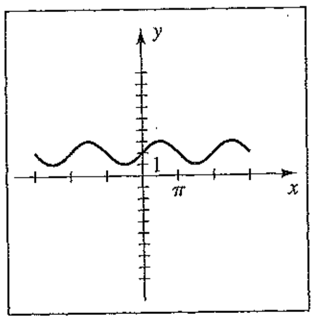

Figure 55

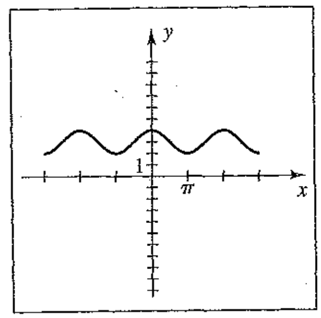

Figure 56

[56] $y = 3 + \cos x$ • shift $y = \cos x$ up 3 units

[57] $y = \cos x - 2$ • shift $y = \cos x$ down 2 units

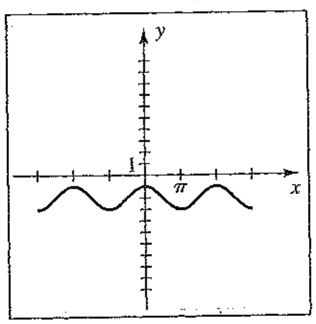

Figure 57

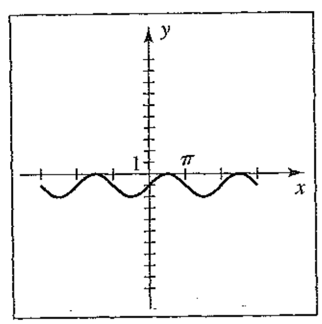

Figure 58

[58] $y = \sin x - 1$ • shift $y = \sin x$ down 1 unit, x-intercepts are at $x = \frac{\pi}{2} + 2\pi n$

[59] $y = 1 + \tan x$ • shift $y = \tan x$ up 1 unit, x-intercepts are at $x = -\frac{\pi}{4} + \pi n$

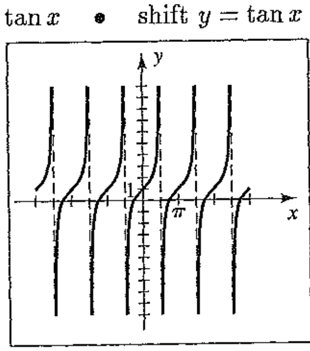

Figure 59

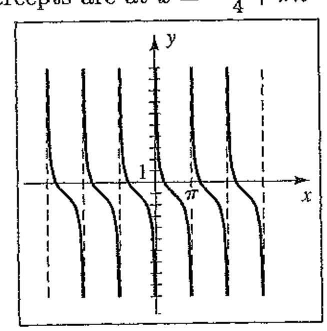

Figure 60

[60] $y = \cot x - 1$ • shift $y = \cot x$ down 1 unit, x-intercepts are at $x = \frac{\pi}{4} + \pi n$

$\boxed{61}$ $y = \sec x - 2$ • shift $y = \sec x$ down 2 units, x-intercepts are at $x = \frac{\pi}{3} + 2\pi n$, $\frac{5\pi}{3} + 2\pi n$

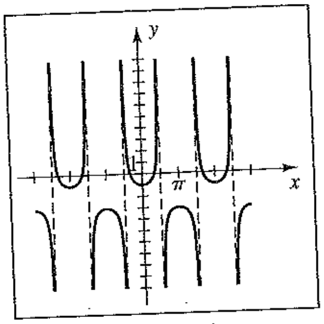

Figure 61

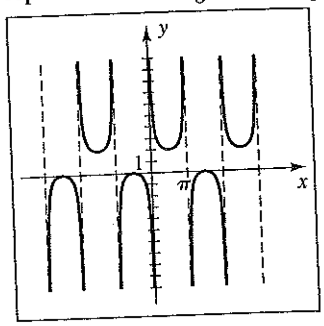

Figure 62

$\boxed{62}$ $y = 1 + \csc x$ • shift $y = \csc x$ up 1 unit, x-intercepts are at $x = \frac{3\pi}{2} + 2\pi n$

$\boxed{63}$ (a) As we move from left to right, the function rises on the intervals

$[-2\pi, -\frac{3\pi}{2})$, $(-\frac{3\pi}{2}, -\pi]$, $[0, \frac{\pi}{2})$, and $(\frac{\pi}{2}, \pi]$.

(b) $[-\pi, -\frac{\pi}{2})$, $(-\frac{\pi}{2}, 0]$, $[\pi, \frac{3\pi}{2})$, $(\frac{3\pi}{2}, 2\pi]$

$\boxed{64}$ (a) $[-\frac{3\pi}{2}, -\pi)$, $(-\pi, -\frac{\pi}{2}]$, $[\frac{\pi}{2}, \pi)$, $(\pi, \frac{3\pi}{2}]$ (b) $(-2\pi, -\frac{3\pi}{2}]$, $[-\frac{\pi}{2}, 0)$, $(0, \frac{\pi}{2}]$, $[\frac{3\pi}{2}, 2\pi)$

$\boxed{65}$ (a) The tangent function increases on *all* intervals on which it is defined.

Between -2π and 2π,

these intervals are $[-2\pi, -\frac{3\pi}{2})$, $(-\frac{3\pi}{2}, -\frac{\pi}{2})$, $(-\frac{\pi}{2}, \frac{\pi}{2})$, $(\frac{\pi}{2}, \frac{3\pi}{2})$, and $(\frac{3\pi}{2}, 2\pi]$.

(b) The tangent function is *never* decreasing on any interval for which it is defined.

$\boxed{66}$ (a) The cotangent function is *never* increasing on any interval for which it is defined.

(b) The cotangent function decreases on *all* intervals on which it is defined.

Between -2π and 2π, these intervals are $(-2\pi, -\pi)$, $(-\pi, 0)$, $(0, \pi)$, and $(\pi, 2\pi)$.

$\boxed{69}$ (a) From $(1, 0)$, move counterclockwise on the unit circle to the point at the tick marked 4. The projection of this point on the y-axis,

approximately -0.7 or -0.8, is the value of $\sin 4$.

(b) From $(1, 0)$, move clockwise 1.2 units to about 5.1.

As in part (a), $\sin(-1.2)$ is about -0.9.

(c) Draw the horizontal line $y = 0.5$.

This line intersects the circle at about 0.5 and 2.6.

$\boxed{70}$ (a) As in part (a) of Exercise 69, $\sin 2$ is about 0.9.

(b) As in part (b) of Exercise 69, $\sin(-2.3)$ is about -0.8.

(c) As in part (c) of Exercise 69,

the horizontal line $y = -0.2$ intersects the circle at about 3.3 and 6.1.

$\boxed{71}$ (a) From (1, 0), move counterclockwise on the unit circle to the point at the tick marked 4. The projection of this point on the x-axis,

approximately -0.6 or -0.7, is the value of $\cos 4$.

(b) Proceeding as in part (a), $\cos(-1.2)$ is about 0.4.

(c) Draw the vertical line $x = -0.6$.

This line intersects the circle at about 2.2 and 4.1.

$\boxed{72}$ (a) As in part (a) of Exercise 71, $\cos 2$ is about -0.5.

(b) As in part (b) of Exercise 71, $\cos(-2.3)$ is about -0.7.

(c) As in part (c) of Exercise 71,

the vertical line $x = 0.2$ intersects the circle at about 1.4 and 4.9.

$\boxed{73}$ (a) Note that midnight occurs when $t = -6$.

Time	Temp.	Humidity	Time	Temp.	Humidity
12 A.M.	60	60	12 P.M.	60	60
3 A.M.	52	74	3 P.M.	68	46
6 A.M.	48	80	6 P.M.	72	40
9 A.M.	52	74	9 P.M.	68	46

(b) Since $T(t) = -12\cos\left(\frac{\pi}{12}t\right) + 60$, its maximum is $60 + 12 = 72$°F at $t = 12$ or 6:00 P.M., and its minimum is $60 - 12 = 48$°F at $t = 0$ or 6:00 A.M. Since $H(t) = 20\cos\left(\frac{\pi}{12}t\right) + 60$, its maximum is $60 + 20 = 80\%$ at $t = 0$ or 6:00 A.M., and its minimum is $60 - 20 = 40\%$ at $t = 12$ or 6:00 P.M.

(c) When the temperature increases, the relative humidity decreases and vice versa. As the temperature cools, the air can hold less moisture and the relative humidity increases. Because of this phenomenon, fog often occurs during the evening hours rather than in the middle of the day.

$\boxed{74}$ (a) Since the elbow joint does not bend and the arm has a constant length, the robotic hand moves in a circular arc. Therefore, its height h is equal to $h = 153\sin\theta + 50 = 153\sin\left(\frac{\pi}{12}t\right) + 50$. If $0 \le \theta \le \pi/2$, then $0 \le t \le 6$.

t (sec)	0	1	2	3	4	5	6
θ (rad)	0	$\pi/12$	$\pi/6$	$\pi/4$	$\pi/3$	$5\pi/12$	$\pi/2$
h (cm)	50.00	89.60	126.50	158.19	182.50	197.79	203.00

(b) From the table, it is obvious that the height of the arm does not increase at a constant rate. Instead, it increases at a slower rate as θ increases at a constant rate. During the first second of time, the hand's height increases by $89.6 - 50 = 39.6$ cm, whereas during the sixth second its height increased by only $203 - 197.79 = 5.21$ cm.

(c) The total distance traveled by the hand is determined by the circular arc with a radius of 153 cm, subtending an angle of $\pi/2$.

Thus, $s = r\theta = 153(\pi/2) = 76.5\pi \approx 240.33$ cm.

$\boxed{75}$ Graph $y = \sin(x^2)$ and $y = 0.5$ on the same coordinate plane. From the graph, we see that $\sin(x^2)$ assumes the value of 0.5 at $x \approx \pm 0.72, \pm 1.62, \pm 2.61, \pm 2.98$.

$[-\pi, \pi, \pi/4]$ by $[-2.09, 2.09]$

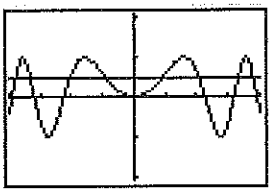

Figure 75

$[0, 25, 2]$ by $[-8.33, 8.33]$

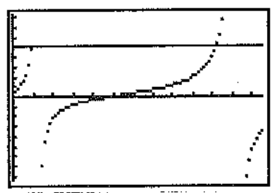

Figure 76

$\boxed{76}$ Graph $y = \tan(\sqrt{x})$ and $y = 5$ on the same coordinate plane. From the graph, we see that $\tan(\sqrt{x})$ assumes the value of 5 at $x \approx 1.89, 20.39$. *Figure 76* was obtained by using Dot Mode. Note that the vertical asymptotes are at $\sqrt{x} = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \frac{\pi^2}{4} \approx 2.47, \frac{9\pi^2}{4} \approx 22.21$.

$\boxed{77}$ We see that the graph of $y = x \sin x$ assumes a maximum value of approximately 1.82 at $x \approx \pm 2.03$, and a minimum value of -4.81 at $x \approx \pm 4.91$.

$[-2\pi, 2\pi, \pi/2]$ by $[-5.19, 3.19]$

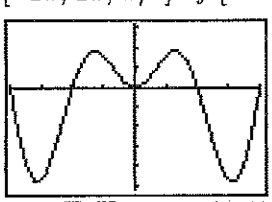

Figure 77

$[-2\pi, 2\pi, \pi/2]$ by $[-4.19, 4.19]$

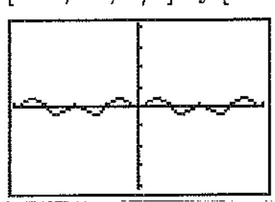

Figure 78

$\boxed{78}$ We see that the graph of $y = \sin^2 x \cos x$ assumes a maximum value of approximately 0.38 at $x \approx \pm 0.96, \pm 5.33$, and a minimum value of -0.38 at $x \approx \pm 2.19, \pm 4.10$.

$\boxed{79}$ As $x \to 0^+$, $f(x) = \dfrac{1 - \cos x}{x} \to 0$.

[−1, 1, 0.5] by [−0.67, 0.67, 0.5]

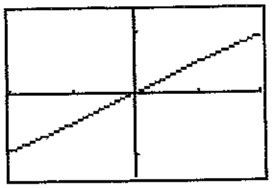

Figure 79

$\boxed{80}$ As $x \to 0^+$, $f(x) = \dfrac{6x - 6\sin x}{x^3} \to 1$.

[−2, 2, 0.5] by [−1.33, 1.33, 0.5]

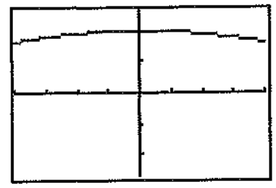

Figure 80

$\boxed{81}$ As $x \to 0^+$, $f(x) = x \cot x \to 1$.

[−2, 2, 0.5] by [−1.33, 1.33, 0.5]

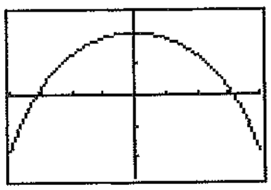

Figure 81

$\boxed{82}$ As $x \to 0^+$, $f(x) = \dfrac{x + \tan x}{\sin x} \to 2$.

[−1.5, 1.5] by [0, 3]

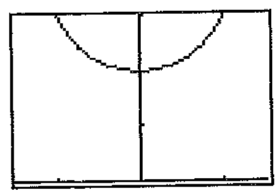

Figure 82

$\boxed{83}$ As $x \to 0^+$, $f(x) = \dfrac{\tan x}{x} \to 1$.

[−1.5, 1.5] by [0, 3]

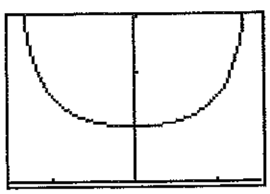

Figure 83

$\boxed{84}$ As $x \to 0^+$, $f(x) = \dfrac{\cos\left(x + \frac{1}{2}\pi\right)}{x} \to -1$.

[−1, 1, 0.5] by [−1.33, 0, 0.5]

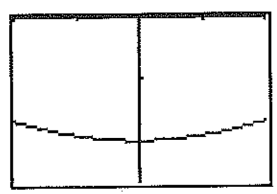

Figure 84

6.4 Exercises

Note: Let θ_C denote the coterminal angle of θ such that $0^\circ \le \theta_C < 360^\circ$ {or $0 \le \theta_C < 2\pi$}. The following formulas are then used in the solutions {on text page 448}.

(1) If θ_C is in QI, then $\theta_R = \theta_C$.

(2) If θ_C is in QII, then $\theta_R = 180^\circ - \theta_C$ {or $\pi - \theta_C$}.

(3) If θ_C is in QIII, then $\theta_R = \theta_C - 180^\circ$ {or $\theta_C - \pi$}.

(4) If θ_C is in QIV, then $\theta_R = 360^\circ - \theta_C$ {or $2\pi - \theta_C$}.

Note: It may be easier to simply draw the angles when explaining the solutions.

$\boxed{1}$ (a) Since 240° is in QIII, $\theta_R = 240^\circ - 180^\circ = 60^\circ$.

(b) Since 340° is in QIV, $\theta_R = 360^\circ - 340^\circ = 20^\circ$.

(c) $\theta_C = -202^\circ + 1(360^\circ) = 158^\circ \in$ QII. $\theta_R = 180^\circ - 158^\circ = 22^\circ$.

(d) $\theta_C = -660^\circ + 2(360^\circ) = 60^\circ \in$ QI. $\theta_R = 60^\circ$.

2 (a) Since 165° is in QII, $\theta_R = 180^\circ - 165^\circ = 15^\circ$.

(b) Since 275° is in QIV, $\theta_R = 360^\circ - 275^\circ = 85^\circ$.

(c) $\theta_C = -110^\circ + 1(360^\circ) = 250^\circ \in$ QIII. $\theta_R = 250^\circ - 180^\circ = 70^\circ$.

(d) $\theta_C = 400^\circ - 1(360^\circ) = 40^\circ \in$ QI. $\theta_R = 40^\circ$.

3 (a) Since $\frac{3\pi}{4}$ is in QII, $\theta_R = \pi - \frac{3\pi}{4} = \frac{\pi}{4}$.

(b) Since $\frac{4\pi}{3}$ is in QIII, $\theta_R = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

(c) $\theta_C = -\frac{\pi}{6} + 1(2\pi) = \frac{11\pi}{6} \in$ QIV. $\theta_R = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

(d) $\theta_C = \frac{9\pi}{4} - 1(2\pi) = \frac{\pi}{4} \in$ QI. $\theta_R = \frac{\pi}{4}$.

4 (a) Since $\frac{7\pi}{4}$ is in QIV, $\theta_R = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$.

(b) Since $\frac{2\pi}{3}$ is in QII, $\theta_R = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

(c) $\theta_C = -\frac{3\pi}{4} + 1(2\pi) = \frac{5\pi}{4} \in$ QIII. $\theta_R = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

(d) $\theta_C = -\frac{23\pi}{6} + 2(2\pi) = \frac{\pi}{6} \in$ QI. $\theta_R = \frac{\pi}{6}$.

5 (a) Since $\frac{\pi}{2} < 3 < \pi$, θ is in QII and $\theta_R = \pi - 3 \approx 0.14$, or 8.1°.

(b) $\theta_C = -2 + 1(2\pi) = 2\pi - 2 \approx 4.28$.

Since $\pi < 4.28 < \frac{3\pi}{2}$, θ_C is in QIII and $\theta_R = (2\pi - 2) - \pi = \pi - 2 \approx 1.14$, or 65.4°.

(c) Since $\frac{3\pi}{2} < 5.5 < 2\pi$, θ is in QIV and $\theta_R = 2\pi - 5.5 \approx 0.78$, or 44.9°.

(d) The number of revolutions formed by θ is $\frac{100}{2\pi} \approx 15.92$, so

$\theta_C = 100 - 15(2\pi) = 100 - 30\pi \approx 5.75$. Since $\frac{3\pi}{2} < 5.75 < 2\pi$,

θ_C is in QIV and $\theta_R = 2\pi - (100 - 30\pi) = 32\pi - 100 \approx 0.53$, or 30.4°.

Alternatively, if your calculator is capable of computing trigonometric functions

of large values, then computing $\sin^{-1}(\sin 100) \approx -0.53 \Rightarrow \theta_R = 0.53$, or 30.4°.

6 (a) Since $\frac{3\pi}{2} < 6 < 2\pi$, θ is in QIV and $\theta_R = 2\pi - 6 \approx 0.28$, or 16.2°.

(b) $\theta_C = -4 + 1(2\pi) = 2\pi - 4 \approx 2.28$.

Since $\frac{\pi}{2} < 2.28 < \pi$, θ_C is in QII and $\theta_R = \pi - (2\pi - 4) = 4 - \pi \approx 0.86$, or 49.2°.

(c) Since $\pi < 4.5 < \frac{3\pi}{2}$, θ is in QIII and $\theta_R = 4.5 - \pi \approx 1.36$, or 77.8°.

(d) As in Exercise 5(d), $\frac{80}{2\pi} \approx 12.73$, so $\theta_C = 80 - 12(2\pi) = 80 - 24\pi \approx 4.60$. Since

$\pi < 4.60 < \frac{3\pi}{2}$, θ_C is in QIII and $\theta_R = (80 - 24\pi) - \pi = 80 - 25\pi \approx 1.46$, or 83.7°.

Alternatively, $\sin^{-1}(\sin 80) \approx -1.46 \Rightarrow \theta_R = 1.46$, or 83.7°.

Note: For the following problems, we use the theorem on reference angles before evaluating.

7 (a) $\sin\frac{2\pi}{3} = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ (b) $\sin\left(-\frac{5\pi}{4}\right) = \sin\frac{3\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

[8] (a) $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$ (b) $\sin(-315^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$

[9] (a) $\cos 150^\circ = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$ (b) $\cos(-60^\circ) = \cos 300^\circ = \cos 60^\circ = \frac{1}{2}$

[10] (a) $\cos\frac{5\pi}{4} = -\cos\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$ (b) $\cos(-\frac{11\pi}{6}) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

[11] (a) $\tan\frac{5\pi}{6} = -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$ (b) $\tan(-\frac{\pi}{3}) = \tan\frac{5\pi}{3} = -\tan\frac{\pi}{3} = -\sqrt{3}$

[12] (a) $\tan 330^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$ (b) $\tan(-225^\circ) = \tan 135^\circ = -\tan 45^\circ = -1$

[13] (a) $\cot 120^\circ = -\cot 60^\circ = -\frac{\sqrt{3}}{3}$ (b) $\cot(-150^\circ) = \cot 210^\circ = \cot 30^\circ = \sqrt{3}$

[14] (a) $\cot\frac{3\pi}{4} = -\cot\frac{\pi}{4} = -1$ (b) $\cot(-\frac{2\pi}{3}) = \cot\frac{4\pi}{3} = \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

[15] (a) $\sec\frac{2\pi}{3} = -\sec\frac{\pi}{3} = -2$ (b) $\sec(-\frac{\pi}{6}) = \sec\frac{11\pi}{6} = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

[16] (a) $\sec 135^\circ = -\sec 45^\circ = -\sqrt{2}$ (b) $\sec(-210^\circ) = \sec 150^\circ = -\sec 30^\circ = -\frac{2}{\sqrt{3}}$

[17] (a) $\csc 240^\circ = -\csc 60^\circ = -\frac{2}{\sqrt{3}}$ (b) $\csc(-330^\circ) = \csc 30^\circ = 2$

[18] (a) $\csc\frac{3\pi}{4} = \csc\frac{\pi}{4} = \sqrt{2}$ (b) $\csc(-\frac{2\pi}{3}) = \csc\frac{4\pi}{3} = -\csc\frac{\pi}{3} = -\frac{2}{\sqrt{3}}$

[19] (a) Using the degree mode on a calculator, $\sin 73^\circ 20' \approx 0.958$.

(b) Using the radian mode on a calculator, $\cos 0.68 \approx 0.778$.

[20] (a) $\cos 38^\circ 30' \approx 0.783$ (b) $\sin 1.48 \approx 0.996$

[21] (a) $\tan 21^\circ 10' \approx 0.387$ (b) $\cot 1.13 \approx 0.472$

[22] (a) $\cot 9^\circ 10' \approx 6.197$ (b) $\tan 0.75 \approx 0.932$

[23] (a) $\sec 67^\circ 50' \approx 2.650$ (b) $\csc 0.32 \approx 3.179$

[24] (a) $\csc 43^\circ 40' \approx 1.448$ (b) $\sec 0.26 \approx 1.035$

[25] (a) Using the degree mode, calculate $\cos^{-1}(0.8620)$ to obtain 30.46° to the nearest one-hundredth of a degree.

(b) Using the answer from part (a), subtract 30 and multiply that result by 60 to obtain $30^\circ 27'$ to the nearest minute.

[26] (a) $\sin\theta = 0.6612 \Rightarrow \theta = \sin^{-1}(0.6612) \approx 41.39^\circ$ (b) $41.39^\circ \approx 41^\circ 23'$

[27] (a) $\tan\theta = 3.7 \Rightarrow \theta = \tan^{-1}(3.7) \approx 74.88^\circ$ (b) $74.88^\circ \approx 74^\circ 53'$

[28] (a) $\cos\theta = 0.8 \Rightarrow \theta = \cos^{-1}(0.8) \approx 36.87^\circ$ (b) $36.87^\circ \approx 36^\circ 52'$

[29] (a) $\sin\theta = 0.4217 \Rightarrow \theta = \sin^{-1}(0.4217) \approx 24.94^\circ$ (b) $24.94^\circ \approx 24^\circ 57'$

[30] (a) $\tan\theta = 4.91 \Rightarrow \theta = \tan^{-1}(4.91) \approx 78.49^\circ$ (b) $78.49^\circ \approx 78^\circ 29'$

[31] (a) $\sec\theta = 4.246 \Rightarrow \cos\theta = \frac{1}{4.246} \Rightarrow \theta = \cos^{-1}(\frac{1}{4.246}) \approx 76.38^\circ$

(b) $76.38^\circ \approx 76^\circ 23'$

[32] (a) $\csc\theta = 11 \Rightarrow \sin\theta = \frac{1}{11} \Rightarrow \theta = \sin^{-1}(\frac{1}{11}) \approx 5.22^\circ$ (b) $5.22^\circ \approx 5^\circ 13'$

$\boxed{33}$ (a) $\sin 98°10' \approx 0.9899$ (b) $\cos 623.7° \approx -0.1097$ (c) $\tan 3 \approx -0.1425$

(d) $\cot 231°40' \approx 0.7907$ (e) $\sec 1175.1° \approx -11.2493$ (f) $\csc 0.82 \approx 1.3677$

$\boxed{34}$ (a) $\sin 496.4° \approx 0.6896$ (b) $\cos 0.65 \approx 0.7961$ (c) $\tan 105°40' \approx -3.5656$

(d) $\cot 1030.2° \approx -0.8451$ (e) $\sec 1.46 \approx 9.0441$ (f) $\csc 320°50' \approx -1.5833$

$\boxed{35}$ (a) Use the degree mode. $\sin\theta = -0.5640 \Rightarrow \theta = \sin^{-1}(-0.5640) \approx -34.3° \Rightarrow$

$\theta_R \approx 34.3°$. Since the sine is negative in QIII and QIV, we use θ_R in those

quadrants. $180° + 34.3° = \underline{214.3°}$ and $360° - 34.3° = \underline{325.7°}$

(b) $\cos\theta = 0.7490 \Rightarrow \theta = \cos^{-1}(0.7490) \approx 41.5°$. $\theta_R \approx 41.5°$, QI: $41.5°$, QIV: $318.5°$

(c) $\tan\theta = 2.798 \Rightarrow \theta = \tan^{-1}(2.798) \approx 70.3°$. $\theta_R \approx 70.3°$, QI: $70.3°$, QIII: $250.3°$

(d) $\cot\theta = -0.9601 \Rightarrow \tan\theta = -\frac{1}{0.9601} \Rightarrow \theta = \tan^{-1}(-\frac{1}{0.9601}) \approx -46.2°$.

$\theta_R \approx 46.2°$, QII: $133.8°$, QIV: $313.8°$

(e) $\sec\theta = -1.116 \Rightarrow \cos\theta = -\frac{1}{1.116} \Rightarrow \theta = \cos^{-1}(-\frac{1}{1.116}) \approx 153.6°$.

$\theta_R \approx 180° - 153.6° = 26.4°$, QII: $153.6°$, QIII: $206.4°$

(f) $\csc\theta = 1.485 \Rightarrow \sin\theta = \frac{1}{1.485} \Rightarrow \theta = \sin^{-1}(\frac{1}{1.485}) \approx 42.3°$.

$\theta_R \approx 42.3°$, QI: $42.3°$, QII: $137.7°$

$\boxed{36}$ (a) $\sin\theta = 0.8225 \Rightarrow \theta = \sin^{-1}(0.8225) \approx 55.3°$. $\theta_R \approx 55.3°$, QI: $55.3°$, QII: $124.7°$

(b) $\cos\theta = -0.6604 \Rightarrow \theta = \cos^{-1}(-0.6604) \approx 131.3°$.

$\theta_R \approx 180° - 131.3° = 48.7°$, QII: $131.3°$, QIII: $228.7°$

(c) $\tan\theta = -1.5214 \Rightarrow \theta = \tan^{-1}(-1.5214) \approx -56.7°$.

$\theta_R \approx 56.7°$, QII: $123.3°$, QIV: $303.3°$

(d) $\cot\theta = 1.3752 \Rightarrow \tan\theta = \frac{1}{1.3752} \Rightarrow \theta = \tan^{-1}(\frac{1}{1.3752}) \approx 36.0°$.

$\theta_R \approx 36.0°$, QI: $36.0°$, QIII: $216.0°$

(e) $\sec\theta = 1.4291 \Rightarrow \cos\theta = \frac{1}{1.4291} \Rightarrow \theta = \cos^{-1}(\frac{1}{1.4291}) \approx 45.6°$.

$\theta_R \approx 45.6°$, QI: $45.6°$, QIV: $314.4°$

(f) $\csc\theta = -2.3179 \Rightarrow \sin\theta = -\frac{1}{2.3179} \Rightarrow \theta = \sin^{-1}(-\frac{1}{2.3179}) \approx -25.6°$.

$\theta_R \approx 25.6°$, QIII: $205.6°$, QIV: $334.4°$

$\boxed{37}$ (a) Use the radian mode. $\sin\theta = 0.4195 \Rightarrow \theta = \sin^{-1}(0.4195) \approx 0.43$.

$\theta_R \approx 0.43$ is one answer. Since the sine is positive in QI and QII,

we also use the reference angle for θ in quadrant II. QII: $\pi - 0.43 \approx 2.71$

(b) $\cos\theta = -0.1207 \Rightarrow \theta = \cos^{-1}(-0.1207) \approx 1.69$ is one answer.

Since 1.69 is in QII, $\theta_R \approx \pi - 1.69 \approx 1.45$.

The cosine is also negative in QIII. QIII: $\pi + 1.45 \approx 4.59$

(c) $\tan\theta = -3.2504 \Rightarrow \theta = \tan^{-1}(-3.2504) \approx -1.27 \Rightarrow \theta_R \approx 1.27$.

QII: $\pi - 1.27 \approx 1.87$, QIV: $2\pi - 1.27 \approx 5.01$

(d) $\cot\theta = 2.6815 \Rightarrow \tan\theta = \frac{1}{2.6815} \Rightarrow \theta = \tan^{-1}(\frac{1}{2.6815}) \approx 0.36 \Rightarrow$

$\theta_{\rm R} \approx 0.36$ is one answer. QIII: $\pi + 0.36 \approx 3.50$

(e) $\sec\theta = 1.7452 \Rightarrow \cos\theta = \frac{1}{1.7452} \Rightarrow \theta = \cos^{-1}(\frac{1}{1.7452}) \approx 0.96 \Rightarrow$

$\theta_{\rm R} \approx 0.96$ is one answer. QIV: $2\pi - 0.96 \approx 5.32$

(f) $\csc\theta = -4.8521 \Rightarrow \sin\theta = -\frac{1}{4.8521} \Rightarrow \theta = \sin^{-1}(-\frac{1}{4.8521}) \approx -0.21 \Rightarrow$

$\theta_{\rm R} \approx 0.21$. QIII: $\pi + 0.21 \approx 3.35$, QIV: $2\pi - 0.21 \approx 6.07$

[38] (a) $\sin\theta = -0.0135 \Rightarrow \theta = \sin^{-1}(-0.0135) \approx -0.01 \Rightarrow \theta_{\rm R} \approx 0.01$.

QIII: $\pi + 0.01 \approx 3.15$; QIV: $2\pi - 0.01 \approx 6.27$

(b) $\cos\theta = 0.9235 \Rightarrow \theta = \cos^{-1}(0.9235) \approx 0.39 \Rightarrow \theta_{\rm R} \approx 0.39$ is one answer.

QIV: $2\pi - 0.39 \approx 5.89$

(c) $\tan\theta = 0.42 \Rightarrow \theta = \tan^{-1}(0.42) \approx 0.40 \Rightarrow \theta_{\rm R} \approx 0.40$ is one answer.

QIII: $\pi + 0.40 \approx 3.54$

(d) $\cot\theta = -2.731 \Rightarrow \tan\theta = -\frac{1}{2.731} \Rightarrow \theta = \tan^{-1}(-\frac{1}{2.731}) \approx -0.35 \Rightarrow$

$\theta_{\rm R} \approx 0.35$. QII: $\pi - 0.35 \approx 2.79$, QIV: $2\pi - 0.35 \approx 5.93$

(e) $\sec\theta = -3.51 \Rightarrow \cos\theta = -\frac{1}{3.51} \Rightarrow \theta = \cos^{-1}(-\frac{1}{3.51}) \approx 1.86$ is one answer.

Since 1.86 is in QII, $\theta_{\rm R} \approx \pi - 1.86 \approx 1.28$. QIII: $\pi + 1.28 \approx 4.42$

(f) $\csc\theta = 1.258 \Rightarrow \sin\theta = \frac{1}{1.258} \Rightarrow \theta = \sin^{-1}(\frac{1}{1.258}) \approx 0.92 \Rightarrow$

$\theta_{\rm R} \approx 0.92$ is one answer. QII: $\pi - 0.92 \approx 2.22$

[39] $\ln I_0 - \ln I = kx\sec\theta \Rightarrow$

$$\ln\frac{I_0}{I} = kx\sec\theta \Rightarrow x = \frac{1}{k\sec\theta}\ln\frac{I_0}{I} = \frac{1}{1.88\sec 12^\circ}\ln 1.72 \approx 0.28\text{ cm.}$$

[40] $\ln\frac{I_0}{I} = kx\sec\theta \Rightarrow \sec\theta = \frac{1}{kx}\ln\frac{I_0}{I} \Rightarrow$

$$\cos\theta = \frac{kx}{\ln(I_0/I)} = \frac{1.88(0.31)}{\ln 2.05} \approx 0.8119 \Rightarrow \theta \approx 35.7^\circ.$$

[41] (a) The solar radiation R will equal R_0 when $\cos\theta = \sin\phi = 1$. This occurs when $\theta = 0^\circ$ and $\phi = 90^\circ$, and corresponds to when the sun is just rising in the east.

(b) The sun located in the southeast corresponds to $\phi = 45^\circ$.

$$R/R_0 = \cos\theta\sin\phi = \cos 60^\circ \sin 45^\circ = \tfrac{1}{2}\cdot\tfrac{\sqrt{2}}{2} = \tfrac{\sqrt{2}}{4} \approx 35\%.$$

[42] (a) $d = 2\pi\left(\dfrac{vR}{0.52\cos\phi}\right)^{1/3} = 2\pi\left(\dfrac{45\cdot 6369}{0.52\cos 48^\circ}\right)^{1/3} \approx 589$ km.

(b) Since $\cos\phi$ decreases as ϕ increases from 0° to 90°, and $\cos\phi$ is in the denominator of the expression, d increases as ϕ increases.

[43] $\sin\theta = \frac{b}{c} \Rightarrow \sin 60^\circ = \frac{b}{18} \Rightarrow b = 18\sin 60^\circ = 18\cdot\frac{\sqrt{3}}{2} = 9\sqrt{3} \approx 15.6$. $\cos\theta = \frac{a}{c} \Rightarrow$

$\cos 60^\circ = \frac{a}{18} \Rightarrow a = 18\cos 60^\circ = 18\cdot\frac{1}{2} = 9$. The hand is located at $(9, 9\sqrt{3})$.

[44] The arm's initial length is $\sqrt{12^2+12^2}=\sqrt{288}$. $\tan\theta_1=\frac{b}{a}=\frac{12}{12}=1 \Rightarrow \theta_1=45^\circ$ is the arm's initial angle. The new arm length is $\sqrt{(-16)^2+10^2}=\sqrt{356}$. The new angle has $\tan\theta_2=\frac{b}{a}=\frac{10}{-16}=-\frac{5}{8}$. Since θ_2 is in quadrant II, $\theta_2\approx 148^\circ$. Thus, the arm must increase its length by $\sqrt{356}-\sqrt{288}\approx 1.9$ inches and rotate $\theta_2-\theta_1\approx 103^\circ$ counterclockwise.

6.5 Exercises

Note: Exer. 1–4: We will refer to $y=\sin x$ as just $\sin x$ ($y=\cos x$ as $\cos x$, etc.).
For the form $y=a\sin bx$, the amplitude is $|a|$ and the period is $\frac{2\pi}{|b|}$.
These are merely listed in the answer along with the values of the x-intercepts.

[1] (a) $y=4\sin x$ • vertically stretch $\sin x$ by a factor of 4 ★ 4, 2π, x-int. @ πn

(b) $y=\sin 4x$ • horizontally compress $\sin x$ by a factor of 4 ★ 1, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{4}n$

(c) $y=\frac{1}{4}\sin x$ • vertically compress $\sin x$ by a factor of 4 ★ $\frac{1}{4}$, 2π, x-int. @ πn

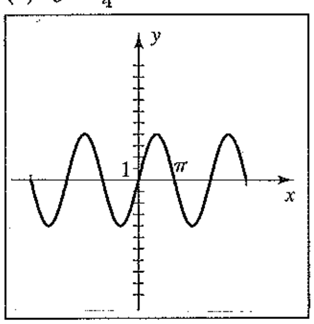

Figure 1(a)

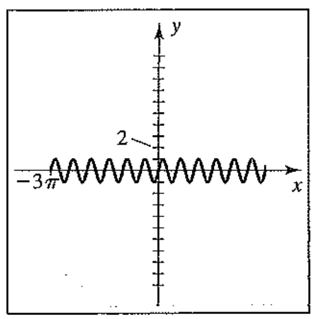

Figure 1(b)

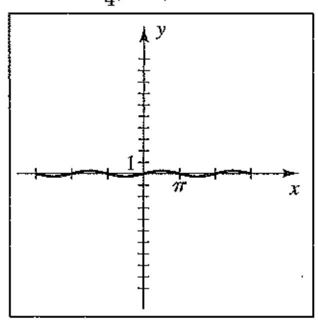

Figure 1(c)

(d) $y=\sin\frac{1}{4}x$ • horizontally stretch $\sin x$ by a factor of 4 ★ 1, 8π, x-int. @ $4\pi n$

(e) $y=2\sin\frac{1}{4}x$ • vertically stretch the graph in part (d) by a factor of 2
★ 2, 8π, x-int. @ $4\pi n$

(f) $y=\frac{1}{2}\sin 4x$ • vertically compress the graph in part (b) by a factor of 2
★ $\frac{1}{2}$, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{4}n$

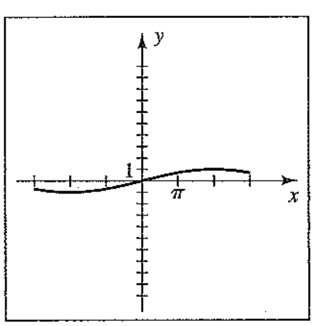

Figure 1(d)

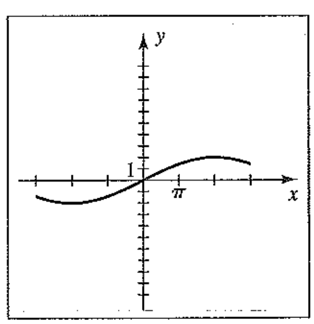

Figure 1(e)

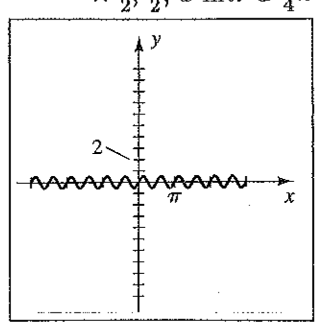

Figure 1(f)

(g) $y = -4\sin x$ • reflect the graph in part (a) through the x-axis
★ 4, 2π, x-int. @ πn

(h) $y = \sin(-4x) = -\sin 4x$ • reflect the graph in part (b) through the x-axis
★ 1, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{4}n$

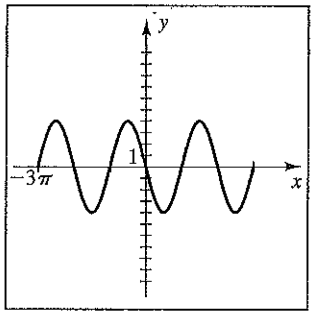

Figure 1(g)

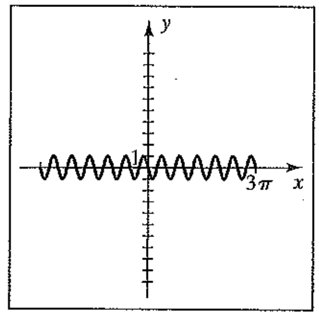

Figure 1(h)

2 (a) $y = 4\cos x$ • vertically stretch $\cos x$ by a factor of 4 ★ 4, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

(b) $y = \cos 4x$ • horizontally compress $\cos x$ by a factor of 4
★ 1, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{8} + \frac{\pi}{4}n$

(c) $y = \frac{1}{4}\cos x$ • vertically compress $\cos x$ by a factor of 4 ★ $\frac{1}{4}$, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

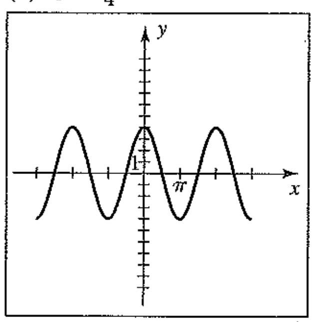

Figure 2(a)

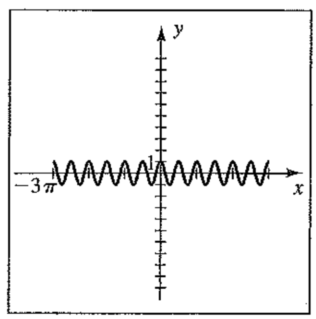

Figure 2(b)

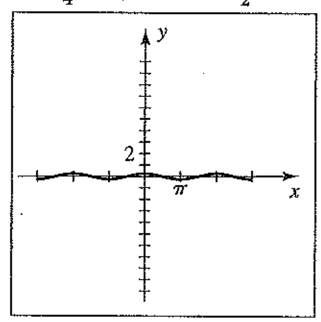

Figure 2(c)

(d) $y = \cos\frac{1}{4}x$ • horizontally stretch $\cos x$ by a factor of 4
★ 1, 8π, x-int. @ $2\pi + 4\pi n$

(e) $y = 2\cos\frac{1}{4}x$ • vertically stretch the graph in part (d) by a factor of 2
★ 2, 8π, x-int. @ $2\pi + 4\pi n$

(f) $y = \frac{1}{2}\cos 4x$ • vertically compress the graph in part (b) by a factor of 2
★ $\frac{1}{2}$, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{8} + \frac{\pi}{4}n$

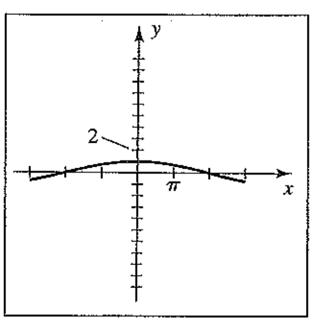

Figure 2(d)

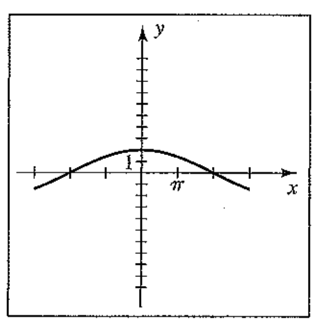

Figure 2(e)

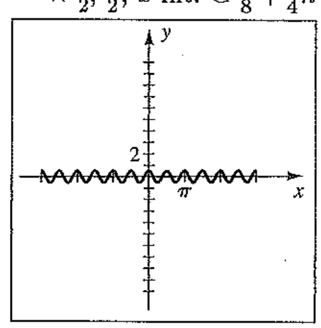

Figure 2(f)

(g) $y = -4\cos x$ • reflect the graph in part (a) through the x-axis

★ 4, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

(h) $y = \cos(-4x) = \cos 4x$ • same as the graph in part (b)

★ 1, $\frac{\pi}{2}$, x-int. @ $\frac{\pi}{8} + \frac{\pi}{4}n$

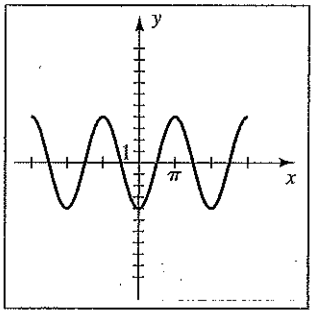

Figure 2(g)

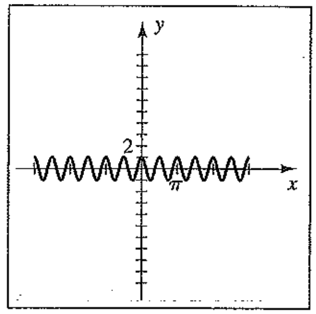

Figure 2(h)

$\boxed{3}$ (a) $y = 3\cos x$ • vertically stretch $\cos x$ by a factor of 3 ★ 3, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

(b) $y = \cos 3x$ • horizontally compress $\cos x$ by a factor of 3

★ 1, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{6} + \frac{\pi}{3}n$

(c) $y = \frac{1}{3}\cos x$ • vertically compress $\cos x$ by a factor of 3 ★ $\frac{1}{3}$, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

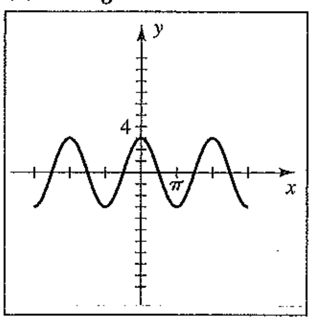

Figure 3(a)

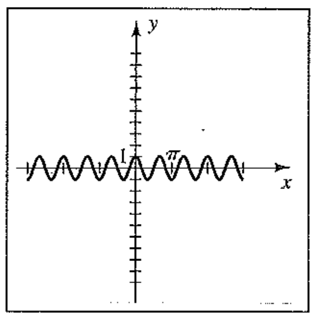

Figure 3(b)

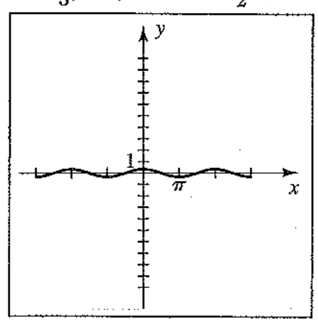

Figure 3(c)

(d) $y = \cos\frac{1}{3}x$ • horizontally stretch $\cos x$ by a factor of 3

★ 1, 6π, x-int. @ $\frac{3\pi}{2} + 3\pi n$

(e) $y = 2\cos\frac{1}{3}x$ • vertically stretch the graph in part (d) by a factor of 2

★ 2, 6π, x-int. @ $\frac{3\pi}{2} + 3\pi n$

(f) $y = \frac{1}{2}\cos 3x$ • vertically compress the graph in part (b) by a factor of 2

★ $\frac{1}{2}$, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{6} + \frac{\pi}{3}n$

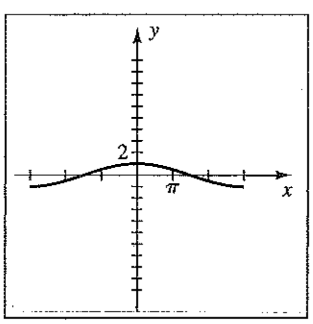

Figure 3(d)

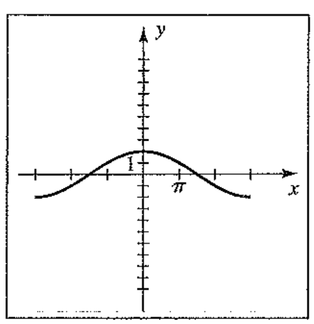

Figure 3(e)

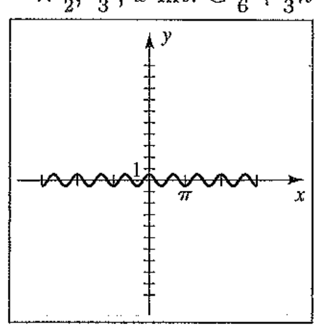

Figure 3(f)

(g) $y = -3\cos x$ • reflect the graph in part (a) through the x-axis
★ 3, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

(h) $y = \cos(-3x) = \cos 3x$ • same as the graph in part (b)
★ 1, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{6} + \frac{\pi}{3}n$

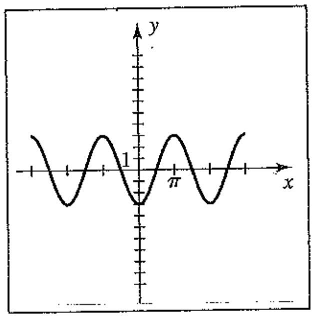

Figure 3(g)

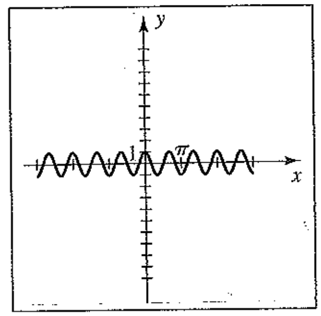

Figure 3(h)

4 (a) $y = 3\sin x$ • vertically stretch $\sin x$ by a factor of 3 ★ 3, 2π, x-int. @ πn

(b) $y = \sin 3x$ • horizontally compress $\sin x$ by a factor of 3 ★ 1, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{3}n$

(c) $y = \frac{1}{3}\sin x$ • vertically compress $\sin x$ by a factor of 3 ★ $\frac{1}{3}$, 2π, x-int. @ πn

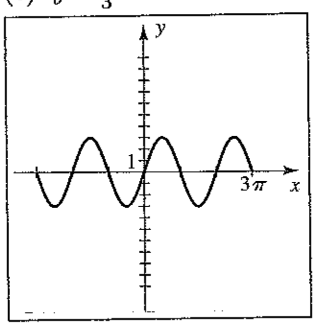

Figure 4(a)

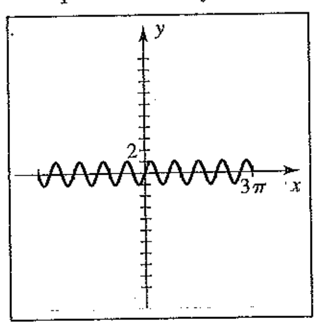

Figure 4(b)

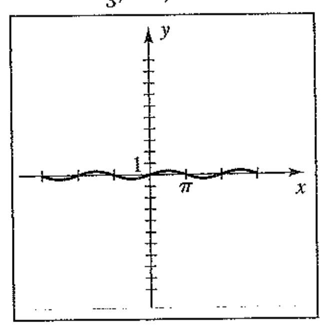

Figure 4(c)

(d) $y = \sin\frac{1}{3}x$ • horizontally stretch $\sin x$ by a factor of 3 ★ 1, 6π, x-int. @ $3\pi n$

(e) $y = 2\sin\frac{1}{3}x$ • vertically stretch the graph in part (d) by a factor of 2
★ 2, 6π, x-int. @ $3\pi n$

(f) $y = \frac{1}{2}\sin 3x$ • vertically compress the graph in part (b) by a factor of 2
★ $\frac{1}{2}$, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{3}n$

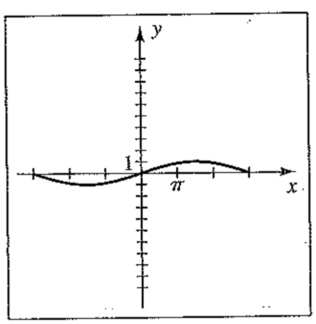

Figure 4(d)

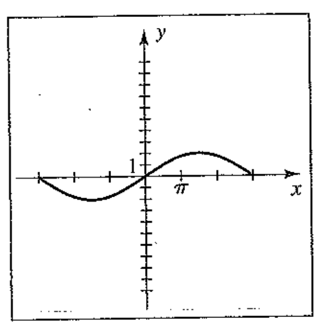

Figure 4(e)

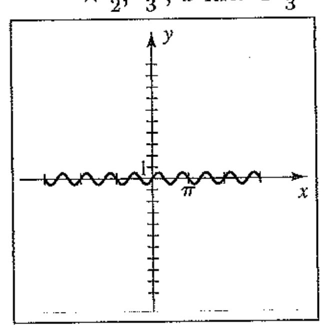

Figure 4(f)

(g) $y = -3\sin x$ • reflect the graph in part (a) through the x-axis

★ 3, 2π, x-int. @ πn

(h) $y = \sin(-3x) = -\sin 3x$ • reflect the graph in part (b) through the x-axis

★ 1, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{3}n$

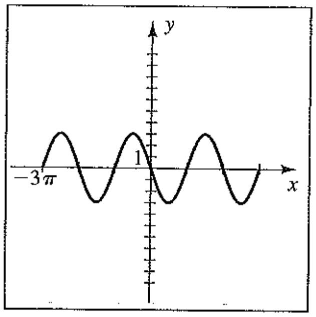

Figure 4(g)

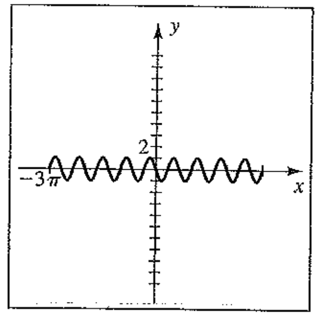

Figure 4(h)

Note: We will write $y = a\sin(bx + c)$ in the form $y = a\sin\left[b\left(x + \frac{c}{b}\right)\right]$. From this form we have the amplitude, $|a|$, the period, $\frac{2\pi}{|b|}$, and the phase shift, $-\frac{c}{b}$. We will also list the interval that corresponds to $[0, 2\pi]$ for the sine functions and $[-\frac{\pi}{2}, \frac{3\pi}{2}]$ for the cosine functions—other intervals could certainly be used. See Exercises 17 and 29 for representative examples.

$\boxed{5}$ $y = \sin(x - \frac{\pi}{2})$ • $0 \le x - \frac{\pi}{2} \le 2\pi \Rightarrow \frac{\pi}{2} \le x \le \frac{5\pi}{2}$ ★ 1, 2π, $\frac{\pi}{2}$, $[\frac{\pi}{2}, \frac{5\pi}{2}]$

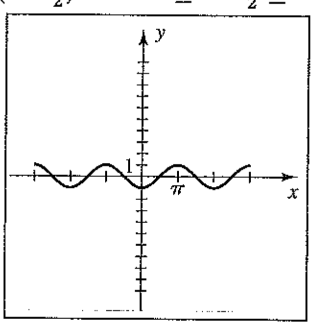

Figure 5

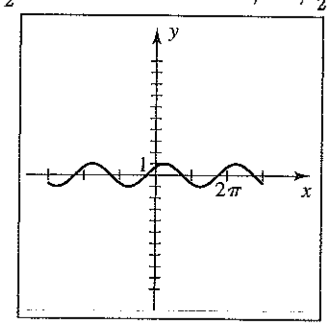

Figure 6

$\boxed{6}$ $y = \sin(x + \frac{\pi}{4})$ • $0 \le x + \frac{\pi}{4} \le 2\pi \Rightarrow -\frac{\pi}{4} \le x \le \frac{7\pi}{4}$ ★ 1, 2π, $-\frac{\pi}{4}$, $[-\frac{\pi}{4}, \frac{7\pi}{4}]$

$\boxed{7}$ $y = 3\sin(x + \frac{\pi}{6})$ • $0 \le x + \frac{\pi}{6} \le 2\pi \Rightarrow -\frac{\pi}{6} \le x \le \frac{11\pi}{6}$ ★ 3, 2π, $-\frac{\pi}{6}$, $[-\frac{\pi}{6}, \frac{11\pi}{6}]$

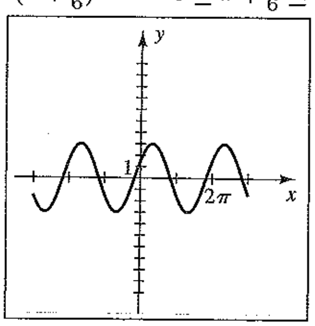

Figure 7

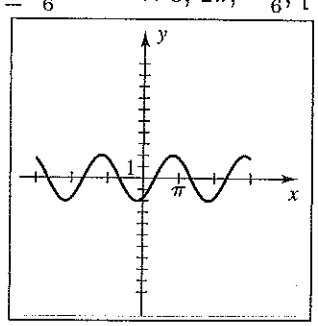

Figure 8

$\boxed{8}$ $y = 2\sin(x - \frac{\pi}{3})$ • $0 \le x - \frac{\pi}{3} \le 2\pi \Rightarrow \frac{\pi}{3} \le x \le \frac{7\pi}{3}$ ★ 2, 2π, $\frac{\pi}{3}$, $[\frac{\pi}{3}, \frac{7\pi}{3}]$

$\boxed{9}$ $y=\cos\left(x+\frac{\pi}{2}\right)$ • $-\frac{\pi}{2}\le x+\frac{\pi}{2}\le\frac{3\pi}{2} \;\Rightarrow\; -\pi\le x\le\pi$ ★ $1,\ 2\pi,\ -\frac{\pi}{2},\ [-\pi,\ \pi]$

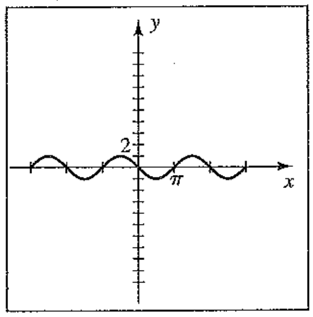

Figure 9

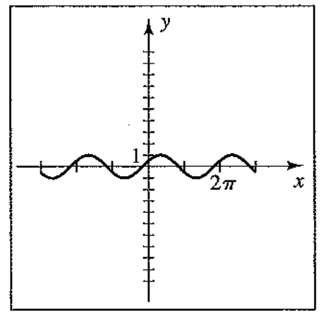

Figure 10

$\boxed{10}$ $y=\cos\left(x-\frac{\pi}{3}\right)$ • $-\frac{\pi}{2}\le x-\frac{\pi}{3}\le\frac{3\pi}{2} \;\Rightarrow\; -\frac{\pi}{6}\le x\le\frac{11\pi}{6}$ ★ $1,\ 2\pi,\ \frac{\pi}{3},\ \left[-\frac{\pi}{6},\ \frac{11\pi}{6}\right]$

$\boxed{11}$ $y=4\cos\left(x-\frac{\pi}{4}\right)$ • $-\frac{\pi}{2}\le x-\frac{\pi}{4}\le\frac{3\pi}{2} \;\Rightarrow\; -\frac{\pi}{4}\le x\le\frac{7\pi}{4}$ ★ $4,\ 2\pi,\ \frac{\pi}{4},\ \left[-\frac{\pi}{4},\ \frac{7\pi}{4}\right]$

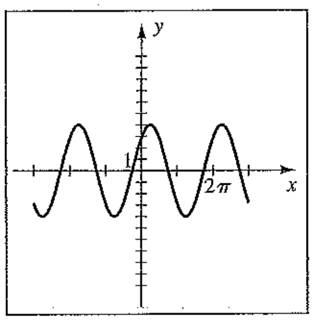

Figure 11

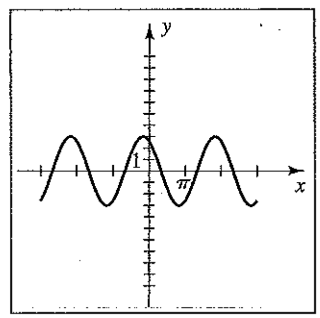

Figure 12

$\boxed{12}$ $y=3\cos\left(x+\frac{\pi}{6}\right)$ • $-\frac{\pi}{2}\le x+\frac{\pi}{6}\le\frac{3\pi}{2} \;\Rightarrow\; -\frac{2\pi}{3}\le x\le\frac{4\pi}{3}$ ★ $3,\ 2\pi,\ -\frac{\pi}{6},\ \left[-\frac{2\pi}{3},\ \frac{4\pi}{3}\right]$

$\boxed{13}$ $y=\sin(2x-\pi)+1=\sin\left[2\left(x-\frac{\pi}{2}\right)\right]+1.$

$0\le 2x-\pi\le 2\pi \;\Rightarrow\; \pi\le 2x\le 3\pi \;\Rightarrow\; \frac{\pi}{2}\le x\le\frac{3\pi}{2}$ ★ $1,\ \pi,\ \frac{\pi}{2},\ \left[\frac{\pi}{2},\ \frac{3\pi}{2}\right]$

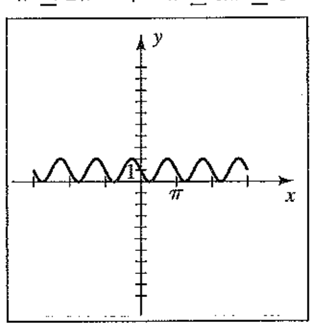

Figure 13

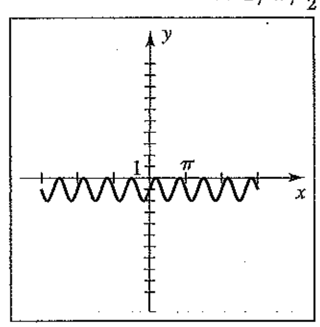

Figure 14

$\boxed{14}$ $y=-\sin(3x+\pi)-1=-\sin\left[3\left(x+\frac{\pi}{3}\right)\right]-1.$

$0\le 3x+\pi\le 2\pi \;\Rightarrow\; -\pi\le 3x\le\pi \;\Rightarrow\; -\frac{\pi}{3}\le x\le\frac{\pi}{3}$ ★ $1,\ \frac{2\pi}{3},\ -\frac{\pi}{3},\ \left[-\frac{\pi}{3},\ \frac{\pi}{3}\right]$

[15] $y = -\cos(3x+\pi) - 2 = -\cos[3(x+\frac{\pi}{3})] - 2.$

$-\frac{\pi}{2} \le 3x+\pi \le \frac{3\pi}{2} \Rightarrow -\frac{3\pi}{2} \le 3x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{6}$ ★ $1, \frac{2\pi}{3}, -\frac{\pi}{3}, [-\frac{\pi}{2}, \frac{\pi}{6}]$

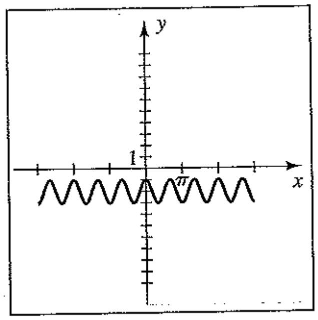

Figure 15

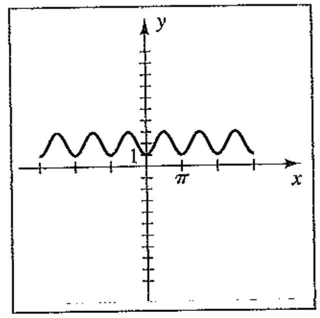

Figure 16

[16] $y = \cos(2x-\pi) + 2 = \cos[2(x-\frac{\pi}{2})] + 2.$

$-\frac{\pi}{2} \le 2x-\pi \le \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} \le 2x \le \frac{5\pi}{2} \Rightarrow \frac{\pi}{4} \le x \le \frac{5\pi}{4}$ ★ $1, \pi, \frac{\pi}{2}, [\frac{\pi}{4}, \frac{5\pi}{4}]$

[17] $y = -2\sin(3x-\pi) = -2\sin[3(x-\frac{\pi}{3})]$. Amplitude $= |-2| = 2$. The first negative sign has the effect of reflecting the graph of $y = 2\sin(3x-\pi)$ through the x-axis. Period $= \frac{2\pi}{|3|} = \frac{2\pi}{3}$, phase shift $= -(-\frac{\pi}{3}) = \frac{\pi}{3}$,

$0 \le 3x-\pi \le 2\pi \Rightarrow \pi \le 3x \le 3\pi \Rightarrow \frac{\pi}{3} \le x \le \pi.$ ★ $2, \frac{2\pi}{3}, \frac{\pi}{3}, [\frac{\pi}{3}, \pi]$

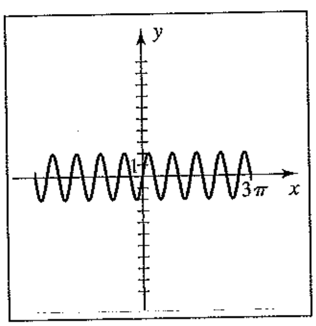

Figure 17

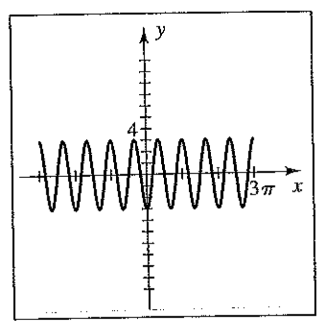

Figure 18

[18] $y = 3\cos(3x-\pi) = 3\cos[3(x-\frac{\pi}{3})]$.

$-\frac{\pi}{2} \le 3x-\pi \le \frac{3\pi}{2} \Rightarrow \frac{\pi}{2} \le 3x \le \frac{5\pi}{2} \Rightarrow \frac{\pi}{6} \le x \le \frac{5\pi}{6}$ ★ $3, \frac{2\pi}{3}, \frac{\pi}{3}, [\frac{\pi}{6}, \frac{5\pi}{6}]$

$\boxed{19}$ $y = \sin\left(\frac{1}{2}x - \frac{\pi}{3}\right) = \sin\left[\frac{1}{2}\left(x - \frac{2\pi}{3}\right)\right]$. $0 \le \frac{1}{2}x - \frac{\pi}{3} \le 2\pi \;\Rightarrow\; \frac{\pi}{3} \le \frac{1}{2}x \le \frac{7\pi}{3} \;\Rightarrow\; \frac{2\pi}{3} \le x \le \frac{14\pi}{3}$

★ $1,\ 4\pi,\ \frac{2\pi}{3},\ \left[\frac{2\pi}{3},\ \frac{14\pi}{3}\right]$

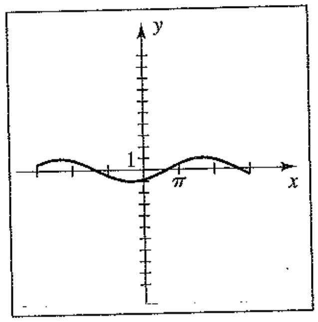

Figure 19

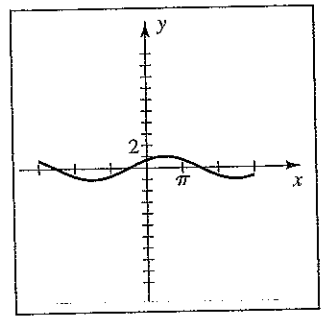

Figure 20

$\boxed{20}$ $y = \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right) = \sin\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{4} \le 2\pi \;\Rightarrow\; -\frac{\pi}{4} \le \frac{1}{2}x \le \frac{7\pi}{4} \;\Rightarrow\; -\frac{\pi}{2} \le x \le \frac{7\pi}{2}$ ★ $1,\ 4\pi,\ -\frac{\pi}{2},\ \left[-\frac{\pi}{2},\ \frac{7\pi}{2}\right]$

$\boxed{21}$ $y = 6\sin \pi x$ • $0 \le \pi x \le 2\pi \;\Rightarrow\; 0 \le x \le 2$ ★ $6,\ 2,\ 0,\ [0,\ 2]$

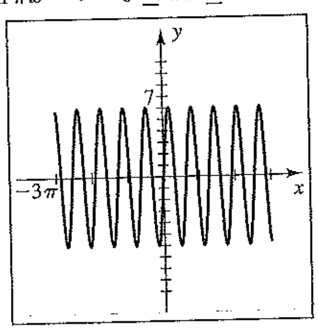

Figure 21

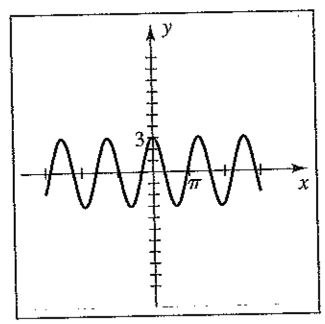

Figure 22

$\boxed{22}$ $y = 3\cos\frac{\pi}{2}x$ • $-\frac{\pi}{2} \le \frac{\pi}{2}x \le \frac{3\pi}{2} \;\Rightarrow\; -1 \le x \le 3$ ★ $3,\ 4,\ 0,\ [-1,\ 3]$

$\boxed{23}$ $y = 2\cos\frac{\pi}{2}x$ • $-\frac{\pi}{2} \le \frac{\pi}{2}x \le \frac{3\pi}{2} \;\Rightarrow\; -1 \le x \le 3$ ★ $2,\ 4,\ 0,\ [-1,\ 3]$

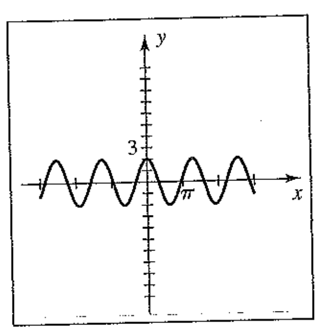

Figure 23

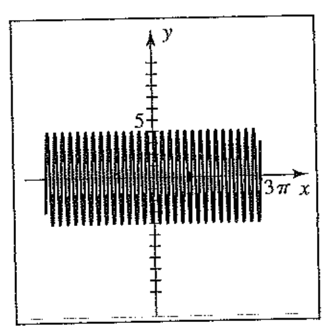

Figure 24

$\boxed{24}$ $y = 4\sin 3\pi x$ • $0 \le 3\pi x \le 2\pi \;\Rightarrow\; 0 \le x \le \frac{2}{3}$ ★ $4,\ \frac{2}{3},\ 0,\ \left[0,\ \frac{2}{3}\right]$

$\boxed{25}$ $y = \frac{1}{2}\sin 2\pi x$ • $0 \le 2\pi x \le 2\pi \Rightarrow 0 \le x \le 1$ ★ $\frac{1}{2}$, 1, 0, [0, 1]

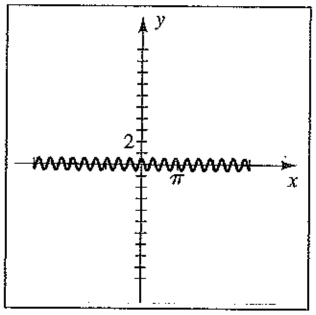

Figure 25

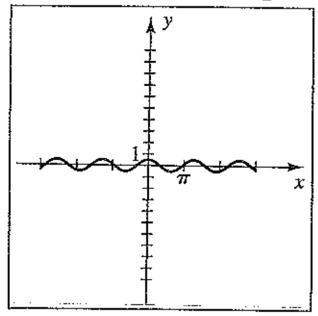

Figure 26

$\boxed{26}$ $y = \frac{1}{2}\cos\frac{\pi}{2}x$ • $-\frac{\pi}{2} \le \frac{\pi}{2}x \le \frac{3\pi}{2} \Rightarrow -1 \le x \le 3$ ★ $\frac{1}{2}$, 4, 0, [−1, 3]

$\boxed{27}$ $y = 5\sin\left(3x - \frac{\pi}{2}\right) = 5\sin\left[3\left(x - \frac{\pi}{6}\right)\right]$.

$0 \le 3x - \frac{\pi}{2} \le 2\pi \Rightarrow \frac{\pi}{2} \le 3x \le \frac{5\pi}{2} \Rightarrow \frac{\pi}{6} \le x \le \frac{5\pi}{6}$ ★ 5, $\frac{2\pi}{3}$, $\frac{\pi}{6}$, $[\frac{\pi}{6}, \frac{5\pi}{6}]$

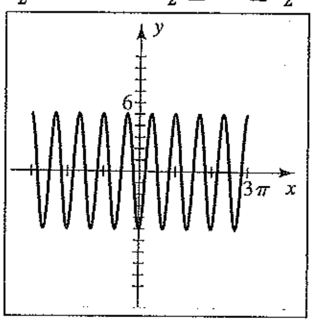

Figure 27

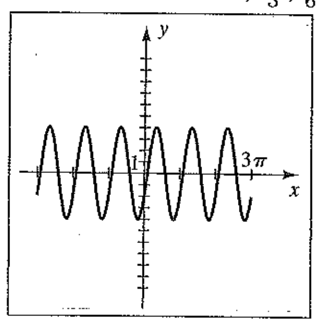

Figure 28

$\boxed{28}$ $y = -4\cos\left(2x + \frac{\pi}{3}\right) = -4\cos\left[2\left(x + \frac{\pi}{6}\right)\right]$.

$-\frac{\pi}{2} \le 2x + \frac{\pi}{3} \le \frac{3\pi}{2} \Rightarrow -\frac{5\pi}{6} \le 2x \le \frac{7\pi}{6} \Rightarrow -\frac{5\pi}{12} \le x \le \frac{7\pi}{12}$ ★ 4, π, $-\frac{\pi}{6}$, $[-\frac{5\pi}{12}, \frac{7\pi}{12}]$

$\boxed{29}$ $y = 3\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) = 3\cos\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$. Amplitude $= |\,3\,| = 3$, period $= \dfrac{2\pi}{|\,1/2\,|} = 4\pi$,

phase shift $= -\left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$, $-\frac{\pi}{2} \le \frac{1}{2}x - \frac{\pi}{4} \le \frac{3\pi}{2} \Rightarrow -\frac{\pi}{4} \le \frac{1}{2}x \le \frac{7\pi}{4} \Rightarrow -\frac{\pi}{2} \le x \le \frac{7\pi}{2}$.

★ 3, 4π, $\frac{\pi}{2}$, $[-\frac{\pi}{2}, \frac{7\pi}{2}]$

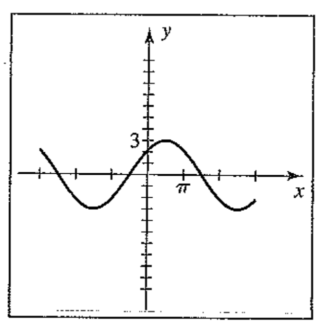

Figure 29

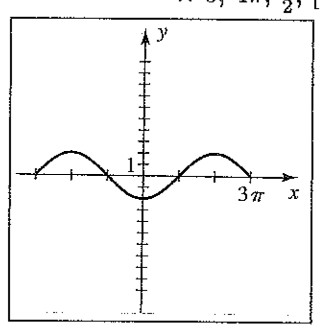

Figure 30

$\boxed{30}$ $y = -2\sin\left(\frac{1}{2}x + \frac{\pi}{2}\right) = -2\sin\left[\frac{1}{2}(x + \pi)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{2} \le 2\pi \Rightarrow -\frac{\pi}{2} \le \frac{1}{2}x \le \frac{3\pi}{2} \Rightarrow -\pi \le x \le 3\pi$ ★ 2, 4π, $-\pi$, $[-\pi, 3\pi]$

[31] $y = -5\cos\left(\frac{1}{3}x + \frac{\pi}{6}\right) = -5\cos\left[\frac{1}{3}\left(x + \frac{\pi}{2}\right)\right]$.

$-\frac{\pi}{2} \le \frac{1}{3}x + \frac{\pi}{6} \le \frac{3\pi}{2} \;\Rightarrow\; -\frac{2\pi}{3} \le \frac{1}{3}x \le \frac{4\pi}{3} \;\Rightarrow\; -2\pi \le x \le 4\pi$ ★ $5,\ 6\pi,\ -\frac{\pi}{2},\ [-2\pi,\ 4\pi]$

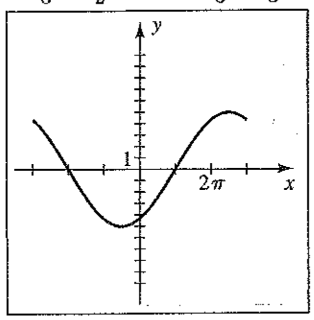

Figure 31

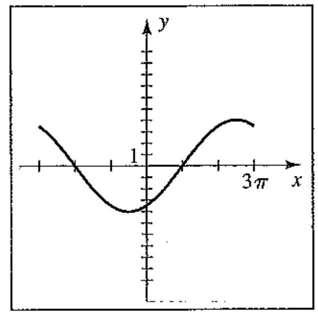

Figure 32

[32] $y = 4\sin\left(\frac{1}{3}x - \frac{\pi}{3}\right) = 4\sin\left[\frac{1}{3}(x - \pi)\right]$.

$0 \le \frac{1}{3}x - \frac{\pi}{3} \le 2\pi \;\Rightarrow\; \frac{\pi}{3} \le \frac{1}{3}x \le \frac{7\pi}{3} \;\Rightarrow\; \pi \le x \le 7\pi$ ★ $4,\ 6\pi,\ \pi,\ [\pi,\ 7\pi]$

[33] $y = 3\cos(\pi x + 4\pi) = 3\cos[\pi(x + 4)]$.

$-\frac{\pi}{2} \le \pi x + 4\pi \le \frac{3\pi}{2} \;\Rightarrow\; -\frac{9\pi}{2} \le \pi x \le -\frac{5\pi}{2} \;\Rightarrow\; -\frac{9}{2} \le x \le -\frac{5}{2}$ ★ $3,\ 2,\ -4,\ [-\frac{9}{2},\ -\frac{5}{2}]$

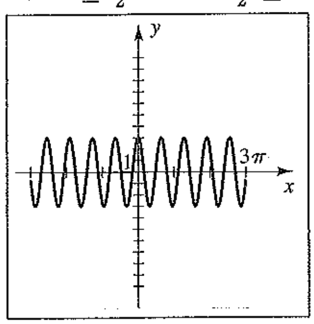

Figure 33

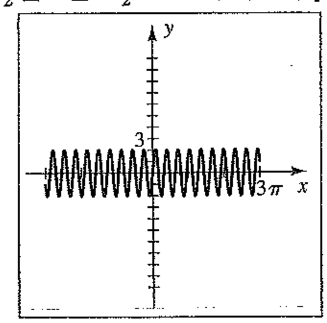

Figure 34

[34] $y = -2\sin(2\pi x + \pi) = -2\sin\left[2\pi\left(x + \frac{1}{2}\right)\right]$.

$0 \le 2\pi x + \pi \le 2\pi \;\Rightarrow\; -\pi \le 2\pi x \le \pi \;\Rightarrow\; -\frac{1}{2} \le x \le \frac{1}{2}$ ★ $2,\ 1,\ -\frac{1}{2},\ [-\frac{1}{2},\ \frac{1}{2}]$

[35] $y = -\sqrt{2}\sin\left(\frac{\pi}{2}x - \frac{\pi}{4}\right) = -\sqrt{2}\sin\left[\frac{\pi}{2}\left(x - \frac{1}{2}\right)\right]$.

$0 \le \frac{\pi}{2}x - \frac{\pi}{4} \le 2\pi \;\Rightarrow\; \frac{\pi}{4} \le \frac{\pi}{2}x \le \frac{9\pi}{4} \;\Rightarrow\; \frac{1}{2} \le x \le \frac{9}{2}$ ★ $\sqrt{2},\ 4,\ \frac{1}{2},\ [\frac{1}{2},\ \frac{9}{2}]$

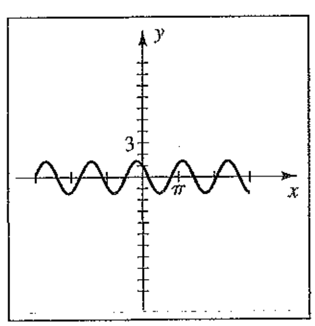

Figure 35

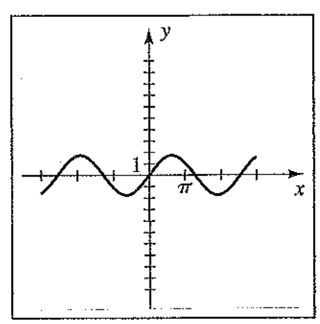

Figure 36

[36] $y = \sqrt{3}\cos\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) = \sqrt{3}\cos\left[\frac{\pi}{4}(x - 2)\right]$.

(continued)

$-\frac{\pi}{2} \le \frac{\pi}{4}x - \frac{\pi}{2} \le \frac{3\pi}{2} \ \Rightarrow \ 0 \le \frac{\pi}{4}x \le 2\pi \ \Rightarrow \ 0 \le x \le 8$ ★ $\sqrt{3}$, 8, 2, [0, 8]

[37] $y = -2\sin(2x - \pi) + 3 = -2\sin\left[2(x - \frac{\pi}{2})\right] + 3.$

$0 \le 2x - \pi \le 2\pi \ \Rightarrow \ \pi \le 2x \le 3\pi \ \Rightarrow \ \frac{\pi}{2} \le x \le \frac{3\pi}{2}$ ★ 2, π, $\frac{\pi}{2}$, $[\frac{\pi}{2}, \frac{3\pi}{2}]$

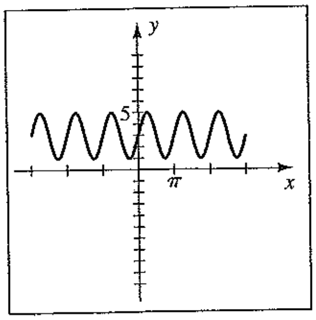

Figure 37

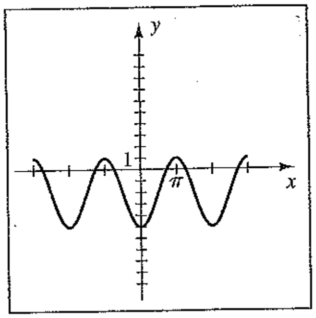

Figure 38

[38] $y = 3\cos(x + 3\pi) - 2$ • $-\frac{\pi}{2} \le x + 3\pi \le \frac{3\pi}{2} \ \Rightarrow \ -\frac{7\pi}{2} \le x \le -\frac{3\pi}{2}$

★ 3, 2π, -3π, $[-\frac{7\pi}{2}, -\frac{3\pi}{2}]$

[39] $y = 5\cos(2x + 2\pi) + 2 = 5\cos\left[2(x + \pi)\right] + 2.$

$-\frac{\pi}{2} \le 2x + 2\pi \le \frac{3\pi}{2} \ \Rightarrow \ -\frac{5\pi}{2} \le 2x \le -\frac{\pi}{2} \ \Rightarrow \ -\frac{5\pi}{4} \le x \le -\frac{\pi}{4}$

★ 5, π, $-\pi$, $[-\frac{5\pi}{4}, -\frac{\pi}{4}]$

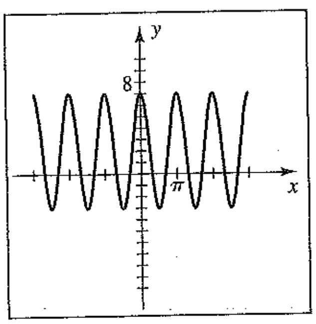

Figure 39

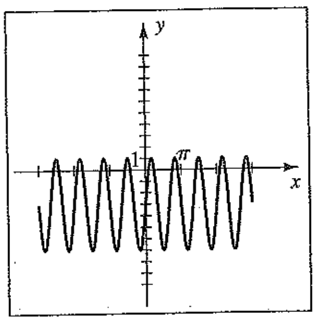

Figure 40

[40] $y = -4\sin(3x - \pi) - 3 = -4\sin\left[3(x - \frac{\pi}{3})\right] - 3.$

$0 \le 3x - \pi \le 2\pi \ \Rightarrow \ \pi \le 3x \le 3\pi \ \Rightarrow \ \frac{\pi}{3} \le x \le \pi$ ★ 4, $\frac{2\pi}{3}$, $\frac{\pi}{3}$, $[\frac{\pi}{3}, \pi]$

[41] (a) The amplitude a is 4 and the period {from $-\pi$ to π} is 2π.

The phase shift is the first negative zero that occurs before a maximum, $-\pi$.

(b) Period $= \frac{2\pi}{b} \ \Rightarrow \ 2\pi = \frac{2\pi}{b} \ \Rightarrow \ b = 1$. Phase shift $= -\frac{c}{b} \ \Rightarrow \ -\pi = -\frac{c}{1} \ \Rightarrow$

$c = \pi$. Hence, $y = a\sin(bx + c) = 4\sin(x + \pi)$.

[42] (a) The amplitude a is 3 and the period {from $-\frac{\pi}{4}$ to $\frac{3\pi}{4}$} is π.

The phase shift is the first negative zero that occurs before a maximum, $-\frac{\pi}{4}$.

(b) Period $= \frac{2\pi}{b} \ \Rightarrow \ \pi = \frac{2\pi}{b} \ \Rightarrow \ b = 2$. Phase shift $= -\frac{c}{b} \ \Rightarrow \ -\frac{\pi}{4} = -\frac{c}{2} \ \Rightarrow$

$c = \frac{\pi}{2}$. Hence, $y = a\sin(bx + c) = 3\sin(2x + \frac{\pi}{2})$.

$\boxed{43}$ (a) The amplitude a is 2 and the period { from -3 to 1 } is 4.

The phase shift is the first negative zero that occurs before a maximum, -3.

(b) Period $= \frac{2\pi}{b} \Rightarrow 4 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{2}$. Phase shift $= -\frac{c}{b} \Rightarrow -3 = -\frac{c}{\pi/2} \Rightarrow$

$c = \frac{3\pi}{2}$. Hence, $y = a \sin(bx + c) = 2\sin\left(\frac{\pi}{2}x + \frac{3\pi}{2}\right)$.

$\boxed{44}$ (a) The amplitude a is 3 and the period { from $-\frac{1}{4}$ to $\frac{3}{4}$ (or 0 to 1) } is 1.

The phase shift is the first negative zero that occurs before a maximum, $-\frac{1}{4}$.

(b) Period $= \frac{2\pi}{b} \Rightarrow 1 = \frac{2\pi}{b} \Rightarrow b = 2\pi$. Phase shift $= -\frac{c}{b} \Rightarrow -\frac{1}{4} = -\frac{c}{2\pi} \Rightarrow$

$c = \frac{\pi}{2}$. Hence, $y = a \sin(bx + c) = 3\sin\left(2\pi x + \frac{\pi}{2}\right)$.

$\boxed{45}$ In the first second, there are 2 complete cycles. Thus, the period is $\frac{1}{2}$.

$$\frac{2\pi}{b} = \frac{1}{2} \Rightarrow b = 4\pi.$$

$\boxed{46}$ $\frac{2\pi}{b} = 24$ hours $\Rightarrow b = \frac{\pi}{12}$. The light intensity would be 0 at $t = 0$ and $t = 12$.

The light intensity is 510 at $t = 6$. $I = 510\sin\left(\frac{\pi}{12}t\right)$

$\boxed{47}$ $\frac{1}{4}$ sec $= \frac{1}{2}$ period $\Rightarrow$ period $= \frac{1}{2}$ sec.

$\frac{2\pi}{b} = \frac{1}{2} \Rightarrow b = 4\pi$ and the amplitude is 8. $a = 8$ and $b = 4\pi \Rightarrow y = 8\sin 4\pi t$.

$\boxed{48}$ (a) Period $= 23 \Rightarrow \frac{2\pi}{b} = 23 \Rightarrow b = \frac{2\pi}{23}$. Similarly, $b = \frac{2\pi}{28}$ and $b = \frac{2\pi}{33}$.

(b) physical: $y = \sin\left(\frac{2\pi}{23}x\right) = \sin\left(\frac{2\pi}{23}\cdot 7670\right) \approx 0.136$, or 13.6%

emotional: $y = \sin\left(\frac{2\pi}{28}x\right) = \sin\left(\frac{2\pi}{28}\cdot 7670\right) \approx -0.434$, or -43.4%

intellectual: $y = \sin\left(\frac{2\pi}{33}x\right) = \sin\left(\frac{2\pi}{33}\cdot 7670\right) \approx 0.458$, or 45.8%

$\boxed{49}$ $f(t) = \frac{1}{2}\cos\left[\frac{\pi}{6}\left(t - \frac{11}{2}\right)\right]$, amplitude $= \frac{1}{2}$, period $= \frac{2\pi}{\pi/6} = 12$, phase shift $= \frac{11}{2}$

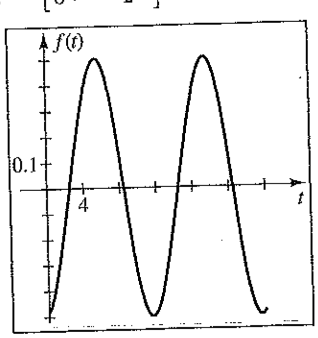

Figure 49

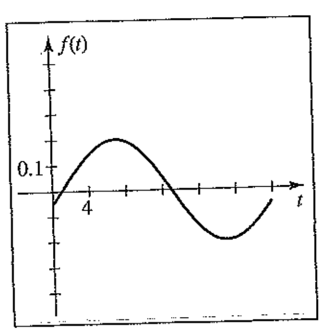

Figure 50

$\boxed{50}$ $f(t) = \frac{1}{5}\cos\left[\frac{\pi}{12}(t - 7)\right]$, amplitude $= \frac{1}{5}$, period $= \frac{2\pi}{\pi/12} = 24$, phase shift $= 7$

[51] $D(t) = 6\sin\left[\frac{2\pi}{365}(t-79)\right] + 12$, amplitude $= 6$, period $= \dfrac{2\pi}{2\pi/365} = 365$,

phase shift $= 79$, range $= \underline{12-6}$ to $\underline{12+6}$ or 6 to 18

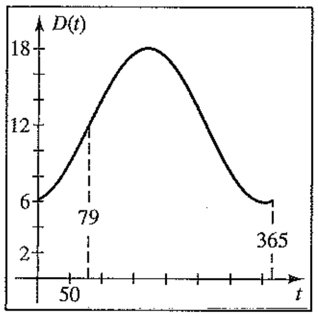

Figure 51

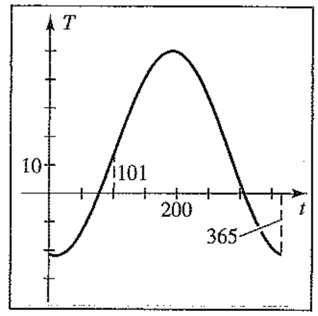

Figure 52

[52] (a) $T(t) = 36\sin\left[\frac{2\pi}{365}(t-101)\right] + 14$, amplitude $= 36$, period $= \dfrac{2\pi}{2\pi/365} = 365$,

phase shift $= 101$, range $= \underline{14-36}$ to $\underline{14+36}$ or -22 to 50

(b) The coldest day will occur when the argument of the sine is $-\frac{\pi}{2}$.

$\{\frac{3\pi}{2}, \frac{7\pi}{2}$, etc. could also be used – they would result in another year's minimum$\}$

$\frac{2\pi}{365}(t-101) = -\frac{\pi}{2} \Rightarrow t - 101 = -\frac{365}{4} \Rightarrow t = 9.75.$

Since $t = 0$ corresponds to January 1, $t = 10$ would correspond to January 11.

[53] The temperature is 20°F at 9:00 A.M. ($t = 0$). It increases to a high of 35°F at 3:00 P.M. ($t = 6$) and then decreases to 20°F at 9:00 P.M. ($t = 12$). It continues to decrease to a low of 5°F at 3:00 A.M. ($t = 18$). It then rises to 20°F at 9:00 A.M. ($t = 24$).

[0, 24, 4] by [0, 40, 4]

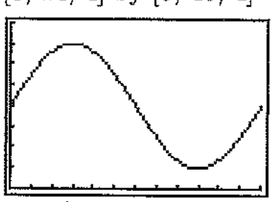

Figure 53

[0, 24, 4] by [40, 120, 20]

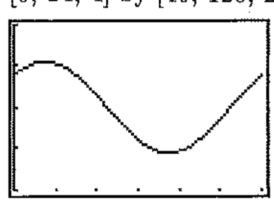

Figure 54

[54] The temperature is approximately 95.6°F at 9:00 A.M. ($t = 0$). It increases to a high of 102°F at noon ($t = 3$) and then decreases to 58°F at midnight ($t = 15$). The temperature then rises to 95.6°F at 9:00 A.M. ($t = 24$).

Note: Exer. 55–58: The period is 24 hours. Thus, $24 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{12}$.

[55] A high of 10°C and a low of -10°C imply that $d = 0$ and $a = 10$. The average temperature of 0°C will occur 6 hours after the low at 4 A.M., which corresponds to $t = 10$. Letting this correspond to the first zero of the sine function, we have

$$f(t) = 10\sin\left[\tfrac{\pi}{12}(t-10)\right] + 0 = 10\sin\left(\tfrac{\pi}{12}t - \tfrac{5\pi}{6}\right) \text{ with } a = 10,\ b = \tfrac{\pi}{12},\ c = -\tfrac{5\pi}{6},\ d = 0.$$

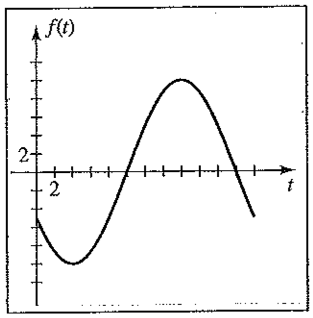

Figure 55

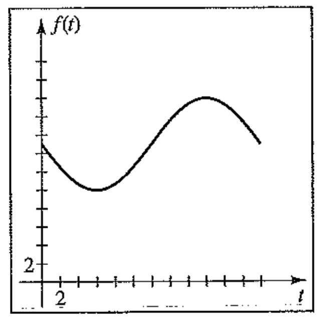

Figure 56

[56] A high of 20°C and a low of 10°C imply that $d = 15$ and $a = 5$. Since $f(t)$ is decreasing at midnight, the average temperature of 15°C (before the high of 20°C) would occur at noon, which corresponds to $t = 12$. Letting this correspond to the first zero of the sine function, we have $f(t) = 5\sin\left[\frac{\pi}{12}(t-12)\right] + 15 \Rightarrow$

$$f(t) = 5\sin\left(\tfrac{\pi}{12}t - \pi\right) + 15 \text{ with } a = 5,\ b = \tfrac{\pi}{12},\ c = -\pi,\ d = 15.$$

[57] A high of 30°C and a low of 10°C imply that $d = 20$ and $a = 10$. The average temperature of 20°C at 9 A.M. corresponds to $t = 9$. Letting this correspond to the first zero of the sine function, we have $f(t) = 10\sin\left[\frac{\pi}{12}(t-9)\right] + 20 \Rightarrow$

$$f(t) = 10\sin\left(\tfrac{\pi}{12}t - \tfrac{3\pi}{4}\right) + 20 \text{ with } a = 10,\ b = \tfrac{\pi}{12},\ c = -\tfrac{3\pi}{4},\ d = 20.$$

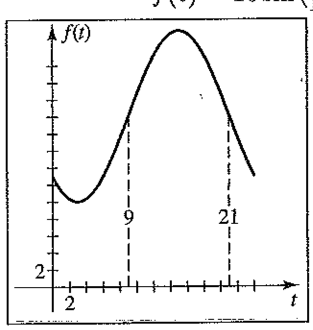

Figure 57

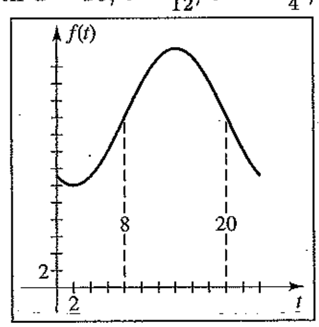

Figure 58

[58] A high of 28°C with an average of 20°C imply that the low is 12°C, $d = 20$ and $a = 8$. It was 20°C at 8 A.M., which corresponds to $t = 8$. Letting this correspond to the first zero of the sine function, we have $f(t) = 8\sin\left[\frac{\pi}{12}(t-8)\right] + 20 \Rightarrow$

$$f(t) = 8\sin\left(\tfrac{\pi}{12}t - \tfrac{2\pi}{3}\right) + 20 \text{ with } a = 8,\ b = \tfrac{\pi}{12},\ c = -\tfrac{2\pi}{3},\ d = 20.$$

59 (b) Since the period is 12 months, $12 = \frac{2\pi}{b} \Rightarrow b = \frac{\pi}{6}$. The maximum precipitation is 6.1 and the minimum is 0.2, so the sine wave is centered vertically at $d = \frac{6.1 + 0.2}{2} = 3.15$ and its amplitude is $a = \frac{6.1 - 0.2}{2} = 2.95$. Since the maximum precipitation occurs at $t = 1$ (January), we must have $bt + c = \frac{\pi}{2} \Rightarrow \frac{\pi}{6}(1) + c = \frac{\pi}{2} \Rightarrow c = \frac{\pi}{3}$. Thus, $P(t) = a\sin(bt + c) + d = 2.95\sin\left(\frac{\pi}{6}t + \frac{\pi}{3}\right) + 3.15$.

[0.5, 24.5, 5] by [−1, 8]

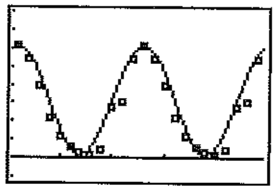

Figure 59

60 (a) See *Figure 60(a & b)*.

(b) The maximum depth is 34.3 feet and the minimum depth is 18 feet. Thus, the amplitude of D is $a = \frac{34.3 - 18}{2} = 8.15$ and it is centered vertically at $d = \frac{34.3 + 18}{2} = 26.15$. The time between maximum depths is approximately 13 hours. Thus, $b = \frac{2\pi}{13}$. The maximum of D occurs at $t = 3$. It follows that $bt + c = \frac{\pi}{2} \Rightarrow \frac{2\pi}{13}(3) + c = \frac{\pi}{2} \Rightarrow c = \frac{\pi}{26}$.

Thus, $D(t) = 8.15\sin\left(\frac{2\pi}{13}t + \frac{\pi}{26}\right) + 26.15$.

(c) Graph $Y_1 = 8.15\sin\left(\frac{2\pi}{13}x + \frac{\pi}{26}\right) + 26.15$ and $Y_2 = 24$ on [0, 30]. $Y_1 < Y_2$ {the depth is less than 24} $\Rightarrow x \in (6.8, 12.2) \cup (19.8, 25.2)$. This corresponds to the time intervals from 6:48 A.M. to 12:12 P.M. and from 7:48 P.M. to 1:12 A.M.

[0, 23, 5] by [0, 50, 10]

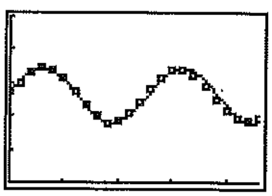

Figure 60(a & b)

[0, 30, 5] by [0, 50, 10]

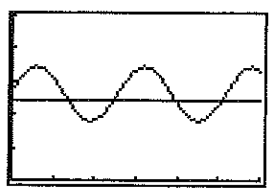

Figure 60(c)

61 (b) Since the period is 12 months, $b = \frac{2\pi}{12} = \frac{\pi}{6}$. From the table, the maximum number of daylight hours is 18.72 and the minimum is 5.88. Thus, the sine wave is centered vertically at $d = \frac{18.72 + 5.88}{2} = 12.3$ and its amplitude is $a = \frac{18.72 - 5.88}{2} = 6.42$. Since the maximum daylight occurs at $t = 7$ (July), we must have $bt + c = \frac{\pi}{2} \Rightarrow \frac{\pi}{6}(7) + c = \frac{\pi}{2} \Rightarrow c = -\frac{2\pi}{3}$.

Thus, $D(t) = a\sin(bt + c) + d = 6.42\sin\left(\frac{\pi}{6}t - \frac{2\pi}{3}\right) + 12.3$. See *Figure 61*.

[0.5, 24.5, 5] by [0, 20, 4]

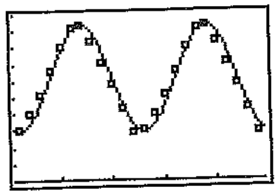

Figure 61

[0.5, 24.5, 4] by [0, 20, 4]

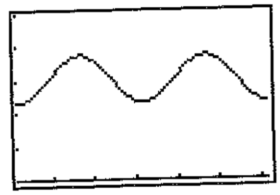

Figure 62

62 (a) We have $a = \dfrac{15.02 - 9.32}{2} = 2.85$ and $d = \dfrac{15.02 + 9.32}{2} = 12.17$. Since the period is 12 months, $b = \frac{2\pi}{12} = \frac{\pi}{6}$. The maximum should occur on June 21 {when $t = 6.7$}. $\frac{\pi}{6}(6.7) + c = \frac{\pi}{2} \Rightarrow c = -\frac{3.7\pi}{6}$. Thus,

$$D(t) = 2.85 \sin\left(\tfrac{\pi}{6}t - \tfrac{3.7\pi}{6}\right) + 12.17.$$

(c) $D(2) \approx 9.96$ hr {true = 10.17} and $D(9) \approx 13.19$ hr {true = 13.08}.

63 As $x \to 0^-$ or as $x \to 0^+$,

y oscillates between -1 and 1 and does not approach a unique value.

[−2, 2, 0.5] by [−1.33, 1.33, 0.5]

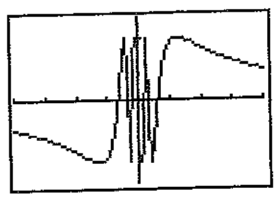

Figure 63

64 As $x \to 0^-$ or as $x \to 0^+$, y appears to approach 0. You should try at least one zoom-in to obtain a better view of the region near the origin, as shown in *Figure 64(b)*.

[−2, 2, 0.5] by [−1.33, 1.33, 0.5]

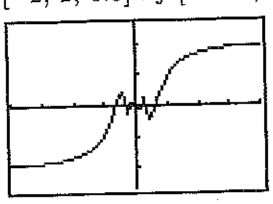

Figure 64(a)

[−0.2, 0.2, 0.05] by [−0.13, 0.13, 0.05]

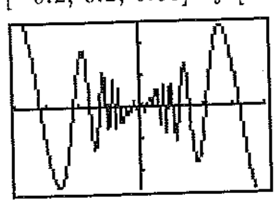

Figure 64(b)

65 As $x \to 0^-$ or as $x \to 0^+$, y appears to approach 2.

[−2, 2, 0.5] by [−0.33, 2.33, 0.5]

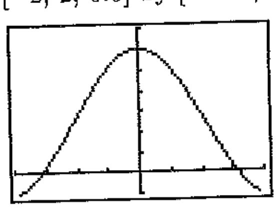

Figure 65

[−2, 2, 0.5] by [−2.2, 2.2, 0.5]

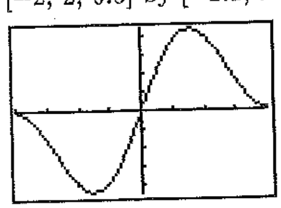

Figure 66

66 As $x \to 0^-$ or as $x \to 0^+$, y appears to approach 0.

$\boxed{67}$ From the graph, we see that there is a horizontal asymptote of $y = 4$.

$[-20, 20, 2]$ by $[-1, 5]$

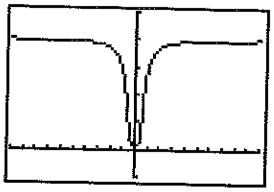

Figure 67(a)

$[-1, 1, 0.25]$ by $[-0.67, 0.67, 0.25]$

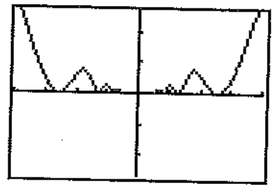

Figure 67(b)

$\boxed{68}$ From the graph, we see that there is a horizontal asymptote of $y = 0$.

$[-20, 20, 2]$ by $[-2, 2]$

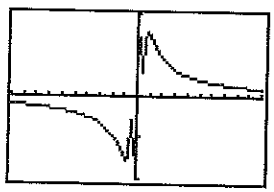

Figure 68

$\boxed{69}$ Graph $Y_1 = \cos 3x$ and $Y_2 = \frac{1}{2}x - \sin x$.

From the graph, Y_1 intersects Y_2 at $x \approx -1.63, -0.45, 0.61, 1.49, 2.42$.

Thus, $\cos 3x \geq \frac{1}{2}x - \sin x$ on $[-\pi, -1.63] \cup [-0.45, 0.61] \cup [1.49, 2.42]$.

$[-\pi, \pi, \pi/4]$ by $[-2.09, 2.09]$

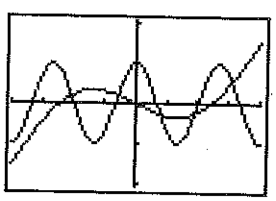

Figure 69

$[-\pi, \pi, \pi/4]$ by $[-2.09, 2.09]$

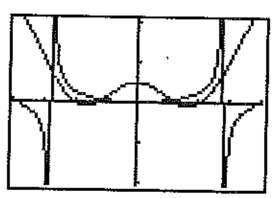

Figure 70

$\boxed{70}$ Graph $Y_1 = \frac{1}{4}\tan\left(\frac{1}{3}x^2\right)$ and $Y_2 = \frac{1}{2}\cos 2x + \frac{1}{5}x^2$. From the graph, Y_1 intersects Y_2 at $x \approx \pm 0.87$. $y = \frac{1}{4}\tan\left(\frac{1}{3}x^2\right)$ is undefined when $\frac{1}{3}x^2 = \frac{\pi}{2}$, or $x = \pm\sqrt{\frac{3\pi}{2}} \approx \pm 2.17$. Thus, $\frac{1}{4}\tan\left(\frac{1}{3}x^2\right) < \frac{1}{2}\cos 2x + \frac{1}{5}x^2$ on $[-\pi, \sqrt{3\pi/2}) \cup (-0.87, 0.87) \cup (\sqrt{3\pi/2}, \pi]$. *Figure 70* was run in Connected Mode—run it in Dot Mode to remove the vertical asymptote.

6.6 Exercises

[1] $y = 4\tan x$ • vertically stretch $\tan x$ by a factor of 4 ★ π

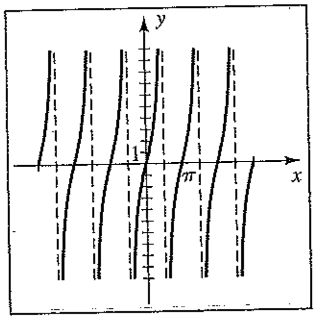

Figure 1

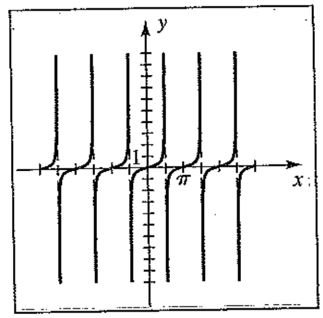

Figure 2

[2] $y = \frac{1}{4}\tan x$ • vertically compress $\tan x$ by a factor of 4 ★ π

[3] $y = 3\cot x$ • vertically stretch $\cot x$ by a factor of 3 ★ π

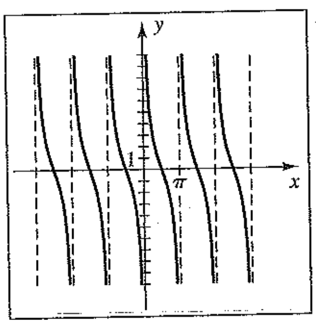

Figure 3

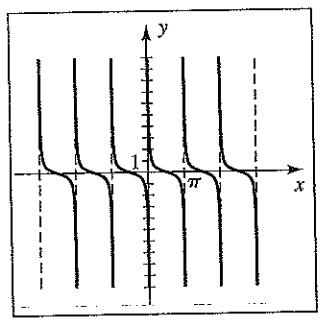

Figure 4

[4] $y = \frac{1}{3}\cot x$ • vertically compress $\cot x$ by a factor of 3 ★ π

[5] $y = 2\csc x$ • vertically stretch $\csc x$ by a factor of 2, $f(\frac{\pi}{2}) = 2$ ★ 2π

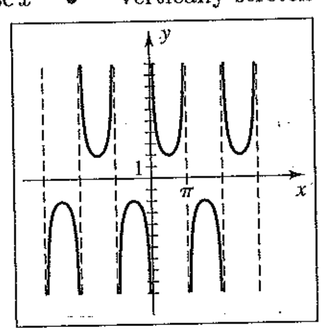

Figure 5

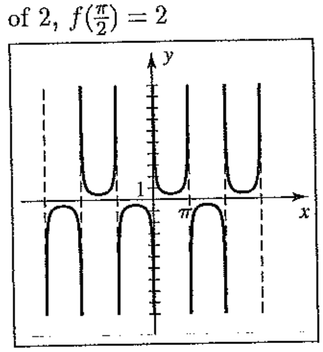

Figure 6

[6] $y = \frac{1}{2}\csc x$ • vertically compress $\csc x$ by a factor of 2, $f(\frac{\pi}{2}) = \frac{1}{2}$ ★ 2π

$\boxed{7}$ $y = 3\sec x$ • vertically stretch $\sec x$ by a factor of 3, $f(0) = 3$ ★ 2π

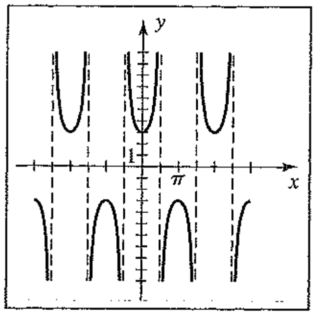

Figure 7

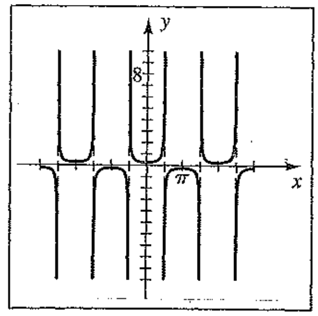

Figure 8

$\boxed{8}$ $y = \frac{1}{4}\sec x$ • vertically compress $\sec x$ by a factor of 4, $f(0) = \frac{1}{4}$ ★ 2π

Note: The vertical asymptotes of each function are denoted by VA @ $x =$.

The periods for the tangent and cotangent graphs are $\pi / |b|$.

The periods for the secant and cosecant graphs are $2\pi / |b|$.

$\boxed{9}$ $y = \tan(x - \frac{\pi}{4})$ • shift $\tan x$ right $\frac{\pi}{4}$ units, VA @ $x = -\frac{\pi}{4} + \pi n$

$-\frac{\pi}{2} \le x - \frac{\pi}{4} \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ ★ π

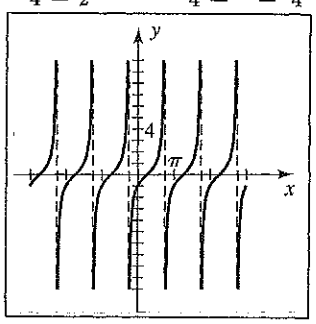

Figure 9

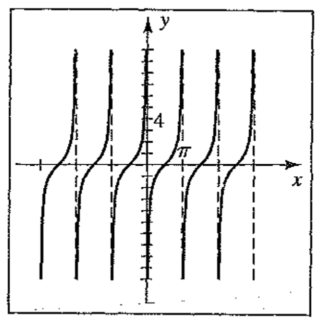

Figure 10

$\boxed{10}$ $y = \tan(x + \frac{\pi}{2})$ • shift $\tan x$ left $\frac{\pi}{2}$ units, VA @ $x = \pi n$

$-\frac{\pi}{2} \le x + \frac{\pi}{2} \le \frac{\pi}{2} \Rightarrow -\pi \le x \le 0$ ★ π

$\boxed{11}$ $y = \tan 2x$ • horizontally compress $\tan x$ by a factor of 2, VA @ $x = -\frac{\pi}{4} + \frac{\pi}{2}n$

$-\frac{\pi}{2} \le 2x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le x \le \frac{\pi}{4}$ ★ $\frac{\pi}{2}$

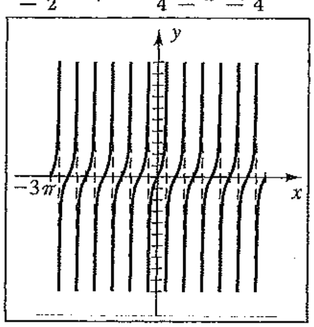

Figure 11

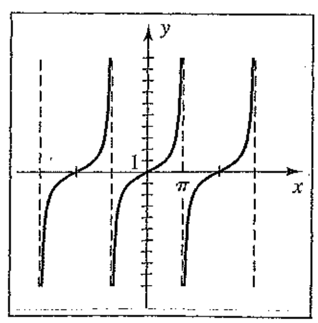

Figure 12

$\boxed{12}$ $y = \tan\frac{1}{2}x$ • horizontally stretch $\tan x$ by a factor of 2, VA @ $x = -\pi + 2\pi n$

$-\frac{\pi}{2} \le \frac{1}{2}x \le \frac{\pi}{2} \Rightarrow -\pi \le x \le \pi$ ★ 2π

[13] $y = \tan\frac{1}{4}x$ • horizontally stretch $\tan x$ by a factor of 4, VA @ $x = -2\pi + 4\pi n$

$-\frac{\pi}{2} \le \frac{1}{4}x \le \frac{\pi}{2} \;\Rightarrow\; -2\pi \le x \le 2\pi$ ★ 4π

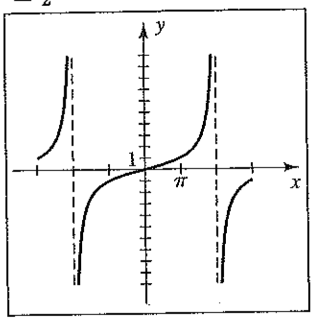

Figure 13

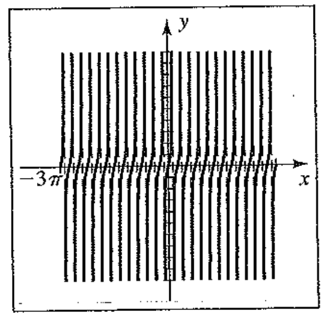

Figure 14

[14] $y = \tan 4x$ • horizontally compress $\tan x$ by a factor of 4, VA @ $x = -\frac{\pi}{8} + \frac{\pi}{4}n$

$-\frac{\pi}{2} \le 4x \le \frac{\pi}{2} \;\Rightarrow\; -\frac{\pi}{8} \le x \le \frac{\pi}{8}$ ★ $\frac{\pi}{4}$

[15] $y = 2\tan\left(2x + \frac{\pi}{2}\right) = 2\tan\left[2\left(x + \frac{\pi}{4}\right)\right]$.

$-\frac{\pi}{2} \le 2x + \frac{\pi}{2} \le \frac{\pi}{2} \;\Rightarrow\; -\pi \le 2x \le 0 \;\Rightarrow\; -\frac{\pi}{2} \le x \le 0$, VA @ $x = \frac{\pi}{2}n$ ★ $\frac{\pi}{2}$

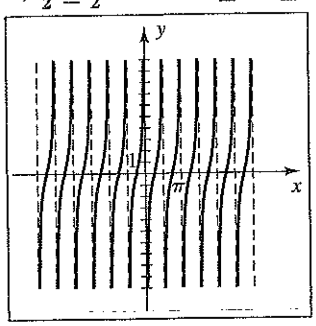

Figure 15

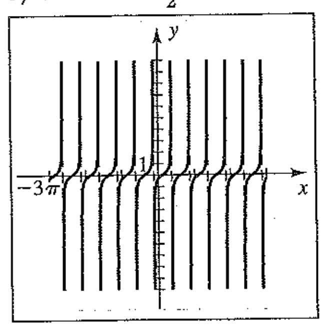

Figure 16

[16] $y = \frac{1}{3}\tan\left(2x - \frac{\pi}{4}\right) = \frac{1}{3}\tan\left[2\left(x - \frac{\pi}{8}\right)\right]$.

$-\frac{\pi}{2} \le 2x - \frac{\pi}{4} \le \frac{\pi}{2} \;\Rightarrow\; -\frac{\pi}{4} \le 2x \le \frac{3\pi}{4} \;\Rightarrow\; -\frac{\pi}{8} \le x \le \frac{3\pi}{8}$, VA @ $x = -\frac{\pi}{8} + \frac{\pi}{2}n$ ★ $\frac{\pi}{2}$

[17] $y = -\frac{1}{4}\tan\left(\frac{1}{2}x + \frac{\pi}{3}\right) = -\frac{1}{4}\tan\left[\frac{1}{2}\left(x + \frac{2\pi}{3}\right)\right]$.

$-\frac{\pi}{2} \le \frac{1}{2}x + \frac{\pi}{3} \le \frac{\pi}{2} \;\Rightarrow\; -\frac{5\pi}{6} \le \frac{1}{2}x \le \frac{\pi}{6} \;\Rightarrow\; -\frac{5\pi}{3} \le x \le \frac{\pi}{3}$, VA @ $x = -\frac{5\pi}{3} + 2\pi n$ ★ 2π

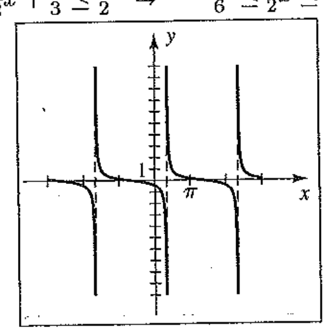

Figure 17

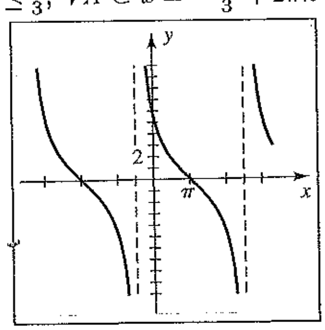

Figure 18

[18] $y = -3\tan\left(\frac{1}{3}x - \frac{\pi}{3}\right) = -3\tan\left[\frac{1}{3}(x - \pi)\right]$.

$-\frac{\pi}{2} \le \frac{1}{3}x - \frac{\pi}{3} \le \frac{\pi}{2} \;\Rightarrow\; -\frac{\pi}{6} \le \frac{1}{3}x \le \frac{5\pi}{6} \;\Rightarrow\; -\frac{\pi}{2} \le x \le \frac{5\pi}{2}$, VA @ $x = -\frac{\pi}{2} + 3\pi n$ ★ 3π

$\boxed{19}$ $y = \cot\left(x - \frac{\pi}{2}\right)$ • $0 \le x - \frac{\pi}{2} \le \pi \Rightarrow \frac{\pi}{2} \le x \le \frac{3\pi}{2}$, *VA* @ $x = \frac{\pi}{2} + \pi n$ ★ π

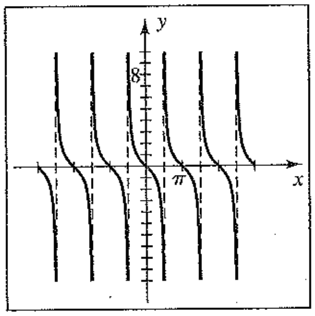

Figure 19

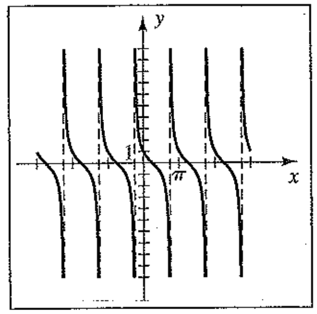

Figure 20

$\boxed{20}$ $y = \cot\left(x + \frac{\pi}{4}\right)$ • $0 \le x + \frac{\pi}{4} \le \pi \Rightarrow -\frac{\pi}{4} \le x \le \frac{3\pi}{4}$, *VA* @ $x = -\frac{\pi}{4} + \pi n$ ★ π

$\boxed{21}$ $y = \cot 2x$ • $0 \le 2x \le \pi \Rightarrow 0 \le x \le \frac{\pi}{2}$, *VA* @ $x = \frac{\pi}{2}n$ ★ $\frac{\pi}{2}$

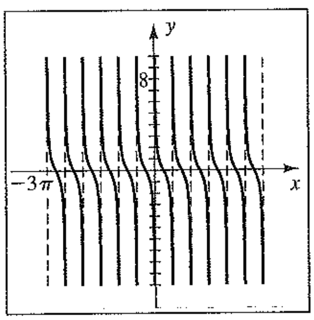

Figure 21

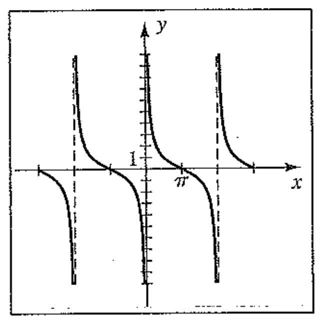

Figure 22

$\boxed{22}$ $y = \cot\frac{1}{2}x$ • $0 \le \frac{1}{2}x \le \pi \Rightarrow 0 \le x \le 2\pi$, *VA* @ $x = 2\pi n$ ★ 2π

$\boxed{23}$ $y = \cot\frac{1}{3}x$ • $0 \le \frac{1}{3}x \le \pi \Rightarrow 0 \le x \le 3\pi$, *VA* @ $x = 3\pi n$ ★ 3π

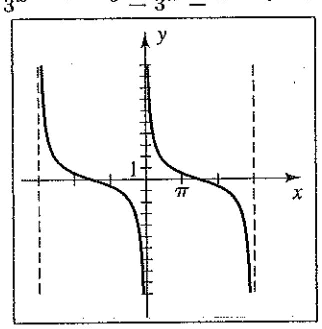

Figure 23

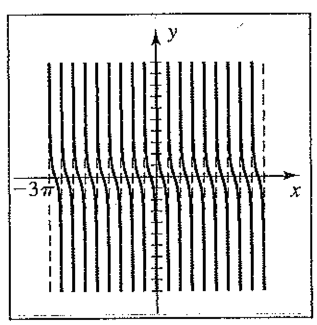

Figure 24

$\boxed{24}$ $y = \cot 3x$ • $0 \le 3x \le \pi \Rightarrow 0 \le x \le \frac{\pi}{3}$, *VA* @ $x = \frac{\pi}{3}n$ ★ $\frac{\pi}{3}$

[25] $y = 2\cot\left(2x + \frac{\pi}{2}\right) = 2\cot\left[2\left(x + \frac{\pi}{4}\right)\right]$.

$0 \le 2x + \frac{\pi}{2} \le \pi \;\Rightarrow\; -\frac{\pi}{2} \le 2x \le \frac{\pi}{2} \;\Rightarrow\; -\frac{\pi}{4} \le x \le \frac{\pi}{4}$, *VA* @ $x = -\frac{\pi}{4} + \frac{\pi}{2}n$ ★ $\frac{\pi}{2}$

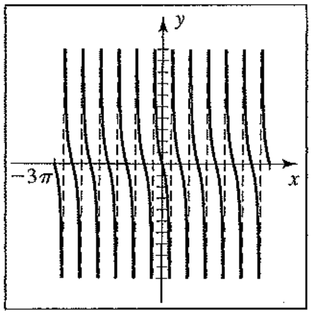

Figure 25

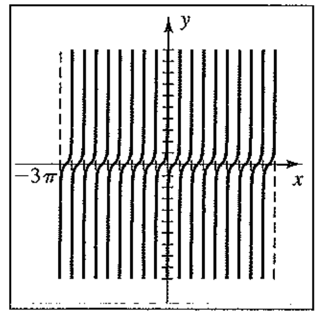

Figure 26

[26] $y = -\frac{1}{3}\cot(3x - \pi) = -\frac{1}{3}\cot\left[3\left(x - \frac{\pi}{3}\right)\right]$.

$0 \le 3x - \pi \le \pi \;\Rightarrow\; \pi \le 3x \le 2\pi \;\Rightarrow\; \frac{\pi}{3} \le x \le \frac{2\pi}{3}$, *VA* @ $x = \frac{\pi}{3} + \frac{\pi}{3}n$ ★ $\frac{\pi}{3}$

[27] $y = -\frac{1}{2}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right) = -\frac{1}{2}\cot\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{4} \le \pi \;\Rightarrow\; -\frac{\pi}{4} \le \frac{1}{2}x \le \frac{3\pi}{4} \;\Rightarrow\; -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, *VA* @ $x = -\frac{\pi}{2} + 2\pi n$ ★ 2π

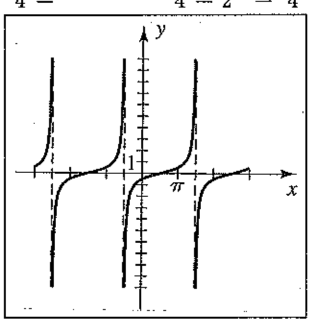

Figure 27

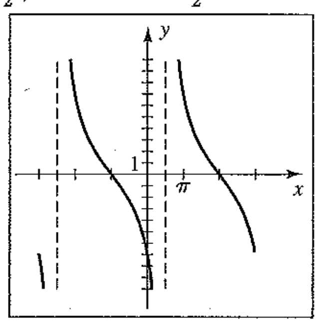

Figure 28

[28] $y = 4\cot\left(\frac{1}{3}x - \frac{\pi}{6}\right) = 4\cot\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{3}x - \frac{\pi}{6} \le \pi \;\Rightarrow\; \frac{\pi}{6} \le \frac{1}{3}x \le \frac{7\pi}{6} \;\Rightarrow\; \frac{\pi}{2} \le x \le \frac{7\pi}{2}$, *VA* @ $x = \frac{\pi}{2} + 3\pi n$ ★ 3π

[29] $y = \sec\left(x - \frac{\pi}{2}\right)$ • $-\frac{\pi}{2} \le x - \frac{\pi}{2} \le \frac{\pi}{2} \;\Rightarrow\; 0 \le x \le \pi$, *VA* @ $x = \pi n$ ★ 2π

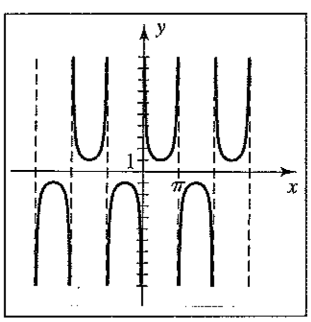

Figure 29

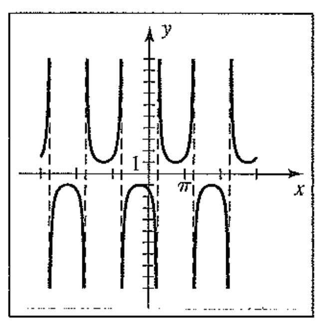

Figure 30

[30] $y = \sec\left(x - \frac{3\pi}{4}\right)$ • $-\frac{\pi}{2} \le x - \frac{3\pi}{4} \le \frac{\pi}{2} \;\Rightarrow\; \frac{\pi}{4} \le x \le \frac{5\pi}{4}$, *VA* @ $x = \frac{\pi}{4} + \pi n$ ★ 2π

$\boxed{31}$ $y = \sec 2x$ • $-\frac{\pi}{2} \le 2x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{4} \le x \le \frac{\pi}{4}$, VA @ $x = -\frac{\pi}{4} + \frac{\pi}{2}n$ ★ π

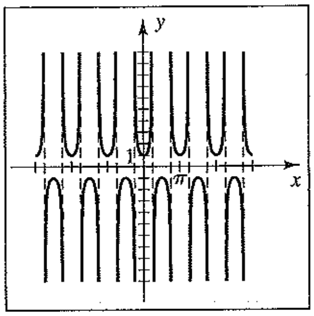

Figure 31

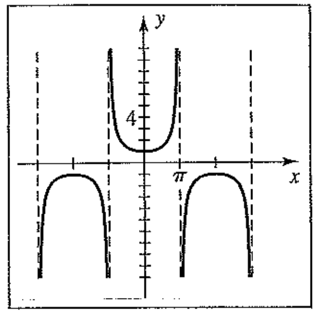

Figure 32

$\boxed{32}$ $y = \sec \frac{1}{2}x$ • $-\frac{\pi}{2} \le \frac{1}{2}x \le \frac{\pi}{2} \Rightarrow -\pi \le x \le \pi$, VA @ $x = -\pi + 2\pi n$ ★ 4π

$\boxed{33}$ $y = \sec \frac{1}{3}x$ • $-\frac{\pi}{2} \le \frac{1}{3}x \le \frac{\pi}{2} \Rightarrow -\frac{3\pi}{2} \le x \le \frac{3\pi}{2}$, VA @ $x = -\frac{3\pi}{2} + 3\pi n$ ★ 6π

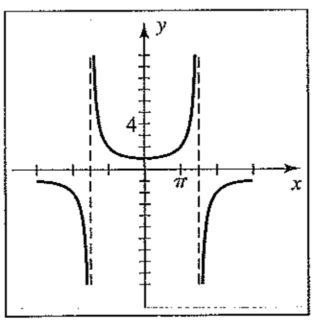

Figure 33

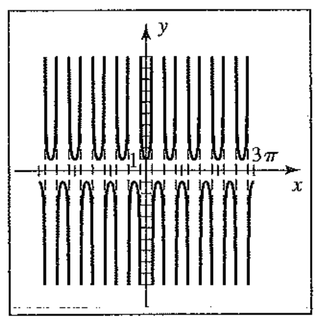

Figure 34

$\boxed{34}$ $y = \sec 3x$ • $-\frac{\pi}{2} \le 3x \le \frac{\pi}{2} \Rightarrow -\frac{\pi}{6} \le x \le \frac{\pi}{6}$, VA @ $x = -\frac{\pi}{6} + \frac{\pi}{3}n$ ★ $\frac{2\pi}{3}$

$\boxed{35}$ $y = 2\sec\left(2x - \frac{\pi}{2}\right) = 2\sec\left[2\left(x - \frac{\pi}{4}\right)\right]$.

$-\frac{\pi}{2} \le 2x - \frac{\pi}{2} \le \frac{\pi}{2} \Rightarrow 0 \le 2x \le \pi \Rightarrow 0 \le x \le \frac{\pi}{2}$, VA @ $x = \frac{\pi}{2}n$ ★ π

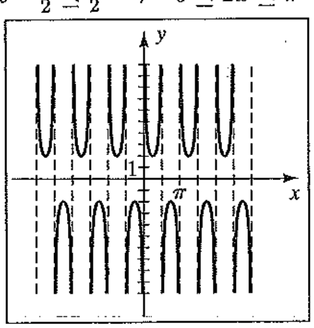

Figure 35

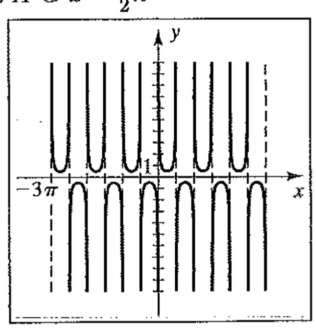

Figure 36

$\boxed{36}$ $y = \frac{1}{2}\sec\left(2x - \frac{\pi}{2}\right) = \frac{1}{2}\sec\left[2\left(x - \frac{\pi}{4}\right)\right]$.

$-\frac{\pi}{2} \le 2x - \frac{\pi}{2} \le \frac{\pi}{2} \Rightarrow 0 \le 2x \le \pi \Rightarrow 0 \le x \le \frac{\pi}{2}$, VA @ $x = \frac{\pi}{2}n$ ★ π

$\boxed{37}$ $y = -\frac{1}{3}\sec\left(\frac{1}{2}x + \frac{\pi}{4}\right) = -\frac{1}{3}\sec\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$.

$-\frac{\pi}{2} \le \frac{1}{2}x + \frac{\pi}{4} \le \frac{\pi}{2} \;\Rightarrow\; -\frac{3\pi}{4} \le \frac{1}{2}x \le \frac{\pi}{4} \;\Rightarrow\; -\frac{3\pi}{2} \le x \le \frac{\pi}{2}$, *VA* @ $x = -\frac{3\pi}{2} + 2\pi n$ ★ 4π

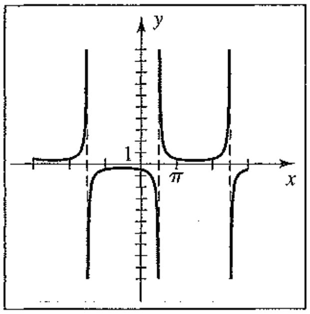

Figure 37

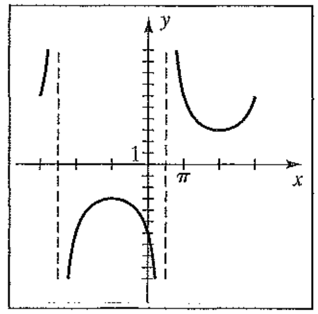

Figure 38

$\boxed{38}$ $y = -3\sec\left(\frac{1}{3}x + \frac{\pi}{3}\right) = -3\sec\left[\frac{1}{3}(x + \pi)\right]$.

$-\frac{\pi}{2} \le \frac{1}{3}x + \frac{\pi}{3} \le \frac{\pi}{2} \;\Rightarrow\; -\frac{5\pi}{6} \le \frac{1}{3}x \le \frac{\pi}{6} \;\Rightarrow\; -\frac{5\pi}{2} \le x \le \frac{\pi}{2}$, *VA* @ $x = -\frac{5\pi}{2} + 3\pi n$ ★ 6π

$\boxed{39}$ $y = \csc\left(x - \frac{\pi}{2}\right)$ • $0 \le x - \frac{\pi}{2} \le \pi \;\Rightarrow\; \frac{\pi}{2} \le x \le \frac{3\pi}{2}$, *VA* @ $x = \frac{\pi}{2} + \pi n$ ★ 2π

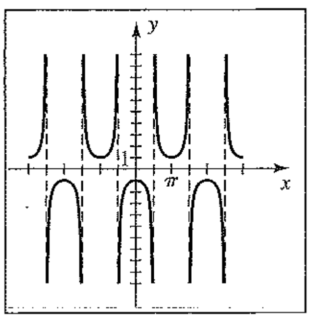

Figure 39

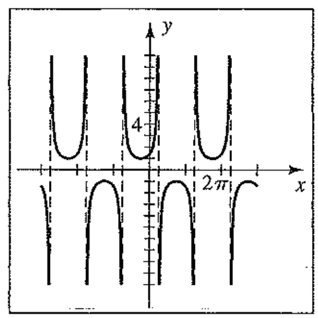

Figure 40

$\boxed{40}$ $y = \csc\left(x + \frac{3\pi}{4}\right)$ • $0 \le x + \frac{3\pi}{4} \le \pi \;\Rightarrow\; -\frac{3\pi}{4} \le x \le \frac{\pi}{4}$, *VA* @ $x = -\frac{3\pi}{4} + \pi n$ ★ 2π

$\boxed{41}$ $y = \csc 2x$ • $0 \le 2x \le \pi \;\Rightarrow\; 0 \le x \le \frac{\pi}{2}$, *VA* @ $x = \frac{\pi}{2}n$ ★ π

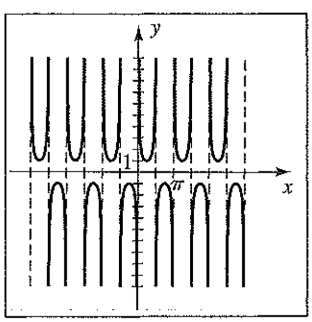

Figure 41

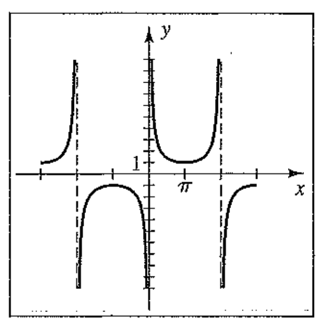

Figure 42

$\boxed{42}$ $y = \csc\frac{1}{2}x$ • $0 \le \frac{1}{2}x \le \pi \;\Rightarrow\; 0 \le x \le 2\pi$, *VA* @ $x = 2\pi n$ ★ 4π

$\boxed{43}$ $y = \csc \frac{1}{3}x \quad \bullet \quad 0 \le \frac{1}{3}x \le \pi \;\Rightarrow\; 0 \le x \le 3\pi,\ VA\ @\ x = 3\pi n$ $\quad\bigstar\ 6\pi$

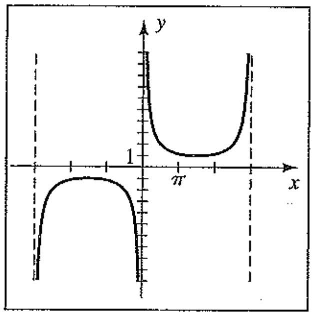

Figure 43

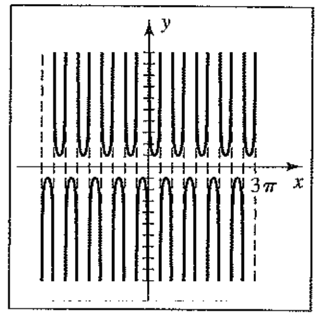

Figure 44

$\boxed{44}$ $y = \csc 3x \quad \bullet \quad 0 \le 3x \le \pi \;\Rightarrow\; 0 \le x \le \frac{\pi}{3},\ VA\ @\ x = \frac{\pi}{3}n$ $\quad\bigstar\ \frac{2\pi}{3}$

$\boxed{45}$ $y = 2\csc\left(2x + \frac{\pi}{2}\right) = 2\csc\left[2\left(x + \frac{\pi}{4}\right)\right]$.

$0 \le 2x + \frac{\pi}{2} \le \pi \;\Rightarrow\; -\frac{\pi}{2} \le 2x \le \frac{\pi}{2} \;\Rightarrow\; -\frac{\pi}{4} \le x \le \frac{\pi}{4},\ VA\ @\ x = -\frac{\pi}{4} + \frac{\pi}{2}n$ $\quad\bigstar\ \pi$

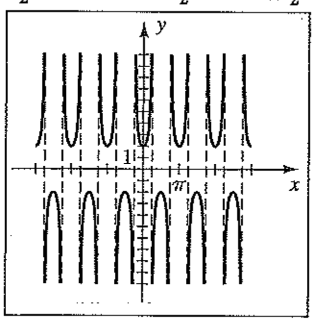

Figure 45

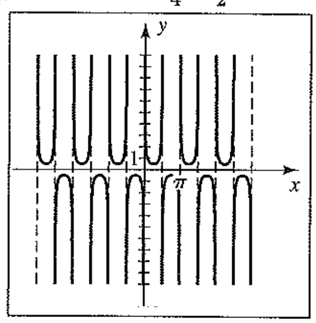

Figure 46

$\boxed{46}$ $y = -\frac{1}{2}\csc(2x - \pi) = -\frac{1}{2}\csc\left[2\left(x - \frac{\pi}{2}\right)\right]$.

$0 \le 2x - \pi \le \pi \;\Rightarrow\; \pi \le 2x \le 2\pi \;\Rightarrow\; \frac{\pi}{2} \le x \le \pi,\ VA\ @\ x = \frac{\pi}{2}n$ $\quad\bigstar\ \pi$

$\boxed{47}$ $y = -\frac{1}{4}\csc\left(\frac{1}{2}x + \frac{\pi}{2}\right) = -\frac{1}{4}\csc\left[\frac{1}{2}(x + \pi)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{2} \le \pi \;\Rightarrow\; -\frac{\pi}{2} \le \frac{1}{2}x \le \frac{\pi}{2} \;\Rightarrow\; -\pi \le x \le \pi,\ VA\ @\ x = -\pi + 2\pi n$ $\quad\bigstar\ 4\pi$

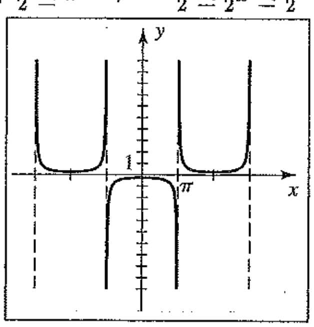

Figure 47

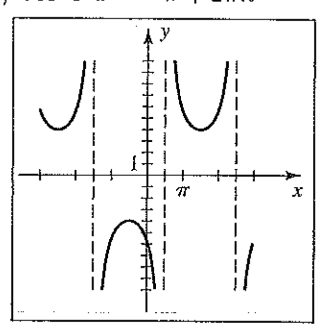

Figure 48

$\boxed{48}$ $y = 4\csc\left(\frac{1}{2}x - \frac{\pi}{4}\right) = 4\csc\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{2}x - \frac{\pi}{4} \le \pi \;\Rightarrow\; \frac{\pi}{4} \le \frac{1}{2}x \le \frac{5\pi}{4} \;\Rightarrow\; \frac{\pi}{2} \le x \le \frac{5\pi}{2},\ VA\ @\ x = \frac{\pi}{2} + 2\pi n$ $\quad\bigstar\ 4\pi$

[49] $y = \tan\frac{\pi}{2}x$ • horizontally stretch $\tan x$ by a factor of $2/\pi$, VA @ $x = -1 + 2n$

$-\frac{\pi}{2} \leq \frac{\pi}{2}x \leq \frac{\pi}{2} \;\Rightarrow\; -1 \leq x \leq 1$ ★ 2

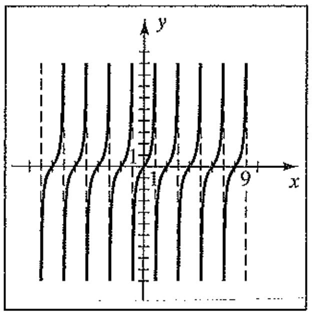

Figure 49

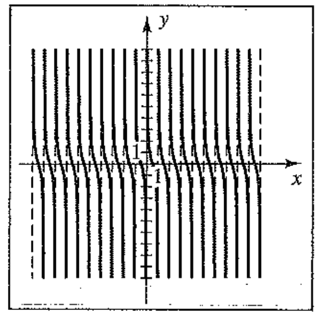

Figure 50

[50] $y = \cot \pi x$ • $0 \leq \pi x \leq \pi \;\Rightarrow\; 0 \leq x \leq 1$, VA @ $x = n$ ★ 1

[51] $y = \csc 2\pi x$ • $0 \leq 2\pi x \leq \pi \;\Rightarrow\; 0 \leq x \leq \frac{1}{2}$, VA @ $x = \frac{1}{2}n$ ★ 1

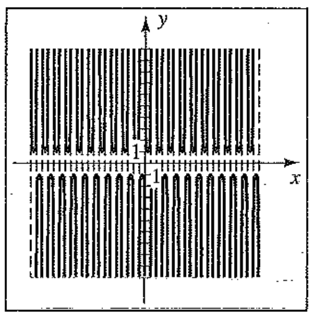

Figure 51

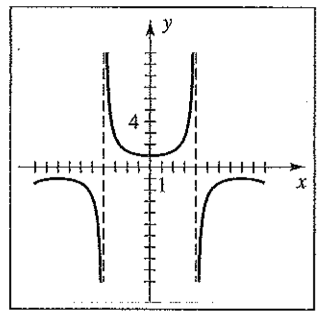

Figure 52

[52] $y = \sec\frac{\pi}{8}x$ • $-\frac{\pi}{2} \leq \frac{\pi}{8}x \leq \frac{\pi}{2} \;\Rightarrow\; -4 \leq x \leq 4$, VA @ $x = -4 + 8n$ ★ 16

[53] Reflecting the graph of $y = \cot x$ through the x-axis, which is $y = -\cot x$, gives us the graph of $y = \tan\left(x + \frac{\pi}{2}\right)$. If we shift this graph to the left (or right), we will obtain the graph of $y = \tan x$. Thus, one equation is $y = -\cot\left(x + \frac{\pi}{2}\right)$.

[54] Shifting the graph of $y = \csc x$ to the left $\frac{\pi}{2}$ units gives us the graph of $y = \sec x$.

Thus, one equation is $y = \csc\left(x + \frac{\pi}{2}\right)$.

[55] $y = |\sin x|$ •

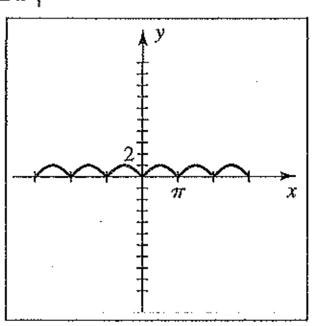

Figure 55

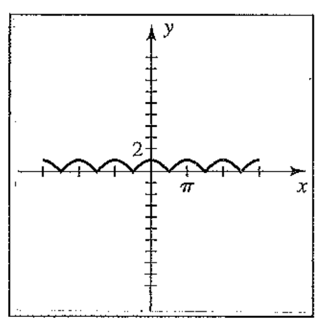

Figure 56

[56] $y = |\cos x|$ •

$\boxed{57}$ $y = |\sin x| + 2$ •

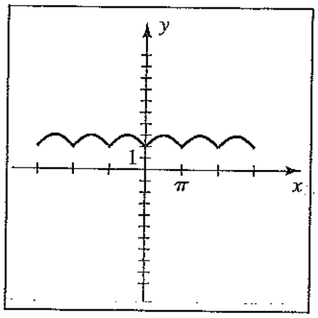

Figure 57

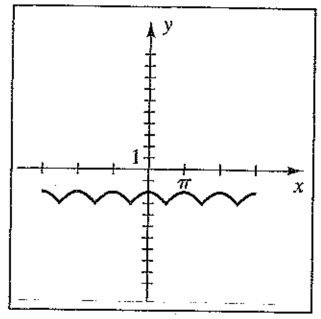

Figure 58

$\boxed{58}$ $y = |\cos x| - 3$ •

$\boxed{59}$ $y = -|\cos x| + 1$ •

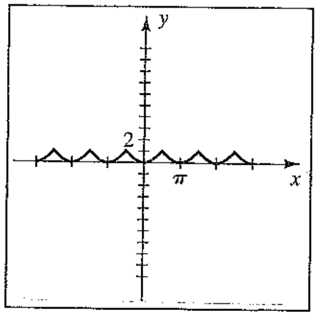

Figure 59

$\boxed{60}$ $y = -|\sin x| - 2$ •

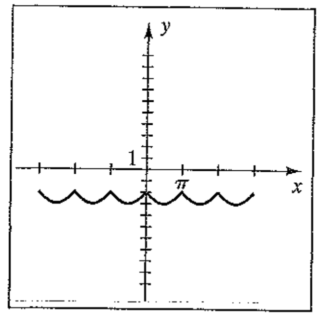

Figure 60

$\boxed{61}$ $y = x + \cos x$ •

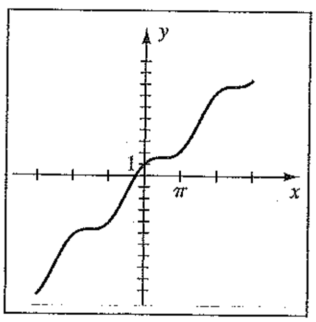

Figure 61

$\boxed{62}$ $y = x - \sin x$ •

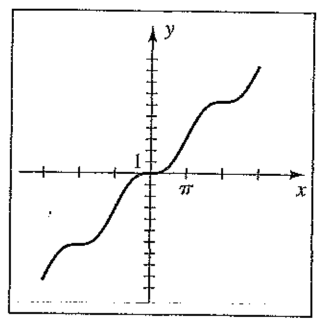

Figure 62

$\boxed{63}$ $y = 2^{-x} \cos x$ •

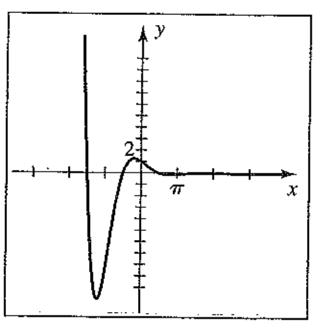

Figure 63

$\boxed{64}$ $y = e^{x} \sin x$ •

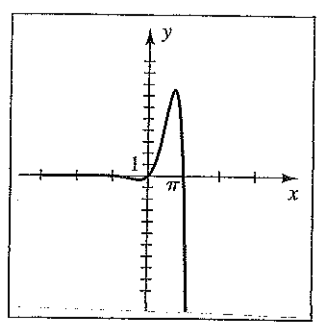

Figure 64

[65] $y = |x| \sin x$ •

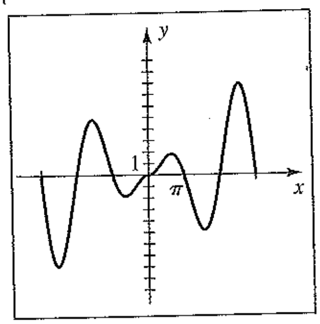

Figure 65

[66] $y = |x| \cos x$ •

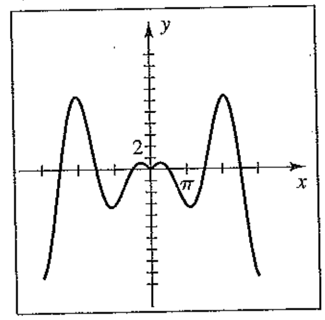

Figure 66

[67] $f(x) = \tan(0.5x)$; $g(x) = \tan[0.5(x + \pi/2)]$ • Since $g(x) = f(x + \pi/2)$, the graph of g can be obtained by shifting the graph of f left a distance of $\frac{\pi}{2}$.

$[-2\pi, 2\pi, \pi/2]$ by $[-4, 4]$

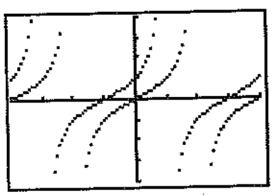

Figure 67

$[-2\pi, 2\pi, \pi/2]$ by $[-4, 4]$

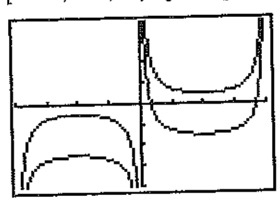

Figure 68

[68] $f(x) = 0.5\csc(0.5x)$; $g(x) = 0.5\csc(0.5x) - 2$ • Since $g(x) = f(x) - 2$, the graph of g can be obtained by shifting the graph of f downward a distance of 2.

[69] $f(x) = 0.5\sec 0.5x$; $g(x) = 0.5\sec[0.5(x - \pi/2)] - 1$ •

Since $g(x) = f(x - \pi/2) - 1$, the graph of g can be obtained by shifting the graph of f horizontally to the right a distance of $\frac{\pi}{2}$ and vertically downward a distance of 1.

$[-2\pi, 2\pi, \pi/2]$ by $[-4, 4]$

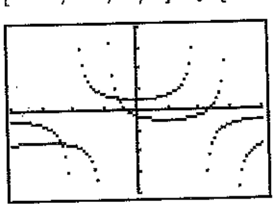

Figure 69

$[-2\pi, 2\pi, \pi/2]$ by $[-4, 4]$

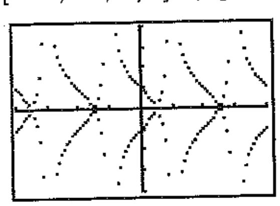

Figure 70

[70] $f(x) = \tan x - 1$; $g(x) = -\tan x + 1$ • Since $g(x) = -f(x)$, the graph of g can be obtained by reflecting the graph of f through the x–axis.

[71] $f(x) = 3\cos 2x$; $\quad g(x) = |3\cos 2x| - 1$ • Since $g(x) = |f(x)| - 1$, the graph of g can be obtained from the graph of f by reflecting it through the x-axis when $f(x) < 0$ and then shifting that graph downward a distance of 1.

[-2π, 2π, $\pi/2$] by [-4, 4]

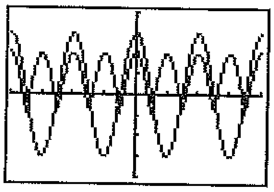

Figure 71

[-2π, 2π, $\pi/2$] by [-4, 4]

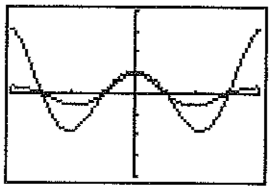

Figure 72

[72] $f(x) = 1.2^{-x}\cos x$; $\quad g(x) = 1.2^{x}\cos x$ •

Since $g(x) = 1.2^{x}\cos x = 1.2^{-(-x)}\cos(-x) = f(-x)$, the graph of g can be obtained from the graph of f by reflecting the graph of f through the y-axis.

[73] The damping factor of $y = e^{-x/4}\sin 4x$ is $e^{-x/4}$.

[-2π, 2π, $\pi/2$] by [-4.19, 4.19]

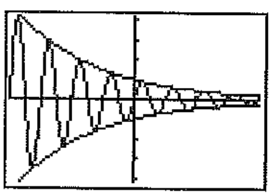

Figure 73

[-2π, 2π, $\pi/2$] by [-4.19, 4.19]

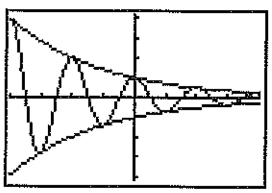

Figure 74

[74] The damping factor of $y = 3^{-x/5}\cos 2x$ is $3^{-x/5}$.

[75] From the graph, we see that the maximum occurs at the approximate coordinates (-2.76, 3.09), and the minimum occurs at the approximate coordinates (1.23, -3.68).

[$-\pi$, π, $\pi/4$] by [-4, 4]

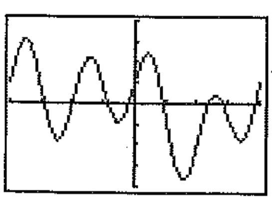

Figure 75

[$-\pi$, π, $\pi/4$] by [-4, 4]

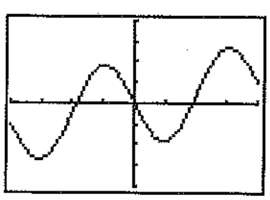

Figure 76

[76] From the graph, we see that the maximum occurs at the approximate coordinates (2.40, 2.68), and the minimum occurs at the approximate coordinates (-2.40, -2.68).

[77] From the graph, we see that f is increasing and one-to-one between

$a \approx -0.70$ and $b \approx 0.12$. Thus, the interval is approximately $[-0.70, 0.12]$.

$[-2, 2]$ by $[-1.33, 1.33]$

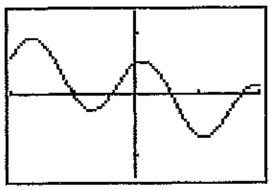

Figure 77

$[-3, 3]$ by $[-1, 3]$

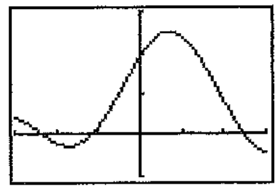

Figure 78

[78] From the graph, we see that f is increasing and one-to-one between

$a \approx -1.70$ and $b \approx 0.70$. Thus, the interval is approximately $[-1.70, 0.70]$.

[79] Graph $Y_1 = \cos(2x-1) + \sin 3x$ and $Y_2 = \sin\frac{1}{3}x + \cos x$.

From the graph, Y_1 intersects Y_2 at $x \approx -1.31, 0.11, 0.95, 2.39$.

Thus, $\cos(2x-1) + \sin 3x \geq \sin\frac{1}{3}x + \cos x$ on $[-\pi, -1.31] \cup [0.11, 0.95] \cup [2.39, \pi]$.

$[-\pi, \pi, \pi/4]$ by $[-2.09, 2.09]$

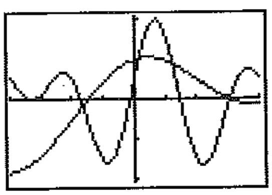

Figure 79

$[-\pi, \pi, \pi/4]$ by $[-2.09, 2.09]$

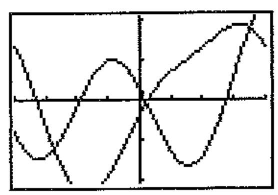

Figure 80

[80] Graph $Y_1 = \frac{1}{2}\cos 2x + 2\cos(x-2)$ and $Y_2 = 2\cos(1.5x+1) + \sin(x-1)$.

From the graph, Y_1 intersects Y_2 at $x \approx -2.16, 0.15, 2.76$. Thus,

$\frac{1}{2}\cos 2x + 2\cos(x-2) < 2\cos(1.5x+1) + \sin(x-1)$ on $(-2.16, 0.15) \cup (2.76, \pi]$.

[81] (a) $\theta = 0 \Rightarrow I = \frac{1}{2}I_0[1 + \cos(\pi \sin 0)] = \frac{1}{2}I_0[1 + \cos(0)] = \frac{1}{2}I_0(2) = I_0$.

(b) $\theta = \pi/3 \Rightarrow I = \frac{1}{2}I_0[1 + \cos(\pi \sin(\pi/3))] \approx 0.044 I_0$.

(c) $\theta = \pi/7 \Rightarrow I = \frac{1}{2}I_0[1 + \cos(\pi \sin(\pi/7))] \approx 0.603 I_0$.

[82] (a) The intensity $I = \frac{1}{2}I_0[1 + \cos(\pi \sin\theta)]$ will be maximum when $\cos(\pi\sin\theta) = 1 \Rightarrow \pi\sin\theta = 0 \Rightarrow \sin\theta = 0 \Rightarrow \theta = 0, \pi$. Thus, the intensity is maximum in the east and west directions. The intensity I will be minimum when $\cos(\pi\sin\theta) = -1 \Rightarrow \pi\sin\theta = \pm\pi \Rightarrow \sin\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$. There is no signal in the north and south directions so that it wouldn't interfere with a radio station at the same wavelength to the north or south.

(b) Graph $Y_1 = \frac{1}{2}[1 + \cos(\pi \sin\theta)]$ and $Y_2 = \frac{1}{3}$. There are four points of intersection on $[0,\ 2\pi)$. They occur at $\theta \approx 0.654,\ 2.488,\ 3.795,\ 5.629$.

$[0,\ 2\pi,\ \pi/2]$ by $[0,\ 1.5,\ 0.5]$

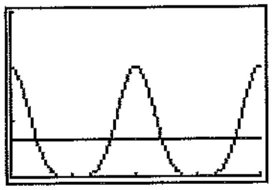

Figure 82

83 (a) The damping factor is $A_0 e^{-\alpha z}$.

(b) The phase shift at depth z_0 is given by $kt - \alpha z_0 = 0 \Rightarrow t = \frac{\alpha}{k} z_0$.

(c) At the surface, $z = 0$. Hence, $S = A_0 \sin kt$ and the amplitude at the surface is A_0. $\text{Amplitude}_{\text{wave}} = \frac{1}{2}\text{Amplitude}_{\text{surface}} \Rightarrow$

$$A_0 e^{-\alpha z} = \tfrac{1}{2} A_0 \Rightarrow e^{-\alpha z} = \tfrac{1}{2} \Rightarrow -\alpha z = \ln\tfrac{1}{2} \Rightarrow z = \frac{-\ln 2}{-\alpha} = \frac{\ln 2}{\alpha}.$$

6.7 Exercises

Note: The missing values are found in terms of the given values.

We could also use proportions to find the remaining parts.

1 $\beta = 90° - \alpha = 60°$. $\tan\alpha = \frac{a}{b} \Rightarrow a = b\tan\alpha = 20(\frac{1}{3}\sqrt{3}) = \frac{20}{3}\sqrt{3}$.

$\sec\alpha = \frac{c}{b} \Rightarrow c = b\sec\alpha = 20(\frac{2}{3}\sqrt{3}) = \frac{40}{3}\sqrt{3}$.

2 $\alpha = 90° - \beta = 45°$. $\cot\beta = \frac{a}{b} \Rightarrow a = b\cot\beta = 35(1) = 35$.

$\csc\beta = \frac{c}{b} \Rightarrow c = b\csc\beta = 35(\sqrt{2}) = 35\sqrt{2}$.

3 $\alpha = 90° - \beta = 45°$. $\cos\beta = \frac{a}{c} \Rightarrow a = c\cos\beta = 30(\frac{1}{2}\sqrt{2}) = 15\sqrt{2}$.

$b = a$ in a $45°$–$45°$–$90°$ $\triangle$.

4 $\beta = 90° - \alpha = 30°$. $\sin\alpha = \frac{a}{c} \Rightarrow a = c\sin\alpha = 6(\frac{1}{2}\sqrt{3}) = 3\sqrt{3}$.

$\cos\alpha = \frac{b}{c} \Rightarrow b = c\cos\alpha = 6(\frac{1}{2}) = 3$.

5 $\tan\alpha = \frac{a}{b} = \frac{5}{5} = 1 \Rightarrow \alpha = 45°$. $\beta = 90° - \alpha = 45°$.

$c = \sqrt{a^2 + b^2} = \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$.

6 $\sin\alpha = \frac{a}{c} = \frac{4\sqrt{3}}{8} = \frac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$. $\beta = 90° - \alpha = 30°$.

$b = \sqrt{c^2 - a^2} = \sqrt{64 - 48} = \sqrt{16} = 4$.

7 $\cos\alpha = \frac{b}{c} = \frac{5\sqrt{3}}{10\sqrt{3}} = \frac{1}{2} \Rightarrow \alpha = 60°$. $\beta = 90° - \alpha = 30°$.

$a = \sqrt{c^2 - b^2} = \sqrt{300 - 75} = \sqrt{225} = 15$.

8 $\cos\alpha = \frac{b}{c} = \frac{7\sqrt{2}}{14} = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45°$. $\beta = 90° - \alpha = 45°$.

$a = \sqrt{c^2 - b^2} = \sqrt{196 - 98} = \sqrt{98} = 7\sqrt{2}$.

$\boxed{9}$ $\beta = 90^\circ - \alpha = 53^\circ$. $\tan\alpha = \frac{a}{b} \Rightarrow a = b\tan\alpha = 24\tan 37^\circ \approx 18$.

$$\sec\alpha = \frac{c}{b} \Rightarrow c = b\sec\alpha = 24\sec 37^\circ \approx 30.$$

$\boxed{10}$ $\alpha = 90^\circ - \beta = 25^\circ 40'$. $\tan\beta = \frac{b}{a} \Rightarrow b = a\tan\beta = 20.1\tan 64^\circ 20' \approx 41.8$.

$$\sec\beta = \frac{c}{a} \Rightarrow c = a\sec\beta = 20.1\sec 64^\circ 20' \approx 46.4.$$

$\boxed{11}$ $\alpha = 90^\circ - \beta = 18^\circ 9'$. $\cot\beta = \frac{a}{b} \Rightarrow a = b\cot\beta = 240.0\cot 71^\circ 51' \approx 78.7$.

$$\csc\beta = \frac{c}{b} \Rightarrow c = b\csc\beta = 240.0\csc 71^\circ 51' \approx 252.6.$$

$\boxed{12}$ $\beta = 90^\circ - \alpha = 58^\circ 50'$. $\cot\alpha = \frac{b}{a} \Rightarrow b = a\cot\alpha = 510\cot 31^\circ 10' \approx 843$.

$$\csc\alpha = \frac{c}{a} \Rightarrow c = a\csc\alpha = 510\csc 31^\circ 10' \approx 985.$$

$\boxed{13}$ $\tan\alpha = \frac{a}{b} = \frac{25}{45} \Rightarrow \alpha \approx 29^\circ$. $\beta = 90^\circ - \alpha \approx 61^\circ$.

$$c = \sqrt{a^2 + b^2} = \sqrt{625 + 2025} = \sqrt{2650} \approx 51.$$

$\boxed{14}$ $\tan\alpha = \frac{a}{b} = \frac{31}{9.0} \Rightarrow \alpha \approx 74^\circ$. $\beta = 90^\circ - \alpha \approx 16^\circ$.

$$c = \sqrt{a^2 + b^2} = \sqrt{961 + 81} = \sqrt{1042} \approx 32.$$

$\boxed{15}$ $\cos\alpha = \frac{b}{c} = \frac{2.1}{5.8} \Rightarrow \alpha \approx 69^\circ$. $\beta = 90^\circ - \alpha \approx 21^\circ$.

$$a = \sqrt{c^2 - b^2} = \sqrt{33.64 - 4.41} = \sqrt{29.23} \approx 5.4.$$

$\boxed{16}$ $\sin\alpha = \frac{a}{c} = \frac{0.42}{0.68} \Rightarrow \alpha \approx 38^\circ$. $\beta = 90^\circ - \alpha \approx 52^\circ$.

$$b = \sqrt{c^2 - a^2} = \sqrt{0.4624 - 0.1764} = \sqrt{0.286} \approx 0.53.$$

$\boxed{17}$ $\cos\alpha = \frac{b}{c} \Rightarrow b = c\cos\alpha$

$\boxed{18}$ $\sin\beta = \frac{b}{c} \Rightarrow b = c\sin\beta$

$\boxed{19}$ $\cot\beta = \frac{a}{b} \Rightarrow a = b\cot\beta$

$\boxed{20}$ $\tan\alpha = \frac{a}{b} \Rightarrow a = b\tan\alpha$

$\boxed{21}$ $\csc\alpha = \frac{c}{a} \Rightarrow c = a\csc\alpha$

$\boxed{22}$ $\sec\beta = \frac{c}{a} \Rightarrow c = a\sec\beta$

$\boxed{23}$ $a^2 + b^2 = c^2 \Rightarrow b^2 = c^2 - a^2 \Rightarrow b = \sqrt{c^2 - a^2}$

$\boxed{24}$ $a^2 + b^2 = c^2 \Rightarrow c = \sqrt{a^2 + b^2}$

$\boxed{25}$ Let h denote the height of the kite and $x = h - 4$.

$$\sin 60^\circ = \frac{x}{500} \Rightarrow x = 500(\tfrac{1}{2}\sqrt{3}) = 250\sqrt{3},\quad h = 250\sqrt{3} + 4 \approx 437 \text{ ft}$$

$\boxed{26}$ $\cot 68^\circ = \frac{x}{15} \Rightarrow x = 15\cot 68^\circ \approx 6.1$ m.

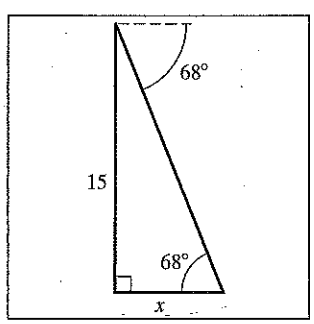

Figure 26

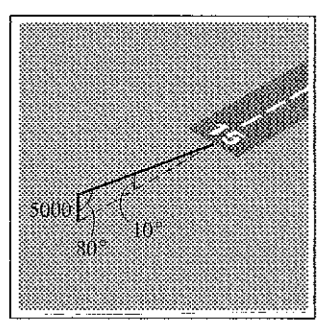

Figure 27

$\boxed{27}$ $\sin 10^\circ = \frac{5000}{x} \Rightarrow x = 5000\csc 10^\circ \approx 28{,}793.85$, or 28,800 ft.

[28] Let l denote the length of the wire. $\cos 58°20' = \frac{40}{l} \Rightarrow l = 40\sec 58°20' \approx 76.2$ m

[29] $\tan 72°40' = \frac{d}{50} \Rightarrow d \approx 160$ m.

[30] (a) $\tan\theta = \frac{h}{d} \Rightarrow h = d\tan\theta$ (b) $h = 1000\tan 59° \approx 1664$ m

[31] Let h denote the altitude. $\sin 75° = \frac{h}{10{,}000} \Rightarrow h \approx 9659$ ft.

[32] Let d denote the distance the plane travels.

$\sin 10° = 15{,}000/d \Rightarrow d = 15{,}000\csc 10°$.

At 250 ft/sec, it will take $\frac{d}{250} = \frac{15{,}000\csc 10°}{250} \approx 345.5$ seconds or 5.76 minutes.

[33] (a) The bridge section is 75 ft. long. $\sin 35° = \frac{d-15}{75} \Rightarrow d = 75\sin 35° + 15 \approx 58$ ft.

(b) Let x be the horizontal distance from the end of a bridge section to a point directly underneath the end of the section. $\cos 35° = \frac{x}{75} \Rightarrow x = 75\cos 35°$.

The distance is $150 - 2x \approx 27$ ft.

[34] The lower section has a horizontal length of $x_1 = 15\cot 25°$ and a slide length of $s_1 = 15\csc 25° \approx 35.5$. The upper section has a horizontal length of $x_2 = 15\cot 35°$ and a slide length of $s_2 = 15\csc 35° \approx 26.2$.

The middle section has a slide length of $s_3 = 100 - x_1 - x_2 \approx 46.4$.

The total slide length is $s_1 + s_2 + s_3 \approx 35.5 + 26.2 + 46.4 = 108.1$ ft.

[35] Let α denote the angle of elevation. $\tan\alpha = \frac{5}{4} \Rightarrow \alpha \approx 51°20'$.

[36] $\sin\alpha = \frac{5}{24} \Rightarrow \alpha \approx 12°$

[37] Let D denote the position of the duck and t the number of seconds required for a direct hit. $\sin\varphi = \frac{\overline{AD}}{\overline{OD}} = \frac{7t}{25t} \Rightarrow \sin\varphi = \frac{7}{25} \Rightarrow \varphi \approx 16.3°$

[38] (a) $\sin\alpha = \frac{4}{9} \Rightarrow \alpha \approx 26.4°$ (b) $\sin 40° = \frac{h}{9} \Rightarrow h \approx 5.8$ m

[39] Let h denote the height of the tower.

$\tan 21°20'24'' = \tan 21.34° = \frac{h}{5280} \Rightarrow h = 5280\tan 21.34° \approx 2063$ ft.

[40] Maximum elongation θ_{max} will occur when the line of sight from the earth to Venus is tangent to the orbit of Venus. Since a tangent line to a circle is perpendicular to a radius, maximum elongation will occur when the angle determined by the sun, Venus, and the earth is 90°. $\sin(\theta_{\text{max}}) = \frac{D_{\text{v}}}{D_{\text{e}}} = \frac{68{,}000{,}000}{91{,}500{,}000} = \frac{68}{91.5} \Rightarrow \theta_{\text{max}} \approx 48°$.

[41] The central angle of a section of the Pentagon has measure $360°/5 = 72°$.

Bisecting that angle, we have an angle of 36° whose opposite side is $\frac{921}{2}$.

The height h is given by $\tan 36° = \frac{\frac{921}{2}}{h} \Rightarrow h = \frac{921}{2\tan 36°}$.

Area $= 5(\frac{1}{2}bh) = 5(\frac{1}{2})(921)\left(\frac{921}{2\tan 36°}\right) \approx 1{,}459{,}379$ ft^2.

[42] The central angle of a section of the octagon has an angle measure of $\frac{360°}{8} = 45°$. Bisecting that angle forms a right triangle with a hypotenuse of 12 cm.

$\sin 22.5° = \frac{x}{12} \Rightarrow x = 12\sin 22.5°$.

There are 8 sides of length $2x$.

Hence, the perimeter is $16x = 192\sin 22.5° \approx 73.5$ cm.

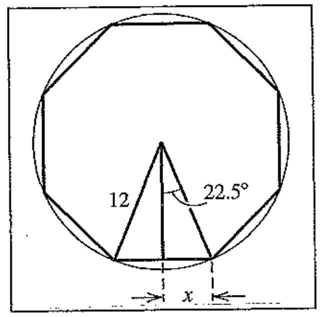

Figure 42

[43] The diagonal of the base is $\sqrt{8^2+6^2} = 10$. $\tan\theta = \frac{4}{10} \Rightarrow \theta \approx 21.8°$

[44] $V = \frac{1}{3}\pi r^2 h \Leftrightarrow 20 = \frac{1}{3}\pi(2)^2 h \Rightarrow h = \frac{15}{\pi}$.

$$\tan\frac{\beta}{2} = \frac{r}{h} = \frac{2}{15/\pi} \Rightarrow \frac{\beta}{2} \approx 22.728°, \text{ and } \beta \approx 45.5°.$$

[45] $\cot 53°30' = \frac{x}{h} \Rightarrow x = h\cot 53°30'$.

$\cot 26°50' = \frac{x+25}{h} \Rightarrow x + 25 = h\cot 26°50' \Rightarrow x = h\cot 26°50' - 25$.

Thus, $h\cot 53°30' = h\cot 26°50' - 25 \Rightarrow$

$$25 = h\cot 26°50' - h\cot 53°30' \Rightarrow h = \frac{25}{\cot 26°50' - \cot 53°30'} \approx 20.2 \text{ m}.$$

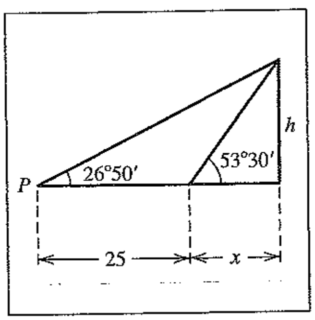

Figure 45

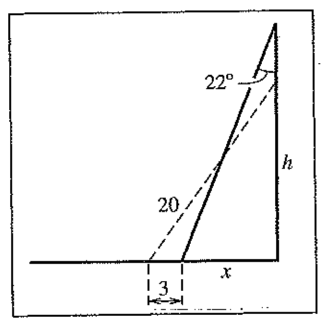

Figure 46

[46] (a) $\sin 22° = \frac{x}{20} \Rightarrow x = 20\sin 22° \approx 7.49$ ft.

(b) $\cos 22° = \frac{h}{20} \Rightarrow h \approx 18.54$ ft, the original height of the ladder.

Let $x = 20\sin 22° + 3$ and then $h = \sqrt{20^2 - x^2} \approx 17.03$ ft.

Hence, the top of the ladder moved from 18.54 to 17.03 or approximately 1.51 ft.

[47] When the angle of elevation is 19°20′, $\tan 19°20' = \frac{h_1}{110} \Rightarrow h_1 = 110\tan 19°20'$.

When the angle of elevation is 31°50′, $\tan 31°50' = \frac{h_2}{110} \Rightarrow h_2 = 110\tan 31°50'$.

$$h_2 - h_1 \approx 68.29 - 38.59 = 29.7 \text{ km}.$$

[48] $\tan 12°50' = \frac{8.2}{y} \Rightarrow y \approx 36.0$. $\tan 31°20' = \frac{x}{y} \Rightarrow x \approx 21.9$.

The height is $x + 8.2$, or 30.1 m.

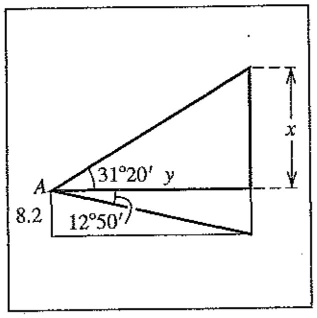

Figure 48

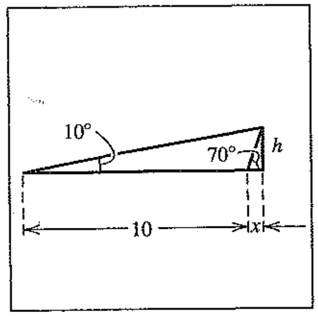

Figure 52

[49] $\sin 65.8° = \frac{r}{r + 380} \Rightarrow r \sin 65.8° + 380 \sin 65.8° = r \Rightarrow$

$r - r \sin 65.8° = 380 \sin 65.8° \Rightarrow r = \frac{380 \sin 65.8°}{1 - \sin 65.8°} \approx 3944$ mi.

[50] Let h denote the length of the antenna and θ the angle opposite the garage.

$\tan\theta = \frac{16}{100} \Rightarrow \theta \approx 9.09°$. $\alpha = 12° + 9.09° = 21.09°$. $\tan 21.09° = \frac{h+16}{100} \Rightarrow$

$h + 16 = 100 \tan 21.09° \Rightarrow h = 100 \tan 21.09° - 16 \approx 22.6$ ft.

[51] Let d be the distance traveled. $\tan 42° = \frac{10{,}000}{d} \Rightarrow d = 10{,}000 \cot 42°$.

Converting to mi/hr, we have $\frac{10{,}000 \cot 42° \text{ ft}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{1 \text{ mile}}{5280 \text{ ft}} \approx 126$ mi/hr.

[52] See *Figure 52*. The motorist travels a distance of 10 km. Now $\tan 10° = \frac{h}{x+10} \Rightarrow$

$x = h \cot 10° - 10$ and $\tan 70° = \frac{h}{x} \Rightarrow x = h \cot 70°$. $h \cot 10° - 10 = h \cot 70° \Rightarrow$

$h = \frac{10}{\cot 10° - \cot 70°} \approx 1.88$ km.

[53] (a) As in Exercise 49, there is a right angle formed on the earth's surface.

Bisecting angle θ and forming a right triangle, we have

$\cos\frac{\theta}{2} = \frac{R}{R+a} = \frac{4000}{26{,}300}$. Thus, $\frac{\theta}{2} \approx 81.25° \Rightarrow \theta \approx 162.5°$.

The percentage of the equator that is within signal range is $\frac{162.5°}{360°} \times 100 \approx 45\%$.

(b) Each satellite has a signal range of more than 120°,

and thus all 3 will cover all points on the equator.

[54] (a) Bisect θ and form a right triangle with

angle $\frac{\theta}{2}$, adjacent side $R - d$, and hypotenuse R.

$\cos\frac{\theta}{2} = \frac{R-d}{R} \Rightarrow R \cos\frac{\theta}{2} = R - d \Rightarrow d = R - R \cos\frac{\theta}{2} \Rightarrow d = R(1 - \cos\frac{\theta}{2})$.

(b) From Exercise 53, $\cos\frac{\theta}{2} = \frac{40}{263}$. The portion of the planet's surface that is within signal range is $\dfrac{\text{surface area of spherical cap}}{\text{surface area of earth}} = \dfrac{2\pi Rd}{4\pi R^2} = \dfrac{d}{2R} = \dfrac{R(1-\cos\frac{\theta}{2})}{2R} =$

$$\frac{1-\cos\frac{\theta}{2}}{2} = \frac{1-\frac{40}{263}}{2} = \frac{223}{526} \approx 0.424 \text{ or } 42.4\%.$$

$\boxed{55}$ Let $x = h - c$. $\sin\alpha = \frac{x}{d} \Rightarrow x = d\sin\alpha$. $h = x + c = d\sin\alpha + c$.

$\boxed{56}$ $\cot\alpha = \frac{x}{d} \Rightarrow x = d\cot\alpha$

$\boxed{57}$ Let x denote the distance from the base of the tower to the closer point.
$\cot\beta = \frac{x}{h} \Rightarrow x = h\cot\beta$. $\cot\alpha = \dfrac{x+d}{h} \Rightarrow x + d = h\cot\alpha \Rightarrow x = h\cot\alpha - d$.
Thus, $h\cot\beta = h\cot\alpha - d \Rightarrow d = h\cot\alpha - h\cot\beta \Rightarrow d = h(\cot\alpha - \cot\beta) \Rightarrow$
$$h = \frac{d}{\cot\alpha - \cot\beta}.$$

$\boxed{58}$ The central angle of a section of an n-sided polygon has an angle measure of $\frac{360°}{n}$.
Bisecting that angle forms a right triangle with a hypotenuse r and opposite side x.
$\sin\left(\frac{1}{2}\cdot\frac{360°}{n}\right) = \sin\left(\frac{180}{n}\right)^{\circ} = \frac{x}{r} \Rightarrow x = r\sin\left(\frac{180}{n}\right)^{\circ}$. There are n sides of length $2x$.
Hence, the perimeter P is $2nx = 2nr\sin\left(\frac{180}{n}\right)^{\circ}$.

$\boxed{59}$ When the angle of elevation is α, $\tan\alpha = \dfrac{h_1}{d} \Rightarrow h_1 = d\tan\alpha$.
When the angle of elevation is β, $\tan\beta = \dfrac{h_2}{d} \Rightarrow h_2 = d\tan\beta$.
$$h = h_2 - h_1 = d\tan\beta - d\tan\alpha = d(\tan\beta - \tan\alpha).$$

$\boxed{60}$ Let x and y denote the sides as labeled in *Figure 48*. We see that $\cot\beta = \frac{y}{d} \Rightarrow$
$y = d\cot\beta$. Also, $\tan\alpha = \frac{x}{y} \Rightarrow x = y\tan\alpha = d\cot\beta\tan\alpha$.
Hence, $h = d + x = d + d\cot\beta\tan\alpha = d(1 + \cot\beta\tan\alpha)$.

$\boxed{61}$ The bearing from P to A is $90° - 20° = 70°$ east of north and is denoted by N70°E.
The bearings for B, C, and D are N40°W, S15°W, and S25°E, respectively.

$\boxed{62}$ The bearings for A, B, C, and D are N15°E, N30°W, S80°W, and S55°E, respectively.

$\boxed{63}$ (a) The ships form a right triangle with legs of 48 miles and 27 miles. The distance between the two ships is $\sqrt{27^2 + 48^2} \approx 55$ miles.

(b) The angle between the side of length 48 and the hypotenuse is found by solving $\tan\alpha = \frac{27}{48}$ for α. Now $\alpha \approx 29°$, so the second ship is approximately $29° + 34° = $ S63°E of the first ship.

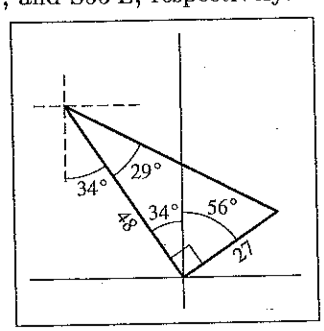

Figure 63

[64] Let F denote the point of the fire.

$\angle BAF = 90^\circ - 35^\circ 50' = 54^\circ 10'$ and $\angle ABF = 90^\circ - 54^\circ 10' = 35^\circ 50'$. Thus,

$\angle BFA = 90^\circ$. If d denotes the distance AF, then $\cos 54^\circ 10' = \frac{d}{5} \Rightarrow d \approx 2.9$ mi.

[65] The plane's flight forms a right triangle with legs 180 miles and 270 miles.

The distance from A to the airplane is then $\sqrt{180^2 + 270^2} \approx 324.5$ mi.

[66] (a) The plane's flight forms two legs (400 mi each) of a right triangle. The hypotenuse of this 45°–45°–90° triangle is $400\sqrt{2}$. The plane is in the direction $153^\circ - 45^\circ = 108^\circ$ from point A (18° below the horizontal). It must take the direction $270^\circ + 18^\circ = 288^\circ$ to get back to A.

(b) At 400 mi/hr, it will take the plane $\sqrt{2}$ hr to travel $400\sqrt{2}$ mi.

[67] Amplitude, 10 cm; period $= \frac{2\pi}{6\pi} = \frac{1}{3}$ sec; frequency $= \frac{6\pi}{2\pi} = 3$ oscillations/sec. The point is at the origin at $t = 0$. It moves upward with decreasing speed, reaching the point with coordinate 10 when $6\pi t = \frac{\pi}{2}$ or $t = \frac{1}{12}$. It then reverses direction and moves downward, gaining speed until it reaches the origin when $6\pi t = \pi$ or $t = \frac{1}{6}$. It continues downward with decreasing speed, reaching the point with coordinate -10 when $6\pi t = \frac{3\pi}{2}$ or $t = \frac{1}{4}$. It then reverses direction and moves upward with increasing speed, returning to the origin when $6\pi t = 2\pi$ or $t = \frac{1}{3}$ to complete one oscillation. Another approach is to simply model this movement in terms of proportions of the sine curve. For one period, the sine increases for $\frac{1}{4}$ period, decreases for $\frac{1}{2}$ period, and increases for its last $\frac{1}{4}$ period.

[68] Amplitude, $\frac{1}{3}$cm; period $= \frac{2\pi}{\pi/4} = 8$ sec; frequency $= \frac{\pi/4}{2\pi} = \frac{1}{8}$oscillation/sec. The point is at $d = \frac{1}{3}$cm when $t = 0$. It then decreases in height until $\frac{\pi}{4}t = \pi$ or $t = 4$ where it obtains a minimum of $d = -\frac{1}{3}$. It then reverses direction and increases to a height of $d = \frac{1}{3}$ when $\frac{\pi}{4}t = 2\pi$ or $t = 8$ to complete one oscillation.

[69] Amplitude, 4 cm; period $= \frac{2\pi}{3\pi/2} = \frac{4}{3}$ sec; frequency $= \frac{3\pi/2}{2\pi} = \frac{3}{4}$oscillation/sec. The point is at $d = 4$ when $t = 0$. It then decreases in height until $\frac{3\pi}{2}t = \pi$ or $t = \frac{2}{3}$ where it obtains a minimum of $d = -4$. It then reverses direction and increases to a height of $d = 4$ when $\frac{3\pi}{2}t = 2\pi$ or $t = \frac{4}{3}$ to complete one oscillation.

[70] Amplitude, 6 cm; period $= \frac{2\pi}{2\pi/3} = 3$ sec; frequency $= \frac{2\pi/3}{2\pi} = \frac{1}{3}$oscillation/sec. The point is at the origin at $t = 0$. It will obtain a maximum height of 6 when $\frac{2\pi}{3}t = \frac{\pi}{2}$ or $t = \frac{3}{4}$. It then decreases in height until it obtains a minimum of -6 when $\frac{2\pi}{3}t = \frac{3\pi}{2}$ or $t = \frac{9}{4}$. It then reverses direction and returns to the origin when $\frac{2\pi}{3}t = 2\pi$ or $t = 3$ to complete one oscillation.

[71] Period $= 3 \Rightarrow \frac{2\pi}{\omega} = 3 \Rightarrow \omega = \frac{2\pi}{3}$. Amplitude $= 5 \Rightarrow a = 5$. $d = 5\cos\frac{2\pi}{3}t$

[72] $\frac{1}{2}$ oscillation per minute $\Rightarrow \frac{\omega}{2\pi} = \frac{1}{2} \Rightarrow \omega = \pi$. Amplitude $= 4 \Rightarrow a = 4$.

$d = 4\sin \pi t$

[73] (a) period $= 30 \Rightarrow \frac{2\pi}{\omega} = 30 \Rightarrow \omega = \frac{\pi}{15}$. When $t = 0$, the wave is at its highest point, thus, we use the cosine function. $y = 25\cos\frac{\pi}{15}t$, where t is in minutes.

(b) 180 ft/sec = 10,800 ft/min.

10,800 ft/min for 30 minutes is a distance of 324,000 ft, or $61\frac{4}{11}$ miles.

[74] (a) $y = a\sin bt = 8\sin\frac{\pi}{6}t \Rightarrow a = 8$ and $b = \frac{\pi}{6}$.

Thus, the wave has an amplitude of 8 ft and its period is $\frac{2\pi}{b} = \frac{2\pi}{\pi/6} = 12$ min.

(b) The wave moves 21 km every 12 min.

The wave's velocity is $\frac{21}{12} = 1.75$ km/min, or, 105 km/hr.

Chapter 6 Review Exercises

[1] $330^\circ \cdot \frac{\pi}{180} = \frac{11 \cdot 30\pi}{6 \cdot 30} = \frac{11\pi}{6}$; $405^\circ \cdot \frac{\pi}{180} = \frac{9 \cdot 45\pi}{4 \cdot 45} = \frac{9\pi}{4}$;

$-150^\circ \cdot \frac{\pi}{180} = -\frac{5 \cdot 30\pi}{6 \cdot 30} = -\frac{5\pi}{6}$; $240^\circ \cdot \frac{\pi}{180} = \frac{4 \cdot 60\pi}{3 \cdot 60} = \frac{4\pi}{3}$;

$36^\circ \cdot \frac{\pi}{180} = \frac{36\pi}{5 \cdot 36} = \frac{\pi}{5}$

[2] $\frac{9\pi}{2} \cdot \left(\frac{180}{\pi}\right)^\circ = \left(\frac{9 \cdot 90 \cdot 2\pi}{2\pi}\right)^\circ = 810^\circ$; $-\frac{2\pi}{3} \cdot \left(\frac{180}{\pi}\right)^\circ = -\left(\frac{2 \cdot 60 \cdot 3\pi}{3\pi}\right)^\circ = -120^\circ$;

$\frac{7\pi}{4} \cdot \left(\frac{180}{\pi}\right)^\circ = \left(\frac{7 \cdot 45 \cdot 4\pi}{4\pi}\right)^\circ = 315^\circ$; $5\pi \cdot \left(\frac{180}{\pi}\right)^\circ = \left(\frac{5 \cdot 180 \cdot \pi}{\pi}\right)^\circ = 900^\circ$;

$\frac{\pi}{5} \cdot \left(\frac{180}{\pi}\right)^\circ = \left(\frac{36 \cdot 5\pi}{5\pi}\right)^\circ = 36^\circ$

[3] (a) $\theta = \frac{s}{r} = \frac{20\text{ cm}}{2\text{ m}} = \frac{20\text{ cm}}{2\,(100)\text{ cm}} = 0.1$ radian

(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(2)^2(0.1) = 0.2\text{ m}^2$

[4] (a) $s = r\theta = (15 \cdot \frac{1}{2})(70 \cdot \frac{\pi}{180}) = \frac{35\pi}{12} \approx 9.16$ cm

(b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}(15 \cdot \frac{1}{2})^2(70 \cdot \frac{\pi}{180}) = \frac{175\pi}{16} \approx 34.4\text{ cm}^2$

[5] $\left(33\frac{1}{3}\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = \frac{200\pi}{3}\,\frac{\text{rad}}{\text{min}}$ and $\left(45\,\frac{\text{rev}}{\text{min}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = 90\pi\,\frac{\text{rad}}{\text{min}}$.

[6] $s = r\theta = (\frac{1}{2} \cdot 12)(\frac{200\pi}{3}) = 400\pi$ in. Linear speed $= 400\pi$ in/min $= \frac{100\pi}{3}$ ft/min.

$s = r\theta = (\frac{1}{2} \cdot 7)(90\pi) = 315\pi$ in. Linear speed $= 315\pi$ in/min $= \frac{105\pi}{4}$ ft/min.

[7] $\sin 60^\circ = \frac{9}{x} \Rightarrow \frac{\sqrt{3}}{2} = \frac{9}{x} \Rightarrow x = 6\sqrt{3}$; $\tan 60^\circ = \frac{9}{y} \Rightarrow \sqrt{3} = \frac{9}{y} \Rightarrow y = 3\sqrt{3}$

[8] $\sin 45^\circ = \frac{x}{7} \Rightarrow \frac{\sqrt{2}}{2} = \frac{x}{7} \Rightarrow x = \frac{7}{2}\sqrt{2}$; $\cos 45^\circ = \frac{y}{7} \Rightarrow \frac{\sqrt{2}}{2} = \frac{y}{7} \Rightarrow y = \frac{7}{2}\sqrt{2}$

[9] $1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1 \Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

[10] $1 + \cot^2\theta = \csc^2\theta \Rightarrow \cot^2\theta = \csc^2\theta - 1 \Rightarrow \cot\theta = \sqrt{\csc^2\theta - 1}$

[11] $\sin\theta\,(\csc\theta - \sin\theta) = \sin\theta\,\csc\theta - \sin^2\theta = 1 - \sin^2\theta = \cos^2\theta$

[12] $\cos\theta(\tan\theta+\cot\theta)=$

$$\cos\theta\cdot\frac{\sin\theta}{\cos\theta}+\cos\theta\cdot\frac{\cos\theta}{\sin\theta}=\sin\theta+\frac{\cos^2\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}=\frac{1}{\sin\theta}=\csc\theta$$

[13] $(\cos^2\theta-1)(\tan^2\theta+1)=(\cos^2\theta-1)(\sec^2\theta)=\cos^2\theta\,\sec^2\theta-\sec^2\theta=1-\sec^2\theta$

[14] $$\frac{\sec\theta-\cos\theta}{\tan\theta}=\frac{\dfrac{1}{\cos\theta}-\cos\theta}{\dfrac{\sin\theta}{\cos\theta}}=\frac{\dfrac{1-\cos^2\theta}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}=\frac{\dfrac{\sin^2\theta}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}}=\frac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}}=\frac{\tan\theta}{\sec\theta}$$

[15] $$\frac{1+\tan^2\theta}{\tan^2\theta}=\frac{1}{\tan^2\theta}+\frac{\tan^2\theta}{\tan^2\theta}=\cot^2\theta+1=\csc^2\theta$$

[16] $$\frac{\sec\theta+\csc\theta}{\sec\theta-\csc\theta}=\frac{\dfrac{1}{\cos\theta}+\dfrac{1}{\sin\theta}}{\dfrac{1}{\cos\theta}-\dfrac{1}{\sin\theta}}=\frac{\dfrac{\sin\theta+\cos\theta}{\cos\theta\sin\theta}}{\dfrac{\sin\theta-\cos\theta}{\cos\theta\sin\theta}}=\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}$$

[17] $$\frac{\cot\theta-1}{1-\tan\theta}=\frac{\dfrac{\cos\theta}{\sin\theta}-1}{1-\dfrac{\sin\theta}{\cos\theta}}=\frac{\dfrac{\cos\theta-\sin\theta}{\sin\theta}}{\dfrac{\cos\theta-\sin\theta}{\cos\theta}}=\frac{(\cos\theta-\sin\theta)\cos\theta}{(\cos\theta-\sin\theta)\sin\theta}=\frac{\cos\theta}{\sin\theta}=\cot\theta$$

[18] $$\frac{1+\sec\theta}{\tan\theta+\sin\theta}=\frac{1+\dfrac{1}{\cos\theta}}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\sin\theta\cos\theta}{\cos\theta}}=\frac{\dfrac{\cos\theta+1}{\cos\theta}}{\dfrac{\sin\theta(1+\cos\theta)}{\cos\theta}}=\frac{1}{\sin\theta}=\csc\theta$$

[19] $$\frac{\tan(-\theta)+\cot(-\theta)}{\tan\theta}=\frac{-\tan\theta-\cot\theta}{\tan\theta}=-\frac{\tan\theta}{\tan\theta}-\frac{\cot\theta}{\tan\theta}=-1-\cot^2\theta=-(1+\cot^2\theta)=-\csc^2\theta$$

[20] $$-\frac{1}{\csc(-\theta)}-\frac{\cot(-\theta)}{\sec(-\theta)}=-\frac{1}{-\csc\theta}-\frac{-\cot\theta}{\sec\theta}=\sin\theta+\frac{\cos\theta/\sin\theta}{1/\cos\theta}=\sin\theta+\frac{\cos^2\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta}=\frac{1}{\sin\theta}=\csc\theta$$

[21] opp $=\sqrt{\text{hyp}^2-\text{adj}^2}=\sqrt{7^2-4^2}=\sqrt{33}.$ ★ $\frac{\sqrt{33}}{7}, \frac{4}{7}, \frac{\sqrt{33}}{4}, \frac{4}{\sqrt{33}}, \frac{7}{4}, \frac{7}{\sqrt{33}}$

[22] (a) $x=30$ and $y=-40 \Rightarrow r=\sqrt{30^2+(-40)^2}=50.$ ★ (a) $-\frac{4}{5}, \frac{3}{5}, -\frac{4}{3}, -\frac{3}{4}, \frac{5}{3}, -\frac{5}{4}$

(b) $2x+3y+6=0 \Leftrightarrow y=-\frac{2}{3}x-2$, so the slope of the given line is $-\frac{2}{3}$.

The line through the origin with that slope is $y=-\frac{2}{3}x$.

If $x=-3$, then $y=2$ and $(-3, 2)$ is a point on the terminal side of θ.

$x=-3$ and $y=2 \Rightarrow r=\sqrt{(-3)^2+2^2}=\sqrt{13}.$

★ (b) $\frac{2}{\sqrt{13}}, -\frac{3}{\sqrt{13}}, -\frac{2}{3}, -\frac{3}{2}, -\frac{\sqrt{13}}{3}, \frac{\sqrt{13}}{2}$

(c) For $\theta=-90^\circ$, choose $x=0$ and $y=-1$. r is 1. ★ (c) -1, 0, U, 0, U, -1

[23] (a) $\sec\theta < 0 \Rightarrow P$ is in QII or QIII. $\sin\theta > 0 \Rightarrow P$ is in QI or QII.

$\therefore P$ is in QII.

(b) $\cot\theta > 0 \Rightarrow P$ is in QI or QIII. $\csc\theta < 0 \Rightarrow P$ is in QIII or QIV.

$\therefore P$ is in QIII.

(c) $\cos\theta > 0 \Rightarrow P$ is in QI or QIV. $\tan\theta < 0 \Rightarrow P$ is in QII or QIV.

$\therefore P$ is in QIV.

[24] (a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = -\frac{4}{3}$; the other values are just the reciprocals

(b) $\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{\csc\theta}{\sec\theta} \Rightarrow -\dfrac{3}{2} = \dfrac{\sqrt{13}/2}{\sec\theta} \Rightarrow \sec\theta = -\dfrac{\sqrt{13}}{3}$

[25] $P(7\pi) = P(\pi) = (-1, 0)$. $P(-\frac{5\pi}{2}) = P(-\frac{\pi}{2}) = (0, -1)$. $P(\frac{9\pi}{2}) = P(\frac{\pi}{2}) = (0, 1)$.

$P(-\frac{3\pi}{4}) = (-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$. $P(18\pi) = P(0) = (1, 0)$. $P(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$.

[26] $P(t + 3\pi) = P(t + \pi) = P(t - \pi) = (\frac{3}{5}, \frac{4}{5})$.

$P(-t) = (-\frac{3}{5}, \frac{4}{5})$. $P(2\pi - t) = P(-t + 2\pi) = (-\frac{3}{5}, \frac{4}{5})$.

[27] (a) $\theta = \frac{5\pi}{4} \Rightarrow \theta_R = \frac{5\pi}{4} - \pi = \frac{\pi}{4}$. $\theta = -\frac{5\pi}{6} \Rightarrow \theta_C = \frac{7\pi}{6}$ and $\theta_R = \frac{7\pi}{6} - \frac{\pi}{6} = \frac{\pi}{6}$.

$\theta = -\frac{9\pi}{8} \Rightarrow \theta_C = \frac{7\pi}{8}$ and $\theta_R = \pi - \frac{7\pi}{8} = \frac{\pi}{8}$.

(b) $\theta = 245^\circ \Rightarrow \theta_R = 245^\circ - 180^\circ = 65^\circ$. $\theta = 137^\circ \Rightarrow \theta_R = 180^\circ - 137^\circ = 43^\circ$.

$\theta = 892^\circ \Rightarrow \theta_C = 172^\circ$ and $\theta_R = 180^\circ - 172^\circ = 8^\circ$.

[28] (a) For $\theta = \frac{9\pi}{2}$, choose $x = 0$ and $y = 1$. $r = 1$. ★ (a) 1, 0, U, 0, U, 1

(b) For $\theta = -\frac{5\pi}{4}$, choose $x = -1$ and $y = 1$. $r = \sqrt{2}$.

★ (b) $\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}, -1, -1, -\sqrt{2}, \sqrt{2}$

(c) For $\theta = 0$, choose $x = 1$ and $y = 0$. $r = 1$. ★ (c) 0, 1, 0, U, 1, U

(d) For $\theta = \frac{11\pi}{6}$, choose $x = \sqrt{3}$ and $y = -1$. $r = 2$.

★ (d) $-\frac{1}{2}, \dfrac{\sqrt{3}}{2}, -\dfrac{\sqrt{3}}{3}, -\sqrt{3}, \dfrac{2}{\sqrt{3}}, -2$

[29] (a) $\cos 225^\circ = -\cos 45^\circ = -\dfrac{\sqrt{2}}{2}$ (b) $\tan 150^\circ = -\tan 30^\circ = -\dfrac{\sqrt{3}}{3}$

(c) $\sin(-\frac{\pi}{6}) = -\sin\frac{\pi}{6} = -\frac{1}{2}$ (d) $\sec\frac{4\pi}{3} = -\sec\frac{\pi}{3} = -2$

(e) $\cot\frac{7\pi}{4} = -\cot\frac{\pi}{4} = -1$ (f) $\csc 300^\circ = -\csc 60^\circ = -\dfrac{2}{\sqrt{3}}$

[30] $\sin\theta = -0.7604 \Rightarrow \theta = \sin^{-1}(-0.7604) \approx -49.5^\circ \Rightarrow \theta_R \approx 49.5^\circ$.

Since the sine is negative in QIII and QIV, and the secant is positive in QIV,

we want the fourth-quadrant angle having $\theta_R = 49.5^\circ$. $360^\circ - 49.5^\circ = 310.5^\circ$

[31] $\tan\theta = 2.7381 \Rightarrow \theta = \tan^{-1}(2.7381) \approx 1.2206 \Rightarrow \theta_R \approx 1.2206$. Since the tangent is positive in QI and QIII, we need to add π to θ_R to find the value in QIII. $\pi + \theta_R \approx 4.3622$

[32] $\sec\theta = 1.6403 \Rightarrow \cos\theta = \frac{1}{1.6403} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{1.6403}\right) \approx 52.44^\circ \Rightarrow \theta_R \approx 52.44^\circ$.

Since the secant is positive in QI and QIV, we need to subtract θ_R from 360° to find the value in QIV. $360^\circ - \theta_R \approx 307.56^\circ$

$\boxed{33}$ $y = 5\cos x$ • vertically stretch $\cos x$ by a factor of 5 ★ 5, 2π, x-int. @ $\frac{\pi}{2} + \pi n$

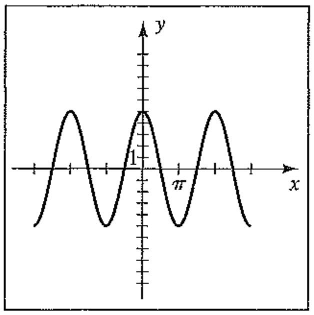

Figure 33

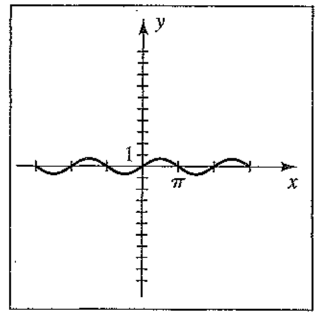

Figure 34

$\boxed{34}$ $y = \frac{2}{3}\sin x$ • vertically compress $\sin x$ by a factor of $\frac{3}{2}$ {or multiply by $\frac{2}{3}$}

★ $\frac{2}{3}$, 2π, x-int. @ πn

$\boxed{35}$ $y = \frac{1}{3}\sin 3x$ • horizontally compress $\sin x$ by a factor of 3 and vertically compress that graph by a factor of 3 ★ $\frac{1}{3}$, $\frac{2\pi}{3}$, x-int. @ $\frac{\pi}{3}n$

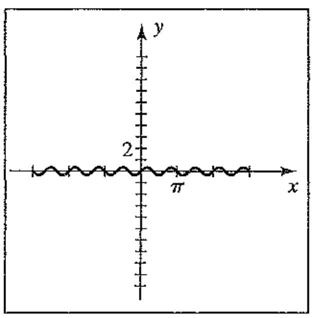

Figure 35

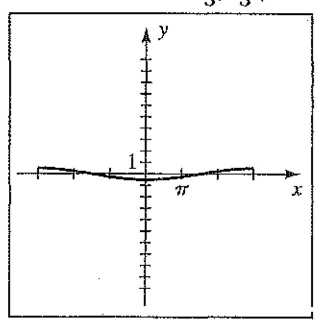

Figure 36

$\boxed{36}$ $y = -\frac{1}{2}\cos\frac{1}{3}x$ • horizontally stretch $\cos x$ by a factor of 3, vertically compress by a factor of 2, and reflect that graph through the x-axis ★ $\frac{1}{2}$, 6π, x-int. @ $\frac{3\pi}{2} + 3\pi n$

$\boxed{37}$ $y = -3\cos\frac{1}{2}x$ • horizontally stretch $\cos x$ by a factor of 2, vertically stretch that graph by a factor of 3, and reflect that graph through the x-axis

★ 3, 4π, x-int. @ $\pi + 2\pi n$

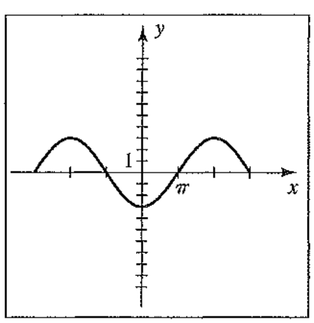

Figure 37

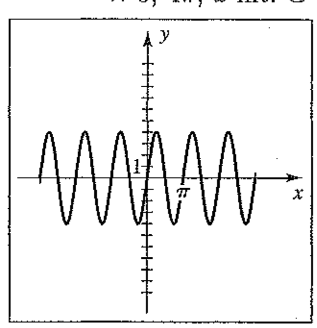

Figure 38

$\boxed{38}$ $y = 4\sin 2x$ • horizontally compress $\sin x$ by a factor of 2 and vertically stretch that graph by a factor of 4 ★ 4, π, x-int. @ $\frac{\pi}{2}n$

[39] $y = 2\sin \pi x$ • horizontally compress $\sin x$ by a factor of π, and vertically stretch that graph by a factor of 2

★ 2, 2, x-int. @ n

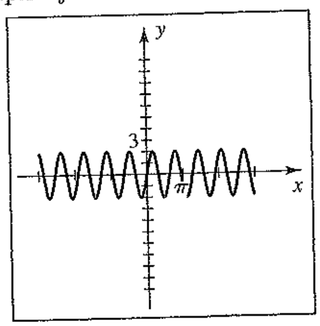

Figure 39

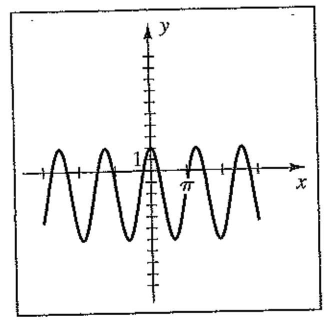

Figure 40

[40] $y = 4\cos\frac{\pi}{2}x - 2$ • horizontally compress $\cos x$ by a factor of $\pi/2$, vertically stretch that graph by a factor of 4, and shift the last graph 2 units down

★ 4, 4, x-int. @ $\frac{2}{3} + 4n$, $\frac{10}{3} + 4n$

Note: Let a denote the amplitude and p the period.

[41] (a) $a = |-1.43| = 1.43$, $\frac{3}{4}p = 1.5 \Rightarrow p = 2$

(b) $b = \frac{2\pi}{p} = \frac{2\pi}{2} = \pi$, $y = 1.43\sin \pi x$

[42] (a) $a = |-3.27| = 3.27$, $\frac{1}{4}p = \frac{3\pi}{4} \Rightarrow p = 3\pi$

(b) $b = \frac{2\pi}{p} = \frac{2\pi}{3\pi} = \frac{2}{3}$, $y = -3.27\sin\frac{2}{3}x$

[43] (a) Since the y-intercept is -3, $a = |-3| = 3$.

The second positive x-intercept is π, so $\frac{3}{4}p = \pi \Rightarrow p = \frac{4\pi}{3}$.

(b) $b = \frac{2\pi}{p} = \frac{2\pi}{4\pi/3} = \frac{3}{2}$, $y = -3\cos\frac{3}{2}x$

[44] (a) Since the y-intercept is 2, $a = |2| = 2$.

The first positive x-intercept is 1, so $\frac{1}{4}p = 1 \Rightarrow p = 4$.

(b) $b = \frac{2\pi}{p} = \frac{2\pi}{4} = \frac{\pi}{2}$, $y = 2\cos\frac{\pi}{2}x$.

[45] $y = 2\sin\left(x - \frac{2\pi}{3}\right)$ • $0 \leq x - \frac{2\pi}{3} \leq 2\pi \Rightarrow \frac{2\pi}{3} \leq x \leq \frac{8\pi}{3}$

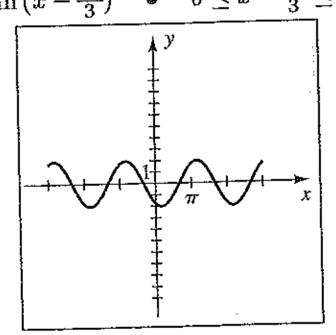

Figure 45

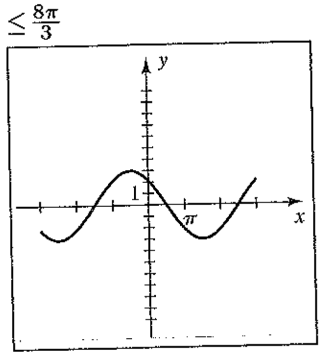

Figure 46

[46] $y = -3\sin\left(\frac{1}{2}x - \frac{\pi}{4}\right) = -3\sin\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$

$0 \leq \frac{1}{2}x - \frac{\pi}{4} \leq 2\pi \Rightarrow \frac{\pi}{4} \leq \frac{1}{2}x \leq \frac{9\pi}{4} \Rightarrow \frac{\pi}{2} \leq x \leq \frac{9\pi}{2}$

[47] $y = -4\cos\left(x + \frac{\pi}{6}\right)$ • $-\frac{\pi}{2} \le x + \frac{\pi}{6} \le \frac{3\pi}{2} \Rightarrow -\frac{2\pi}{3} \le x \le \frac{4\pi}{3}$

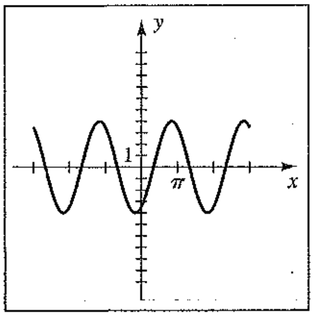

Figure 47

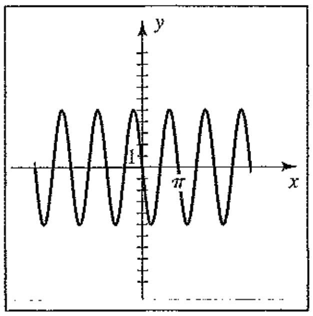

Figure 48

[48] $y = 5\cos\left(2x + \frac{\pi}{2}\right) = 5\cos\left[2\left(x + \frac{\pi}{4}\right)\right]$.

$-\frac{\pi}{2} \le 2x + \frac{\pi}{2} \le \frac{3\pi}{2} \Rightarrow -\pi \le 2x \le \pi \Rightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

[49] $y = 2\tan\left(\frac{1}{2}x - \pi\right) = 2\tan\left[\frac{1}{2}(x - 2\pi)\right]$.

$-\frac{\pi}{2} \le \frac{1}{2}x - \pi \le \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \le \frac{1}{2}x \le \frac{3\pi}{2} \Rightarrow \pi \le x \le 3\pi$, *VA* @ $x = \pi + 2\pi n$

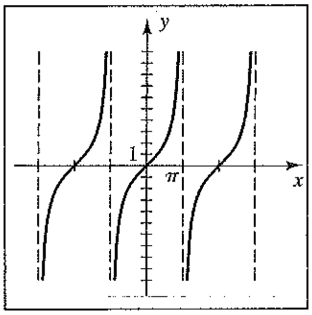

Figure 49

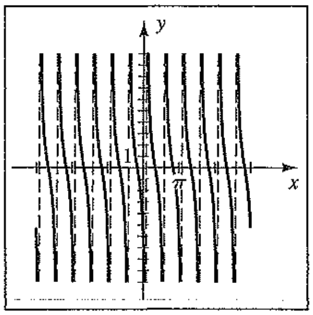

Figure 50

[50] $y = -3\tan\left(2x + \frac{\pi}{3}\right) = -3\tan\left[2\left(x + \frac{\pi}{6}\right)\right]$.

$-\frac{\pi}{2} \le 2x + \frac{\pi}{3} \le \frac{\pi}{2} \Rightarrow -\frac{5\pi}{6} \le 2x \le \frac{\pi}{6} \Rightarrow -\frac{5\pi}{12} \le x \le \frac{\pi}{12}$, *VA* @ $x = -\frac{5\pi}{12} + \frac{\pi}{2}n$

[51] $y = -4\cot\left(2x - \frac{\pi}{2}\right) = -4\cot\left[2\left(x - \frac{\pi}{4}\right)\right]$.

$0 \le 2x - \frac{\pi}{2} \le \pi \Rightarrow \frac{\pi}{2} \le 2x \le \frac{3\pi}{2} \Rightarrow \frac{\pi}{4} \le x \le \frac{3\pi}{4}$, *VA* @ $x = \frac{\pi}{4} + \frac{\pi}{2}n$

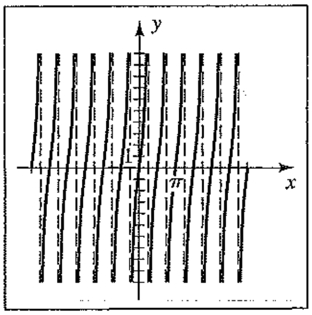

Figure 51

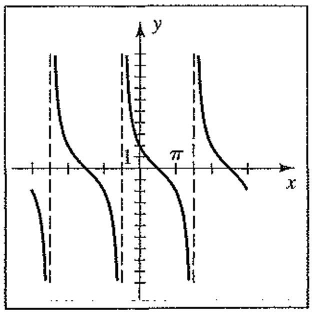

Figure 52

[52] $y = 2\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right) = 2\cot\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{4} \le \pi \Rightarrow -\frac{\pi}{4} \le \frac{1}{2}x \le \frac{3\pi}{4} \Rightarrow -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, *VA* @ $x = -\frac{\pi}{2} + 2\pi n$

[53] $y = \sec\left(\frac{1}{2}x + \pi\right) = \sec\left[\frac{1}{2}(x + 2\pi)\right]$.

$-\frac{\pi}{2} \le \frac{1}{2}x + \pi \le \frac{\pi}{2} \;\Rightarrow\; -\frac{3\pi}{2} \le \frac{1}{2}x \le -\frac{\pi}{2} \;\Rightarrow\; -3\pi \le x \le -\pi$, VA @ $x = -3\pi + 2\pi n$

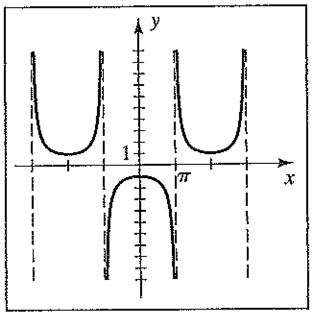

Figure 53

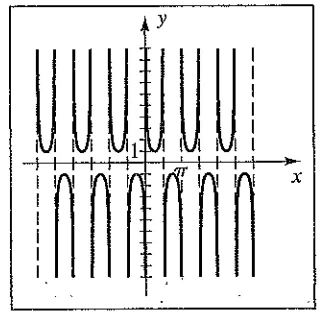

Figure 54

[54] $y = \sec\left(2x - \frac{\pi}{2}\right) = \sec\left[2\left(x - \frac{\pi}{4}\right)\right]$.

$-\frac{\pi}{2} \le 2x - \frac{\pi}{2} \le \frac{\pi}{2} \;\Rightarrow\; 0 \le 2x \le \pi \;\Rightarrow\; 0 \le x \le \frac{\pi}{2}$, VA @ $x = \frac{\pi}{2}n$

[55] $y = \csc\left(2x - \frac{\pi}{4}\right) = \csc\left[2\left(x - \frac{\pi}{8}\right)\right]$.

$0 \le 2x - \frac{\pi}{4} \le \pi \;\Rightarrow\; \frac{\pi}{4} \le 2x \le \frac{5\pi}{4} \;\Rightarrow\; \frac{\pi}{8} \le x \le \frac{5\pi}{8}$, VA @ $x = \frac{\pi}{8} + \frac{\pi}{2}n$

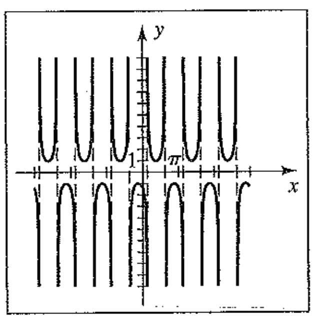

Figure 55

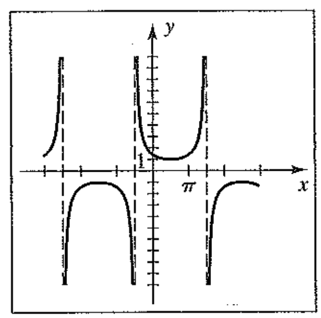

Figure 56

[56] $y = \csc\left(\frac{1}{2}x + \frac{\pi}{4}\right) = \csc\left[\frac{1}{2}\left(x + \frac{\pi}{2}\right)\right]$.

$0 \le \frac{1}{2}x + \frac{\pi}{4} \le \pi \;\Rightarrow\; -\frac{\pi}{4} \le \frac{1}{2}x \le \frac{3\pi}{4} \;\Rightarrow\; -\frac{\pi}{2} \le x \le \frac{3\pi}{2}$, VA @ $x = -\frac{\pi}{2} + 2\pi n$

[57] $\alpha = 90^\circ - \beta = 30^\circ$. $\cot\beta = \frac{a}{b} \;\Rightarrow\; a = b\cot\beta = 40\left(\frac{1}{3}\sqrt{3}\right) \approx 23$.

$\csc\beta = \frac{c}{b} \;\Rightarrow\; c = b\csc\beta = 40\left(\frac{2}{3}\sqrt{3}\right) \approx 46$.

[58] $\beta = 90^\circ - \alpha = 35^\circ 20'$. $\tan\alpha = \frac{a}{b} \;\Rightarrow\; a = b\tan\alpha = 220\tan 54^\circ 40' \approx 310$.

$\sec\alpha = \frac{c}{b} \;\Rightarrow\; c = b\sec\alpha = 220\sec 54^\circ 40' \approx 380$.

[59] $\tan\alpha = \frac{a}{b} = \frac{62}{25} \;\Rightarrow\; \alpha \approx 68^\circ$. $\beta = 90^\circ - \alpha \approx 22^\circ$.

$c = \sqrt{a^2 + b^2} = \sqrt{3844 + 625} = \sqrt{4469} \approx 67$.

[60] $\sin\alpha = \frac{a}{c} = \frac{9.0}{41} \;\Rightarrow\; \alpha \approx 13^\circ$. $\beta = 90^\circ - \alpha \approx 77^\circ$.

$b = \sqrt{c^2 - a^2} = \sqrt{1681 - 81} = \sqrt{1600} = 40$.

$\boxed{61}$ (a) $\left(\frac{545 \text{ rev}}{1 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ sec}}\right) = \frac{109\pi}{6}$ rad/sec ≈ 57 rad/sec

(b) $d = 22.625$ ft $\Rightarrow$ $C = \pi d = 22.625\pi$ ft.

$$\left(\frac{22.625\pi \text{ ft}}{1 \text{ rev}}\right)\left(\frac{545 \text{ rev}}{1 \text{ min}}\right)\left(\frac{1 \text{ mile}}{5280 \text{ ft}}\right)\left(\frac{60 \text{ min}}{1 \text{ hour}}\right) \approx 440.2 \text{ mi/hr}$$

$\boxed{62}$ Let h denote the height of the tower. $\tan 79.2^\circ = \frac{h}{200} \Rightarrow h \approx 1048$ ft.

$\boxed{63}$ $\Delta f = \frac{2fv}{c} \Rightarrow v = \frac{c(\Delta f)}{2f} = \frac{186{,}000 \times 10^8}{2 \times 10^{14}} = 0.093$ mi/sec

$\boxed{64}$ The angle φ has an adjacent side of $\frac{1}{2}(230$ m$)$, or 115 m. $\tan\varphi = \frac{147}{115} \Rightarrow \varphi \approx 52^\circ$.

$\boxed{65}$ Let d denote the distance from Venus to the sun.

$$\sin 47^\circ = \frac{d}{92{,}900{,}000} \Rightarrow d \approx 67{,}942{,}759 \text{ mi., or approximately } 67{,}900{,}000 \text{ mi.}$$

$\boxed{66}$ The depth of the cone is 4 inches and its slant height is 5 inches.

Thus, $4^2 + r^2 = 5^2 \Rightarrow r = 3$ inches. The circumference of the rim of the cone is $2\pi r = 6\pi$. On the circle, $\theta = \frac{s}{r} = \frac{6\pi}{5}$ radians $= 216^\circ$.

$\boxed{67}$ Let A denote the point at the 36° angle. Let Q denote the highest point of the mountain and P denote Q's projection such that $\angle APQ = 90^\circ$. Let x denote the distance from the left end of the mountain to P and let y denote the distance from the right end of the mountain to P. $\cot 36^\circ = \frac{x+200}{260} \Rightarrow x = 260\cot 36^\circ - 200$.

$$\cot 47^\circ = \frac{y+150}{260} \Rightarrow y = 260\cot 47^\circ - 150. \quad x + y \approx 250 \text{ ft.}$$

$\boxed{68}$ (a) Let h denote the height of the building and x the distance between the two buildings. $\tan 59^\circ = \frac{h-50}{x}$ and $\tan 62^\circ = \frac{h}{x} \Rightarrow h = x\tan 59^\circ + 50$ and

$$h = x\tan 62^\circ \Rightarrow x\tan 62^\circ - x\tan 59^\circ = 50 \Rightarrow x = \frac{50}{\tan 62^\circ - \tan 59^\circ} \approx 231.0 \text{ ft.}$$

(b) From part (a), $h = x\tan 62^\circ \approx 434.5$ ft.

$\boxed{69}$ (a) Let $x = \overline{QR}$. $\cot\beta = \frac{x}{h} \Rightarrow x = h\cot\beta$. $\cot\alpha = \frac{d+x}{h} \Rightarrow x = h\cot\alpha - d$.

Thus, $h\cot\beta = h\cot\alpha - d \Rightarrow d = h(\cot\alpha - \cot\beta) \Rightarrow h = \frac{d}{\cot\alpha - \cot\beta}$.

(b) $d = 2$ miles, $\alpha = 15^\circ$, and $\beta = 20^\circ \Rightarrow h = \frac{2}{\cot 15^\circ - \cot 20^\circ} \approx 2.03$, or 2 miles.

$\boxed{70}$ (a) Extend the two boundary lines for h {call these l_{top} and l_{bottom}} to the right until they intersect a line l extended down from the front edge of the building. Let x denote the distance from the intersection of the incline and l_{bottom} to l and y the distance on l from l_{top} to the lower left corner of the building.

$\cos\alpha = \frac{x}{d} \Rightarrow x = d\cos\alpha$. $\sin\alpha = \frac{h+y}{d} \Rightarrow y = d\sin\alpha - h$.

$\tan\theta = \frac{y+T}{x} \Rightarrow T = x\tan\theta - y = d\cos\alpha\tan\theta - d\sin\alpha + h \Rightarrow$

$$T = h + d(\cos\alpha\tan\theta - \sin\alpha).$$

(b) $T = 6 + 50(\cos 15^\circ \tan 31.4^\circ - \sin 15^\circ) \approx 6 + 50(0.3308) \approx 22.54$ ft.

$\boxed{71}$ (a) $\cos\theta = \frac{15}{s} \Rightarrow s = \frac{15}{\cos\theta}$. $E = \frac{5000\cos\theta}{s^2} = \frac{5000\cos\theta}{225/\cos^2\theta} = \frac{200}{9}\cos^3\theta$.

$$\theta = 30^\circ \Rightarrow E = \tfrac{200}{9}(\tfrac{1}{2}\sqrt{3})^3 = \tfrac{200}{9}(\tfrac{3}{8}\sqrt{3}) = \tfrac{25}{3}\sqrt{3} \approx 14.43 \text{ ft-candles}$$

(b) $E = \frac{1}{2}E_{\text{max}} \Rightarrow \frac{200}{9}\cos^3\theta = \frac{1}{2}(\frac{200}{9}\cos^3 0^\circ) \Rightarrow \cos^3\theta = \frac{1}{2} \Rightarrow \cos\theta = \sqrt[3]{\frac{1}{2}} \Rightarrow$

$$\theta \approx 37.47^\circ$$

$\boxed{72}$ (a) Let $x = \overline{PT}$ and $y = \overline{QT}$. Now $x^2 + d^2 = y^2$, $h = x\sin\alpha$, and $h = y\sin\beta$.

$$d^2 = y^2 - x^2 = \frac{h^2}{\sin^2\beta} - \frac{h^2}{\sin^2\alpha} = \frac{h^2(\sin^2\alpha - \sin^2\beta)}{\sin^2\alpha\,\sin^2\beta} \Rightarrow h^2 = \frac{d^2\sin^2\alpha\,\sin^2\beta}{\sin^2\alpha - \sin^2\beta} \Rightarrow$$

$$h = \frac{d\sin\alpha\,\sin\beta}{\sqrt{\sin^2\alpha - \sin^2\beta}}.$$

(b) $\alpha = 30^\circ$, $\beta = 20^\circ$, and $d = 10 \Rightarrow h = \dfrac{10\sin 30^\circ \sin 20^\circ}{\sqrt{\sin^2 30^\circ - \sin^2 20^\circ}} \approx 4.69$ miles.

$\boxed{73}$ (a) Let d denote the distance from the end of the bracket to the wall.

$$\cos 30^\circ = \frac{d}{85.5} \Rightarrow d = (85.5)(\tfrac{1}{2}\sqrt{3}) \approx 74.05 \text{ in.}$$

(b) Let y denote the side opposite 30°. $\sin 30^\circ = \dfrac{y}{85.5} \Rightarrow y = 42.75$ in.

Thus, the distance from the ceiling to the top of the screen is the

(length of the bracket $+\,y\,-$ one-half the height of the screen) $=$

$$(18'' + 42.75'' - 36'') = 24.75''.$$

$\boxed{74}$ (a) $\sin\theta = \dfrac{\frac{1}{2}x}{a} \Rightarrow x = 2a\sin\theta$. The area of one face is $\frac{1}{2}$(base)(height) $= \frac{1}{2}xa$.

$$S = 4(\tfrac{1}{2}ax) = 2ax = 2a(2a\sin\theta) = 4a^2\sin\theta.$$

(b) $\cos\theta = \frac{y}{a} \Rightarrow y = a\cos\theta$.

$$V = \tfrac{1}{3}(\text{base area})(\text{height}) = \tfrac{1}{3}x^2y = \tfrac{1}{3}(2a\sin\theta)^2(a\cos\theta) = \tfrac{4}{3}a^3\sin^2\theta\cos\theta.$$

$\boxed{75}$ (a) Let $\theta = \angle PCQ$. Since $\angle BPC = 90^\circ$, $\cos\theta = \dfrac{R}{R+h} \Rightarrow$

$$R + h = R\sec\theta \Rightarrow h = R\sec\theta - R. \text{ Since } \theta = \tfrac{s}{R},\ h = R\sec\tfrac{s}{R} - R.$$

(b) $h = R\left(\sec\frac{s}{R} - 1\right) = 4000\left(\sec\frac{50}{4000} - 1\right) \approx 0.31252 \text{ mi} \approx 1650 \text{ ft}$.

$\boxed{76}$ $y = 1 - 1\cos(\frac{1}{2}\pi x/10) = -\cos(\frac{\pi}{20}x) + 1$.

For $0 \le x \le 10$,

$0 \le \frac{\pi}{20}x \le \frac{\pi}{2} \Rightarrow$

$1 \ge \cos(\frac{\pi}{20}x) \ge 0 \Rightarrow$

$-1 \le -\cos(\frac{\pi}{20}x) \le 0 \Rightarrow$

$0 \le -\cos(\frac{\pi}{20}x) + 1 \le 1 \Rightarrow$

$0 \le y \le 1$.

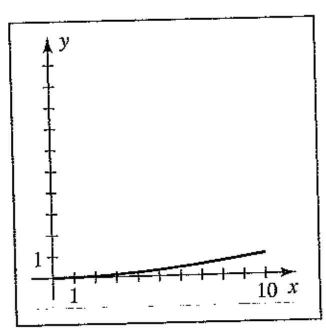

Figure 76

[77] The range of temperatures is 0.6°F, so $a = 0.3$. $\frac{2\pi}{b} = 24 \Rightarrow b = \frac{\pi}{12}$. The average temperature occurs at 11 A.M., 6 hours before the high at 5 P.M., which corresponds to $t = 11$. The argument of the sine is then $\frac{\pi}{12}(t-11)$, or $\frac{\pi}{12}t - \frac{11\pi}{12}$. Thus,

$$y = 98.6 + (0.3)\sin\left(\tfrac{\pi}{12}t - \tfrac{11\pi}{12}\right) \text{ \{or equivalently, } y = 98.6 + (0.3)\sin\left(\tfrac{\pi}{12}t + \tfrac{13\pi}{12}\right) \text{ \}.}$$

[78] (a) $p = \frac{2\pi}{\pi/6} = 12$ months

(b) The highest temperature will occur when the argument of the sine is $\frac{\pi}{2}$.

$\frac{\pi}{6}(t-3) = \frac{\pi}{2} \Rightarrow t-3=3 \Rightarrow t=6$ months.

This is July 1st and the temperature is 20.8°C, or 69.44°F.

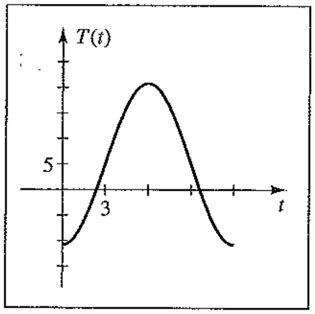

Figure 78

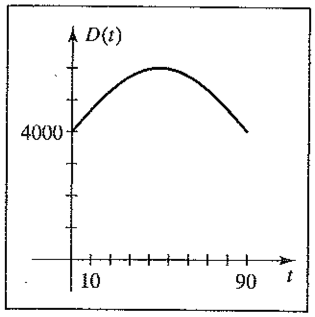

Figure 79

[79] (a) $p = \frac{2\pi}{\pi/90} = 180$ days

(b) As in Exercise 78, $\frac{\pi}{90}t = \frac{\pi}{2} \Rightarrow t = 45$ days into summer.

[80] (a) The cork is in simple harmonic motion. At $t = 0$, its height is 13 ft.

It decreases until $t = 1$, reaching a minimum of 11 ft.

It then increases, reaching a maximum of 13 ft at $t = 2$.

(b) From part (a), the cork is rising for $1 \le t \le 2$.

Chapter 6 Discussion Exercises

[1] On the TI-82/83 with $a = 15$, there is an indication that there are 15 sine waves on each side of the y-axis, but the minimums and maximums do not get to -1 and 1, respectively. With $a = 30$, the number of sine waves is undetectable—there simply aren't enough pixels for any degree of clarity. With $a = 45$, there are 2 sine waves on each side of the y-axis—there should be 45!

[2] Divide 10^{k+1} by 2π, subtract the greatest integer of that result from itself, leaving only a fractional part of 2π radians. Multiply that result by 2π to give you a number between 0 and 2π, and then take the sine of that value. On the TI-82/83, $k = 11$. To find $\sin(10^{12})$, enter 10^{12}, divide by 2π, take fPart Ans to obtain .89 {fPart is under the NUM submenu from the MATH key}, multiply by 2π, and take the sine of that result. Thus, $\sin(10^{12}) \approx \sin(5.59203) \approx -0.6374$.

[3] The sum on the left-side of the equation can never be greater than 3—no solutions.

[4] A discussion should bring out the following comments:

(1) the trigonometric functions are all periodic functions,

(2) periodic functions can't be one-to-one functions,

(3) for a function to have an inverse it must be one-to-one;

and come up with the conclusion that any one of the trigonometric functions cannot have an inverse unless its domain is restricted.

[5] The graph of $y_1 = x$, $y_2 = \sin x$, and $y_3 = \tan x$ in the suggested viewing rectangle, $[-0.1,\ 0.1]$ by $[-0.1,\ 0.1]$, indicates that their values are very close to each other near $x = 0$—in fact, the graphs of the functions are indistinguishable. Creating a table of values on the order of 10^{-10} also shows that all three functions are nearly equal.

[6] (a) Consider the track as a unit circle. The 2 km can then be thought of as an angle measurement of 2 radians. Thus, $x = \cos 2 \approx -0.4161$ and $y = \sin 2 \approx 0.9093$.

(b) Since the diameter is 2 km, the radius is 1 km and the circumference is 2π km. $\frac{500}{2\pi} \approx 79.577472$ revolutions. Subtracting the 79 whole laps and multiplying the remainder by 2π we obtain $(2\pi)(0.577472) \approx 3.6283607$ radians. Thus, $x \approx \cos(3.6283607) \approx -0.8838$ and $y \approx \sin(3.6283607) \approx -0.4678$. Alternatively, $x = \cos 500 \approx -0.8838$ and $y = \sin 500 \approx -0.4678$.

7 (a) S is at $(0, -1)$ on the rectangular coordinate system. Starting at S, subtract 1 from the 2 km to get to the circular portion of the track. Now consider $t = (1 + \frac{3\pi}{2})$ as the radian measurement of the track in Discussion Exercise 6. $\{\frac{3\pi}{2}$ to get to the bottom of the circle and 1 to make the second km.$\}$ $x = \cos t + 1 \approx 1.8415$ and $y = \sin t \approx -0.5403$.

(b) The perimeter of the track is $(4 + 2\pi)$ km. $\frac{500}{4+2\pi} \approx 48.623066$ laps. $(4 + 2\pi)(0.623066) \approx 6.4071052$. To determine where this places us on the track, start at S and subtract 1 $\{$to get to $(1, -1)\}$, subtract π $\{$to get to $(1, 1)\}$, and subtract 2 $\{$to get to $(-1, 1)\}$. This is about 0.26551259. Now consider $t = (0.26551259 + \frac{\pi}{2})$ as the radian measurement of the track in Discussion Exercise 6. $x \approx \cos t - 1 \approx -1.2624$ and $y \approx \sin t \approx 0.9650$.

8 (a) $\omega = \frac{5000 \text{ rev}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} \times \frac{2\pi \text{ radians}}{1 \text{ revolution}} = \frac{500\pi}{3} \approx 523.6$ radian/sec.

(b) Since the depth is 18, we have $d = 18$. The radius is 5 inches, so $a = 5$. Thus, $D(t) = a\cos(\omega t + c) + d = 5\cos\left(\frac{500\pi}{3}t + c\right) + 18$. Since the point is initially at a depth of 23 inches, $D = 23$ when $t = 0$. Substituting, we have $23 = 5\cos(c) + 18 \Rightarrow \cos c = 1 \Rightarrow c = 0$. Hence, $D(t) = 5\cos\left(\frac{500\pi}{3}t\right) + 18$.

(c) Graph $Y_1 = 5\cos\left(\frac{500\pi}{3}x\right) + 18$ on the interval $[0, 0.12]$. From the graph, we see that the propeller completes approximately 10 revolutions in 0.12 seconds.

[0, 0.12, 0.01] by [0, 25, 5]

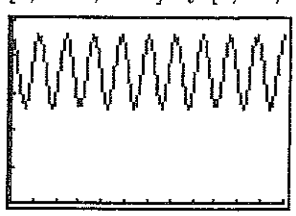

Figure 8