Chapter 1: Fundamental Concepts of Algebra

1.1 Exercises

$\boxed{1}$ (a) Since x and y have opposite signs, the product xy is negative.

(b) Since $x^2 > 0$ and $y > 0$, $x^2y > 0$.

(c) Since $x < 0$ and $y > 0$, $\frac{x}{y} < 0$, and $\frac{x}{y} + x < 0$.

(d) Since $y > 0$ and $x < 0$, $y - x > 0$.

$\boxed{2}$ (a) Since x and y have opposite signs, the quotient $\frac{x}{y}$ is negative.

(b) Since $x < 0$ and $y^2 > 0$, $xy^2 < 0$.

(c) Since $x - y < 0$ and $xy < 0$, $\dfrac{x-y}{xy} > 0$.

(d) Since $y > 0$ and $y - x > 0$, $y(y - x) > 0$.

$\boxed{3}$ (a) Since -7 is to the left of -4 on a coordinate line, $-7 \boxed{<} -4$.

(b) Using a calculator, we see that $\frac{\pi}{2} \approx 1.5708$. Hence, $\frac{\pi}{2} \boxed{>} 1.57$.

(c) $\sqrt{225} \boxed{=} 15$

$\boxed{4}$ (a) Since -3 is to the right of -5 on a coordinate line, $-3 \boxed{>} -5$.

(b) Using a calculator, we see that $\frac{\pi}{4} \approx 0.7854$. Hence, $\frac{\pi}{4} \boxed{<} 0.8$.

(c) $\sqrt{289} \boxed{=} 17$

$\boxed{5}$ (a) Since $\frac{1}{11} = 0.\overline{09}$, $\frac{1}{11} \boxed{>} 0.09$. (b) Since $\frac{2}{3} = 0.\overline{6}$, $\frac{2}{3} \boxed{>} 0.6666$.

(c) Since $\frac{22}{7} = 3.\overline{142857}$ and $\pi \approx 3.141593$, $\frac{22}{7} \boxed{>} \pi$.

$\boxed{6}$ (a) Since $\frac{1}{7} = 0.\overline{142857}$, $\frac{1}{7} \boxed{<} 0.143$. (b) Since $\frac{5}{6} = 0.8\overline{3}$, $\frac{5}{6} \boxed{>} 0.833$.

(c) Since $\sqrt{2} \approx 1.4142$, $\sqrt{2} \boxed{>} 1.4$.

$\boxed{7}$ (a) x is negative $\Leftrightarrow$ $x < 0$ (b) y is nonnegative $\Leftrightarrow$ $y \geq 0$

(c) q is less than or equal to π $\Leftrightarrow$ $q \leq \pi$

(d) d is between 4 and 2 $\Leftrightarrow$ $2 < d < 4$ (e) t is not less than 5 $\Leftrightarrow$ $t \geq 5$

(f) The negative of z is not greater than 3 $\Leftrightarrow$ $-z \leq 3$

(g) The quotient of p and q is at most 7 $\Leftrightarrow$ $\dfrac{p}{q} \leq 7$

(h) The reciprocal of w is at least 9 $\Leftrightarrow$ $\dfrac{1}{w} \geq 9$

(i) The absolute value of x is greater than 7 $\Leftrightarrow$ $|x| > 7$

$\boxed{8}$ (a) b is positive $\Leftrightarrow$ $b > 0$ (b) s is nonpositive $\Leftrightarrow$ $s \leq 0$

(c) w is greater than or equal to -4 $\Leftrightarrow$ $w \geq -4$

(d) c is between $\frac{1}{5}$ and $\frac{1}{3}$ $\Leftrightarrow$ $\frac{1}{5} < c < \frac{1}{3}$ (e) p is not greater than -2 $\Leftrightarrow$ $p \leq -2$

(f) The negative of m is not less than -2 $\Leftrightarrow$ $-m \geq -2$

(g) The quotient of r and s is at least $\frac{1}{5}$ $\Leftrightarrow$ $\frac{r}{s} \geq \frac{1}{5}$

(h) The reciprocal of f is at most 14 $\Leftrightarrow$ $\dfrac{1}{f} \leq 14$

(i) The absolute value of x is less than 4 $\Leftrightarrow$ $|x| < 4$

$\boxed{9}$ (a) $|-3-2| = |-5| = -(-5)$ {since $-5 < 0$} $= 5$

(b) $|-5| - |2| = -(-5) - 2 = 5 - 2 = 3$

(c) $|7| + |-4| = 7 + [-(-4)] = 7 + 4 = 11$

$\boxed{10}$ (a) $|-11+1| = |-10| = -(-10)$ {since $-10 < 0$} $= 10$

(b) $|6| - |-3| = 6 - [-(-3)] = 6 - 3 = 3$

(c) $|8| + |-9| = 8 + [-(-9)] = 8 + 9 = 17$

$\boxed{11}$ (a) $(-5)\,|3-6| = (-5)\,|-3| = (-5)[-(-3)] = (-5)(3) = -15$

(b) $|-6|/(-2) = -(-6)/(-2) = 6/(-2) = -3$

(c) $|-7| + |4| = -(-7) + 4 = 7 + 4 = 11$

$\boxed{12}$ (a) $(4)\,|6-7| = (4)\,|-1| = (4)[-(-1)] = (4)(1) = 4$

(b) $5/|-2| = 5/[-(-2)] = 5/2$

(c) $|-1| + |-9| = -(-1) + [-(-9)] = 1 + 9 = 10$

$\boxed{13}$ (a) Since $(4-\pi)$ is positive, $|4-\pi| = 4-\pi$.

(b) Since $(\pi-4)$ is negative, $|\pi-4| = -(\pi-4) = 4-\pi$.

(c) Since $(\sqrt{2}-1.5)$ is negative, $|\sqrt{2}-1.5| = -(\sqrt{2}-1.5) = 1.5-\sqrt{2}$.

$\boxed{14}$ (a) Since $(\sqrt{3}-1.7)$ is positive, $|\sqrt{3}-1.7| = \sqrt{3}-1.7$.

(b) Since $(1.7-\sqrt{3})$ is negative, $|1.7-\sqrt{3}| = -(1.7-\sqrt{3}) = \sqrt{3}-1.7$.

(c) $\left|\frac{1}{5}-\frac{1}{3}\right| = \left|\frac{3}{15}-\frac{5}{15}\right| = \left|-\frac{2}{15}\right| = -(-\frac{2}{15}) = \frac{2}{15}$

$\boxed{15}$ (a) $d(A, B) = |7-3| = |4| = 4$ (b) $d(B, C) = |-5-7| = |-12| = 12$

(c) $d(C, B) = d(B, C) = 12$ (d) $d(A, C) = |-5-3| = |-8| = 8$

$\boxed{16}$ (a) $d(A, B) = |-2-(-6)| = |4| = 4$ (b) $d(B, C) = |4-(-2)| = |6| = 6$

(c) $d(C, B) = d(B, C) = 6$ (d) $d(A, C) = |4-(-6)| = |10| = 10$

$\boxed{17}$ (a) $d(A, B) = |1-(-9)| = |10| = 10$ (b) $d(B, C) = |10-1| = |9| = 9$

(c) $d(C, B) = d(B, C) = 9$ (d) $d(A, C) = |10-(-9)| = |19| = 19$

$\boxed{18}$ (a) $d(A, B) = |-4-8| = |-12| = 12$ (b) $d(B, C) = |-1-(-4)| = |3| = 3$

(c) $d(C, B) = d(B, C) = 3$ (d) $d(A, C) = |-1-8| = |-9| = 9$

Note: Exer. 19–24: Since $|a| = |-a|$, the answers could have a different form.

For example, $|-3-x| \geq 8$ is equivalent to $|x+3| \geq 8$.

$\boxed{19}$ $d(A, B) = |7-x| \;\Rightarrow\; |7-x| < 5$

$\boxed{20}$ $d(A, B) = |-\sqrt{2}-x| \;\Rightarrow\; |-\sqrt{2}-x| > 1$

$\boxed{21}$ $d(A, B) = |-3-x| \;\Rightarrow\; |-3-x| \geq 8$

$\boxed{22}$ $d(A, B) = |4-x| \;\Rightarrow\; |4-x| \leq 2$

$\boxed{23}$ $d(A, B) = |x-4| \;\Rightarrow\; |x-4| \leq 3$

$\boxed{24}$ $d(A, B) = |x-(-2)| = |x+2| \;\Rightarrow\; |x+2| \geq 2$

Note: Exer. 25–32: Have students substitute a permissible value for the letter to first test if the expression inside the absolute value symbol is positive or negative.

$\boxed{25}$ Pick an arbitrary value for x that is less than -3, say -5.

Since $3 + (-5) = -2$ is negative, we conclude that if $x < -3$, then $3 + x$ is negative.

Hence, $|\,3 + x\,| = -(3 + x) = -x - 3$.

$\boxed{26}$ If $x > 5$, then $5 - x < 0$, and $|\,5 - x\,| = -(5 - x) = x - 5$.

$\boxed{27}$ If $x < 2$, then $2 - x > 0$, and $|\,2 - x\,| = 2 - x$.

$\boxed{28}$ If $x \geq -7$, then $7 + x \geq 0$, and $|\,7 + x\,| = 7 + x$.

$\boxed{29}$ If $a < b$, then $a - b < 0$, and $|\,a - b\,| = -(a - b) = b - a$.

$\boxed{30}$ If $a > b$, then $a - b > 0$, and $|\,a - b\,| = a - b$.

$\boxed{31}$ Since $x^2 + 4 > 0$ for every x, $|\,x^2 + 4\,| = x^2 + 4$.

$\boxed{32}$ Since $-x^2 - 1 < 0$ for every x, $|\,-x^2 - 1\,| = -(-x^2 - 1) = x^2 + 1$.

$\boxed{33}$ LS $= \dfrac{ab + ac}{a} = \frac{ab}{a} + \frac{ac}{a} = b + c \boxed{\neq}$ RS $(b + ac)$.

$\boxed{34}$ From Exercise 33, LS$\boxed{=}$RS.

$\boxed{35}$ LS $= \dfrac{b + c}{a} = \frac{b}{a} + \frac{c}{a}\boxed{=}$RS.

$\boxed{36}$ LS $= \dfrac{a + c}{b + d} = \dfrac{a}{b + d} + \dfrac{c}{b + d}\boxed{\neq}$RS $\left(\frac{a}{b} + \frac{c}{d}\right)$.

$\boxed{37}$ LS $= (a \div b) \div c = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}$. RS $= a \div (b \div c) = a \div \frac{b}{c} = a \cdot \frac{c}{b} = \frac{ac}{b}$. LS$\boxed{\neq}$RS

$\boxed{38}$ LS $= (a - b) - c = a - b - c$. RS $= a - (b - c) = a - b + c$. LS$\boxed{\neq}$RS

$\boxed{39}$ LS $= \dfrac{a - b}{b - a} = \dfrac{-(b - a)}{b - a} = -1\boxed{=}$RS.

$\boxed{40}$ LS $= -(a + b) = -a - b\boxed{\neq}$RS $(-a + b)$.

$\boxed{41}$ (a) $\left|3.2^2 - \sqrt{3.15}\right| \approx 8.4652$

(b) $\sqrt{(15.6 - 1.5)^2 + (4.3 - 5.4)^2} \approx 14.1428$

$\boxed{42}$ (a) $\dfrac{3.42 - 1.29}{5.83 + 2.64} \approx 0.2515$

(b) $\pi^3 \approx 31.0063$

$\boxed{43}$ (a) $\dfrac{1.2 \times 10^3}{3.1 \times 10^2 + 1.52 \times 10^3} \approx 0.6557 = 6.557 \times 10^{-1}$ *Note:* For the TI-83 Plus, use 1.2E3/(3.1E2 + 1.52E3), where E is obtained by pressing $\boxed{\text{2nd}}\,\boxed{\text{EE}}$.

(b) $(1.23 \times 10^{-4}) + \sqrt{4.5 \times 10^3} \approx 67.08 = 6.708 \times 10^1$

$\boxed{44}$ (a) $\sqrt{|3.45 - 1.2 \times 10^4| + 10^5} \approx 334.7 = 3.347 \times 10^2$

(b) $(1.791 \times 10^2) \times (9.84 \times 10^3) = 1{,}762{,}344 \approx 1.762 \times 10^6$

$\boxed{45}$ Construct a right triangle with sides of lengths $\sqrt{2}$ and 1. The hypotenuse will have length $\sqrt{3}$. Next construct a right triangle with sides of lengths $\sqrt{3}$ and $\sqrt{2}$.

The hypotenuse will have length $\sqrt{5}$.

$\boxed{46}$ Use $C = 2\pi r$ with $r = 1$, 2, and 10 to obtain 2π, 4π, and 20π units from the origin.

$\boxed{47}$ The large rectangle has area $a(b + c)$.

The sum of the areas of the two small rectangles is $ab + ac$.

[48] $x_1 = \frac{3}{2}$ and $n = 2 \Rightarrow x_2 = \frac{1}{2}\left(x_1 + \frac{n}{x_1}\right) = \frac{1}{2}\left(\frac{3}{2} + \frac{2}{\frac{3}{2}}\right) = \frac{1}{2}(\frac{3}{2} + \frac{4}{3}) = \frac{1}{2}(\frac{17}{6}) = \frac{17}{12}$.

$$x_3 = \frac{1}{2}\left(x_2 + \frac{2}{x_2}\right) = \frac{1}{2}\left(\frac{17}{12} + \frac{2}{\frac{17}{12}}\right) = \frac{1}{2}(\frac{17}{12} + \frac{24}{17}) = \frac{1}{2}(\frac{577}{204}) = \frac{577}{408}$$

[49] (a) $427{,}000 = 4.27 \times 10^5$ (b) $0.000\ 000\ 098 = 9.8 \times 10^{-8}$

(c) $810{,}000{,}000 = 8.1 \times 10^8$

[50] (a) $85{,}200 = 8.52 \times 10^4$ (b) $0.000\ 005\ 5 = 5.5 \times 10^{-6}$

(c) $24{,}900{,}000 = 2.49 \times 10^7$

[51] (a) $8.3 \times 10^5 = 830{,}000$ (b) $2.9 \times 10^{-12} = 0.000\ 000\ 000\ 002\ 9$

(c) $5.63 \times 10^8 = 563{,}000{,}000$

[52] (a) $2.3 \times 10^7 = 23{,}000{,}000$ (b) $7.01 \times 10^{-9} = 0.000\ 000\ 007\ 01$

(c) $1.23 \times 10^{10} = 12{,}300{,}000{,}000$

[53] $0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 001\ 7 = 1.7 \times 10^{-24}$

[54] $9.1 \times 10^{-31} = 0.000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 91$

[55] $\frac{186{,}000 \text{ miles}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot 1 \text{ year} \approx 5.87 \times 10^{12}$ mi

[56] (a) 100 billion $= 100{,}000{,}000{,}000 = 1 \times 10^{11}$

(b) $d \approx (100{,}000 \text{ yr})\left(5.87 \times 10^{12} \frac{\text{mi}}{\text{yr}}\right) = 5.87 \times 10^{17}$ mi

[57] $\dfrac{\frac{1.01 \text{ grams}}{\text{mole}}}{\frac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}} \cdot 1 \text{ atom} = \frac{1.01 \text{ grams}}{6.02 \times 10^{23}} \approx 0.1678 \times 10^{-23} \text{ g} = 1.678 \times 10^{-24}$ g

[58] $(2.5 \text{ million})(0.00035\%) = (2.5 \times 10^6)(3.5 \times 10^{-6}) = 8.75 \approx 9$ halibut

[59] $\frac{24 \text{ frames}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot 48 \text{ hours} = 4.1472 \times 10^6$ frames

[60] $\frac{2 \times 10^{11} \text{ calculations}}{\text{second}} \cdot \frac{60 \text{ seconds}}{1 \text{ minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hour}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot 60 \text{ days} =$

1.0368×10^{18} calculations

[61] (a) $1 \text{ ft}^2 = 144 \text{ in.}^2 \Rightarrow 144 \text{ in.}^2 \times 1.4 \text{ lb/in.}^2 = 201.6$ lb.

(b) $40 \times 8 = 320 \text{ ft}^2 = 46{,}080 \text{ in.}^2$; $46{,}080 \times 1.4 = 64{,}512$ lb;

$64{,}512 \text{ lb}/(2000 \text{ lb/ton}) = 32.256$ tons

[62] (a) We start with 400 adults, 150 yearlings, and 200 calves {total = 750}

Number of *Adults* = surviving adults + surviving yearlings

$= (0.90)(400) + (0.80)(150) = \underline{480}$

Number of *Yearlings* = surviving calves

$= (0.75)(200) = \underline{150}$

Number of *Calves* = number of female adults

$= (0.50)(480) = \underline{240}$

(b) 75% of last spring's calves equal the number of this year's yearlings (150), so the number of *calves* is <u>200</u>.

The number of calves is equal to the number of adult females and this is one-half of the number of adults, so the number of *adults* is <u>400</u>.

90% of these (360) are part of the 400 adults this year. The other 40 adults represent 80% of last year's yearlings, so the number of *yearlings* is <u>50</u>.

1.2 Exercises

$\boxed{1}$ $(-\frac{2}{3})^4 = \frac{16}{81}$

$\boxed{2}$ $(-3)^3 = -27 = \frac{-27}{1}$

$\boxed{3}$ $\dfrac{2^{-3}}{3^{-2}} = \dfrac{3^2}{2^3} = \dfrac{9}{8}$

$\boxed{4}$ $\dfrac{2^0 + 0^2}{2+0} = \dfrac{1+0}{2} = \dfrac{1}{2}$

$\boxed{5}$ $-2^4 + 3^{-1} = -16 + \frac{1}{3} = -\frac{48}{3} + \frac{1}{3} = \frac{-47}{3}$

$\boxed{6}$ $(-\frac{3}{2})^4 - 2^{-4} = \frac{81}{16} - \frac{1}{16} = \frac{80}{16} = \frac{5}{1}$

$\boxed{7}$ $16^{-3/4} = 1/16^{3/4} = 1/(\sqrt[4]{16})^3 = 1/2^3 = \frac{1}{8}$

$\boxed{8}$ $9^{5/2} = (\sqrt{9})^5 = 3^5 = \frac{243}{1}$

$\boxed{9}$ $(-0.008)^{2/3} = (\sqrt[3]{-0.008})^2 = (-0.2)^2 = 0.04 = \frac{4}{100} = \frac{1}{25}$

$\boxed{10}$ $(0.008)^{-2/3} = 1/(0.008)^{2/3} = 1/(\sqrt[3]{0.008})^2 = 1/(0.2)^2 = 1/(0.04) = \frac{25}{1}$

$\boxed{11}$ $(\frac{1}{2}x^4)(16x^5) = (\frac{1}{2} \cdot 16)x^{4+5} = 8x^9$

$\boxed{12}$ $(-3x^{-2})(4x^4) = (-3 \cdot 4)x^{-2+4} = -12x^2$

$\boxed{13}$ $\dfrac{(2x^3)(3x^2)}{(x^2)^3} = \dfrac{6x^5}{x^6} = \dfrac{6}{x}$

$\boxed{14}$ $\dfrac{(2x^2)^3}{4x^4} = \dfrac{8x^6}{4x^4} = 2x^2$

$\boxed{15}$ $(\frac{1}{6}a^5)(-3a^2)(4a^7) = -2a^{14}$

$\boxed{16}$ $(-4b^3)(\frac{1}{6}b^2)(-9b^4) = 6b^9$

$\boxed{17}$ $\dfrac{(6x^3)^2}{(2x^2)^3} \cdot (3x^2)^0 = \dfrac{36x^6}{8x^6} \cdot 1 = \dfrac{9}{2}$

$\boxed{18}$ $\dfrac{(3y^3)(2y^2)^2}{(y^4)^3} \cdot (y^3)^0 = \dfrac{(3y^3)(4y^4)}{y^{12}} \cdot 1 = \dfrac{12y^7}{y^{12}} = \dfrac{12}{y^5}$

$\boxed{19}$ $(3u^7v^3)(4u^4v^{-5}) = 12u^{11}v^{-2} = \dfrac{12u^{11}}{v^2}$

$\boxed{20}$ $(x^2yz^3)(-2xz^2)(x^3y^{-2}) = -2x^6y^{-1}z^5 = \dfrac{-2x^6z^5}{y}$

$\boxed{21}$ $(8x^4y^{-3})(\frac{1}{2}x^{-5}y^2) = 4x^{-1}y^{-1} = \frac{4}{xy}$

$\boxed{22}$ $\left(\dfrac{4a^2b}{a^3b^2}\right)\left(\dfrac{5a^2b}{2b^4}\right) = \dfrac{20a^4b^2}{2a^3b^6} = \dfrac{10a}{b^4}$

$\boxed{23}$ $(\frac{1}{3}x^4y^{-3})^{-2} = (\frac{1}{3})^{-2}x^{-8}y^6 = 3^2x^{-8}y^6 = \dfrac{9y^6}{x^8}$

$\boxed{24}$ $(-2xy^2)^5\left(\dfrac{x^7}{8y^3}\right) = (-32x^5y^{10})\left(\dfrac{x^7}{8y^3}\right) = -4x^{12}y^7$

$\boxed{25}$ $(3y^3)^4(4y^2)^{-3} = 81y^{12} \cdot 4^{-3}y^{-6} = 81y^6 \cdot \frac{1}{64} = \frac{81}{64}y^6$

$\boxed{26}$ $(-3a^2b^{-5})^3 = -27a^6b^{-15} = -\dfrac{27a^6}{b^{15}}$

$\boxed{27}$ $(-2r^4s^{-3})^{-2} = (-2)^{-2}r^{-8}s^6 = \dfrac{s^6}{(-2)^2r^8} = \dfrac{s^6}{4r^8}$

$\boxed{28}$ $(2x^2y^{-5})(6x^{-3}y)(\frac{1}{3}x^{-1}y^3) = 4x^{-2}y^{-1} = \dfrac{4}{x^2y}$

$\boxed{29}$ $(5x^2y^{-3})(4x^{-5}y^4) = 20x^{-3}y = \dfrac{20y}{x^3}$

$\boxed{30}$ $(-2r^2s)^5(3r^{-1}s^3)^2 = (-32r^{10}s^5)(9r^{-2}s^6) = -288r^8s^{11}$

$\boxed{31}$ $\left(\dfrac{3x^5y^4}{x^0y^{-3}}\right)^2 = \dfrac{9x^{10}y^8}{y^{-6}} = 9x^{10}y^{14}$

$\boxed{32}$ $(4a^2b)^4\left(\dfrac{-a^3}{2b}\right)^2 = (256a^8b^4)\left(\dfrac{a^6}{4b^2}\right) = 64a^{14}b^2$

$\boxed{33}$ $(4a^{3/2})(2a^{1/2}) = 8a^{4/2} = 8a^2$

$\boxed{34}$ $(-6x^{7/5})(2x^{8/5}) = -12x^{15/5} = -12x^3$

$\boxed{35}$ $(3x^{5/6})(8x^{2/3}) = 24x^{(5/6)+(4/6)} = 24x^{9/6} = 24x^{3/2}$

$\boxed{36}$ $(8r)^{1/3}(2r^{1/2}) = (2r^{1/3})(2r^{1/2}) = 4r^{(2/6)+(3/6)} = 4r^{5/6}$

$\boxed{37}$ $(27a^6)^{-2/3} = 27^{-2/3}a^{-12/3} = \dfrac{1}{(\sqrt[3]{27})^2a^4} = \dfrac{1}{9a^4}$

$\boxed{38}$ $(25z^4)^{-3/2} = 25^{-3/2}z^{-12/2} = \dfrac{1}{(\sqrt{25})^3z^6} = \dfrac{1}{125z^6}$

$\boxed{39}$ $(8x^{-2/3})x^{1/6} = 8x^{(-4/6)+(1/6)} = 8x^{-3/6} = \dfrac{8}{x^{1/2}}$

$\boxed{40}$ $(3x^{1/2})(-2x^{5/2}) = -6x^3$

$\boxed{41}$ $\left(\dfrac{-8x^3}{y^{-6}}\right)^{2/3} = \dfrac{(-2)^2x^2}{(y^{-2})^2} = \dfrac{4x^2}{y^{-4}} = 4x^2y^4$

$\boxed{42}$ $\left(\dfrac{-y^{3/2}}{y^{-1/3}}\right)^3 = \dfrac{-y^{9/2}}{y^{-1}} = -y^{11/2}$

$\boxed{43}$ $\left(\dfrac{x^6}{9y^{-4}}\right)^{-1/2} = \dfrac{x^{-3}}{9^{-1/2}y^2} = \dfrac{9^{1/2}x^{-3}}{y^2} = \dfrac{3}{x^3y^2}$

$\boxed{44}$ $\left(\dfrac{c^{-4}}{16d^8}\right)^{3/4} = \dfrac{c^{-3}}{(\sqrt[4]{16})^3d^6} = \dfrac{c^{-3}}{8d^6} = \dfrac{1}{8c^3d^6}$

$\boxed{45}$ $\dfrac{(x^6y^3)^{-1/3}}{(x^4y^2)^{-1/2}} = \dfrac{x^{-2}y^{-1}}{x^{-2}y^{-1}} = 1$

$\boxed{46}$ $a^{4/3}a^{-3/2}a^{1/6} = a^{(8/6)-(9/6)+(1/6)} = a^{0/6} = a^0 = 1$

$\boxed{47}$ $\sqrt[4]{x^3} = (x^3)^{1/4} = x^{3/4}$

$\boxed{48}$ $\sqrt[3]{x^5} = (x^5)^{1/3} = x^{5/3}$

$\boxed{49}$ $\sqrt[3]{(a+b)^2} = [(a+b)^2]^{1/3} = (a+b)^{2/3}$

$\boxed{50}$ $\sqrt{a+\sqrt{b}} = (a+b^{1/2})^{1/2}$

$\boxed{51}$ $\sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

$\boxed{52}$ $\sqrt[3]{r^3-s^3} = (r^3-s^3)^{1/3}$

$\boxed{53}$ (a) $4x^{3/2} = 4x^1x^{1/2} = 4x\sqrt{x}$ (b) $(4x)^{3/2} = (4x)^1\,4^{1/2}x^{1/2} = 8x\sqrt{x}$

$\boxed{54}$ (a) $4 + x^{3/2} = 4 + x^1x^{1/2} = 4 + x\sqrt{x}$

(b) $(4+x)^{3/2} = (4+x)^1(4+x)^{1/2} = (4+x)\sqrt{4+x}$

$\boxed{55}$ (a) $8 - y^{1/3} = 8 - \sqrt[3]{y}$ (b) $(8-y)^{1/3} = \sqrt[3]{8-y}$

$\boxed{56}$ (a) $8y^{1/3} = 8\sqrt[3]{y}$ (b) $(8y)^{1/3} = 8^{1/3}y^{1/3} = 2\sqrt[3]{y}$

$\boxed{57}$ $\sqrt{81} = \sqrt{9^2} = 9$

$\boxed{58}$ $\sqrt[3]{-125} = \sqrt[3]{(-5)^3} = -5$

$\boxed{59}$ $\sqrt[5]{-64} = \sqrt[5]{-32}\,\sqrt[5]{2} = \sqrt[5]{(-2)^5}\,\sqrt[5]{2} = -2\sqrt[5]{2}$

$\boxed{60}$ $\sqrt[4]{256} = \sqrt[4]{4^4} = 4$

$\boxed{61}$ $\dfrac{1}{\sqrt[3]{2}} = \dfrac{1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{1}{2}\sqrt[3]{4}$

$\boxed{62}$ $\sqrt{\frac{1}{7}} = \sqrt{\frac{1}{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \frac{1}{7}\sqrt{7}$

$\boxed{63}$ $\sqrt{9x^{-4}y^6} = 3x^{-2}y^3 = \dfrac{3y^3}{x^2}$

$\boxed{64}$ $\sqrt{16a^8b^{-2}} = 4a^4b^{-1} = \dfrac{4a^4}{b}$

$\boxed{65}$ $\sqrt[3]{8a^6b^{-3}} = 2a^2b^{-1} = \dfrac{2a^2}{b}$

$\boxed{66}$ $\sqrt[4]{81r^5s^8} = \sqrt[4]{3^4r^4s^8}\,\sqrt[4]{r} = 3rs^2\sqrt[4]{r}$

$\boxed{67}$ $\sqrt{\dfrac{3x}{2y^3}} = \sqrt{\dfrac{3x}{2y^3}} \cdot \dfrac{\sqrt{2y}}{\sqrt{2y}} = \dfrac{1}{2y^2}\sqrt{6xy}$

$\boxed{68}$ $\sqrt{\dfrac{1}{3x^3y}} = \sqrt{\dfrac{1}{3x^3y}} \cdot \dfrac{\sqrt{3xy}}{\sqrt{3xy}} = \dfrac{1}{3x^2y}\sqrt{3xy}$

$\boxed{69}$ $\sqrt[3]{\dfrac{2x^4y^4}{9x}} = \sqrt[3]{\dfrac{2x^4y^4}{9x}} \cdot \dfrac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}} = \dfrac{\sqrt[3]{x^6y^3}\,\sqrt[3]{6y}}{3x} = \dfrac{x^2y\,\sqrt[3]{6y}}{3x} = \dfrac{xy}{3}\sqrt[3]{6y}$

$\boxed{70}$ $\sqrt[3]{\dfrac{3x^2y^5}{4x}} = \sqrt[3]{\dfrac{3x^2y^5}{4x}} \cdot \dfrac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} = \dfrac{\sqrt[3]{x^3y^3}\,\sqrt[3]{6xy^2}}{2x} = \dfrac{xy\,\sqrt[3]{6xy^2}}{2x} = \dfrac{y}{2}\sqrt[3]{6xy^2}$

$\boxed{71}$ $\sqrt[4]{\dfrac{5x^8y^3}{27x^2}} = \sqrt[4]{\dfrac{5x^8y^3}{27x^2}} \cdot \dfrac{\sqrt[4]{3x^2}}{\sqrt[4]{3x^2}} = \dfrac{\sqrt[4]{x^8}\,\sqrt[4]{15x^2y^3}}{3x} = \dfrac{x^2\,\sqrt[4]{15x^2y^3}}{3x} = \dfrac{x}{3}\sqrt[4]{15x^2y^3}$

$\boxed{72}$ $\sqrt[4]{\dfrac{x^7y^{12}}{125x}} = \sqrt[4]{\dfrac{x^7y^{12}}{125x}} \cdot \dfrac{\sqrt[4]{5x^3}}{\sqrt[4]{5x^3}} = \dfrac{\sqrt[4]{x^8y^{12}}\,\sqrt[4]{5x^2}}{5x} = \dfrac{x^2y^3\,\sqrt[4]{5x^2}}{5x} = \dfrac{xy^3}{5}\sqrt[4]{5x^2}$

$\boxed{73}$ $\sqrt[5]{\dfrac{5x^7y^2}{8x^3}} = \sqrt[5]{\dfrac{5x^7y^2}{8x^3}} \cdot \dfrac{\sqrt[5]{4x^2}}{\sqrt[5]{4x^2}} = \dfrac{\sqrt[5]{x^5}\,\sqrt[5]{20x^4y^2}}{2x} = \dfrac{x\,\sqrt[5]{20x^4y^2}}{2x} = \frac{1}{2}\sqrt[5]{20x^4y^2}$

$\boxed{74}$ $\sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} = \sqrt[5]{\dfrac{3x^{11}y^3}{9x^2}} \cdot \dfrac{\sqrt[5]{27x^3}}{\sqrt[5]{27x^3}} = \dfrac{\sqrt[5]{x^{10}}\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x^2\,\sqrt[5]{81x^4y^3}}{3x} = \dfrac{x}{3}\sqrt[5]{81x^4y^3}$

$\boxed{75}$ $\sqrt[4]{(3x^5y^{-2})^4} = 3x^5y^{-2} = \dfrac{3x^5}{y^2}$

$\boxed{76}$ $\sqrt[6]{(2u^{-3}v^4)^6} = 2u^{-3}v^4 = \dfrac{2v^4}{u^3}$

$\boxed{77}$ $\sqrt[5]{\dfrac{8x^3}{y^4}}\,\sqrt[5]{\dfrac{4x^4}{y^2}} = \sqrt[5]{\dfrac{8x^3}{y^4}}\,\sqrt[5]{\dfrac{4x^4}{y^2}} \cdot \dfrac{\sqrt[5]{y^4}}{\sqrt[5]{y^4}} = \dfrac{\sqrt[5]{32x^5}\,\sqrt[5]{x^2y^4}}{y^2} = \dfrac{2x}{y^2}\sqrt[5]{x^2y^4}$

$\boxed{78}$ $\sqrt{5xy^7}\,\sqrt{10x^3y^3} = \sqrt{25x^4y^{10}}\,\sqrt{2} = 5x^2y^5\sqrt{2}$

$\boxed{79}$ $\sqrt[3]{3t^4v^2}\,\sqrt[3]{-9t^{-1}v^4} = \sqrt[3]{-27t^3v^6} = -3tv^2$

$\boxed{80}$ $\sqrt[3]{(2r-s)^3} = 2r - s$

$\boxed{81}$ $\sqrt{x^6y^4} = \sqrt{(x^3)^2(y^2)^2} = |\,x^3\,|\,|\,y^2\,| = |\,x^3\,|\,y^2$

$\boxed{82}$ $\sqrt{x^4y^{10}} = \sqrt{(x^2)^2(y^5)^2} = |\,x^2\,|\,|\,y^5\,| = x^2\,|\,y^5\,|$

$\boxed{83}$ $\sqrt[4]{x^8(y-1)^{12}} = \sqrt[4]{(x^2)^4((y-1)^3)^4} = |\,x^2\,|\,|\,(y-1)^3\,| = x^2\,|\,(y-1)^3\,|$, or $x^2(y-1)^2\,|\,(y-1)\,|$

$\boxed{84}$ $\sqrt[4]{(x+2)^{12}y^4} = \sqrt[4]{((x+2)^3)^4y^4} = |\,(x+2)^3\,|\,|\,y\,|$, or $(x+2)^2\,|\,(x+2)y\,|$

$\boxed{85}$ $(a^r)^2 = a^{2r}\,\boxed{\neq}\,a^{(r^2)}$ since $2r \neq r^2$ for *all* values of r; for example, let $r = 1$.

$\boxed{86}$ Squaring the right side gives us $(a+1)^2 = a^2 + 2a + 1$. Squaring the left side gives us $a^2 + 1$. $a^2 + 2a + 1\,\boxed{\neq}\,a^2 + 1$ for *all* values of a; for example, let $a = 1$.

87 $(ab)^{xy} = a^{xy}b^{xy} \boxed{\neq} a^x b^y$ for *all* values of x and y; for example, let $x = 1$ and $y = 2$.

88 $\sqrt{a^r} = (a^r)^{1/2} = (a^{1/2})^r \boxed{=} (\sqrt{a})^r$

89 $\sqrt[n]{\frac{1}{c}} = \left(\frac{1}{c}\right)^{1/n} = \frac{1^{1/n}}{c^{1/n}} \boxed{=} \frac{1}{\sqrt[n]{c}}$

90 $\frac{1}{a^k} = a^{-k} \boxed{\neq} a^{1/k}$ since $-k \neq 1/k$ for *all* values of k; for example, let $k = 1$.

91 (a) $(-3)^{2/5} = [(-3)^2]^{1/5} = 9^{1/5} \approx 1.5518$

(b) $(-5)^{4/3} = [(-5)^4]^{1/3} = 625^{1/3} \approx 8.5499$

92 (a) $(-1.2)^{3/7} = [(-1.2)^3]^{1/7} = (-1.728)^{1/7} \approx -1.0813$

(b) $(-5.08)^{7/3} = [(-5.08)^7]^{1/3} \approx (-87{,}306.38)^{1/3} \approx -44.3624$

93 (a) $\sqrt{\pi + 1} \approx 2.0351$ (b) $\sqrt[3]{15.1} + 5^{1/4} \approx 3.9670$

94 (a) $(2.6 - 1.9)^{-2} \approx 2.0408$ (b) $5^{\sqrt{7}} \approx 70.6807$

95 $\$200(1.04)^{180} \approx \$232{,}825.78$

96 $h = 1454$ ft $\Rightarrow$ $d = 1.2\sqrt{h} = 1.2\sqrt{1454} \approx 45.8$ mi

97 $W = 230$ kg $\Rightarrow$ $L = 0.46\sqrt[3]{W} = 0.46\sqrt[3]{230} \approx 2.82$ m

98 $L = 25$ ft $\Rightarrow$ $W = 0.0016L^{2.43} = 0.0016(25)^{2.43} \approx 3.99$ tons

99 $b = 75$ and $w = 180$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b - 35}} = \dfrac{180}{\sqrt[3]{75 - 35}} \approx 52.6$.

$b = 120$ and $w = 250$ $\Rightarrow$ $W = \dfrac{w}{\sqrt[3]{b - 35}} = \dfrac{250}{\sqrt[3]{120 - 35}} \approx 56.9$.

It is interesting to note that the 75-kg lifter can lift 2.4 times his/her body weight and the 120-kg lifter can lift approximately 2.08 times his/her body weight, but *the formula ranks the 120-kg lifter as the superior lifter.*

100 (a) $h = 72$ in. and $w = 175$ lb $\Rightarrow$

$$S = (0.1091)w^{0.425}h^{0.725} = (0.1091)(175)^{0.425}(72)^{0.725} \approx 21.76 \text{ ft}^2.$$

(b) $h = 66$ in. $\Rightarrow$ $S_1 = (0.1091)w^{0.425}(66)^{0.725}$. A 10% increase in weight would be represented by $1.1w$ and thus $S_2 = (0.1091)(1.1w)^{0.425}(66)^{0.725}$.

$S_2/S_1 = (1.1)^{0.425} \approx 1.04$, which represents a 4% increase in S.

101 $W = 0.1166h^{1.7}$

Height	64	65	66	67	68	69	70	71
Weight	137	141	145	148	152	156	160	164
Height	72	73	74	75	76	77	78	79
Weight	168	172	176	180	184	188	192	196

[102] $W = 0.1049h^{1.7}$

Height	60	61	62	63	64	65	66	67
Weight	111	114	117	120	123	127	130	133
Height	68	69	70	71	72	73	74	75
Weight	137	140	144	147	151	154	158	162

1.3 Exercises

[1] $(3x^3 + 4x^2 - 7x + 1) + (9x^3 - 4x^2 - 6x) = 12x^3 - 13x + 1$

[2] $(7x^3 + 2x^2 - 11x) + (-3x^3 - 2x^2 + 5x - 3) = 4x^3 - 6x - 3$

[3] $(4x^3 + 5x - 3) - (3x^3 + 2x^2 + 5x - 7) = x^3 - 2x^2 + 4$

[4] $(6x^3 - 2x^2 + x - 2) - (8x^2 - x - 2) = 6x^3 - 10x^2 + 2x$

[5] $(2x + 5)(3x - 7) = (2x)(3x) + (2x)(-7) + (5)(3x) + (5)(-7) =$
$6x^2 - 14x + 15x - 35 = 6x^2 + x - 35$

[6] $(3x - 4)(2x + 9) = (3x)(2x) + (3x)(9) + (-4)(2x) + (-4)(9) =$
$6x^2 + 27x - 8x - 36 = 6x^2 + 19x - 36$

[7] $(5x + 7y)(3x + 2y) = (5x)(3x) + (5x)(2y) + (7y)(3x) + (7y)(2y) =$
$15x^2 + 10xy + 21xy + 14y^2 = 15x^2 + 31xy + 14y^2$

[8] $(4x - 3y)(x - 5y) = (4x)(x) + (4x)(-5y) + (-3y)(x) + (-3y)(-5y) =$
$4x^2 - 20xy - 3xy + 15y^2 = 4x^2 - 23xy + 15y^2$

[9] $(2u + 3)(u - 4) + 4u(u - 2) = (2u^2 - 5u - 12) + (4u^2 - 8u) = 6u^2 - 13u - 12$

[10] $(3u - 1)(u + 2) + 7u(u + 1) = (3u^2 + 5u - 2) + (7u^2 + 7u) = 10u^2 + 12u - 2$

[11] $(3x + 5)(2x^2 + 9x - 5) = 3x(2x^2 + 9x - 5) + 5(2x^2 + 9x - 5) =$
$(6x^3 + 27x^2 - 15x) + (10x^2 + 45x - 25) = 6x^3 + 37x^2 + 30x - 25$

[12] $(7x - 4)(x^3 - x^2 + 6) = 7x(x^3 - x^2 + 6) + (-4)(x^3 - x^2 + 6) =$
$(7x^4 - 7x^3 + 42x) + (-4x^3 + 4x^2 - 24) = 7x^4 - 11x^3 + 4x^2 + 42x - 24$

[13] $(t^2 + 2t - 5)(3t^2 - t + 2) = t^2(3t^2 - t + 2) + 2t(3t^2 - t + 2) + (-5)(3t^2 - t + 2) =$
$(3t^4 - t^3 + 2t^2) + (6t^3 - 2t^2 + 4t) + (-15t^2 + 5t - 10) = 3t^4 + 5t^3 - 15t^2 + 9t - 10$

[14] $(r^2 - 8r - 2)(-r^2 + 3r - 1)$
$= r^2(-r^2 + 3r - 1) + (-8r)(-r^2 + 3r - 1) + (-2)(-r^2 + 3r - 1)$
$= (-r^4 + 3r^3 - r^2) + (8r^3 - 24r^2 + 8r) + (2r^2 - 6r + 2) = -r^4 + 11r^3 - 23r^2 + 2r + 2$

[15] $(x + 1)(2x^2 - 2)(x^3 + 5) = 2\,[(x + 1)(x^2 - 1)](x^3 + 5) = 2(x^3 + x^2 - x - 1)(x^3 + 5) =$
$2(x^6 + x^5 - x^4 + 4x^3 + 5x^2 - 5x - 5) = 2x^6 + 2x^5 - 2x^4 + 8x^3 + 10x^2 - 10x - 10$

[16] $(2x - 1)(x^2 - 5)(x^3 - 1) = (2x^3 - x^2 - 10x + 5)(x^3 - 1) =$
$2x^6 - x^5 - 10x^4 + 3x^3 + x^2 + 10x - 5$

$\boxed{17}$ $\dfrac{8x^2y^3 - 10x^3y}{2x^2y} = \dfrac{8x^2y^3}{2x^2y} - \dfrac{10x^3y}{2x^2y} = 4y^2 - 5x$

$\boxed{18}$ $\dfrac{6a^3b^3 - 9a^2b^2 + 3ab^4}{3ab^2} = \dfrac{6a^3b^3}{3ab^2} - \dfrac{9a^2b^2}{3ab^2} + \dfrac{3ab^4}{3ab^2} = 2a^2b - 3a + b^2$

$\boxed{19}$ $\dfrac{3u^3v^4 - 2u^5v^2 + (u^2v^2)^2}{u^3v^2} = \dfrac{3u^3v^4}{u^3v^2} - \dfrac{2u^5v^2}{u^3v^2} + \dfrac{u^4v^4}{u^3v^2} = 3v^2 - 2u^2 + uv^2$

$\boxed{20}$ $\dfrac{6x^2yz^3 - xy^2z}{xyz} = \dfrac{6x^2yz^3}{xyz} - \dfrac{xy^2z}{xyz} = 6xz^2 - y$

$\boxed{21}$ $(2x + 3y)(2x - 3y) = (2x)^2 - (3y)^2 = 4x^2 - 9y^2$

$\boxed{22}$ $(5x + 4y)(5x - 4y) = (5x)^2 - (4y)^2 = 25x^2 - 16y^2$

$\boxed{23}$ $(x^2 + 2y)(x^2 - 2y) = (x^2)^2 - (2y)^2 = x^4 - 4y^2$

$\boxed{24}$ $(3x + y^3)(3x - y^3) = (3x)^2 - (y^3)^2 = 9x^2 - y^6$

$\boxed{25}$ $(x^2 + 9)(x^2 - 4) = x^4 - 4x^2 + 9x^2 - 36 = x^4 + 5x^2 - 36$

$\boxed{26}$ $(x^2 + 1)(x^2 - 16) = x^4 - 16x^2 + x^2 - 16 = x^4 - 15x^2 - 16$

$\boxed{27}$ $(3x + 2y)^2 = (3x)^2 + 2(3x)(2y) + (2y)^2 = 9x^2 + 12xy + 4y^2$

$\boxed{28}$ $(5x - 4y)^2 = (5x)^2 - 2(5x)(4y) + (4y)^2 = 25x^2 - 40xy + 16y^2$

$\boxed{29}$ $(x^2 - 3y^2)^2 = (x^2)^2 - 2(x^2)(3y^2) + (3y^2)^2 = x^4 - 6x^2y^2 + 9y^4$

$\boxed{30}$ $(2x^2 + 5y^2)^2 = (2x^2)^2 + 2(2x^2)(5y^2) + (5y^2)^2 = 4x^4 + 20x^2y^2 + 25y^4$

$\boxed{31}$ $(x + 2)^2(x - 2)^2 = [(x + 2)(x - 2)]^2 = (x^2 - 4)^2 = (x^2)^2 - 2(x^2)(4) + (4)^2 =$
$x^4 - 8x^2 + 16$

$\boxed{32}$ $(x + y)^2(x - y)^2 = [(x + y)(x - y)]^2 = (x^2 - y^2)^2 = (x^2)^2 - 2(x^2)(y^2) + (y^2)^2 =$
$x^4 - 2x^2y^2 + y^4$

$\boxed{33}$ $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = x - y$

$\boxed{34}$ $(\sqrt{x} + \sqrt{y})^2(\sqrt{x} - \sqrt{y})^2 = [(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})]^2 = (x - y)^2 = x^2 - 2xy + y^2$

$\boxed{35}$ $(x^{1/3} - y^{1/3})(x^{2/3} + x^{1/3}y^{1/3} + y^{2/3})$
$= x^{1/3}(x^{2/3} + x^{1/3}y^{1/3} + y^{2/3}) - y^{1/3}(x^{2/3} + x^{1/3}y^{1/3} + y^{2/3})$
$= x + x^{2/3}y^{1/3} + x^{1/3}y^{2/3} - x^{2/3}y^{1/3} - x^{1/3}y^{2/3} - y = x - y$

$\boxed{36}$ $(x^{1/3} + y^{1/3})(x^{2/3} - x^{1/3}y^{1/3} + y^{2/3})$
$= x^{1/3}(x^{2/3} - x^{1/3}y^{1/3} + y^{2/3}) + y^{1/3}(x^{2/3} - x^{1/3}y^{1/3} + y^{2/3})$
$= x - x^{2/3}y^{1/3} + x^{1/3}y^{2/3} + x^{2/3}y^{1/3} - x^{1/3}y^{2/3} + y = x + y$

$\boxed{37}$ $(x - 2y)^3 = (x)^3 - 3(x)^2(2y) + 3(x)(2y)^2 - (2y)^3 = x^3 - 6x^2y + 12xy^2 - 8y^3$

$\boxed{38}$ $(x + 3y)^3 = (x)^3 + 3(x)^2(3y) + 3(x)(3y)^2 + (3y)^3 = x^3 + 9x^2y + 27xy^2 + 27y^3$

$\boxed{39}$ $(2x + 3y)^3 = (2x)^3 + 3(2x)^2(3y) + 3(2x)(3y)^2 + (3y)^3 = 8x^3 + 36x^2y + 54xy^2 + 27y^3$

$\boxed{40}$ $(3x - 4y)^3 = (3x)^3 - 3(3x)^2(4y) + 3(3x)(4y)^2 - (4y)^3 = 27x^3 - 108x^2y + 144xy^2 - 64y^3$

Note: Exer. 41–44: Treat these as "the sum of the squares plus twice the product of all possible pairs of terms," that is, $(x+y+z)^2 = x^2+y^2+z^2+2xy+2xz+2yz$.

$\boxed{41}$ $(a+b-c)^2 = a^2+b^2+c^2+2ab-2ac-2bc$

$\boxed{42}$ $(x^2+x+1)^2 = (x^2)^2+x^2+1+2x^3+2x^2+2x = x^4+2x^3+3x^2+2x+1$

$\boxed{43}$ $(2x+y-3z)^2 = 4x^2+y^2+9z^2+4xy-12xz-6yz$

$\boxed{44}$ $(x-2y+3z)^2 = x^2+4y^2+9z^2-4xy+6xz-12yz$

$\boxed{45}$ $rs+4st = s(r+4t)$

$\boxed{46}$ $4u^2-2uv = 2u(2u-v)$

$\boxed{47}$ $3a^2b^2-6a^2b = 3a^2b(b-2)$

$\boxed{48}$ $10xy+15xy^2 = 5xy(2+3y)$

$\boxed{49}$ $3x^2y^3-9x^3y^2 = 3x^2y^2(y-3x)$

$\boxed{50}$ $16x^5y^2+8x^3y^3 = 8x^3y^2(2x^2+y)$

$\boxed{51}$ $15x^3y^5-25x^4y^2+10x^6y^4 = 5x^3y^2(3y^3-5x+2x^3y^2)$

$\boxed{52}$ $121r^3s^4+77r^2s^4-55r^4s^3 = 11r^2s^3(11rs+7s-5r^2)$

$\boxed{53}$ $8x^2-53x-21 = (8x+3)(x-7)$

$\boxed{54}$ $7x^2+10x-8 = (7x-4)(x+2)$

$\boxed{55}$ x^2+3x+4 is irreducible

$\boxed{56}$ $3x^2-4x+2$ is irreducible

$\boxed{57}$ $6x^2+7x-20 = (3x-4)(2x+5)$

$\boxed{58}$ $12x^2-x-6 = (3x+2)(4x-3)$

$\boxed{59}$ $12x^2-29x+15 = (3x-5)(4x-3)$

$\boxed{60}$ $21x^2+41x+10 = (3x+5)(7x+2)$

$\boxed{61}$ $4x^2-20x+25 = (2x-5)(2x-5) = (2x-5)^2$

$\boxed{62}$ $9x^2+24x+16 = (3x+4)(3x+4) = (3x+4)^2$

$\boxed{63}$ $25z^2+30z+9 = (5z+3)(5z+3) = (5z+3)^2$

$\boxed{64}$ $16z^2-56z+49 = (4z-7)(4z-7) = (4z-7)^2$

$\boxed{65}$ $45x^2+38xy+8y^2 = (5x+2y)(9x+4y)$

$\boxed{66}$ $50x^2+45xy-18y^2 = (5x+6y)(10x-3y)$

$\boxed{67}$ $36r^2-25t^2 = (6r)^2-(5t)^2 = (6r+5t)(6r-5t)$

$\boxed{68}$ $81r^2-16t^2 = (9r)^2-(4t)^2 = (9r+4t)(9r-4t)$

$\boxed{69}$ $z^4-64w^2 = (z^2)^2-(8w)^2 = (z^2+8w)(z^2-8w)$

$\boxed{70}$ $9y^4-121x^2 = (3y^2)^2-(11x)^2 = (3y^2+11x)(3y^2-11x)$

$\boxed{71}$ $x^4-4x^2 = x^2(x^2-4) = x^2(x^2-2^2) = x^2(x+2)(x-2)$

$\boxed{72}$ $x^3-25x = x(x^2-25) = x(x^2-5^2) = x(x+5)(x-5)$

$\boxed{73}$ x^2+25 is irreducible

$\boxed{74}$ $4x^2+9$ is irreducible

$\boxed{75}$ $75x^2-48y^2 = 3(25x^2-16y^2) = 3\left[(5x)^2-(4y)^2\right] = 3(5x+4y)(5x-4y)$

$\boxed{76}$ $64x^2-36y^2 = 4(16x^2-9y^2) = 4\left[(4x)^2-(3y)^2\right] = 4(4x+3y)(4x-3y)$

$\boxed{77}$ $64x^3+27 = (4x)^3+(3)^3 = (4x+3)\left[(4x)^2-(4x)(3)+(3)^2\right] = (4x+3)(16x^2-12x+9)$

$\boxed{78}$ $125x^3-8 = (5x)^3-(2)^3 = (5x-2)\left[(5x)^2+(5x)(2)+(2)^2\right] = (5x-2)(25x^2+10x+4)$

$\boxed{79}$ $64x^3-y^6 = (4x)^3-(y^2)^3 = (4x-y^2)\left[(4x)^2+(4x)(y^2)+(y^2)^2\right] =$

$(4x-y^2)(16x^2+4xy^2+y^4)$

$\boxed{80}$ $216x^9 + 125y^3 = (6x^3)^3 + (5y)^3 = (6x^3 + 5y)\,[(6x^3)^2 - (6x^3)(5y) + (5y)^2] =$
$(6x^3 + 5y)(36x^6 - 30x^3y + 25y^2)$

$\boxed{81}$ $343x^3 + y^9 = (7x)^3 + (y^3)^3 = (7x + y^3)\,[(7x)^2 - (7x)(y^3) + (y^3)^2] =$
$(7x + y^3)(49x^2 - 7xy^3 + y^6)$

$\boxed{82}$ $x^6 - 27y^3 = (x^2)^3 - (3y)^3 = (x^2 - 3y)\,[(x^2)^2 + (x^2)(3y) + (3y)^2] =$
$(x^2 - 3y)(x^4 + 3x^2y + 9y^2)$

$\boxed{83}$ $125 - 27x^3 = (5)^3 - (3x)^3 = (5 - 3x)\,[(5)^2 + (5)(3x) + (3x)^2] =$
$(5 - 3x)(25 + 15x + 9x^2)$

$\boxed{84}$ $x^3 + 64 = (x)^3 + (4)^3 = (x + 4)\,[(x)^2 - (x)(4) + (4)^2] = (x + 4)(x^2 - 4x + 16)$

$\boxed{85}$ $2ax - 6bx + ay - 3by = 2x(a - 3b) + y(a - 3b) = (2x + y)(a - 3b)$

$\boxed{86}$ $2ay^2 - axy + 6xy - 3x^2 = ay(2y - x) + 3x(2y - x) = (ay + 3x)(2y - x)$

$\boxed{87}$ $3x^3 + 3x^2 - 27x - 27 = 3(x^3 + x^2 - 9x - 9) =$
$3[x^2(x + 1) - 9(x + 1)] = 3(x^2 - 9)(x + 1) = 3(x + 3)(x - 3)(x + 1)$

$\boxed{88}$ $5x^3 + 10x^2 - 20x - 40 = 5(x^3 + 2x^2 - 4x - 8) = 5[x^2(x + 2) - 4(x + 2)] =$
$5(x^2 - 4)(x + 2) = 5(x + 2)(x - 2)(x + 2) = 5(x + 2)^2(x - 2)$

$\boxed{89}$ $x^4 + 2x^3 - x - 2 = x^3(x + 2) - 1(x + 2) = (x^3 - 1)(x + 2) = (x - 1)(x + 2)(x^2 + x + 1)$

$\boxed{90}$ $x^4 - 3x^3 + 8x - 24 = x^3(x - 3) + 8(x - 3) = (x^3 + 8)(x - 3) =$
$(x + 2)(x - 3)(x^2 - 2x + 4)$

$\boxed{91}$ $a^3 - a^2b + ab^2 - b^3 = a^2(a - b) + b^2(a - b) = (a^2 + b^2)(a - b)$

$\boxed{92}$ $6w^8 + 17w^4 + 12 = (2w^4 + 3)(3w^4 + 4)$

$\boxed{93}$ $a^6 - b^6 = (a^3)^2 - (b^3)^2 = (a^3 + b^3)(a^3 - b^3) = (a + b)(a - b)(a^2 - ab + b^2)(a^2 + ab + b^2)$

$\boxed{94}$ $x^8 - 16 = (x^4)^2 - 4^2 = (x^4 + 4)(x^4 - 4) = (x^4 + 4)(x^2 + 2)(x^2 - 2)$

$\boxed{95}$ $x^2 + 4x + 4 - 9y^2 = (x + 2)^2 - (3y)^2 = (x + 2 + 3y)(x + 2 - 3y)$

$\boxed{96}$ $x^2 - 4y^2 - 6x + 9 = (x^2 - 6x + 9) - 4y^2 = (x - 3)^2 - (2y)^2 = (x - 3 + 2y)(x - 3 - 2y)$

$\boxed{97}$ $y^2 - x^2 + 8y + 16 = (y^2 + 8y + 16) - x^2 = (y + 4)^2 - (x)^2 = (y + 4 + x)(y + 4 - x)$

$\boxed{98}$ $y^2 + 9 - 6y - 4x^2 = (y^2 - 6y + 9) - 4x^2 = (y - 3)^2 - (2x)^2 = (y - 3 + 2x)(y - 3 - 2x)$

$\boxed{99}$ $y^6 + 7y^3 - 8 = (y^3 + 8)(y^3 - 1) = (y + 2)(y^2 - 2y + 4)(y - 1)(y^2 + y + 1)$

$\boxed{100}$ $8c^6 + 19c^3 - 27 = (8c^3 + 27)(c^3 - 1) = (2c + 3)(4c^2 - 6c + 9)(c - 1)(c^2 + c + 1)$

$\boxed{101}$ $x^{16} - 1 = (x^8 + 1)(x^8 - 1) = (x^8 + 1)(x^4 + 1)(x^4 - 1) =$
$(x^8 + 1)(x^4 + 1)(x^2 + 1)(x^2 - 1) = (x^8 + 1)(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$

$\boxed{102}$ $4x^3 + 4x^2 + x = x(4x^2 + 4x + 1) = x(2x + 1)(2x + 1) = x(2x + 1)^2$

$\boxed{103}$ Area of I is $(x - y)x$, area of II is $(x - y)y$,
and $A = x^2 - y^2 = (x - y)x + (x - y)y = (x - y)(x + y)$.

$\boxed{104}$ Volume of I is $x^2(x-y)$, volume of II is $xy(x-y)$, and volume of III is $y^2(x-y)$.

$$V = x^3 - y^3 = x^2(x-y) + xy(x-y) + y^2(x-y) = (x-y)(x^2+xy+y^2).$$

$\boxed{105}$ (a) For the 25-year-old female, use

$C_f = 66.5 + 13.8w + 5h - 6.8y$ with $w = 59$, $h = 163$, and $y = 25$.

$C_f = 66.5 + 13.8(59) + 5(163) - 6.8(25) = 1525.7$ calories

For the 55-year-old male, use

$C_m = 655 + 9.6w + 1.9h - 4.7y$ with $w = 75$, $h = 178$, and $y = 55$.

$C_m = 655 + 9.6(75) + 1.9(178) - 4.7(55) = 1454.7$ calories

(b) As people age they require fewer calories. The coefficients of w and h are positive because large people require more calories.

1.4 Exercises

$\boxed{1}$ $\frac{3}{50} + \frac{7}{30} = \frac{3}{2\cdot 5^2} + \frac{7}{2\cdot 3\cdot 5} = \frac{3\cdot 3 + 7\cdot 5}{2\cdot 3\cdot 5^2} = \frac{9+35}{2\cdot 3\cdot 5^2} = \frac{44}{2\cdot 3\cdot 5^2} = \frac{22}{3\cdot 5^2} = \frac{22}{75}$

$\boxed{2}$ $\frac{4}{63} + \frac{5}{42} = \frac{4}{3^2\cdot 7} + \frac{5}{2\cdot 3\cdot 7} = \frac{4\cdot 2 + 5\cdot 3}{2\cdot 3^2\cdot 7} = \frac{8+15}{2\cdot 3^2\cdot 7} = \frac{23}{2\cdot 3^2\cdot 7} = \frac{23}{126}$

$\boxed{3}$ $\frac{5}{24} - \frac{3}{20} = \frac{5}{2^3\cdot 3} - \frac{3}{2^2\cdot 5} = \frac{5\cdot 5 - 3(2\cdot 3)}{2^3\cdot 3\cdot 5} = \frac{25-18}{2^3\cdot 3\cdot 5} = \frac{7}{2^3\cdot 3\cdot 5} = \frac{7}{120}$

$\boxed{4}$ $\frac{11}{54} - \frac{7}{72} = \frac{11}{2\cdot 3^3} - \frac{7}{2^3\cdot 3^2} = \frac{11\cdot 2^2 - 7\cdot 3}{2^3\cdot 3^3} = \frac{44-21}{2^3\cdot 3^3} = \frac{23}{2^3\cdot 3^3} = \frac{23}{216}$

$\boxed{5}$ $\frac{2x^2+7x+3}{2x^2-7x-4} = \frac{(2x+1)(x+3)}{(2x+1)(x-4)} = \frac{x+3}{x-4}$

$\boxed{6}$ $\frac{2x^2+9x-5}{3x^2+17x+10} = \frac{(x+5)(2x-1)}{(x+5)(3x+2)} = \frac{2x-1}{3x+2}$

$\boxed{7}$ $\frac{y^2-25}{y^3-125} = \frac{(y+5)(y-5)}{(y-5)(y^2+5y+25)} = \frac{y+5}{y^2+5y+25}$

$\boxed{8}$ $\frac{y^2-9}{y^3+27} = \frac{(y+3)(y-3)}{(y+3)(y^2-3y+9)} = \frac{y-3}{y^2-3y+9}$

$\boxed{9}$ $\frac{12+r-r^2}{r^3+3r^2} = \frac{(3+r)(4-r)}{r^2(r+3)} = \frac{4-r}{r^2}$

$\boxed{10}$ $\frac{10+3r-r^2}{r^4+2r^3} = \frac{(2+r)(5-r)}{r^3(r+2)} = \frac{5-r}{r^3}$

$\boxed{11}$ $\frac{9x^2-4}{3x^2-5x+2}\cdot\frac{9x^4-6x^3+4x^2}{27x^4+8x} = \frac{(3x+2)(3x-2)}{(3x-2)(x-1)}\cdot\frac{x^2(9x^2-6x+4)}{x(3x+2)(9x^2-6x+4)} = \frac{x}{x-1}$

$\boxed{12}$ $\frac{4x^2-9}{2x^2+7x+6}\cdot\frac{4x^4+6x^3+9x^2}{8x^7-27x^4} = \frac{(2x+3)(2x-3)}{(2x+3)(x+2)}\cdot\frac{x^2(4x^2+6x+9)}{x^4(2x-3)(4x^2+6x+9)} = \frac{1}{x^2(x+2)}$

$\boxed{13}$ $\frac{5a^2+12a+4}{a^4-16} \div \frac{25a^2+20a+4}{a^2-2a} = \frac{(5a+2)(a+2)}{(a^2+4)(a+2)(a-2)}\cdot\frac{a(a-2)}{(5a+2)(5a+2)} =$

$$\frac{a}{(a^2+4)(5a+2)}$$

14 $$\frac{a^3-8}{a^2-4} \div \frac{a}{a^3+8} = \frac{(a-2)(a^2+2a+4)}{(a+2)(a-2)} \cdot \frac{(a+2)(a^2-2a+4)}{a} = \frac{(a^2+2a+4)(a^2-2a+4)}{a}$$

15 $$\frac{6}{x^2-4} - \frac{3x}{x^2-4} = \frac{6-3x}{x^2-4} = \frac{3(2-x)}{(x+2)(x-2)} = \frac{-3}{x+2}$$

16 $$\frac{15}{x^2-9} - \frac{5x}{x^2-9} = \frac{15-5x}{x^2-9} = \frac{5(3-x)}{(x+3)(x-3)} = \frac{-5}{x+3}$$

17 $$\frac{2}{3s+1} - \frac{9}{(3s+1)^2} = \frac{2(3s+1)-9}{(3s+1)^2} = \frac{6s-7}{(3s+1)^2}$$

18 $$\frac{4}{(5s-2)^2} + \frac{s}{5s-2} = \frac{4+s(5s-2)}{(5s-2)^2} = \frac{5s^2-2s+4}{(5s-2)^2}$$

19 $$\frac{2}{x} + \frac{3x+1}{x^2} - \frac{x-2}{x^3} = \frac{2x^2+(3x+1)x-x+2}{x^3} = \frac{5x^2+2}{x^3}$$

20 $$\frac{5}{x} - \frac{2x-1}{x^2} + \frac{x+5}{x^3} = \frac{5x^2-x(2x-1)+x+5}{x^3} = \frac{3x^2+2x+5}{x^3}$$

21 $$\frac{3t}{t+2} + \frac{5t}{t-2} - \frac{40}{t^2-4} = \frac{3t(t-2)+5t(t+2)-40}{t^2-4} = \frac{8t^2+4t-40}{t^2-4} = \frac{4(2t+5)(t-2)}{(t+2)(t-2)} = \frac{4(2t+5)}{t+2}$$

22 $$\frac{t}{t+3} + \frac{4t}{t-3} - \frac{18}{t^2-9} = \frac{t(t-3)+4t(t+3)-18}{t^2-9} = \frac{5t^2+9t-18}{t^2-9} = \frac{(5t-6)(t+3)}{(t+3)(t-3)} = \frac{5t-6}{t-3}$$

23 $$\frac{4x}{3x-4} + \frac{8}{3x^2-4x} + \frac{2}{x} = \frac{4x(x)+8+2(3x-4)}{x(3x-4)} = \frac{4x^2+6x}{x(3x-4)} = \frac{2x(2x+3)}{x(3x-4)} = \frac{2(2x+3)}{3x-4}$$

24 $$\frac{12x}{2x+1} - \frac{3}{2x^2+x} + \frac{5}{x} = \frac{12x(x)-3+5(2x+1)}{x(2x+1)} = \frac{12x^2+10x+2}{x(2x+1)} = \frac{2(6x^2+5x+1)}{x(2x+1)} = \frac{2(2x+1)(3x+1)}{x(2x+1)} = \frac{2(3x+1)}{x}$$

25 $$\frac{2x}{x+2} - \frac{8}{x^2+2x} + \frac{3}{x} = \frac{2x(x)-8+3(x+2)}{x(x+2)} = \frac{2x^2+3x-2}{x(x+2)} = \frac{(2x-1)(x+2)}{x(x+2)} = \frac{2x-1}{x}$$

26 $$\frac{5x}{2x+3} - \frac{6}{2x^2+3x} + \frac{2}{x} = \frac{5x(x)-6+2(2x+3)}{x(2x+3)} = \frac{5x^2+4x}{x(2x+3)} = \frac{x(5x+4)}{x(2x+3)} = \frac{5x+4}{2x+3}$$

27 $$\frac{p^4+3p^3-8p-24}{p^3-2p^2-9p+18} = \frac{p^3(p+3)-8(p+3)}{p^2(p-2)-9(p-2)} = \frac{(p^3-8)(p+3)}{(p^2-9)(p-2)} = \frac{(p-2)(p^2+2p+4)(p+3)}{(p+3)(p-3)(p-2)} = \frac{p^2+2p+4}{p-3}$$

28 $$\frac{2ac+bc-6ad-3bd}{6ac+2ad+3bc+bd} = \frac{c(2a+b)-3d(2a+b)}{2a(3c+d)+b(3c+d)} = \frac{(c-3d)(2a+b)}{(2a+b)(3c+d)} = \frac{c-3d}{3c+d}$$

$\boxed{29}$ $3+\frac{5}{u}+\frac{2u}{3u+1}=\frac{3u(3u+1)+5(3u+1)+2u(u)}{u(3u+1)}=\frac{11u^2+18u+5}{u(3u+1)}$

$\boxed{30}$ $4+\frac{2}{u}-\frac{3u}{u+5}=\frac{4u(u+5)+2(u+5)-3u(u)}{u(u+5)}=\frac{u^2+22u+10}{u(u+5)}$

$\boxed{31}$ $\frac{2x+1}{x^2+4x+4}-\frac{6x}{x^2-4}+\frac{3}{x-2}=\frac{(2x+1)(x-2)-6x(x+2)+3(x^2+4x+4)}{(x+2)^2(x-2)}=$

$\frac{-x^2-3x+10}{(x+2)^2(x-2)}=-\frac{x^2+3x-10}{(x+2)^2(x-2)}=-\frac{(x+5)(x-2)}{(x+2)^2(x-2)}=-\frac{x+5}{(x+2)^2}$

$\boxed{32}$ $\frac{2x+6}{x^2+6x+9}+\frac{5x}{x^2-9}+\frac{7}{x-3}=\frac{2}{x+3}+\frac{5x}{x^2-9}+\frac{7}{x-3}=\frac{2(x-3)+5x+7(x+3)}{x^2-9}=$

$\frac{14x+15}{x^2-9}$

$\boxed{33}$ $\dfrac{\frac{b}{a}-\frac{a}{b}}{\frac{1}{a}-\frac{1}{b}}=\dfrac{\left(\frac{b}{a}-\frac{a}{b}\right)\cdot ab}{\left(\frac{1}{a}-\frac{1}{b}\right)\cdot ab}=\frac{b^2-a^2}{b-a}=\frac{(b+a)(b-a)}{b-a}=a+b$

$\boxed{34}$ $\dfrac{\frac{1}{x+2}-3}{\frac{4}{x}-x}=\dfrac{\frac{1-3(x+2)}{x+2}}{\frac{4-x(x)}{x}}=\dfrac{\frac{-3x-5}{x+2}}{\frac{4-x^2}{x}}=\frac{-(3x+5)x}{(x+2)(2+x)(2-x)}=\frac{x(3x+5)}{(x-2)(x+2)^2}$

$\boxed{35}$ $\dfrac{\frac{x}{y^2}-\frac{y}{x^2}}{\frac{1}{y^2}-\frac{1}{x^2}}=\dfrac{\left(\frac{x}{y^2}-\frac{y}{x^2}\right)\cdot x^2y^2}{\left(\frac{1}{y^2}-\frac{1}{x^2}\right)\cdot x^2y^2}=\frac{x^3-y^3}{x^2-y^2}=\frac{(x-y)(x^2+xy+y^2)}{(x+y)(x-y)}=\frac{x^2+xy+y^2}{x+y}$

$\boxed{36}$ $\dfrac{\frac{r}{s}+\frac{s}{r}}{\frac{r^2}{s^2}-\frac{s^2}{r^2}}=\dfrac{\left(\frac{r}{s}+\frac{s}{r}\right)\cdot r^2s^2}{\left(\frac{r^2}{s^2}-\frac{s^2}{r^2}\right)\cdot r^2s^2}=\frac{r^3s+rs^3}{r^4-s^4}=\frac{rs(r^2+s^2)}{(r^2+s^2)(r^2-s^2)}=\frac{rs}{r^2-s^2}$

$\boxed{37}$ $\dfrac{y^{-1}+x^{-1}}{(xy)^{-1}}=\dfrac{\frac{1}{y}+\frac{1}{x}}{\frac{1}{xy}}=\dfrac{\left(\frac{1}{y}+\frac{1}{x}\right)\cdot xy}{\left(\frac{1}{xy}\right)\cdot xy}=\frac{x+y}{1}=x+y$

$\boxed{38}$ $\dfrac{y^{-2}-x^{-2}}{y^{-2}+x^{-2}}=\dfrac{\frac{1}{y^2}-\frac{1}{x^2}}{\frac{1}{y^2}+\frac{1}{x^2}}=\dfrac{\left(\frac{1}{y^2}-\frac{1}{x^2}\right)\cdot x^2y^2}{\left(\frac{1}{y^2}+\frac{1}{x^2}\right)\cdot x^2y^2}=\frac{x^2-y^2}{x^2+y^2}$

$\boxed{39}$ $\dfrac{\frac{5}{x+1}+\frac{2x}{x+3}}{\frac{x}{x+1}+\frac{7}{x+3}}=\dfrac{\frac{5(x+3)+2x(x+1)}{(x+1)(x+3)}}{\frac{x(x+3)+7(x+1)}{(x+1)(x+3)}}=\frac{5x+15+2x^2+2x}{x^2+3x+7x+7}=\frac{2x^2+7x+15}{x^2+10x+7}$

$\boxed{40}$ $\dfrac{\frac{3}{w}-\frac{6}{2w+1}}{\frac{5}{w}+\frac{8}{2w+1}}=\dfrac{\frac{3(2w+1)-6w}{w(2w+1)}}{\frac{5(2w+1)+8w}{w(2w+1)}}=\frac{6w+3-6w}{10w+5+8w}=\frac{3}{18w+5}$

[41] $$\frac{\frac{3}{x-1}-\frac{3}{a-1}}{x-a}=\frac{\frac{3(a-1)-3(x-1)}{(x-1)(a-1)}}{x-a}=\frac{3a-3x}{(x-1)(a-1)(x-a)}=\frac{3(a-x)}{(x-1)(a-1)(x-a)}=-\frac{3}{(x-1)(a-1)}$$

[42] $$\frac{\frac{x+2}{x}-\frac{a+2}{a}}{x-a}=\frac{\frac{a(x+2)-x(a+2)}{ax}}{x-a}=\frac{2a-2x}{ax(x-a)}=\frac{2(a-x)}{ax(x-a)}=-\frac{2}{ax}$$

[43] $$\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}=\frac{2xh+h^2-3h}{h}=\frac{h(2x+h-3)}{h}=2x+h-3$$

[44] $$\frac{(x+h)^3+5(x+h)-(x^3+5x)}{h}=\frac{3x^2h+3xh^2+h^3+5h}{h}=\frac{h(3x^2+3xh+h^2+5)}{h}=3x^2+3xh+h^2+5$$

[45] $$\frac{\frac{1}{(x+h)^3}-\frac{1}{x^3}}{h}=\frac{\frac{x^3-(x+h)^3}{(x+h)^3x^3}}{h}=\frac{x^3-(x+h)^3}{hx^3(x+h)^3}=$$
$$\frac{[x-(x+h)][x^2+x(x+h)+(x+h)^2]}{hx^3(x+h)^3}=\frac{-h(3x^2+3xh+h^2)}{hx^3(x+h)^3}=-\frac{3x^2+3xh+h^2}{x^3(x+h)^3}$$

[46] $$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\frac{\frac{x-(x+h)}{(x+h)x}}{h}=\frac{-h}{hx(x+h)}=\frac{-1}{x(x+h)}$$

[47] $$\frac{\frac{4}{3x+3h-1}-\frac{4}{3x-1}}{h}=\frac{\frac{4(3x-1)-4(3x+3h-1)}{(3x+3h-1)(3x-1)}}{h}=\frac{12x-4-12x-12h+4}{h(3x+3h-1)(3x-1)}=\frac{-12h}{h(3x+3h-1)(3x-1)}=\frac{-12}{(3x+3h-1)(3x-1)}$$

[48] $$\frac{\frac{5}{2x+2h+3}-\frac{5}{2x+3}}{h}=\frac{\frac{5(2x+3)-5(2x+2h+3)}{(2x+2h+3)(2x+3)}}{h}=\frac{10x+15-10x-10h-15}{h(2x+2h+3)(2x+3)}=\frac{-10h}{h(2x+2h+3)(2x+3)}=\frac{-10}{(2x+2h+3)(2x+3)}$$

[49] $$\frac{\sqrt{t}+5}{\sqrt{t}-5}=\frac{\sqrt{t}+5}{\sqrt{t}-5}\cdot\frac{\sqrt{t}+5}{\sqrt{t}+5}=\frac{t+10\sqrt{t}+25}{t-25}$$

[50] $$\frac{\sqrt{t}-4}{\sqrt{t}+4}=\frac{\sqrt{t}-4}{\sqrt{t}+4}\cdot\frac{\sqrt{t}-4}{\sqrt{t}-4}=\frac{t-8\sqrt{t}+16}{t-16}$$

[51] $$\frac{81x^2-16y^2}{3\sqrt{x}-2\sqrt{y}}=\frac{81x^2-16y^2}{3\sqrt{x}-2\sqrt{y}}\cdot\frac{3\sqrt{x}+2\sqrt{y}}{3\sqrt{x}+2\sqrt{y}}=\frac{(9x+4y)(9x-4y)(3\sqrt{x}+2\sqrt{y})}{9x-4y}=(9x+4y)(3\sqrt{x}+2\sqrt{y})$$

[52] $$\frac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}=\frac{16x^2-y^2}{2\sqrt{x}-\sqrt{y}}\cdot\frac{2\sqrt{x}+\sqrt{y}}{2\sqrt{x}+\sqrt{y}}=\frac{(4x+y)(4x-y)(2\sqrt{x}+\sqrt{y})}{4x-y}=(4x+y)(2\sqrt{x}+\sqrt{y})$$

$\boxed{53}$ $$\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}}=\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}}\cdot\frac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}=\frac{\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2}}{a-b}$$

$\boxed{54}$ $$\frac{1}{\sqrt[3]{x}+\sqrt[3]{y}}=\frac{1}{\sqrt[3]{x}+\sqrt[3]{y}}\cdot\frac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}=\frac{\sqrt[3]{x^2}-\sqrt[3]{xy}+\sqrt[3]{y^2}}{x+y}$$

$\boxed{55}$ $$\frac{\sqrt{a}-\sqrt{b}}{a^2-b^2}=\frac{\sqrt{a}-\sqrt{b}}{a^2-b^2}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}=\frac{a-b}{(a+b)(a-b)(\sqrt{a}+\sqrt{b})}=\frac{1}{(a+b)(\sqrt{a}+\sqrt{b})}$$

$\boxed{56}$ $$\frac{\sqrt{b}+\sqrt{c}}{b^2-c^2}=\frac{\sqrt{b}+\sqrt{c}}{b^2-c^2}\cdot\frac{\sqrt{b}-\sqrt{c}}{\sqrt{b}-\sqrt{c}}=\frac{b-c}{(b+c)(b-c)(\sqrt{b}-\sqrt{c})}=\frac{1}{(b+c)(\sqrt{b}-\sqrt{c})}$$

$\boxed{57}$ $$\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}=\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\frac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}=$$
$$\frac{(2x+2h+1)-(2x+1)}{h(\sqrt{2(x+h)+1}+\sqrt{2x+1})}=\frac{2}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}$$

$\boxed{58}$ $$\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}=\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}=\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}=$$
$$\frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$\boxed{59}$ $$\frac{\sqrt{1-x-h}-\sqrt{1-x}}{h}=\frac{\sqrt{1-x-h}-\sqrt{1-x}}{h}\cdot\frac{\sqrt{1-x-h}+\sqrt{1-x}}{\sqrt{1-x-h}+\sqrt{1-x}}=$$
$$\frac{(1-x-h)-(1-x)}{h(\sqrt{1-x-h}+\sqrt{1-x})}=\frac{-1}{\sqrt{1-x-h}+\sqrt{1-x}}$$

$\boxed{60}$ $$\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}=\frac{\sqrt[3]{x+h}-\sqrt[3]{x}}{h}\cdot\frac{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}=$$
$$\frac{(x+h)-x}{h(\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2})}=\frac{1}{\sqrt[3]{(x+h)^2}+\sqrt[3]{x(x+h)}+\sqrt[3]{x^2}}$$

$\boxed{61}$ $$\frac{4x^2-x+5}{x^{2/3}}=\frac{4x^2}{x^{2/3}}-\frac{x}{x^{2/3}}+\frac{5}{x^{2/3}}=4x^{4/3}-x^{1/3}+5x^{-2/3}$$

$\boxed{62}$ $$\frac{x^2+4x-6}{\sqrt{x}}=\frac{x^2}{\sqrt{x}}+\frac{4x}{\sqrt{x}}-\frac{6}{\sqrt{x}}=x^{3/2}+4x^{1/2}-6x^{-1/2}$$

$\boxed{63}$ $$\frac{(x^2+2)^2}{x^5}=\frac{x^4+4x^2+4}{x^5}=\frac{x^4}{x^5}+\frac{4x^2}{x^5}+\frac{4}{x^5}=x^{-1}+4x^{-3}+4x^{-5}$$

$\boxed{64}$ $$\frac{(\sqrt{x}-3)^2}{x^3}=\frac{x-6\sqrt{x}+9}{x^3}=\frac{x}{x^3}-\frac{6\sqrt{x}}{x^3}+\frac{9}{x^3}=x^{-2}-6x^{-5/2}+9x^{-3}$$

Note: You may wish to demonstrate the 3 techniques shown in Example 9 with one of these simpler expressions in 65–68. Exercises 65–82 are worked using the factoring concept given as the third method of simplification in Example 9.

$\boxed{65}$ $x^{-3}+x^2=x^{-3}(1+x^5)=\dfrac{1+x^5}{x^3}$

$\boxed{66}$ $x^{-4}-x=x^{-4}(1-x^5)=\dfrac{1-x^5}{x^4}$

$\boxed{67}$ $x^{-1/2}-x^{3/2}=x^{-1/2}(1-x^2)=\dfrac{1-x^2}{x^{1/2}}$

$\boxed{68}$ $x^{-2/3}+x^{7/3}=x^{-2/3}(1+x^3)=\dfrac{1+x^3}{x^{2/3}}$

$\boxed{69}$ $(2x^2-3x+1)(4)(3x+2)^3(3)+(3x+2)^4(4x-3)=$
$$(3x+2)^3[12(2x^2-3x+1)+(3x+2)(4x-3)]=(3x+2)^3(36x^2-37x+6)$$

$\boxed{70}$ $(6x-5)^3(2)(x^2+4)(2x)+(x^2+4)^2(3)(6x-5)^2(6)=$
$$2(6x-5)^2(x^2+4)[2x(6x-5)+9(x^2+4)]=2(x^2+4)(6x-5)^2(21x^2-10x+36)$$

$\boxed{71}$ $(x^2-4)^{1/2}(3)(2x+1)^2(2)+(2x+1)^3(\tfrac{1}{2})(x^2-4)^{-1/2}(2x)=$
$$(x^2-4)^{-1/2}(2x+1)^2[6(x^2-4)+x(2x+1)]=\frac{(2x+1)^2(8x^2+x-24)}{(x^2-4)^{1/2}}$$

$\boxed{72}$ $(3x+2)^{1/3}(2)(4x-5)(4)+(4x-5)^2(\tfrac{1}{3})(3x+2)^{-2/3}(3)=$
$$(3x+2)^{-2/3}(4x-5)[8(3x+2)+(4x-5)]=\frac{(4x-5)(28x+11)}{(3x+2)^{2/3}}$$

$\boxed{73}$ $(3x+1)^6(\tfrac{1}{2})(2x-5)^{-1/2}(2)+(2x-5)^{1/2}(6)(3x+1)^5(3)=$
$$(3x+1)^5(2x-5)^{-1/2}[(3x+1)+18(2x-5)]=\frac{(3x+1)^5(39x-89)}{(2x-5)^{1/2}}$$

$\boxed{74}$ $(x^2+9)^4(-\tfrac{1}{3})(x+6)^{-4/3}+(x+6)^{-1/3}(4)(x^2+9)^3(2x)=$
$$(\tfrac{1}{3})(x^2+9)^3(x+6)^{-4/3}[-(x^2+9)+24x(x+6)]=\frac{(x^2+9)^3(23x^2+144x-9)}{3(x+6)^{4/3}}$$

$\boxed{75}$ $\dfrac{(6x+1)^3(27x^2+2)-(9x^3+2x)(3)(6x+1)^2(6)}{(6x+1)^6}=$
$$\frac{(6x+1)^2[(6x+1)(27x^2+2)-18(9x^3+2x)]}{(6x+1)^6}=\frac{27x^2-24x+2}{(6x+1)^4}$$

$\boxed{76}$ $\dfrac{(x^2-1)^4(2x)-x^2(4)(x^2-1)^3(2x)}{(x^2-1)^8}=\dfrac{(2x)(x^2-1)^3[(x^2-1)-4x^2]}{(x^2-1)^8}=\dfrac{-2x(3x^2+1)}{(x^2-1)^5}$

$\boxed{77}$ $\dfrac{(x^2+2)^3(2x)-x^2(3)(x^2+2)^2(2x)}{[(x^2+2)^3]^2}=\dfrac{(x^2+2)^2(2x)\big[(x^2+2)^1-x^2(3)\big]}{(x^2+2)^6}=$
$$\frac{2x(x^2+2-3x^2)}{(x^2+2)^4}=\frac{2x(2-2x^2)}{(x^2+2)^4}=\frac{4x(1-x^2)}{(x^2+2)^4}$$

$\boxed{78}$ $\dfrac{(x^2-5)^4(3x^2)-x^3(4)(x^2-5)^3(2x)}{[(x^2-5)^4]^2}=\dfrac{(x^2-5)^3(x^2)\big[(x^2-5)^1(3)-(x)(4)(2x)\big]}{(x^2-5)^8}=$
$$\frac{x^2(3x^2-15-8x^2)}{(x^2-5)^5}=\frac{x^2(-5x^2-15)}{(x^2-5)^5}=-\frac{5x^2(x^2+3)}{(x^2-5)^5}$$

$\sqrt{9+16} = \sqrt{9} + \sqrt{16}$

[79] $\dfrac{(x^2+4)^{1/3}(3)-(3x)(\frac{1}{3})(x^2+4)^{-2/3}(2x)}{[(x^2+4)^{1/3}]^2} = \dfrac{(x^2+4)^{-2/3}[3(x^2+4)-2x^2]}{(x^2+4)^{2/3}} = \dfrac{x^2+12}{(x^2+4)^{4/3}}$

[80] $\dfrac{(1-x^2)^{1/2}(2x)-x^2(\frac{1}{2})(1-x^2)^{-1/2}(-2x)}{[(1-x^2)^{1/2}]^2} = \dfrac{x(1-x^2)^{-1/2}[2(1-x^2)+x^2]}{(1-x^2)} = \dfrac{x(2-x^2)}{(1-x^2)^{3/2}}$

[81] $\dfrac{(4x^2+9)^{1/2}(2)-(2x+3)(\frac{1}{2})(4x^2+9)^{-1/2}(8x)}{[(4x^2+9)^{1/2}]^2} =$

$$\dfrac{(4x^2+9)^{-1/2}[2(4x^2+9)-4x(2x+3)]}{(4x^2+9)} = \dfrac{18-12x}{(4x^2+9)^{3/2}} = \dfrac{6(3-2x)}{(4x^2+9)^{3/2}}$$

[82] $\dfrac{(3x+2)^{1/2}(\frac{1}{3})(2x+3)^{-2/3}(2)-(2x+3)^{1/3}(\frac{1}{2})(3x+2)^{-1/2}(3)}{[(3x+2)^{1/2}]^2} =$

$$\dfrac{(\frac{1}{3})(\frac{1}{2})(3x+2)^{-1/2}(2x+3)^{-2/3}[4(3x+2)-9(2x+3)]}{3x+2} = -\dfrac{6x+19}{6(3x+2)^{3/2}(2x+3)^{2/3}}$$

[83] Table $Y_1 = \dfrac{113x^3+280x^2-150x}{22x^3+77x^2-100x-350}$ and $Y_2 = \dfrac{3x}{2x+7}+\dfrac{4x^2}{1.1x^2-5}$.

x	Y_1	Y_2
1	−0.6923	−0.6923
2	−26.12	−26.12
3	8.0392	8.0392
4	5.8794	5.8794
5	5.3268	5.3268

The values for Y_1 and Y_2 agree. Therefore, the two expressions might be equal.

[84] Table $Y_1 = \dfrac{20x^2+41x+31}{10x^3+10x^2}$ and $Y_2 = \dfrac{1}{x}+\dfrac{1}{x+1}+\dfrac{3.2}{x^2}$.

x	Y_1	Y_2
1	4.6	4.7
2	1.6083	1.6333
3	0.92778	0.93889
4	0.64375	0.65
5	0.49067	0.49467

The values for Y_1 and Y_2 do not agree. Therefore, the two expressions are not equal.

Chapter 1 Review Exercises

[1] (a) $(\frac{2}{3})(-\frac{5}{8}) = -\frac{1}{3}\cdot\frac{5}{4} = -\frac{5}{12}$

(b) $\frac{3}{4}+\frac{6}{5} = \frac{15}{20}+\frac{24}{20} = \frac{39}{20}$

(c) $\frac{5}{8}-\frac{6}{7} = \frac{35}{56}-\frac{48}{56} = -\frac{13}{56}$

(d) $\frac{3}{4}\div\frac{6}{5} = \frac{3}{4}\cdot\frac{5}{6} = \frac{1}{4}\cdot\frac{5}{2} = \frac{5}{8}$

[2] (a) Since -0.1 is to the left of -0.001 on a coordinate line, $-0.1 \boxed{<} -0.001$.

(b) Since $\sqrt{9} = 3$ and 3 is to the right of -3 on a coordinate line, $\sqrt{9} \boxed{>} -3$.

(c) Since $\frac{1}{6} = 0.1\overline{6}$, $\frac{1}{6} \boxed{>} 0.166$.

[3] (a) x is negative $\Leftrightarrow$ $x < 0$ (b) a is between $\frac{1}{2}$ and $\frac{1}{3}$ $\Leftrightarrow$ $\frac{1}{3} < a < \frac{1}{2}$

(c) The absolute value of x is not greater than 4 $\Leftrightarrow$ $|x| \leq 4$

[4] (a) $|-7| = -(-7) = 7$ (b) $\dfrac{|-5|}{-5} = \dfrac{-(-5)}{-5} = \dfrac{5}{-5} = -1$

(c) $|3^{-1} - 2^{-1}| = \left|\frac{1}{3} - \frac{1}{2}\right| = \left|\frac{2}{6} - \frac{3}{6}\right| = \left|-\frac{1}{6}\right| = -(-\frac{1}{6}) = \frac{1}{6}$

[5] (a) $d(A, C) = |-3 - (-8)| = |5| = 5$ (b) $d(C, A) = d(A, C) = 5$

(c) $d(B, C) = |-3 - 4| = |-7| = -(-7) = 7$

[6] (a) $(x+y)^2 = x^2 + 2xy + y^2 \boxed{\neq} x^2 + y^2$ for every nonzero x and nonzero y.

(b) $\dfrac{1}{\sqrt{x+y}} = \dfrac{1}{\sqrt{x}} + \dfrac{1}{\sqrt{y}}$ is not true if $x = y = 1$.

(c) $\dfrac{1}{\sqrt{c} - \sqrt{d}} = \dfrac{1}{\sqrt{c} - \sqrt{d}} \cdot \dfrac{\sqrt{c} + \sqrt{d}}{\sqrt{c} + \sqrt{d}} \boxed{=} \dfrac{\sqrt{c} + \sqrt{d}}{c - d}$

[7] (a) $93{,}700{,}000{,}000 = 9.37 \times 10^{10}$ (b) $0.000\ 004\ 02 = 4.02 \times 10^{-6}$

[8] (a) $6.8 \times 10^7 = 68{,}000{,}000$ (b) $7.3 \times 10^{-4} = 0.000\ 73$

[9] If $x \leq -3$, then $x + 3 \leq 0$, and $|x+3| = -(x+3) = -x - 3$.

[10] If $2 < x < 3$, then $x - 2 > 0$ and $x - 3 < 0$. Thus, $(x-2)(x-3) < 0$ and

$|(x-2)(x-3)| = -(x-2)(x-3)$, or, equivalently, $(2-x)(x-3)$.

[11] $-3^2 + 2^0 + 27^{-2/3} = -9 + 1 + \dfrac{1}{(\sqrt[3]{27})^2} = -8 + \dfrac{1}{3^2} = -\dfrac{72}{9} + \dfrac{1}{9} = \dfrac{-71}{9}$

[12] $(\frac{1}{2})^0 - 1^2 + 16^{-3/4} = 1 - 1 + \dfrac{1}{(\sqrt[4]{16})^3} = 0 + \dfrac{1}{2^3} = \dfrac{1}{8}$

[13] $(3a^2b)^2(2ab^3) = (9a^4b^2)(2ab^3) = 18a^5b^5$ [14] $\dfrac{6r^3y^2}{2r^5y} = \dfrac{3y}{r^2}$

[15] $\dfrac{(3x^2y^{-3})^{-2}}{x^{-5}y} = \dfrac{3^{-2}x^{-4}y^6}{x^{-5}y} = \dfrac{x^5y^5}{3^2x^4} = \dfrac{xy^5}{9}$ [16] $\left(\dfrac{a^{2/3}b^{3/2}}{a^2b}\right)^6 = \dfrac{a^4b^9}{a^{12}b^6} = \dfrac{b^3}{a^8}$

[17] $(-2p^2q)^3\left(\dfrac{p}{4q^2}\right)^2 = (-8p^6q^3)\left(\dfrac{p^2}{16q^4}\right) = -\dfrac{p^8}{2q}$

[18] $c^{-4/3}c^{3/2}c^{1/6} = c^{(-8+9+1)/6} = c^{2/6} = c^{1/3}$

[19] $\left(\dfrac{xy^{-1}}{\sqrt{z}}\right)^4 \div \left(\dfrac{x^{1/3}y^2}{z}\right)^3 = \dfrac{x^4y^{-4}}{z^2} \cdot \dfrac{z^3}{xy^6} = \dfrac{x^3z}{y^{10}}$

[20] $\left(\dfrac{-64x^3}{z^6y^9}\right)^{2/3} = \dfrac{(\sqrt[3]{-64})^2x^2}{z^4y^6} = \dfrac{16x^2}{z^4y^6}$

$\boxed{21}$ $\left[(a^{2/3}b^{-2})^3\right]^{-1} = (a^2b^{-6})^{-1} = a^{-2}b^6 = \dfrac{b^6}{a^2}$

$\boxed{22}$ $\dfrac{(3u^2v^5w^{-4})^3}{(2uv^{-3}w^2)^4} = \dfrac{27u^6v^{15}w^{-12}}{16u^4v^{-12}w^8} = \dfrac{27u^2v^{27}}{16w^{20}}$

$\boxed{23}$ $\dfrac{r^{-1}+s^{-1}}{(rs)^{-1}} = \left(\frac{1}{r}+\frac{1}{s}\right)\cdot rs = s + r$

$\boxed{24}$ $(u+v)^3(u+v)^{-2} = (u+v)^1 = u+v$

$\boxed{25}$ $s^{5/2}s^{-4/3}s^{-1/6} = s^{(15-8-1)/6} = s^{6/6} = s^1 = s$

$\boxed{26}$ $x^{-2} - y^{-1} = \dfrac{1}{x^2} - \dfrac{1}{y} = \dfrac{y-x^2}{x^2y}$

$\boxed{27}$ $\sqrt[3]{(x^4y^{-1})^6} = (x^4y^{-1})^{6/3} = x^8y^{-2} = \dfrac{x^8}{y^2}$

$\boxed{28}$ $\sqrt[3]{8x^5y^3z^4} = \sqrt[3]{8x^3y^3z^3}\sqrt[3]{x^2z} = 2xyz\sqrt[3]{x^2z}$

$\boxed{29}$ $\dfrac{1}{\sqrt[3]{4}} = \dfrac{1}{\sqrt[3]{4}}\cdot\dfrac{\sqrt[3]{2}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{8}} = \frac{1}{2}\sqrt[3]{2}$

$\boxed{30}$ $\sqrt{\dfrac{a^2b^3}{c}} = \dfrac{\sqrt{a^2b^3}}{\sqrt{c}}\cdot\dfrac{\sqrt{c}}{\sqrt{c}} = \dfrac{\sqrt{a^2b^2}\sqrt{bc}}{c} = \dfrac{ab}{c}\sqrt{bc}$

$\boxed{31}$ $\sqrt[3]{4x^2y}\sqrt[3]{2x^5y^2} = \sqrt[3]{8x^6y^3}\sqrt[3]{x} = 2x^2y\sqrt[3]{x}$

$\boxed{32}$ $\sqrt[4]{(-4a^3b^2c)^2} = \sqrt[4]{16a^6b^4c^2} = \sqrt[4]{2^4a^4b^4}\sqrt[4]{a^2c^2} = 2ab\sqrt[4]{(ac)^2} = 2ab\sqrt{ac}$

$\boxed{33}$ $\dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}}-1\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1}{\sqrt{t}}-\dfrac{\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1}{\sqrt{t}}\left(\dfrac{1-\sqrt{t}}{\sqrt{t}}\right) = \dfrac{1-\sqrt{t}}{t}$

$\boxed{34}$ $\sqrt{\sqrt[3]{(c^3d^6)^4}} = \sqrt[6]{c^{12}d^{24}} = c^2d^4$

$\boxed{35}$ $\dfrac{\sqrt{12x^4y}}{\sqrt{3x^2y^5}} = \sqrt{\dfrac{12x^4y}{3x^2y^5}} = \sqrt{\dfrac{4x^2}{y^4}} = \dfrac{2x}{y^2}$

$\boxed{36}$ $\sqrt[3]{(a+2b)^3} = a+2b$

$\boxed{37}$ $\sqrt[3]{\dfrac{1}{2\pi^2}} = \dfrac{1}{\sqrt[3]{2\pi^2}}\cdot\dfrac{\sqrt[3]{4\pi}}{\sqrt[3]{4\pi}} = \dfrac{\sqrt[3]{4\pi}}{\sqrt[3]{8\pi^3}} = \dfrac{1}{2\pi}\sqrt[3]{4\pi}$

$\boxed{38}$ $\sqrt[3]{\dfrac{x^2}{9y}} = \sqrt[3]{\dfrac{x^2}{9y}}\cdot\dfrac{\sqrt[3]{3y^2}}{\sqrt[3]{3y^2}} = \dfrac{\sqrt[3]{3x^2y^2}}{\sqrt[3]{27y^3}} = \dfrac{1}{3y}\sqrt[3]{3x^2y^2}$

$\boxed{39}$ $\dfrac{1-\sqrt{x}}{1+\sqrt{x}} = \dfrac{1-\sqrt{x}}{1+\sqrt{x}}\cdot\dfrac{1-\sqrt{x}}{1-\sqrt{x}} = \dfrac{1-2\sqrt{x}+x}{1-x}$

$\boxed{40}$ $\dfrac{1}{\sqrt{a}+\sqrt{a-2}} = \dfrac{1}{\sqrt{a}+\sqrt{a-2}}\cdot\dfrac{\sqrt{a}-\sqrt{a-2}}{\sqrt{a}-\sqrt{a-2}} = \dfrac{\sqrt{a}-\sqrt{a-2}}{a-(a-2)} = \dfrac{\sqrt{a}-\sqrt{a-2}}{2}$

$\boxed{41}$ $\dfrac{81x^2-y^2}{3\sqrt{x}+\sqrt{y}} = \dfrac{81x^2-y^2}{3\sqrt{x}+\sqrt{y}}\cdot\dfrac{3\sqrt{x}-\sqrt{y}}{3\sqrt{x}-\sqrt{y}} = \dfrac{(9x+y)(9x-y)(3\sqrt{x}-\sqrt{y})}{9x-y} =$

$(9x+y)(3\sqrt{x}-\sqrt{y})$

$\boxed{42}$ $\dfrac{3+\sqrt{x}}{3-\sqrt{x}} = \dfrac{3+\sqrt{x}}{3-\sqrt{x}}\cdot\dfrac{3+\sqrt{x}}{3+\sqrt{x}} = \dfrac{x+6\sqrt{x}+9}{9-x}$

$\boxed{43}$ $(3x^3-4x^2+x-7)+(x^4-2x^3+3x^2+5) = x^4+x^3-x^2+x-2$

$\boxed{44}$ $(4z^4 - 3z^2 + 1) - z(z^3 + 4z^2 - 4) = 4z^4 - 3z^2 + 1 - z^4 - 4z^3 + 4z =$
$3z^4 - 4z^3 - 3z^2 + 4z + 1$

$\boxed{45}$ $(x + 4)(x + 3) - (2x - 1)(x - 5) = (x^2 + 7x + 12) - (2x^2 - 11x + 5) = -x^2 + 18x + 7$

$\boxed{46}$ $(4x - 5)(2x^2 + 3x - 7) = (4x)(2x^2 + 3x - 7) + (-5)(2x^2 + 3x - 7) =$
$(8x^3 + 12x^2 - 28x) + (-10x^2 - 15x + 35) = 8x^3 + 2x^2 - 43x + 35$

$\boxed{47}$ $(3y^3 - 2y^2 + y + 4)(y^2 - 3) = (3y^3 - 2y^2 + y + 4)y^2 + (3y^3 - 2y^2 + y + 4)(-3) =$
$(3y^5 - 2y^4 + y^3 + 4y^2) + (-9y^3 + 6y^2 - 3y - 12) = 3y^5 - 2y^4 - 8y^3 + 10y^2 - 3y - 12$

$\boxed{48}$ $(3x + 2)(x - 5)(5x + 4) = (3x + 2)(5x^2 - 21x - 20) =$
$(15x^3 - 63x^2 - 60x) + (10x^2 - 42x - 40) = 15x^3 - 53x^2 - 102x - 40$

$\boxed{49}$ $(a - b)(a^3 + a^2b + ab^2 + b^3) = (a^4 + a^3b + a^2b^2 + ab^3) - (a^3b + a^2b^2 + ab^3 + b^4) =$
$a^4 - b^4$

$\boxed{50}$ $\dfrac{9p^4q^3 - 6p^2q^4 + 5p^3q^2}{3p^2q^2} = \dfrac{9p^4q^3}{3p^2q^2} - \dfrac{6p^2q^4}{3p^2q^2} + \dfrac{5p^3q^2}{3p^2q^2} = 3p^2q - 2q^2 + \frac{5}{3}p$

$\boxed{51}$ $(3a - 5b)(2a + 7b) = 6a^2 + 11ab - 35b^2$

$\boxed{52}$ $(4r^2 - 3s)^2 = (4r^2)^2 - 2(4r^2)(3s) + (3s)^2 = 16r^4 - 24r^2s + 9s^2$

$\boxed{53}$ $(13a^2 + 4b)(13a^2 - 4b) = (13a^2)^2 - (4b^2)^2 = 169a^4 - 16b^2$

$\boxed{54}$ $(a^3 - a^2)^2 = (a^3)^2 - 2(a^3)(a^2) + (a^2)^2 = a^6 - 2a^5 + a^4$

$\boxed{55}$ $(2a + b)^3 = (2a)^3 + 3(2a)^2(b) + 3(2a)(b)^2 + (b)^3 = 8a^3 + 12a^2b + 6ab^2 + b^3$

$\boxed{56}$ $(c^2 - d^2)^3 = (c^2)^3 - 3(c^2)^2(d^2) + 3(c^2)(d^2)^2 - (d^2)^3 = c^6 - 3c^4d^2 + 3c^2d^4 - d^6$

$\boxed{57}$ $(3x + 2y)^2(3x - 2y)^2 = [(3x + 2y)(3x - 2y)]^2 = (9x^2 - 4y^2)^2 = 81x^4 - 72x^2y^2 + 16y^4$

$\boxed{58}$ $(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)$

$\boxed{59}$ $60xw + 70w = 10w(6x + 7)$

$\boxed{60}$ $2r^4s^3 - 8r^2s^5 = 2r^2s^3(r^2 - 4s^2) = 2r^2s^3(r + 2s)(r - 2s)$

$\boxed{61}$ $28x^2 + 4x - 9 = (14x + 9)(2x - 1)$

$\boxed{62}$ $16a^4 + 24a^2b^2 + 9b^4 = (4a^2 + 3b^2)(4a^2 + 3b^2) = (4a^2 + 3b^2)^2$

$\boxed{63}$ $2wy + 3yx - 8wz - 12zx = y(2w + 3x) - 4z(2w + 3x) = (y - 4z)(2w + 3x)$

$\boxed{64}$ $2c^3 - 12c^2 + 3c - 18 = 2c^2(c - 6) + 3(c - 6) = (2c^2 + 3)(c - 6)$

$\boxed{65}$ $8x^3 + 64y^3 = 8(x^3 + 8y^3) = 8[(x)^3 + (2y)^3] = 8(x + 2y)(x^2 - 2xy + 4y^2)$

$\boxed{66}$ $u^3v^4 - u^6v = u^3v(v^3 - u^3) = u^3v(v - u)(v^2 + uv + u^2)$

$\boxed{67}$ $p^8 - q^8 = (p^4)^2 - (q^4)^2 = (p^4 + q^4)(p^4 - q^4) = (p^4 + q^4)(p^2 + q^2)(p^2 - q^2) =$
$(p^4 + q^4)(p^2 + q^2)(p + q)(p - q)$

$\boxed{68}$ $x^4 - 8x^3 + 16x^2 = x^2(x^2 - 8x + 16) = x^2(x - 4)(x - 4) = x^2(x - 4)^2$

$\boxed{69}$ $w^6 + 1 = (w^2)^3 + (1)^3 = (w^2 + 1)(w^4 - w^2 + 1)$

$\boxed{70}$ $3x + 6 = 3(x + 2)$

$\boxed{71}$ $x^2 + 36$ is irreducible

$\boxed{72}$ $x^2 - 49y^2 - 14x + 49 = (x^2 - 14x + 49) - 49y^2 = (x-7)^2 - (7y)^2 =$
$$(x - 7 + 7y)(x - 7 - 7y)$$

$\boxed{73}$ $x^5 - 4x^3 + 8x^2 - 32 = x^3(x^2 - 4) + 8(x^2 - 4) = (x^3 + 8)(x^2 - 4) =$
$$(x+2)(x^2 - 2x + 4)(x+2)(x-2) = (x-2)(x+2)^2(x^2 - 2x + 4)$$

$\boxed{74}$ $4x^4 + 12x^3 + 20x^2 = 4x^2(x^2 + 3x + 5)$

$\boxed{75}$ $\dfrac{6x^2 - 7x - 5}{4x^2 + 4x + 1} = \dfrac{(3x-5)(2x+1)}{(2x+1)(2x+1)} = \dfrac{3x-5}{2x+1}$

$\boxed{76}$ $\dfrac{r^3 - t^3}{r^2 - t^2} = \dfrac{(r-t)(r^2 + rt + t^2)}{(r+t)(r-t)} = \dfrac{r^2 + rt + t^2}{r+t}$

$\boxed{77}$ $\dfrac{6x^2 - 5x - 6}{x^2 - 4} \div \dfrac{2x^2 - 3x}{x+2} = \dfrac{(3x+2)(2x-3)}{(x+2)(x-2)} \cdot \dfrac{x+2}{x(2x-3)} = \dfrac{3x+2}{x(x-2)}$

$\boxed{78}$ $\dfrac{2}{4x-5} - \dfrac{5}{10x+1} = \dfrac{2(10x+1) - 5(4x-5)}{(4x-5)(10x+1)} = \dfrac{27}{(4x-5)(10x+1)}$

$\boxed{79}$ $\dfrac{7}{x+2} + \dfrac{3x}{(x+2)^2} - \dfrac{5}{x} = \dfrac{7(x)(x+2) + 3x(x) - 5(x+2)^2}{x(x+2)^2} =$
$$\frac{7x^2 + 14x + 3x^2 - 5x^2 - 20x - 20}{x(x+2)^2} = \frac{5x^2 - 6x - 20}{x(x+2)^2}$$

$\boxed{80}$ $\dfrac{x + x^{-2}}{1 + x^{-2}} = \dfrac{x + \dfrac{1}{x^2}}{1 + \dfrac{1}{x^2}} = \dfrac{\left(x + \dfrac{1}{x^2}\right) \cdot x^2}{\left(1 + \dfrac{1}{x^2}\right) \cdot x^2} = \dfrac{x^3 + 1}{x^2 + 1}$

$\boxed{81}$ $\dfrac{1}{x} - \dfrac{2}{x^2 + x} - \dfrac{3}{x+3} = \dfrac{1(x+1)(x+3) - 2(x+3) - 3x(x+1)}{x(x+1)(x+3)} =$
$$\frac{x^2 + 4x + 3 - 2x - 6 - 3x^2 - 3x}{x(x+1)(x+3)} = \frac{-2x^2 - x - 3}{x(x+1)(x+3)}$$

$\boxed{82}$ $(a^{-1} + b^{-1})^{-1} = \left(\frac{1}{a} + \frac{1}{b}\right)^{-1} = \left(\dfrac{b+a}{ab}\right)^{-1} = \dfrac{ab}{a+b}$

$\boxed{83}$ $\dfrac{x + 2 - \dfrac{3}{x+4}}{\dfrac{x}{x+4} + \dfrac{1}{x+4}} = \dfrac{\dfrac{(x+2)(x+4) - 3}{x+4}}{\dfrac{x+1}{x+4}} = \dfrac{x^2 + 6x + 5}{x+1} = \dfrac{(x+1)(x+5)}{x+1} = x + 5$

$\boxed{84}$ $\dfrac{\dfrac{x}{x+2} - \dfrac{4}{x+2}}{x - 3 - \dfrac{6}{x+2}} = \dfrac{\dfrac{x-4}{x+2}}{\dfrac{(x-3)(x+2) - 6}{x+2}} = \dfrac{x-4}{x^2 - x - 12} = \dfrac{x-4}{(x+3)(x-4)} = \dfrac{1}{x+3}$

$\boxed{85}$ $(x^2+1)^{3/2}(4)(x+5)^3 + (x+5)^4(\frac{3}{2})(x^2+1)^{1/2}(2x) =$
$$(x^2+1)^{1/2}(x+5)^3[4(x^2+1) + 3x(x+5)] = (x^2+1)^{1/2}(x+5)^3(7x^2 + 15x + 4)$$

$\boxed{86}$ $\dfrac{(4 - x^2)(\frac{1}{3})(6x+1)^{-2/3}(6) - (6x+1)^{1/3}(-2x)}{(4-x^2)^2} =$
$$\frac{(6x+1)^{-2/3}[2(4-x^2) + 2x(6x+1)]}{(4-x^2)^2} = \frac{10x^2 + 2x + 8}{(6x+1)^{2/3}(4-x^2)^2} = \frac{2(5x^2 + x + 4)}{(6x+1)^{2/3}(4-x^2)^2}$$

$\boxed{87}$ $(5.5 \text{ liters})\left(10^6 \dfrac{\text{mm}^3}{\text{liter}}\right)\left(5 \times 10^6 \dfrac{\text{cells}}{\text{mm}^3}\right) = 2.75 \times 10^{13}$ red blood cells

$\boxed{88}$ $\dfrac{70 \text{ (or 90) beats}}{\text{minute}} \cdot \dfrac{60 \text{ minutes}}{1 \text{ hour}} \cdot \dfrac{24 \text{ hours}}{1 \text{ day}} \cdot \dfrac{365 \text{ days}}{1 \text{ year}} \cdot 80 \text{ years} =$

2.94336×10^9 (or 3.78432×10^9) beats

$\boxed{89}$ $h = 86$ cm and $w = 13$ kg $\Rightarrow$

$$S = (0.007184)w^{0.425}h^{0.725} = (0.007184)(13)^{0.425}(86)^{0.725} \approx 0.54 \text{ m}^2.$$

$\boxed{90}$ $p = 40$ dyne/cm^2 and $v = 60$ cm^3 $\Rightarrow$ $c = pv^{-1.4} = 40(60)^{-1.4} \approx 0.13$ dyne-cm.

Chapter 1 Discussion Exercises

$\boxed{1}$ 1 gallon ≈ 0.13368 ft^3 is a conversion factor that would help. The volume of the tank is 10,000 gallons ≈ 1336.8 ft^3. Use $V = \frac{4}{3}\pi r^3$ to determine the radius $r \approx 6.833$ ft and then use $S = 4\pi r^2$ to find the surface area—about 586.85 ft^2.

$\boxed{2}$ Squaring the right side gives us $(a+b)^2 = a^2 + 2ab + b^2$. Squaring the left side gives us $a^2 + b^2$. Now $a^2 + 2ab + b^2$ will equal $a^2 + b^2$ only if $2ab = 0$. The expression $2ab$ equals zero only if either $a = 0$ or $b = 0$.

$\boxed{3}$ We first need to determine the term that needs to be added and subtracted. If we add and subtract $10x$, we will obtain the square of a binomial—i.e., $(x^2 + 10x + 25) - 10x = (x+5)^2 - 10x$. We can now factor this expression as the difference of two squares, $(x+5)^2 - 10x = (x + 5 + \sqrt{10x})(x + 5 - \sqrt{10x})$.

$\boxed{4}$ The first expression can be evaluated at $x = 1$, whereas the second expression is undefined at $x = 1$.

$\boxed{5}$ They get close to the ratio of leading coefficients as x gets larger.

$\boxed{6}$ $\dfrac{3x^2 - 5x - 2}{x^2 - 4} = \dfrac{(3x+1)(x-2)}{(x+2)(x-2)} = \dfrac{3x+1}{x+2}$. Evaluating the original expression and the simplified expression with any $x \neq \pm 2$ gives us the same value. This evaluation does not prove that the expressions are equal for any value of x other than the one selected. The simplification proves that the expressions are equal for all values of x except $x = 2$.

[7] Follow the algebraic simplification given.

1 Write down his/her age.	Denote the age with x.
2 Multiply it by 2.	$2x$
3 Add 5.	$2x+5$
4 Multiply this sum by 50.	$50(2x+5)=100x+250$
5 Subtract 365.	$(100x+250)-365=100x-115$
6 Add his/her height (in inches).	$100x-115+y$, where y is the height
7 Add 115.	$100x-115+y+115=100x+y$

As a specific example, suppose the age is 21 and the height is 68. The number obtained by following the steps is $100x+y=2168$ and we can see that the first two digits of the result equal the age and the last two digits equal the height.

[8]
$$\begin{aligned}
V_{\text{out}} &= I_{\text{in}}\left(-\frac{RXi}{R-Xi}\right)=\frac{V_{\text{in}}}{Z_{\text{in}}}\left(-\frac{RXi}{R-Xi}\right) && \{\text{definition of } I_{\text{in}}\}\\
&= \frac{V_{\text{in}}}{\frac{R^2-X^2-3RXi}{R-Xi}}\left(-\frac{RXi}{R-Xi}\right) && \{\text{definition of } Z_{\text{in}}\}\\
&= \frac{V_{\text{in}}(R-Xi)}{R^2-X^2-3RXi}\left(-\frac{RXi}{R-Xi}\right)\\
&= -\frac{RXi}{R^2-X^2-3RXi}(V_{\text{in}})\\
&= -\frac{RRi}{R^2-R^2-3RRi}(V_{\text{in}}) && \{\text{let } X=R\}\\
&= -\frac{R^2i}{-3R^2i}(V_{\text{in}})=\frac{1}{3}V_{\text{in}}
\end{aligned}$$