Chapter 3: Functions and Graphs

3.1 Exercises

[1]

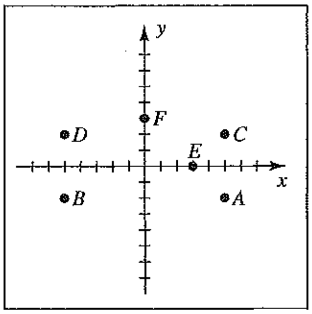

Figure 1

[2]

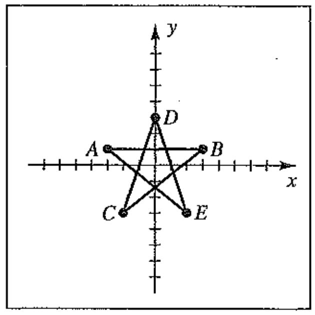

Figure 2

[3] The set of all points of the form $(a,\ a)$ is the line bisecting quadrants I and III.

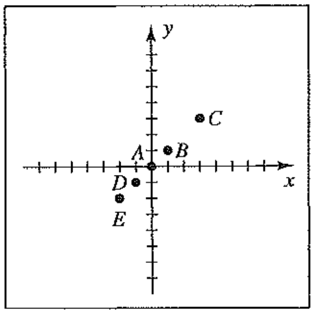

Figure 3

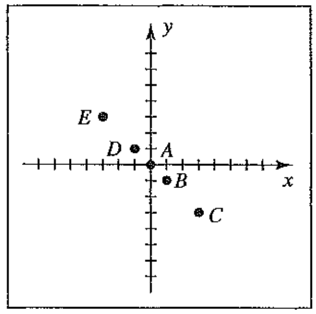

Figure 4

[4] The set of all points of the form $(a,\ -a)$ is the line bisecting quadrants II and IV.

[5] The points are $A(3,\ 3)$, $B(-3,\ 3)$, $C(-3,\ -3)$, $D(3,\ -3)$, $E(3,\ 0)$, and $F(0,\ 3)$.

[6] The points are $A(0,\ 4)$, $B(-4,\ 0)$, $C(0,\ -4)$, $D(4,\ 0)$, $E(2,\ 2)$, and $F(-2,\ -2)$.

[7] (a) $x = -2$ is the line parallel to the y-axis that intersects the x-axis at $(-2,\ 0)$.

(b) $y = 3$ is the line parallel to the x-axis that intersects the y-axis at $(0,\ 3)$.

(c) $x \geq 0$ is the set of all points to the right of and on the y-axis.

(d) $xy > 0$ is the set of all points in quadrants I and III
{ x and y have the same sign }.

(e) $y < 0$ is the set of all points below the x-axis.

(f) $x = 0$ is the set of all points on the y-axis.

[8] (a) $y = -2$ is the line parallel to the x-axis that intersects the y-axis at $(0,\ -2)$.

(b) $x = -4$ is the line parallel to the y-axis that intersects the x-axis at $(-4,\ 0)$.

(c) $x/y < 0$ is the set of all points in quadrants II and IV.
{ x and y have opposite signs }.

(d) $xy = 0$ is the set of all points on the x-axis or y-axis.

(e) $y > 1$ is the set of all points
above the line parallel to the x-axis which intersects the y-axis at $(0,\ 1)$.

(f) $y = 0$ is the set of all points on the x-axis.

$\boxed{9}$ (a) $A(4,\ -3),\ B(6,\ 2) \ \Rightarrow\ d(A,\ B) = \sqrt{(6-4)^2 + [2-(-3)]^2} = \sqrt{4+25} = \sqrt{29}$

(b) $M_{AB} = \left(\frac{4+6}{2},\ \frac{-3+2}{2}\right) = (5,\ -\frac{1}{2})$

$\boxed{10}$ (a) $A(-2,\ -5),\ B(4,\ 6) \ \Rightarrow$

$$d(A,\ B) = \sqrt{[4-(-2)]^2 + [6-(-5)]^2} = \sqrt{36+121} = \sqrt{157}$$

(b) $M_{AB} = \left(\frac{-2+4}{2},\ \frac{-5+6}{2}\right) = (1,\ \frac{1}{2})$

$\boxed{11}$ (a) $A(-5,\ 0),\ B(-2,\ -2) \ \Rightarrow$

$$d(A,\ B) = \sqrt{[-2-(-5)]^2 + (-2-0)^2} = \sqrt{9+4} = \sqrt{13}$$

(b) $M_{AB} = \left(\frac{-5+(-2)}{2},\ \frac{0+(-2)}{2}\right) = (-\frac{7}{2},\ -1)$

$\boxed{12}$ (a) $A(6,\ 2),\ B(6,\ -2) \ \Rightarrow\ d(A,\ B) = \sqrt{(6-6)^2 + (-2-2)^2} = \sqrt{0+16} = 4$

(b) $M_{AB} = \left(\frac{6+6}{2},\ \frac{2+(-2)}{2}\right) = (6,\ 0)$

$\boxed{13}$ (a) $A(7,\ -3),\ B(3,\ -3) \ \Rightarrow\ d(A,\ B) = \sqrt{(3-7)^2 + [-3-(-3)]^2} = \sqrt{16+0} = 4$

(b) $M_{AB} = \left(\frac{7+3}{2},\ \frac{-3+(-3)}{2}\right) = (5,\ -3)$

$\boxed{14}$ (a) $A(-4,\ 7),\ B(0,\ -8) \ \Rightarrow$

$$d(A,\ B) = \sqrt{[0-(-4)]^2 + (-8-7)^2} = \sqrt{16+225} = \sqrt{241}$$

(b) $M_{AB} = \left(\frac{-4+0}{2},\ \frac{7+(-8)}{2}\right) = (-2,\ -\frac{1}{2})$

$\boxed{15}$ Show that $d(A,\ C)^2 = d(A,\ B)^2 + d(B,\ C)^2$; that is, $(\sqrt{130})^2 = (\sqrt{98})^2 + (\sqrt{32})^2$.

$$\text{Area} = \tfrac{1}{2}bh = \tfrac{1}{2}(\sqrt{32})(\sqrt{98}) = \tfrac{1}{2}(4\sqrt{2})(7\sqrt{2}) = 28.$$

$\boxed{16}$ Show that $d(A,\ B)^2 = d(A,\ C)^2 + d(B,\ C)^2$; that is, $(\sqrt{145})^2 = (\sqrt{29})^2 + (\sqrt{116})^2$.

$$\text{Area} = \tfrac{1}{2}bh = \tfrac{1}{2}(\sqrt{29})(\sqrt{116}) = \tfrac{1}{2}(\sqrt{29})(2\sqrt{29}) = 29.$$

$\boxed{17}$ Show that $d(A,\ B) = d(B,\ C) = d(C,\ D) = d(D,\ A)$ {each is $\sqrt{29}$} and

$$d(A,\ C)^2 = d(A,\ B)^2 + d(B,\ C)^2 \ \{d(A,\ C) = \sqrt{58}\}$$

$\boxed{18}$ Show that $d(A,\ D) = d(B,\ C)$ {each is $\sqrt{45}$} and $d(A,\ B) = d(C,\ D)$ {each is $\sqrt{17}$}

$\boxed{19}$ Let $B = (x,\ y)$. $A(-3,\ 8) \ \Rightarrow\ M_{AB} = \left(\frac{-3+x}{2},\ \frac{8+y}{2}\right)$. $M_{AB} = C(5,\ -10) \ \Rightarrow$

$$-3 + x = 2(5) \text{ and } 8 + y = 2(-10) \ \Rightarrow\ x = 13 \text{ and } y = -28. \ \ B = (13,\ -28).$$

$\boxed{20}$ If Q is the midpoint of segment AB, then the midpoint of QB is the point that is three-fourths of the way from $A(5,\ -8)$ to $B(-6,\ 2)$.

$Q = M_{AB} = \left(\frac{5+(-6)}{2},\ \frac{-8+2}{2}\right) = (-\frac{1}{2},\ -3)$.

$$M_{QB} = \left(\frac{(-1/2)+(-6)}{2},\ \frac{-3+2}{2}\right) = (-\tfrac{13}{4},\ -\tfrac{1}{2}).$$

[21] Similar to Example 3, we must show that $d(A, C) = d(B, C) = \sqrt{145}$.

[22] Show that $d(A, C) = d(B, C) = \sqrt{125}$.

[23] We must have $d(A, P) = d(B, P)$.

$$\sqrt{(x+4)^2 + (y+3)^2} = \sqrt{(x-6)^2 + (y-1)^2} \Rightarrow$$
$$x^2 + 8x + 16 + y^2 + 6y + 9 = x^2 - 12x + 36 + y^2 - 2y + 1 \Rightarrow$$
$$8x + 6y + 25 = -12x - 2y + 37 \Rightarrow 20x + 8y = 12 \Rightarrow 5x + 2y = 3$$

[24] We must have $d(A, P) = d(B, P)$.

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-5)^2 + (y+4)^2} \Rightarrow$$
$$x^2 + 6x + 9 + y^2 - 4y + 4 = x^2 - 10x + 25 + y^2 + 8y + 16 \Rightarrow$$
$$6x - 4y + 13 = -10x + 8y + 41 \Rightarrow 16x - 12y = 28 \Rightarrow 4x - 3y = 7$$

[25] $d(P, O) = 5 \Rightarrow \sqrt{(x-0)^2 + (y-0)^2} = 5 \Rightarrow \sqrt{x^2 + y^2} = 5$.

This is a circle of radius 5 with center at the origin.

[26] $d(C, P) = r \Rightarrow \sqrt{(x-h)^2 + (y-k)^2} = r$.

This is a circle of radius r and center (h, k).

[27] Let $Q(0, y)$ be an arbitrary point on the y-axis.

$6 = d(P, Q) \Rightarrow 6 = \sqrt{(0-5)^2 + (y-3)^2} \Rightarrow 36 = 25 + y^2 - 6y + 9 \Rightarrow$

$y^2 - 6y - 2 = 0 \Rightarrow y = 3 \pm \sqrt{11}$. The points are $(0, 3 + \sqrt{11})$ and $(0, 3 - \sqrt{11})$.

[28] Let $Q(x, 0)$ be an arbitrary point on the x-axis.

$5 = d(P, Q) \Rightarrow 5 \Rightarrow \sqrt{(x+2)^2 + (0-4)^2} \Rightarrow 25 = x^2 + 4x + 4 + 16 \Rightarrow$

$x^2 + 4x - 5 = 0 \Rightarrow (x+5)(x-1) = 0 \Rightarrow x = -5, 1$.

The points are $(1, 0)$ and $(-5, 0)$.

[29] $5 = \sqrt{(2a-1)^2 + (a-3)^2} \Rightarrow 25 = 4a^2 - 4a + 1 + a^2 - 6a + 9 \Rightarrow$

$5a^2 - 10a - 15 = 0 \Rightarrow a^2 - 2a - 3 = 0 \Rightarrow (a-3)(a+1) = 0 \Rightarrow a = 3, -1$.

Since the y-coordinate is negative in the third quadrant, $a = -1$,

and $(2a, a) = (-2, -1)$.

[30] $3 = \sqrt{(a+2)^2 + (a-1)^2} \Rightarrow 9 = a^2 + 4a + 4 + a^2 - 2a + 1 \Rightarrow$

$0 = 2a^2 + 2a - 4 \Rightarrow a^2 + a - 2 = 0 \Rightarrow (a+2)(a-1) = 0 \Rightarrow a = -2, 1$.

The points are $(-2, -2)$ and $(1, 1)$.

[31] $d(P, Q) > \sqrt{26} \Rightarrow \sqrt{(5-a)^2 + (2a-3)^2} > \sqrt{26} \Rightarrow$

$25 - 10a + a^2 + 4a^2 - 12a + 9 > 26 \Rightarrow 5a^2 - 22a + 8 > 0 \Rightarrow$

$(5a-2)(a-4) > 0 \Rightarrow a < \frac{2}{5}$ or $a > 4$.

Use a sign diagram to establish the final answer.

$\boxed{32}$ $d(A,\ P) + d(B,\ P) = 5 \ \Rightarrow\ \sqrt{(x+2)^2 + y^2} + \sqrt{(x-2)^2 + y^2} = 5 \ \Rightarrow$

$\sqrt{x^2 + 4x + 4 + y^2} = 5 - \sqrt{x^2 - 4x + 4 + y^2} \ \Rightarrow$

$x^2 + 4x + 4 + y^2 = 25 - 10\sqrt{x^2 - 4x + 4 + y^2} + x^2 - 4x + 4 + y^2 \ \Rightarrow$

$10\sqrt{x^2 - 4x + 4 + y^2} = 25 - 8x \ \Rightarrow\ 100(x^2 - 4x + 4 + y^2) = 625 - 400x + 64x^2 \ \Rightarrow$

$36x^2 + 100y^2 = 225$

$\boxed{33}$ Let M be the midpoint of the hypotenuse. Then $M = (\frac{1}{2}a,\ \frac{1}{2}b)$.

Show that $d(A,\ M) = d(B,\ M) = d(O,\ M) = \frac{1}{2}\sqrt{a^2 + b^2}$.

$\boxed{34}$ Let $D(a,\ b+c)$ be the fourth vertex as shown in *Figure 34*. We need to show that the midpoint of OD is the same as the midpoint of AC.

$M_{OD} = \left(\frac{0+a}{2},\ \frac{0+b+c}{2}\right) = \left(\frac{a}{2},\ \frac{b+c}{2}\right)$ and

$M_{CA} = \left(\frac{0+a}{2},\ \frac{c+b}{2}\right) = \left(\frac{a}{2},\ \frac{b+c}{2}\right)$.

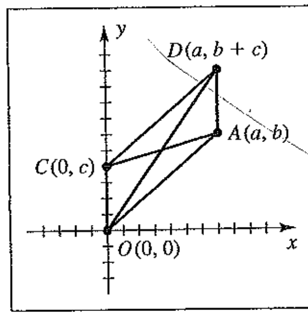

Figure 34

$\boxed{35}$ Plot $A(-5,\ -3.5)$, $B(-2,\ 2)$, $C(1,\ 0.5)$, $D(4,\ 1)$, and $E(7,\ 2.5)$.

$[-10,\ 10]$ by $[-10,\ 10]$

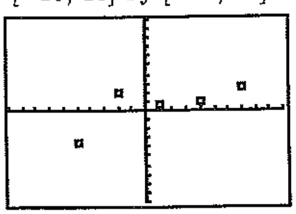

Figure 35

$[-12,\ 12]$ by $[-8,\ 8]$

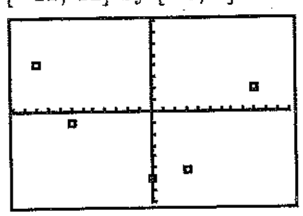

Figure 36

$\boxed{36}$ Plot $A(-10,\ 4)$, $B(-7,\ -1.1)$, $C(0,\ -6)$, $D(3,\ -5.1)$, and $E(9,\ 2.1)$.

$\boxed{37}$ (a) Plot (1990, 54,871,330), (1991, 55,786,390), (1992, 57,211,600), (1993, 58,834,440), and (1994, 59,332,200).

(b) The number of cable subscribers is increasing each year. $\{54\text{E}6 = 54 \times 10^6\}$

[1988, 1996] by [54E6, 61E6, E6]

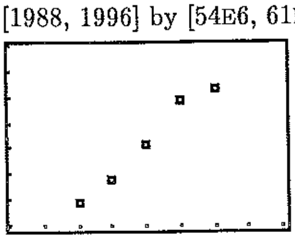

Figure 37

[1895, 2000, 10] by [0, 3000, 1000]

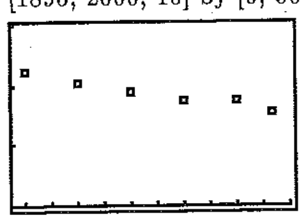

Figure 38

$\boxed{38}$ (a) Plot (1900, 2226), (1920, 2042), (1940, 1878), (1960, 1763), (1980, 1745), and (1993, 1556).

(b) Find the midpoint of (1920, 2042) and (1940, 1878).

$\left(\frac{1920+1940}{2}, \frac{2042+1878}{2}\right) = (1930, 1960)$.

The midpoint formula predicts 1960 daily newspapers published in the year 1930 compared to the actual value of 1942 daily newspapers.

3.2 Exercises

$\boxed{1}$ To find the x-intercept, let $y = 0$ in $y = 2x - 3$, and solve for x. We get 1.5.
To find the y-intercept, let $x = 0$ in $y = 2x - 3$, and solve for y. We get -3.

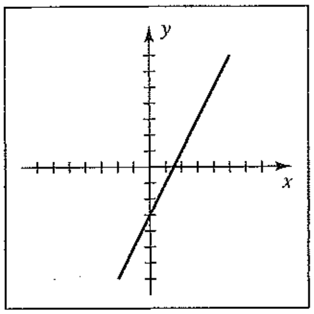

Figure 1

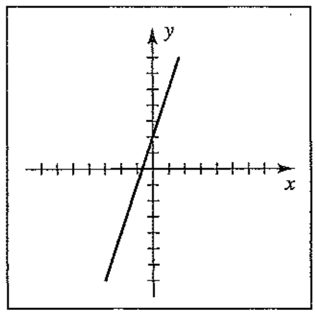

Figure 2

$\boxed{2}$ $y = 3x + 2$ • x-intercept $-\frac{2}{3}$; y-intercept 2

$\boxed{3}$ $y = -x + 1$ • x-intercept 1; y-intercept 1

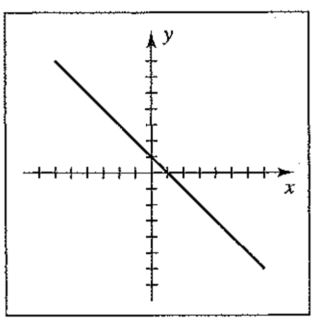

Figure 3

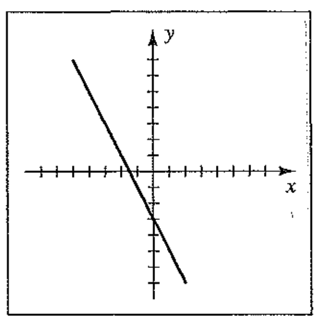

Figure 4

$\boxed{4}$ $y = -2x - 3$ • x-intercept -1.5; y-intercept -3

[5] $y = -4x^2$ • x-intercept 0; y-intercept 0

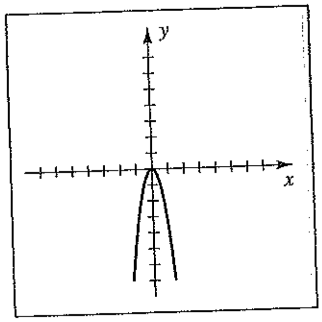

Figure 5

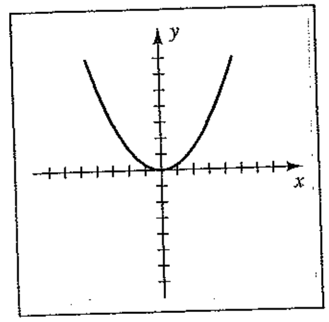

Figure 6

[6] $y = \frac{1}{3}x^2$ • x-intercept 0; y-intercept 0

[7] $y = 2x^2 - 1$ • x-intercepts $\pm\sqrt{\frac{1}{2}} = \pm\frac{1}{2}\sqrt{2}$; y-intercept -1

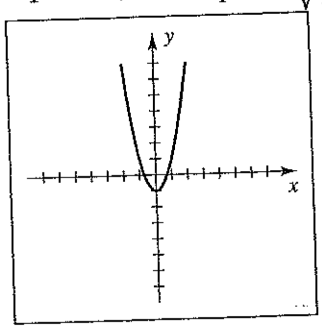

Figure 7

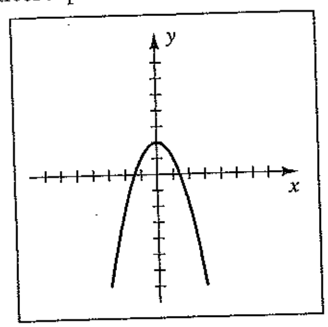

Figure 8

[8] $y = -x^2 + 2$ • x-intercepts $\pm\sqrt{2}$; y-intercept 2

[9] $x = \frac{1}{4}y^2$ • x-intercept 0; y-intercept 0

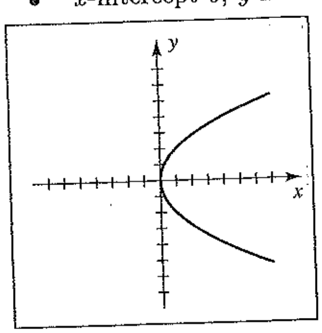

Figure 9

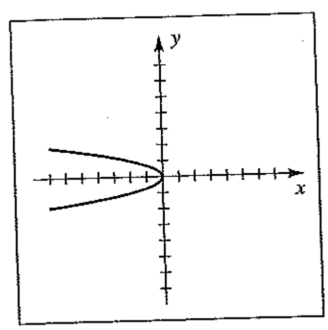

Figure 10

[10] $x = -2y^2$ • x-intercept 0; y-intercept 0

[11] $x = -y^2 + 3$ • x-intercept 3; y-intercepts $\pm\sqrt{3}$

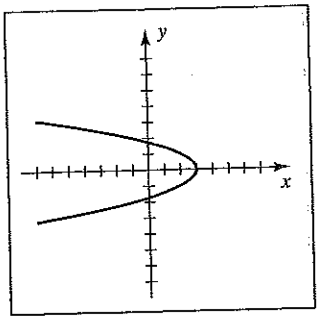

Figure 11

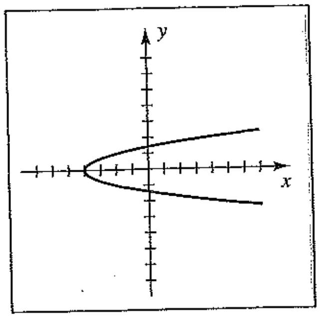

Figure 12

[12] $x = 2y^2 - 4$ • x-intercept -4; y-intercepts $\pm\sqrt{2}$

[13] $y = -\frac{1}{2}x^3$ • x-intercept 0; y-intercept 0

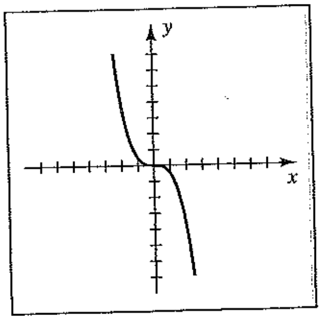

Figure 13

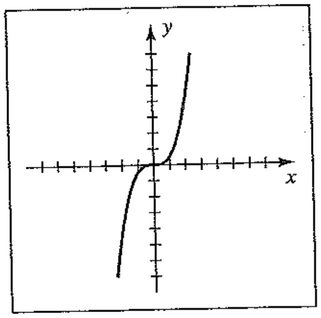

Figure 14

[14] $y = \frac{1}{2}x^3$ • x-intercept 0; y-intercept 0

[15] $y = x^3 - 8$ • x-intercept 2; y-intercept -8

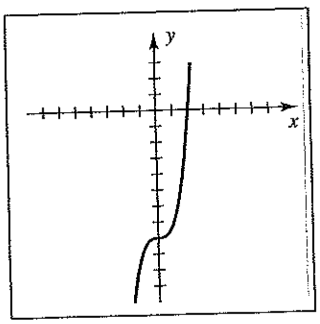

Figure 15

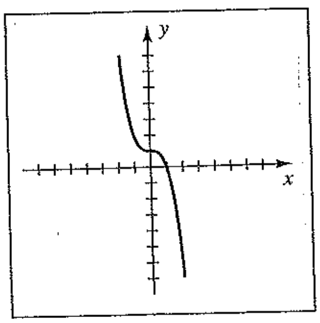

Figure 16

[16] $y = -x^3 + 1$ • x-intercept 1; y-intercept 1

$\boxed{17}$ $y = \sqrt{x}$ • x-intercept 0; y-intercept 0

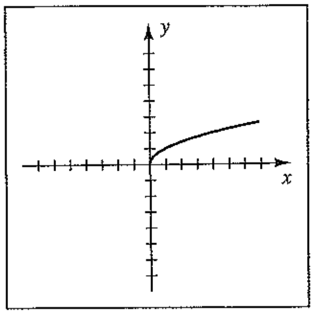

Figure 17

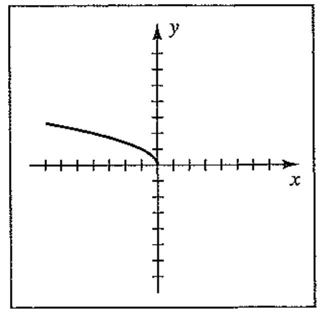

Figure 18

$\boxed{18}$ $y = \sqrt{-x}$ • x-intercept 0; y-intercept 0

$\boxed{19}$ $y = \sqrt{x} - 4$ • x-intercept 16; y-intercept -4

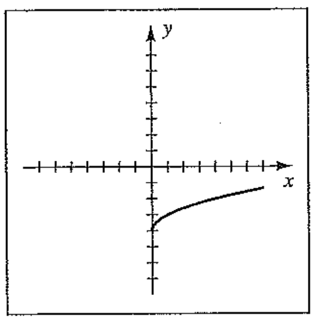

Figure 19

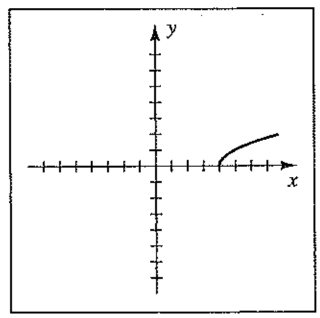

Figure 20

$\boxed{20}$ $y = \sqrt{x-4}$ • x-intercept 4; y-intercept: None

$\boxed{21}$ (a) 5, 7 (b) 9, 11 (c) 13

$\boxed{22}$ (a) 6, 8 (b) 10, 12 (c) 14

$\boxed{23}$ $x^2 + y^2 = 11$ is a circle of radius $\sqrt{11}$ with center at the origin.

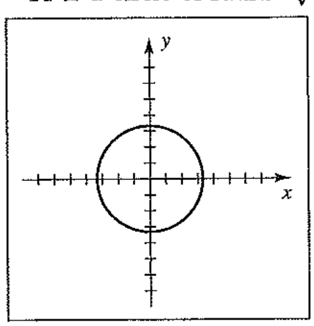

Figure 23

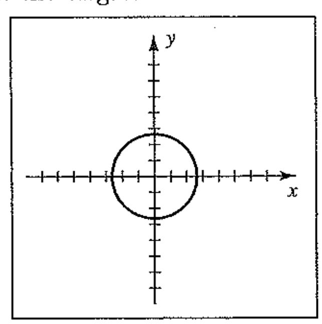

Figure 24

$\boxed{24}$ $x^2 + y^2 = 7$ is a circle of radius $\sqrt{7}$ with center at the origin.

[25] $(x+3)^2+(y-2)^2=9$ is a circle of radius $r=\sqrt{9}=3$ with center $C(-3,\ 2)$.

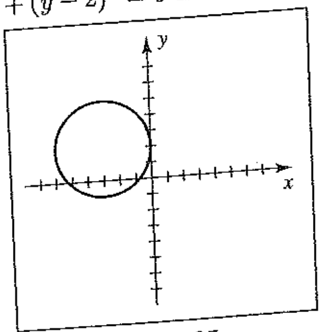

Figure 25

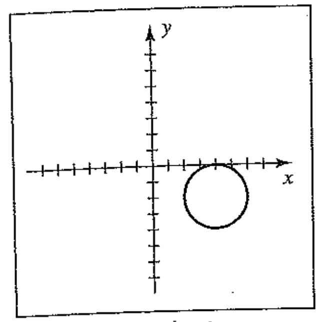

Figure 26

[26] $(x-4)^2+(y+2)^2=4$ is a circle of radius $r=\sqrt{4}=2$ with center $C(4,\ -2)$.

[27] $(x+3)^2+y^2=16$ is a circle of radius $r=\sqrt{16}=4$ with center $C(-3,\ 0)$.

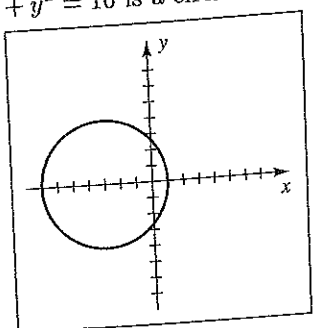

Figure 27

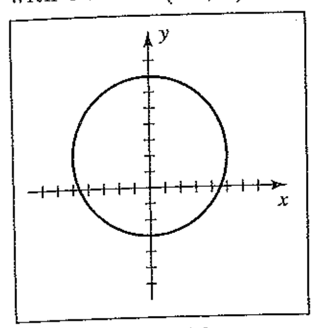

Figure 28

[28] $x^2+(y-2)^2=25$ is a circle of radius $r=\sqrt{25}=5$ with center $C(0,\ 2)$.

[29] $4x^2+4y^2=25 \;\Rightarrow\; x^2+y^2=\frac{25}{4}$ is a circle of radius $r=\sqrt{\frac{25}{4}}=\frac{5}{2}$ with center $C(0,\ 0)$.

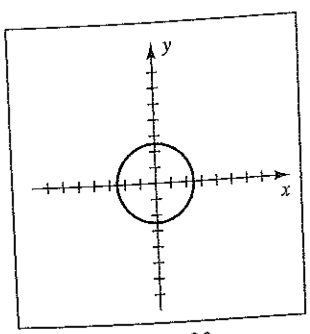

Figure 29

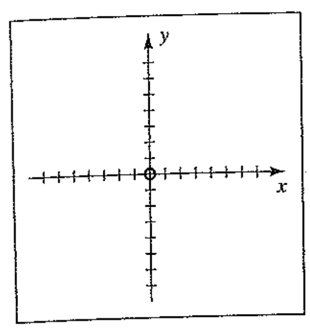

Figure 30

[30] $9x^2+9y^2=1 \;\Rightarrow\; x^2+y^2=\frac{1}{9}$ is a circle of radius $r=\sqrt{\frac{1}{9}}=\frac{1}{3}$ with center $C(0,\ 0)$.

31 As in Example 9, $y = -\sqrt{16 - x^2}$ is the lower half of the circle $x^2 + y^2 = 16$.

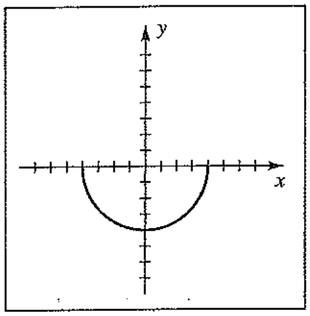

Figure 31

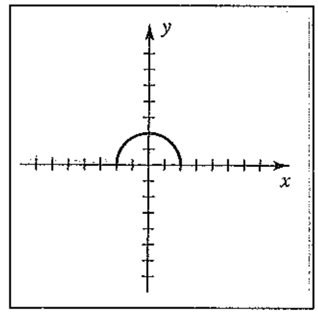

Figure 32

32 $y = \sqrt{4 - x^2}$ is the upper half of the circle $x^2 + y^2 = 4$.

33 $x = \sqrt{9 - y^2}$ is the right half of the circle $x^2 + y^2 = 9$.

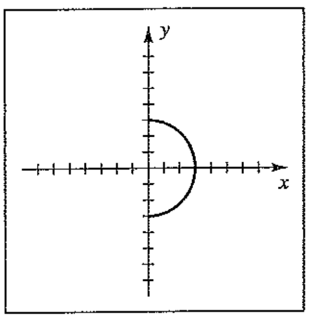

Figure 33

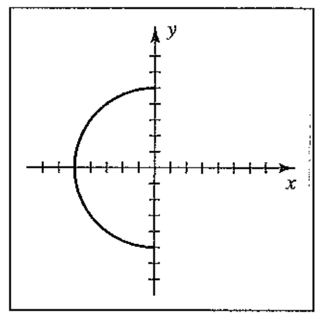

Figure 34

34 $x = -\sqrt{25 - y^2}$ is the left half of the circle $x^2 + y^2 = 25$.

35 Center $C(2, -3)$, radius 5 • $(x-2)^2 + (y+3)^2 = 5^2 = 25$

36 Center $C(-4, 1)$, radius 3 • $(x+4)^2 + (y-1)^2 = 3^2 = 9$

37 Center $C(\frac{1}{4}, 0)$, radius $\sqrt{5}$ • $(x - \frac{1}{4})^2 + y^2 = (\sqrt{5})^2 = 5$

38 Center $C(\frac{3}{4}, -\frac{2}{3})$, radius $3\sqrt{2}$ • $(x - \frac{3}{4})^2 + (y + \frac{2}{3})^2 = (3\sqrt{2})^2 = 18$

39 An equation of a circle with center $C(-4, 6)$ is $(x+4)^2 + (y-6)^2 = r^2$.

Letting $x = 1$ and $y = 2$ yields $5^2 + (-4)^2 = r^2 \Rightarrow r^2 = 41$. $(x+4)^2 + (y-6)^2 = 41$

40 An equation of a circle with center at the origin is $x^2 + y^2 = r^2$.

Letting $x = 4$ and $y = -7$ yields $4^2 + (-7)^2 = r^2 \Rightarrow r^2 = 65$. $x^2 + y^2 = 65$

41 The circle is tangent to the y-axis and has center $C(-3, 6)$.

Its radius, 3, is the distance from the y-axis to the x-value of the center.

An equation is $(x+3)^2 + (y-6)^2 = 9$.

42 The circle is tangent to the x-axis and has center $C(4, -1)$.

Its radius, 1, is the distance from the x-axis to the y-value of the center.

An equation is $(x-4)^2 + (y+1)^2 = 1$.

$\boxed{43}$ Since the radius is 4 and $C(h, k)$ is in QII, $h = -4$ and $k = 4$.

An equation is $(x+4)^2 + (y-4)^2 = 16$.

$\boxed{44}$ Since the radius is 3 and $C(h, k)$ is in QIV, $h = 3$ and $k = -3$.

An equation is $(x-3)^2 + (y+3)^2 = 9$.

$\boxed{45}$ The center of the circle is the midpoint M of $A(4, -3)$ and $B(-2, 7)$. $M = (1, 2)$.

The radius of the circle is $\frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{136} = \sqrt{34}$.

An equation is $(x-1)^2 + (y-2)^2 = 34$.

$\boxed{46}$ As in the solution to Exercise 45, $M_{AB} = (-1, 4)$ and

$r = \frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{80} = \sqrt{20}$. An equation is $(x+1)^2 + (y-4)^2 = 20$.

$\boxed{47}$ $x^2 + y^2 - 4x + 6y - 36 = 0 \Rightarrow x^2 - 4x + \underline{\ 4\ } + y^2 + 6y + \underline{\ 9\ } = 36 + \underline{\ 4\ } + \underline{\ 9\ } \Rightarrow$

$(x-2)^2 + (y+3)^2 = 49$. $C(2, -3)$; $r = 7$

$\boxed{48}$ $x^2 + y^2 + 8x - 10y + 37 = 0 \Rightarrow$

$x^2 + 8x + \underline{\ 16\ } + y^2 - 10y + \underline{\ 25\ } = -37 + \underline{\ 16\ } + \underline{\ 25\ } \Rightarrow$

$(x+4)^2 + (y-5)^2 = 4$. $C(-4, 5)$; $r = 2$

$\boxed{49}$ $x^2 + y^2 + 4y - 117 = 0 \Rightarrow x^2 + y^2 + 4y + \underline{\ 4\ } = 117 + \underline{\ 4\ } \Rightarrow x^2 + (y+2)^2 = 121$.

$C(0, -2)$; $r = 11$

$\boxed{50}$ $x^2 + y^2 - 10x + 18 = 0 \Rightarrow x^2 - 10x + \underline{\ 25\ } + y^2 = -18 + \underline{\ 25\ } \Rightarrow (x-5)^2 + y^2 = 7$.

$C(5, 0)$; $r = \sqrt{7}$

$\boxed{51}$ $2x^2 + 2y^2 - 12x + 4y - 15 = 0 \Rightarrow x^2 - 6x + \underline{\ 9\ } + y^2 + 2y + \underline{\ 1\ } = \frac{15}{2} + \underline{\ 9\ } + \underline{\ 1\ } \Rightarrow$

$(x-3)^2 + (y+1)^2 = \frac{35}{2}$. $C(3, -1)$; $r = \frac{1}{2}\sqrt{70}$

$\boxed{52}$ $9x^2 + 9y^2 + 12x - 6y + 4 = 0 \Rightarrow x^2 + \frac{4}{3}x + \underline{\frac{4}{9}} + y^2 - \frac{2}{3}y + \underline{\frac{1}{9}} = -\frac{4}{9} + \underline{\frac{4}{9}} + \underline{\frac{1}{9}} \Rightarrow$

$(x+\frac{2}{3})^2 + (y-\frac{1}{3})^2 = \frac{1}{9}$. $C(-\frac{2}{3}, \frac{1}{3})$; $r = \frac{1}{3}$

$\boxed{53}$ $x^2 + y^2 + 4x - 2y + 5 = 0 \Rightarrow x^2 + 4x + \underline{\ 4\ } + y^2 - 2y + \underline{\ 1\ } = -5 + \underline{\ 4\ } + \underline{\ 1\ } \Rightarrow$

$(x+2)^2 + (y-1)^2 = 0$. $C(-2, 1)$; $r = 0$ (a point)

$\boxed{54}$ $x^2 + y^2 - 6x + 4y + 13 = 0 \Rightarrow x^2 - 6x + \underline{\ 9\ } + y^2 + 4y + \underline{\ 4\ } = -13 + \underline{\ 9\ } + \underline{\ 4\ } \Rightarrow$

$(x-3)^2 + (y+2)^2 = 0$. $C(3, -2)$; $r = 0$ (a point)

$\boxed{55}$ $x^2 + y^2 - 2x - 8y + 19 = 0 \Rightarrow$

$x^2 - 2x + \underline{\ 1\ } + y^2 - 8y + \underline{\ 16\ } = -19 + \underline{\ 1\ } + \underline{\ 16\ } \Rightarrow$

$(x-1)^2 + (y-4)^2 = -2$. This is not a circle since r^2 cannot equal -2.

$\boxed{56}$ $x^2 + y^2 + 4x + 6y + 16 = 0 \Rightarrow x^2 + 4x + \underline{\ 4\ } + y^2 + 6y + \underline{\ 9\ } = -16 + \underline{\ 4\ } + \underline{\ 9\ } \Rightarrow$

$(x+2)^2 + (y+3)^2 = -3$. This is not a circle since r^2 cannot equal -3.

[57] To obtain equations for the upper and lower halves, we solve the given equation for y in terms of x. $x^2+y^2=36 \Rightarrow y^2=36-x^2 \Rightarrow y=\pm\sqrt{36-x^2}$.

The upper half is $y=\sqrt{36-x^2}$ and the lower half is $y=-\sqrt{36-x^2}$.

To obtain equations for the right and left halves, we solve for x in terms of y.

$x^2+y^2=36 \Rightarrow x^2=36-y^2 \Rightarrow x=\pm\sqrt{36-y^2}$.

The right half is $x=\sqrt{36-y^2}$ and the left half is $x=-\sqrt{36-y^2}$.

[58] $(x+3)^2+y^2=64 \Rightarrow y^2=64-(x+3)^2 \Rightarrow y=\pm\sqrt{64-(x+3)^2}$.

$(x+3)^2+y^2=64 \Rightarrow (x+3)^2=64-y^2 \Rightarrow$

$x+3=\pm\sqrt{64-y^2} \Rightarrow x=-3\pm\sqrt{64-y^2}$.

[59] $(x-2)^2+(y+1)^2=49 \Rightarrow (y+1)^2=49-(x-2)^2 \Rightarrow$

$y+1=\pm\sqrt{49-(x-2)^2} \Rightarrow y=-1\pm\sqrt{49-(x-2)^2}$.

$(x-2)^2+(y+1)^2=49 \Rightarrow (x-2)^2=49-(y+1)^2 \Rightarrow$

$x-2=\pm\sqrt{49-(y+1)^2} \Rightarrow x=2\pm\sqrt{49-(y+1)^2}$.

[60] $(x-3)^2+(y-5)^2=4 \Rightarrow (y-5)^2=4-(x-3)^2 \Rightarrow$

$y-5=\pm\sqrt{4-(x-3)^2} \Rightarrow y=5\pm\sqrt{4-(x-3)^2}$.

$(x-3)^2+(y-5)^2=4 \Rightarrow (x-3)^2=4-(y-5)^2 \Rightarrow$

$x-3=\pm\sqrt{4-(y-5)^2} \Rightarrow x=3\pm\sqrt{4-(y-5)^2}$.

[61] From the figure, we see that the diameter of the circle is $1-(-7)=8$ units, so the radius is $\frac{1}{2}(8)=4$. The center (h, k) is at the average of the values shown; that is, $h=\dfrac{-7+1}{2}=-3$ and $k=\dfrac{-2+6}{2}=2$. Using the standard form of an equation of a circle, $(x-h)^2+(y-k)^2=r^2$, we have $[x-(-3)]^2+(y-2)^2=4^2$, or

$(x+3)^2+(y-2)^2=4^2$ {or use 16}.

[62] diameter $=4-(-2)=6$; radius $=3$; $h=\dfrac{-2+4}{2}=1$ and $k=\dfrac{-5+1}{2}=-2$

An equation is $(x-1)^2+[y-(-2)]^2=3^2$, or $(x-1)^2+(y+2)^2=3^2$ {or use 9}.

[63] The figure shows the lower semicircle of a circle centered at the origin having radius 4. The circle has equation $x^2+y^2=4^2$. We want the lower semicircle, so we must have *negative* y-values, indicating that we should solve for y and use the negative sign. $x^2+y^2=4^2 \Rightarrow y^2=4^2-x^2 \Rightarrow y=\pm\sqrt{4^2-x^2}$, so $y=-\sqrt{4^2-x^2}$ is the desired equation.

[64] $x^2+y^2=3^2 \Rightarrow x^2=3^2-y^2 \Rightarrow x=\pm\sqrt{3^2-y^2}$, so $x=-\sqrt{3^2-y^2}$ is the desired equation since we want *negative* x-values.

[65] We need to determine if the distance from P to C is *less than* r, *greater than* r, or *equal to* r and hence, P will be *inside* the circle, *outside* the circle, or *on* the circle, respectively.

(a) $P(2,\ 3),\ C(4,\ 6) \Rightarrow d(P,\ C)=\sqrt{4+9}=\sqrt{13}<r\ \{r=4\} \Rightarrow P$ is *inside* C.

(b) $P(4,\ 2),\ C(1,\ -2) \Rightarrow d(P,\ C)=\sqrt{9+16}=5=r\ \{r=5\} \Rightarrow P$ is *on* C.

(c) $P(-3,\ 5),\ C(2,\ 1) \Rightarrow d(P,\ C)=\sqrt{25+16}=\sqrt{41}>r\ \{r=6\} \Rightarrow$

P is *outside* C.

[66] (a) $P(3,\ 8),\ C(-2,\ -4) \Rightarrow d(P,\ C)=\sqrt{25+144}=13=r\ \{r=13\} \Rightarrow$

P is *on* C.

(b) $P(-2,\ 5),\ C(3,\ 7) \Rightarrow d(P,\ C)=\sqrt{25+4}=\sqrt{29}<r\ \{r=6\} \Rightarrow$

P is *inside* C.

(c) $P(1,\ -2),\ C(6,\ -7) \Rightarrow d(P,\ C)=\sqrt{25+25}=\sqrt{50}>r\ \{r=7\} \Rightarrow$

P is *outside* C.

[67] (a) To find the x-intercepts, let $y=0$ and solve the resulting equation for x.

$$x^2-4x+4=0 \Rightarrow (x-2)^2=0 \Rightarrow x=2$$

(b) To find the y-intercepts, let $x=0$ and solve the resulting equation for y.

$$y^2-6y+4=0 \Rightarrow y=\frac{6\pm\sqrt{36-16}}{2}=3\pm\sqrt{5}.$$

[68] (a) $y=0 \Rightarrow x^2-10x+13=0 \Rightarrow x=\dfrac{10\pm\sqrt{100-52}}{2}=5\pm2\sqrt{3}$

(b) $x=0 \Rightarrow y^2+4y+13=0 \Rightarrow y=\dfrac{-4\pm\sqrt{16-52}}{2}$.

The negative discriminant implies that there are no real solutions to the equation and hence, no y-intercepts.

[69] $x^2+y^2+4x-6y+4=0 \Leftrightarrow (x+2)^2+(y-3)^2=9$. This is a circle with center $C(-2,\ 3)$ and radius 3. The circle we want has the same center, $C(-2,\ 3)$, and radius that is equal to the distance from C to $P(2,\ 6)$.

$d(P,\ C)=\sqrt{16+9}=5$ and an equation is $(x+2)^2+(y-3)^2=25$.

[70] By the Pythagorean theorem, the two stations are $d=\sqrt{100^2+80^2}\approx128.06$ miles apart. The sum of their radii, $80+50=130$, is greater than d, indicating that the circles representing their broadcast ranges do overlap.

[71] The equation of circle C_2 is $(x-h)^2+(y-2)^2=2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5-2$ $\{C_2$ radius $-\ C_1$ radius$\}=3$ and sides of length 2 and h. Thus, $h^2+2^2=3^2 \Rightarrow h=\sqrt{5}$.

[72] The equation of circle C_2 is $(x-h)^2+(y-3)^2=2^2$. If we draw a line from the origin to the center of C_2, we form a right triangle with hypotenuse $5+2$ {C_2 radius + C_1 radius} = 7 and sides of length 3 and h. Thus, $h^2+3^2=7^2 \Rightarrow h=\sqrt{40}$.

[73] The graph of y_1 is *below* the graph of y_2 to the left of $x=-3$ and to the right of $x=2$. Writing the x-values in interval notation gives us $(-\infty, -3) \cup (2, \infty)$. Note that the y-values play no role in writing the answer in interval notation.

[74] $y_1 < y_2$ between $x=-8$ and $x=8$, so the interval is $(-8, 8)$.

[75] $y_1 < y_2$ between $x=-1$ and $x=1$, excluding $x=0$ since $y_1=y_2$ at that value, so the interval is $(-1, 0) \cup (0, 1)$.

[76] $y_1 < y_2$ to the left of $x=-8$, between $x=-1$ and $x=1$, and to the right of $x=8$, so the interval notation is $(-\infty, -8) \cup (-1, 1) \cup (8, \infty)$.

[77] The viewing rectangles significantly affect the shape of the circle. Viewing rectangle (2) results in a graph that most looks like a circle.

$[-2, 2]$ by $[-2, 2]$

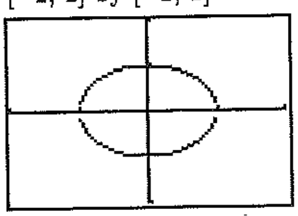

Figure 77(1)

$[-3, 3]$ by $[-2, 2]$

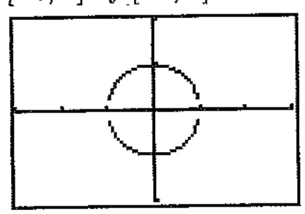

Figure 77(2)

$[-2, 2]$ by $[-5, 5]$

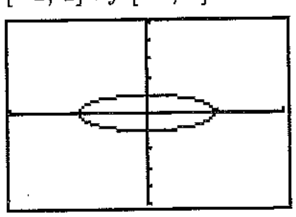

Figure 77(3)

$[-5, 5]$ by $[-2, 2]$

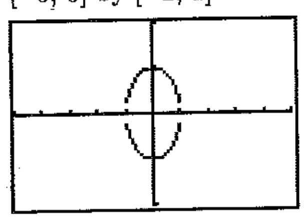

Figure 77(4)

[78] (a) From the graph there are two x–intercepts and two y–intercepts.

(b) Using the free–moving cursor, one can conclude that $|x| + |y| < 5$ is true whenever the point (x, y) is located *inside* the diamond shape.

$[-5, 5]$ by $[-5, 5]$

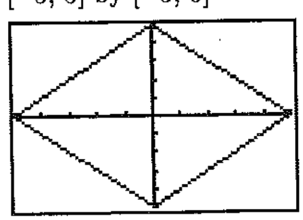

Figure 78

79 Assign $x^3 - \frac{9}{10}x^2 - \frac{43}{25}x + \frac{24}{25}$ to Y_1. After trying a standard viewing rectangle, we see that the x-intercepts are near the origin and we choose the viewing rectangle $[-6, 6]$ by $[-4, 4]$. This is simply one choice, not necessarily the best choice. For most calculator exercises, we have selected viewing rectangles that are in a $3:2$ proportion (horizontal : vertical) to maintain a true proportion. From the graph, there are three x-intercepts. Use a root feature to determine that they are -1.2, 0.5, and 1.6.

$[-6, 6]$ by $[-4, 4]$

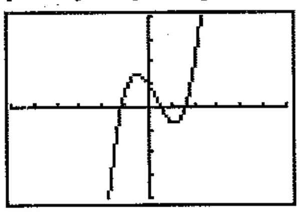

Figure 79

$[-6, 6]$ by $[-4, 4]$

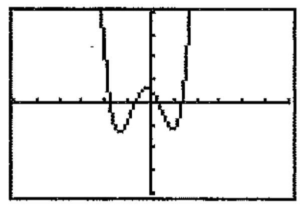

Figure 80

80 From the graph, there are four x-intercepts.

They are approximately -1.8, -0.7, 0.3 and 1.35.

81 Make the assignments $Y_1 = x^3 + x$, $Y_2 = \sqrt{1 - x^2}$, and $Y_3 = -Y_2$.

From the graph, there are two points of intersection.

They are approximately $(0.6, 0.8)$ and $(-0.6, -0.8)$.

$[-3, 3]$ by $[-2, 2]$

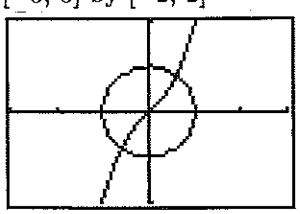

Figure 81

$[-3, 3]$ by $[-2, 2]$

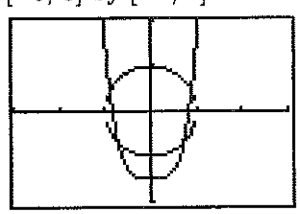

Figure 82

82 Make the assignments $Y_1 = 3x^4 - \frac{3}{2}$, $Y_2 = \sqrt{1 - x^2}$, and $Y_3 = -Y_2$.

From the graph, there are four points of intersection.

They are approximately $(\pm 0.9, 0.4)$ and $(\pm 0.7, -0.7)$.

83 Depending on the type of graphing utility used, you may need to solve for y first. $x^2 + (y-1)^2 = 1 \ \Rightarrow\ y = 1 \pm \sqrt{1 - x^2}$; $(x - \frac{5}{4})^2 + y^2 = 1 \ \Rightarrow\ y = \pm\sqrt{1 - (x - \frac{5}{4})^2}$. Make the assignments $Y_1 = \sqrt{1 - x^2}$, $Y_2 = 1 + Y_1$, $Y_3 = 1 - Y_1$, $Y_4 = \sqrt{1 - (x - \frac{5}{4})^2}$, and $Y_5 = -Y_4$. If a Y_5 is not available, you will need to use other function assignments or alternate methods. For example, on the TI-81, you can graph Y_5 by using DrawF $-Y_4$. Be sure to "turn off" Y_1 before graphing. From the graph, there are two points of intersection. See *Figure 83* on the next page.

They are approximately $(0.999, 0.968)$ and $(0.251, 0.032)$.

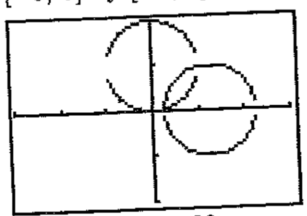

Figure 83

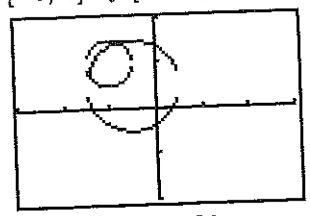

Figure 84

[84] $(x+1)^2+(y-1)^2=\frac{1}{4} \Rightarrow y=1\pm\sqrt{\frac{1}{4}-(x+1)^2}$; $(x+\frac{1}{2})^2+(y-\frac{1}{2})^2=1 \Rightarrow$ $y=\frac{1}{2}\pm\sqrt{1-(x+\frac{1}{2})^2}$. From the graph, there are two points of intersection. They are approximately $(-0.79, 1.46)$ and $(-1.46, 0.79)$.

[85] The cars are initially 4 miles apart. Their distance decreases to 0 when they meet on the highway after 2 minutes. Then, their distance starts to increase until it is 4 miles after a total of 4 minutes.

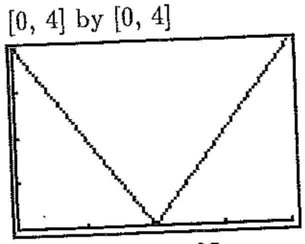

Figure 85

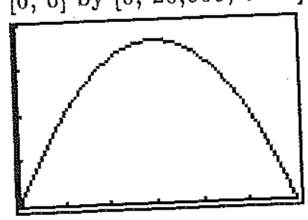

Figure 86

[86] At noon on Sunday the pool is empty since when $x=0$, $A=0$. It is then filled with water, until at noon on Wednesday $(x=3)$, it contains 18,000 gallons. It is then drained until at noon on Saturday $(x=6)$, it is empty again.

[87] (a) $v=1087\sqrt{(20+273)/273}\approx 1126$ ft/sec.

(b) Algebraically: $v=1087\sqrt{\dfrac{T+273}{273}} \Rightarrow 1000=1087\sqrt{\dfrac{T+273}{273}} \Rightarrow$

$$T=\frac{1000^2\times 273}{1087^2}-273\approx -42^{\circ}\mathrm{C}.$$

Graphically: Graph $\mathrm{Y}_1=1087\sqrt{(T+273)/273}$ and $\mathrm{Y}_2=1000$. At the point of their intersection, $T\approx -42^{\circ}\mathrm{C}$.

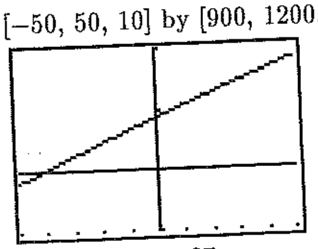

Figure 87

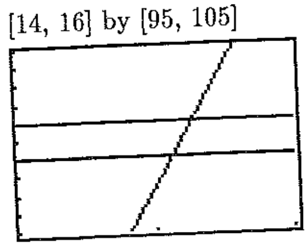

Figure 88

[88] The horizontal lines intersect the graph of A at $s \approx 15.12$, 15.27. Thus, if $15.12 \leq s \leq 15.27$, then $99 \leq A \leq 101$. See *Figure 88.*

3.3 Exercises

[1] $A(-3, 2)$, $B(5, -4)$ $\Rightarrow$ $m_{AB} = \dfrac{(-4) - 2}{5 - (-3)} = \dfrac{-6}{8} = -\dfrac{3}{4}$

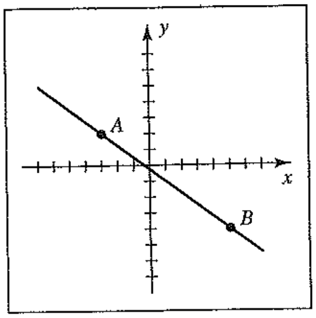

Figure 1

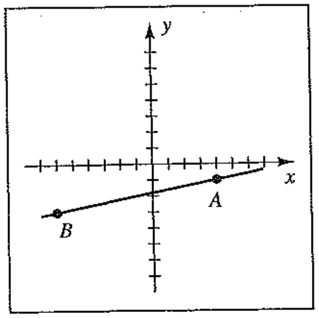

Figure 2

[2] $A(4, -1)$, $B(-6, -3)$ $\Rightarrow$ $m_{AB} = \dfrac{-3 + 1}{-6 - 4} = \dfrac{-2}{-10} = \dfrac{1}{5}$

[3] $A(2, 5)$, $B(-7, 5)$ $\Rightarrow$ $m_{AB} = \dfrac{5 - 5}{-7 - 2} = \dfrac{0}{-9} = 0$

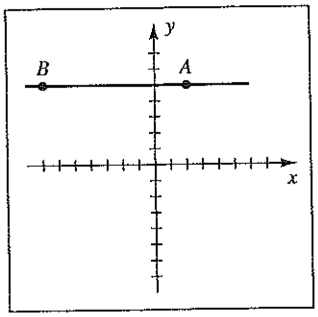

Figure 3

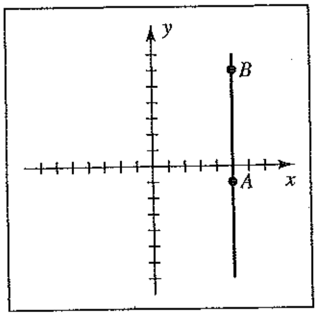

Figure 4

[4] $A(5, -1)$, $B(5, 6)$ $\Rightarrow$ $m_{AB} = \dfrac{6 + 1}{5 - 5} = \dfrac{7}{0}$ $\Rightarrow$ m is undefined

[5] $A(-3, 2)$, $B(-3, 5)$ $\Rightarrow$ $m_{AB} = \dfrac{5 - 2}{-3 - (-3)} = \dfrac{3}{0}$ $\Rightarrow$ m is undefined

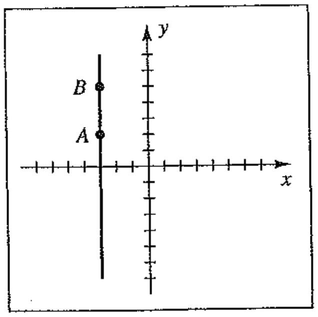

Figure 5

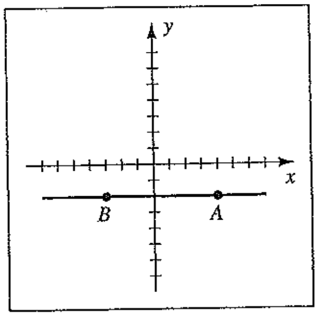

Figure 6

[6] $A(4, -2)$, $B(-3, -2)$ $\Rightarrow$ $m_{AB} = \dfrac{-2 - (-2)}{-3 - 4} = \dfrac{0}{-7} = 0$

[7] Show that the slopes of opposite sides are equal.

$A(-3, 1)$, $B(5, 3)$, $C(3, 0)$, $D(-5, -2)$ $\Rightarrow$ $m_{AB} = \frac{1}{4} = m_{DC}$ and $m_{DA} = \frac{3}{2} = m_{CB}$.

[8] Show that the slopes of one pair of opposite sides are equal.

$A(2, 3)$, $B(5, -1)$, $C(0, -6)$, $D(-6, 2)$ $\Rightarrow$ $m_{AB} = -\frac{4}{3} = m_{CD}$.

[9] Show that the slopes of opposite sides are equal (parallel lines) and the slopes of two adjacent sides are negative reciprocals (perpendicular lines). $A(6, 15)$, $B(11, 12)$, $C(-1, -8)$, $D(-6, -5)$ $\Rightarrow$ $m_{DA} = \frac{5}{3} = m_{CB}$ and $m_{AB} = -\frac{3}{5} = m_{DC}$.

[10] Show that adjacent sides are perpendicular.

$A(1, 4)$, $B(6, -4)$, $C(-15, -6)$ $\Rightarrow$ $m_{AB} = -\frac{8}{5}$ and $m_{AC} = \frac{5}{8}$.

[11] $A(-1, -3)$ is 5 units to the left and 5 units down from $B(4, 2)$. D will have the same relative position from $C(-7, 5)$; that is, $(-7-5, 5-5) = (-12, 0)$.

[12] Let $E = M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$, $F = M_{BC} = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$,

$G = M_{CD} = \left(\frac{x_3 + x_4}{2}, \frac{y_3 + y_4}{2}\right)$, and $H = M_{AD} = \left(\frac{x_1 + x_4}{2}, \frac{y_1 + y_4}{2}\right)$.

The slopes of opposite sides are equal (or lines are vertical).

$$m_{EF} = m_{GH} = \frac{y_3 - y_1}{x_3 - x_1} \text{ and } m_{FG} = m_{EH} = \frac{y_4 - y_2}{x_4 - x_2}$$

[13] $m = 3, -2, \frac{2}{3}, -\frac{1}{4}$ •

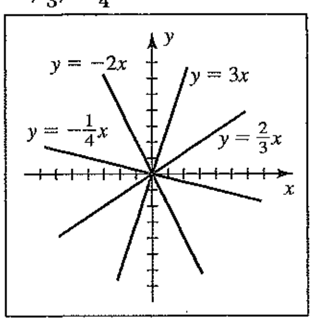

Figure 13

[14] $m = 5, -3, \frac{1}{2}, -\frac{1}{3}$ •

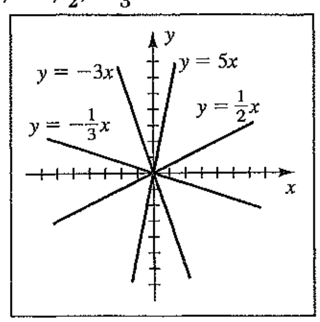

Figure 14

[15] $P(3, 1)$; $m = \frac{1}{2}, -1, -\frac{1}{5}$ •

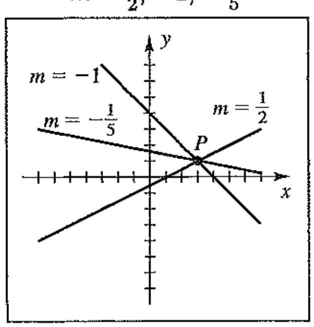

Figure 15

[16] $P(-2, 4)$; $m = 1, -2, -\frac{1}{2}$ •

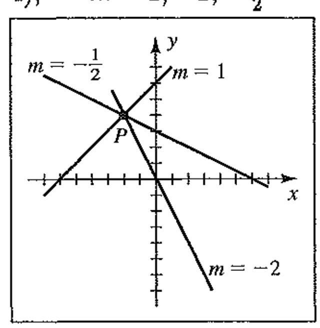

Figure 16

$\boxed{17}$ From the figure, the slope of one of the lines is $\dfrac{\Delta y}{\Delta x} = \dfrac{5}{4}$, so the slopes are $\pm\dfrac{5}{4}$.
Using the point-slope form for the equation of a line with slope $m = \pm\frac{5}{4}$ and point $(x_1, y_1) = (2, -3)$ gives us $y - (-3) = \pm\frac{5}{4}(x - 2)$, or $y + 3 = \pm\frac{5}{4}(x - 2)$.

$\boxed{18}$ $m = \dfrac{\Delta y}{\Delta x} = \dfrac{3}{4}$, so the slopes are $\pm\dfrac{3}{4}$. Using the point $(x_1, y_1) = (-1, 2)$ gives us $y - 2 = \pm\frac{3}{4}[x - (-1)]$, or $y - 2 = \pm\frac{3}{4}(x + 1)$.

$\boxed{19}$ $y = x + 3$, $\quad y = x + 1$, $\quad y = -x + 1$ •

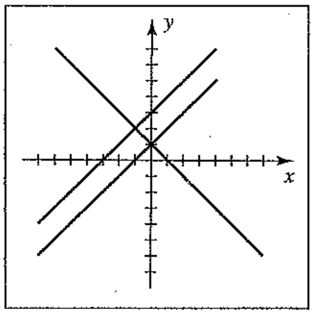

Figure 19

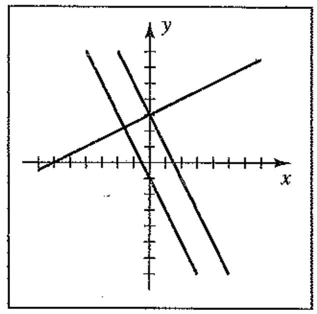

Figure 20

$\boxed{20}$ $y = -2x - 1$, $\quad y = -2x + 3$, $\quad y = \frac{1}{2}x + 3$ •

$\boxed{21}$ (a) Parallel to the y-axis implies the equation is of the form $x = k$.
The x-value of $A(5, -2)$ is 5, hence $x = 5$ is the equation.

(b) Perpendicular to the y-axis implies the equation is of the form $y = k$.
The y-value of $A(5, -2)$ is -2, hence $y = -2$ is the equation.

$\boxed{22}$ (a) The line through $A(-4, 2)$ and parallel to the x-axis is $y = 2$.

(b) The line through $A(-4, 2)$ and perpendicular to the x-axis is $x = -4$.

$\boxed{23}$ Using the point-slope form, the equation of the line through $A(5, -3)$ with slope -4
is $y + 3 = -4(x - 5) \Rightarrow y + 3 = -4x + 20 \Rightarrow 4x + y = 17$.

$\boxed{24}$ $A(-1, 4)$; slope $\frac{2}{3} \Rightarrow y - 4 = \frac{2}{3}(x + 1) \Rightarrow 3(y - 4) = 2(x + 1) \Rightarrow$
$3y - 12 = 2x + 2 \Rightarrow 2x - 3y = -14$.

$\boxed{25}$ $A(4, 0)$; slope $-3 \Rightarrow y - 0 = -3(x - 4) \Rightarrow y = -3x + 12 \Rightarrow 3x + y = 12$.

$\boxed{26}$ $A(0, -2)$; slope $5 \Rightarrow y + 2 = 5(x - 0) \Rightarrow y + 2 = 5x \Rightarrow 5x - y = 2$.

$\boxed{27}$ $A(4, -5)$, $B(-3, 6) \Rightarrow m_{AB} = -\frac{11}{7}$.
$y + 5 = -\frac{11}{7}(x - 4) \Rightarrow 7y + 35 = -11x + 44 \Rightarrow 11x + 7y = 9$.

$\boxed{28}$ $A(-1, 6)$, $B(5, 0) \Rightarrow m = -1$. $y - 0 = -1(x - 5) \Rightarrow y = -x + 5 \Rightarrow x + y = 5$.

$\boxed{29}$ $5x - 2y = 4 \Leftrightarrow y = \frac{5}{2}x - 2$. Using the same slope, $\frac{5}{2}$, with $A(2, -4)$,
gives us $y + 4 = \frac{5}{2}(x - 2) \Rightarrow 2y + 8 = 5x - 10 \Rightarrow 5x - 2y = 18$.

$\boxed{30}$ $x + 3y = 1 \Leftrightarrow y = -\frac{1}{3}x + \frac{1}{3}$. Using the same slope, $-\frac{1}{3}$, with $A(-3, 5)$,
gives us $y - 5 = -\frac{1}{3}(x + 3) \Rightarrow 3y - 15 = -x - 3 \Rightarrow x + 3y = 12$.

[54] (a) $L = at + b$ and $L = 24$ when $t = 0 \Rightarrow L = at + 24$.

$L = 53$ when $t = 7 \Rightarrow 53 = 7a + 24 \Rightarrow a = \frac{29}{7}$ and $L = \frac{29}{7}t + 24$.

(b) From part (a), the slope is $\frac{29}{7}$ ft/month $= \frac{29}{210}$ ft/day ≈ 1.657 inches/day.

(c) $W = at + b$ and $W = 3$ when $t = 0 \Rightarrow W = at + 3$.

$W = 23$ when $t = 7 \Rightarrow 23 = 7a + 3 \Rightarrow a = \frac{20}{7}$ and $W = \frac{20}{7}t + 3$.

(d) From part (c), the slope is $\frac{20}{7}$ tons/month $= \frac{2}{21}$ tons/day ≈ 190.476 pounds/day.

[55] (a) $y = mx = \dfrac{\text{change in } y \text{ from the beginning of the season}}{\text{change in } x \text{ from the beginning of the season}}(x) = \dfrac{5-0}{14-0}x = \dfrac{5}{14}x$.

(b) $x = 162 \Rightarrow y = \frac{5}{14}(162) \approx 58$.

[56] (a) $y = mx = \dfrac{\text{change in } y \text{ for the year}}{\text{change in } x \text{ for the year}}(x) = \dfrac{18{,}000 - 0}{(31 + 28 + 24) - 0}x = \dfrac{18{,}000}{83}x$.

(b) $x = 365 \Rightarrow y = \frac{18{,}000}{83}(365) \approx 79{,}157$.

[57] (a) Using the slope-intercept form, $W = mt + b = mt + 10$.

$W = 30$ when $t = 3 \Rightarrow 30 = 3m + 10 \Rightarrow m = \frac{20}{3}$ and $W = \frac{20}{3}t + 10$.

(b) $t = 6 \Rightarrow W = \frac{20}{3}(6) + 10 \Rightarrow W = 50$ lb

(c) $W = 70 \Rightarrow 70 = \frac{20}{3}t + 10 \Rightarrow 60 = \frac{20}{3}t \Rightarrow t = 9$ years old

(d) The graph has endpoints at (0, 10) and (12, 90).

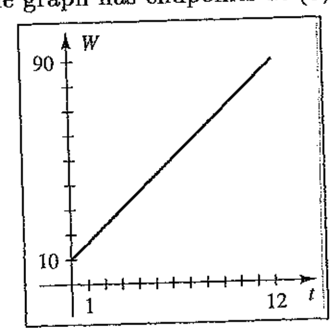

Figure 57

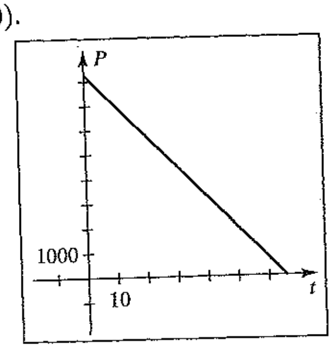

Figure 58

[58] (a) Using the slope-intercept form, $P = mt + b = mt + 8250$.

$P = 8125$ when $t = 1$ {after one payment} $\Rightarrow 8125 = m + 8250 \Rightarrow$

$m = -125$ and $P = -125t + 8250$.

(b) $P = 5000 \Rightarrow -125t + 8250 = 5000 \Rightarrow -125t = -3250 \Rightarrow t = 26$ months

(c) $\frac{8250}{125} = 66$ payments. The graph has endpoints at (0, 8250) and (66, 0).

[59] Using (10, 2480) and (25, 2440),

we have $H - 2440 = \dfrac{2440 - 2480}{25 - 10}(T - 25)$, or $H = -\frac{8}{3}T + \dfrac{7520}{3}$.

[60] (a) Using (1800, 100) and (5000, 40), we have

$P - 40 = \dfrac{40 - 100}{5000 - 1800}(h - 5000)$, or $P = -\dfrac{3}{160}h + \dfrac{535}{4}$ for $1800 \le h \le 5000$.

(b) $h = 2400 \Rightarrow P = -\frac{3}{160}(2400) + \frac{535}{4} = \frac{355}{4}$, or 88.75%.

$\boxed{61}$ (a) Using the slope-intercept form with $m = 0.032$ and $b = 13.5$,

we have $T = 0.032t + 13.5$.

(b) $t = 2000 - 1915 = 85 \Rightarrow T = 0.032(85) + 13.5 = 16.22°\text{C}$.

$\boxed{62}$ (a) Using (1870, 11.8) and (1969, 13.5), we have $T - 13.5 = \dfrac{13.5 - 11.8}{1969 - 1870}(t - 1969)$.

(b) $T = 12.5 \Rightarrow -1 = \dfrac{13.5 - 11.8}{1969 - 1870}(t - 1969) \Rightarrow -\dfrac{99}{1.7} = t - 1969 \Rightarrow$

$t \approx 1910.76$, or during the year 1910.

$\boxed{63}$ (a) Expenses = (\$1000) + (5% of R) + (\$2600) + (50% of R) $\Rightarrow E = 0.55R + 3600$.

(b) Profit = Revenue − Expenses $\Rightarrow P = R - (0.55R + 3600) \Rightarrow$

$P = 0.45R - 3600$.

(c) *Break even* means P would be 0. $P = 0 \Rightarrow 0 = 0.45R - 3600 \Rightarrow$

$0.45R = 3600 \Rightarrow R = 3600(\frac{100}{45}) = \$8000/\text{month}$

$\boxed{64}$ (a) $a = 100$ gives us $y = \frac{25}{6}(t + 1)$

with endpoints $(0, \frac{25}{6})$ and $(12, \frac{325}{6})$,

and $y = 8t$ with endpoints (0, 0) and (12, 96).

(b) $\frac{1}{24}(t+1)a = \frac{2}{25}ta \Rightarrow \frac{1}{24}(t+1) = \frac{2}{25}t \Rightarrow$

$25(t+1) = 24(2t) \Rightarrow 25t + 25 = 48t \Rightarrow$

$25 = 23t \Rightarrow t = \frac{25}{23}$ yr ≈ 13 months

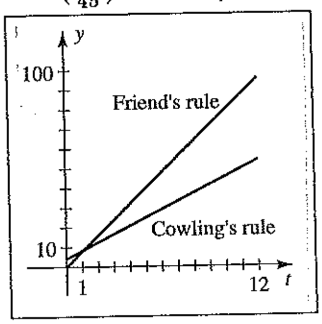

Figure 64

$\boxed{65}$ To determine if a target is hit, set $y = 0$ and solve for x.

(a) $y - 2 = -1(x - 1) \Rightarrow x + y = 3$. $y = 0 \Rightarrow x = 3$ and a creature is hit.

(b) $y - \frac{5}{3} = -\frac{4}{9}(x - \frac{3}{2}) \Rightarrow 4x + 9y = 21$. $y = 0 \Rightarrow x = 5.25$ and no creature is hit.

$\boxed{66}$ (a) $C = F$ and $C = \frac{5}{9}(F - 32) \Rightarrow F = \frac{5}{9}(F - 32) \Rightarrow 9F = 5F - 160 \Rightarrow$

$4F = -160 \Rightarrow F = -40$.

(b) $F = 2C$ and $C = \frac{5}{9}(F - 32) \Rightarrow C = \frac{5}{9}(2C - 32) \Rightarrow 9C = 10C - 160 \Rightarrow$

$C = 160$ and hence, $F = 320$.

$\boxed{67}$ $s = \dfrac{v_2 - v_1}{h_2 - h_1} \Rightarrow 0.07 = \dfrac{v_2 - 22}{185 - 0} \Rightarrow v_2 = 22 + 0.07(185) = 34.95$ mi/hr.

$\boxed{68}$ From Exercise 67, the average wind shear is $s = \dfrac{v_2 - v_1}{h_2 - h_1}$.

We know $v_1 = 32$ at $h_1 = 20$. We need to find v_2 at $h_2 = 200$. $\dfrac{v_1}{v_2} = \left(\dfrac{h_1}{h_2}\right)^P \Rightarrow$

$$v_2 = v_1\left(\frac{h_2}{h_1}\right)^P = 32\left(\frac{200}{20}\right)^{0.13}. \text{ Thus, } s = \frac{32(10^{0.13}) - 32}{200 - 20} \approx 0.062 \text{ (mi/hr)/ft.}$$

$\boxed{69}$ The slope of AB is $\dfrac{-1.11905 - (-1.3598)}{-0.55 - (-1.3)} = 0.321$. Similarly, the slopes of BC and CD are also 0.321. Therefore, the points all lie on the same line. Since the common slope is 0.321, let $a = 0.321$. $y = 0.321x + b \Rightarrow -1.3598 = 0.321(-1.3) + b \Rightarrow$ $b = -0.9425$. Thus, the points are linearly related by the equation

$$y = 0.321x - 0.9425.$$

70 The slopes of AB and BC are both -0.44, whereas the slope of CD is approximately -1.107. Thus, the points do not lie on the same line.

71 $x - 3y = -58 \Leftrightarrow y = (x + 58)/3$ and $3x - y = -70 \Leftrightarrow y = 3x + 70$. Assign $(x + 58)/3$ to Y_1 and $y = 3x + 70$ to Y_2. Using a standard viewing rectangle, we don't see the lines. Zooming out gives us an indication where the lines intersect and by using an intersect feature, we find that the lines intersect at $(-19, 13)$.

[−30, 3, 2] by [−2, 20, 2]

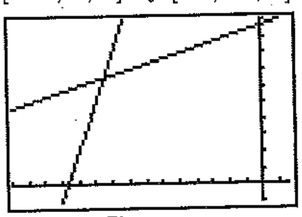

Figure 71

[−12, 12] by [−2, 14]

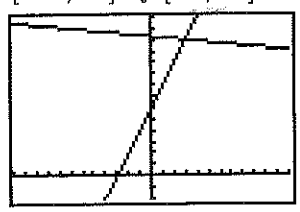

Figure 72

72 $x + 10y = 123 \Leftrightarrow y = (-x + 123)/10$ and $2x - y = -6 \Leftrightarrow y = 2x + 6$. Assign $(-x + 123)/10$ to Y_1 and $y = 2x + 6$ to Y_2. Similar to Exercise 71, the lines intersect at $(3, 12)$.

73 From the graph, we can see that the points of intersection are $A(-0.8, -0.6)$, $B(4.8, -3.4)$, and $C(2, 5)$. The lines intersecting at A are perpendicular since they have slopes of 2 and $-\frac{1}{2}$. Since $d(A, B) = \sqrt{39.2}$ and $d(A, C) = \sqrt{39.2}$, the triangle is isosceles. Thus, the polygon is a right isosceles triangle.

[−15, 15] by [−10, 10]

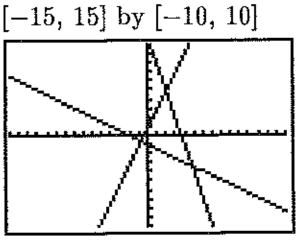

Figure 73

[−3, 3] by [−2, 2]

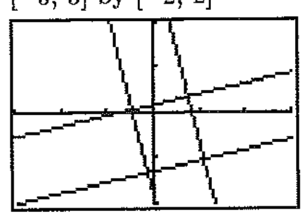

Figure 74

74 The equations of the lines can be rewritten as $y = \frac{1}{4.2}x + 0.17$, $y = -4.2x - 1.9$, $y = \frac{1}{4.2}x - 1.3$, and $y = -4.2x + 3.5$. From the graph, we can see that the points of intersection are approximately $A(0.75, 0.35)$, $B(1.08, -1.04)$, $C(-0.14, -1.33)$, and $D(-0.47, 0.059)$. The first and third lines are parallel as are the second and fourth lines. In addition, these pairs of lines are perpendicular to each other since their slopes are -4.2 and $\frac{1}{4.2}$. Since $d(A, B) \approx 1.43$ and $d(A, D) \approx 1.25$, it is not a square. Thus, the polygon is a rectangle.

[75] The data appear to be linear. Using the two arbitrary points $(-7, -25)$ and $(4.6, 12.2)$, the slope of the line is $\frac{12.2-(-25)}{4.6-(-7)} \approx 3.2$. An equation of the line is $y+25 = 3.2(x+7) \Rightarrow y = 3.2x - 2.6$. If we find the regression line on a calculator, we get the model $y \approx 3.20303x - 2.51553$.

[−8, 5] by [−27, 15, 5]

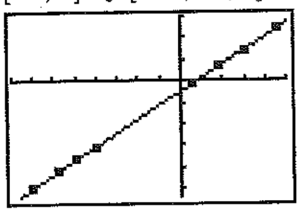

Figure 75

[−1, 8] by [−1, 5]

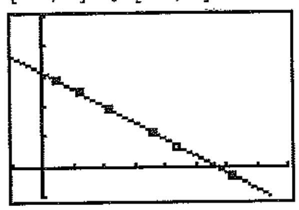

Figure 76

[76] The data appear to be linear. Using the two arbitrary points $(0.4, 2.88)$ and $(6.2, -0.3)$, the slope of the line is $\frac{-0.3-2.88}{6.2-0.4} \approx -0.55$. An equation of the line is $y - 2.88 = -0.55(x-0.4) \Rightarrow y = -0.55x + 3.1$. The regression line is $y \approx -0.54905x + 3.09882$.

[77] (a) [1980, 1988] by [300, 625, 100]

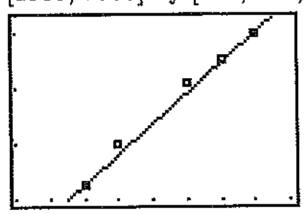

Figure 77

(b) To find a first approximation for the line use the arbitrary points (1982, 325) and (1987, 600). The resulting line is $y = 55x - 108{,}685$. Adjustments may be made to this equation. The regression line is $y \approx 54.01163x - 106{,}714.477$.

(c) Let $y = 55x - 108{,}685$. When $x = 1984$, $y = 435$ and when $x = 1995$, $y = 1040$.

[78] (a) [1950, 1985, 5] by [225, 240, 5]

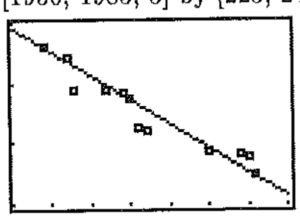

Figure 78

(b) For an approximation for the line, we'll use the first and last points given; that is, (1954, 238.0) and (1981, 227.3). These points determine the line

$$(T - 238.0) = \frac{227.3 - 238.0}{1981 - 1954}(Y - 1954) \Rightarrow T = -0.3963Y + 1012.36.$$

The regression line is $y \approx -0.35173x + 924.37981$.

(c) $T = -0.3963(1985) + 1012.36 = 225.7$ seconds, which is 0.6 second fast.

(d) The slope of the line is approximately -0.4. This means that *on the average*, the record time for the mile has decreased by 0.4 sec/yr.

3.4 Exercises

1 $f(x) = -x^2 - x - 4 \Rightarrow f(-2) = -4 + 2 - 4 = -6$, $f(0) = -4$, and $f(4) = -24$.

2 $f(x) = -x^3 - x^2 + 3 \Rightarrow f(-3) = 27 - 9 + 3 = 21$, $f(0) = 3$, and $f(2) = -9$.

3 $f(x) = \sqrt{x-4} - 3x \Rightarrow f(4) = -12$, $f(8) = -22$, and $f(13) = -36$.

4 $f(x) = \dfrac{x}{x-3} \Rightarrow f(-2) = \frac{2}{5}$, $f(0) = 0$, and $f(3)$ is undefined.

5 (a) $f(x) = 5x - 2 \Rightarrow f(a) = 5(a) - 2 = 5a - 2$ (b) $f(-a) = 5(-a) - 2 = -5a - 2$

(c) $-f(a) = -1 \cdot (5a - 2) = -5a + 2$ (d) $f(a+h) = 5(a+h) - 2 = 5a + 5h - 2$

(e) $f(a) + f(h) = (5a - 2) + (5h - 2) = 5a + 5h - 4$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(5a + 5h - 2) - (5a - 2)}{h} = \dfrac{5h}{h} = 5$

6 (a) $f(x) = 3 - 4x \Rightarrow f(a) = 3 - 4(a) = 3 - 4a$ (b) $f(-a) = 3 - 4(-a) = 3 + 4a$

(c) $-f(a) = -1 \cdot (3 - 4a) = 4a - 3$ (d) $f(a+h) = 3 - 4(a+h) = 3 - 4a - 4h$

(e) $f(a) + f(h) = (3 - 4a) + (3 - 4h) = 6 - 4a - 4h$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(3 - 4a - 4h) - (3 - 4a)}{h} = \dfrac{-4h}{h} = -4$

7 (a) $f(x) = -x^2 + 4 \Rightarrow f(a) = -(a)^2 + 4 = -a^2 + 4$

(b) $f(-a) = -(-a)^2 + 4 = -a^2 + 4$ (c) $-f(a) = -1 \cdot (-a^2 + 4) = a^2 - 4$

(d) $f(a+h) = -(a+h)^2 + 4 = -(a^2 + 2ah + h^2) + 4 = -a^2 - 2ah - h^2 + 4$

(e) $f(a) + f(h) = (-a^2 + 4) + (-h^2 + 4) = -a^2 - h^2 + 8$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(-a^2 - 2ah - h^2 + 4) - (-a^2 + 4)}{h} = \dfrac{-2ah - h^2}{h} = \dfrac{h(-2a - h)}{h}$
$= -2a - h$

8 (a) $f(x) = 3 - x^2 \Rightarrow f(a) = 3 - (a)^2 = 3 - a^2$

(b) $f(-a) = 3 - (-a)^2 = 3 - a^2$ (c) $-f(a) = -1 \cdot (3 - a^2) = -3 + a^2$

(d) $f(a+h) = 3 - (a+h)^2 = 3 - (a^2 + 2ah + h^2) = 3 - a^2 - 2ah - h^2$

(e) $f(a) + f(h) = (3 - a^2) + (3 - h^2) = 6 - a^2 - h^2$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(3 - a^2 - 2ah - h^2) - (3 - a^2)}{h} = \dfrac{-2ah - h^2}{h} = \dfrac{h(-2a - h)}{h}$
$= -2a - h$

9 (a) $f(x) = x^2 - x + 3 \Rightarrow f(a) = (a)^2 - (a) + 3 = a^2 - a + 3$

(b) $f(-a) = (-a)^2 - (-a) + 3 = a^2 + a + 3$

(c) $-f(a) = -1 \cdot (a^2 - a + 3) = -a^2 + a - 3$

(d) $f(a+h) = (a+h)^2 - (a+h) + 3 = a^2 + 2ah + h^2 - a - h + 3$

(e) $f(a) + f(h) = (a^2 - a + 3) + (h^2 - h + 3) = a^2 + h^2 - a - h + 6$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(a^2 + 2ah + h^2 - a - h + 3) - (a^2 - a + 3)}{h} = \dfrac{2ah + h^2 - h}{h} =$
$\dfrac{h(2a + h - 1)}{h} = 2a + h - 1$

$\boxed{10}$ (a) $f(x) = 2x^2 + 3x - 7 \Rightarrow f(a) = 2(a)^2 + 3(a) - 7 = 2a^2 + 3a - 7$

(b) $f(-a) = 2(-a)^2 + 3(-a) - 7 = 2a^2 - 3a - 7$

(c) $-f(a) = -1 \cdot (2a^2 + 3a - 7) = -2a^2 - 3a + 7$

(d) $f(a+h) = 2(a+h)^2 + 3(a+h) - 7 = 2(a^2 + 2ah + h^2) + 3a + 3h - 7 =$
$2a^2 + 4ah + 2h^2 + 3a + 3h - 7$

(e) $f(a) + f(h) = (2a^2 + 3a - 7) + (2h^2 + 3h - 7) = 2a^2 + 2h^2 + 3a + 3h - 14$

(f) $\dfrac{f(a+h) - f(a)}{h} = \dfrac{(2a^2 + 4ah + 2h^2 + 3a + 3h - 7) - (2a^2 + 3a - 7)}{h} =$

$\dfrac{4ah + 2h^2 + 3h}{h} = \dfrac{h(4a + 2h + 3)}{h} = 4a + 2h + 3$

$\boxed{11}$ (a) $g\left(\frac{1}{a}\right) = 4\left(\frac{1}{a}\right)^2 = \dfrac{4}{a^2}$ (b) $\dfrac{1}{g(a)} = \dfrac{1}{4(a)^2} = \dfrac{1}{4a^2}$

(c) $g(\sqrt{a}) = 4(\sqrt{a})^2 = 4a$ (d) $\sqrt{g(a)} = \sqrt{4a^2} = 2\,|\,a\,| = 2a$ since $a > 0$

$\boxed{12}$ (a) $g\left(\frac{1}{a}\right) = 2\left(\frac{1}{a}\right) - 5 = \frac{2}{a} - 5 = \dfrac{2 - 5a}{a}$ (b) $\dfrac{1}{g(a)} = \dfrac{1}{2(a) - 5} = \dfrac{1}{2a - 5}$

(c) $g(\sqrt{a}) = 2(\sqrt{a}) - 5 = 2\sqrt{a} - 5$ (d) $\sqrt{g(a)} = \sqrt{2a - 5}$

$\boxed{13}$ (a) $g\left(\frac{1}{a}\right) = \dfrac{2(1/a)}{(1/a)^2 + 1} = \dfrac{2/a}{1/a^2 + 1} \cdot \dfrac{a^2}{a^2} = \dfrac{2a}{1 + a^2} = \dfrac{2a}{a^2 + 1}$

(b) $\dfrac{1}{g(a)} = \dfrac{1}{\dfrac{2a}{a^2 + 1}} = \dfrac{a^2 + 1}{2a}$ (c) $g(\sqrt{a}) = \dfrac{2\sqrt{a}}{(\sqrt{a})^2 + 1} = \dfrac{2\sqrt{a}}{a + 1}$

(d) $\sqrt{g(a)} = \sqrt{\dfrac{2a}{a^2 + 1}} \cdot \dfrac{\sqrt{a^2 + 1}}{\sqrt{a^2 + 1}} = \dfrac{\sqrt{2a^3 + 2a}}{a^2 + 1}$

$\boxed{14}$ (a) $g\left(\frac{1}{a}\right) = \dfrac{(\frac{1}{a})^2}{\frac{1}{a} + 1} = \dfrac{\frac{1}{a^2}}{\frac{1 + a}{a}} \cdot \dfrac{a^2}{a^2} = \dfrac{1}{a(a+1)}$ (b) $\dfrac{1}{g(a)} = \dfrac{1}{\dfrac{a^2}{a + 1}} = \dfrac{a + 1}{a^2}$

(c) $g(\sqrt{a}) = \dfrac{(\sqrt{a})^2}{\sqrt{a} + 1} \cdot \dfrac{\sqrt{a} - 1}{\sqrt{a} - 1} = \dfrac{a(\sqrt{a} - 1)}{a - 1}$

(d) $\sqrt{g(a)} = \sqrt{\dfrac{a^2}{a + 1}} \cdot \sqrt{\dfrac{a + 1}{a + 1}} = \dfrac{a\sqrt{a + 1}}{a + 1}$

$\boxed{15}$ All vertical lines intersect the graph in at most one point,
so the graph *is* the graph of a function because it passes the Vertical Line Test.

$\boxed{16}$ At least one vertical line intersects the graph in more than one point,
so the graph *is not* the graph of a function because it fails the Vertical Line Test.

$\boxed{17}$ The domain D is the set of x-values; that is, $D = [-4, 1] \cup [2, 4)$. Note that the solid dots on the figure correspond to using brackets {including}, whereas the open dot corresponds to using parentheses {excluding}. The range R is the set of y-values; that is, $R = [-3, 3)$.

$\boxed{18}$ $D = \{x\text{-values}\} = [-4, 4)$. $R = \{y\text{-values}\} = (-3, -1) \cup [1, 3]$.

$\boxed{19}$ (a) $[-3, 4]$ (b) $[-2, 2]$ (c) $f(1) = 0$

(d) $f(x) = 1 \Rightarrow x = -1, \frac{1}{2}, 2$ (e) $f(x) > 1 \Rightarrow x \in (-1, \frac{1}{2}) \cup (2, 4]$

$\boxed{20}$ (a) $[-5, 7]$ (b) $[-1, 2]$ (c) $f(1) = -1$

(d) $f(x) = 1 \Rightarrow x = -3, -1, 3, 5$ (e) $f(x) > 1 \Rightarrow x \in (-3, -1) \cup (3, 5)$

$\boxed{21}$ $f(x) = \sqrt{2x+7}$ • $2x + 7 \geq 0 \Rightarrow x \geq -\frac{7}{2} \Leftrightarrow [-\frac{7}{2}, \infty)$

$\boxed{22}$ $f(x) = \sqrt{8-3x}$ • $8 - 3x \geq 0 \Rightarrow x \leq \frac{8}{3} \Leftrightarrow (-\infty, \frac{8}{3}]$

$\boxed{23}$ $f(x) = \sqrt{9-x^2}$ • $9 - x^2 \geq 0 \Rightarrow 3 \geq |x| \Rightarrow -3 \leq x \leq 3 \Leftrightarrow [-3, 3]$

$\boxed{24}$ $f(x) = \sqrt{x^2-25}$ • $x^2 - 25 \geq 0 \Rightarrow |x| \geq 5 \Rightarrow x \geq 5$ or $x \leq -5 \Leftrightarrow$

$(-\infty, -5] \cup [5, \infty)$

$\boxed{25}$ $f(x) = \dfrac{x+1}{x^3-4x}$ • $x^3 - 4x = 0 \Rightarrow x(x+2)(x-2) = 0 \Leftrightarrow \mathbb{R} - \{\pm 2, 0\}$

$\boxed{26}$ $f(x) = \dfrac{4x}{6x^2+13x-5}$ •

$6x^2 + 13x - 5 = 0 \Rightarrow (2x+5)(3x-1) = 0 \Leftrightarrow \mathbb{R} - \{-\frac{5}{2}, \frac{1}{3}\}$

$\boxed{27}$ $f(x) = \dfrac{\sqrt{2x-3}}{x^2-5x+4}$ •

$x^2 - 5x + 4 = 0 \Rightarrow (x-1)(x-4) = 0 \Rightarrow x = 1, 4;\ 2x - 3 \geq 0 \Rightarrow x \geq \frac{3}{2}$.

The domain is $[\frac{3}{2}, \infty)$, excluding 4, or, equivalently, $[\frac{3}{2}, 4) \cup (4, \infty)$.

$\boxed{28}$ $f(x) = \dfrac{\sqrt{4x-3}}{x^2-4}$ • $x^2 - 4 = 0 \Rightarrow (x+2)(x-2) = 0 \Rightarrow x = \pm 2;$

$4x - 3 \geq 0 \Rightarrow x \geq \frac{3}{4} \Leftrightarrow [\frac{3}{4}, 2) \cup (2, \infty)$

$\boxed{29}$ $f(x) = \dfrac{x-4}{\sqrt{x-2}}$ • $x - 2 > 0 \Rightarrow x > 2 \Leftrightarrow (2, \infty)$

$\boxed{30}$ $f(x) = \dfrac{1}{(x-3)\sqrt{x+3}}$ • $x + 3 > 0 \Rightarrow x > -3\ \{x \neq 3\} \Leftrightarrow (-3, 3) \cup (3, \infty)$

$\boxed{31}$ $f(x) = \sqrt{x+2} + \sqrt{2-x}$ • $x + 2 \geq 0 \Rightarrow x \geq -2;\ 2 - x \geq 0 \Rightarrow x \leq 2$.

The domain is the intersection of $x \geq -2$ and $x \leq 2$; that is, $[-2, 2]$.

$\boxed{32}$ $f(x) = \sqrt{(x-2)(x-6)}$ •

$(x-2)(x-6) \geq 0 \Rightarrow x \leq 2$ or $x \geq 6$ {use a sign diagram} $\Rightarrow (-\infty, 2] \cup [6, \infty)$

$\boxed{33}$ (a) $D = \{x\text{-values}\} = [-5, -3) \cup (-1, 1] \cup (2, 4]$; $R = \{y\text{-values}\} = \{-3\} \cup [-1, 4]$.

Note that the notation for including the single value -3 in R uses braces.

(b) ***f* is increasing** on an interval if it goes up as we move from left to right, so f is increasing on $[-4, -3) \cup [3, 4]$.

***f* is decreasing** on an interval if it goes down as we move from left to right, so f is decreasing on $[-5, -4] \cup (2, 3]$. Note that the values $x = -4$ and $x = 3$ are in intervals that are listed as increasing and in intervals that are listed as decreasing.

(continued)

f is constant on an interval if the y-values do not change, so f is constant on $(-1, 1]$.

$\boxed{34}$ (a) $D = \{ x\text{-values} \} = [-5, -3) \cup (-2, -1] \cup [0, 2) \cup (3, 5]$;

$R = \{ y\text{-values} \} = [-3, 3] \cup \{ 4 \}$

(b) **f is increasing** on $[-5, -3) \cup (3, 4]$. **f is decreasing** on $[0, 2)$.

f is constant on $(-2, -1] \cup [4, 5]$.

$\boxed{35}$ The graph of the function is increasing on $(-\infty, -3]$ and is decreasing on $[-3, 2]$, so there must be a high point at $x = -3$. Now the graph of the function is increasing on $[2, \infty)$, so there must be a low point at $x = 2$.

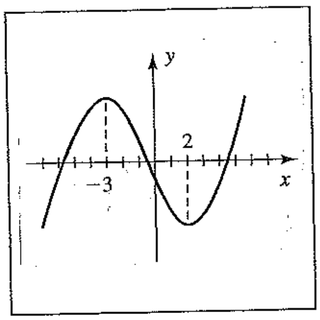

Figure 35

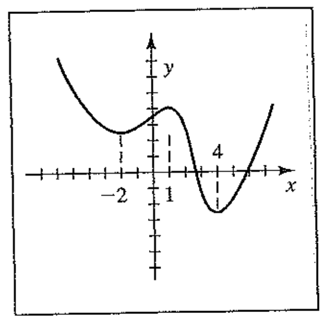

Figure 36

$\boxed{36}$ The graph of the function is decreasing on $(-\infty, -2]$ and $[1, 4]$, and is increasing on $[-2, 1]$ and $[4, \infty)$.

$\boxed{37}$ (b) $D = (-\infty, \infty)$, $R = (-\infty, \infty)$ (c) Increasing on $(-\infty, \infty)$

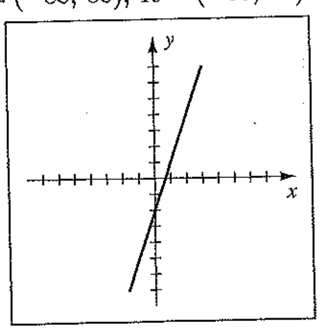

Figure 37

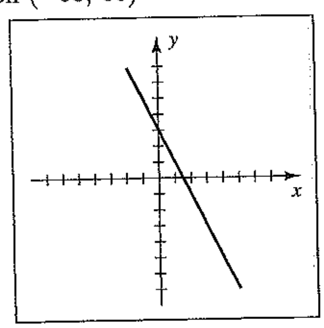

Figure 38

$\boxed{38}$ (b) $D = (-\infty, \infty)$, $R = (-\infty, \infty)$ (c) Decreasing on $(-\infty, \infty)$

$\boxed{39}$ (b) $D = (-\infty, \infty)$, $R = (-\infty, 4]$ (c) Increasing on $(-\infty, 0]$, decreasing on $[0, \infty)$

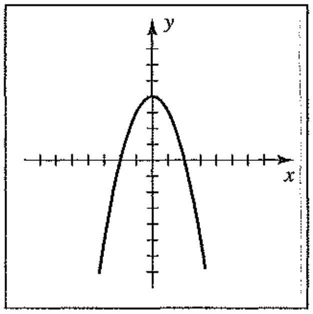

Figure 39

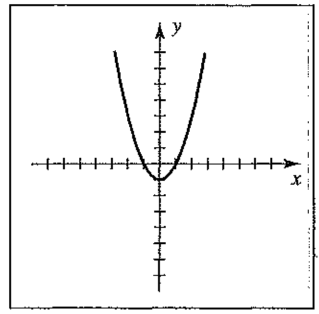

Figure 40

$\boxed{40}$ (b) $D = (-\infty, \infty)$, $R = [-1, \infty)$ (c) Decreasing on $(-\infty, 0]$, increasing on $[0, \infty)$

$\boxed{41}$ (b) $D = [-4, \infty)$, $R = [0, \infty)$ (c) Increasing on $[-4, \infty)$

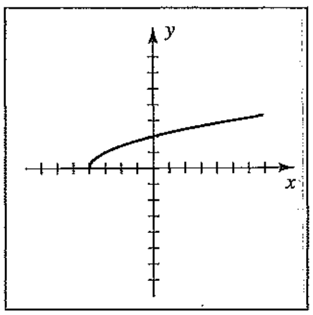

Figure 41

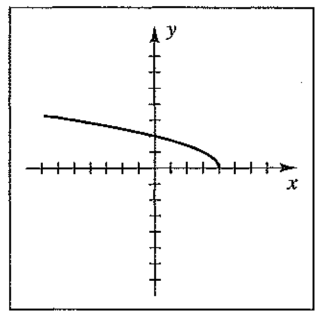

Figure 42

$\boxed{42}$ (b) $D = (-\infty, 4]$, $R = [0, \infty)$ (c) Decreasing on $(-\infty, 4]$

$\boxed{43}$ (b) $D = (-\infty, \infty)$, $R = \{-2\}$ (c) Constant on $(-\infty, \infty)$

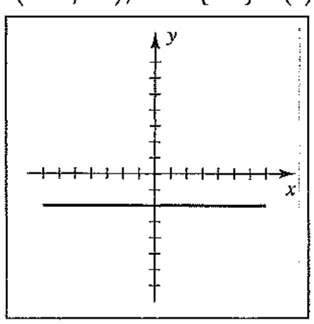

Figure 43

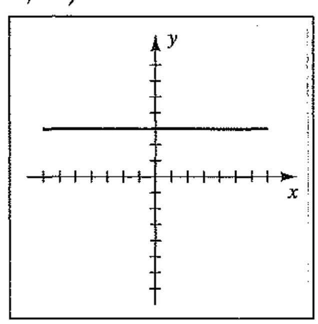

Figure 44

$\boxed{44}$ (b) $D = (-\infty, \infty)$, $R = \{3\}$ (c) Constant on $(-\infty, \infty)$

[45] (b) $D = [-6, 6]$, $R = [-6, 0]$ (c) Decreasing on $[-6, 0]$, increasing on $[0, 6]$

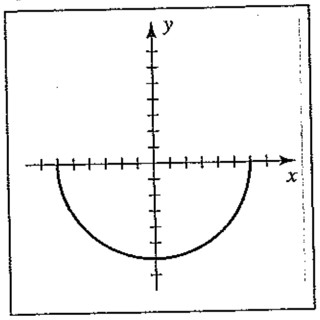

Figure 45

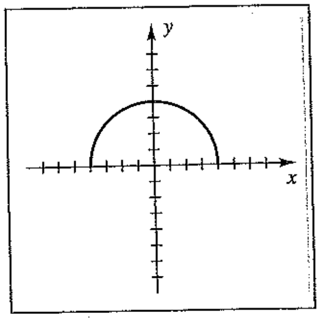

Figure 46

[46] (b) $D = [-4, 4]$, $R = [0, 4]$ (c) Increasing on $[-4, 0]$, decreasing on $[0, 4]$

[47] $f(x) = x^2 - 3x$, so $f(2) = 2^2 - 3(2) = 4 - 6 = -2$.

$$\frac{f(2+h)-f(2)}{h} = \frac{[(2+h)^2 - 3(2+h)] - (-2)}{h} = \frac{4+4h+h^2-6-3h+2}{h} = \frac{h+h^2}{h} = \frac{h(h+1)}{h} = h+1$$

[48] $f(x) = -2x^2 + 3$, so $f(2) = -2(2)^2 + 3 = -2 \cdot 4 + 3 = -8 + 3 = -5$.

$$\frac{f(2+h)-f(2)}{h} = \frac{[-2(2+h)^2+3] - (-5)}{h} = \frac{-2(4+4h+h^2)+3+5}{h} = \frac{-8-8h-2h^2+8}{h} = \frac{-8h-2h^2}{h} = \frac{h(-8-2h)}{h} = -2h-8$$

[49] $$\frac{f(x+h)-f(x)}{h} = \frac{[(x+h)^2+5]-[x^2+5]}{h} = \frac{(x^2+2xh+h^2+5)-(x^2+5)}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = 2x+h$$

[50] $$\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \frac{\dfrac{x^2-(x+h)^2}{(x+h)^2x^2}}{h} = \frac{x^2-(x^2+2xh+h^2)}{hx^2(x+h)^2} = \frac{-2xh-h^2}{hx^2(x+h)^2} = -\frac{h(2x+h)}{hx^2(x+h)^2} = -\frac{2x+h}{x^2(x+h)^2}$$

[51] $$\frac{f(x)-f(a)}{x-a} = \frac{\sqrt{x-3}-\sqrt{a-3}}{x-a} = \frac{\sqrt{x-3}-\sqrt{a-3}}{x-a} \cdot \frac{\sqrt{x-3}+\sqrt{a-3}}{\sqrt{x-3}+\sqrt{a-3}} = \frac{(x-3)-(a-3)}{(x-a)(\sqrt{x-3}+\sqrt{a-3})} = \frac{x-a}{(x-a)(\sqrt{x-3}+\sqrt{a-3})} = \frac{1}{\sqrt{x-3}+\sqrt{a-3}}$$

[52] $$\frac{f(x)-f(a)}{x-a} = \frac{(x^3-2)-(a^3-2)}{x-a} = \frac{x^3-a^3}{x-a} = \frac{(x-a)(x^2+ax+a^2)}{x-a} = x^2+ax+a^2$$

[53] As in Example 7, $a = \dfrac{2-1}{3-(-3)} = \dfrac{1}{6}$ and f has the form $f(x) = \frac{1}{6}x + b$.

$f(3) = \frac{1}{6}(3) + b = \frac{1}{2} + b$. But $f(3) = 2$, so $\frac{1}{2} + b = 2 \Rightarrow b = \frac{3}{2}$, and $f(x) = \frac{1}{6}x + \frac{3}{2}$.

$\boxed{54}$ $a = \dfrac{-2-7}{4-(-2)} = -\frac{9}{6} = -\frac{3}{2} \Rightarrow f(x) = -\frac{3}{2}x + b.$ $f(-2) = -\frac{3}{2}(-2) + b = 3 + b$ and

$f(-2) = 7 \Rightarrow 3 + b = 7 \Rightarrow b = 4$, and $f(x) = -\frac{3}{2}x + 4.$

Note: For Exercises 55–64, a good question to consider is "Given a particular value of x, can a unique value of y be found?" If the answer is yes, the value of y (general formula) is given. If no, two ordered pairs satisfying the relation having x in the first position are given.

$\boxed{55}$ $2y = x^2 + 5 \Rightarrow y = \dfrac{x^2+5}{2}$, a function

$\boxed{56}$ $x = 3y + 2 \Rightarrow x - 2 = 3y \Rightarrow y = \frac{x-2}{3}$, a function

$\boxed{57}$ $x^2 + y^2 = 4 \Rightarrow y^2 = 4 - x^2 \Rightarrow y = \pm\sqrt{4-x^2}$, not a function, $(0, \pm 2)$

$\boxed{58}$ $y^2 - x^2 = 1 \Rightarrow y^2 = 1 + x^2 \Rightarrow y = \pm\sqrt{1+x^2}$, not a function, $(0, \pm 1)$

$\boxed{59}$ $y = 3$ is a function since for any x,

$(x, 3)$ is the only ordered pair in W having x in the first position.

$\boxed{60}$ $x = 3$ is not a function, $(3, 0)$ and $(3, 1)$

$\boxed{61}$ Many ordered pairs with x-coordinate 0 satisfy $xy = 0$.

Two such ordered pairs are $(0, 0)$ and $(0, 1)$. Not a function

$\boxed{62}$ $x + y = 0 \Rightarrow y = -x$, a function

$\boxed{63}$ $|y| = |x| \Rightarrow \pm y = \pm x \Rightarrow y = \pm x$, not a function, $(1, \pm 1)$

$\boxed{64}$ Many ordered pairs with x-coordinate 3 (or any other number) satisfy $y < x$.

Two such ordered pairs are $(3, 1)$ and $(3, 2)$. Not a function

$\boxed{65}$ $V = lwh \Rightarrow V(x) = (30-2x)(20-2x)(x) = 4x(15-x)(10-x)$

{ V or $V(x)$ may be used, many of the text answers are given in function notation }

$\boxed{66}$ $S = 2\pi rh + 2(2\pi r^2) = 2\pi r(10) + 4\pi r^2 = 20\pi r + 4\pi r^2 = 4\pi r(5+r)$

$\boxed{67}$ (a) $A = 500 \Rightarrow xy = 500 \Rightarrow y = \frac{500}{x}$

(b) $P =$ Linear feet of wall $= x + 2(y) + 2(x-3) = 3x + 2\left(\frac{500}{x}\right) - 6.$

$C = 100P = 300x + \dfrac{100{,}000}{x} - 600.$

$\boxed{68}$ (a) $V = lwh \Rightarrow 6 = xy(1.5) \Rightarrow xy = 4 \Rightarrow y = \frac{4}{x}$

(b) Surface area $S = xy + 2(1.5)x + 2(1.5)y = x\left(\frac{4}{x}\right) + 3x + 3\left(\frac{4}{x}\right) = 4 + 3x + \frac{12}{x}$

$\boxed{69}$ $S(h) = 6(h-25) + 100 = 6h - 150 + 100 = 6h - 50.$

$\boxed{70}$ $T(x) = \dfrac{125{,}000 \text{ BTUs}}{1 \text{ gallon of gas}} \cdot x \text{ gallons} \cdot \dfrac{\$0.342}{1{,}000{,}000 \text{ BTU}} = \$0.04275x.$

[71] (a) Using (6, 48) and (7, 50.5), we have

$y - 48 = \frac{50.5 - 48}{7 - 6}(t - 6)$, or $y = 2.5t + 33$.

(b) The slope represents the yearly increase in height, 2.5 in./yr.

(c) $t = 10 \Rightarrow y = 2.5(10) + 33 = 58$ in.

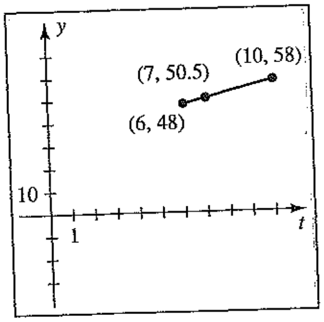

Figure 71

[72] Let A denote the area of the contamination. A is linearly related to t, so $A = at + b$.

$A = 0$ when $t = 0 \Rightarrow b = 0$. $A = 40{,}000$ when $t = 40 \Rightarrow a = 1000$.

Thus, $A = 1000t$. Since the contamination is circular, $A = \pi r^2$.

Hence, $\pi r^2 = 1000t \Rightarrow r = \sqrt{\frac{1000t}{\pi}}$.

[73] The height of the balloon is $2t$. Using the Pythagorean theorem,

$d^2 = 100^2 + (2t)^2 \Rightarrow d^2 = 2^2(50)^2 + 2^2t^2 \Rightarrow d = 2\sqrt{t^2 + 2500}$.

[74] (a) By the Pythagorean theorem, $x^2 + y^2 = 15^2 \Rightarrow y = \sqrt{225 - x^2}$.

(b) $\mathcal{A} = \frac{1}{2}bh = \frac{1}{2}xy = \frac{1}{2}x\sqrt{225 - x^2}$. The domain of this function is $-15 \leq x \leq 15$; however, only $0 < x < 15$ will form triangles.

[75] (a) $(CT)^2 + (PT)^2 = (PC)^2 \Rightarrow r^2 + y^2 = (h + r)^2 \Rightarrow$

$r^2 + y^2 = h^2 + 2hr + r^2 \Rightarrow y^2 = h^2 + 2hr \; \{y > 0\} \Rightarrow y(h) = \sqrt{h^2 + 2hr}$

(b) $y = \sqrt{(200)^2 + 2(4000)(200)} = \sqrt{(200)^2(1 + 40)} = 200\sqrt{41} \approx 1280.6$ mi

[76] (a) The dimensions L, 50, and $x - 2$ form a right triangle 2 feet off the ground.

Thus, $L^2 = 50^2 + (x - 2)^2 \Rightarrow L = \sqrt{2500 + (x - 2)^2}$.

(b) $L = 75 \Rightarrow 75^2 = 50^2 + (x - 2)^2 \Rightarrow x - 2 = \pm\sqrt{3125} \Rightarrow$

$x = 25\sqrt{5} + 2 \approx 57.9$ ft

[77] Let y be the distance from the control booth to the beginning of the runway.

Then $y^2 = 300^2 + 20^2$ and, in a different plane, $d^2 = y^2 + x^2$.

Solving for d, $d = \sqrt{y^2 + x^2} = \sqrt{90{,}400 + x^2}$.

[78] $\text{Time}_{\text{total}} = \text{Time}_{\text{rowing}} + \text{Time}_{\text{walking}}$ {use $t = d/r$ and $d(A, P) = 6 - x$} $\Rightarrow$

$$T = \frac{\sqrt{2^2 + (6 - x)^2}}{3} + \frac{x}{5} \Rightarrow T = \frac{\sqrt{x^2 - 12x + 40}}{3} + \frac{x}{5}$$

79 (b) The maximum y-value of 0.75 occurs when $x \approx 0.55$ and
the minimum y-value of -0.75 occurs when $x \approx -0.55$.

Therefore, the range of f is approximately $[-0.75, 0.75]$.

(c) f is decreasing on $[-2, -0.55]$ and on $[0.55, 2]$. f is increasing on $[-0.55, 0.55]$.

$[-2, 2]$ by $[-2, 2]$

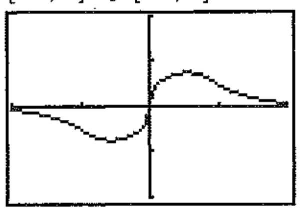

Figure 79

$[-1, 1]$ by $[-1, 1]$

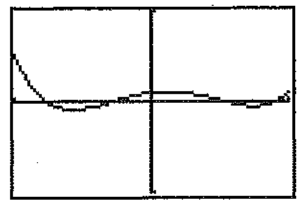

Figure 80

80 (b) The maximum y-value of 0.5 occurs when $x = -1$ and
the minimum y-value of -0.094 occurs when $x \approx -0.56$.

Therefore, the range of f is approximately $[-0.094, 0.5]$.

(c) f is decreasing on $[-1, -0.56]$ and on $[0.12, 0.75]$.

f is increasing on $[-0.56, 0.12]$ and on $[0.75, 1]$.

81 (b) The maximum y-value of 1 occurs when $x = 0$ and
the minimum y-value of -1.03 occurs when $x \approx 1.06$.

Therefore, the range of f is approximately $[-1.03, 1]$.

(c) f is decreasing on $[0, 1.06]$. f is increasing on $[-0.7, 0]$ and on $[1.06, 1.4]$.

$[-0.7, 1.4]$ by $[-1.1, 1]$

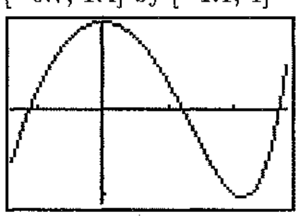

Figure 81

$[-4, 4]$ by $[-4, 4]$

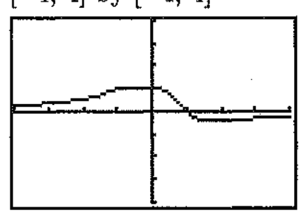

Figure 82

82 (b) The maximum y-value of 1.08 occurs when $x \approx -0.69$ and
the minimum y-value of -0.42 occurs when $x \approx 1.78$.

Therefore, the range of f is approximately $[-0.42, 1.08]$.

(c) f is decreasing on $[-0.69, 1.78]$. f is increasing on $[-4, -0.69]$ and on $[1.78, 4]$.

[83] For each of (a)–(e), an assignment to Y_1, an appropriate viewing rectangle, and the solutions are listed.

(a) $Y_1 = (x\hat{}5)\hat{}(1/3)$, VR: [−40, 40, 10] by [−40, 40, 10], $x = 8$

(b) $Y_1 = (x\hat{}4)\hat{}(1/3)$, VR: [−40, 40, 10] by [−40, 40, 10], $x = \pm 8$

(c) $Y_1 = (x\hat{}2)\hat{}(1/3)$, VR: [−40, 40, 10] by [−40, 40, 10], no real solutions

(d) $Y_1 = (x\hat{}3)\hat{}(1/4)$, VR: [0, 800, 100] by [0, 200, 100], $x = 625$

(e) $Y_1 = (x\hat{}3)\hat{}(1/2)$, VR: [−30, 30, 10] by [−30, 30, 10], no real solutions

[84] (a) $Y_1 = (x\hat{}3)\hat{}(1/5)$, VR: [−250, 250, 100] by [−30, 30, 10], $x = -243$

(b) $Y_1 = (x\hat{}2)\hat{}(1/3)$, VR: [−130, 130, 50] by [0, 30, 10], $x = \pm 125$

(c) $Y_1 = (x\hat{}4)\hat{}(1/3)$, VR: [−50, 50, 10] by [−50, 50, 10], no real solutions

(d) $Y_1 = (x\hat{}3)\hat{}(1/2)$, VR: [−5, 30, 5] by [−5, 30, 5], $x = 9$

(e) $Y_1 = (x\hat{}3)\hat{}(1/4)$, VR: [−10, 10,] by [−10, 10], no real solutions

[85] (a) There are $95 \times 63 = 5985$ total pixels in the screen.

(b) If a function is graphed in dot mode, only one pixel in each column of pixels on the screen can be darkened. Therefore, there are at most 95 pixels darkened.

Note: In connected mode this may not be true.

[86] (a)

[0, 75, 10] by [0, 600, 100]

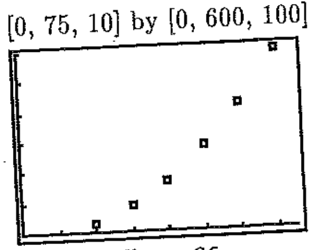

Figure 86

(b) The data and plot show that stopping distance is not a linear function of the speed. The distance required to stop a car traveling at 30 mi/hr is 86 ft whereas the distance required to stop a car traveling at 60 mi/hr is 414 ft. $\frac{414}{86} \approx 4.81$ rather than double.

(c) If you double the speed of a car, it requires almost *five* times the stopping distance. If stopping distance were a linear function of speed, doubling the speed would require approximately twice the stopping distance.

[87] (a) First, we must determine an equation of the line that passes through the points (1985, 11,450) and (1994, 20,021).

$$y - 11{,}450 = \frac{20{,}021 - 11{,}450}{1994 - 1985}(x - 1985) = \frac{2857}{3}(x - 1985) \Rightarrow$$

$y = \frac{2857}{3}x - \frac{5{,}636{,}795}{3}$. Thus, let $f(x) = \frac{2857}{3}x - \frac{5{,}636{,}795}{3}$ and graph f.

See *Figure 87(a)*.

(b) The average annual increase in the price paid for a new car is equal to the slope: $\frac{2857}{3} \approx \$952.33$.

(c) Graph $y = \frac{2857}{3}x - \frac{5{,}636{,}795}{3}$ and $y = 25{,}000$ on the same coordinate axes. Their point of intersection is approximately (1999.2, 25,000). Thus, according to this model, in the year 1999 the average price paid for a new car was \$25,000.

[1984, 2005, 5] by [1E4, 3E4, 1E4]

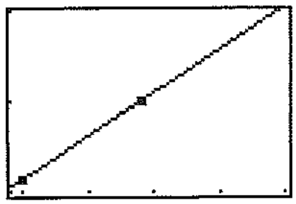

Figure 87(a)

[1984, 2005, 5] by [1E4, 3E4, 1E4]

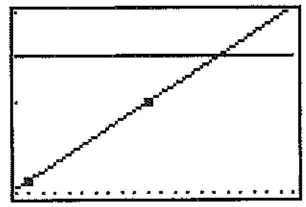

Figure 87(c)

3.5 Exercises

$\boxed{1}$ Since f is an even function, $f(-x) = f(x)$.

From the table, we see that $f(2) = 7$, so $f(-2) = 7$.

Since g is an odd function, $g(-x) = -g(x)$.

From the table, we see that $g(2) = -6$, so $g(-2) = -(-6) = 6$.

$\boxed{2}$ Since f is an even function, $f(-x) = f(x)$.

From the table, we see that $f(3) = -5$, so $f(-3) = -5$.

Since g is an odd function, $g(-x) = -g(x)$.

From the table, we see that $g(3) = 15$, so $g(-3) = -15$.

$\boxed{3}$ $f(x) = 5x^3 + 2x \Rightarrow f(-x) = 5(-x)^3 + 2(-x) = -5x^3 - 2x = -(5x^3 + 2x) = -f(x)$, so f is odd

$\boxed{4}$ $f(x) = |\,x\,| - 3 \Rightarrow f(-x) = |\,-x\,| - 3 = |\,x\,| - 3 = f(x)$, so f is even

$\boxed{5}$ $f(x) = 3x^4 + 2x^2 - 5 \Rightarrow f(-x) = 3(-x)^4 + 2(-x)^2 - 5 = 3x^4 + 2x^2 - 5 = f(x)$, so f is even

$\boxed{6}$ $f(x) = 7x^5 - 4x^3 \Rightarrow$

$f(-x) = 7(-x)^5 - 4(-x)^3 = -7x^5 + 4x^3 = -(7x^5 - 4x^3) = -f(x)$, so f is odd

$\boxed{7}$ $f(x) = 8x^3 - 3x^2 \Rightarrow f(-x) = 8(-x)^3 - 3(-x)^2 = -8x^3 - 3x^2 \neq \pm f(x)$, so f is neither even nor odd

$\boxed{8}$ $f(x) = 12 \Rightarrow f(-x) = 12 = f(x)$, so f is even

$\boxed{9}$ $f(x) = \sqrt{x^2+4} \Rightarrow f(-x) = \sqrt{(-x)^2+4} = \sqrt{x^2+4} = f(x)$, so f is even

$\boxed{10}$ $f(x) = 3x^2 - 5x + 1 \Rightarrow f(-x) = 3(-x)^2 - 5(-x) + 1 = 3x^2 + 5x + 1 \neq \pm f(x)$, so f is neither even nor odd

$\boxed{11}$ $f(x) = \sqrt[3]{x^3 - x} \Rightarrow f(-x) = \sqrt[3]{(-x)^3 - (-x)} = \sqrt[3]{-x^3 + x} = \sqrt[3]{-1(x^3 - x)} =$

$\sqrt[3]{-1}\,\sqrt[3]{x^3 - x} = -\sqrt[3]{x^3 - x} = -f(x)$, so f is odd

$\boxed{12}$ $f(x) = x^3 - \frac{1}{x} \Rightarrow f(-x) = (-x)^3 - \frac{1}{-x} = -x^3 + \frac{1}{x} = -\left(x^3 - \frac{1}{x}\right) = -f(x)$, so f is odd

$\boxed{13}$ $f(x) = |x| + c$, $c = -3, 1, 3$ • Shift $g(x) = |x|$ down 3, up 1, and up 3 units, respectively.

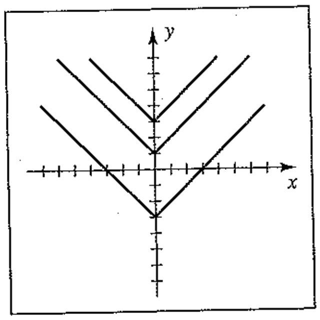

Figure 13

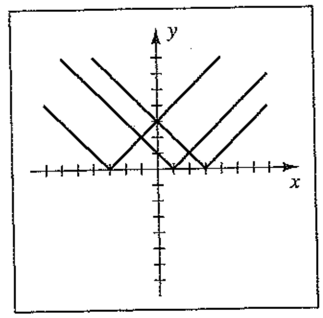

Figure 14

$\boxed{14}$ $f(x) = |x - c|$, $c = -3, 1, 3$ • Shift $g(x) = |x|$ left 3, right 1, and right 3 units, respectively.

$\boxed{15}$ $f(x) = -x^2 + c$, $c = -4, 2, 4$ • Shift $g(x) = -x^2$ down 4, up 2, up 4, respectively.

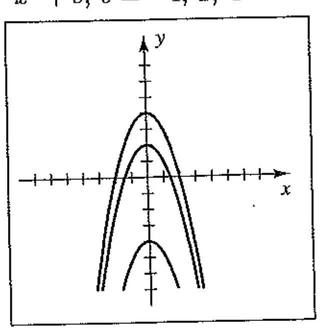

Figure 15

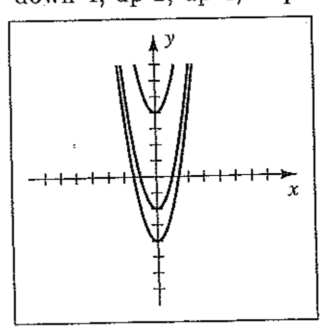

Figure 16

$\boxed{16}$ $f(x) = 2x^2 - c$, $c = -4, 2, 4$ • Shift $g(x) = 2x^2$ up 4, down 2, down 4, respectively.

[17] $f(x) = 2\sqrt{x} + c$, $c = -3, 0, 2$ • Shift $g(x) = 2\sqrt{x}$ down 3, up 2, respectively.

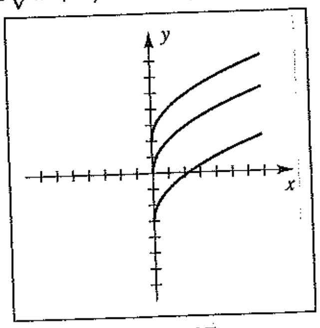

Figure 17

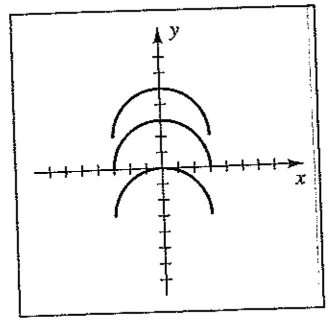

Figure 18

[18] $f(x) = \sqrt{9 - x^2} + c$, $c = -3, 0, 2$ •

Shift $g(x) = \sqrt{9 - x^2}$ down 3, up 2, respectively.

[19] $f(x) = \frac{1}{2}\sqrt{x - c}$, $c = -2, 0, 3$ • Shift $g(x) = \frac{1}{2}\sqrt{x}$ left 2, right 3, respectively.

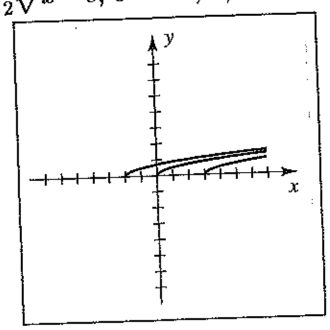

Figure 19

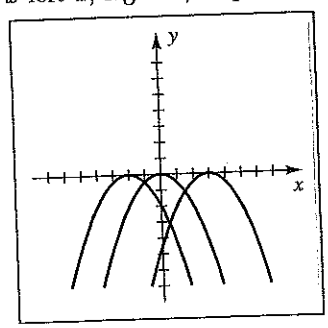

Figure 20

[20] $f(x) = -\frac{1}{2}(x - c)^2$, $c = -2, 0, 3$ • Shift $g(x) = -\frac{1}{2}x^2$ left 2, right 3, respectively.

[21] $f(x) = c\sqrt{4 - x^2}$, $c = -2, 1, 3$ •

For $c = -2$, reflect $g(x) = \sqrt{4 - x^2}$ through the x-axis and vertically stretch it by a factor of 2. For $c = 3$, vertically stretch g by a factor of 3.

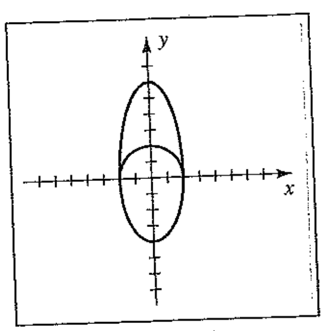

Figure 21

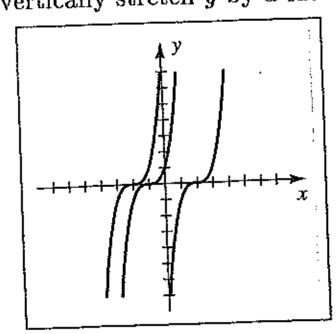

Figure 22

[22] $f(x) = (x + c)^3$, $c = -2, 1, 2$ •

Shift $g(x) = x^3$ right 2, left 1, and left 2, respectively.

[23] $f(x) = cx^3$, $c = -\frac{1}{3}, 1, 2$ • For $c = -\frac{1}{3}$, reflect $g(x) = x^3$ through the x-axis and vertically compress it by a factor of $1/(1/3) = 3$.

For $c = 2$, vertically stretch g by a factor of 2.

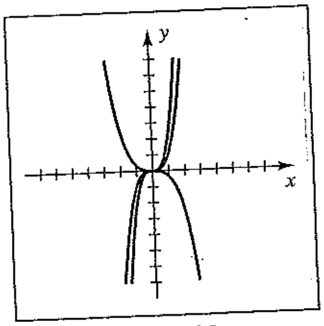

Figure 23

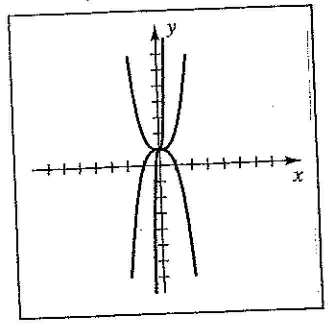

Figure 24

[24] $f(x) = (cx)^3 + 1$, $c = -1, 1, 4$ • For $c = -1$, reflect $g(x) = x^3 + 1$ through the y-axis. For $c = 4$, horizontally compress g by a factor of 4 { this could also be considered as a vertical stretch by a factor of $4^3 = 64$ }.

[25] $f(x) = \sqrt{cx} - 1$, $c = -1, \frac{1}{9}, 4$ •
For $c = -1$, reflect $g(x) = \sqrt{x} - 1$ through the y-axis. For $c = \frac{1}{9}$, horizontally stretch g by a factor $1/(1/9) = 9$ { x-intercept changes from 1 to 9 }.
For $c = 4$, horizontally compress g by a factor 4 { x-intercept changes from 1 to $\frac{1}{4}$ }.

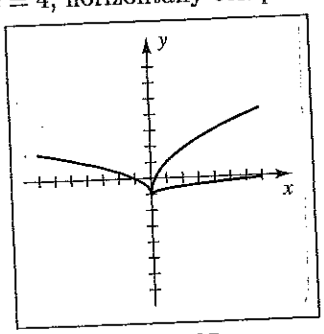

Figure 25

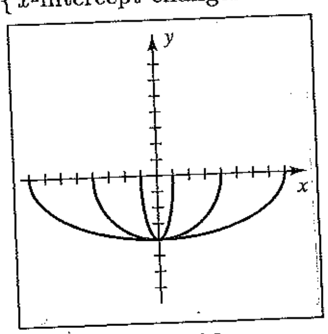

Figure 26

[26] $f(x) = -\sqrt{16 - (cx)^2}$, $c = 1, \frac{1}{2}, 4$ • For $c = \frac{1}{2}$, horizontally stretch $g(x) = -\sqrt{16 - x^2}$ by a factor of $1/(1/2) = 2$ { x-intercepts change from ± 4 to ± 8 }.
For $c = 4$, horizontally compress g by a factor of 4 { x-intercepts change from ± 4 to ± 1 }.

[27] $P(0, 5)$; $y = f(x+2) - 1$

{ $x + 2$ [subtract 2 from the x-coordinate] } $\rightarrow (-2, 5)$

{ -1 [subtract 1 from the y-coordinate] } $\rightarrow (-2, 4)$

[28] $P(3, -1)$; $y = 2f(x) + 4$

{ $\times 2$ [multiply the y-coordinate by 2] } $\rightarrow (3, -2)$

{ $+4$ [add 4 to the y-coordinate] } $\rightarrow (3, 2)$

$\boxed{29}$ $P(3, -2)$; $y = 2f(x-4)+1$

{ $x-4$ [add 4 to the x-coordinate] } $\rightarrow (7, -2)$

{ $\times 2$ [multiply the y-coordinate by 2] } $\rightarrow (7, -4)$

{ $+1$ [add 1 to the y-coordinate] } $\rightarrow (7, -3)$

$\boxed{30}$ $P(-2, 4)$; $y = \frac{1}{2}f(x-3)+3$

{ $x-3$ [add 3 to the x-coordinate] } $\rightarrow (1, 4)$

{ $\times\frac{1}{2}$ [multiply the y-coordinate by $\frac{1}{2}$] } $\rightarrow (1, 2)$

{ $+3$ [add 3 to the y-coordinate] } $\rightarrow (1, 5)$

$\boxed{31}$ $P(3, 9)$; $y = \frac{1}{3}f(\frac{1}{2}x)-1$

{ $\frac{1}{2}x$ [multiply the x-coordinate by 2] } $\rightarrow (6, 9)$

{ $\times\frac{1}{3}$ [multiply the y-coordinate by $\frac{1}{3}$] } $\rightarrow (6, 3)$

{ -1 [subtract 1 from the y-coordinate] } $\rightarrow (6, 2)$

$\boxed{32}$ $P(-2, 1)$; $y = -3f(2x)-5$

{ $2x$ [divide the x-coordinate by 2] } $\rightarrow (-1, 1)$

{ $\times -3$ [multiply the y-coordinate by -3] } $\rightarrow (-1, -3)$

{ -5 [subtract 5 from the y-coordinate] } $\rightarrow (-1, -8)$

$\boxed{33}$ For $y = f(x-2)+3$, the graph of f is shifted 2 units to the right and 3 units up.

$\boxed{34}$ For $y = 3f(x-1)$, the graph of f is shifted 1 unit to the right and stretched vertically by a factor of 3.

$\boxed{35}$ For $y = f(-x)-2$, the graph of f is reflected through the y-axis and shifted 2 units down.

$\boxed{36}$ For $y = -f(x+4)$, the graph of f is shifted 4 units to the left and reflected through the x-axis.

$\boxed{37}$ For $y = -\frac{1}{2}f(x)$, the graph of f is compressed vertically by a factor of 2 and reflected through the x-axis.

$\boxed{38}$ For $y = f(\frac{1}{2}x)-3$, the graph of f is stretched horizontally by a factor of 2 and shifted 3 units down.

$\boxed{39}$ For $y = -2f(\frac{1}{3}x)$, the graph of f is stretched horizontally by a factor of 3, stretched vertically by a factor of 2, and reflected through the x-axis.

$\boxed{40}$ For $y = \frac{1}{3}|f(x)|$, the part of the graph of f below the x-axis is reflected through the x-axis and that graph is compressed vertically by a factor of 3.

[41] (a) $y = f(x+3)$ • shift f left 3 units

(b) $y = f(x-3)$ • shift f right 3 units

(c) $y = f(x)+3$ • shift f up 3 units

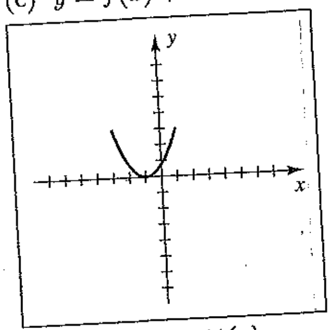

Figure 41(a)

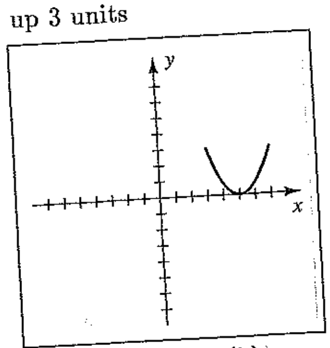

Figure 41(b)

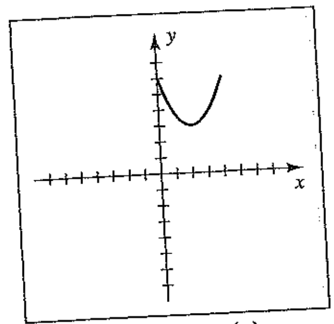

Figure 41(c)

(d) $y = f(x)-3$ • shift f down 3 units

(e) $y = -3f(x)$ • reflect f through the x-axis and vertically stretch it by a factor of 3

(f) $y = -\frac{1}{3}f(x)$ • reflect f through the x-axis and vertically compress it by a factor of $1/(1/3) = 3$

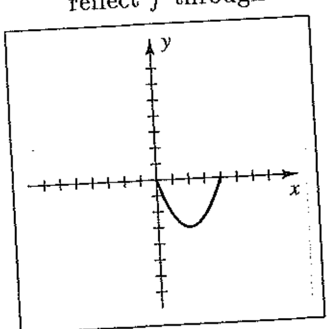

Figure 41(d)

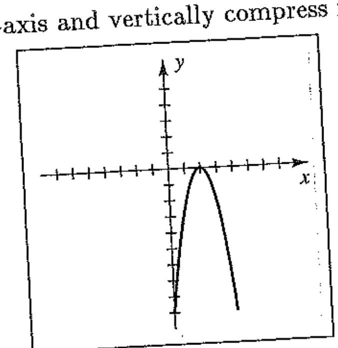

Figure 41(e)

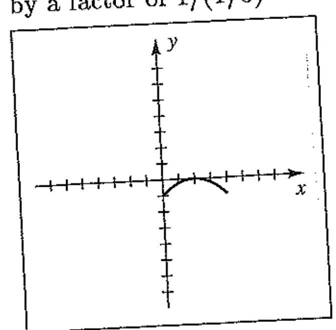

Figure 41(f)

(g) $y = f(-\frac{1}{2}x)$ • reflect f through the y-axis and horizontally stretch it by a factor of $1/(1/2) = 2$

(h) $y = f(2x)$ • horizontally compress f by a factor of 2

(i) $y = -f(x+2)-3$ • reflect f about the x-axis, shift it left 2 units and down 3

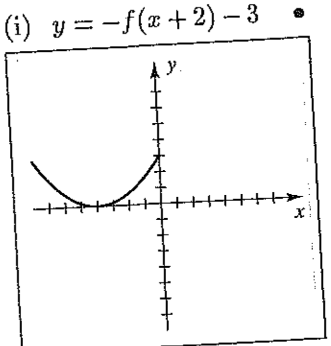

Figure 41(g)

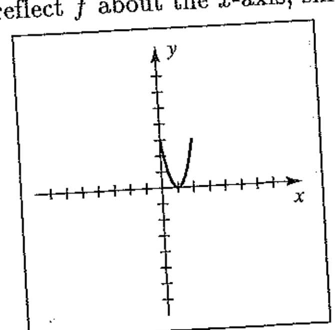

Figure 41(h)

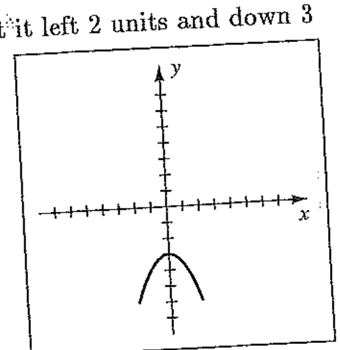

Figure 41(i)

(j) $y = f(x-2)+3$ • shift f right 2 units and up 3

(k) $y = |\,f(x)\,|$ • since no portion of the graph lies below the x-axis, the graph is unchanged

(l) $y = f(\,|\,x\,|\,)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates

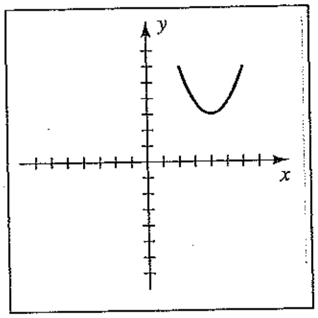

Figure 41(j)

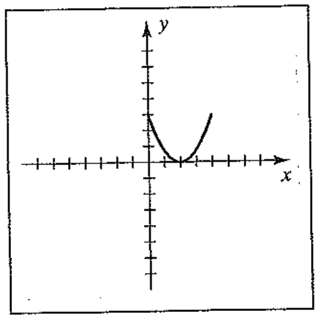

Figure 41(k)

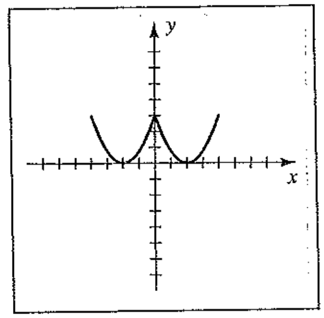

Figure 41(l)

$\boxed{42}$ (a) $y = f(x-2)$ • shift f right 2 units

(b) $y = f(x+2)$ • shift f left 2 units

(c) $y = f(x)-2$ • shift f down 2 units

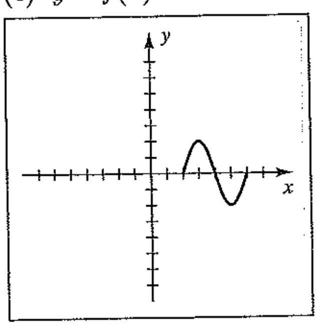

Figure 42(a)

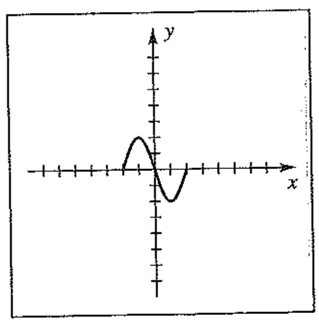

Figure 42(b)

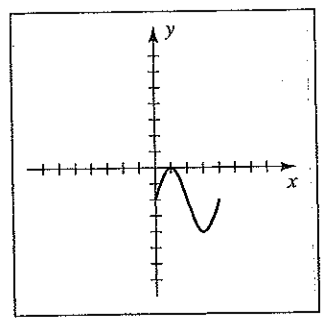

Figure 42(c)

(d) $y = f(x)+2$ • shift f up 2 units

(e) $y = -2f(x)$ •

reflect f through the x-axis and vertically stretch it by a factor of 2

(f) $y = -\frac{1}{2}f(x)$ •

reflect f through the x-axis and vertically compress it by a factor of $1/(1/2) = 2$

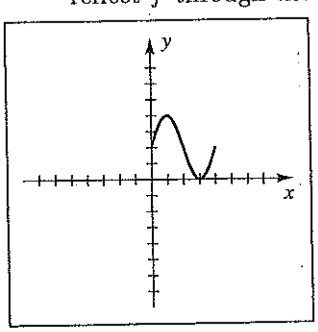

Figure 42(d)

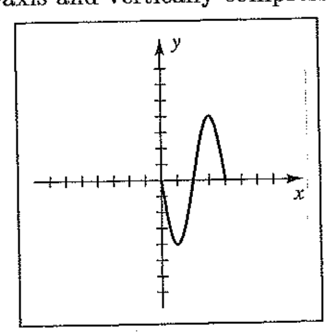

Figure 42(e)

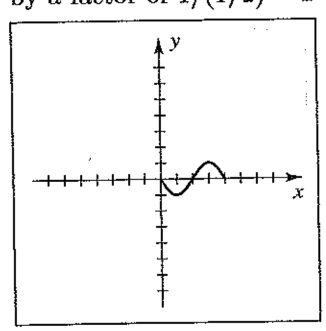

Figure 42(f)

(g) $y = f(-2x)$ • reflect f through the y-axis and horizontally compress it by a factor of 2

(h) $y = f(\frac{1}{2}x)$ • horizontally stretch f by a factor of $1/(1/2) = 2$

(i) $y = -f(x+4) - 2$ • reflect f about the x-axis, shift it left 4 units and down 2

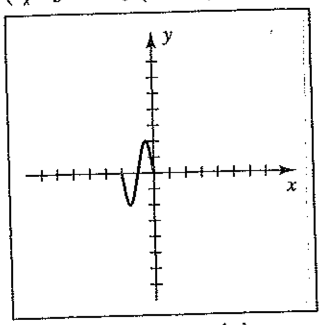

Figure 42(g)

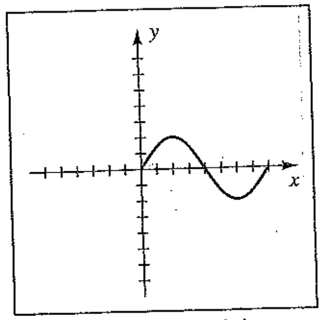

Figure 42(h)

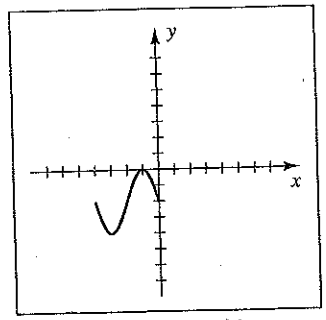

Figure 42(i)

(j) $y = f(x-4) + 2$ • shift f right 4 units and up 2

(k) $y = |\, f(x) \,|$ • reflect the portion of the graph below the x-axis through the x-axis.

(l) $y = f(\,|\, x \,|\,)$ • include the reflection of the given graph through the y-axis since all points have positive x-coordinates

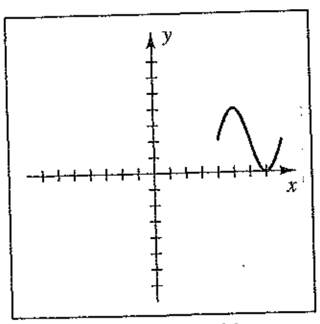

Figure 42(j)

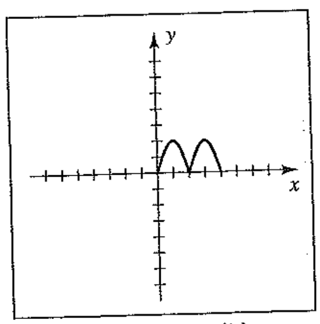

Figure 42(k)

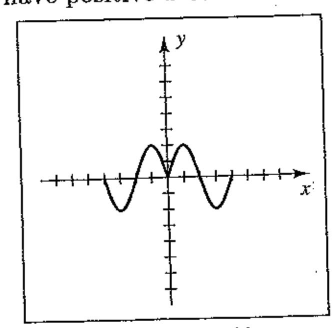

Figure 42(l)

$\boxed{43}$ (a) f is shifted left 9 units and up 1 $\Rightarrow$ $y = f(x+9) + 1$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 7 units and down 1 $\Rightarrow$ $y = -f(x+7) - 1$

$\boxed{44}$ (a) f is shifted left 1 unit and up 1 $\Rightarrow$ $y = f(x+1) + 1$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$ {or $y = f(-x)$}

(c) f is reflected through the x-axis and shifted right 2 units $\Rightarrow$ $y = -f(x-2)$

$\boxed{45}$ (a) f is shifted left 4 units $\Rightarrow$ $y = f(x+4)$

(b) f is shifted up 1 unit $\Rightarrow$ $y = f(x) + 1$

(c) f is reflected through the y-axis $\Rightarrow$ $y = f(-x)$

[46] (a) f is shifted right 2 units and up 2 $\Rightarrow$ $y = f(x-2)+2$

(b) f is reflected through the x-axis $\Rightarrow$ $y = -f(x)$

(c) f is reflected through the x-axis and shifted left 4 units and up 2 $\Rightarrow$

$$y = -f(x+4)+2$$

[47] $f(x) = \begin{cases} 3 & \text{if } x \le -1 \\ -2 & \text{if } x > -1 \end{cases}$

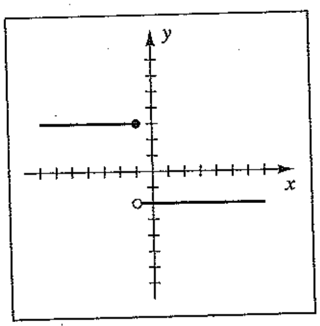

Figure 47

[48] $f(x) = \begin{cases} -1 & \text{if } x \text{ is an integer} \\ -2 & \text{if } x \text{ is not an integer} \end{cases}$

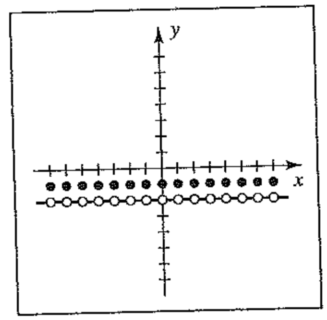

Figure 48

[49] $f(x) = \begin{cases} 3 & \text{if } x < -2 \\ -x+1 & \text{if } |x| \le 2 \\ -3 & \text{if } x > 2 \end{cases}$

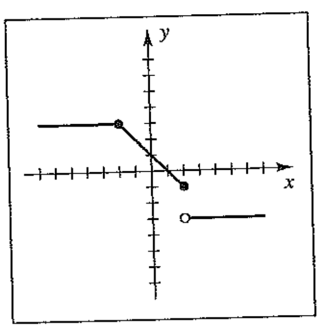

Figure 49

[50] $f(x) = \begin{cases} -2x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1 \\ -2 & \text{if } x \ge 1 \end{cases}$

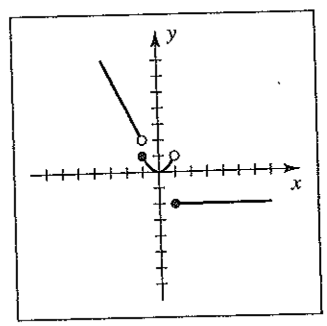

Figure 50

[51] $f(x) = \begin{cases} x+2 & \text{if } x \le -1 \\ x^3 & \text{if } |x| < 1 \\ -x+3 & \text{if } x \ge 1 \end{cases}$

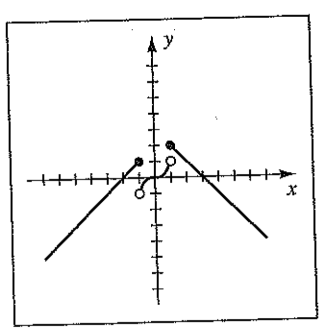

Figure 51

[52] $f(x) = \begin{cases} x-3 & \text{if } x \le -2 \\ -x^2 & \text{if } -2 < x < 1 \\ -x+4 & \text{if } x \ge 1 \end{cases}$

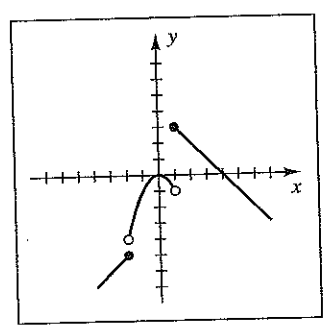

Figure 52

$\boxed{53}$ (a) $f(x) = [\![x-3]\!]$ • shift $g(x) = [\![x]\!]$ right 3 units

(b) $f(x) = [\![x]\!] - 3$ • shift g down 3 units, which is the same graph as in part (a).

(c) $f(x) = 2[\![x]\!]$ • vertically stretch g by a factor of 2

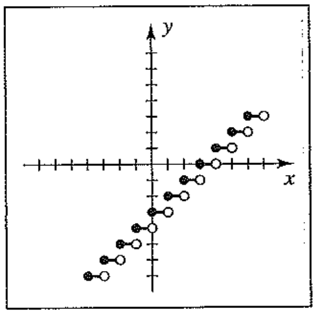

Figure 53(a)

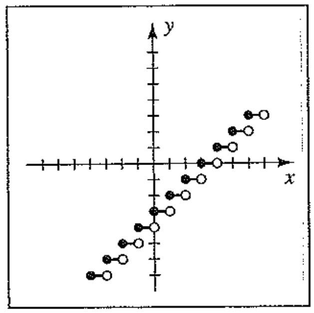

Figure 53(b)

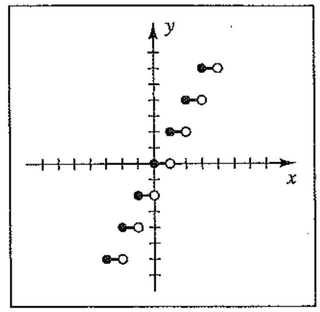

Figure 53(c)

(d) $f(x) = [\![2x]\!]$ • horizontally compress g by a factor of 2

(e) $f(x) = [\![-x]\!]$ • reflect g through the y-axis

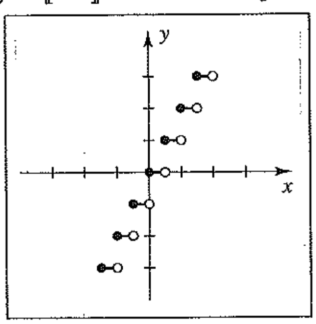

Figure 53(d)

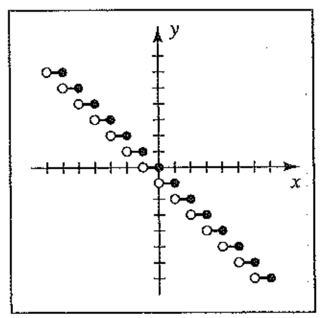

Figure 53(e)

$\boxed{54}$ (a) $f(x) = [\![x+2]\!]$ • shift $g(x) = [\![x]\!]$ left 2 units

(b) $f(x) = [\![x]\!] + 2$ • shift g up 2 units, which is the same graph as in part (a).

(c) $f(x) = \frac{1}{2}[\![x]\!]$ • vertically compress g by a factor of $1/(1/2) = 2$

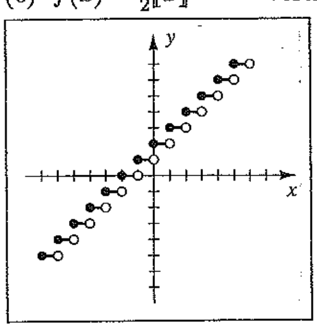

Figure 54(a)

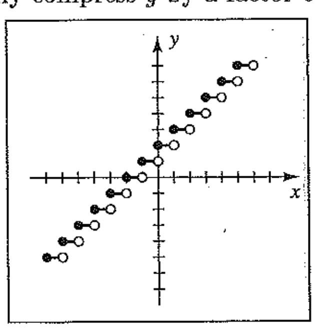

Figure 54(b)

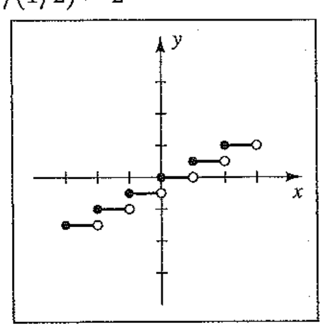

Figure 54(c)

(d) $f(x) = [\![\frac{1}{2}x]\!]$ • horizontally stretch g by a factor of $1/(1/2) = 2$

(e) $f(x) = -[\![-x]\!]$ • reflect g through the y-axis and through the x-axis

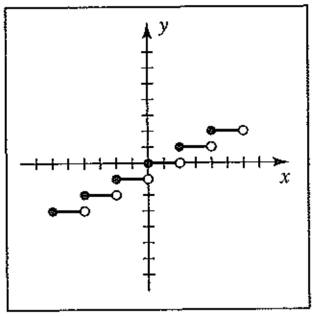

Figure 54(d)

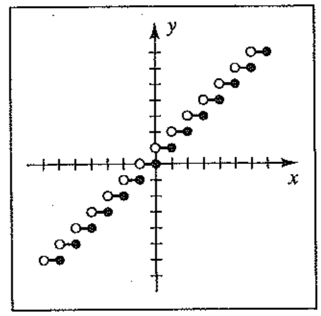

Figure 54(e)

$\boxed{55}$ The graph of $x = y^2$ is not the graph of a function because if $x > 0$,

two different points on the graph have x-coordinate x.

$\boxed{56}$ The graph of $x = -|y|$ is not the graph of a function because if $x < 0$,

two different points on the graph have x-coordinate x.

$\boxed{57}$ Reflect each portion of the graph that is below the x-axis through the x-axis.

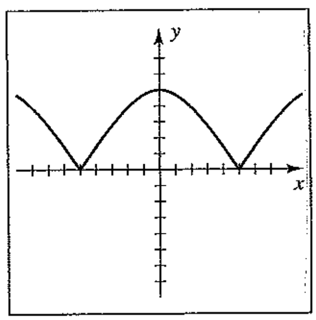

Figure 57

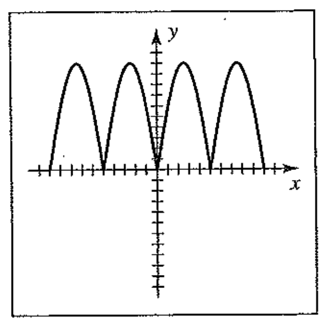

Figure 58

$\boxed{58}$ Same as Exercise 57.

$\boxed{59}$ $y = |9 - x^2|$ • First sketch $y = 9 - x^2$,

then reflect the portions of the graph below the x-axis through the x-axis.

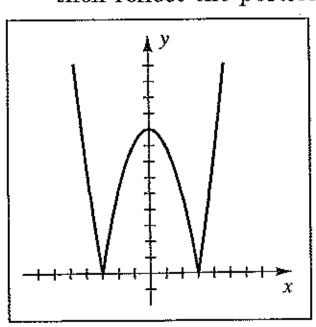

Figure 59

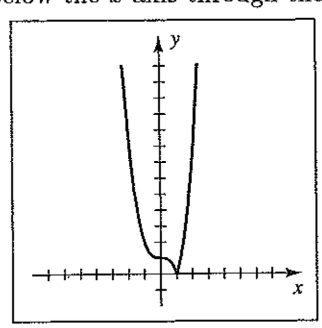

Figure 60

$\boxed{60}$ $y = |x^3 - 1|$ •

[61] $y = | \sqrt{x} - 1 |$ •

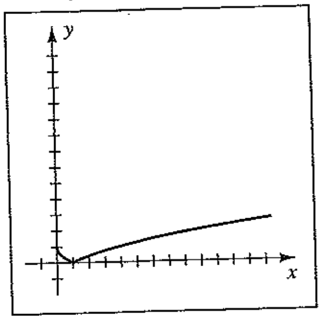

Figure 61

[62] $y = | \, |x| - 1 \, |$ •

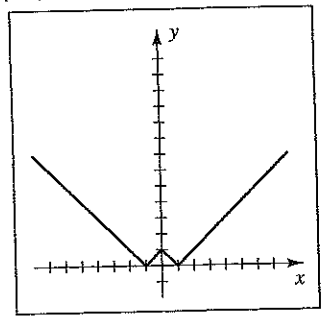

Figure 62

[63] (a) For $y = -2f(x)$, multiply the y-coordinates by -2. $D = [-2, 6]$, $R = [-16, 8]$

(b) For $y = f(\frac{1}{2}x)$, multiply the x-coordinates by 2. $D = [-4, 12]$, $R = [-4, 8]$

(c) For $y = f(x-3)+1$, add 3 to the x-coordinates and add 1 to the y-coordinates. $D = [1, 9]$, $R = [-3, 9]$

(d) For $y = f(x+2)-3$, subtract 2 from the x-coordinates and subtract 3 from the y-coordinates. $D = [-4, 4]$, $R = [-7, 5]$

(e) For $y = f(-x)$, multiply all x-coordinates by -1. $D = [-6, 2]$, $R = [-4, 8]$

(f) For $y = -f(x)$, multiply all y-coordinates by -1. $D = [-2, 6]$, $R = [-8, 4]$

(g) $y = f(|x|)$ • Graphically, we can reflect all points with positive x-coordinates through the y-axis, so the domain $[-2, 6]$ becomes $[-6, 6]$. Algebraically, we are replacing x with $|x|$, so $-2 \le x \le 6$ becomes $-2 \le |x| \le 6$, which is equivalent $|x| \le 6$, or, equivalently, $-6 \le x \le 6$. The range stays the same because of the given assumptions: $f(2) = 8$ and $f(6) = -4$; that is, the full range is taken on for $x \ge 0$. Note that the range could not be determined if $f(-2)$ was equal to 8. $D = [-6, 6]$, $R = [-8, 4]$

(h) $y = |f(x)|$ • The points with y-coordinates having values from -4 to 0 will have values from 0 to 4, so the range will be $[0, 8]$. $D = [-2, 6]$, $R = [0, 8]$

[64] (a) For $y = \frac{1}{2}f(x)$, multiply the y-coordinates by $\frac{1}{2}$. $D = [-6, -2]$, $R = [-5, -2]$

(b) For $y = f(2x)$, multiply the x-coordinates by $\frac{1}{2}$. $D = [-3, -1]$, $R = [-10, -4]$

(c) For $y = f(x-2)+5$, add 2 to the x-coordinates and add 5 to the y-coordinates. $D = [-4, 0]$, $R = [-5, 1]$

(d) For $y = f(x+4)-1$, subtract 4 from the x-coordinates and subtract 1 from the y-coordinates. $D = [-10, -6]$, $R = [-11, -5]$

(e) For $y = f(-x)$, negate all x-coordinates. $D = [2, 6]$, $R = [-10, -4]$

(f) For $y = -f(x)$, negate all y-coordinates. $D = [-6, -2]$, $R = [4, 10]$

(g) For $y = f(\,|\,x\,|\,)$, there is no graph since the domain of f consists of only negative values, -6 to -2, and $|\,x\,|$ is never negative.

(h) For $y = |\,f(x)\,|$, the negative y-coordinates having values from -10 to -4 will have values from 4 to 10. $D = [-6, -2]$, $R = [4, 10]$

$\boxed{65}$ If $0 \le x \le 20{,}000$, then $T(x) = 0.15x$. If $x > 20{,}000$, then the tax is 15% of the first 20,000, which is 3000, plus 20% of the amount over 20,000. We may summarize and simplify as follows:

$$T(x) = \begin{cases} 0.15x & \text{if } 0 \le x \le 20{,}000 \\ 3000 + 0.20(x - 20{,}000) & \text{if } x > 20{,}000 \end{cases} = \begin{cases} 0.15x & \text{if } 0 \le x \le 20{,}000 \\ 0.20x - 1000 & \text{if } x > 20{,}000 \end{cases}$$

$\boxed{66}$ If $0 \le x \le 500{,}000$, then $T(x) = 0.01x$. If $x > 500{,}000$, then the tax is 1% of the first 500,000, which is 5000, plus 1.25% of the amount over 500,000.

$$\begin{aligned} T(x) &= \begin{cases} 0.01x & \text{if } 0 \le x \le 500{,}000 \\ 5000 + 0.0125(x - 500{,}000) & \text{if } x > 500{,}000 \end{cases} \\ &= \begin{cases} 0.01x & \text{if } 0 \le x \le 500{,}000 \\ 0.0125x - 1250 & \text{if } x > 500{,}000 \end{cases} \end{aligned}$$

$\boxed{67}$ The author receives \$1.20 on the first 10,000 copies, \$1.50 on the next 5000, and \$1.80 on each additional copy.

$$\begin{aligned} R(x) &= \begin{cases} 1.20x & \text{if } 0 \le x \le 10{,}000 \\ 12{,}000 + 1.50(x - 10{,}000) & \text{if } 10{,}000 < x \le 15{,}000 \\ 19{,}500 + 1.80(x - 15{,}000) & \text{if } x > 15{,}000 \end{cases} \\ &= \begin{cases} 1.20x & \text{if } 0 \le x \le 10{,}000 \\ 1.50x - 3000 & \text{if } 10{,}000 < x \le 15{,}000 \\ 1.80x - 7500 & \text{if } x > 15{,}000 \end{cases} \end{aligned}$$

$\boxed{68}$ The cost for 1000 kWh is \$57.70 and the cost for 5000 kWh is $\$57.70 + 4000(\$0.0532) = \$270.50$.

$$\begin{aligned} C(x) &= \begin{cases} 0.0577x & \text{if } 0 \le x \le 1000 \\ 57.70 + 0.0532(x - 1000) & \text{if } 1000 < x \le 5000 \\ 270.50 + 0.0511(x - 5000) & \text{if } x > 5000 \end{cases} \\ &= \begin{cases} 0.0577x & \text{if } 0 \le x \le 1000 \\ 4.50 + 0.0532x & \text{if } 1000 < x \le 5000 \\ 15.00 + 0.0511x & \text{if } x > 5000 \end{cases} \end{aligned}$$

$\boxed{69}$ From a graph, we see that the solutions of $|\,1.3x + 2.8\,| = 1.2x + 5$ are approximately -3.12 and 22.00. It turns out that these are the exact solutions. The solution is $(-3.12, 22.00)$.

[70] The solutions of $|\,0.3x\,| - 2 = 2.2 - 0.63x^2$ are approximately ± 2.35 {by symmetry}. The solution is $(-\infty, -2.35) \cup (2.35, \infty)$.

[71] The solutions of $|\,1.2x^2 - 10.8\,| = 1.36x + 4.08$ are -3 and approximately 1.87 and 4.13. The solution is $(-\infty, -3) \cup (-3, 1.87) \cup (4.13, \infty)$.

[72] The solutions of $|\,\sqrt{16 - x^2} - 3\,| = 0.12x^2 - 0.3$ are approximately ± 3.60, ± 2.25 {by symmetry}. The solution is $(-3.60, -2.25) \cup (2.25, 3.60)$.

[73] Since $g(x) = f(x) + 4$, the graph of g can be obtained by shifting the graph of f upward a distance of 4.

[−12, 12] by [−8, 8]

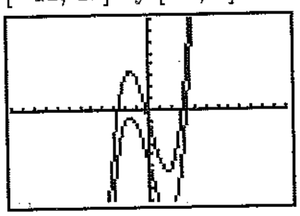

Figure 73

[−12, 12] by [−8, 8]

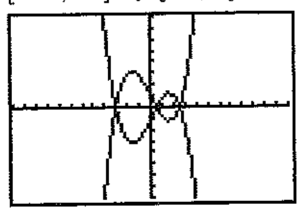

Figure 74

[74] Since $g(x) = -f(x)$, the graph of g can be obtained reflecting the graph of f through the x–axis.

[75] Since $g(x) = f(\frac{1}{2}x)$, the graph of g can be obtained by stretching the graph of f horizontally by a factor of 2.

[−12, 12] by [−8, 8]

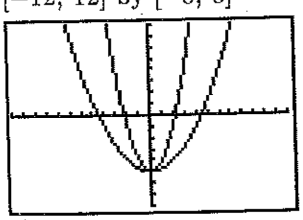

Figure 75

[−12, 12] by [−8, 8]

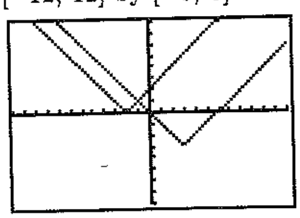

Figure 76

[76] Since $g(x) = f(x - 5) - 3$, the graph of g can be obtained by shifting the graph of f horizontally to the right a distance of 5 and vertically downward a distance of 3.

[77] Since $g(x) = |\,f(x)\,|$, the graph of g is the same as the graph of f if f is non-negative. If $f(x) < 0$, then the graph of f will be reflected through the x–axis.

[−12, 12] by [−8, 8]

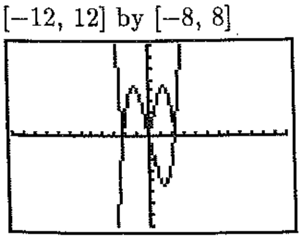

Figure 77

[−12, 12] by [−8, 8]

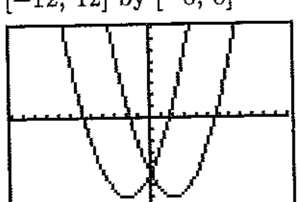

Figure 78

[78] Since $g(x) = f(-x)$, the graph of g can be obtained from the graph of f by reflecting the graph of f through the y-axis. See *Figure 78.*

[79] (a) Option I gives $C_1 = 4(\$29.95) + \$0.25(500 - 200) = 119.80 + 75.00 = \$194.80$.

Option II gives $C_2 = 4(\$39.95) + \$0.15(500) = 159.80 + 75.00 = \$234.80$.

(b) Let x represent the mileage. The cost function for Option I is the piecewise linear function

$$C_1(x) = \begin{cases} 119.80 & \text{if } 0 \le x \le 200 \\ 119.80 + 0.25(x - 200) & \text{if } x > 200 \end{cases}$$

Option II is the linear function $C_2(x) = 159.8 + 0.15x$ for $x \ge 0$.

(c) Let $C_1 = Y_1$ and $C_2 = Y_2$.

Table $Y_1 = 119.80 + 0.25(x - 200)*(x > 200)$ and $Y_2 = 159.80 + 0.15x$

x	Y_1	Y_2	x	Y_1	Y_2
100	119.8	174.8	700	244.8	264.8
200	119.8	189.8	800	269.8	279.8
300	144.8	204.8	900	294.8	294.8
400	169.8	219.8	1000	319.8	309.8
500	194.8	234.8	1100	344.8	324.8
600	219.8	249.8	1200	369.8	339.8

(d) From the table, we see that the options are equal in cost for $x = 900$ miles. Option I is preferable if $x \in [0, 900)$ and Option II is preferable if $x > 900$.

[80] (a) Since 1 mi = 5,280 ft and each car requires $(12 + d)$ ft, it follows directly that the bridge can hold $[\![5280/(12 + d)]\!]$ cars.

The greatest integer function is necessary since a fraction of a car is not allowed.

(b) Since the bridge is 1 mile long, the car "density" is $\dfrac{5280}{12 + d}$ cars/mi. If each car is moving at v mi/hr, then the flow rate is $F = [\![5280v/(12 + d)]\!]$ cars/hr.

3.6 Exercises

[1] $V(-3, 1) \Rightarrow y = a[x - (-3)]^2 + 1 \Rightarrow y = a(x + 3)^2 + 1$

[2] $V(4, -2) \Rightarrow y = a(x - 4)^2 - 2$

[3] $V(0, -3) \Rightarrow y = a(x - 0)^2 - 3 \Rightarrow y = ax^2 - 3$

[4] $V(-2, 0) \Rightarrow y = a[x - (-2)]^2 + 0 \Rightarrow y = a(x + 2)^2$

[5] $f(x) = -x^2 - 4x - 8 = -(x^2 + 4x + \underline{4}) - 8 + \underline{4} = -(x + 2)^2 - 4$

[6] $f(x) = x^2 - 6x + 11 = x^2 - 6x + \underline{9} + 11 - \underline{9} = (x - 3)^2 + 2$

[7] $f(x) = 2x^2 - 12x + 22 \Rightarrow \frac{1}{2}f(x) = x^2 - 6x + \underline{9} + 11 - \underline{9} = (x - 3)^2 + 2 \Rightarrow$

$f(x) = 2(x - 3)^2 + 4$

[8] $f(x) = 5x^2 + 20x + 17 \;\Rightarrow\; \frac{1}{5}f(x) = x^2 + 4x + \underline{\;4\;} + \frac{17}{5} - \underline{\;4\;} = (x+2)^2 - \frac{3}{5} \;\Rightarrow$

$f(x) = 5(x+2)^2 - 3$

[9] $f(x) = -3x^2 - 6x - 5 \;\Rightarrow\; -\frac{1}{3}f(x) = x^2 + 2x + \underline{\;1\;} + \frac{5}{3} - \underline{\;1\;} = (x+1)^2 + \frac{2}{3} \;\Rightarrow$

$f(x) = -3(x+1)^2 - 2$

[10] $f(x) = -4x^2 + 16x - 13 \;\Rightarrow\; -\frac{1}{4}f(x) = x^2 - 4x + \underline{\;4\;} + \frac{13}{4} - \underline{\;4\;} = (x-2)^2 - \frac{3}{4} \;\Rightarrow$

$f(x) = -4(x-2)^2 + 3$

[11] $f(x) = -\frac{3}{4}x^2 + 9x - 34 \;\Rightarrow\; -\frac{4}{3}f(x) = x^2 - 12x + \frac{136}{3} =$

$x^2 - 12x + \underline{\;36\;} + \frac{136}{3} - \underline{\;36\;} = (x-6)^2 + \frac{28}{3} \;\Rightarrow\; f(x) = -\frac{3}{4}(x-6)^2 - 7$

[12] $f(x) = \frac{2}{5}x^2 - \frac{12}{5}x + \frac{23}{5} \;\Rightarrow$

$\frac{5}{2}f(x) = x^2 - 6x + \frac{23}{2} = x^2 - 6x + \underline{\;9\;} + \frac{23}{2} - \underline{\;9\;} = (x-3)^2 + \frac{5}{2} \;\Rightarrow$

$f(x) = \frac{2}{5}(x-3)^2 + 1$

[13] (a) $x^2 - 4x = 0 \;\Rightarrow\; x = \dfrac{4 \pm \sqrt{16 - 0}}{2} = 0,\ 4$

(b) *Note:* Encourage students to recognize that the x-coordinate of the vertex, $-b/(2a)$, is easily seen in part (a).

$f(x) = x^2 - 4x \;\Rightarrow\; -\dfrac{b}{2a} = -\dfrac{-4}{2(1)} = 2.$ $f(2) = -4$ is a minimum since $a > 0$.

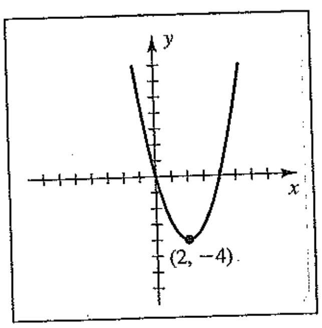

Figure 13

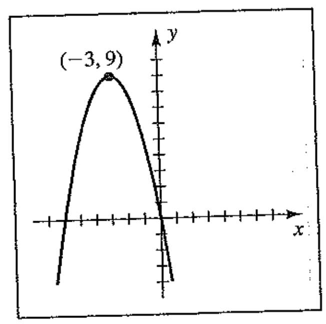

Figure 14

[14] (a) $-x^2 - 6x = 0 \;\Rightarrow\; x = \dfrac{6 \pm \sqrt{36 - 0}}{-2} = -6,\ 0$

(b) $f(x) = -x^2 - 6x \;\Rightarrow\; -\dfrac{b}{2a} = -\dfrac{-6}{2(-1)} = -3.$

$f(-3) = 9$ is a maximum since $a < 0$.

$\boxed{15}$ (a) $-12x^2+11x+15=0 \;\Rightarrow\; x=\dfrac{-11\pm\sqrt{121+720}}{-24}=\dfrac{-11\pm 29}{-24}=-\dfrac{3}{4},\,\dfrac{5}{3}$

(b) $f(x)=-12x^2+11x+15 \;\Rightarrow\; -\dfrac{b}{2a}=-\dfrac{11}{2(-12)}=\frac{11}{24}.$

$f(\frac{11}{24})=\frac{841}{48}\approx 17.52$ is a maximum since $a<0$.

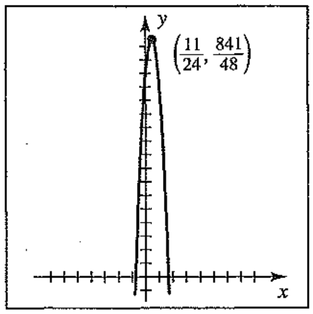

Figure 15

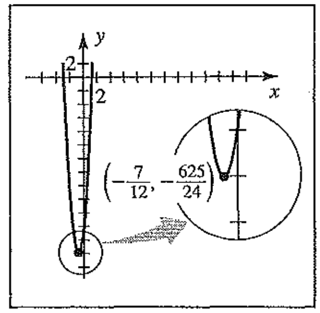

Figure 16

$\boxed{16}$ (a) $6x^2+7x-24=0 \;\Rightarrow\; x=\dfrac{-7\pm\sqrt{49+576}}{12}=\dfrac{-7\pm 25}{12}=-\dfrac{8}{3},\,\dfrac{3}{2}$

(b) $f(x)=6x^2+7x-24 \;\Rightarrow\; -\dfrac{b}{2a}=-\dfrac{7}{2(6)}=-\dfrac{7}{12}.$

$f(-\frac{7}{12})=-\frac{625}{24}\approx -26.04$ is a minimum since $a>0$.

$\boxed{17}$ (a) $9x^2+24x+16=0 \;\Rightarrow\; x=\dfrac{-24\pm\sqrt{576-576}}{18}=\dfrac{-24}{18}=-\dfrac{4}{3}$

(b) $f(x)=9x^2+24x+16 \;\Rightarrow\; -\dfrac{b}{2a}=-\dfrac{24}{2(9)}=-\frac{4}{3}.$

$f(-\frac{4}{3})=0$ is a minimum since $a>0$.

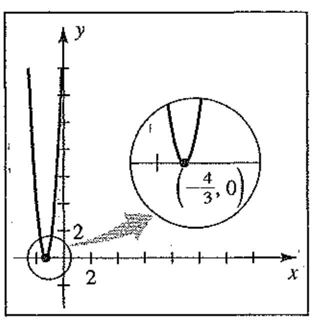

Figure 17

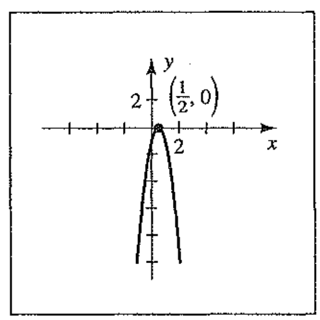

Figure 18

$\boxed{18}$ (a) $-4x^2+4x-1=0 \;\Rightarrow\; x=\dfrac{-4\pm\sqrt{16-16}}{-8}=\dfrac{-4}{-8}=\dfrac{1}{2}$

(b) $f(x)=-4x^2+4x-1 \;\Rightarrow\; -\dfrac{b}{2a}=-\dfrac{4}{2(-4)}=\frac{1}{2}.$

$f(\frac{1}{2})=0$ is a maximum since $a<0$.

$\boxed{19}$ (a) $x^2+4x+9=0 \Rightarrow x=\dfrac{-4\pm\sqrt{16-36}}{2}$. There are no x-intercepts.

(b) $f(x)=x^2+4x+9 \Rightarrow -\dfrac{b}{2a}=-\dfrac{4}{2(1)}=-2.$

$f(-2)=5$ is a minimum since $a>0$.

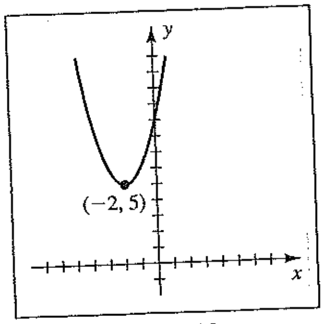

Figure 19

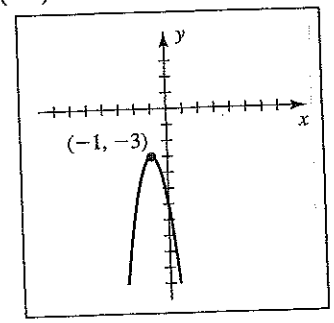

Figure 20

$\boxed{20}$ (a) $-3x^2-6x-6=0 \Rightarrow x=\dfrac{6\pm\sqrt{36-72}}{-6}$. There are no x-intercepts.

(b) $f(x)=-3x^2-6x-6 \Rightarrow -\dfrac{b}{2a}=-\dfrac{-6}{2(-3)}=-1.$

$f(-1)=-3$ is a maximum since $a<0$.

$\boxed{21}$ (a) $-2x^2+20x-43=0 \Rightarrow x=\dfrac{-20\pm\sqrt{400-344}}{-4}=5\pm\frac{1}{2}\sqrt{14}\approx 6.87,\ 3.13$

(b) $f(x)=-2x^2+20x-43 \Rightarrow -\dfrac{b}{2a}=-\dfrac{20}{2(-2)}=5.$

$f(5)=7$ is a maximum since $a<0$.

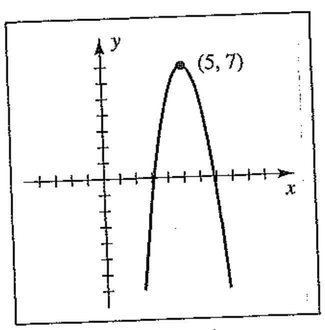

Figure 21

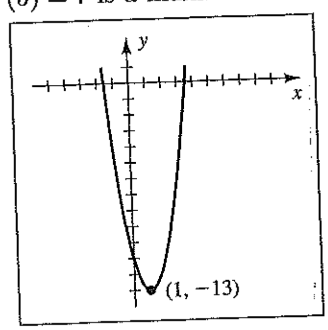

Figure 22

$\boxed{22}$ (a) $2x^2-4x-11=0 \Rightarrow x=\dfrac{4\pm\sqrt{16+88}}{4}=1\pm\frac{1}{2}\sqrt{26}\approx 3.55,\ -1.55$

(b) $f(x)=2x^2-4x-11 \Rightarrow -\dfrac{b}{2a}=-\dfrac{-4}{2(2)}=1.$

$f(1)=-13$ is a minimum since $a>0$.

$\boxed{23}$ $V(4,-1) \Rightarrow y=a(x-4)^2-1$. $x=0, y=1 \Rightarrow 1=a(0-4)^2-1 \Rightarrow$

$2=16a \Rightarrow a=\frac{1}{8}$. Hence, $y=\frac{1}{8}(x-4)^2-1$.

$\boxed{24}$ $V(2,4) \Rightarrow y=a(x-2)^2+4$. $x=0, y=0 \Rightarrow 0=a(0-2)^2+4 \Rightarrow$

$-4=4a \Rightarrow a=-1$. Hence, $y=-(x-2)^2+4$.

[25] $V(-2,\ 4) \Rightarrow y = a(x+2)^2 + 4$. $x = 1,\ y = 0 \Rightarrow 0 = a(1+2)^2 + 4 \Rightarrow$
$-4 = 9a \Rightarrow a = -\frac{4}{9}$. Hence, $y = -\frac{4}{9}(x+2)^2 + 4$.

[26] $V(-1,\ -2) \Rightarrow y = a(x+1)^2 - 2$. $x = 2,\ y = 3 \Rightarrow 3 = a(2+1)^2 - 2 \Rightarrow$
$5 = 9a \Rightarrow a = \frac{5}{9}$. Hence, $y = \frac{5}{9}(x+1)^2 - 2$.

[27] From the figure, the x-intercepts are -2 and 4, so the equation must have the form $y = a[x-(-2)](x-4) = a(x+2)(x-4)$. To find the value of a, use the point $(2,\ 4)$.
$4 = a(2+2)(2-4) \Rightarrow 4 = a(4)(-2) \Rightarrow -8a = 4 \Rightarrow a = -\frac{1}{2}$,
so the equation is $y = -\frac{1}{2}(x+2)(x-4)$.

[28] $y = a(x+4)(x-6)$. $x = 4,\ y = -4 \Rightarrow -4 = a(8)(-2) \Rightarrow a = \frac{1}{4}$,
so the equation is $y = \frac{1}{4}(x+4)(x-6)$.

[29] $V(0,\ -2) \Rightarrow (h,\ k) = (0,\ -2)$. $x = 3,\ y = 25 \Rightarrow 25 = a(3-0)^2 - 2 \Rightarrow$
$27 = 9a \Rightarrow a = 3$. Hence, $y = 3(x-0)^2 - 2$, or $y = 3x^2 - 2$.

[30] $V(0,\ 5) \Rightarrow (h,\ k) = (0,\ 5)$. $x = 2,\ y = -3 \Rightarrow -3 = a(2-0)^2 + 5 \Rightarrow$
$-8 = 4a \Rightarrow a = -2$. Hence, $y = -2(x-0)^2 + 5$, or $y = -2x^2 + 5$.

[31] $V(3,\ 5) \Rightarrow y = a(x-3)^2 + 5$. $x = 0,\ y = 0 \Rightarrow 0 = a(0-3)^2 + 5 \Rightarrow$
$-5 = 9a \Rightarrow a = -\frac{5}{9}$. Hence, $y = -\frac{5}{9}(x-3)^2 + 5$.

[32] $V(4,\ -7) \Rightarrow y = a(x-4)^2 - 7$. $x = -4,\ y = 0 \Rightarrow 0 = a(-4-4)^2 - 7 \Rightarrow$
$7 = 64a \Rightarrow a = \frac{7}{64}$. Hence, $y = \frac{7}{64}(x-4)^2 - 7$.

[33] $V(1,\ 4) \Rightarrow y = a(x-1)^2 + 4$. $x = -3,\ y = 0 \Rightarrow 0 = a(-3-1)^2 + 4 \Rightarrow$
$-4 = 16a \Rightarrow a = -\frac{1}{4}$. Hence, $y = -\frac{1}{4}(x-1)^2 + 4$.

[34] $V(4,\ -48) \Rightarrow y = a(x-4)^2 - 48$. $x = 0,\ y = 0 \Rightarrow 0 = a(0-4)^2 - 48 \Rightarrow$
$48 = 16a \Rightarrow a = 3$. Hence, $y = 3(x-4)^2 - 48$.

[35] Let d denote the distance between the parabola and the line.
$d = (\text{parabola}) - (\text{line}) = (-2x^2 + 4x + 3) - (x - 2) = -2x^2 + 3x + 5$.
This relation is quadratic and the x-value of its maximum value is

$$-\frac{b}{2a} = -\frac{3}{2(-2)} = \frac{3}{4}. \text{ Thus, maximum } d = -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) + 5 = \frac{49}{8} = 6.125.$$

[36] As in #35, $d = (\text{line}) - (\text{parabola}) = (-x+3) - (2x^2 + 8x + 4) = -2x^2 - 9x - 1$, and

$$-\frac{b}{2a} = -\frac{-9}{2(-2)} = -\frac{9}{4}. \text{ Thus, maximum } d = -2\left(-\frac{9}{4}\right)^2 - 9\left(-\frac{9}{4}\right) - 1 = \frac{73}{8} = 9.125.$$

Note: The applied problems can be solved using a variety of methods.

[37] The vertex is located at $h = \frac{-b}{2a} = \frac{-2.867}{2(-0.058)} \approx 24.72$ km. Since $a < 0$,
this will produce a maximum value.

[38] The vertex is located at $h = \frac{-b}{2a} = \frac{-3.811}{2(-0.078)} \approx 24.43$ km. Since $a < 0$,
this will produce a maximum value.

[39] Since the x-intercepts are 0 and 21, the maximum will occur halfway between them; that is, when the infant weighs 10.5 lb.

[40] (a) M will be a maximum when $v = \frac{-b}{2a} = \frac{-5/2}{2(-1/30)} = \frac{75}{2}$, or 37.5 mi/hr.

(b) $v = \frac{75}{2} \Rightarrow M = -\frac{1}{30}(\frac{75}{2})^2 + \frac{5}{2}(\frac{75}{2}) = -\frac{375}{8} + \frac{375}{4} = \frac{375}{8} = 46.875$ mi/gal.

[41] (a) s will be a maximum when $t = \frac{-b}{2a} = \frac{-144}{2(-16)} = \frac{9}{2}$. $s(\frac{9}{2}) = 424$ ft.

(b) When $t = 0$, $s(t) = 100$ ft, which is the height of the building.

[42] (a) $s = 0$ when $t = 12 \Rightarrow 0 = -16(12)^2 + v_0(12) \Rightarrow v_0 = 192$ ft/sec.

(b) Since the total flight is 12 seconds, the maximum height will occur when $t = 6$.

$s(6) = -16(6)^2 + 192(6) = 576$ ft.

[43] Let x and $40 - x$ denote the numbers, and their product P is $x(40 - x)$.

P has zeros at 0 and 40 and is a maximum (since $a < 0$) when $x = \frac{0 + 40}{2} = 20$.

The product will be a maximum when both numbers are 20.

[44] Let x and $x - 40$ denote the numbers, and their product P is $x(x - 40)$.

P has zeros at 0 and 40 and is a minimum (since $a > 0$) when $x = \frac{0 + 40}{2} = 20$.

The product will be a minimum for $x = 20$ and $x - 40 = -20$.

[45] (a) Perimeter $= 1000 \Rightarrow 3x + 4y = 1000 \Rightarrow y = 250 - \frac{3}{4}x$

(b) $A = xy = x(250 - \frac{3}{4}x) = -\frac{3}{4}x^2 + 250x$

(c) A will be a maximum when $x = \frac{-b}{2a} = \frac{-250}{2(-3/4)} = \frac{500}{3} = 166\frac{2}{3}$ ft.

$y = 250 - \frac{3}{4}(\frac{500}{3}) = 125$ ft.

[46] Represent the perimeter of the field by $2y + 4x = 1000$. $A = xy = x(500 - 2x)$.

A has zeros at 0 and 250 and will be a maximum when $x = \frac{0 + 250}{2} = 125$ yd.

$y = 500 - 2(125) = 250$ yd. The dimensions should be

125 yd by 250 yd with intermediate fences parallel to the short side.

[47] The parabola has vertex $V(\frac{9}{2}, 3)$. Hence, the equation has the form $y = a(x - \frac{9}{2})^2 + 3$.

Using the point (9, 0) {or (0, 0)}, we have $0 = a(9 - \frac{9}{2})^2 + 3 \Rightarrow a = -\frac{4}{27}$.

Thus, the path may be described by $y = -\frac{4}{27}(x - \frac{9}{2})^2 + 3$.

[48] (a) Since (0, 15) is on the graph, $c = 15$. Substituting (175, 0) for (x, y) yields

$$0 = a(175)^2 + 175 + 15 \Rightarrow a = -\frac{190}{175^2} \text{ and } y = -\frac{190}{175^2}x^2 + x + 15.$$

(b) y will be a maximum when $x = \frac{-b}{2a} = \frac{-1}{2(-190/175^2)} = \frac{175^2}{380} \approx 80.59$.

The corresponding y-value is $\frac{8405}{152} \approx 55.3$ ft.

[49] (a) Since the vertex is at (0, 10), an equation for the parabola is $y = ax^2 + 10$.

Substituting (200, 90) for (x, y) yields $a = \frac{1}{500}$ and $y = \frac{1}{500}x^2 + 10$.

(b) The cables are spaced 40 ft apart. Letting $x = 40$, 80, 120, and 160 gives us a total length of $10 + 2(\frac{66}{5} + \frac{114}{5} + \frac{194}{5} + \frac{306}{5}) = 282$ ft.

[50] (a) Substituting $x = 0$ and $m = 0$ into $m = 2ax + b$ yields $b = 0$.

Substituting $x = 800$ and $m = \frac{1}{5}$ into $m = 2ax$ yields $a = \frac{1}{8000}$, hence, $y = \frac{1}{8000}x^2$.

(b) Substitute $x = 800$ into $y = \frac{1}{8000}x^2$ to get $y = 80$. Thus, $B = (800, 80)$.

[51] An equation describing the doorway is $y = ax^2 + 9$. Since the doorway is 6 feet wide at the base, $x = 3$ when $y = 0 \Rightarrow 0 = 9a + 9 \Rightarrow a = -1$. Thus, the equation is $y = -x^2 + 9$. To fit an 8 foot high box through the doorway, we must find x when $y = 8$. $y = 8 \Rightarrow 8 = -x^2 + 9 \Rightarrow x = \pm 1$.

Hence, the box can only be 2 feet wide.

[52] (a) $P = 24 \Rightarrow 2x + 2y = 24 \Rightarrow y = 12 - x$

(b) $A = xy = x(12 - x)$

(c) A is zero at 0 and 12 and will be a maximum when $x = \dfrac{0 + 12}{2} = 6$.

Thus, the maximum value of A occurs if the rectangle is a square.

[53] Let x denote the number of pairs of shoes that are ordered.

$$A(x) = \begin{cases} 40x & \text{if } x < 50 \\ (40 - 0.04x)x & \text{if } 50 \le x \le 600 \end{cases}$$

The maximum value of the first part of A is (\$40)(49) = \$1960. For the second part of A, $A = -0.04x^2 + 40x$ has a maximum when $x = \dfrac{-b}{2a} = \dfrac{-40}{2(-0.04)} = 500$ pairs.

$A(500) = 10{,}000 > 1960$, so $x = 500$ produces a maximum for both parts of A.

[54] Let x denote the number of people in the group. The discount per person is $0.50(x - 30)$. The amount of money taken in by the agency may be expressed as:

$$A(x) = \begin{cases} 60x & \text{if } x \le 30 \\ [60 - 0.50(x - 30)]x & \text{if } 30 < x \le 90 \end{cases}$$

The maximum value of the first part of A is (\$60)(30) = \$1800. For the second part of A, $A = (75 - \frac{1}{2}x)x$ has a maximum when $x = 75$ {the x-intercepts are 0 and 150}. This value corresponds to each person receiving a discount of \$22.50 and hence, paying \$37.50 for the tour. $A(75) = 2812.50 > 1800$, so $x = 75$ produces a maximum for both parts of A.

$\boxed{55}$ (a) Let y denote the number of \$1 decreases in the monthly charge.

$R(y) = (\#\text{ of customers})(\text{monthly charge per customer})$
$= (5000 + 500y)(20 - y)$
$= 500(10 + y)(20 - y)$

Now let x denote the monthly charge, which is $20 - y$.

R becomes $500[10 + (20 - x)](x) = 500x(30 - x)$.

(b) R has x-intercepts at 0 and 30, and must have its vertex halfway between them at $x = 15$.

Note that this gives us $y = 5$, and we have 7500 customers for a revenue of \$112,500.

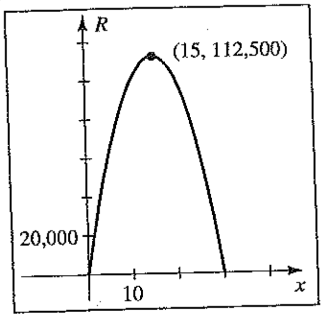

Figure 55

$\boxed{56}$ Let x denote the number of \$10 increases in rent and $M(x)$ the monthly income. The number of occupied apartments is $180 - 5x$ and the rent per apartment is $300 + 10x$. $M(x) = (\#\text{ of occupied apartments})(\text{rent per apartment}) = (180 - 5x)(300 + 10x) = 5 \cdot 10(36 - x)(30 + x)$. The x-intercepts of M are -30 and 36. Hence, the maximum of M will occur when $x = \frac{-30 + 36}{2} = 3$.

The rent charged should be $\$300 + \$10(3) = \$330$.

$\boxed{57}$ From the graph, there are three points of intersection.

Their coordinates are approximately $(-0.57, 0.64)$, $(0.02, -0.27)$, and $(0.81, -0.41)$.

[−3, 3] by [−2, 2]

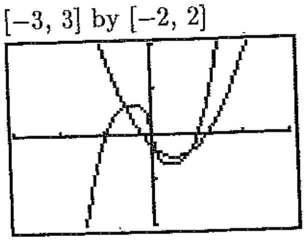

Figure 57

[−6, 6] by [−4, 4]

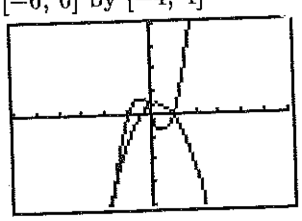

Figure 58

$\boxed{58}$ From the graph, there are three points of intersection.

Their coordinates are approximately $(-1.61, -2.99)$, $(-0.05, 0.37)$, and $(0.98, -0.06)$.

$\boxed{59}$ Since $a > 0$, all parabolas open upward. From the graph, we can see that smaller values of a result in the parabola opening wider while larger values of a result in the parabola becoming narrower. See *Figure 59*.

[−8, 4] by [−1, 7]

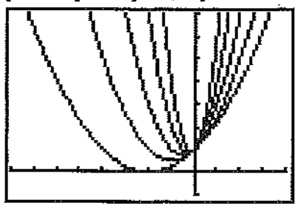

Figure 59

[−6, 6] by [−2, 6]

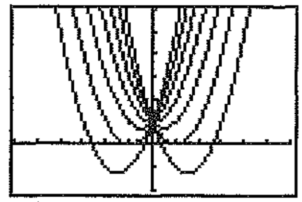

Figure 60

60 As $|b|$ increases, the graph of each parabola shifts downward. Negative values of b shift the parabola to the right while positive values of b shift the parabola to the left.

61 (a) Let January correspond to 1, February to 2, ... , and December to 12.

(b) Let $f(x) = a(x-h)^2 + k$. The vertex appears to occur near (7, 0.8) Thus, $h = 7$ and $k = 0.8$. Using trial and error, a reasonable value for a is 0.17. Thus, let $f(x) = 0.17(x-7)^2 + 0.8$. {From the TI-83 Plus, the quadratic regression equation is $y \approx 0.1476x^2 - 1.9951x + 7.8352$.}

(c) $f(4) = 2.33$, compared to the actual value of 2.4 in.

[0.5, 12] by [0, 8]

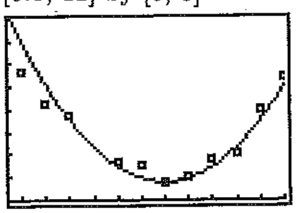

Figure 61

[1980, 1995] by [5, 15]

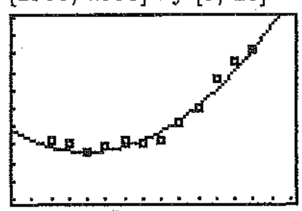

Figure 62

62 (a) Overall, the data decreases slightly and then starts to increase, although there is an unexpected decrease in 1987.

(b) Start by choosing a vertex. The lowest data point is (1984, 7.6), so let $h = 1984$ and $k = 7.6$ in the equation $f(x) = a(x-h)^2 + k$. Choosing one more point will determine the parabola.

$$f(1993) = 13.3 \quad\Rightarrow\quad 13.3 = a(1993-1984)^2 + 7.6 \quad\Rightarrow\quad a \approx 0.07.$$

Let $f(x) = 0.07(x-1984)^2 + 7.6$. f can be adjusted to give a slightly better fit.

63 (a) The equation of the line passing through $A(-800, -48)$ and $B(-500, 0)$ is $y = \frac{4}{25}x + 80$. The equation of the line passing through $D(500, 0)$ and $E(800, -48)$ is $y = -\frac{4}{25}x + 80$. Let $y = a(x-h)^2 + k$ be the equation of the parabola passing through the points $B(-500, 0)$, $C(0, 40)$, and $D(500, 0)$. The vertex is located at (0, 40) so $y = a(x-0)^2 + 40$. Since $D(500, 0)$ is on the graph, $0 = a(500-0)^2 + 40 \Rightarrow a = -\frac{1}{6250}$ and $y = -\frac{1}{6250}x^2 + 40$. Thus, let

$$f(x) = \begin{cases} \frac{4}{25}x + 80 & \text{if } -800 \le x < -500 \\ -\frac{1}{6250}x^2 + 40 & \text{if } -500 \le x \le 500 \\ -\frac{4}{25}x + 80 & \text{if } 500 < x \le 800 \end{cases}$$

(b) Graph the equations:

$Y_1 = (4/25*x + 80)/(x < -500)$,
$Y_2 = (-1/6250*x\hat{\ }2 + 40)/(x \ge -500 \text{ and } x \le 500)$,
$Y_3 = (-4/25*x + 80)/(x > 500)$

[−800, 800, 100] by [−100, 200, 100]

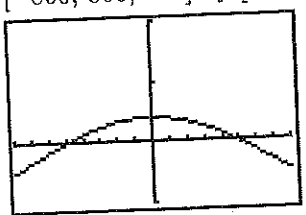

Figure 63

[−500, 2000, 500] by [0, 800, 100]

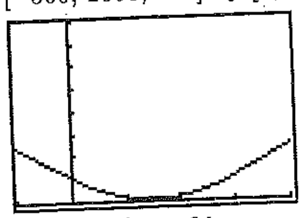

Figure 64

64 (a) The equation of the line passing through $A(-500,\ 243\frac{1}{3})$ and $B(0,\ 110)$ is $y = -\frac{4}{15}x + 110$. The equation of the line passing through $D(1500,\ 110)$ and $E(2000,\ 243\frac{1}{3})$ is $y = \frac{4}{15}x - 290$. Let $y = a(x-h)^2 + k$ be the equation of the parabola passing through the points $B(0,\ 110)$, $C(750,\ 10)$, and $D(1500,\ 110)$. The vertex is located at $(750,\ 10)$ so $y = a(x-750)^2 + 10$. Since $D(1500,\ 110)$ is on the graph, $110 = a(1500-750)^2 + 10 \Rightarrow a = \frac{1}{5625}$; $y = \frac{1}{5625}(x-750)^2 + 10$. Thus, let

$$f(x) = \begin{cases} -\frac{4}{15}x + 110 & \text{if } -500 \le x < 0 \\ \frac{1}{5625}(x-750)^2 + 10 & \text{if } 0 \le x \le 1500 \\ \frac{4}{15}x - 290 & \text{if } 1500 < x \le 2000 \end{cases}$$

(b) Graph the equations:

$Y_1 = (-4/15*x + 110)/(x < 0)$,
$Y_2 = (1/5625*(x - 750)\hat{\ }2 + 10)/(x \ge 0 \text{ and } x \le 1500)$,
$Y_3 = (4/15*x - 290)/(x > 1500)$

65 (a) f must have zeros of 0 and 150. Thus, $f(x) = a(x-0)(x-150)$. Also, f will have a maximum of 100 occurring at $x = 75$. (The vertex will be midway between the zeros of f.) $a(75-0)(75-150) = 100 \Rightarrow a = \frac{100}{(75)(-75)} = -\frac{4}{225}$. $f(x) = -\frac{4}{225}(x)(x-150) = -\frac{4}{225}x^2 + \frac{8}{3}x$.

(c) The value of k affects both the distance and the height traveled by the object. The distance and height decrease by a factor of $\frac{1}{k}$ when $k > 1$ and increase by a factor of $\frac{1}{k}$ when $0 < k < 1$.

[0, 180, 50] by [0, 120, 50]

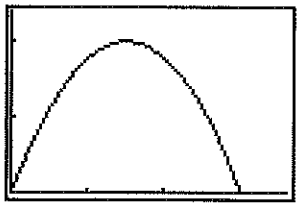

Figure 65(b)

[0, 600, 50] by [0, 400, 50]

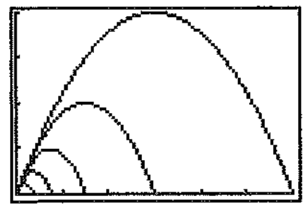

Figure 65(c)

3.7 Exercises

1 (a) $(f+g)(3) = f(3) + g(3) = 6 + 9 = 15$ (b) $(f-g)(3) = f(3) - g(3) = 6 - 9 = -3$

(c) $(fg)(3) = f(3) \cdot g(3) = 6 \cdot 9 = 54$ (d) $(f/g)(3) = f(3)/g(3) = 6/9 = \frac{2}{3}$

2 (a) $(f+g)(3) = f(3) + g(3) = -9 + 5 = -4$

(b) $(f-g)(3) = f(3) - g(3) = -9 - 5 = -14$

(c) $(fg)(3) = f(3) \cdot g(3) = -9 \cdot 5 = -45$ (d) $(f/g)(3) = f(3)/g(3) = -9/5 = -\frac{9}{5}$

3 (a) $(f+g)(x) = f(x) + g(x) = (x^2+2) + (2x^2-1) = 3x^2 + 1;$

$(f-g)(x) = f(x) - g(x) = (x^2+2) - (2x^2-1) = 3 - x^2;$

$(fg)(x) = f(x) \cdot g(x) = (x^2+2) \cdot (2x^2-1) = 2x^4 + 3x^2 - 2;$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+2}{2x^2-1}$

(b) The domain of $f+g$, $f-g$, and fg is the set of all real numbers, $\mathbb{R}$.

(c) The domain of f/g is the same as in (b), except we must exclude the zeros of g.

Hence, the domain of f/g is all real numbers except $\pm\frac{1}{2}\sqrt{2}$.

4 (a) $(f+g)(x) = f(x) + g(x) = (x^2+x) + (x^2-3) = 2x^2 + x - 3;$

$(f-g)(x) = f(x) - g(x) = (x^2+x) - (x^2-3) = x + 3;$

$(fg)(x) = f(x) \cdot g(x) = (x^2+x) \cdot (x^2-3) = x^4 + x^3 - 3x^2 - 3x;$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+x}{x^2-3}$

(b) $\mathbb{R}$ (c) All real numbers except $\pm\sqrt{3}$

5 (a) $(f+g)(x) = f(x) + g(x) = \sqrt{x+5} + \sqrt{x+5} = 2\sqrt{x+5};$

$(f-g)(x) = f(x) - g(x) = \sqrt{x+5} - \sqrt{x+5} = 0;$

$(fg)(x) = f(x) \cdot g(x) = \sqrt{x+5} \cdot \sqrt{x+5} = x + 5;$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x+5}}{\sqrt{x+5}} = 1$

(b) $[-5, \infty)$ (c) $(-5, \infty)$

$\boxed{6}$ (a) $(f+g)(x) = f(x) + g(x) = \sqrt{3-2x} + \sqrt{x+4}$;
$(f-g)(x) = f(x) - g(x) = \sqrt{3-2x} - \sqrt{x+4}$;
$(fg)(x) = f(x)\cdot g(x) = \sqrt{3-2x}\cdot\sqrt{x+4} = \sqrt{(3-2x)(x+4)}$;
$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{3-2x}}{\sqrt{x+4}} = \sqrt{\dfrac{3-2x}{x+4}}$

(b) $[-4, \frac{3}{2}]$ (c) $(-4, \frac{3}{2}]$

$\boxed{7}$ (a) $(f+g)(x) = f(x) + g(x) = \dfrac{2x}{x-4} + \dfrac{x}{x+5} = \dfrac{2x(x+5) + x(x-4)}{(x-4)(x+5)} = \dfrac{3x^2+6x}{(x-4)(x+5)}$;

$(f-g)(x) = f(x) - g(x) = \dfrac{2x}{x-4} - \dfrac{x}{x+5} = \dfrac{2x(x+5) - x(x-4)}{(x-4)(x+5)} = \dfrac{x^2+14x}{(x-4)(x+5)}$;

$(fg)(x) = f(x)\cdot g(x) = \dfrac{2x}{x-4}\cdot\dfrac{x}{x+5} = \dfrac{2x^2}{(x-4)(x+5)}$;

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{2x/(x-4)}{x/(x+5)} = \dfrac{2(x+5)}{x-4}$

(b) All real numbers except -5 and 4 (c) All real numbers except -5, 0, and 4

$\boxed{8}$ (a) $(f+g)(x) = f(x) + g(x) = \dfrac{x}{x-2} + \dfrac{3x}{x+4} = \dfrac{x(x+4) + 3x(x-2)}{(x-2)(x+4)} = \dfrac{4x^2-2x}{(x-2)(x+4)}$;

$(f-g)(x) = f(x) - g(x) = \dfrac{x}{x-2} - \dfrac{3x}{x+4} = \dfrac{x(x+4) - 3x(x-2)}{(x-2)(x+4)} = \dfrac{-2x^2+10x}{(x-2)(x+4)}$;

$(fg)(x) = f(x)\cdot g(x) = \dfrac{x}{x-2}\cdot\dfrac{3x}{x+4} = \dfrac{3x^2}{(x-2)(x+4)}$;

$\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x/(x-2)}{3x/(x+4)} = \dfrac{x+4}{3(x-2)}$

(b) All real numbers except -4 and 2 (c) All real numbers except -4, 0, and 2

$\boxed{9}$ (a) $(f\circ g)(x) = f(g(x)) = f(-x^2) = 2(-x^2) - 1 = -2x^2 - 1$
(b) $(g\circ f)(x) = g(f(x)) = g(2x-1) = -(2x-1)^2 = -(4x^2-4x+1) = -4x^2+4x-1$
(c) $(f\circ f)(x) = f(f(x)) = f(2x-1) = 2(2x-1) - 1 = (4x-2) - 1 = 4x-3$
(d) $(g\circ g)(x) = g(g(x)) = g(-x^2) = -(-x^2)^2 = -(x^4) = -x^4$

$\boxed{10}$ (a) $(f\circ g)(x) = f(g(x)) = f(x-1) = 3(x-1)^2 = 3(x^2-2x+1) = 3x^2-6x+3$
(b) $(g\circ f)(x) = g(f(x)) = g(3x^2) = (3x^2) - 1 = 3x^2 - 1$
(c) $(f\circ f)(x) = f(f(x)) = f(3x^2) = 3(3x^2)^2 = 3(9x^4) = 27x^4$
(d) $(g\circ g)(x) = g(g(x)) = g(x-1) = (x-1) - 1 = x-2$

Note: Let $h(x) = (f\circ g)(x) = f(g(x))$ and $k(x) = (g\circ f)(x) = g(f(x))$.
$h(-2)$ and $k(3)$ could be worked two ways, as in Example 3 in the text.

$\boxed{11}$ (a) $h(x) = f(3x+7) = 2(3x+7) - 5 = 6x+9$
(b) $k(x) = g(2x-5) = 3(2x-5) + 7 = 6x-8$
(c) Using the result from part (a), $h(-2) = 6(-2) + 9 = -12 + 9 = -3$.
(d) Using the result from part (b), $k(3) = 6(3) - 8 = 18 - 8 = 10$.

$\boxed{12}$ (a) $h(x) = f(6x - 1) = 5(6x - 1) + 2 = 30x - 3$

(b) $k(x) = g(5x + 2) = 6(5x + 2) - 1 = 30x + 11$

(c) Using the result from part (a), $h(-2) = 30(-2) - 3 = -60 - 3 = -63$.

(d) Using the result from part (b), $k(3) = 30(3) + 11 = 90 + 11 = 101$.

$\boxed{13}$ (a) $h(x) = f(5x) = 3(5x)^2 + 4 = 75x^2 + 4$

(b) $k(x) = g(3x^2 + 4) = 5(3x^2 + 4) = 15x^2 + 20$

(c) $h(-2) = 75(-2)^2 + 4 = 300 + 4 = 304$

(d) $k(3) = 15(3)^2 + 20 = 135 + 20 = 155$

$\boxed{14}$ (a) $h(x) = f(4x^2) = 3(4x^2) - 1 = 12x^2 - 1$

(b) $k(x) = g(3x - 1) = 4(3x - 1)^2 = 4(9x^2 - 6x + 1) = 36x^2 - 24x + 4$

(c) $h(-2) = 12(-2)^2 - 1 = 48 - 1 = 47$

(d) $k(3) = 36(3)^2 - 24(3) + 4 = 324 - 72 + 4 = 256$

$\boxed{15}$ (a) $h(x) = f(2x - 1) = 2(2x - 1)^2 + 3(2x - 1) - 4 = 8x^2 - 2x - 5$

(b) $k(x) = g(2x^2 + 3x - 4) = 2(2x^2 + 3x - 4) - 1 = 4x^2 + 6x - 9$

(c) $h(-2) = 8(-2)^2 - 2(-2) - 5 = 32 + 4 - 5 = 31$

(d) $k(3) = 4(3)^2 + 6(3) - 9 = 36 + 18 - 9 = 45$

$\boxed{16}$ (a) $h(x) = f(3x^2 - x + 2) = 5(3x^2 - x + 2) - 7 = 15x^2 - 5x + 3$

(b) $k(x) = g(5x - 7) = 3(5x - 7)^2 - (5x - 7) + 2 = 75x^2 - 215x + 156$

(c) $h(-2) = 15(-2)^2 - 5(-2) + 3 = 60 + 10 + 3 = 73$

(d) $k(3) = 75(3)^2 - 215(3) + 156 = 675 - 645 + 156 = 186$

$\boxed{17}$ (a) $h(x) = f(2x^3 - 5x) = 4(2x^3 - 5x) = 8x^3 - 20x$

(b) $k(x) = g(4x) = 2(4x)^3 - 5(4x) = 128x^3 - 20x$

(c) $h(-2) = 8(-2)^3 - 20(-2) = -64 + 40 = -24$

(d) $k(3) = 128(3)^3 - 20(3) = 3456 - 60 = 3396$

$\boxed{18}$ (a) $h(x) = f(3x) = (3x)^3 + 2(3x)^2 = 27x^3 + 18x^2$

(b) $k(x) = g(x^3 + 2x^2) = 3(x^3 + 2x^2) = 3x^3 + 6x^2$

(c) $h(-2) = 27(-2)^3 + 18(-2)^2 = -216 + 72 = -144$

(d) $k(3) = 3(3)^3 + 6(3)^2 = 81 + 54 = 135$

$\boxed{19}$ (a) $h(x) = f(-7) = |-7| = 7$ (b) $k(x) = g(|x|) = -7$

(c) $h(-2) = 7$ since $h(\text{any value}) = 7$ (d) $k(3) = -7$ since $k(\text{any value}) = -7$

$\boxed{20}$ (a) $h(x) = f(x^2) = 5$ (b) $k(x) = g(5) = (5)^2 = 25$

(c) $h(-2) = 5$ since $h(\text{any value}) = 5$ (d) $k(3) = 25$ since $k(\text{any value}) = 25$

[21] (a) $h(x) = f(\sqrt{x+2}) = (\sqrt{x+2})^2 - 3(\sqrt{x+2}) = x + 2 - 3\sqrt{x+2}$. The domain of $f \circ g$ is the set of all x in the domain of g, $x \geq -2$, such that $g(x)$ is in the domain of f. Since the domain of f is $\mathbb{R}$, any value of $g(x)$ is in its domain. Thus, the domain is all x such that $x \geq -2$. Note that *both* $g(x)$ *and* $f(g(x))$ are defined for x in $[-2, \infty)$.

(b) $k(x) = g(x^2 - 3x) = \sqrt{(x^2 - 3x) + 2} = \sqrt{x^2 - 3x + 2}$. The domain of $g \circ f$ is the set of all x in the domain of f, $\mathbb{R}$, such that $f(x)$ is in the domain of g. Since the domain of g is $x \geq -2$, we must solve $f(x) \geq -2$. $x^2 - 3x \geq -2 \Rightarrow x^2 - 3x + 2 \geq 0 \Rightarrow (x-1)(x-2) \geq 0 \Rightarrow x \in (-\infty, 1] \cup [2, \infty)$ {use a sign diagram}. Thus, the domain is all x such that $x \in (-\infty, 1] \cup [2, \infty)$. Note that *both* $f(x)$ *and* $g(f(x))$ are defined for x in $(-\infty, 1] \cup [2, \infty)$.

[22] (a) $h(x) = f(x^2 + 2x) = \sqrt{(x^2 + 2x) - 15} = \sqrt{x^2 + 2x - 15}$.
Domain of $g = \mathbb{R}$. Domain of $f = [15, \infty)$. $g(x) \geq 15 \Rightarrow x^2 + 2x \geq 15 \Rightarrow$
$x^2 + 2x - 15 \geq 0 \Rightarrow (x+5)(x-3) \geq 0 \Rightarrow x \in (-\infty, -5] \cup [3, \infty)$.

(b) $k(x) = g(\sqrt{x-15}) = (\sqrt{x-15})^2 + 2(\sqrt{x-15}) = x - 15 + 2\sqrt{x-15}$.
Domain of $f = [15, \infty)$. Domain of $g = \mathbb{R}$. Since $f(x)$ is always in the domain of g, the domain of $g \circ f$ is the same as the domain of f, $[15, \infty)$.

[23] (a) $h(x) = f(\sqrt{3x}) = (\sqrt{3x})^2 - 4 = 3x - 4$.
Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$. Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

(b) $k(x) = g(x^2 - 4) = \sqrt{3(x^2 - 4)} = \sqrt{3x^2 - 12}$.
Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.
$f(x) \geq 0 \Rightarrow x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4 \Rightarrow |x| \geq 2 \Rightarrow x \in (-\infty, -2] \cup [2, \infty)$.

[24] (a) $h(x) = f(\sqrt{x}) = -(\sqrt{x})^2 + 1 = -x + 1$.
Domain of $g = [0, \infty)$. Domain of $f = \mathbb{R}$. Since $g(x)$ is always in the domain of f, the domain of $f \circ g$ is the same as the domain of g, $[0, \infty)$.

(b) $k(x) = g(-x^2 + 1) = \sqrt{-x^2 + 1}$. Domain of $f = \mathbb{R}$. Domain of $g = [0, \infty)$.
$f(x) \geq 0 \Rightarrow -x^2 + 1 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow |x| \leq 1 \Rightarrow x \in [-1, 1]$.

[25] (a) $h(x) = f(\sqrt{x+5}) = \sqrt{\sqrt{x+5} - 2}$. Domain of $g = [-5, \infty)$.
Domain of $f = [2, \infty)$. $g(x) \geq 2 \Rightarrow \sqrt{x+5} \geq 2 \Rightarrow x + 5 \geq 4 \Rightarrow$
$x \geq -1$ or $x \in [-1, \infty)$.

(b) $k(x) = g(\sqrt{x-2}) = \sqrt{\sqrt{x-2} + 5}$. Domain of $f = [2, \infty)$.
Domain of $g = [-5, \infty)$. $f(x) \geq -5 \Rightarrow \sqrt{x-2} \geq -5$. This is always true since the result of a square root is nonnegative. The domain is $[2, \infty)$.

$\boxed{26}$ (a) $h(x) = f(\sqrt{x+2}) = \sqrt{3 - \sqrt{x+2}}$. Domain of $g = [-2, \infty)$.

Domain of $f = (-\infty, 3]$. $g(x) \leq 3 \Rightarrow \sqrt{x+2} \leq 3 \Rightarrow x + 2 \leq 9 \Rightarrow x \leq 7$.

We must remember that $x \geq -2$, hence, $-2 \leq x \leq 7$.

(b) $k(x) = g(\sqrt{3-x}) = \sqrt{\sqrt{3-x}+2}$. Domain of $f = (-\infty, 3]$.

Domain of $g = [-2, \infty)$. $f(x) \geq -2 \Rightarrow \sqrt{3-x} \geq -2$. This is always true since the result of a square root is nonnegative. The domain is $(-\infty, 3]$.

$\boxed{27}$ (a) $h(x) = f(\sqrt{x^2-16}) = \sqrt{3 - \sqrt{x^2-16}}$. Domain of $g = (-\infty, -4] \cup [4, \infty)$.

Domain of $f = (-\infty, 3]$. $g(x) \leq 3 \Rightarrow \sqrt{x^2-16} \leq 3 \Rightarrow x^2 - 16 \leq 9 \Rightarrow$

$x^2 \leq 25 \Rightarrow x \in [-5, 5]$. But $|x| \geq 4$ from the domain of g.

Hence, the domain of $f \circ g$ is $[-5, -4] \cup [4, 5]$.

(b) $k(x) = g(\sqrt{3-x}) = \sqrt{(\sqrt{3-x})^2 - 16} = \sqrt{3-x-16} = \sqrt{-x-13}$.

Domain of $f = (-\infty, 3]$. Domain of $g = (-\infty, -4] \cup [4, \infty)$.

$f(x) \geq 4$ $\{f(x)$ cannot be less than $0\}$ $\Rightarrow \sqrt{3-x} \geq 4 \Rightarrow 3 - x \geq 16 \Rightarrow$

$x \leq -13$.

$\boxed{28}$ (a) $h(x) = f(\sqrt[3]{x-5}) = (\sqrt[3]{x-5})^3 + 5 = x - 5 + 5 = x$.

Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$. All values of $g(x)$ are in the domain of f.

Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g(x^3 + 5) = \sqrt[3]{(x^3+5) - 5} = \sqrt[3]{x^3} = x$.

Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$. All values of $f(x)$ are in the domain of g.

Hence, the domain of $g \circ f$ is $\mathbb{R}$.

$\boxed{29}$ (a) $h(x) = f\left(\dfrac{2x-5}{3}\right) = \dfrac{3\left(\dfrac{2x-5}{3}\right)+5}{2} = \dfrac{2x-5+5}{2} = \dfrac{2x}{2} = x$.

Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R}$. All values of $g(x)$ are in the domain of f.

Hence, the domain of $f \circ g$ is $\mathbb{R}$.

(b) $k(x) = g\left(\dfrac{3x+5}{2}\right) = \dfrac{2\left(\dfrac{3x+5}{2}\right)-5}{3} = \dfrac{3x+5-5}{3} = \dfrac{3x}{3} = x$.

Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R}$. All values of $f(x)$ are in the domain of g.

Hence, the domain of $g \circ f$ is $\mathbb{R}$.

$\boxed{30}$ (a) $h(x) = f(x-1) = \dfrac{1}{(x-1)-1} = \dfrac{1}{x-2}$. Domain of $g = \mathbb{R}$. Domain of $f = \mathbb{R} - \{1\}$.

$g(x) \neq 1 \Rightarrow x - 1 \neq 1 \Rightarrow x \neq 2$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{2\}$.

(b) $k(x) = g\left(\dfrac{1}{x-1}\right) = \dfrac{1}{x-1} - 1 = \dfrac{1-(x-1)}{x-1} = \dfrac{2-x}{x-1}$.

Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R}$.

All values of $f(x)$ are in the domain of g. The domain of $g \circ f$ is $\mathbb{R} - \{1\}$.

$\boxed{31}$ (a) $h(x) = f\left(\frac{1}{x^3}\right) = \left(\frac{1}{x^3}\right)^2 = \frac{1}{x^6}$. Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R}$.

All values of $g(x)$ are in the domain of f. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

(b) $k(x) = g(x^2) = \frac{1}{(x^2)^3} = \frac{1}{x^6}$. Domain of $f = \mathbb{R}$. Domain of $g = \mathbb{R} - \{0\}$.

All values of $f(x)$ are in the domain of g except for 0.

Since f is 0 when x is 0, the domain of $f \circ g$ is $\mathbb{R} - \{0\}$.

$\boxed{32}$ (a) $h(x) = f\left(\frac{3}{x}\right) = \frac{3/x}{(3/x) - 2} \cdot \frac{x}{x} = \frac{3}{3 - 2x}$.

Domain of $g = \mathbb{R} - \{0\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \neq 2 \;\Rightarrow\; \frac{3}{x} \neq 2 \;\Rightarrow\; x \neq \frac{3}{2}$. Hence, the domain of $f \circ g$ is $\mathbb{R} - \{0, \frac{3}{2}\}$.

(b) $k(x) = g\left(\frac{x}{x-2}\right) = \frac{3}{x/(x-2)} = \frac{3x - 6}{x}$.

Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{0\}$.

$f(x) \neq 0 \;\Rightarrow\; \frac{x}{x-2} \neq 0 \;\Rightarrow\; x \neq 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \{0, 2\}$.

$\boxed{33}$ (a) $h(x) = f\left(\frac{x-3}{x-4}\right) = \frac{\frac{x-3}{x-4} - 1}{\frac{x-3}{x-4} - 2} \cdot \frac{x-4}{x-4} = \frac{x - 3 - 1(x-4)}{x - 3 - 2(x-4)} = \frac{1}{5-x}$.

Domain of $g = \mathbb{R} - \{4\}$. Domain of $f = \mathbb{R} - \{2\}$.

$g(x) \neq 2 \;\Rightarrow\; \frac{x-3}{x-4} \neq 2 \;\Rightarrow\; x - 3 \neq 2x - 8 \;\Rightarrow\; x \neq 5$.

The domain is $\mathbb{R} - \{4, 5\}$.

(b) $k(x) = g\left(\frac{x-1}{x-2}\right) = \frac{\frac{x-1}{x-2} - 3}{\frac{x-1}{x-2} - 4} \cdot \frac{x-2}{x-2} = \frac{x - 1 - 3(x-2)}{x - 1 - 4(x-2)} = \frac{-2x+5}{-3x+7}$.

Domain of $f = \mathbb{R} - \{2\}$. Domain of $g = \mathbb{R} - \{4\}$.

$f(x) \neq 4 \;\Rightarrow\; \frac{x-1}{x-2} \neq 4 \;\Rightarrow\; x - 1 \neq 4x - 8 \;\Rightarrow\; x \neq \frac{7}{3}$.

The domain is $\mathbb{R} - \{2, \frac{7}{3}\}$.

$\boxed{34}$ (a) $h(x) = f\left(\frac{x-5}{x+4}\right) = \frac{\frac{x-5}{x+4} + 2}{\frac{x-5}{x+4} - 1} \cdot \frac{x+4}{x+4} = \frac{x - 5 + 2(x+4)}{x - 5 - 1(x+4)} = \frac{3x+3}{-9} = \frac{-x-1}{3}$.

Domain of $g = \mathbb{R} - \{-4\}$. Domain of $f = \mathbb{R} - \{1\}$.

$g(x) \neq 1 \;\Rightarrow\; \frac{x-5}{x+4} \neq 1 \;\Rightarrow\; x - 5 \neq x + 4$. This is always true —

so no additional values need to be excluded, and thus, the domain is $\mathbb{R} - \{-4\}$.

(b) $k(x) = g\left(\frac{x+2}{x-1}\right) = \frac{\frac{x+2}{x-1} - 5}{\frac{x+2}{x-1} + 4} \cdot \frac{x-1}{x-1} = \frac{x + 2 - 5(x-1)}{x + 2 + 4(x-1)} = \frac{-4x+7}{5x-2}$.

Domain of $f = \mathbb{R} - \{1\}$. Domain of $g = \mathbb{R} - \{-4\}$.

$f(x) \neq -4 \;\Rightarrow\; \frac{x+2}{x-1} \neq -4 \;\Rightarrow\; x + 2 \neq -4x + 4 \;\Rightarrow\; x \neq \frac{2}{5}$.

The domain is $\mathbb{R} - \{\frac{2}{5}, 1\}$.

$\boxed{35}$ $(f \circ g)(x) = f(g(x)) = f(x+3) = (x+3)^2 - 2$. $(f \circ g)(x) = 0 \Rightarrow$
$$(x+3)^2 - 2 = 0 \Rightarrow (x+3)^2 = 2 \Rightarrow x+3 = \pm\sqrt{2} \Rightarrow x = -3 \pm \sqrt{2}$$

$\boxed{36}$ $(f \circ g)(x) = f(g(x)) = f(2x-1) = (2x-1)^2 - (2x-1) - 2 = 4x^2 - 6x$.
$$(f \circ g)(x) = 0 \Rightarrow 4x^2 - 6x = 0 \Rightarrow 2x(2x-3) = 0 \Rightarrow x = 0, \tfrac{3}{2}$$

$\boxed{37}$ (a) $(f \circ g)(6) = f(g(6)) = f(8) = 5$ (b) $(g \circ f)(6) = g(f(6)) = g(7) = 6$

(c) $(f \circ f)(6) = f(f(6)) = f(7) = 6$ (d) $(g \circ g)(6) = g(g(6)) = g(8) = 5$

(e) $(f \circ g)(9) = f(g(9)) = f(4)$, but $f(4)$ cannot be determined from the table.

$\boxed{38}$ (a) $(T \circ S)(1) = T(S(1)) = T(0) = 2$ (b) $(S \circ T)(1) = S(T(1)) = S(3) = 2$

(c) $(T \circ T)(1) = T(T(1)) = T(3) = 0$ (d) $(S \circ S)(1) = S(S(1)) = S(0) = 1$

(e) $(T \circ S)(4) = T(S(4)) = T(5)$, but $T(5)$ cannot be determined from the table.

$\boxed{39}$ $(D \circ R)(x) = D(R(x)) = D(20x) =$
$$\sqrt{400 + (20x)^2} = \sqrt{400 + 400x^2} = \sqrt{400(1+x^2)} = 20\sqrt{x^2+1}$$

$\boxed{40}$ $(S \circ D)(t) = S(D(t)) = S(2t+5) = 4\pi(2t+5)^2$

$\boxed{41}$ $(fg)(-x) = f(-x)g(-x) = -f(x)g(x) = -(fg)(x) \Rightarrow fg$ is an odd function.

$\boxed{42}$ If f is even and odd, then $f(-x) = f(x)$ and $f(-x) = -f(x)$.

Thus, $f(x) = -f(x) \Rightarrow 2f(x) = 0 \Rightarrow f(x) = 0$.

Hence, $f(x) = 0$ is a function that is both even and odd.

$\boxed{43}$ (ROUND2 ∘ SSTAX)(437.21) = ROUND2(SSTAX(437.21))
= ROUND2(0.0715 · 437.21)
= ROUND2(31.260515) = 31.26

$\boxed{44}$ (a) (CHR ∘ ORD)("C") = CHR(ORD("C")) = CHR(67) = "C"

(b) CHR(ORD("A") + 3) = CHR(65 + 3) = CHR(68) = "D"

$\boxed{45}$ $r = 6t$ and $A = \pi r^2 \Rightarrow A(t) = \pi(6t)^2 = 36\pi t^2$ ft^2.

$\boxed{46}$ $V = \frac{4}{3}\pi r^3 \Rightarrow r^3 = \frac{3V}{4\pi} \Rightarrow r = \sqrt[3]{\frac{3V}{4\pi}}$. $V = \frac{9}{2}\pi t \Rightarrow r(t) = \sqrt[3]{\frac{27t}{8}} = \frac{3}{2}\sqrt[3]{t}$ ft.

$\boxed{47}$ $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{3V}{\pi}}$. $V = 243\pi t \Rightarrow r(t) = \sqrt[3]{729t} = 9\sqrt[3]{t}$ ft.

$\boxed{48}$ $x^2 + x^2 = y^2 \Rightarrow y^2 = 2x^2 \Rightarrow y = \sqrt{2}x$.
$$y^2 + x^2 = d^2 \Rightarrow d^2 = 3x^2 \Rightarrow d(x) = \sqrt{3}x.$$

$\boxed{49}$ Let l denote the length of the rope.

At $t = 0$, $l = 20$. At time t, $l = 20 + 5t$. $h^2 + 20^2 = l^2 \Rightarrow$
$$h(t) = \sqrt{(20+5t)^2 - 20^2} = \sqrt{25t^2 + 200t} = \sqrt{25(t^2+8t)} = 5\sqrt{t^2+8t}.$$

$\boxed{50}$ The triangle has sides of length 28, 50, and $\sqrt{28^2 + 50^2} = \sqrt{3284} = 2\sqrt{821}$.

Let $y = h - 2$. Using similar triangles and the fact that $d = 2t$,
$$\frac{y}{d} = \frac{28}{2\sqrt{821}} \Rightarrow y = \frac{14}{\sqrt{821}}d \Rightarrow h(t) = \frac{28}{\sqrt{821}}t + 2.$$

[51] From Exercise 77 of Section 3.4, $d = \sqrt{90{,}400 + x^2}$. Let $x = 500 + 150t$.

Thus, $d(t) = \sqrt{90{,}400 + (500 + 150t)^2} = 10\sqrt{225t^2 + 1500t + 3404}$.

[52] Consider the cable to be a right circular cylinder.

$A = 2\pi rh = \pi dh = 1200\pi d \; \{h = 1200 \text{ inches}\} \Rightarrow d = \frac{A}{1200\pi}$.

$A = -750t$, so $d(t) = \text{original value} + \text{change} = 4 - \frac{750t}{1200\pi} = 4 - \frac{5}{8\pi}t$ inches.

[53] $y = (x^2 + 3x)^{1/3}$ • Suppose you were to find the value of y if x was equal to 3. Using a calculator, you might compute the value of $x^2 + 3x$ first, and then raise that result to the $\frac{1}{3}$ power. Thus, we would choose $y = u^{1/3}$ and $u = x^2 + 3x$.

[54] For $y = \sqrt[4]{x^4 - 16}$, choose $u = x^4 - 16$ and $y = \sqrt[4]{u}$.

[55] For $y = \frac{1}{(x-3)^4}$, choose $u = x - 3$ and $y = 1/u^4 = u^{-4}$.

[56] For $y = 4 + \sqrt{x^2 + 1}$, choose $u = x^2 + 1$ and $y = 4 + \sqrt{u}$.

[57] For $y = (x^4 - 2x^2 + 5)^5$, choose $u = x^4 - 2x^2 + 5$ and $y = u^5$.

[58] For $y = \frac{1}{(x^2 + 3x - 5)^3}$, choose $u = x^2 + 3x - 5$ and $y = 1/u^3 = u^{-3}$.

[59] For $y = \frac{\sqrt{x+4} - 2}{\sqrt{x+4} + 2}$, there is not a "simple" choice for y as in previous exercises.

One choice for u is $u = x + 4$. Then y would be $\frac{\sqrt{u} - 2}{\sqrt{u} + 2}$.

Another choice for u is $u = \sqrt{x+4}$. Then y would be $\frac{u-2}{u+2}$.

[60] For $y = \frac{\sqrt[3]{x}}{1 + \sqrt[3]{x}}$, choose $u = \sqrt[3]{x}$ and $y = \frac{u}{1+u}$.

[61] $(f \circ g)(x) = f(g(x)) =$

$$f(x^3 + 1) = \sqrt{x^3+1} - 1 = \left(\sqrt{x^3+1} - 1\right) \times \frac{\sqrt{x^3+1}+1}{\sqrt{x^3+1}+1} = \frac{x^3}{\sqrt{x^3+1}+1}.$$

Thus, $(f \circ g)(0.0001) \approx \frac{(10^{-4})^3}{2} = 5 \times 10^{-13}$.

[62] $f(1.12) \approx 0.321170$, $g(1.12) \approx 0.280105$, $f(5.2) \approx 4.106542$, and $f(f(5.2)) \approx 3.014835$

$$\Rightarrow \frac{(f+g)(1.12) - (f/g)(1.12)}{[(f \circ f)(5.2)]^2} = \frac{[f(1.12) + g(1.12)] - f(1.12)/g(1.12)}{\{f[f(5.2)]\}^2} \approx -0.059997$$

[63] *Note:* Point out to students that Y_2 is actually f, Y_1 is the function we are replacing x with in f, and Y_3 is accepting Y_2's output values as its input values. Hence, Y_3 is a function of a function of a function.

(a) $y = -2f(x)$; $Y_1 = x$, graph $Y_3 = -2Y_2$ {turn off Y_1 and Y_2};

see *Figure 63(a)* on the next page. $D = [-2, 6]$, $R = [-16, 8]$

Note: The graphs often do not show the correct endpoints—you need to change the viewing rectangle or zoom in to actually view them on the screen.

$[-12, 12, 2]$ by $[-16, 8, 2]$

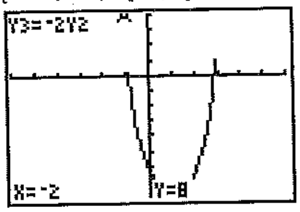

Figure 63(a)

$[-12, 12, 2]$ by $[-16, 8, 2]$

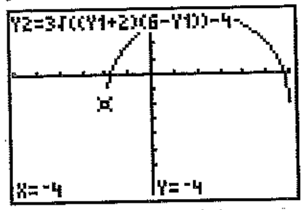

Figure 63(b)

(b) $y = f(\frac{1}{2}x)$; $\mathrm{Y}_1 = 0.5x$, graph Y_2; $D = [-4, 12]$, $R = [-4, 8]$

(c) $y = f(x-3)+1$; $\mathrm{Y}_1 = x-3$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 + 1$; $D = [1, 9]$, $R = [-3, 9]$

$[-12, 12, 2]$ by $[-6, 10, 2]$

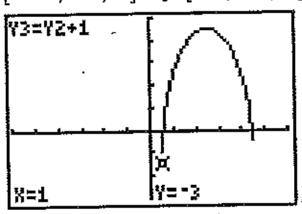

Figure 63(c)

$[-12, 12, 2]$ by $[-6, 10, 2]$

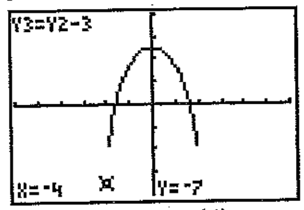

Figure 63(d)

(d) $y = f(x+2)-3$; $\mathrm{Y}_1 = x+2$, graph $\mathrm{Y}_3 = \mathrm{Y}_2 - 3$; $D = [-4, 4]$, $R = [-7, 5]$

(e) $y = f(-x)$; $\mathrm{Y}_1 = -x$, graph Y_2; $D = [-6, 2]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-8, 8, 2]$

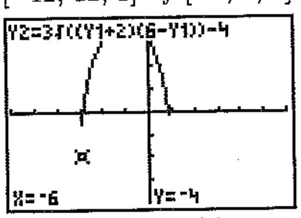

Figure 63(e)

$[-12, 12, 2]$ by $[-8, 8, 2]$

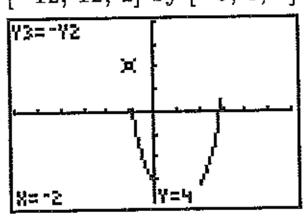

Figure 63(f)

(f) $y = -f(x)$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = -\mathrm{Y}_2$; $D = [-2, 6]$, $R = [-8, 4]$

(g) $y = f(|x|)$; $\mathrm{Y}_1 = \mathrm{abs}\, x$, graph Y_2; $D = [-6, 6]$, $R = [-4, 8]$

$[-12, 12, 2]$ by $[-8, 8, 2]$

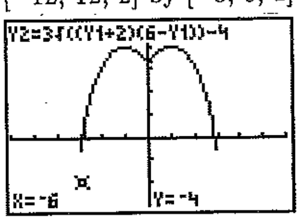

Figure 63(g)

$[-2, 6]$ by $[0, 8]$

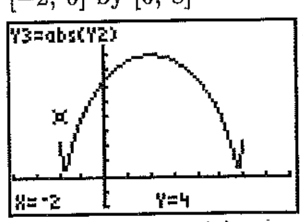

Figure 63(h)

(h) $y = |f(x)|$; $\mathrm{Y}_1 = x$, graph $\mathrm{Y}_3 = \mathrm{abs}\,\mathrm{Y}_2$; $D = [-2, 6]$, $R = [0, 8]$

[64] (a) $y = \frac{1}{2}f(x)$; $Y_1 = x$, graph $Y_3 = 0.5Y_2$; $D = [-6, -2]$, $R = [-5, -2]$

$[-10, 6]$ by $[-11, 10]$

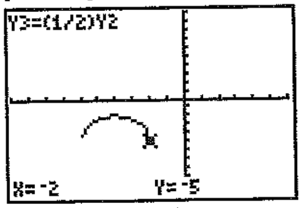

Figure 64(a)

$[-10, 6]$ by $[-11, 10]$

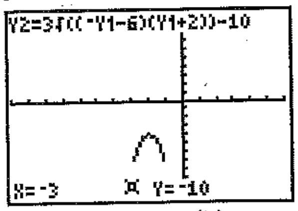

Figure 64(b)

(b) $y = f(2x)$; $Y_1 = 2x$, graph Y_2; $D = [-3, -1]$, $R = [-10, -4]$

(c) $y = f(x-2)+5$; $Y_1 = x-2$, graph $Y_3 = Y_2 + 5$; $D = [-4, 0]$, $R = [-5, 1]$

$[-10, 6]$ by $[-11, 10]$

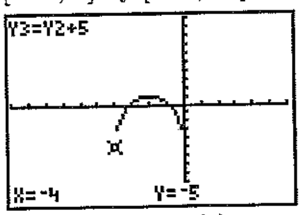

Figure 64(c)

$[-10, 6]$ by $[-11, 10]$

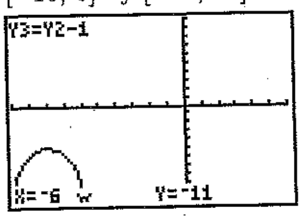

Figure 64(d)

(d) $y = f(x+4)-1$; $Y_1 = x+4$, graph $Y_3 = Y_2 - 1$; $D = [-10, -6]$, $R = [-11, -5]$

(e) $y = f(-x)$; $Y_1 = -x$, graph Y_2; $D = [2, 6]$, $R = [-10, -4]$

$[-10, 6]$ by $[-11, 10]$

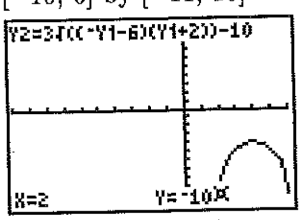

Figure 64(e)

$[-10, 6]$ by $[-11, 10]$

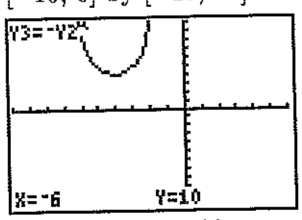

Figure 64(f)

(f) $y = -f(x)$; $Y_1 = x$, graph $Y_3 = -Y_2$; $D = [-6, -2]$, $R = [4, 10]$

(g) $y = f(|x|)$; No graph

(h) $y = |f(x)|$; $Y_1 = x$,

graph $Y_3 = \text{abs}\, Y_2$;

$D = [-6, -2]$, $R = [4, 10]$

$[-10, 6]$ by $[-11, 10]$

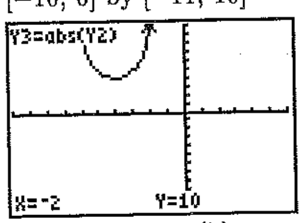

Figure 64(h)

Chapter 3 Review Exercises

[1] If $y/x < 0$, then y and x must have opposite signs, and hence,

the set consists of all points in quadrants II and IV.

$\boxed{2}$ Show that $d(A, B)^2 + d(A, C)^2 = d(B, C)^2$; that is, $(\sqrt{80})^2 + (\sqrt{5})^2 = (\sqrt{85})^2$.

Area $= \frac{1}{2}bh = \frac{1}{2}(\sqrt{80})(\sqrt{5}) = \frac{1}{2}(4\sqrt{5})(\sqrt{5}) = 10$.

$\boxed{3}$ (a) $P(-5, 9)$, $Q(-8, -7)$ $\Rightarrow$

$$d(P, Q) = \sqrt{[-8-(-5)]^2 + (-7-9)^2} = \sqrt{9+256} = \sqrt{265}.$$

(b) $P(-5, 9)$, $Q(-8, -7)$ $\Rightarrow$ $M_{PQ} = \left(\dfrac{-5+(-8)}{2}, \dfrac{9+(-7)}{2}\right) = (-\frac{13}{2}, 1)$.

(c) Let $R = (x, y)$. $Q = M_{PR}$ $\Rightarrow$ $(-8, -7) = \left(\dfrac{-5+x}{2}, \dfrac{9+y}{2}\right)$ $\Rightarrow$

$-5 + x = -16$ and $9 + y = -14$ $\Rightarrow$ $R = (-11, -23)$.

$\boxed{4}$ Let $Q(0, y)$ be an arbitrary point on the y-axis. $13 = d(P, Q)$ $\Rightarrow$
$13 = \sqrt{(0-12)^2 + (y-6)^2}$ $\Rightarrow$ $169 = 144 + y^2 - 12y + 36$ $\Rightarrow$
$y^2 - 12y + 11 = 0$ $\Rightarrow$ $(y-1)(y-11) = 0$ $\Rightarrow$ $y = 1, 11$.

The points are $(0, 1)$ and $(0, 11)$.

$\boxed{5}$ $d(P, Q) < 3$ $\Rightarrow$ $\sqrt{(-2-a)^2 + (a-1)^2} < 3$ $\Rightarrow$ $4 + 4a + a^2 + a^2 - 2a + 1 < 9$ $\Rightarrow$
$2a^2 + 2a - 4 < 0$ $\Rightarrow$ $a^2 + a - 2 < 0$ $\Rightarrow$ $(a+2)(a-1) < 0$ $\Rightarrow$ $-2 < a < 1$.

Use a sign diagram to establish the final answer.

$\boxed{6}$ The equation of a circle with center $C(7, -4)$ is $(x-7)^2 + (y+4)^2 = r^2$.
Letting $x = -3$ and $y = 3$ yields $(-10)^2 + 7^2 = r^2$ $\Rightarrow$ $r^2 = 149$.

An equation is $(x-7)^2 + (y+4)^2 = 149$.

$\boxed{7}$ The center of the circle is the midpoint of $A(8, 10)$ and $B(-2, -14)$.

$M_{AB} = \left(\dfrac{8+(-2)}{2}, \dfrac{10+(-14)}{2}\right) = (3, -2)$. The radius of the circle is

$\frac{1}{2} \cdot d(A, B) = \frac{1}{2}\sqrt{(-2-8)^2 + (-14-10)^2} = \frac{1}{2}\sqrt{100+576} = \frac{1}{2} \cdot 26 = 13$.

An equation is $(x-3)^2 + (y+2)^2 = 13^2 = 169$.

$\boxed{8}$ We need to solve the equation for x. $(x+2)^2 + y^2 = 9$ $\Rightarrow$
$(x+2)^2 = 9 - y^2$ $\Rightarrow$ $x + 2 = \pm\sqrt{9-y^2}$ $\Rightarrow$ $x = -2 \pm \sqrt{9-y^2}$.

Choose the term with the minus sign for the left half.

$\boxed{9}$ $C(11, -5)$, $D(-8, 6)$ $\Rightarrow$ $m_{CD} = \dfrac{6-(-5)}{-8-11} = \dfrac{11}{-19} = -\dfrac{11}{19}$.

$\boxed{10}$ Show that the slopes of one pair of opposite sides are equal.

$A(-3, 1)$, $B(1, -1)$, $C(4, 1)$, and $D(3, 5)$ $\Rightarrow$ $m_{AD} = \frac{2}{3} = m_{BC}$.

$\boxed{11}$ (a) $6x + 2y + 5 = 0$ $\Leftrightarrow$ $y = -3x - \frac{5}{2}$. Using the same slope, -3, with $A(\frac{1}{2}, -\frac{1}{3})$,

we have $y + \frac{1}{3} = -3(x - \frac{1}{2})$ $\Rightarrow$ $6y + 2 = -18x + 9$ $\Rightarrow$ $18x + 6y = 7$.

(b) Using the negative reciprocal of -3 for the slope,

$y + \frac{1}{3} = \frac{1}{3}(x - \frac{1}{2})$ $\Rightarrow$ $6y + 2 = 2x - 1$ $\Rightarrow$ $2x - 6y = 3$.

$\boxed{12}$ Solving for y gives us: $8x + 3y - 24 = 0$ $\Leftrightarrow$ $3y = -8x + 24$ $\Leftrightarrow$ $y = -\frac{8}{3}x + 8$

$\boxed{13}$ The radius of the circle is the distance from the line $x = 4$ to the x-value of the center $C(-5, -1)$; $r = 4 - (-5) = 9$. An equation is $(x + 5)^2 + (y + 1)^2 = 81$.

$\boxed{14}$ $x^2 + y^2 - 4x + 10y + 26 = 0 \ \Rightarrow$

$x^2 - 4x + \underline{\ 4\ } + y^2 + 10y + \underline{\ 25\ } = -26 + \underline{\ 4\ } + \underline{\ 25\ } \ \Rightarrow \ (x-2)^2 + (y+5)^2 = 3 \ \Rightarrow$

$C(2, -5)$. We want the equation of the line through $(-3, 0)$ and $(2, -5)$.

$$y - 0 = \tfrac{-5-0}{2+3}(x+3) \ \Rightarrow \ y = -1(x+3) \ \Rightarrow \ x + y = -3.$$

$\boxed{15}$ $P(4, -3)$ with $m = 5 \ \Rightarrow \ y + 3 = 5(x-4) \ \Rightarrow \ y + 3 = 5x - 20 \ \Rightarrow \ 5x - y = 23$.

$\boxed{16}$ $A(-1, 2)$ and $B(3, -4) \ \Rightarrow \ M_{AB} = (1, -1)$ and $m_{AB} = -\frac{3}{2}$.

$$y + 1 = \tfrac{2}{3}(x-1) \ \Rightarrow \ 3y + 3 = 2x - 2 \ \Rightarrow \ 2x - 3y = 5.$$

$\boxed{17}$ $x^2 + y^2 - 12y + 31 = 0 \ \Rightarrow \ x^2 + y^2 - 12y + \underline{\ 36\ } = -31 + \underline{\ 36\ } \ \Rightarrow \ x^2 + (y-6)^2 = 5$.

$$C(0, 6);\ r = \sqrt{5}$$

$\boxed{18}$ $4x^2 + 4y^2 + 24x - 16y + 39 = 0 \ \Rightarrow$

$x^2 + 6x + \underline{\ 9\ } + y^2 - 4y + \underline{\ 4\ } = -\frac{39}{4} + \underline{\ 9\ } + \underline{\ 4\ } \ \Rightarrow \ (x+3)^2 + (y-2)^2 = \frac{13}{4}$.

$$C(-3, 2);\ r = \tfrac{1}{2}\sqrt{13}$$

$\boxed{19}$ (a) $f(x) = \dfrac{x}{\sqrt{x+3}} \ \Rightarrow \ f(1) = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$ (b) $f(-1) = -\dfrac{1}{\sqrt{2}}$ (c) $f(0) = \dfrac{0}{\sqrt{3}} = 0$

(d) $f(-x) = \dfrac{-x}{\sqrt{-x+3}} = -\dfrac{x}{\sqrt{3-x}}$ (e) $-f(x) = -1 \cdot f(x) = -\dfrac{x}{\sqrt{x+3}}$

(f) $f(x^2) = \dfrac{x^2}{\sqrt{x^2+3}}$ (g) $[f(x)]^2 = \left(\dfrac{x}{\sqrt{x+3}}\right)^2 = \dfrac{x^2}{x+3}$

$\boxed{20}$ $f(x) = \dfrac{-32(x^2-4)}{(9-x^2)^{5/3}} \ \Rightarrow \ f(4) = \dfrac{(-)(+)}{(-)} = +$. $f(4)$ is positive.

$\boxed{21}$ $f(x) = \dfrac{-2(x^2-20)(5-x)}{(6-x^2)^{4/3}} \ \Rightarrow \ f(4) = \dfrac{(-)(-)(+)}{(+)} = +$. $f(4)$ is positive.

$\boxed{22}$ (a) $3x - 4 \geq 0 \ \Rightarrow \ x \geq \frac{4}{3}$; $D = [\frac{4}{3}, \infty)$.

Since y is the result of a square root, $y \geq 0$; $R = [0, \infty)$.

(b) $D =$ All real numbers except -3.

Since y is the square of the nonzero term $\frac{1}{x+3}$, $y > 0$; $R = (0, \infty)$.

$\boxed{23}$
$$\begin{aligned}
\frac{f(a+h) - f(a)}{h} &= \frac{\left[-(a+h)^2 + (a+h) + 5\right] - \left[-a^2 + a + 5\right]}{h} \\
&= \frac{-a^2 - 2ah - h^2 + a + h + 5 + a^2 - a - 5}{h} \\
&= \frac{-2ah - h^2 + h}{h} \\
&= \frac{h(-2a - h + 1)}{h} = -2a - h + 1
\end{aligned}$$

[24] $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\dfrac{1}{a+h+2}-\dfrac{1}{a+2}}{h} = \dfrac{\dfrac{(a+2)-(a+h+2)}{(a+h+2)(a+2)}}{h} = \dfrac{-h}{(a+h+2)(a+2)h} =$

$-\dfrac{1}{(a+h+2)(a+2)}$

[25] $f(x) = ax + b$ is the desired form. $a = \frac{7-2}{3-1} = \frac{5}{2}$. $f(x) = \frac{5}{2}x + b \Rightarrow$

$f(1) = \frac{5}{2} + b$, but $f(1) = 2$, so $\frac{5}{2} + b = 2$, and $b = -\frac{1}{2}$. Thus, $f(x) = \frac{5}{2}x - \frac{1}{2}$.

[26] (a) $f(x) = \sqrt[3]{x^3+4x} \Rightarrow$

$f(-x) = \sqrt[3]{(-x)^3+4(-x)} = \sqrt[3]{-1(x^3+4x)} = -\sqrt[3]{x^3+4x} = -f(x)$, so f is odd

(b) $f(x) = \sqrt[3]{3x^2-x^3} \Rightarrow$

$f(-x) = \sqrt[3]{3(-x)^2-(-x)^3} = \sqrt[3]{3x^2+x^3} \neq \pm f(x)$, so f is neither even nor odd

(c) $f(x) = \sqrt[3]{x^4+3x^2+5} \Rightarrow$

$f(-x) = \sqrt[3]{(-x)^4+3(-x)^2+5} = \sqrt[3]{x^4+3x^2+5} = f(x)$, so f is even

[27] $x + 5 = 0 \Leftrightarrow x = -5$, a vertical line; x-intercept -5; y-intercept: None

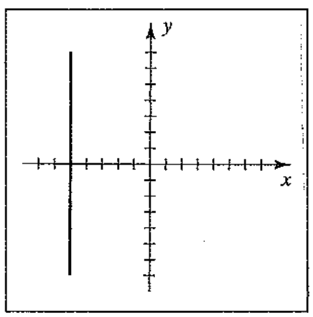

Figure 27

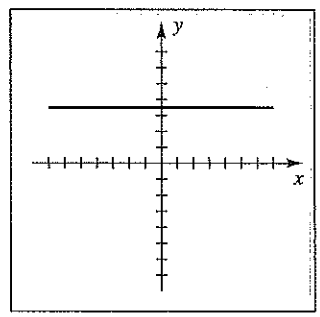

Figure 28

[28] $2y - 7 = 0 \Leftrightarrow y = \frac{7}{2}$, a horizontal line; x-intercept: None; y-intercept 3.5

[29] $2y + 5x - 8 = 0 \Leftrightarrow y = -\frac{5}{2}x + 4$, a line with slope $-\frac{5}{2}$ and y-intercept 4;

x-intercept 1.6

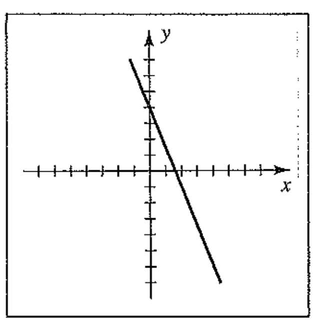

Figure 29

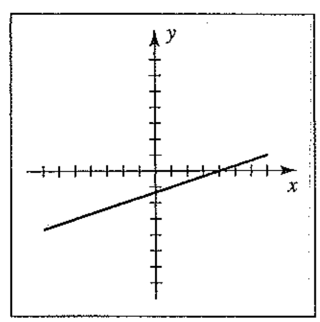

Figure 30

[30] $x = 3y + 4 \Leftrightarrow y = \frac{1}{3}x - \frac{4}{3}$, a line with slope $\frac{1}{3}$ and y-intercept $-\frac{4}{3}$; x-intercept 4

[31] $9y + 2x^2 = 0 \Leftrightarrow y = -\frac{2}{9}x^2$, a parabola opening down; x- and y-intercept 0

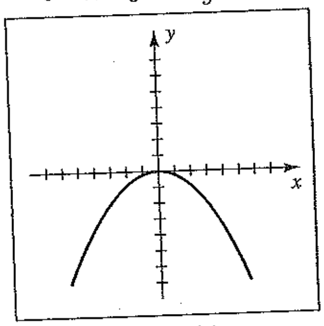

Figure 31

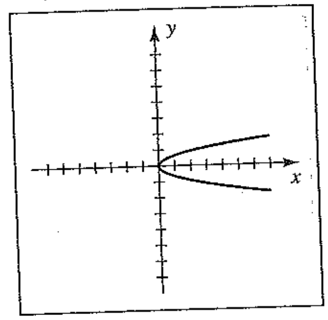

Figure 32

[32] $3x - 7y^2 = 0 \Leftrightarrow x = \frac{7}{3}y^2$, a parabola opening to the right; x- and y-intercept 0

[33] $y = \sqrt{1 - x}$; x-intercept 1; y-intercept 1

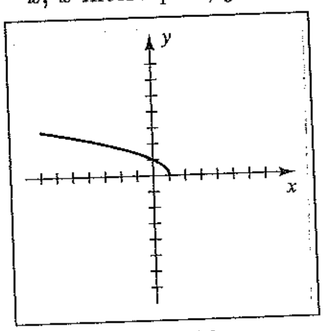

Figure 33

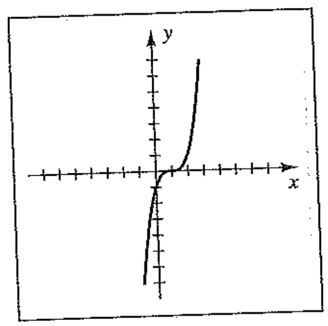

Figure 34

[34] $y = (x-1)^3$, shift $y = x^3$ right one unit; x-intercept 1; y-intercept -1

[35] $y^2 = 16 - x^2 \Leftrightarrow x^2 + y^2 = 16$; x-intercepts ± 4; y-intercepts ± 4

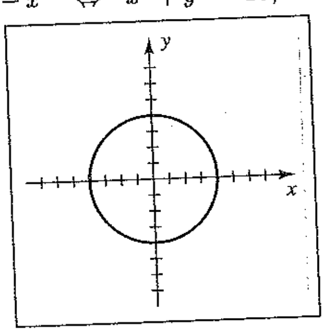

Figure 35

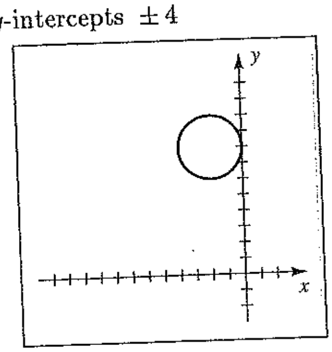

Figure 36

[36] $x^2 + y^2 + 4x - 16y + 64 = 0 \Rightarrow$

$x^2 + 4x + \underline{\ 4\ } + y^2 - 16y + \underline{\ 64\ } = -64 + \underline{\ 4\ } + \underline{\ 64\ } \Rightarrow (x+2)^2 + (y-8)^2 = 4$;

$C(-2, 8)$, $r = \sqrt{4} = 2$; x-intercept: None; y-intercept 8

[37] $x^2+y^2-8x=0 \Leftrightarrow x^2-8x+\underline{\ 16\ }+y^2=\underline{\ 16\ } \Leftrightarrow (x-4)^2+y^2=16;$

$C(4,\ 0),\ r=\sqrt{16}=4$; x-intercepts 0 and 8; y-intercept 0

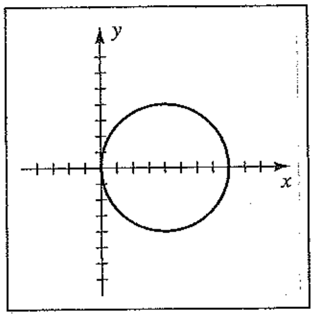

Figure 37

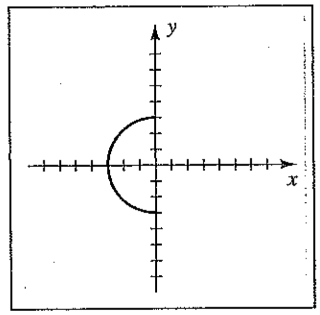

Figure 38

[38] $x=-\sqrt{9-y^2}$ is the left half of the circle $x^2+y^2=9$;

x-intercept -3; y-intercepts ± 3

[39] $y=(x-3)^2-2$ has vertex $(3,\ -2)$; x-intercepts $3\pm\sqrt{2}$; y-intercept 7

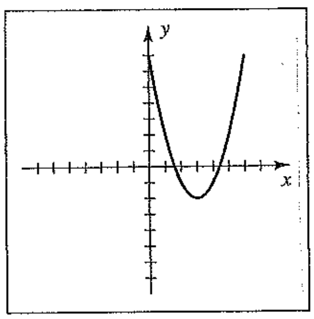

Figure 39

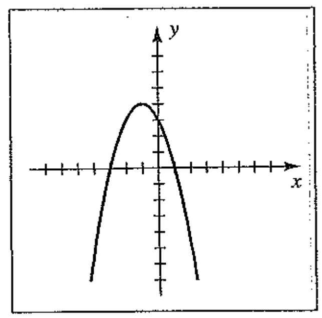

Figure 40

[40] $y=-x^2-2x+3=-(x^2+2x+\underline{\ 1\ })+3+\underline{\ 1\ }=-(x+1)^2+4;\ V(-1,\ 4);$

x-intercepts -3 and 1; y-intercept 3

[41] The radius of the large circle is 3 and the radius of the small circle is 1. Since the center of the small circle is 4 units from the origin and lies on the line $y=x$, we must have $x^2+y^2=4^2 \Rightarrow x^2+x^2=16 \Rightarrow 2x^2=16 \Rightarrow x^2=8 \Rightarrow x=\sqrt{8}$. The center of the small circle is $(\sqrt{8},\ \sqrt{8})$.

[42] The graph of $y=-f(x-2)$ is the graph of $y=f(x)$ shifted to the right 2 units and reflected through the x-axis.

[43] (b) $D = \mathbb{R}$; $R = \mathbb{R}$ (c) Decreasing on $(-\infty, \infty)$

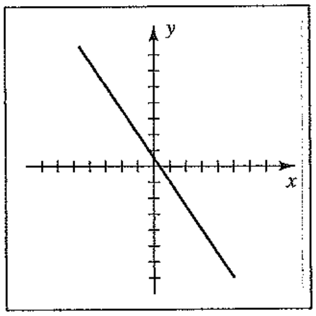

Figure 43

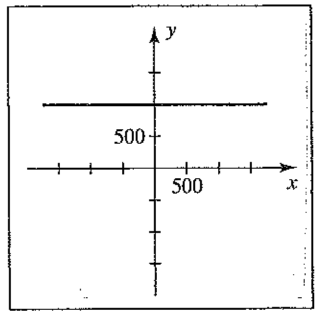

Figure 44

[44] (b) $D = \mathbb{R}$; $R = \{1000\}$ (c) Constant on $(-\infty, \infty)$

[45] (b) $D = \mathbb{R}$; $R = [0, \infty)$ (c) Decreasing on $(-\infty, -3]$, increasing on $[-3, \infty)$

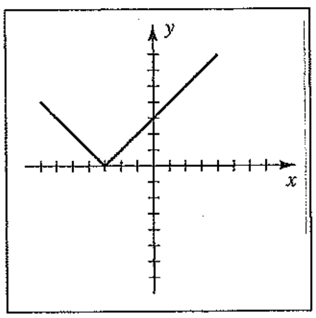

Figure 45

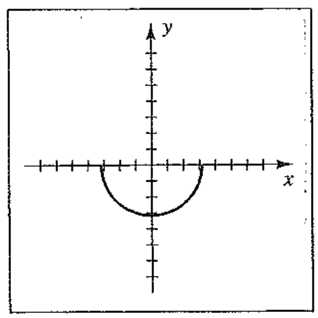

Figure 46

[46] (b) $D = [-\sqrt{10}, \sqrt{10}]$; $R = [-\sqrt{10}, 0]$

(c) Decreasing on $[-\sqrt{10}, 0]$, increasing on $[0, \sqrt{10}]$

[47] (b) $D = [-1, \infty)$; $R = (-\infty, 1]$ (c) Decreasing on $[-1, \infty)$

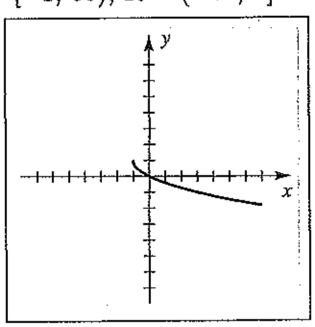

Figure 47

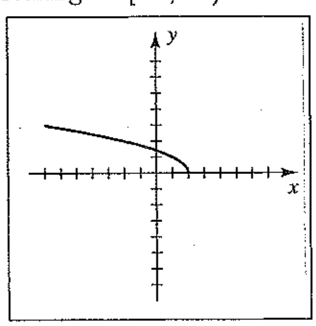

Figure 48

[48] (b) $D = (-\infty, 2]$; $R = [0, \infty)$ (c) Decreasing on $(-\infty, 2]$

$\boxed{49}$ (b) $D = \mathbb{R}$; $R = (-\infty, 9]$ (c) Increasing on $(-\infty, 0]$, decreasing on $[0, \infty)$

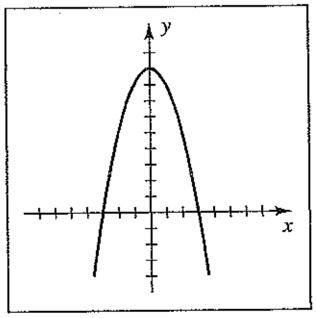

Figure 49

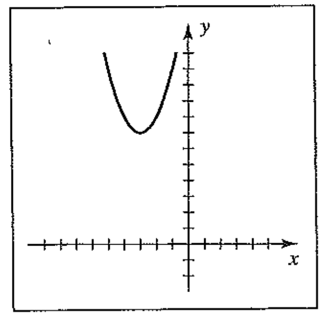

Figure 50

$\boxed{50}$ $f(x) = x^2 + 6x + 16 = x^2 + 6x + 9 + 7 = (x+3)^2 + 7$. (b) $D = \mathbb{R}$; $R = [7, \infty)$

(c) Decreasing on $(-\infty, -3]$, increasing on $[-3, \infty)$

$\boxed{51}$ (b) $D = \mathbb{R}$; $R = [0, \infty)$

(c) Decreasing on $(-\infty, 0]$, increasing on $[0, 2]$, constant on $[2, \infty)$

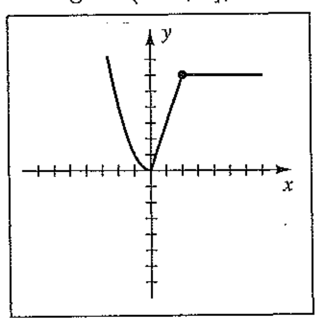

Figure 51

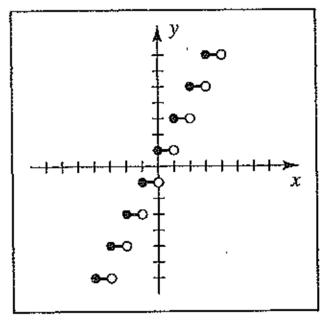

Figure 52

$\boxed{52}$ (b) $D = \mathbb{R}$; $R = \{\ldots, -3, -1, 1, 3, \ldots\}$

(c) Constant on $[n, n+1)$, where n is any integer

$\boxed{53}$ (a) $y = \sqrt{x}$ (b) $y = \sqrt{x+4}$ (c) $y = \sqrt{x} + 4$

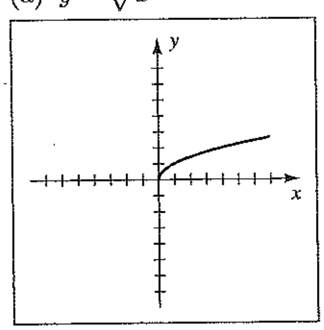

Figure 53(a)

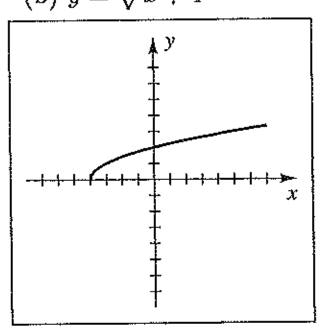

Figure 53(b)

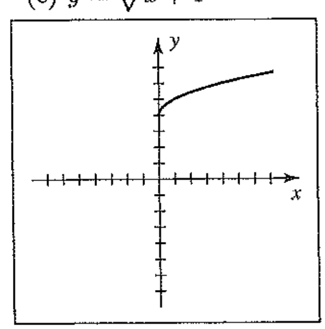

Figure 53(c)

(d) $y = 4\sqrt{x}$

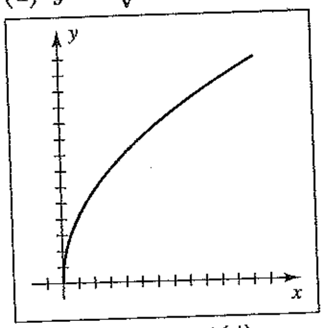

Figure 53(d)

(e) $y = \frac{1}{4}\sqrt{x}$

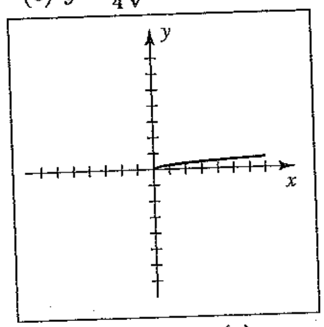

Figure 53(e)

(f) $y = -\sqrt{x}$

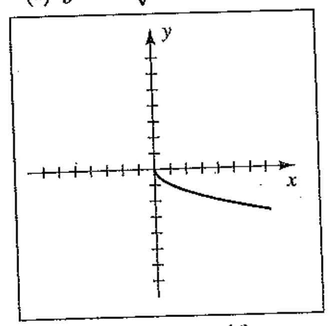

Figure 53(f)

54 (a) $y = f(x-2)$ • shift f right 2 units

(b) $y = f(x) - 2$ • shift f down 2 units

(c) $y = f(-x)$ • reflect f through the y-axis

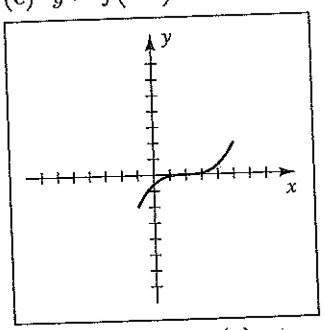

Figure 54(a)

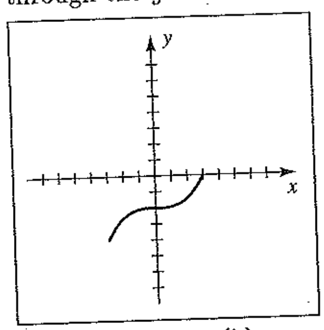

Figure 54(b)

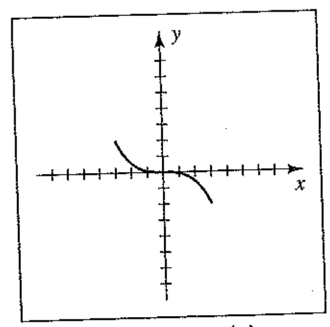

Figure 54(c)

(d) $y = f(2x)$ • horizontally compress f by a factor of 2

(e) $y = f(\frac{1}{2}x)$ • horizontally stretch f by a factor of $1/(1/2) = 2$

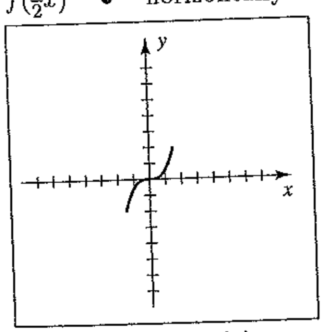

Figure 54(d)

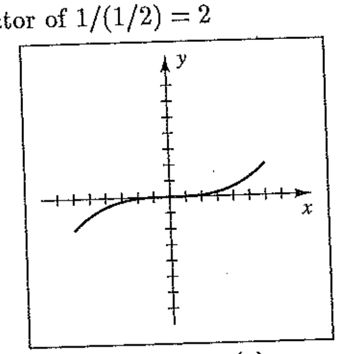

Figure 54(e)

(f) $y = |f(x)|$ • reflect the portion of the graph below the x-axis through the x-axis.

(g) $y = f(|x|)$ • include the reflection of all points with positive x-coordinates through the y-axis—results in the same graph as in part (f).

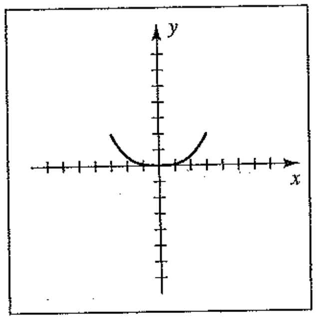

Figure 54(f)

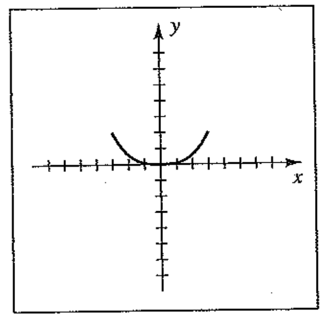

Figure 54(g)

[55] Using the intercept form, an equation is $\frac{x}{5} + \frac{y}{-2} = 1$, or, equivalently, $2x - 5y = 10$.

[56] The midpoint of $(-7, 1)$ and $(3, 1)$ is $(-2, 1)$. This point is 5 units from either of the given points. An equation is $(x+2)^2 + (y-1)^2 = 5^2 = 25$.

[57] $V(2, -4)$ and $P(-2, 4)$ with $y = a(x-h)^2 + k \;\Rightarrow\; 4 = a(-2-2)^2 - 4 \;\Rightarrow\; 8 = 16a \;\Rightarrow\; a = \frac{1}{2}$. An equation is $y = \frac{1}{2}(x-2)^2 - 4$.

[58] The graph could be made by taking $y = |x|$ and reflecting it through the x-axis, and then shifting that graph to the right 2 units and down 1 unit. $y = -|x-2| - 1$

[59] $f(x) = 5x^2 + 30x + 49 \;\Rightarrow\; -\frac{b}{2a} = -\frac{30}{2(5)} = -3$. $f(-3) = 4$ is a minimum since $a > 0$.

[60] $f(x) = -3x^2 + 30x - 82 \;\Rightarrow\; -\frac{b}{2a} = -\frac{30}{2(-3)} = 5$. $f(5) = -7$ is a maximum since $a < 0$.

[61] $f(x) = -12(x+1)^2 - 37$ is in the standard form. $f(-1) = -37$ is a maximum.

[62] $f(x) = 3(x+2)(x-10)$ has x-intercepts at -2 and 10. The vertex is halfway between them at $x = 4$. $f(4) = 3 \cdot 6 \cdot (-6) = -108$ is a minimum.

[63] $f(x) = -2x^2 + 12x - 14 = -2(x^2 - 6x + \underline{\;9\;}) - 14 + 18 = -2(x-3)^2 + 4$.

[64] $V(3, -2) \;\Rightarrow\; (h, k) = (3, -2)$ in $y = a(x-h)^2 + k$.
$x = 5,\ y = 4 \;\Rightarrow\; 4 = a(5-3)^2 - 2 \;\Rightarrow\; 6 = 4a \;\Rightarrow\; a = \frac{3}{2}$. Hence, $y = \frac{3}{2}(x-3)^2 - 2$.

[65] The domain of $f(x) = \sqrt{4 - x^2}$ is $[-2, 2]$. The domain of $g(x) = \sqrt{x}$ is $[0, \infty)$.

(a) The domain of fg is the intersection of those two domains, $[0, 2]$.

(b) The domain of f/g is the same as that of fg, excluding any values that make g equal to 0. Thus, the domain of f/g is $(0, 2]$.

[66] (a) $f(x) = 8x - 1$ and $g(x) = \sqrt{x-2} \;\Rightarrow\; (f \circ g)(2) = f(g(2)) = f(0) = -1$

(b) $(g \circ f)(2) = g(f(2)) = g(15) = \sqrt{13}$

$\boxed{67}$ (a) $(f \circ g)(x) = f(g(x)) = 2(3x+2)^2 - 5(3x+2) + 1 = 18x^2 + 9x - 1$

(b) $(g \circ f)(x) = g(f(x)) = 3(2x^2 - 5x + 1) + 2 = 6x^2 - 15x + 5$

$\boxed{68}$ (a) $(f \circ g)(x) = f(g(x)) = \sqrt{3\left(\frac{1}{x^2}\right) + 2} = \sqrt{\frac{3 + 2x^2}{x^2}}$

(b) $(g \circ f)(x) = g(f(x)) = \frac{1}{(\sqrt{3x+2})^2} = \frac{1}{3x+2}$

$\boxed{69}$ (a) $h(x) = f(\sqrt{x-3}) = \sqrt{25 - (\sqrt{x-3})^2} = \sqrt{25 - (x-3)} = \sqrt{28 - x}$.

Domain of $g = [3, \infty)$. Domain of $f = [-5, 5]$.

$g(x) \leq 5$ { $g(x)$ cannot be less than 0 } $\Rightarrow$ $\sqrt{x-3} \leq 5$ $\Rightarrow$

$x - 3 \leq 25$ $\Rightarrow$ $x \leq 28$. $[3, \infty) \cap (-\infty, 28] = [3, 28]$

(b) $k(x) = g(\sqrt{25 - x^2}) = \sqrt{\sqrt{25 - x^2} - 3}$.

Domain of $f = [-5, 5]$. Domain of $g = [3, \infty)$.

$f(x) \geq 3$ $\Rightarrow$ $\sqrt{25 - x^2} \geq 3$ $\Rightarrow$ $25 - x^2 \geq 9$ $\Rightarrow$ $x^2 \leq 16$ $\Rightarrow$ $x \in [-4, 4]$.

$\boxed{70}$ (a) $h(x) = f\left(\frac{2}{x}\right) = \frac{2/x}{3(2/x) + 2} \cdot \frac{x}{x} = \frac{2}{6 + 2x} = \frac{1}{x+3}$. Domain of $g = \mathbb{R} - \{0\}$.

Domain of $f = \mathbb{R} - \{-\frac{2}{3}\}$. $g(x) \neq -\frac{2}{3}$ $\Rightarrow$ $\frac{2}{x} \neq -\frac{2}{3}$ $\Rightarrow$ $x \neq -3$.

Hence, the domain of $f \circ g$ is $\mathbb{R} - \{-3, 0\}$.

(b) $k(x) = g\left(\frac{x}{3x+2}\right) = \frac{2}{x/(3x+2)} = \frac{6x+4}{x}$.

Domain of $f = \mathbb{R} - \{-\frac{2}{3}\}$. Domain of $g = \mathbb{R} - \{0\}$.

$f(x) \neq 0$ $\Rightarrow$ $\frac{x}{3x+2} \neq 0$ $\Rightarrow$ $x \neq 0$. Hence, the domain of $g \circ f$ is $\mathbb{R} - \{-\frac{2}{3}, 0\}$.

$\boxed{71}$ For $y = \sqrt[3]{x^2 - 5x}$, choose $u = x^2 - 5x$ and $y = \sqrt[3]{u}$.

$\boxed{72}$ The slope of the ramp should be between $\frac{1}{12}$ and $\frac{1}{20}$. If the rise of the ramp is 3 feet, then the run should be between $3 \times 12 = 36$ ft and $3 \times 20 = 60$ ft. The range of the ramp lengths should be from $L = \sqrt{3^2 + 36^2} \approx 36.1$ ft to $L = \sqrt{3^2 + 60^2} \approx 60.1$ ft.

$\boxed{73}$ (a) 2008 corresponds to $t = 2008 - 1948 = 60$. $d = 181 + 1.065(60) = 244.9 \approx 245$ ft

(b) $d = 265$ $\Rightarrow$ $265 = 181 + 1.065t$ $\Rightarrow$ $t = \frac{265 - 181}{1.065} \approx 78.9$ yr,

which corresponds to $79 + 1948 = 2027$ or Olympic year 2028.

$\boxed{74}$ (a) $V = at + b$ is the desired form. $V = 89{,}000$ when $t = 0$ $\Rightarrow$ $V = at + 89{,}000$.

$V = 125{,}000$ when $t = 6$ $\Rightarrow$ $125{,}000 = 6a + 89{,}000$ $\Rightarrow$ $a = \frac{36{,}000}{6} = 6000$ and

hence, $V = 6000t + 89{,}000$.

(b) $V = 103{,}000$ $\Rightarrow$ $103{,}000 = 6000t + 89{,}000$ $\Rightarrow$ $t = \frac{7}{3}$, or $2\frac{1}{3}$.

$\boxed{75}$ (a) $F = aC + b$ is the desired form. $F = 32$ when $C = 0$ $\Rightarrow$ $F = aC + 32$. $F = 212$

when $C = 100$ $\Rightarrow$ $212 = 100a + 32$ $\Rightarrow$ $a = \frac{180}{100} = \frac{9}{5}$ and hence, $F = \frac{9}{5}C + 32$.

(b) If C increases 1°, F increases $(\frac{9}{5})^\circ$, or 1.8°.

[76] (a) $C_1(x) = \left(1.25 \dfrac{\text{dollars}}{\text{gallon}}\right) \div \left(20 \dfrac{\text{miles}}{\text{gallon}}\right) \cdot x \text{ miles} = \dfrac{1.25}{20}x = 0.0625x$, or $\dfrac{1}{16}x$.

(b) After the tune-up, the gasoline mileage will be 10% more than 20 mi/gal; that is, 22 mi/gal. $C_2(x) = \frac{1.25}{22}x + 50 = \frac{5}{88}x + 50 \approx 0.0568x + 50$.

(c) $C_2 < C_1 \Rightarrow \left[\frac{5}{88}x + 50 < \frac{1}{16}x\right] \cdot 16(11) \Rightarrow 10x + 8800 < 11x \Rightarrow$ $x > 8800$ miles.

[77] Surface area $S = 2(4)(x) + (4)(y) = 8x + 4y$. Cost $C = 2(8x) + 5(4y) = 16x + 20y$.

(a) $C = 400 \Rightarrow 16x + 20y = 400 \Rightarrow 20y = -16x + 400 \Rightarrow y = -\frac{4}{5}x + 20$

(b) $V = lwh = (y)(4)(x) = 4xy = 4x(-\frac{4}{5}x + 20)$

[78] $V = \pi r^2 h$ and $V = 24\pi \Rightarrow h = \dfrac{24}{r^2}$. $S = \pi r^2 + 2\pi rh = \pi r^2 + 2\pi r \cdot \dfrac{24}{r^2} = \pi r^2 + \frac{48\pi}{r}$.

$$C = (0.30)(\pi r^2) + (0.10)(\tfrac{48\pi}{r}) = \frac{3\pi r^2}{10} + \frac{48\pi}{10r} = \frac{3\pi(r^3 + 16)}{10r}.$$

[79] (a) $V = (10 \text{ ft}^3 \text{ per minute})(t \text{ minutes}) = 10t$

(b) The height and length of the bottom triangular region are in the proportion 6–60, or 1–10, and the length is 10 times the height. When $0 \le h \le 6$, the volume is $V = (\text{cross sectional area})(\text{pool width}) = \frac{1}{2}bh(40) = \frac{1}{2}(10h)(h)(40) = 200h^2 \text{ ft}^3$. When $6 < h \le 9$, the triangular region is full and

$$V = 200(6)^2 + (h - 6)(80)(40) = 7200 + 3200(h - 6).$$

(c) $10t = 200h^2 \Rightarrow h = \sqrt{t/20}$;

$$0 \le h \le 6 \Rightarrow 0 \le \sqrt{t/20} \le 6 \Rightarrow 0 \le t/20 \le 36 \Rightarrow 0 \le t \le 720.$$

$10t = 7200 + 3200(h - 6) \Rightarrow h - 6 = \dfrac{t - 720}{320} \Rightarrow h = 6 + \dfrac{t - 720}{320}$;

$6 < h \le 9 \Rightarrow 6 < 6 + \dfrac{t - 720}{320} \le 9 \Rightarrow 0 < \dfrac{t - 720}{320} \le 3 \Rightarrow$

$$0 < t - 720 \le 960 \Rightarrow 720 < t \le 1680.$$

[80] (a) Using similar triangles, $\frac{r}{x} = \frac{2}{4} \Rightarrow r = \frac{1}{2}x$.

(b) $\text{Volume}_{\text{cone}} + \text{Volume}_{\text{cup}} = \text{Volume}_{\text{total}} \Rightarrow \frac{1}{3}\pi r^2 h + \pi r^2 h = 5 \Rightarrow$

$$\tfrac{1}{3}\pi(\tfrac{1}{2}x)^2(x) + \pi(2)^2(y) = 5 \Rightarrow 5 - \tfrac{\pi}{12}x^3 = 4\pi y \Rightarrow y = \tfrac{5}{4\pi} - \tfrac{1}{48}x^3$$

[81] (a) $\dfrac{y}{b} = \dfrac{y + h}{a} \Rightarrow ay = by + bh \Rightarrow y(a - b) = bh \Rightarrow y = \dfrac{bh}{a - b}$

(b) $V = \frac{1}{3}\pi a^2(y + h) - \frac{1}{3}\pi b^2 y = \frac{\pi}{3}\left[(a^2 - b^2)y + a^2 h\right] =$

$$\tfrac{\pi}{3}\left[(a^2 - b^2)\frac{bh}{a - b} + a^2 h\right] = \tfrac{\pi}{3}h\left[(a + b)b + a^2\right] = \tfrac{\pi}{3}h(a^2 + ab + b^2)$$

(c) $a = 6,\ b = 3,\ V = 600 \Rightarrow \frac{\pi}{3}h(6^2 + 6 \cdot 3 + 3^2) = 600 \Rightarrow h = \frac{1800}{63\pi} = \frac{200}{7\pi} \approx 9.1 \text{ ft}$

[82] Let t denote the time (in hr) after 1:00 P.M. If the starting point for ship B is the origin, then the locations of A and B are $-30+15t$ and $-10t$, respectively. Using the Pythagorean theorem, $d^2 = (-30+15t)^2 + (-10t)^2 = 325t^2 - 900t + 900$. The time at which the distance between the ships is minimal is the same as the time at which the square of the distance between the ships is minimal.

Thus, $t = -\frac{b}{2a} = -\frac{-900}{2(325)} = \frac{18}{13}$, or about 2:23 P.M.

[83] Let r denote the radius of the semicircles and x the length of the rectangle.

Perimeter = half-mile $\Rightarrow$ $2x + 2\pi r = \frac{1}{2}$ $\Rightarrow$ $x = -\pi r + \frac{1}{4}$.

$A = 2rx = 2r(-\pi r + \frac{1}{4}) = -2\pi r^2 + \frac{1}{2}r$. The maximum value of A occurs when

$r = -\frac{b}{2a} = -\frac{1/2}{2(-2\pi)} = \frac{1}{8\pi}$ mi. $x = -\pi\left(\frac{1}{8\pi}\right) + \frac{1}{4} = \frac{1}{8}$ mi.

[84] (a) $g = 32$ $\Rightarrow$ $f(t) = -16t^2 + 16t$. Solving $f(t) = 0$ gives us $-16t(t-1)$ $\Rightarrow$

$t = 0, 1$. The player is in the air for 1 second.

(b) $t = -\frac{b}{2a} = -\frac{16}{2(-16)} = \frac{1}{2}$. $f(\frac{1}{2}) = 4$ $\Rightarrow$ the player jumps 4 feet high.

(c) $g = \frac{32}{6}$ $\Rightarrow$ $f(t) = -\frac{8}{3}t^2 + 16t$. Solving $f(t) = 0$ yields $t = 0$ or 6.

The player would be in the air for 6 seconds on the moon.

$t = -\frac{b}{2a} = -\frac{16}{2(-8/3)} = 3$. $f(3) = 24$ $\Rightarrow$ the player jumps 24 feet high.

[85] (a) Solving $-0.016x^2 + 1.6x = \frac{1}{5}x$ for x represents the intersection between the parabola and the line. $-0.08x^2 + 8x = x$ $\Rightarrow$ $7x - \frac{8}{100}x^2 = 0$ $\Rightarrow$

$x(7 - \frac{8}{100}x) = 0$ $\Rightarrow$ $x = 0, \frac{175}{2}$. The rocket lands at $(\frac{175}{2}, \frac{35}{2}) = (87.5, 17.5)$.

(b) The *difference* d between the parabola and the line is to be maximized here.

$d = (-0.016x^2 + 1.6x) - (\frac{1}{5}x) = -0.016x^2 + 1.4x$. d obtains a maximum when

$x = -\frac{b}{2a} = -\frac{1.4}{2(-0.016)} = 43.75$. The maximum height of the rocket

above the ground is $d = -0.016(43.75)^2 + 1.4(43.75) = 30.625$ units.

Chapter 3 Discussion Exercises

[1] Graphs of equations of the form $y = x^{p/q}$, where $x \geq 0$, and p and q are positive integers all pass through (0, 0) and (1, 1). If $p/q < 1$, the graph is above $y = x$ for $0 \leq x \leq 1$ and below $y = x$ for $x \geq 1$. The closer p/q is to 1, the closer $y = x^{p/q}$ is to $y = x$. If $p/q > 1$, the graph is below $y = x$ for $0 \leq x \leq 1$ and above $y = x$ for $x \geq 1$.

[2] (a) About the x-axis • replace y with $-y$: $-y = \frac{1}{2}x - 3 \Rightarrow g(x) = -\frac{1}{2}x + 3$

(b) About the y-axis • replace x with $-x$: $y = \frac{1}{2}(-x) - 3 \Rightarrow g(x) = -\frac{1}{2}x - 3$

(c) About the line $y = 2$ • by examining the graphs of $f(x) = \frac{1}{2}x - 3$ and $y = 2$, we observe that the slope of the reflected line should be $-\frac{1}{2}$ and the y-intercept should be 5 units above $y = 2$ (since the y-intercept of f is 5 units below 2). Hence, $g(x) = -\frac{1}{2}x + 7$.

(d) About the line $x = 3$ • similar to part (c), the slope of g is $-\frac{1}{2}$. The x-intercept of f, 6, is 3 units to the right of $x = 3$, so the x-intercept of g should be 3 units to the left of $x = 3$. Hence, $g(x) = -\frac{1}{2}x$.

Note: An interesting generalization can be made for problems of the form of those in parts (a)–(d) which would be a worthwhile exploratory exercise in itself. It goes as follows: If the graph of $y = f(x)$ is reflected about the line $y = k$ (or $x = k$), how can you obtain the equation of the new graph? Answer: For $y = k$, replace y with $2k - y$; for $x = k$, replace x with $2k - x$.

[3] For the graph of $g(x) = \sqrt{f(x)}$, where $f(x) = ax^2 + bx + c$, consider 2 cases:

(1) $\mathbf{(a > 0)}$ If f has 0 or 1 x-intercept(s), the domain of g is $\mathbb{R}$ and its range is $[\sqrt{k}, \infty)$, where k is the y-value of the vertex of f. If f has 2 x-intercepts (say x_1 and x_2 with $x_1 < x_2$), then the domain of g is $(-\infty, x_1] \cup [x_2, \infty)$ and its range is $[0, \infty)$. The general shape is similar to the v-shape of the graph of $y = \sqrt{a}\,|\,x\,|$.

(2) $\mathbf{(a < 0)}$ If f has no x-intercepts, there is no graph of g. If f has 1 x-intercept, the graph of g consists of that point. If f has 2 x-intercepts, the domain of g is $[x_1, x_2]$ and the range is $[0, \sqrt{k}]$. The shape of g is that of the top half of an oval.

The main advantage of graphing g as a composition (say $\mathrm{Y}_1 = f$ and $\mathrm{Y}_2 = \sqrt{\mathrm{Y}_1}$ on a graphing calculator) is to observe the relationship between the range of f and the domain of g.

[4] $f(x) = ax^2 + bx + c \Rightarrow$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left[a(x+h)^2 + b(x+h) + c\right] - \left[ax^2 + bx + c\right]}{h}$$

$$= \frac{ax^2 + 2ahx + ah^2 + bx + bh + c - ax^2 - bx - c}{h}$$

$$= \frac{2ahx + ah^2 + bh}{h} = \frac{h(2ax + ah + b)}{h} = 2ax + ah + b$$

5 The expression $2x+h+6$ represents the slope of the line between the points $P(x,\ f(x))$ and $Q(x+h,\ f(x+h))$. If $h=0$, then $2x+6$ represents the slope of the tangent line at the point $P(x,\ f(x))$.

6 To determine the x-coordinate of R,
we want to start at x_1 and go $\frac{m}{n}$ of the way to x_2. We could write this as

$$x_3 = x_1 + \tfrac{m}{n}\Delta x = x_1 + \tfrac{m}{n}(x_2 - x_1) = x_1 + \tfrac{m}{n}x_2 - \tfrac{m}{n}x_1 = \left(1 - \tfrac{m}{n}\right)x_1 + \tfrac{m}{n}x_2.$$

Similarly, $y_3 = \left(1 - \frac{m}{n}\right)y_1 + \frac{m}{n}y_2$.

7 The values of the x-intercepts (if they exist) are found by using the quadratic formula $\left(x = -\dfrac{b}{2a} \pm \dfrac{\sqrt{b^2-4ac}}{2a}\right)$. Hence, the distance d from the axis of symmetry, $x = -\dfrac{b}{2a}$, to either x-intercept is $d = \dfrac{\sqrt{b^2-4ac}}{2\,|\,a\,|}$ and $d^2 = \dfrac{b^2-4ac}{4a^2}$. From page 217, the y-coordinate of the vertex is $h = c - \dfrac{b^2}{4a} = \dfrac{4ac-b^2}{4a}$, so

$$\frac{h}{d^2} = \frac{\dfrac{4ac-b^2}{4a}}{\dfrac{b^2-4ac}{4a^2}} = -\frac{4a^2(4ac-b^2)}{4a(b^2-4ac)} = -a.$$

Thus, $h = -ad^2$. Note that this relationship also reveals a connection between the discriminant D and the y-coordinate of the vertex, namely $h = -D^2/(4a)$.

8 The graph from Exercise 54(e) of Section 3.5 ($y = -[\![-x]\!]$) illustrates the concept of one of the most common billing methods with the open and closed endpoints reversed from those of the greatest integer function. Starting with $y = -[\![-x]\!]$ and adjusting for jumps every 15 minutes gives us $y = -[\![-x/15]\!]$. Since each quarter-hour charge is \$20, we multiply by 20 to obtain $y = -20[\![-x/15]\!]$. Because of the initial \$40 charge, we must add 40 to obtain the function $f(x) = 40 - 20[\![-x/15]\!]$.

9 $D = 0.0833x^2 - 0.4996x + 3.5491 \;\Rightarrow\; 0.0833x^2 - 0.4996x + (3.5491 - D) = 0.$

Solving for x with the quadratic formula yields

$$x = \frac{0.4996 \pm \sqrt{(-0.4996)^2 - 4(0.0833)(3.5491 - D)}}{2(0.0833)}, \text{ or, equivalently,}$$

$$x = \frac{4996 \pm \sqrt{33{,}320{,}000D - 93{,}295{,}996}}{1666}.$$

From *Figure 9* (a graph of D), we see that $3 \le x \le 15$ corresponds to the right half of the parabola. Hence, we choose the plus sign in the equation for x.

[−20, 20, 2] by [0, 40, 2]

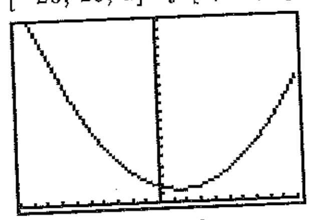

Figure 9

10 (a) Let January correspond to 1, February to 2, ... , and December to 12.

[0.5, 12.5] by [0, 5]

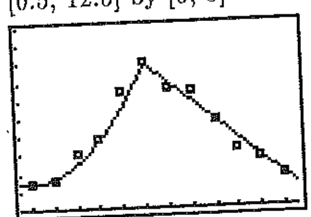

Figure 10

(b) The data points are (approximately) parabolic on the interval [1, 6] and linear on [6, 12]. Let $f_1(x) = a(x - h)^2 + k$ on [1, 6] and $f_2(x) = mx + b$ on [6, 12]. On [1, 6], let the vertex $(h, k) = (1, 0.7)$. Since (6, 4) is on the graph of f_1, $f_1(6) = a(6 - 1)^2 + 0.7 = 4 \;\Rightarrow\; a = 0.132$. Thus, $f_1(x) = 0.132(x - 1)^2 + 0.7$ on [1, 6]. Now, let $f_2(x) = mx + b$ pass through the points (6, 4) and (12, 0.9). An equation of this line is approximately $(y - 4) = -0.517(x - 6)$. Thus, let $f_2(x) = -0.517x + 7.102$ on [6, 12].

$$f(x) = \begin{cases} 0.132(x-1)^2 + 0.7 & \text{if } 1 \le x \le 6 \\ -0.517x + 7.102 & \text{if } 6 < x \le 12 \end{cases}$$

(c) To plot the piecewise function, let $Y_1 = (0.132(x - 1)\hat{}2 + 0.7)/(x \le 6)$ and $Y_2 = (-0.517x + 7.102)/(x > 6)$. These assignments use the concept of Boolean division. For example, when $(x \le 6)$ is false, the expression Y_1 will be undefined (division by 0) and the calculator will not plot any values.