Chapter 4: Polynomial and Rational Functions

4.1 Exercises

$\boxed{1}$ $f(x) = 2x^3 + c$ (a) $c = 3$ (b) $c = -3$ •

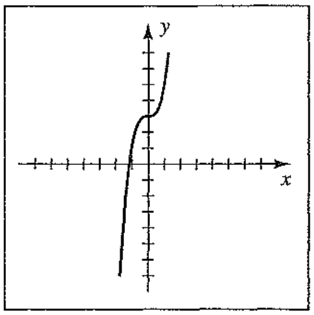

Figure 1(a)

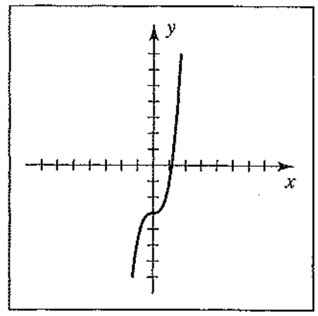

Figure 1(b)

$\boxed{2}$ $f(x) = -2x^3 + c$ (a) $c = -2$ (b) $c = 2$ •

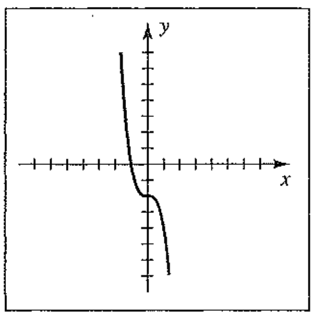

Figure 2(a)

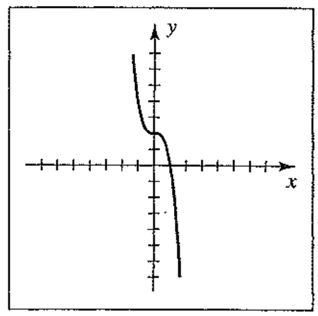

Figure 2(b)

$\boxed{3}$ $f(x) = ax^3 + 2$ (a) $a = 2$ (b) $a = -\frac{1}{3}$ •

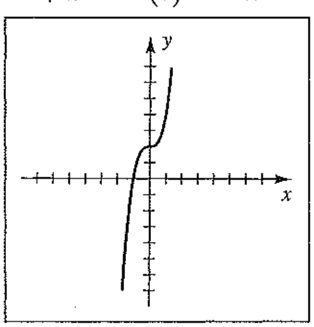

Figure 3(a)

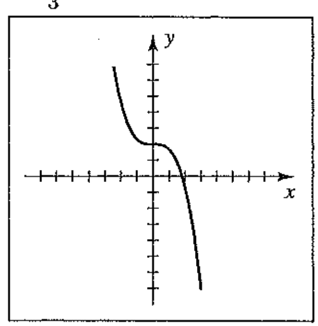

Figure 3(b)

$\boxed{4}$ $f(x) = ax^3 - 3$ (a) $a = -2$ (b) $a = \frac{1}{4}$

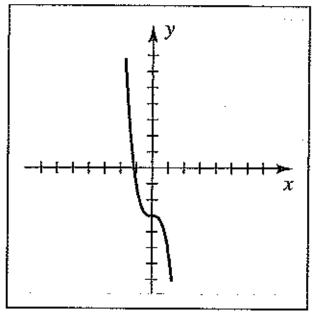

Figure 4(a)

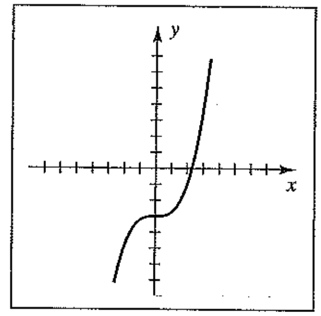

Figure 4(b)

$\boxed{5}$ $f(x) = x^3 - 4x^2 + 3x - 2$; $a = 3$, $b = 4$; $f(3) = -2 < 0$, $f(4) = 10 > 0$

By the intermediate value theorem for polynomial functions,

f takes on every value between -2 and 10 in the interval [3, 4], namely, 0.

$\boxed{6}$ $f(x) = 2x^3 + 5x^2 - 3$; $a = -3$, $b = -2$; $f(-3) = -12 < 0$, $f(-2) = 1 > 0$

$\boxed{7}$ $f(x) = -x^4 + 3x^3 - 2x + 1$; $a = 2$, $b = 3$; $f(2) = 5 > 0$, $f(3) = -5 < 0$

$\boxed{8}$ $f(x) = 2x^4 + 3x - 2$; $a = \frac{1}{2}$, $b = \frac{3}{4}$; $f(\frac{1}{2}) = -\frac{3}{8} < 0$, $f(\frac{3}{4}) = \frac{113}{128} > 0$

$\boxed{9}$ $f(x) = x^5 + x^3 + x^2 + x + 1$; $a = -\frac{1}{2}$, $b = -1$; $f(-\frac{1}{2}) = \frac{19}{32} > 0$, $f(-1) = -1 < 0$

$\boxed{10}$ $f(x) = x^5 - 3x^4 - 2x^3 + 3x^2 - 9x - 6$; $a = 3$, $b = 4$; $f(3) = -60 < 0$, $f(4) = 134 > 0$

$\boxed{11}$ (a) The graph has x-intercepts at -1, 1, and 2—so $(x+1)$, $(x-1)$, and $(x-2)$ are factors in its equation. It is the graph of a cubic. **C**

(b) The graph has x-intercepts at -1, 1, and 2. It is the graph of a quartic. **D**

(c) The graph has x-intercepts at 0 and 2. It is the graph of a cubic with a negative leading coefficient. **B**

(d) The graph has x-intercepts at 0 and 2. It is the graph of a cubic with a positive leading coefficient. **A**

$\boxed{12}$ (a) The graph has x-intercepts at -2, -1, and 1—so $(x+2)$, $(x+1)$, and $(x-1)$ are factors in its equation. It is the graph of a quartic. **D**

(b) The graph has x-intercepts at 0 and 1. It is the graph of a cubic. **A**

(c) The graph has x-intercepts at -2 and 0. It is the graph of a cubic with a negative leading coefficient. **B**

(d) The graph has x-intercepts at -2, -1, and 3. It is the graph of a cubic with a positive leading coefficient. **C**

$\boxed{13}$ $f(x) = \frac{1}{4}x^3 - 2 = \frac{1}{4}(x^3 - 8) = \frac{1}{4}(x-2)(x^2+2x+4)$; $f(x) > 0$ if $x > 2$, $f(x) < 0$ if $x < 2$

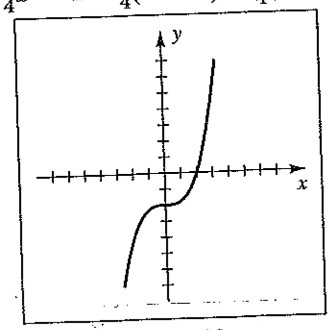

Figure 13

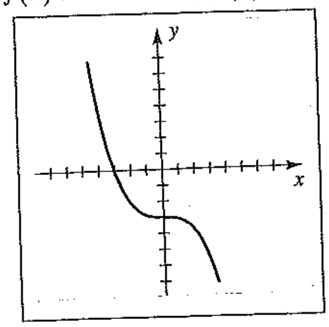

Figure 14

$\boxed{14}$ $f(x) = -\frac{1}{9}x^3 - 3 = -\frac{1}{9}(x^3 + 27) = -\frac{1}{9}(x+3)(x^2-3x+9)$;

$f(x) > 0$ if $x < -3$, $f(x) < 0$ if $x > -3$

$\boxed{15}$ $f(x) = -\frac{1}{16}x^4 + 1 = -\frac{1}{16}(x^4 - 16) = -\frac{1}{16}(x^2+4)(x+2)(x-2)$;

$f(x) > 0$ if $|x| < 2$, $f(x) < 0$ if $|x| > 2$

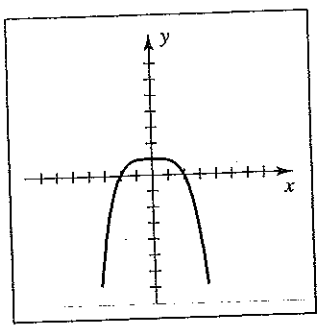

Figure 15

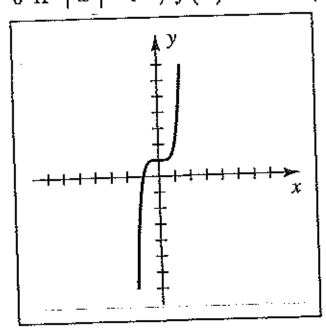

Figure 16

$\boxed{16}$ $f(x) = x^5 + 1 = (x+1)(x^4 - x^3 + x^2 - x + 1)$; $f(x) > 0$ if $x > -1$, $f(x) < 0$ if $x < -1$

$\boxed{17}$ $f(x) = x^4 - 4x^2 = x^2(x+2)(x-2)$; $f(x) > 0$ if $|x| > 2$, $f(x) < 0$ if $0 < |x| < 2$

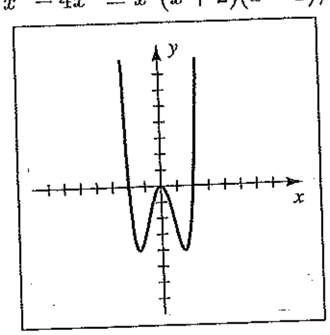

Figure 17

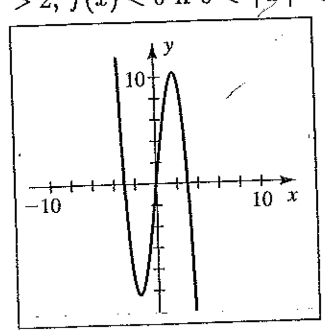

Figure 18

$\boxed{18}$ $f(x) = 9x - x^3 = x(3+x)(3-x)$;

$f(x) > 0$ if $x < -3$ or $0 < x < 3$, $f(x) < 0$ if $-3 < x < 0$ or $x > 3$

[19] $f(x) = -x^3 + 3x^2 + 10x = -x(x^2 - 3x - 10) = -x(x+2)(x-5)$;

$f(x) > 0$ if $x < -2$ or $0 < x < 5$, $f(x) < 0$ if $-2 < x < 0$ or $x > 5$

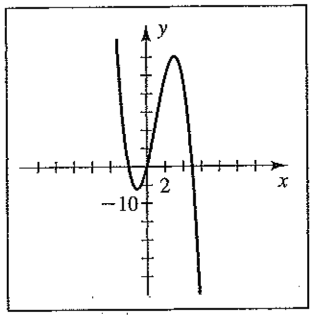

Figure 19

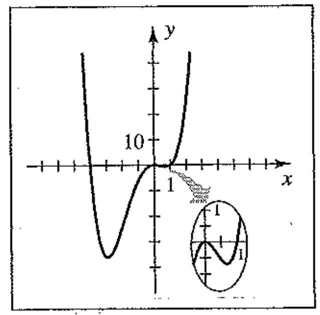

Figure 20

[20] $f(x) = x^4 + 3x^3 - 4x^2 = x^2(x^2 + 3x - 4) = x^2(x+4)(x-1)$;

$f(x) > 0$ if $x < -4$ or $x > 1$, $f(x) < 0$ if $-4 < x < 0$ or $0 < x < 1$

[21] $f(x) = \frac{1}{6}(x+2)(x-3)(x-4)$;

$f(x) > 0$ if $-2 < x < 3$ or $x > 4$, $f(x) < 0$ if $x < -2$ or $3 < x < 4$

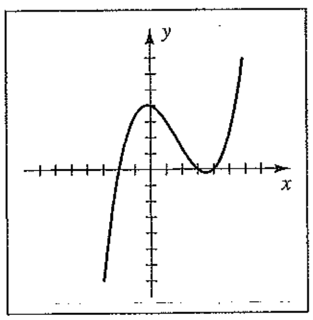

Figure 21

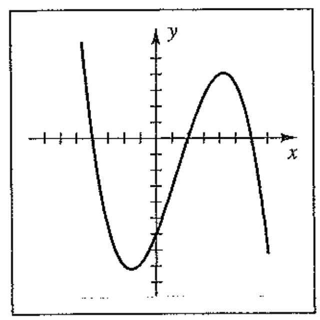

Figure 22

[22] $f(x) = -\frac{1}{8}(x+4)(x-2)(x-6)$;

$f(x) > 0$ if $x < -4$ or $2 < x < 6$, $f(x) < 0$ if $-4 < x < 2$ or $x > 6$

[23] $f(x) = x^3 + 2x^2 - 4x - 8 = x^2(x+2) - 4(x+2) = (x+2)^2(x-2)$;

$f(x) > 0$ if $x > 2$, $f(x) < 0$ if $x < -2$ or $|x| < 2$

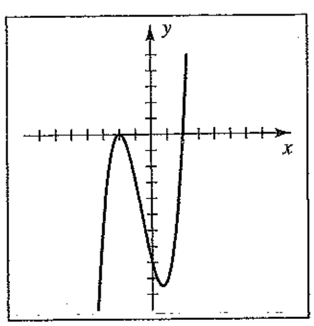

Figure 23

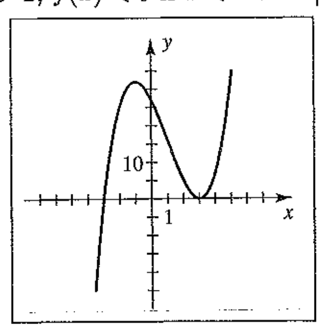

Figure 24

[24] $f(x) = x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3) = (x-3)^2(x+3)$;

$f(x) > 0$ if $|x| < 3$ or $x > 3$, $f(x) < 0$ if $x < -3$

$\boxed{25}$ $f(x) = x^4 - 6x^2 + 8 = (x^2 - 2)(x + 2)(x - 2)$;

$f(x) > 0$ if $|x| > 2$ or $|x| < \sqrt{2}$, $f(x) < 0$ if $\sqrt{2} < |x| < 2$

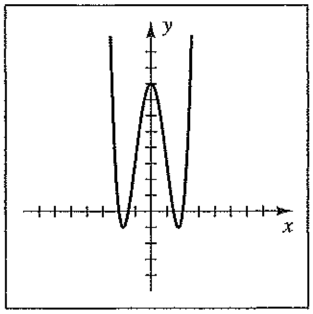

Figure 25

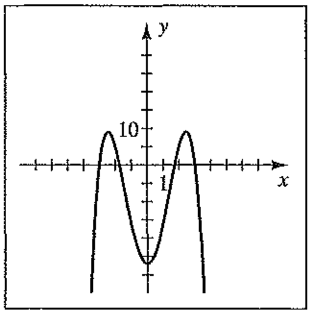

Figure 26

$\boxed{26}$ $f(x) = -x^4 + 12x^2 - 27 = (-x^2 + 3)(x^2 - 9) = -(x^2 - 3)(x + 3)(x - 3)$;

$f(x) > 0$ if $\sqrt{3} < |x| < 3$, $f(x) < 0$ if $|x| > 3$ or $|x| < \sqrt{3}$

$\boxed{27}$ $f(x) = x^2(x + 2)(x - 1)^2(x - 2)$;

$f(x) > 0$ if $|x| > 2$, $f(x) < 0$ if $|x| < 2$, $x \neq 0$, $x \neq 1$

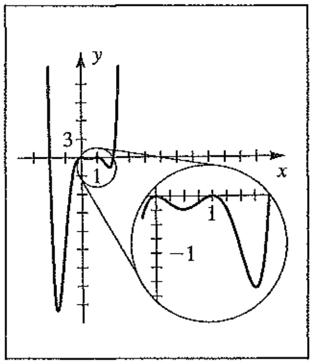

Figure 27

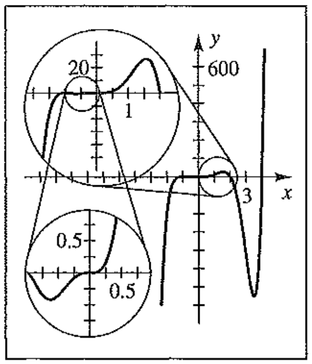

Figure 28

$\boxed{28}$ $f(x) = x^3(x + 1)^2(x - 2)(x - 4)$;

$f(x) > 0$ if $0 < x < 2$ or $x > 4$, $f(x) < 0$ if $x < 0$, $x \neq -1$ or $2 < x < 4$

$\boxed{29}$ The sign of $f(x)$ is positive on $(-\infty, -4)$, so the graph of f must be above the x-axis on that interval. The sign of $f(x)$ is negative on $(-4, 0)$, so the graph of f must cross the x-axis at $x = -4$ and be below the x-axis on the interval $(-4, 0)$. The sign of $f(x)$ is negative on $(0, 1)$, so the graph of f must touch the x-axis at $x = 0$ and then fall below the x-axis on $(0, 1)$. The sign of $f(x)$ is positive on $(1, 3)$, so the graph of f must cross the x-axis at $x = 1$ and be above the x-axis on that interval. The sign of $f(x)$ is negative on $(3, \infty)$, so the graph of f must cross the x-axis at $x = 3$ and be below the x-axis on the interval $(3, \infty)$. See *Figure 29*.

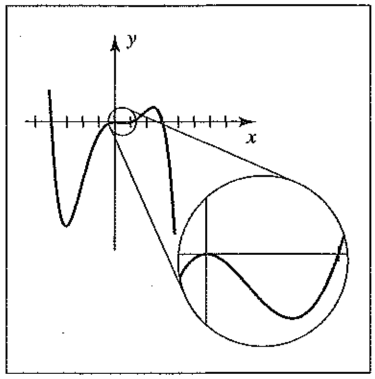

Figure 29

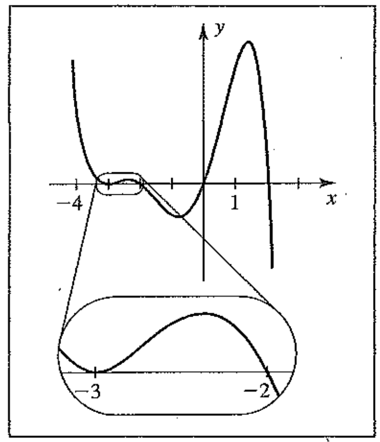

Figure 30

$\boxed{30}$ The graph of f is above the x-axis on $(-\infty,\ -2)$, touching the axis at $x = -3$. The graph is below the x-axis on $(-2,\ 0)$, above the x-axis on $(0,\ 2)$, and below the x-axis on $(2,\ \infty)$.

$\boxed{31}$ (a) The graph of $f(x) = (x-a)(x-b)(x-c)$, where $a < 0 < b < c$, must have one negative zero, a, and two positive zeros, b and c. The general shape is that of a cubic polynomial.

(b) The y-intercept is $f(0) = (-a)(-b)(-c) = -abc$.

(c) The solution to $f(x) < 0$ is $(-\infty,\ a) \cup (b,\ c)$; that is, where the graph of f is below the x-axis.

(d) The solution to$f(x) \geq 0$ is $[a,\ b] \cup [c,\ \infty)$; that is, where the graph of f is above or on the x-axis.

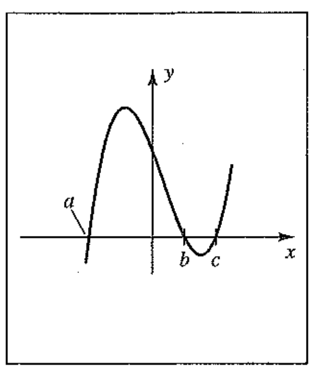

Figure 31

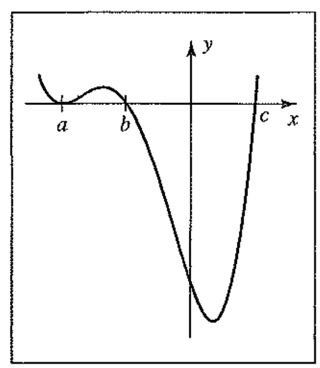

Figure 32

$\boxed{32}$ (a) The graph of $f(x) = (x-a)^2(x-b)(x-c)$, where $a < b < 0 < c$, must have two negative zeros, a and b, and one positive zero, c. The general shape is that of a quartic polynomial.

(b) The y-intercept is $f(0) = (-a)^2(-b)(-c) = a^2bc$.

(c) The solution to $f(x) > 0$ is $(-\infty,\ a) \cup (a,\ b) \cup (c,\ \infty)$; that is, where the graph of f is above the x-axis.

(d) The solution to $f(x) \le 0$ is $\{a\} \cup [b,\ c]$; that is, where the graph of f is below or on the x-axis.

[33] If n is even, then $(-x)^n = x^n$ and hence $f(-x) = f(x)$. Thus, f is an even function.

[34] If n is odd, then $(-x)^n = -x^n$ and hence $f(-x) = -f(x)$. Thus, f is an odd function.

[35] $f(-1) = -4 - 6k$ and $f(-1) = 4 \ \Rightarrow\ -4 - 6k = 4 \ \Rightarrow\ 6k = -8 \ \Rightarrow\ k = -\frac{4}{3}$.

[36] $f(2) = 8k + 4 - 2k + 2$ and $f(2) = 12 \ \Rightarrow\ 6k + 6 = 12 \ \Rightarrow\ 6k = 6 \ \Rightarrow\ k = 1$.

[37] $f(2) = 16k - 32$ and $f(2) = 0 \ \Rightarrow\ 16k - 32 = 0 \ \Rightarrow\ k = 2$.

$f(x) = x^3 - 2x^2 - 16x + 32 = x^2(x-2) - 16(x-2) = (x+4)(x-4)(x-2)$.

The other two zeros are ± 4.

[38] $f(-2) = 2k - 8$ and $f(-2) = 0 \ \Rightarrow\ 2k - 8 = 0 \ \Rightarrow\ 2k = 8 \ \Rightarrow\ k = 4$.

$f(x) = x^3 - 3x^2 - 4x + 12 = x^2(x-3) - 4(x-3) = (x-3)(x^2-4) =$

$(x-3)(x+2)(x-2)$. The other two zeros are 2 and 3.

[39] $P(x) = \frac{1}{2}(5x^3 - 3x) = \frac{1}{2}x(5x^2 - 3)$.

$P(x) > 0$ on $(-\frac{1}{5}\sqrt{15},\ 0)$ and $(\frac{1}{5}\sqrt{15},\ \infty)$.

$P(x) < 0$ on $(-\infty,\ -\frac{1}{5}\sqrt{15})$ and $(0,\ \frac{1}{5}\sqrt{15})$.

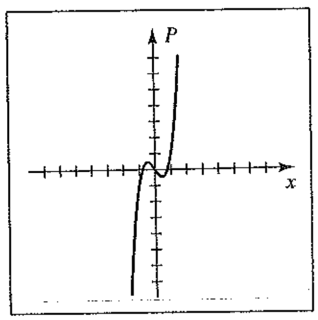

Figure 39

[40] $x^2 = \dfrac{8 \pm 4\sqrt{2}}{16} = \dfrac{2 \pm \sqrt{2}}{4} \ \Rightarrow\ x = \pm\frac{1}{2}\sqrt{2 \pm \sqrt{2}} \approx \pm 0.92,\ \pm 0.38;$

$f(x) > 0$ if $|\,x\,| < \frac{1}{2}\sqrt{2 - \sqrt{2}}$ or $|\,x\,| > \frac{1}{2}\sqrt{2 + \sqrt{2}}$

[41] (a) $V(x) = lwh = (30 - x - x)(20 - x - x)x = x(20 - 2x)(30 - 2x) =$

$4x(10 - x)(15 - x) = 4x(x - 10)(x - 15)$.

(b) $V(x) > 0$ on $(0,\ 10)$ and $(15,\ \infty)$. Allowable values for x are in $(0,\ 10)$.

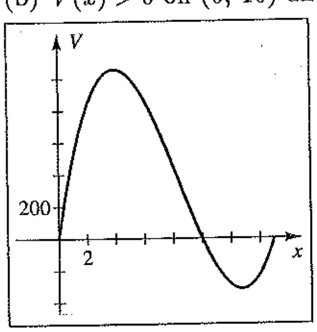

Figure 41

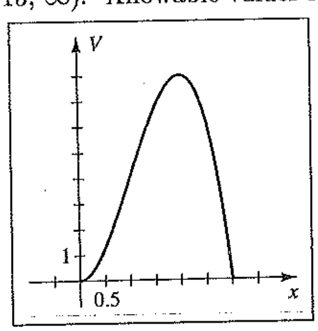

Figure 42

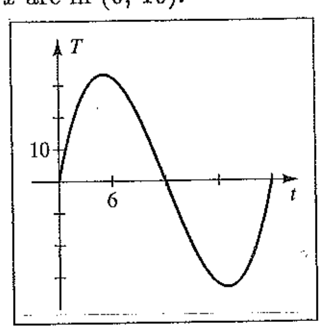

Figure 43

[42] (a) Since we are disregarding the thickness of the lumber, 4 boards are y feet long and 8 boards are x feet long. Total length $= 24 \Rightarrow 4y + 8x = 24 \Rightarrow$
$y = 6 - 2x$. $V = x^2y = x^2(6-2x) = 6x^2 - 2x^3$.

(b) From part (a), $V = 6x^2 - 2x^3$, or, equivalently, $V = -2x^2(x-3)$. See *Figure 42*.

[43] (a) $T = \frac{1}{20}t(t-12)(t-24) = 0 \Rightarrow t = 0, 12, 24$. $T > 0$ for
$0 < t < 12$ {6 A.M. to 6 P.M.}; $T < 0$ for $12 < t < 24$ {6 P.M. to 6 A.M.}.

(b) See *Figure 43*.

(c) 12 noon corresponds to $t = 6$, $T(6) = 32.4 > 32\,^\circ\mathrm{F}$ and $T(7) = 29.75 < 32\,^\circ\mathrm{F}$

[44] (a) At the end of the board, $s = 10$.
Letting $d = 1$ and $L = 10$ yields $1 = 100c(20) \Rightarrow c = \frac{1}{2000}$.

(b) $s = 6.5 \Rightarrow d = (\frac{1}{2000})(6.5)^2[3(10) - 6.5] \approx 0.4964 < \frac{1}{2}$.
$s = 6.6 \Rightarrow d = (\frac{1}{2000})(6.6)^2[3(10) - 6.6] \approx 0.5097 > \frac{1}{2}$.

[45] (a)
$$\begin{aligned} N(t) &= -t^4 + 21t^2 + 100 \\ &= -(t^4 - 21t^2 - 100) \\ &= -(t^2 - 25)(t^2 + 4) \\ &= -(t+5)(t-5)(t^2+4) \end{aligned}$$
If $t > 0$, then $N(t) > 0$ for $0 < t < 5$.

(b) The population becomes extinct when $N = 0$.
This occurs after 5 years.

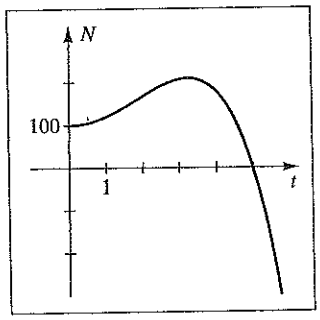

Figure 45

[46] (a) $R = 0 \Rightarrow -4t^3 + 42t = 0 \Rightarrow -2t(2t^2 - 21) = 0 \Rightarrow t = \frac{1}{2}\sqrt{42}$ {for $t > 0$}.
The population ceases to grow after $\frac{1}{2}\sqrt{42} \approx 3.24$ years.

(b) $R > 0$ if t is in the interval $(0, \frac{1}{2}\sqrt{42})$.

[47] (a) $f(x) = 2x^4$, $g(x) = 2x^4 - 5x^2 + 1$, $h(x) = 2x^4 + 5x^2 - 1$, $k(x) = 2x^4 - x^3 + 2x$

x	$f(x)$	$g(x)$	$h(x)$	$k(x)$
-60	25,920,000	25,902,001	25,937,999	26,135,880
-40	5,120,000	5,112,001	5,127,999	5,183,920
-20	320,000	318,001	321,999	327,960
20	320,000	318,001	321,999	312,040
40	5,120,000	5,112,001	5,127,999	5,056,080
60	25,920,000	25,902,001	25,937,999	25,704,120

(b) As $|x|$ becomes large, the function values become similar.

(c) The term with the highest power of x: $2x^4$.

[48] (a) $f(x) = -3x^3$, $g(x) = -3x^3 - x^2 + 1$, $h(x) = -3x^3 + x^2 - 1$, $k(x) = -3x^3 - 2x^2 + 2x$

$[-2, 2]$ by $[-2, 2]$

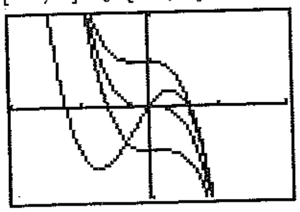

Figure 48(1)

$[-10, 10]$ by $[-10, 10]$

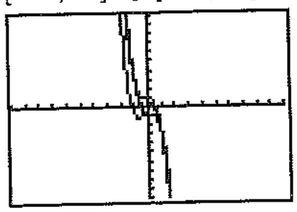

Figure 48(2)

$[-50, 50, 10]$ by $[-5E3, 5E3, 1E3]$

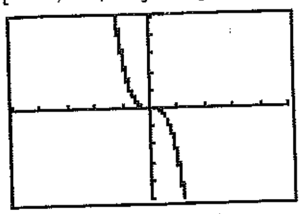

Figure 48(3)

$[-100, 100, 10]$ by $[-5E5, 5E5, 1E5]$

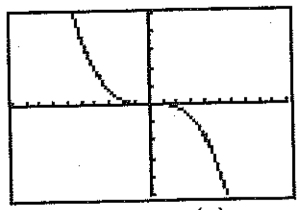

Figure 48(4)

(b) As the viewing rectangle increases in size, the graphs look alike.

(c) Their end behavior is similar because their highest degree term is $-3x^3$. This term determines the shape of the graph when $|x|$ is large.

[49] (a) $[-9, 9]$ by $[-6, 6]$

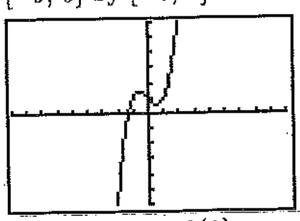

Figure 49(1)

$[-9, 9]$ by $[-6, 6]$

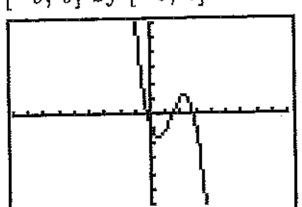

Figure 49(2)

$[-9, 9]$ by $[-6, 6]$

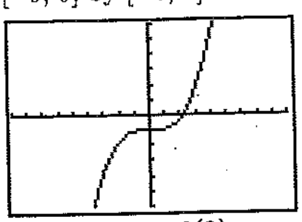

Figure 49(3)

$[-9, 9]$ by $[-6, 6]$

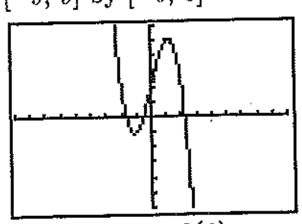

Figure 49(4)

(b) (1) $f(x) = x^3 - x + 1$ • As x approaches ∞, $f(x)$ approaches ∞; as x approaches $-\infty$, $f(x)$ approaches $-\infty$

(2) $f(x) = -x^3 + 4x^2 - 3x - 1$ • As x approaches ∞, $f(x)$ approaches $-\infty$; as x approaches $-\infty$, $f(x)$ approaches ∞

(3) $f(x) = 0.1x^3 - 1$ • As x approaches ∞, $f(x)$ approaches ∞; as x approaches $-\infty$, $f(x)$ approaches $-\infty$

(4) $f(x) = -x^3 + 4x + 2$ • As x approaches ∞, $f(x)$ approaches $-\infty$; as x approaches $-\infty$, $f(x)$ approaches ∞

(c) For the cubic function $f(x) = ax^3 + bx^2 + cx + d$ with $a > 0$, $f(x)$ approaches ∞ as x approaches ∞ and $f(x)$ approaches $-\infty$ as x approaches $-\infty$. With $a < 0$, $f(x)$ approaches $-\infty$ as x approaches ∞ and $f(x)$ approaches ∞ as x approaches $-\infty$.

50 (a) $[-9, 9]$ by $[-6, 6]$

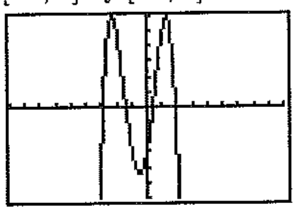

Figure 50(1)

$[-9, 9]$ by $[-6, 6]$

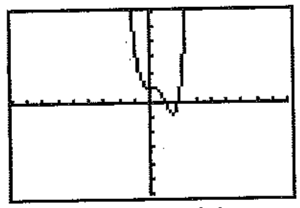

Figure 50(2)

$[-9, 9]$ by $[-6, 6]$

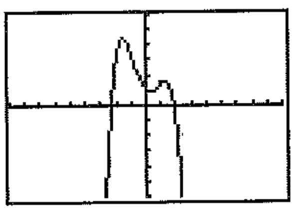

Figure 50(3)

$[-9, 9]$ by $[-6, 6]$

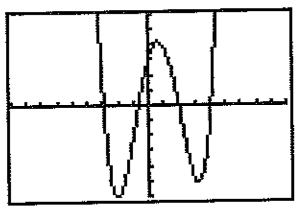

Figure 50(4)

(b) (1) $f(x) = -x^4 - 2x^3 + 5x^2 + 6x - 3$ • As $x \to \pm\infty$, $f(x) \to -\infty$

(2) $f(x) = x^4 - 2x^3 + 1$ • As $x \to \pm\infty$, $f(x) \to \infty$

(3) $f(x) = -\frac{1}{2}x^4 + 2x^2 - x + 1$ • As $x \to \pm\infty$, $f(x) \to -\infty$

(4) $f(x) = \frac{1}{5}x^4 - \frac{1}{2}x^3 - \frac{7}{3}x^2 + \frac{7}{2}x + 3$ • As $x \to \pm\infty$, $f(x) \to \infty$

(c) For the fourth-degree polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ with $a > 0$, $f(x)$ approaches ∞ as $|x|$ approaches ∞ and with $a < 0$, $f(x)$ approaches $-\infty$ as $|x|$ approaches ∞.

51 From the graph, f has three zeros. They are approximately -1.89, 0.49, and 1.20.

$[-4.5, 4.5]$ by $[-3, 3]$

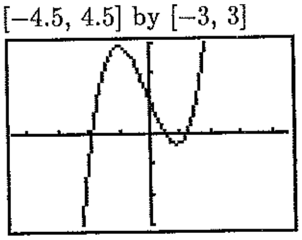

Figure 51

$[-4.5, 4.5]$ by $[-3, 3]$

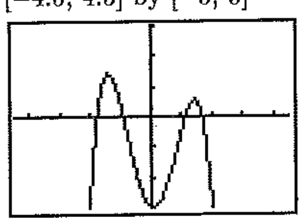

Figure 52

52 From the graph, f has four zeros.

They are approximately -1.78, -0.91, 1.11, and 1.67.

[53] From the graph, f has three zeros. They are approximately -1.88, 0.35, and 1.53.

$[-4.5, 4.5]$ by $[-3, 3]$

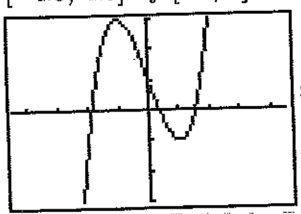

Figure 53

$[-4.5, 4.5]$ by $[-3, 3]$

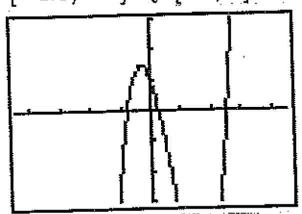

Figure 54

[54] From the graph, f has three zeros. They are approximately -0.77, 0.26, and 2.52.

[55] If $f(x) = x^3 + 5x - 2$ and $k = 1$, then $f(x) > k$ on $(0.56, \infty)$.

$[-4.5, 4.5]$ by $[-3, 3]$

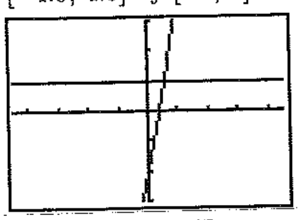

Figure 55

$[-4.5, 4.5]$ by $[-2, 4]$

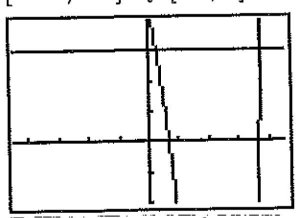

Figure 56

[56] If $f(x) = x^4 - 4x^3 + 3x^2 - 8x + 5$ and $k = 3$, then $f(x) > k$ on $(-\infty, 0.27) \cup (3.73, \infty)$.

[57] If $f(x) = x^5 - 2x^2 + 2$ and $k = -2$, then $f(x) > k$ on $(-1.10, \infty)$.

$[-4.5, 4.5]$ by $[-3, 3]$

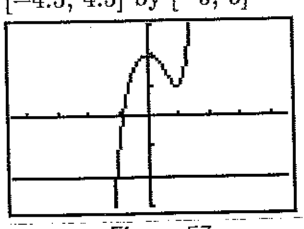

Figure 57

$[-4.5, 4.5]$ by $[-3, 3]$

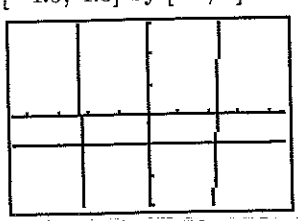

Figure 58

[58] If $f(x) = x^4 - 2x^3 + 10x - 26$ and $k = -1$, then $f(x) > k$ on $(-\infty, -2.24) \cup (2.24, \infty)$.

[59] From the graph, there are three points of intersection.

Their coordinates are approximately $(-1.29, -0.77)$, $(0.085, 2.66)$, and $(1.36, -0.42)$.

$[-4.5, 4.5]$ by $[-2, 4]$

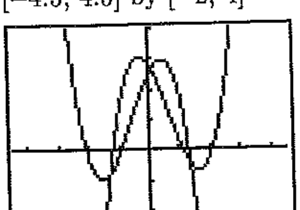

Figure 59

$[-5.5, 5]$ by $[-3, 4]$

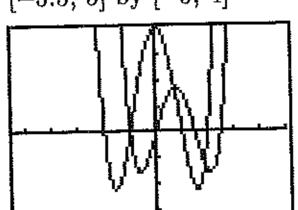

Figure 60

[60] From the graph, there are three points of intersection. One is $(-1, 0)$.

The others are approximately $(0.71, 1.72)$ and $(1.87, -1.25)$.

[61] (a) [1975, 1995] by [20, 40]

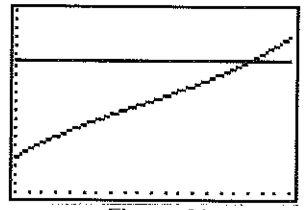

Figure 61

(b) Graph $Y_1 = 0.0014(x-1975)^3 - 0.0388(x-1975)^2 + 0.8783(x-1975) + 23.82$ and $Y_2 = 34.4$. Their graphs intersect when $x \approx 1991.9792 \approx 1992$. There were 34.4 million recipients in 1992.

[62] (a) The number of preschool children decreased during 1970 to 1976, but since then, it has increased.

[0, 25, 5] by [3E5, 8E5, 1E5]

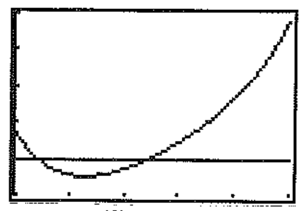

Figure 62

(b) $f(x) = 4.363x^4 - 236.3x^3 + 5527x^2 - 46{,}519x + 475{,}913 \Rightarrow f(6) \approx 350{,}385$

(c) Graph $Y_1 = f(x)$ and $Y_2 = 400{,}000$. There are two points of intersection at $x \approx 2.12, 12.30$. Thus, there were approximately 400,000 participants during the years 1972 and 1982.

4.2 Exercises

[1–8]	$f(x) =$ dividend;	$p(x) =$ divisor •	★ quotient; remainder
[1]	$f(x) = 2x^4 - x^3 - 3x^2 + 7x - 12$;	$p(x) = x^2 - 3$ •	★ $2x^2 - x + 3$; $4x - 3$
[2]	$f(x) = 3x^4 + 2x^3 - x^2 - x - 6$;	$p(x) = x^2 + 1$ •	★ $3x^2 + 2x - 4$; $-3x - 2$
[3]	$f(x) = 3x^3 + 2x - 4$;	$p(x) = 2x^2 + 1$ •	★ $\frac{3}{2}x$; $\frac{1}{2}x - 4$
[4]	$f(x) = 3x^3 - 5x^2 - 4x - 8$;	$p(x) = 2x^2 + x$ •	★ $\frac{3}{2}x - \frac{13}{4}$; $-\frac{3}{4}x - 8$
[5]	$f(x) = 7x + 2$;	$p(x) = 2x^2 - x - 4$ •	★ 0; $7x + 2$
[6]	$f(x) = -5x^2 + 3$;	$p(x) = x^3 - 3x + 9$ •	★ 0; $-5x^2 + 3$
[7]	$f(x) = 9x + 4$;	$p(x) = 2x - 5$ •	★ $\frac{9}{2}$; $\frac{53}{2}$
[8]	$f(x) = 7x^2 + 3x - 10$;	$p(x) = x^2 - x + 10$ •	★ 7; $10x - 80$

[9] Dividing $f(x) = 3x^3 - x^2 + 5x - 4$ by $x - 2$ using either long division or synthetic division yields a remainder of 26.

$\boxed{10}$ Divide $f(x) = 2x^3 + 4x^2 - 3x - 1$ by $x - 3$ to obtain a remainder of 80.

$\boxed{11}$ Divide $f(x) = x^4 - 6x^2 + 4x - 8$ by $x + 3$ to obtain a remainder of 7.

$\boxed{12}$ Divide $f(x) = x^4 + 3x^2 - 12$ by $x + 2$ to obtain a remainder of 16.

$\boxed{13}$ Since $f(-3) = 0$, $x + 3$ is a factor of $f(x) = x^3 + x^2 - 2x + 12$.

$\boxed{14}$ Since $f(2) = 0$, $x - 2$ is a factor of $f(x) = x^3 + x^2 - 11x + 10$.

$\boxed{15}$ Since $f(-2) = 0$, $x + 2$ is a factor of $f(x) = x^{12} - 4096$.

$\boxed{16}$ Since $f(2) = 0$, $x - 2$ is a factor of $f(x) = x^4 - 3x^3 - 2x^2 + 5x + 6$.

Note: In Exercises 17–20, let $a = 1$.

$\boxed{17}$ f has degree 3 with zeros -2, 0, 5 $\Rightarrow$

$$f(x) = a\left[x - (-2)\right](x - 0)(x - 5) = x(x + 2)(x - 5) = x(x^2 - 3x - 10) = x^3 - 3x^2 - 10x$$

$\boxed{18}$ f has degree 3 with zeros ± 2, 3 $\Rightarrow$

$$f(x) = a(x + 2)(x - 2)(x - 3) = (x^2 - 4)(x - 3) = x^3 - 3x^2 - 4x + 12$$

$\boxed{19}$ f has degree 4 with zeros -2, ± 1, 4 $\Rightarrow$

$$f(x) = a(x + 2)(x + 1)(x - 1)(x - 4) = (x^2 - 1)(x^2 - 2x - 8) = x^4 - 2x^3 - 9x^2 + 2x + 8$$

$\boxed{20}$ f has degree 4 with zeros -3, 0, 1, 5 $\Rightarrow$

$$f(x) = a(x + 3)(x)(x - 1)(x - 5) = x(x^2 + 2x - 3)(x - 5) = x^4 - 3x^3 - 13x^2 + 15x$$

$\boxed{21}$
$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & 7 \end{array}$$
The synthetic division indicates that the quotient is $2x^2 + x + 6$ and the remainder is 7.

$\boxed{22}$ $3x^3 - 4x^2 - x + 8$; $x + 4$ • ★ $3x^2 - 16x + 63$; -244

$\boxed{23}$ $x^3 - 8x - 5$; $x + 3$ • ★ $x^2 - 3x + 1$; -8

$\boxed{24}$ $5x^3 - 6x^2 + 15$; $x - 4$ • ★ $5x^2 + 14x + 56$; 239

$\boxed{25}$ $3x^5 + 6x^2 + 7$; $x + 2$ • ★ $3x^4 - 6x^3 + 12x^2 - 18x + 36$; -65

$\boxed{26}$ $-2x^4 + 10x - 3$; $x - 3$ • ★ $-2x^3 - 6x^2 - 18x - 44$; -135

$\boxed{27}$ $4x^4 - 5x^2 + 1$; $x - \frac{1}{2}$ • ★ $4x^3 + 2x^2 - 4x - 2$; 0

$\boxed{28}$ $9x^3 - 6x^2 + 3x - 4$; $x - \frac{1}{3}$ • ★ $9x^2 - 3x + 2$; $-\frac{10}{3}$

$\boxed{29}$
$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 6 & 27 & 69 \\ \hline & 2 & 9 & 23 & 73 \end{array}$$
The synthetic division indicates that $f(3) = 73$.

$\boxed{30}$ $f(x) = -x^3 + 4x^2 + x$; $c = -2$ • $f(-2) = 22$

$\boxed{31}$ $f(x) = 0.3x^3 + 0.04x - 0.034$; $c = -0.2$ • $f(-0.2) = -0.0444$

$\boxed{32}$ $f(x) = 8x^5 - 3x^2 + 7$; $c = \frac{1}{2}$ • $f(\frac{1}{2}) = \frac{13}{2}$

$\boxed{33}$ $f(x) = x^2 + 3x - 5$; $c = 2 + \sqrt{3}$ • $f(2 + \sqrt{3}) = 8 + 7\sqrt{3}$

$\boxed{34}$ $f(x) = x^3 - 3x^2 - 8$; $c = 1 + \sqrt{2}$ • $f(1 + \sqrt{2}) = -10 - \sqrt{2}$

$\boxed{35}$ $f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4;$ $c = -2$ • ★ $f(-2) = 0$

$\boxed{36}$ $f(x) = 4x^3 - 9x^2 - 8x - 3;$ $c = 3$ • ★ $f(3) = 0$

$\boxed{37}$ $f(x) = 4x^3 - 6x^2 + 8x - 3;$ $c = \frac{1}{2}$ • ★ $f(\frac{1}{2}) = 0$

$\boxed{38}$ $f(x) = 27x^4 - 9x^3 + 3x^2 + 6x + 1;$ $c = -\frac{1}{3}$ • ★ $f(-\frac{1}{3}) = 0$

$\boxed{39}$ $f(-2) = k^2 - 8k + 15$. This remainder must be zero if $f(x)$ is to be divisible by $x + 2$.

$k^2 - 8k + 15 = 0 \Rightarrow k = 3, 5.$

$\boxed{40}$ $f(1) = k^2 - 4k + 3$. As in the previous exercise, $k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3.$

$\boxed{41}$ $f(c) = 3c^4 + c^2 + 5 \geq 5\ \forall c \in \mathbb{R}$; that is, the remainder cannot be zero.

$\boxed{42}$ $f(c) = -c^4 - 3c^2 - 2 \leq -2\ \forall c \in \mathbb{R}$; that is, the remainder cannot be zero.

$\boxed{43}$ $f(x) = 3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6 \Rightarrow f(-1) = 3 - 5 - 4 - 2 - 6 = -14.$

{ change sign on coefficients of odd powers of x }

$\boxed{44}$ If $f(x) = x^n - y^n$, then $f(y) = y^n - y^n = 0$. Hence, $x - y$ is a factor of f.

$\boxed{45}$ If $f(x) = x^n - y^n$ and n is even, then $f(-y) = (-y)^n - (y)^n = y^n - y^n = 0$.

Hence, $x + y$ is a factor of $x^n - y^n$.

$\boxed{46}$ If $f(x) = x^n + y^n$ and n is odd, then $f(-y) = (-y)^n + y^n = -y^n + y^n = 0$.

Hence, $x + y$ is a factor of $x^n + y^n$.

$\boxed{47}$ (a) $V = \pi r^2 h = \pi x^2(6 - x)$

(b) The volume of the cylinder of radius 1 and altitude 5 is $\pi(1)^2 5 = 5\pi$.

$5\pi = \pi x^2(6 - x) \Rightarrow x^3 - 6x^2 + 5 = 0 \Rightarrow (x - 1)(x^2 - 5x - 5) = 0 \Rightarrow$

$x = 1, \dfrac{5 \pm \sqrt{45}}{2}$. $\dfrac{5 + \sqrt{45}}{2} \approx 5.85$ would be an allowable value of x.

The point P is $P(x, y) = (\frac{1}{2}(5 + \sqrt{45}), \frac{1}{2}(7 - \sqrt{45}))$.

$\boxed{48}$ The width, depth, and diameter form a right triangle.

$w^2 + d^2 = 2^2 \Rightarrow d^2 = 4 - w^2 = \frac{7}{4}$ for $w = \frac{3}{2}$. $S = kwd^2 = k(\frac{3}{2})(\frac{7}{4}) = \frac{21}{8}k$. Now

$\frac{21}{8}k = kw(4 - w^2) \Rightarrow 8w^3 - 32w + 21 = 0 \Rightarrow (w - \frac{3}{2})(8w^2 + 12w - 14) = 0 \Rightarrow$

$w = \frac{3}{2}, \dfrac{-3 \pm \sqrt{37}}{4}$. $\dfrac{\sqrt{37} - 3}{4} \approx 0.77$ would be an allowable value of w.

$\boxed{49}$ (a) $A = lw = (2x)(y) = 2x(4 - x^2) = 8x - 2x^3$

(b) $A = 6 \Rightarrow 6 = 8x - 2x^3 \Rightarrow x^3 - 4x + 3 = 0 \Rightarrow (x - 1)(x^2 + x - 3) = 0 \Rightarrow$

$x = 1, \dfrac{-1 \pm \sqrt{13}}{2}$. $\dfrac{\sqrt{13} - 1}{2}$ would be an allowable value of x.

The base would then be $\sqrt{13} - 1 \approx 2.61$.

$\boxed{50}$ (a) Let $\frac{3}{2} - 2r$ denote the length of the right circular cylinder portion of the capsule.

$V = \frac{4}{3}\pi r^3 + \pi r^2(\frac{3}{2} - 2r) = \pi r^2(\frac{4}{3}r + \frac{3}{2} - 2r) = \pi r^2(\frac{3}{2} - \frac{2}{3}r)$.

(b) The tablet has volume $\pi(\frac{1}{2})^2 \cdot \frac{1}{3} = \frac{\pi}{12}$. $\frac{\pi}{12} = \pi r^2(\frac{3}{2} - \frac{2}{3}r) \Rightarrow$
$8r^3 - 18r^2 + 1 = 0 \Rightarrow (4r-1)(2r^2 - 4r - 1) = 0 \Rightarrow r = \frac{1}{4}, 1 \pm \frac{1}{2}\sqrt{6}$.
$\frac{1}{4}$ cm is the only allowable solution.

[51] If $f(x) = x^8 - 7.9x^5 - 0.8x^4 + x^3 + 1.2x - 9.81$,
then the remainder is $f(0.21) \approx -9.55$.

[−1, 1] by [−20, 0, 2]

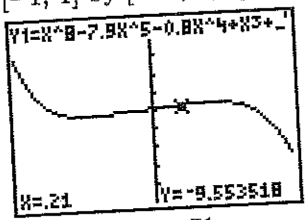

Figure 51

[−1, 1] by [−20, 0, 2]

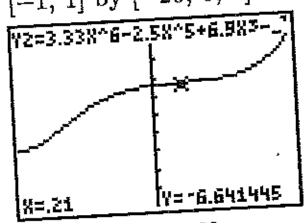

Figure 52

[52] If $f(x) = 3.33x^6 - 2.5x^5 + 6.9x^3 - 4.1x^2 + 1.22x - 6.78$,
then the remainder is $f(0.21) \approx -6.64$.

[53] $f(1.6) = -2k^4 + 2.56k^3 + 3.2k + 4.096$. Graph $y = -2k^4 + 2.56k^3 + 3.2k + 4.096$ (that is, $y = -2x^4 + 2.56x^3 + 3.2x + 4.096$). From the graph, we see that $y = 0$ when $k \approx -0.75, 1.96$. Thus, if k assumes either of these values, $f(1.6) = 0$ and f will be divisible by $x - 1.6$ by the factor theorem.

[−9, 9] by [−3, 9]

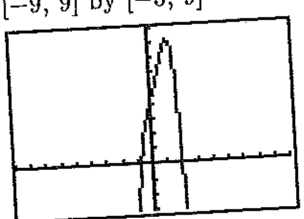

Figure 53

[−4.5, 4.5] by [−3, 3]

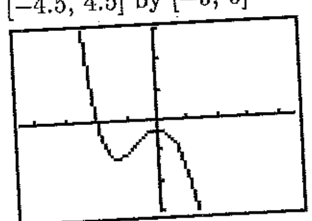

Figure 54

[54] $f(-0.4) = -0.064k^5 - 0.4k^3 - 1.2k^2 - 0.336$.
Graph $y = -0.064k^5 - 0.4k^3 - 1.2k^2 - 0.336$. From the graph, we see that $y = 0$ when $k \approx -1.98$. Thus, if k assumes this value, $f(-0.4) = 0$ and f will be divisible by $x + 0.4$ by the factor theorem.

4.3 Exercises

[1] $f(x) = a(x+1)(x-2)(x-3)$;
$f(-2) = a(-1)(-4)(-5) = 80 \Rightarrow -20a = 80 \Rightarrow a = -4$ ★ $-4x^3 + 16x^2 - 4x - 24$

[2] $f(x) = a(x+5)(x-2)(x-4)$;
$f(3) = a(8)(1)(-1) = -24 \Rightarrow -8a = -24 \Rightarrow a = 3$ ★ $3x^3 - 3x^2 - 66x + 120$

[3] $f(x) = a(x+4)(x-3)(x)$;

$f(2) = a(6)(-1)(2) = -36 \;\Rightarrow\; -12a = -36 \;\Rightarrow\; a = 3$ ★ $3x^3 + 3x^2 - 36x$

[4] $f(x) = a(x+3)(x+2)(x)$;

$f(-4) = a(-1)(-2)(-4) = 16 \;\Rightarrow\; -8a = 16 \;\Rightarrow\; a = -2$ ★ $-2x^3 - 10x^2 - 12x$

[5] $f(x) = a(x+2i)(x-2i)(x-3)$; $f(1) = a(1+2i)(1-2i)(-2) = 20 \;\Rightarrow$

$-10a = 20 \;\Rightarrow\; a = -2$ ★ $-2x^3 + 6x^2 - 8x + 24$

[6] $f(x) = a(x+3i)(x-3i)(x-4)$; $f(-1) = a(-1+3i)(-1-3i)(-5) = 50 \;\Rightarrow$

$-50a = 50 \;\Rightarrow\; a = -1$ ★ $-x^3 + 4x^2 - 9x + 36$

[7] $f(x) = a(x+4)^2(x-3)^2 = (x^2 + x - 12)^2 \;\{a = 1\} = x^4 + 2x^3 - 23x^2 - 24x + 144$

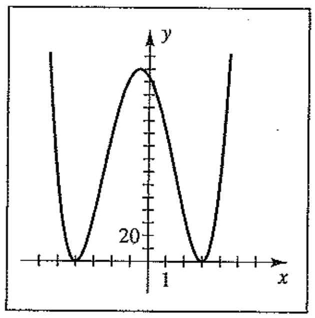

Figure 7

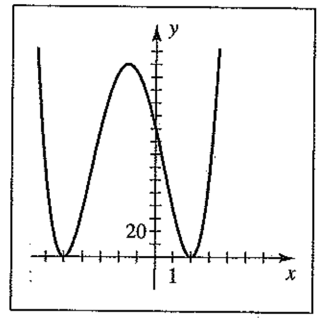

Figure 8

[8] $f(x) = a(x+5)^2(x-2)^2 = (x^2 + 3x - 10)^2 \;\{a = 1\} = x^4 + 6x^3 - 11x^2 - 60x + 100.$

Note that the figure can be obtained by shifting *Figure 7* left 1 unit.

[9] $f(x) = a(x)^3(x-3)^3$ so $f(2) = a(8)(-1) = -8a$. But $f(2) = -24$, so $-8a = -24$, or,

$a = 3$. $f(x) = 3(x)^3(x^3 - 9x^2 + 27x - 27) = 3x^6 - 27x^5 + 81x^4 - 81x^3$.

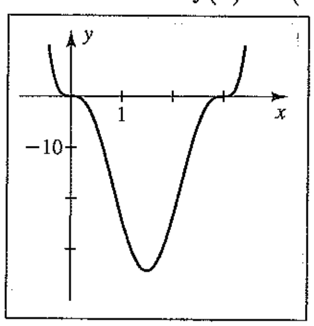

Figure 9

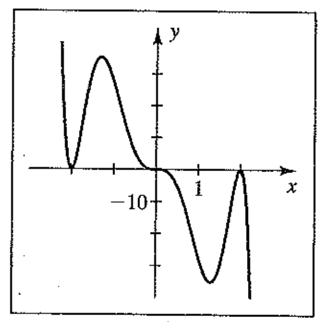

Figure 10

[10] $f(x) = a(x)^3(x+2)^2(x-2)^2$ so $f(-1) = a(-1)(1)(9) = -9a$.

But $f(-1) = 27$, so $-9a = 27$, or, $a = -3$.

$$f(x) = -3x^3(x^2-4)^2 = -3x^3(x^4 - 8x^2 + 16) = -3x^7 + 24x^5 - 48x^3.$$

[11] The graph has x-intercepts at -1, $\frac{3}{2}$, 3, and $f(0) = \frac{7}{2}$.

$f(x) = a(x+1)(x-\frac{3}{2})(x-3)$; $f(0) = a(1)(-\frac{3}{2})(-3) = \frac{7}{2} \;\Rightarrow\; \frac{9}{2}a = \frac{7}{2} \;\Rightarrow\; a = \frac{7}{9}$.

$$f(x) = \tfrac{7}{9}(x+1)(x-\tfrac{3}{2})(x-3)$$

$\boxed{12}$ The graph has x-intercepts at 0, 1, 3, 5, and $f(-1) = 4$.

$f(x) = a(x)(x-1)(x-3)(x-5)$; $f(-1) = a(-1)(-2)(-4)(-6) = 4 \Rightarrow$

$48a = 4 \Rightarrow a = \frac{1}{12}$. $\quad f(x) = \frac{1}{12}x(x-1)(x-3)(x-5)$

$\boxed{13}$ 3 is a zero of multiplicity one, 1 is a zero of multiplicity two, and $f(0) = 3$.

$f(x) = a(x-1)^2(x-3)$; $f(0) = a(1)(-3) = 3 \Rightarrow a = -1$. $f(x) = -1(x-1)^2(x-3)$.

$\boxed{14}$ 4 is a zero of multiplicity one, 2 is a zero of multiplicity two, and $f(1) = -3$.

$f(x) = a(x-2)^2(x-4)$; $f(1) = a(1)(-3) = -3 \Rightarrow a = 1$. $f(x) = 1(x-2)^2(x-4)$.

$\boxed{15}$ ★ $-\frac{2}{3}$ (multiplicity 1); 0 (multiplicity 2); $\frac{5}{2}$ (multiplicity 3)

$\boxed{16}$ ★ -1 (multiplicity 4); 0 (multiplicity 1); $\frac{7}{3}$ (multiplicity 2)

$\boxed{17}$ $f(x) = 4x^5 + 12x^4 + 9x^3 = x^3(2x+3)^2$ ★ $-\frac{3}{2}$ (multiplicity 2); 0 (multiplicity 3)

$\boxed{18}$ ★ $\pm\frac{1}{2}\sqrt{5}$ (each of multiplicity 2)

$\boxed{19}$ $f(x) = (x^2 + x - 12)^3(x^2-9)^2 = (x+4)^3(x-3)^3(x+3)^2(x-3)^2$

★ -4 (multiplicity 3); -3 (multiplicity 2); 3 (multiplicity 5)

$\boxed{20}$ $f(x) = (6x^2 + 7x - 5)^4(4x^2-1)^2 = (3x+5)^4(2x-1)^4(2x+1)^2(2x-1)^2$

★ $-\frac{5}{3}$ (multiplicity 4); $-\frac{1}{2}$ (multiplicity 2); $\frac{1}{2}$ (multiplicity 6)

$\boxed{21}$ $f(x) = x^4 + 7x^2 - 144 = (x^2+16)(x+3)(x-3)$

★ $\pm 4i$, ± 3 (each of multiplicity 1)

$\boxed{22}$ $f(x) = x^4 + 21x^2 - 100 = (x^2+25)(x+2)(x-2)$

★ $\pm 5i$, ± 2 (each of multiplicity 1)

$\boxed{23}$ Using synthetic division, $f(x) = x^4 + 7x^3 + 13x^2 - 3x - 18 =$

$(x+3)(x^3+4x^2+x-6) = (x+3)^2(x^2+x-2) = (x+3)^2(x+2)(x-1)$.

$\boxed{24}$ $f(x) = x^4 - 9x^3 + 22x^2 - 32 = (x-4)^2(x-2)(x+1)$

$\boxed{25}$ $f(x) = x^6 - 4x^5 + 5x^4 - 5x^2 + 4x - 1 = (x-1)^5(x+1)$

$\boxed{26}$ $f(x) = x^5 + x^4 - 6x^3 - 14x^2 - 11x - 3 = (x+1)^4(x-3)$

Note: For the following exercises, let $f(x)$ denote the polynomial, P, the number of sign changes in $f(x)$, and N, the number of sign changes in $f(-x)$. The types of possible solutions are listed in the order positive, negative, nonreal complex.

$\boxed{27}$ There are 3 sign changes in $f(x) = 4x^3 - 6x^2 + x - 3$, so P = 3. There are no sign changes in $f(-x) = -4x^3 - 6x^2 - x - 3$, so N = 0, and there are no negative solutions of the given equation. The equation has either 3 positive solutions or 1 positive solution along with 2 nonreal complex solutions. ★ 3, 0, 0 or 1, 0, 2

$\boxed{28}$ $5x^3 - 6x - 4 = 0$ • P = 1, N = 2 ★ 1, 2, 0 or 1, 0, 2

$\boxed{29}$ $4x^3 + 2x^2 + 1 = 0$ • P = 0, N = 1 ★ 0, 1, 2

$\boxed{30}$ $3x^3 - 4x^2 + 3x + 7 = 0$ • P = 2, N = 1 ★ 2, 1, 0 or 0, 1, 2

[31] $3x^4+2x^3-4x+2=0$ • $P=2, N=2$ ★ 2, 2, 0; 2, 0, 2; 0, 2, 2; 0, 0, 4

[32] $2x^4-x^3+x^2-3x+4=0$ • $P=4, N=0$ ★ 4, 0, 0; 2, 0, 2; 0, 0, 4

[33] $x^5+4x^4+3x^3-4x+2=0$ • $P=2, N=3$ ★ 2, 3, 0; 2, 1, 2; 0, 3, 2; 0, 1, 4

[34] $2x^6+5x^5+2x^2-3x+4=0$ • $P=2, N=2$ ★ 2, 2, 2; 2, 0, 4; 0, 2, 4; 0, 0, 6

[35] From the graph of $f(x)=x^3-4x^2-5x+7$, we see that the bounds given by the first theorem (5 and -2) are indeed the smallest and largest integers that are upper and lower bounds.

[36] From the graph of $f(x)=2x^3-5x^2+4x-8$, we see that the least integer upper bound is 3 and the greatest integer lower bound is 2. According to the first theorem, the greatest *negative* integer lower bound is -1 (the first theorem gives no information about a greatest *positive* integer lower bound).

[37] From the graph of $f(x)=x^4-x^3-2x^2+3x+6$, we see that there are no real zeros. The first theorem gives us upper and lower bounds of 2 and -2, respectively.

[38] Similar to Exercise 35 with
$f(x)=2x^4-9x^3-8x-10$, upper bound 5, and lower bound -1.

[39] Similar to Exercise 35 with
$f(x)=2x^5-13x^3+2x-5$, upper bound 3, and lower bound -3.

[40] Similar to Exercise 36 with $f(x)=3x^5+2x^4-x^3-8x^2-7$, upper bound 2, and lower bound -1. From the graph of f, the lower bound is 1.

[41] $f(x)=a(x+1)^2(x-1)(x-2)^3$.

$$f(0)=a(1)(-1)(-8)=8a \text{ and } f(0)=-2 \Rightarrow a=-\tfrac{1}{4}.$$

[42] $f(x)=a(x+2)(x+1)(x-2)^2$. $f(0)=a(2)(1)(4)=8a$ and $f(0)=1 \Rightarrow a=\frac{1}{8}$.

[43] (a) $f(x)=a(x+3)^3(x+1)(x-2)^2$.

(b) $a=1$ and $x=0 \Rightarrow f(0)=1(3)^3(1)(-2)^2=108$.

[44] (a) $f(x)=a(x+2)^3(x-3)^2$.

(b) $a=-1$ and $x=0 \Rightarrow f(0)=-1(2)^3(-3)^2=-72$.

[45] From the graph, we see that $f(x)=x^5-16.75x^3+12.75x^2+49.5x-54$ has zeros of -4, -2, 1.5, and 3. There is a double root at 1.5. Since the leading coefficient of f is 1, we have $f(x)=1(x+4)(x+2)(x-1.5)^2(x-3)$.

[−5, 5] by [−150, 150, 25]

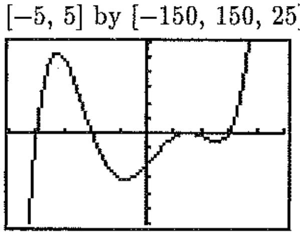

Figure 45

[−5, 5] by [−100, 100, 25]

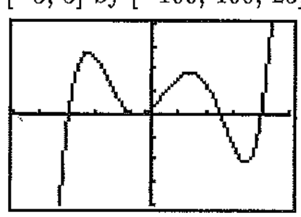

Figure 46

$\boxed{46}$ From the graph, we see that $f(x) = x^5 - 2.5x^4 - 12.75x^3 + 19.625x^2 + 27.625x + 7.5$ has zeros of -3, -0.5, 2.5, and 4. There is a double root at -0.5. Since the leading coefficient of f is 1, we have $f(x) = (x+3)(x+0.5)^2(x-2.5)(x-4)$.

$\boxed{47}$ Since the zeros are -2, 1, 2, and 3, the polynomial must have the form

$$f(x) = a(x+2)(x-1)(x-2)(x-3).$$

Now, $f(0) = a(2)(-1)(-2)(-3) = -12a$ and $f(0) = -24 \Rightarrow -12a = -24 \Rightarrow a = 2$. Let $f(x) = 2(x+2)(x-1)(x-2)(x-3)$. Since f has been completely determined, we must check the remaining data point(s). $f(-1) = 2(1)(-2)(-3)(-4) = -48 \neq -52$.

Thus, a fourth-degree polynomial *does not* fit the data points.

$\boxed{48}$ $f(x) = a(x)(x+3)(x+1)(x-2)(x-3)$. $f(-2) = a(-2)(1)(-1)(-4)(-5) = 40a$ and $f(-2) = 5 \Rightarrow 40a = 5 \Rightarrow a = \frac{1}{8}$. Let $f(x) = \frac{1}{8}x(x+3)(x+1)(x-2)(x-3)$. $f(1) = \frac{1}{8}(1)(4)(2)(-1)(-2) = 2$.

Thus, a fifth-degree polynomial *does* fit the data points.

$\boxed{49}$ $f(x) = a(x-2)(x-5.2)(x-10.1)$. $f(1.1) = a(-33.21) = -49.815 \Rightarrow a = 1.5$. Let $f(x) = 1.5(x-2)(x-5.2)(x-10.1)$. $f(3.5) = 25.245$ and $f(6.4) = -29.304$.

Thus, a third-degree polynomial *does* fit the data points.

$\boxed{50}$ $f(x) = a(x-1.25)(x-2)(x-6.5)(x-10)$. $f(2.5) = a(18.75) = 56.25 \Rightarrow a = 3$. Let $f(x) = 3(x-1.25)(x-2)(x-6.5)(x-10)$. $f(2.5) = 56.25$ and $f(3) = 128.625$. However, $f(9) = -406.875 \neq -307.75$.

Thus, a fourth-degree polynomial *does not* fit the data points.

$\boxed{51}$ The zeros are 0, 5, 19, 24, and $f(12) = 10$. $f(t) = a(t)(t-5)(t-19)(t-24)$;
$f(12) = a(12)(7)(-7)(-12) = 10 \Rightarrow 7056a = 10 \Rightarrow a = \frac{5}{3528}$.

$$f(t) = \tfrac{5}{3528}t(t-5)(t-19)(t-24)$$

$\boxed{52}$ $f(x) = a(x-c_1)(x-c_2)(x-c_3)$;

$$f(c) = a(c-c_1)(c-c_2)(c-c_3) = 1 \Rightarrow a = \frac{1}{(c-c_1)(c-c_2)(c-c_3)}$$

$$f(x) = \frac{1}{(c-c_1)(c-c_2)(c-c_3)}(x-c_1)(x-c_2)(x-c_3)$$

$\boxed{53}$ The graph of f does not cross the x-axis at a zero of even multiplicity, but does cross the x-axis at a zero of odd multiplicity. The higher the multiplicity of a zero, the more horizontal the graph of f is near that zero. See *Figure 53*.

[−3, 3] by [−2, 2]

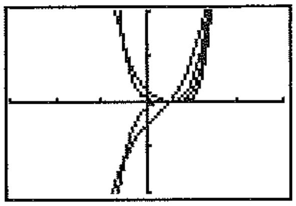

Figure 53

[−3, 3] by [−2, 2]

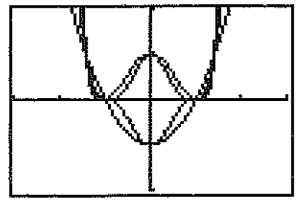

Figure 54

54 The conclusions from Exercise 53 do not change when there is more than one zero.

55 From the graph of f there are two zeros. They are -1.2 and 1.1. The zero at -1.2 has even multiplicity and the zero at 1.1 has odd multiplicity. Since f has degree 3, the zero at -1.2 must have multiplicity 2 and the zero at 1.1 has multiplicity 1.

[−3, 3] by [−3, 1]

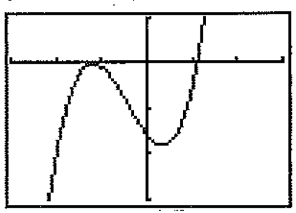

Figure 55

[−3, 3] by [−2, 2]

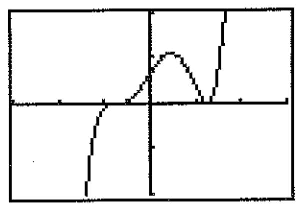

Figure 56

56 From the graph of f there are two zeros. They are -0.75 and 1.25. The zero at -0.75 has odd multiplicity and the zero at 1.25 has even multiplicity. Since f has degree 5, there are two possibilities for the multiplicity: -0.75 has multiplicity 1 and 1.25 has multiplicity 4, or -0.75 has multiplicity 3 and 1.25 has multiplicity 2. By careful inspection we can see that the graph of f levels off as it crosses the x-axis at -0.75. This means that the multiplicity of this zero is greater than 1. (See Exercise 53.) Thus, -0.75 has multiplicity 3 and 1.25 has multiplicity 2.

57 From the graph of $A(t) = -\frac{1}{2400}t^3 + \frac{1}{20}t^2 + \frac{7}{6}t + 340$, we see that $A = 400$ when $t \approx 27.1$. Thus, the carbon dioxide concentration will be 400 in $1980 + 27.1 = 2007.1$, or, during the year 2007.

[0, 60, 10] by [0, 600, 100]

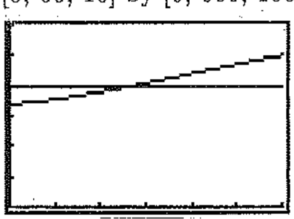

Figure 57

[0, 60, 10] by [−2, 2]

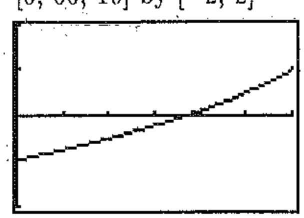

Figure 58

58 From the graph of $T(t) = \frac{21}{5{,}000{,}000}t^3 - \frac{127}{1{,}000{,}000}t^2 + \frac{1293}{50{,}000}t - 1$, we see that $T = 0$ when $t \approx 37.1$. Thus, the average temperature will have increased by 1°C in $1980 + 37.1 = 2017.1$, or, during the year 2017.

59 (a) Graphing the data and the functions show that the best fit is $h(x)$.

(b) Since the temperature changes sign between April and May and between October and November, an average temperature of 0°F occurs when $4 \leq x \leq 5$ and $10 \leq x \leq 11$.

(c) Finding the zeros of h between 1 and 12 gives us $x \approx 4.02$, 10.53.

[0.5, 12.5] by [−30, 50, 10]

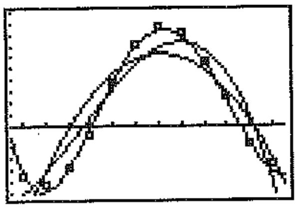

Figure 59

[0.5, 12.5] by [−20, 70, 10]

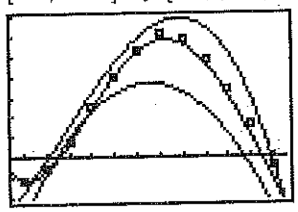

Figure 60

60 (a) Graphing the data and the functions show that the best fit is $h(x)$.

(b) Since the temperature changes sign between February and March and between November and December, an average temperature of 0°F occurs when $2 \leq x \leq 3$ and $11 \leq x \leq 12$.

(c) Finding the zeros of h between 1 and 12 gives us $x \approx 2.54$, 11.42.

61 Let $r = 6$ and $k = 0.7$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = \frac{604.8\pi}{3} - 6\pi x^2 + \frac{1}{3}\pi x^3$ and determine the positive zeros. There are two zeros located at $x \approx 7.64$, 15.47. Since the sphere floats, it will not sink deeper than twice the radius, which is 12 centimeters. Thus, the pine sphere will sink approximately 7.64 centimeters into the water.

[−20, 20, 5] by [−800, 800, 100]

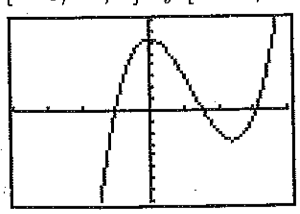

Figure 61

[−20, 20, 5] by [−1000, 1000, 100]

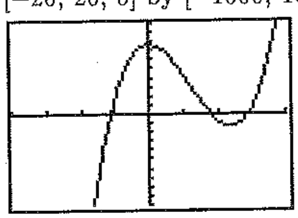

Figure 62

62 Let $r = 6$ and $k = 0.85$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = \frac{734.4\pi}{3} - 6\pi x^2 + \frac{1}{3}\pi x^3$ and determine the positive zeros. There are two zeros located at $x \approx 9.07$, 14.51. Since the sphere floats, it will not sink deeper than twice the radius, which is 12. Thus, the oak sphere will sink approximately 9.07 centimeters into the water. This is slightly deeper than the pine sphere because the density of oak wood is greater than that of pine.

$\boxed{63}$ Let $r = 6$ and $k = 1$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = 288\pi - 6\pi x^2 + \frac{1}{3}\pi x^3$ and determine the positive zero. There is a zero at $x = 12$. This means that the entire sphere is just submerged. The sphere has the same density as water and neither sinks nor floats, much like a balloon filled with water.

[−20, 20, 5] by [−1000, 1000, 100]

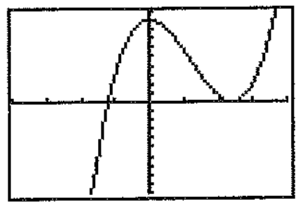

Figure 63

4.4 Exercises

$\boxed{1}$ Since $3 + 2i$ is a root, so is $3 - 2i$. Thus, the polynomial is of the form $[x - (3 + 2i)][x - (3 - 2i)] = x^2 - 6x + 13$ {from the discussion on the top of p. 282}.

$\boxed{2}$ $[x - (-4 + 3i)][x - (-4 - 3i)] = x^2 + 8x + 25$

$\boxed{3}$ $(x - 2)[x - (-2 - 5i)][x - (-2 + 5i)] = (x - 2)(x^2 + 4x + 29)$

$\boxed{4}$ $(x + 3)[x - (1 - 7i)][x - (1 + 7i)] = (x + 3)(x^2 - 2x + 50)$

$\boxed{5}$ $x(x + 1)[x - (3 + i)][x - (3 - i)] = x(x + 1)(x^2 - 6x + 10)$

$\boxed{6}$ $x(x - 2)[x - (-2 - i)][x - (-2 + i)] = x(x - 2)(x^2 + 4x + 5)$

$\boxed{7}$ $[x - (4 + 3i)][x - (4 - 3i)][x - (-2 + i)][x - (-2 - i)] = (x^2 - 8x + 25)(x^2 + 4x + 5)$

$\boxed{8}$ $[x - (3 + 5i)][x - (3 - 5i)][x - (-1 - i)][x - (-1 + i)] = (x^2 - 6x + 34)(x^2 + 2x + 2)$

$\boxed{9}$ $[x - (-2i)][x - (2i)][x - (1 - i)][x - (1 + i)] = x(x^2 + 4)(x^2 - 2x + 2)$

$\boxed{10}$ $[x - (3i)][x - (-3i)][x - (4 + i)][x - (4 - i)] = x(x^2 + 9)(x^2 - 8x + 17)$

Note: Show that none of the possible rational roots listed satisfy the equation in 11–14.

$\boxed{11}$ $x^3 + 3x^2 - 4x + 6 = 0$ • ★ $\pm 1, \pm 2, \pm 3, \pm 6$

$\boxed{12}$ $3x^3 - 4x^2 + 7x + 5 = 0$ • ★ $\pm 1, \pm\frac{1}{3}, \pm 5, \pm\frac{5}{3}$

$\boxed{13}$ $x^5 - 3x^3 + 4x^2 + x - 2 = 0$ • ★ $\pm 1, \pm 2$

$\boxed{14}$ $2x^5 + 3x^3 + 7 = 0$ • ★ $\pm 1, \pm\frac{1}{2}, \pm 7, \pm\frac{7}{2}$

$\boxed{15}$ $x^3 - x^2 - 10x - 8 = 0$ • ★ $-2, -1, 4$

$\boxed{16}$ $x^3 + x^2 - 14x - 24 = 0$ • ★ $-3, -2, 4$

$\boxed{17}$ $2x^3 - 3x^2 - 17x + 30 = 0$ • ★ $-3, 2, \frac{5}{2}$

$\boxed{18}$ $12x^3 + 8x^2 - 3x - 2 = 0$ • ★ $-\frac{2}{3}, \pm\frac{1}{2}$

$\boxed{19}$ $x^4 + 3x^3 - 30x^2 - 6x + 56 = 0$ • ★ $-7, \pm\sqrt{2}, 4$

$\boxed{20}$ $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$ • ★ -1 (multiplicity 2), $\frac{1}{3}, 2, 3$

[21] $6x^5 + 19x^4 + x^3 - 6x^2 = 0$ • ★ $-3, -\frac{2}{3}, 0$ (multiplicity 2), $\frac{1}{2}$

[22] $6x^4 + 5x^3 - 17x^2 - 6x = 0$ • ★ $-2, -\frac{1}{3}, 0, \frac{3}{2}$

[23] $8x^3 + 18x^2 + 45x + 27 = 0$ • ★ $-\frac{3}{4}, -\frac{3}{4} \pm \frac{3}{4}\sqrt{7}\,i$

[24] $3x^3 - x^2 + 11x - 20 = 0$ • ★ $\frac{4}{3}, -\frac{1}{2} \pm \frac{1}{2}\sqrt{19}\,i$

[25] $f(x) = 6x^5 - 23x^4 + 24x^3 + x^2 - 12x + 4$ has zeros at $-\frac{2}{3}, \frac{1}{2}$, 1 (mult. 2), and 2.

Thus, $f(x) = 6(x + \frac{2}{3})(x - \frac{1}{2})(x - 1)^2(x - 2) = (3x + 2)(2x - 1)(x - 1)^2(x - 2)$.

[26] $f(x) = -6x^5 + 5x^4 + 14x^3 - 8x^2 - 8x + 3$ has zeros at -1 (mult. 2), $\frac{1}{3}$, 1, and $\frac{3}{2}$.

Thus, $f(x) = -6(x + 1)^2(x - \frac{1}{3})(x - 1)(x - \frac{3}{2}) = (x + 1)^2(3x - 1)(1 - x)(2x - 3)$.

[27] From the graph, we see that $f(x) = 2x^3 - 25.4x^2 + 3.02x + 24.75$ has zeros of approximately -0.9, 1.1, and 12.5. Since the leading coefficient of f is 2, we have $f(x) = 2(x + 0.9)(x - 1.1)(x - 12.5)$.

[−5, 15, 5] by [−600, 100, 100]

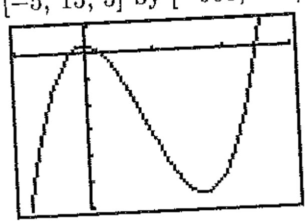

Figure 27

[−5, 5] by [−15, 15, 5]

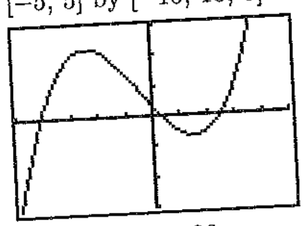

Figure 28

[28] From the graph, we see that $f(x) = 0.5x^3 + 0.65x^2 - 5.365x + 1.5375$ has zeros of approximately -4.1, 0.3, and 2.5. Since the leading coefficient of f is 0.5, we have $f(x) = 0.5(x + 4.1)(x - 0.3)(x - 2.5)$.

[29] No. If i is a root, then $-i$ is also a root. Hence, the polynomial would have factors $x - 1$, $x + 1$, $x - i$, $x + i$ and therefore would be of degree greater than 3.

[30] The theorem applies only to polynomials with real coefficients whereas the polynomial in question has nonreal complex coefficients.

[31] Since n is odd and nonreal complex zeros occur in conjugate pairs for polynomials with real coefficients, there must be at least one real zero.

[32] By the theorem on rational zeros, r is of the form c/d, where c is a factor of a_0 and d is a factor of a_n. Since a_n is 1, its divisors are ± 1.

Thus, r will be an integer and a factor of a_0.

[33] (a) $V(x) = x(20 - 2x)(30 - 2x) = 1000 \Rightarrow 4x^3 - 100x^2 + 600x - 1000 = 0 \Rightarrow$ $4(x - 5)\left[x - (10 - 5\sqrt{2})\right]\left[x - (10 + 5\sqrt{2})\right] = 0$. The allowable range from Exercise 41 of Section 4.1 was (0, 10), so discard $10 + 5\sqrt{2}$.

The two boxes having volume 1000 in^3 have dimensions

⟦A⟧ $5 \times 10 \times 20$ and ⟦B⟧ $(10 - 5\sqrt{2}) \times (10\sqrt{2}) \times (10 + 10\sqrt{2})$.

(b) The surface area function is

$S(x) = (20 - 2x)(30 - 2x) + 2(x)(20 - 2x) + 2(x)(30 - 2x) = -4x^2 + 600.$

$S(5) = 500$ and $S(10 - 5\sqrt{2}) = 400\sqrt{2} \approx 565.7$ so box ⟦A⟧ has less surface area.

$\boxed{34}$ From Exercise 42 in §4.1, $V(x) = x^2(6 - 2x)$. $V(x) = 4 \Rightarrow x^3 - 3x^2 + 2 = 0 \Rightarrow$

$(x - 1)(x^2 - 2x - 2) = 0 \Rightarrow \{x > 0\}\ x = 1, 1 + \sqrt{3}.$

$\boxed{35}$ (a) The sides of the triangle are x, $x + 1$, and $\sqrt{2x + 1}$.

$A = \frac{1}{2}bh \Rightarrow 30 = \frac{1}{2}x\sqrt{2x+1} \Rightarrow 2x^3 + x^2 - 3600 = 0.$

(b) There is one sign change in $f(x) = 2x^3 + x^2 - 3600$.

By Descartes' rule of signs there is one positive real root.

Synthetically dividing 13 into f, we obtain a third row of 2, 27, 351, and 963.

These are all positive so 13 is an upper bound for the zeros of f.

(c) $f(x) = 0 \Rightarrow (x - 12)(2x^2 + 25x + 300) = 0.$

The legs of the triangle are 12 and 5, and the hypotenuse is 13.

$\boxed{36}$ $V(x) = 27\pi \Rightarrow 10\pi x^2 + \frac{4}{3}\pi x^3 = 27\pi \Rightarrow 4x^3 + 30x^2 - 81 = 0 \Rightarrow$

$(2x - 3)(2x^2 + 18x + 27) = 0.$

$x = \frac{3}{2}$ is the only positive solution, so the radius is 1.5 ft.

$\boxed{37}$ (a) $\text{Volume}_{\text{total}} = \text{Volume}_{\text{cube}} + \text{Volume}_{\text{roof}}$

$= x^3 + \frac{1}{2}bhx = x^3 + \frac{1}{2}(x)(6 - x)(x) = x^3 + \frac{1}{2}x^2(6 - x).$

(b) Volume $= 80 \Rightarrow x^3 + 6x^2 - 160 = 0 \Rightarrow (x - 4)(x^2 + 10x + 40) = 0.$

The length of the side is 4 ft.

$\boxed{38}$ Form a triangle with the center pole and the midpoint of any side.

The hypotenuse (call it y) is the height of one of the triangular sides with base x.

$(\frac{1}{2}x)^2 + (8)^2 = y^2 \Rightarrow y = \sqrt{64 + \frac{1}{4}x^2}$. $\text{Area}_{\text{total}} = \text{Area}_{\text{base}} + \text{Area}_{\text{4 sides}} =$

$(\text{side})(\text{side}) + 4(\frac{1}{2})(\text{base})(\text{height}) = x^2 + 4(\frac{1}{2})(x)\sqrt{64 + \frac{1}{4}x^2} = 384 \Rightarrow$

$147{,}456 - 768x^2 + x^4 = 4x^2(64 + \frac{1}{4}x^2) \Rightarrow x^2 = 144 \Rightarrow x = 12.$

$\boxed{39}$ $x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 0.62 = -1 \Leftrightarrow$

$x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 1.62 = 0.$

The graph of $y = x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 1.62$ intersects the x-axis three times. The zeros at -1.5 and 1.2 have even multiplicity (since the graph is tangent to the x-axis at these points) and the zero at -0.5 has odd multiplicity (since the graph crosses the x-axis at this point). Since the equation has degree 5, the only possibility is that the zeros at -1.5 and 1.2 have multiplicity 2 and the zero at -0.5 has multiplicity 1. Thus, the equation has no nonreal solutions. See *Figure 39.*

[−4.5, 4.5] by [−3, 3]

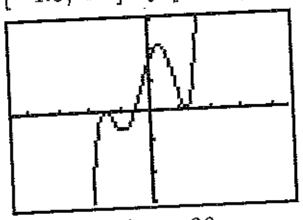

Figure 39

[−4.5, 4.5] by [−3, 3]

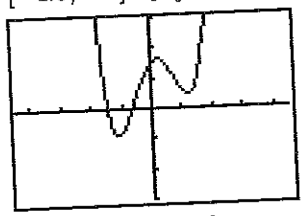

Figure 40

[40] $x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 3.5 = 2 \Leftrightarrow x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 1.5 = 0$. The graph of $y = x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 1.5$ crosses the x-axis twice. There are two real zeros and they have odd multiplicity. By careful inspection we can see that the graph does not level off at either zero. Therefore, the zeros must have multiplicity 1. (See Exercise 53 in §4.3.) Since the equation has degree 4, there are two nonreal solutions.

[41] Graph $y = x^4 + 1.4x^3 + 0.44x^2 - 0.56x - 0.96$.

From the graph, zeros are located at −1.2 and 0.8. Using synthetic division,

$\dfrac{x^4 + 1.4x^3 + 0.44x^2 - 0.56x - 0.96}{x + 1.2} = x^3 + 0.2x^2 + 0.2x - 0.8$ and

$\dfrac{x^3 + 0.2x^2 + 0.2x - 0.8}{x - 0.8} = x^2 + x + 1$. The zeros of $x^2 + x + 1$ are $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

Thus, the solutions to the equation are -1.2, 0.8, $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

[−4.5, 4.5] by [−3, 3]

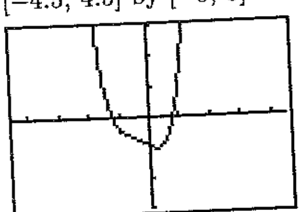

Figure 41

[−6.5, 7] by [−3, 6]

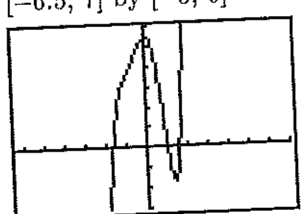

Figure 42

[42] Graph $y = x^5 + 1.1x^4 - 2.62x^3 - 4.72x^2 - 0.2x + 5.44$.

From the graph, zeros are located at −1.7, 1, and 1.6. Using synthetic division,

$\dfrac{x^5 + 1.1x^4 - 2.62x^3 - 4.72x^2 - 0.2x + 5.44}{x + 1.7} = x^4 - 0.6x^3 - 1.6x^2 - 2x + 3.2$,

$\dfrac{x^4 - 0.6x^3 - 1.6x^2 - 2x + 3.2}{x - 1} = x^3 + 0.4x^2 - 1.2x - 3.2$, and

$\dfrac{x^3 + 0.4x^2 - 1.2x - 3.2}{x - 1.6} = x^2 + 2x + 2$. The zeros of $x^2 + 2x + 2$ are $-1 \pm i$.

Thus, the solutions to the equation are -1.7, 1, 1.6, $-1 \pm i$.

[43] From the graph, we see that $D(h) = 0.4$ when $h \approx 10{,}200$.

Thus, the density of the atmosphere is 0.4 kg/m^3 at 10,200 m.

[0, 3E4, 2E3] by [0, 1.2, 0.2]

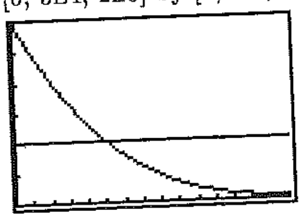

Figure 43

[0, 1000, 100] by [0, 5]

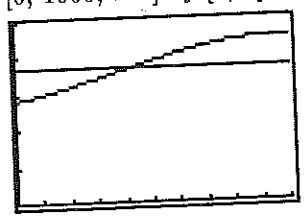

Figure 44

[44] From the graph, we see that $D(h) = 3.7$ when $h \approx 418$. Thus, the density of the earth is 3.7 g/cm^3 at 418 m. (The graphs also intersect at $h \approx -674$ and $h \approx 1394$. However, these values are not in the domain of D.)

4.5 Exercises

[1] (b) $D =$ all nonzero real numbers; $R = D$ (c) Decreasing on $(-\infty, 0)$ and on $(0, \infty)$

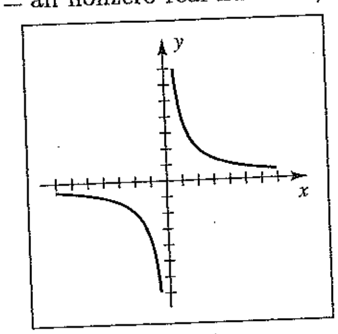

Figure 1

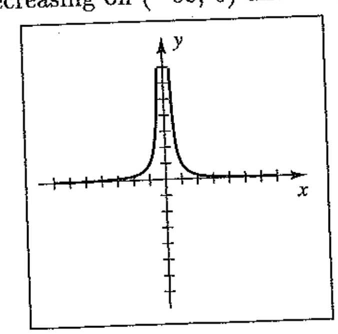

Figure 2

[2] (b) $D =$ all nonzero real numbers; $R = (0, \infty)$

(c) Increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

[3] $f(x) = \dfrac{-2(x+5)(x-6)}{(x-3)(x-6)} = \dfrac{-2(x+5)}{(x-3)}$ if $x \neq 6$ • Note that $x - 6$ appears in both the numerator and denominator, so there is a hole at $x = 6$. The y-value for the hole can be found by substituting 6 for x in the rest of the function. $\dfrac{-2(6+5)}{(6-3)} = \dfrac{-2(11)}{3} = -\dfrac{22}{3}$, so the hole is at $(6, -\frac{22}{3})$. There is one other zero of the denominator, namely 3, so there is a vertical asymptote of $x = 3$. The degrees of the numerator and denominator are the same, namely 2, so the ratio of leading coefficients gives us the value of the horizontal asymptote. In this case, it is $y = \frac{-2}{1} = -2$.

[4] $f(x) = \dfrac{2(x+4)(x+2)}{5(x+2)(x-1)} = \dfrac{2(x+4)}{5(x-1)}$ if $x \neq -2$ • There is a hole at $x = -2$. Its y-value is $\frac{2(2)}{5(-3)} = -\frac{4}{15}$. The vertical asymptote is $x = 1$ and the horizontal asymptote is $y = \frac{2}{5}$.

[5] There is a hole at $x = -2$, so $(x+2)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x = 1$, so $(x-1)$ must be a factor in the denominator. There is an x-intercept at -3, so $(x+3)$ must be a factor in the numerator. The horizontal asymptote is $y = 2$, so the ratio of leading coefficients must be 2. Combining this information gives us this possibility: $f(x) = \dfrac{2(x+3)(x+2)}{(x-1)(x+2)}$

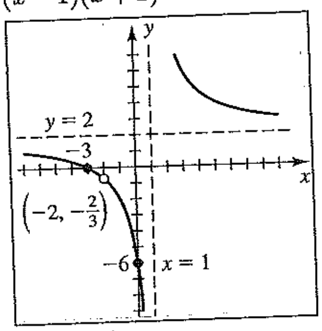

Figure 5

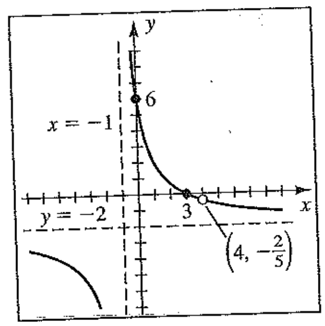

Figure 6

[6] There is a hole at $x = 4$, so $(x-4)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x = -1$, so $(x+1)$ must be a factor in the denominator. There is an x-intercept at 3, so $(x-3)$ must be a factor in the numerator. The horizontal asymptote is $y = -2$, so the ratio of leading coefficients must be -2. Combining this information gives us this possibility: $f(x) = \dfrac{-2(x-3)(x-4)}{(x+1)(x-4)}$

[7] $f(x) = \dfrac{3}{x-4}$ • VA of $x = 4$

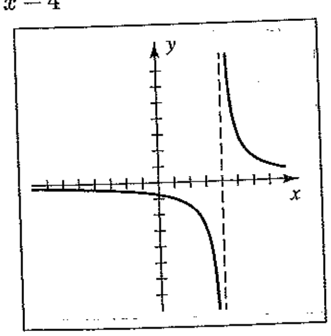

Figure 7

[8] $f(x) = \dfrac{-3}{x+3}$ • VA of $x = -3$

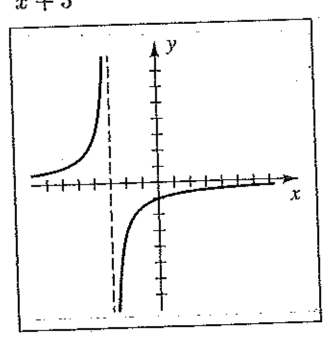

Figure 8

$\boxed{9}$ $f(x) = \dfrac{-3x}{x+2}$ • VA of $x = -2$

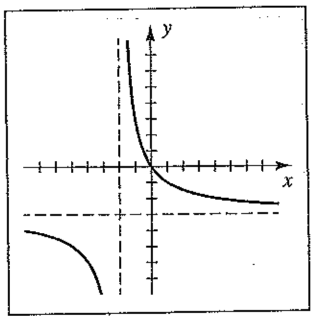

Figure 9

$\boxed{10}$ $f(x) = \dfrac{4x}{2x-5}$ • VA of $x = \frac{5}{2}$

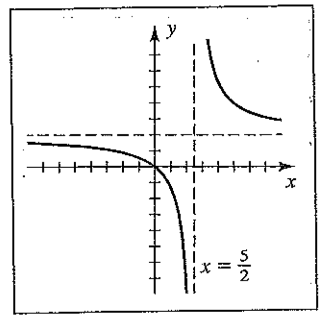

Figure 10

$\boxed{11}$ $f(x) = \dfrac{4x-1}{2x+3}$ • VA of $x = -\frac{3}{2}$

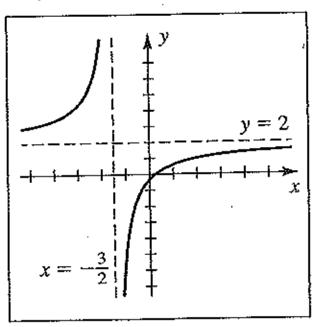

Figure 11

$\boxed{12}$ $f(x) = \dfrac{5x+3}{3x-7}$ • VA of $x = \frac{7}{3}$

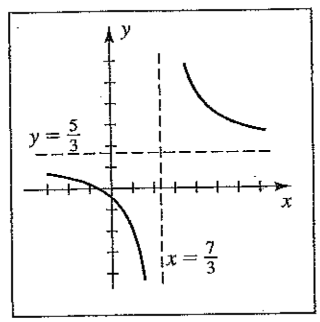

Figure 12

$\boxed{13}$ $f(x) = \dfrac{(4x-1)(x-2)}{(2x+3)(x-2)}$ •

VA of $x = -\frac{3}{2}$; hole at $x = 2$

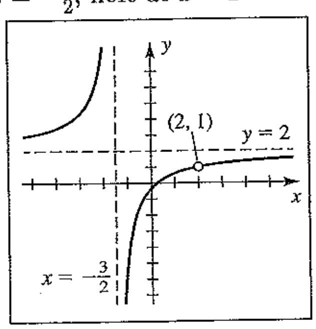

Figure 13

$\boxed{14}$ $f(x) = \dfrac{(5x+3)(x+1)}{(3x-7)(x+1)}$ •

VA of $x = \frac{7}{3}$; hole at $x = -1$

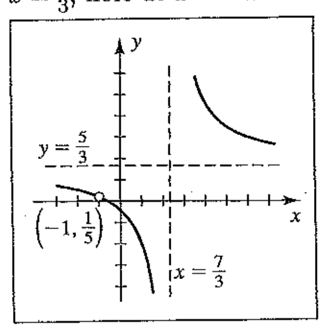

Figure 14

Note: Let I denote the point of intersection between the function and its horizontal or oblique asymptote. See Exercises 22 and 36 for more detailed work on finding I.

$\boxed{15}$ $f(x) = \dfrac{x-2}{x^2-x-6} = \dfrac{x-2}{(x+2)(x-3)}$; $I = (2, 0)$

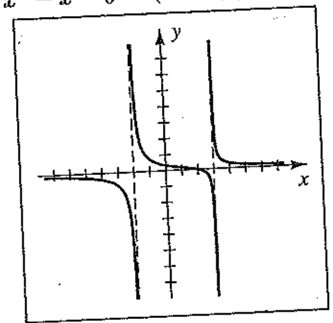

Figure 15

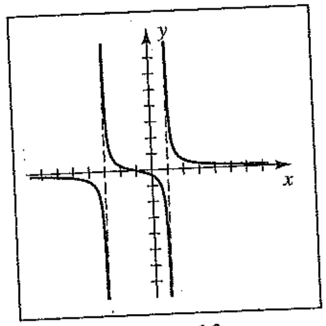

Figure 16

$\boxed{16}$ $f(x) = \dfrac{x+1}{x^2+2x-3} = \dfrac{x+1}{(x+3)(x-1)}$; $I = (-1, 0)$

$\boxed{17}$ $f(x) = \dfrac{-4}{(x-2)^2}$ •

$\boxed{18}$ $f(x) = \dfrac{2}{(x+1)^2}$ •

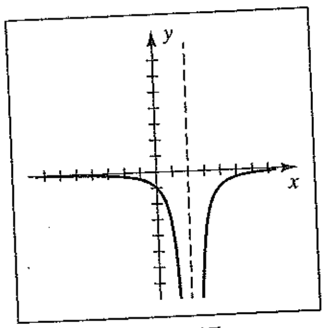

Figure 17

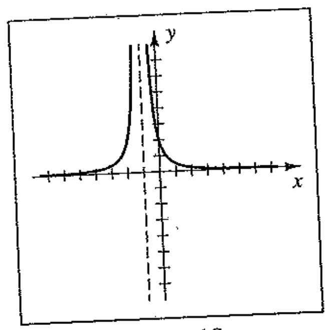

Figure 18

$\boxed{19}$ $f(x) = \dfrac{x-3}{x^2-1} = \dfrac{x-3}{(x+1)(x-1)}$; $I = (3, 0)$

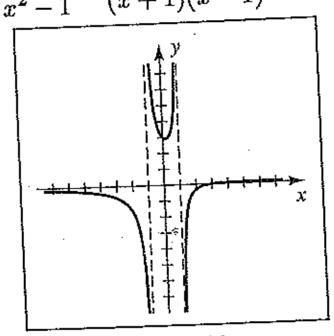

Figure 19

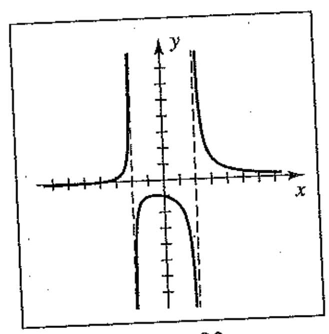

Figure 20

$\boxed{20}$ $f(x) = \dfrac{x+4}{x^2-4} = \dfrac{x+4}{(x+2)(x-2)}$; $I = (-4, 0)$

[21] $f(x)=\dfrac{2x^2-2x-4}{x^2+x-12}=\dfrac{2(x+1)(x-2)}{(x+4)(x-3)}$; $I=(5,\ 2)$

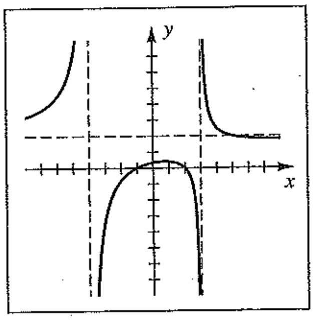

Figure 21

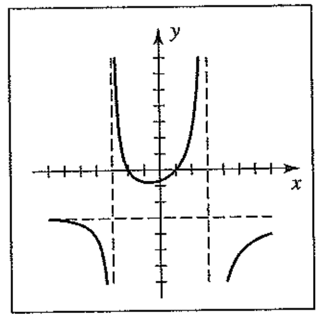

Figure 22

[22] $f(x)=\dfrac{-3x^2-3x+6}{x^2-9}=\dfrac{-3(x+2)(x-1)}{(x+3)(x-3)}$

$f(x)=-3 \;\Rightarrow\; \dfrac{-3x^2-3x+6}{x^2-9}=-3 \;\Rightarrow\; -3x^2-3x+6=-3x^2+27 \;\Rightarrow$
$-3x=21 \;\Rightarrow\; x=-7,\ I=(-7,\ -3).$

[23] $f(x)=\dfrac{-x^2-x+6}{x^2+3x-4}=\dfrac{-1(x+3)(x-2)}{(x+4)(x-1)}$; $I=(-1,\ -1)$

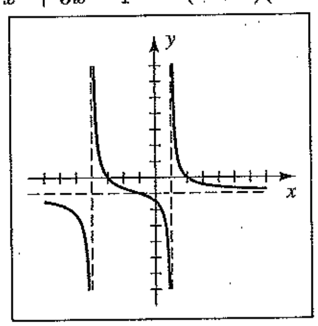

Figure 23

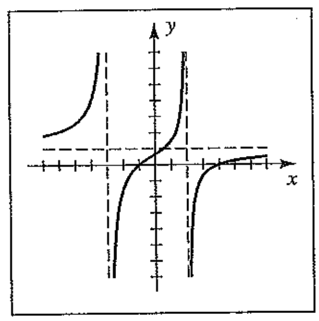

Figure 24

[24] $f(x)=\dfrac{x^2-3x-4}{x^2+x-6}=\dfrac{(x+1)(x-4)}{(x+3)(x-2)}$; $I=(\frac{1}{2},\ 1)$

[25] $f(x)=\dfrac{3x^2-3x-36}{x^2+x-2}=\dfrac{3(x+3)(x-4)}{(x+2)(x-1)}$; $I=(-5,\ 3)$

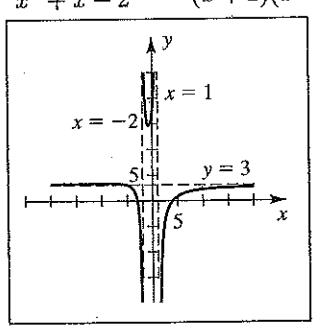

Figure 25

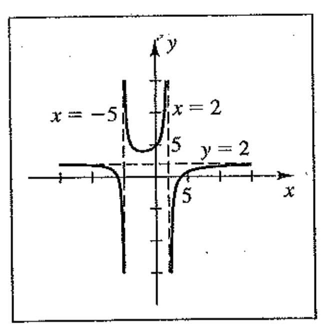

Figure 26

[26] $f(x)=\dfrac{2x^2+4x-48}{x^2+3x-10}=\dfrac{2(x+6)(x-4)}{(x+5)(x-2)}$; $I=(-14,\ 2)$

$\boxed{27}$ $f(x) = \dfrac{-2x^2 + 10x - 12}{x^2 + x} = \dfrac{-2(x-2)(x-3)}{(x+1)(x)}$; $I = (1, -2)$

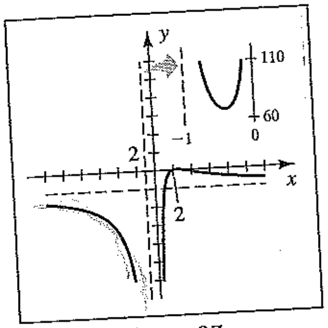

Figure 27

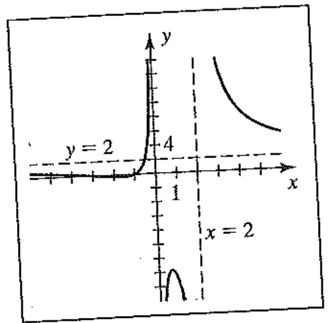

Figure 28

$\boxed{28}$ $f(x) = \dfrac{2x^2 + 8x + 6}{x^2 - 2x} = \dfrac{2(x+3)(x+1)}{(x)(x-2)}$; $I = (-\frac{1}{2}, 2)$

$\boxed{29}$ $f(x) = \dfrac{x-1}{x^3 - 4x} = \dfrac{x-1}{(x+2)(x)(x-2)}$; $I = (1, 0)$

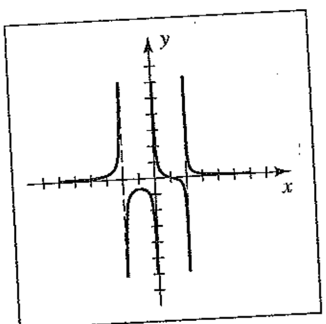

Figure 29

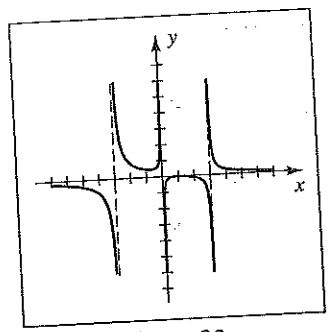

Figure 30

$\boxed{30}$ $f(x) = \dfrac{x^2 - 2x + 1}{x^3 - 9x} = \dfrac{(x-1)^2}{(x+3)(x)(x-3)}$; $I = (1, 0)$

$\boxed{31}$ $f(x) = \dfrac{-3x^2}{x^2+1}$; f is an even function

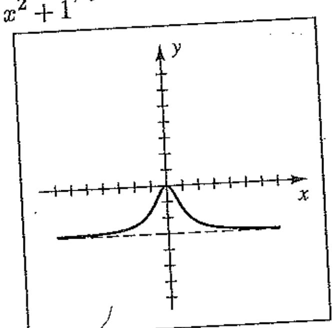

Figure 31

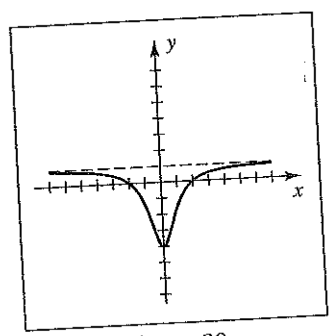

Figure 32

$\boxed{32}$ $f(x) = \dfrac{x^2 - 4}{x^2 + 1} = \dfrac{(x+2)(x-2)}{x^2+1}$; f is an even function

[33] $f(x) = \dfrac{x^2 - x - 6}{x+1} = \dfrac{(x+2)(x-3)}{x+1} = x - 2 - \dfrac{4}{x+1}$.

The expression $\dfrac{4}{x+1} \to 0$ as $x \to \pm\infty$, so $y = x - 2$ is an oblique asymptote for f.

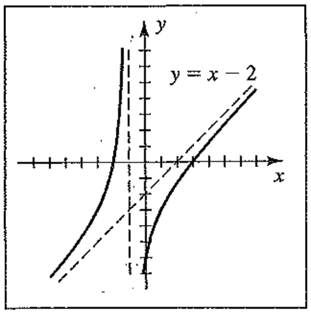

Figure 33

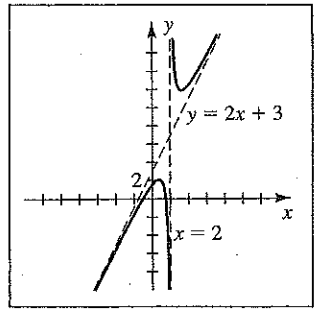

Figure 34

[34] $f(x) = \dfrac{2x^2 - x - 3}{x-2} = \dfrac{(x+1)(2x-3)}{x-2} = 2x + 3 + \dfrac{3}{x-2}$

[35] $f(x) = \dfrac{8 - x^3}{2x^2} = \dfrac{(2-x)(4+2x+x^2)}{2x^2} = -\frac{1}{2}x + \dfrac{8}{2x^2}$

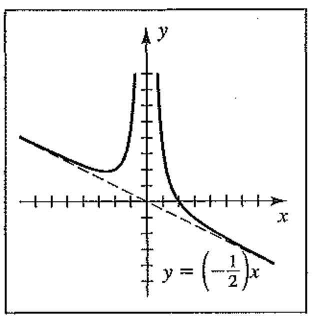

Figure 35

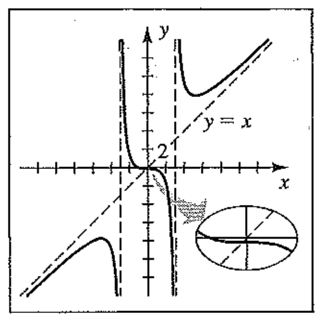

Figure 36

[36] $f(x) = \dfrac{x^3 + 1}{x^2 - 9} = \dfrac{(x+1)(x^2 - x + 1)}{(x+3)(x-3)} = x + \dfrac{9x+1}{x^2-9}$

$f(x) = x \;\Rightarrow\; \dfrac{x^3+1}{x^2-9} = x \;\Rightarrow\; x^3 + 1 = x^3 - 9x \;\Rightarrow\; x = -\frac{1}{9},\ I = (-\frac{1}{9}, -\frac{1}{9})$.

$\boxed{37}$ $f(x)=\dfrac{2x^2+x-6}{x^2+3x+2}=\dfrac{(x+2)(2x-3)}{(x+2)(x+1)}=\dfrac{2x-3}{x+1}$ for $x\neq -2$;

To determine the value of y when $x=-2$, substitute -2 into $\dfrac{2x-3}{x+1}$ to get 7.
There is a hole in the graph at $(-2, 7)$.

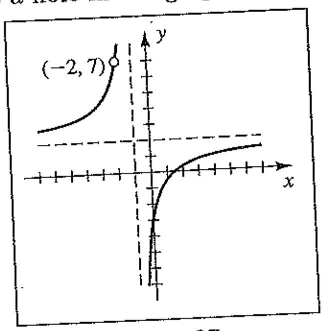

Figure 37

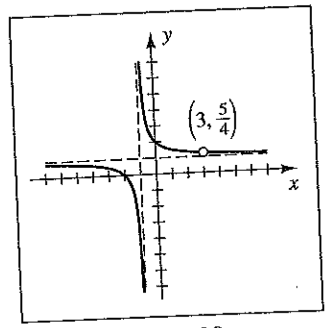

Figure 38

$\boxed{38}$ $f(x)=\dfrac{x^2-x-6}{x^2-2x-3}=\dfrac{(x+2)(x-3)}{(x+1)(x-3)}=\dfrac{x+2}{x+1}$ for $x\neq 3$; hole at $(3, \frac{5}{4})$.

$\boxed{39}$ $f(x)=\dfrac{x-1}{1-x^2}=\dfrac{x-1}{(1+x)(1-x)}=\dfrac{-1}{x+1}$ for $x\neq 1$; hole at $(1, -\frac{1}{2})$

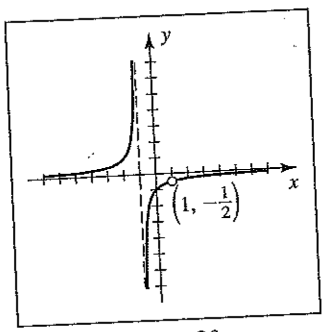

Figure 39

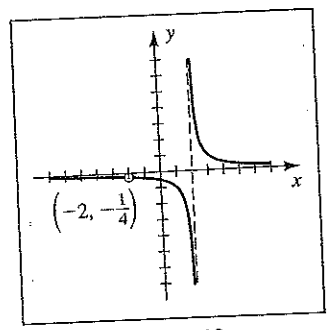

Figure 40

$\boxed{40}$ $f(x)=\dfrac{x+2}{x^2-4}=\dfrac{x+2}{(x+2)(x-2)}=\dfrac{1}{x-2}$ for $x\neq -2$; hole at $(-2, -\frac{1}{4})$

$\boxed{41}$ $f(x)=\dfrac{x^2+x-2}{x+2}=\dfrac{(x+2)(x-1)}{x+2}=x-1$ for $x\neq -2$; hole at $(-2, -3)$

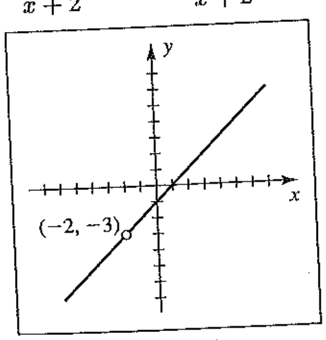

Figure 41

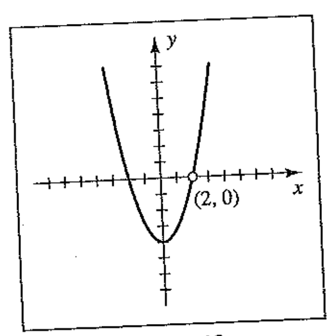

Figure 42

$\boxed{42}$ $f(x)=\dfrac{x^3-2x^2-4x+8}{x-2}=\dfrac{(x^2-4)(x-2)}{x-2}=x^2-4$ for $x\neq 2$; hole at $(2, 0)$

[43] $f(x) = \dfrac{x^2+4x+4}{x^2+3x+2} = \dfrac{(x+2)^2}{(x+1)(x+2)} = \dfrac{x+2}{x+1}$ for $x \neq -2$.

Note that the hole is on the x-axis at $(-2,\ 0)$.

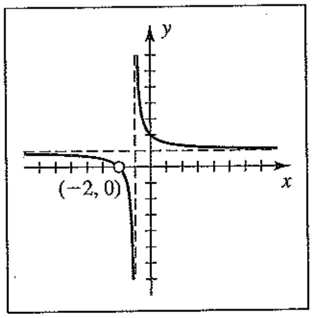

Figure 43

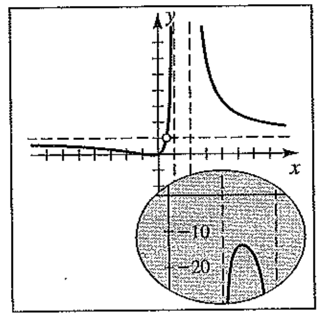

Figure 44

[44] $f(x) = \dfrac{(x^2+x)(2x-1)}{(x^2-3x+2)(2x-1)} = \dfrac{x(x+1)(2x-1)}{(x-1)(x-2)(2x-1)} = \dfrac{x(x+1)}{(x-1)(x-2)}$ for $x \neq \frac{1}{2}$.

Note that the hole is on the horizontal asymptote at $(\frac{1}{2},\ 1)$.

[45] $f(x) = \dfrac{-1(x-3)}{x-4} = \dfrac{3-x}{x-4}$

[46] $f(x) = \dfrac{a(x-2)}{x(x+2)}$; $f(3) = 1$ and $f(3) = \dfrac{a(1)}{3(5)} = \frac{a}{15} \Rightarrow a = 15$; $f(x) = \dfrac{15x-30}{x^2+2x}$

[47] $f(x) = \dfrac{a(x+1)(x-2)}{(x-1)(x+3)(x-2)}$; $f(0) = -2$ and $f(0) = \dfrac{a(1)}{(-1)(3)} = \frac{a}{-3} \Rightarrow a = 6$;

$$f(x) = \frac{6(x+1)(x-2)}{(x-1)(x+3)(x-2)} = \frac{6x^2-6x-12}{x^3-7x+6}$$

[48] $f(x) = \dfrac{2(x+2)(x-1)x}{(x+1)(x-3)x} = \dfrac{2x^3+2x^2-4x}{x^3-2x^2-3x}$

[49] (a) The radius of the outside cylinder is $(r+0.5)$ ft and its height is $(h+1)$ ft.

Since the volume is 16π ft^3, we have $16\pi = \pi(r+0.5)^2(h+1) \Rightarrow$

$$h = \frac{16}{(r+0.5)^2} - 1.$$

(b) $V(r) = \pi r^2 h = \pi r^2\left[\dfrac{16}{(r+0.5)^2} - 1\right]$

(c) r and h must both be positive. $h > 0 \Rightarrow \dfrac{16}{(r+0.5)^2} - 1 > 0 \Rightarrow$

$16 > (r+0.5)^2 \Rightarrow |r+0.5| < 4 \Rightarrow -4.5 < r < 3.5$. The last inequality combined with $r > 0$ means that the excluded values are $r \le 0$ and $r \ge 3.5$.

[50] $a = 100$ and $y = ta/(t+12) \Rightarrow y = 100t/(t+12)$.

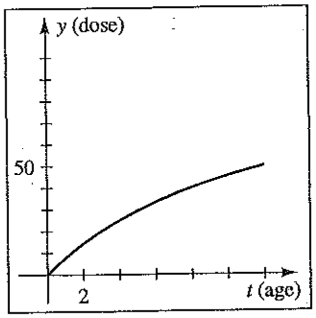

Figure 50

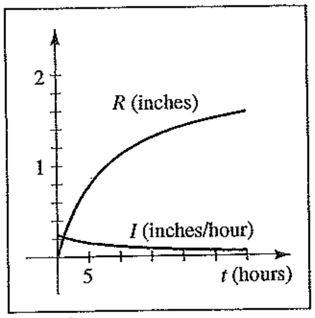

Figure 52

[51] (a) Since 5 gallons of water flow into the tank each minute, $V(t) = 50 + 5t$. Since each additional gallon of water contains 0.1 lb of salt, $A(t) = 5(0.1)t = 0.5t$.

(b) $c(t) = \dfrac{A(t)}{V(t)} = \dfrac{0.5t}{50+5t} = \dfrac{t}{10t+100}$ lb/gal

(c) As $t \to \infty$, $c(t) \to 0.1$ lb. of salt per gal.

[52] (a) As t increases, the total number of inches of rain approaches the constant a.

(b) Note that at the start of the storm, $I = 0.25$ and $R = 0$, i.e., the intensity is greater than the accumulation. See *Figure 52.*

[53] (a) $R > S \Rightarrow \dfrac{4500\,S}{S+500} > S \Rightarrow \dfrac{S(S-4000)}{S+500} < 0 \; \{S > 0\} \Rightarrow 0 < S < 4000$

(b) The greatest possible number of offspring that survive to maturity is 4500, the horizontal asymptote value. 90% of 4500 is 4050.

$R = 4050 \Rightarrow 4050 = \dfrac{4500\,S}{S+500} \Rightarrow 4050S + 2{,}025{,}000 = 4500S \Rightarrow$
$2{,}025{,}000 = 450S \Rightarrow S = 4500.$

(c) 80% of 4500 is 3600. $R = 3600 \Rightarrow S = 2000$.

(d) A 125% increase $\left\{\dfrac{4500-2000}{2000} \times 100\right\}$ in the number S of spawners produces only a 12.5% increase $\left\{\dfrac{4050-3600}{3600} \times 100\right\}$ in the number R of offspring surviving to maturity.

[54] (a) $x = 20 \Rightarrow D \approx 229.4$ and $x = 25 \Rightarrow D = 189.1$; the density decreases

(b) As $x \to \infty$, $D \to 0$.

The density gets closer to 0 as the distance from the center increases.

(c) $D > 400 \Rightarrow \dfrac{5000x}{x^2+36} > 400 \Rightarrow$

$25x > 2(x^2+36) \left\{\text{multiply by } \tfrac{1}{200}(x^2+36)\text{, which is positive}\right\} \Rightarrow$

$2x^2 - 25x + 72 < 0 \Rightarrow (2x-9)(x-8) < 0 \Rightarrow 4.5 < x < 8$

55 Assign $20x^2+80x+72$ to Y_1, $10x^2+40x+41$ to Y_2, and Y_1/Y_2 to Y_3. Zoom-in around $(-2, -8)$ to confirm that this a low point and that there is not a vertical asymptote at $x=-2$. To determine the vertical asymptotes, graph Y_2 only {turn off Y_3}, and look for its zeros. If these values are not zeros of the numerator, then they are the values of the vertical asymptotes. No vertical asymptotes in this case.

$[-9, 3]$ by $[-9, 3]$

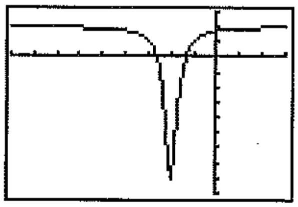

Figure 55

$[-5, 9]$ by $[-1, 13]$

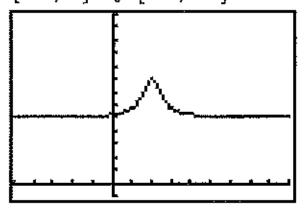

Figure 56

56 Similar to Exercise 55, there is a high point at $(2, 8)$. No vertical asymptotes.

57 $f(x)=\dfrac{(x-1)^2}{(x-0.999)^2}$ • Note that a standard viewing rectangle gives the horizontal line $y=1$. *Figure 57* was obtained by using Dot mode. An equation of the vertical asymptote is $x=0.999$.

$[0.7, 1.3, 0.1]$ by $[0.8, 1.2, 0.1]$

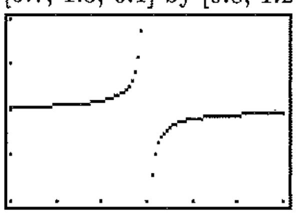

Figure 57

$[-4, 8]$ by $[-1, 7]$

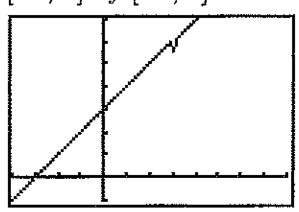

Figure 58

58 $f(x)=\dfrac{x^2-9.01}{x-3}$ • Note that a standard viewing rectangle gives a line, but we recognize that there is a vertical asymptote at $x=3$ since $x-3$ is a factor of the denominator but not of the numerator.

59 (a) The graph of g is the horizontal line $y=1$ with holes at $x=0, \pm 1, \pm 2, \pm 3$.
The TI-85/86 shows a small break in the line to indicate a hole.

(b) The graph of h is the graph of p with holes at $x=0, \pm 1, \pm 2, \pm 3$.

60 (a) The graph of f is that of a seventh-degree polynomial with zeros at $x=0, \pm 1, \pm 2, \pm 3$. Its sign changes at each zero.

(b) The graph of k has vertical asymptotes at $x=0, \pm 1, \pm 2, \pm 3$. Its sign changes at each asymptote. The values of k are reciprocals of the values of f—so as f gets larger, k gets smaller, and vice versa. Try graphing k with a viewing rectangle of $[-4, 4]$ by $[-0.5, 0.5]$.

[61] (a) GPA with additional credits = desired GPA $\Rightarrow$ $\dfrac{48(2.75)+y(4.0)}{48+y}=x$ $\Rightarrow$

$132+4y=48x+xy$ $\Rightarrow$ $132-48x=xy-4y$ $\Rightarrow$ $132-48x=y(x-4)$ $\Rightarrow$

$$y=\frac{132-48x}{x-4}$$

(b)

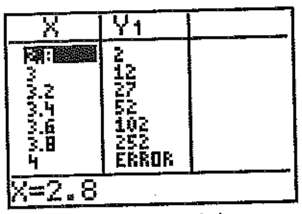

Figure 61(b)

(c) [2, 4] by [0, 1000, 100]

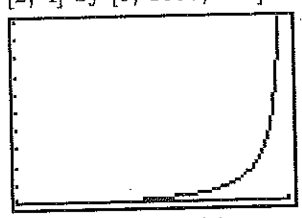

Figure 61(c)

(d) The vertical asymptote of the graph is $x=4$.

(e) Regardless of the number of additional credit hours obtained at 4.0, a cumulative GPA of 4.0 is not attainable.

4.6 Exercises

[1] $u=kv$; $12=k(30)$ $\Rightarrow$ $k=\frac{2}{5}$

[2] $s=kt$; $18=k(10)$ $\Rightarrow$ $k=\frac{9}{5}$

[3] $r=k\frac{s}{t}$; $7=\dfrac{k(-2)}{4}$ $\Rightarrow$ $k=-14$

[4] $w=k\dfrac{z}{\sqrt{u}}$; $6=\dfrac{k(2)}{3}$ $\Rightarrow$ $k=9$

[5] $y=k\dfrac{x^2}{z^3}$; $25=\dfrac{k(25)}{27}$ $\Rightarrow$ $k=27$

[6] $q=\dfrac{k}{x+y}$; $1.4=\dfrac{k}{0.5+0.7}$ $\Rightarrow$ $k=1.68$

[7] $z=kx^2y^3$; $16=k(49)(-8)$ $\Rightarrow$ $k=-\frac{2}{49}$

[8] $r=k\dfrac{sv}{p^3}$; $40=\dfrac{k(2)(3)}{125}$ $\Rightarrow$ $k=\frac{2500}{3}$

[9] $y=k\dfrac{x}{z^2}$; $16=\dfrac{k(4)}{9}$ $\Rightarrow$ $k=36$

[10] $y=k\dfrac{x}{r+s}$; $2=\dfrac{k(3)}{5+7}$ $\Rightarrow$ $k=8$

[11] $y=k\dfrac{\sqrt{x}}{z^3}$; $5=\dfrac{k(3)}{8}$ $\Rightarrow$ $k=\frac{40}{3}$

[12] $y=k\dfrac{x^2}{\sqrt{z}}$; $10=\dfrac{k(25)}{4}$ $\Rightarrow$ $k=\frac{8}{5}$

[13] (a) $P=kd$

(b) $118=k(2)$ $\Rightarrow$ $k=59$

(c) $P=59(5)=295\ \text{lb/ft}^2$

(d) $P=59d$ is a simple linear relationship. The graph is a line with slope 59 and y-intercept 0.

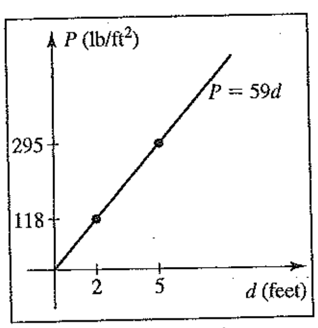

Figure 13(d)

$\boxed{14}$ (a) $F = kx$

(b) $4 = k(0.3) \Rightarrow k = \frac{40}{3}$

(c) $F = \frac{40}{3}(1.5) = 20$ lb

(d) $F = \frac{40}{3}x$ is a simple linear relationship. The graph is a line with slope $\frac{40}{3}$ and y-intercept 0.

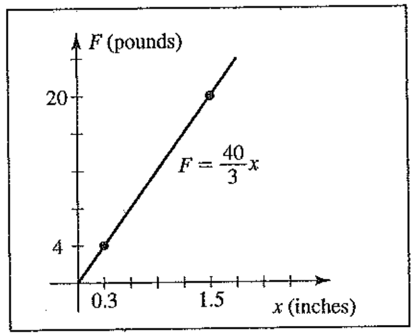

Figure 14(d)

$\boxed{15}$ (a) $R = k\frac{l}{d^2} = \frac{kl}{d^2}$

(b) $25 = \frac{k(100)}{(0.01)^2} \Rightarrow k = \frac{1}{40{,}000}$

(c) $R = \frac{50}{(40{,}000)(0.015)^2} = \frac{50}{9}$ ohms

(d) $R = \frac{kl}{d^2} = \frac{(1/40{,}000)(100)}{d^2} = \frac{1}{400d^2}$.

The graph of R for $d > 0$ has a vertical asymptote of $R = 0$ and a horizontal asymptote of $d = 0$.

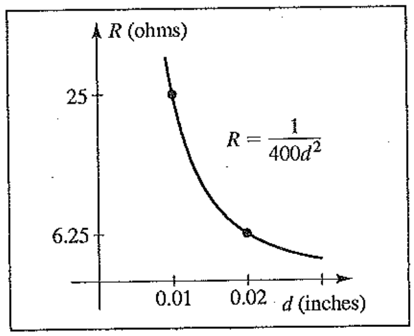

Figure 15(d)

$\boxed{16}$ (a) $I = \frac{k}{d^2}$

(b) $1{,}000{,}000 = \frac{k}{(50)^2} \Rightarrow k = 2.5 \times 10^9$

(c) $I = \frac{2.5 \times 10^9}{(5280)^2} \approx 89.7$ candlepower

(d) $I = \frac{2.5 \times 10^9}{d^2}$ has a VA of $I = 0$ and a HA of $d = 0$.

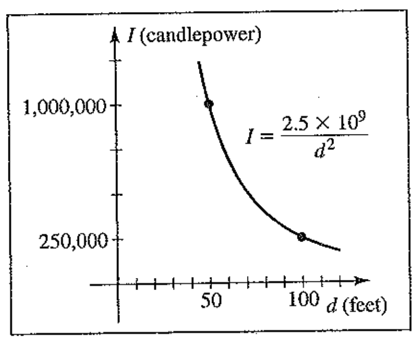

Figure 16(d)

$\boxed{17}$ (a) $P = k\sqrt{l}$

(b) $1.5 = k\sqrt{2} \Rightarrow k = \frac{3}{4}\sqrt{2}$

(c) $P = \frac{3}{4}\sqrt{2}(\sqrt{6}) = \frac{3}{2}\sqrt{3}$ sec

$\boxed{18}$ (a) $C = 2\pi r \Leftrightarrow r = \frac{C}{2\pi}$. $V = \pi r^2 L = \pi\left(\frac{C}{2\pi}\right)^2 L = \frac{1}{4\pi}C^2 L$.

(b) $V = \frac{1}{4\pi}(22)^2(27) = \frac{3267}{\pi} \approx 1039.9$ cm^3, or 1040 cm^3

$\boxed{19}$ (a) $T = kd^{3/2}$

(b) $365 = k(93)^{3/2} \Rightarrow k = \frac{365}{(93)^{3/2}}$

(c) $T = \frac{365}{(93)^{3/2}} \cdot (67)^{3/2} \approx 223.2$ days

[20] (a) $R = kv^2$ (b) $150 = k(70)^2 \Rightarrow k = \frac{3}{98}$

(c) $R = \frac{3}{98}(80)^2 = \frac{9600}{49} \approx 195.9$ ft

[21] (a) $V = k\sqrt{L}$ (b) $35 = k\sqrt{50} \Rightarrow k = \frac{7}{2}\sqrt{2}$

(c) $V = \frac{7}{2}\sqrt{2}(\sqrt{150}) = 35\sqrt{3} \approx 60.6$ mi/hr

[22] (a) $F = k\dfrac{Q_1Q_2}{d^2} = \dfrac{kQ_1Q_2}{d^2}$

(b) $F = \dfrac{kQ_1Q_2}{(\frac{1}{4}d)^2} = 16\left(\dfrac{kQ_1Q_2}{d^2}\right)$, the force F is multiplied by 16

[23] (a) $W = kh^3$ (b) $200 = k(6)^3 \Rightarrow k = \frac{25}{27}$

(c) $W = \frac{25}{27}(\frac{11}{2})^3 \approx 154.1$ lb, or 154 lb

[24] (a) $V = k\dfrac{nT}{P} = \dfrac{knT}{P}$ (b) $V = \dfrac{k(2n)(\frac{1}{2}T)}{(\frac{1}{2}P)} = 2\left(\dfrac{knT}{P}\right)$, i.e., V is doubled.

[25] (a) $F = kPr^4 \Rightarrow P = \dfrac{F}{kr^4}$ under normal conditions

(b) $3F = kP(1.1r)^4 \Rightarrow P = \dfrac{3F}{(1.1)^4kr^4} \approx 2.05\left(\dfrac{F}{kr^4}\right)$, or about 2.05 times as hard

[26] $T = kn$; $10 = k(300) \Rightarrow k = \frac{1}{30}$; $200 = \frac{1}{30}n \Rightarrow n = 6000$

[27] Let k be the constant of variation. Then, $C = \dfrac{kDE}{Vt} \Rightarrow D = \left(\dfrac{Ct}{k}\right)\dfrac{V}{E}$, where $\dfrac{Ct}{k}$ is constant. If V is twice its original value and E is $\frac{4}{5}$ of its original value, then

D becomes $\dfrac{2}{4/5} = \dfrac{5}{2} = 250\%$ of its original value. Thus, D increases by 250%.

[28] $C = \dfrac{kDE}{Vt}$. D increases to $\dfrac{13}{10}$ of its original value, V decreases to $\dfrac{9}{10}$ of its original value, and t increases to $\dfrac{16}{10}$ of its original value. C becomes $\dfrac{(13/10)}{(9/10)(16/10)} = \dfrac{65}{72} \approx$

90.3% of its original value. Thus, C decreases by approximately 9.7%.

[29] The square of the distance from the origin to the point (x, y) is $x^2 + y^2$. $d = \dfrac{k}{x^2 + y^2}$.

If (x_1, y_1) is the new point that has density d_1, then

$$d_1 = \frac{k}{x_1^2 + y_1^2} = \frac{k}{(\frac{1}{3}x)^2 + (\frac{1}{3}y)^2} = \frac{k}{\frac{1}{9}x^2 + \frac{1}{9}y^2} = \frac{k}{\frac{1}{9}(x^2 + y^2)} = 9 \cdot \frac{k}{x^2 + y^2} = 9d.$$

The density d is multiplied by 9.

[30] The distance from the origin to the point (x, y) is $\sqrt{x^2 + y^2}$. $T = k/\sqrt{x^2 + y^2}$.

$T = 20$ and $P(3, 4) \Rightarrow 20 = k/5 \Rightarrow k = 100$.

$T = 100/\sqrt{24^2 + 7^2} = 100/25 = 4$°C.

[31] $\dfrac{y}{x} = \dfrac{0.72}{0.6} = \dfrac{1.44}{1.2} = \dfrac{5.04}{4.2} = \dfrac{8.52}{7.1} = \dfrac{11.16}{9.3} = 1.2.$

Thus, y varies directly as x with constant of variation $k = 1.2$, and $y = 1.2x$.

[32] $xy = -5.3$ for each data point.

Thus, y varies inversely as x with constant of variation $k = -5.3$, and $y = -\dfrac{5.3}{x}$.

$\boxed{33}$ $x^2y = -10.1$ for each data point.

Thus, y varies inversely as x^2 with constant of variation $k = -10.1$, and $y = -\dfrac{10.1}{x^2}$.

$\boxed{34}$ $y/x^3 = 2.67$ for each data point. Thus,

y varies directly as the cube of x with constant of variation $k = 2.67$, and $y = 2.67x^3$.

$\boxed{35}$ (a) $D = kS^{2.3} \Rightarrow k = \dfrac{D}{S^{2.3}}$. Using the 6 data points: $\dfrac{33}{20^{2.3}} \approx 0.0336$;

$\dfrac{86}{30^{2.3}} \approx 0.0344$; $\dfrac{167}{40^{2.3}} \approx 0.0345$; $\dfrac{278}{50^{2.3}} \approx 0.0344$; $\dfrac{414}{60^{2.3}} \approx 0.0337$; $\dfrac{593}{70^{2.3}} \approx 0.0338$.

Let $k = 0.034$. Thus, $D = 0.034S^{2.3}$.

(b) Graph the data together with $Y_1 = 0.034x^{2.3}$.

[0, 75, 10] by [0, 600, 100]

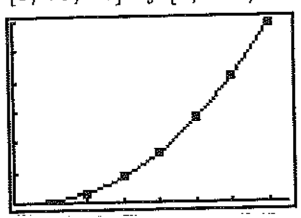

Figure 35

Chapter 4 Review Exercises

$\boxed{1}$ Shift $y = x^3$ left 2 units. $f(x) > 0$ if $x > -2$, $f(x) < 0$ if $x < -2$.

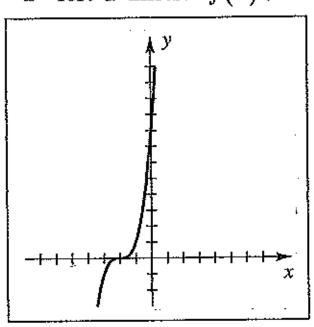

Figure 1

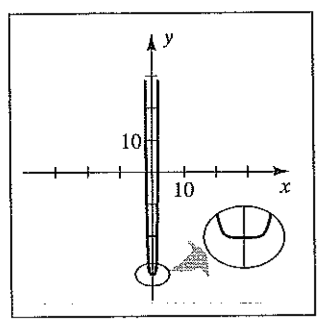

Figure 2

$\boxed{2}$ Shift $y = x^6$ down 32 units.

$f(x) > 0$ if $x < -\sqrt[6]{32}$ or $x > \sqrt[6]{32}$, $f(x) < 0$ if $-\sqrt[6]{32} < x < \sqrt[6]{32}$.

[3] $f(x) = -\frac{1}{4}(x+2)(x-1)^2(x-3)$ has zeros at -2, 1 (multiplicity 2), and 3.
$f(x) > 0$ if $-2 < x < 1$ or $1 < x < 3$, $f(x) < 0$ if $x < -2$ or $x > 3$.

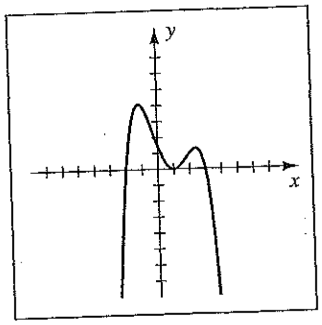

Figure 3

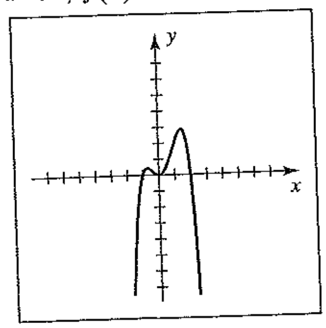

Figure 4

[4] $f(x) = 2x^2 + x^3 - x^4 = x^2(2 + x - x^2) = -x^2(x-2)(x+1)$.
$f(x) > 0$ if $-1 < x < 0$ or $0 < x < 2$, $f(x) < 0$ if $x < -1$ or $x > 2$.

[5] $f(x) = x^3 + 2x^2 - 8x = x(x^2 + 2x - 8) = x(x+4)(x-2)$.
$f(x) > 0$ if $-4 < x < 0$ or $x > 2$, $f(x) < 0$ if $x < -4$ or $0 < x < 2$.

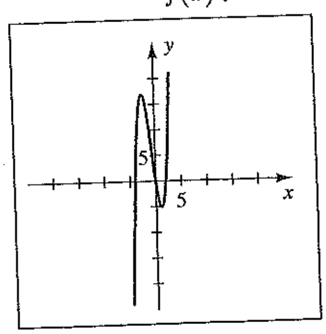

Figure 5

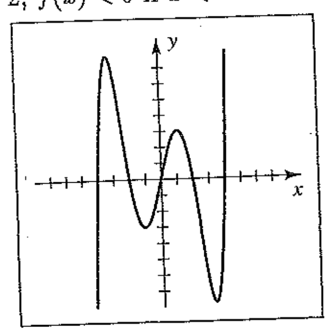

Figure 6

[6] $f(x) = \frac{1}{15}(x^5 - 20x^3 + 64x) = \frac{1}{15}x(x^4 - 20x^2 + 64) = \frac{1}{15}x(x^2-4)(x^2-16) =$
$\frac{1}{15}x(x+2)(x-2)(x+4)(x-4)$. $f(x) > 0$ if $-4 < x < -2$, $0 < x < 2$, or $x > 4$,
$f(x) < 0$ if $x < -4$, $-2 < x < 0$, or $2 < x < 4$.

[7] $f(0) = -9 < 100$ and $f(10) = 561 > 100$. By the intermediate value theorem for polynomial functions, f takes on every value between -9 and 561. Hence, there is at least one real number a in $[0, 10]$ such that $f(a) = 100$.

[8] Let $f(x) = x^5 - 3x^4 - 2x^3 - x + 1$. $f(0) = 1 > 0$ and $f(1) = -4 < 0$. By the intermediate value theorem for polynomial functions, f takes on every value between -4 and 1. Hence, there is at least one real number a in $[0, 1]$ such that $f(a) = 0$.

[9] $f(x) = 3x^5 - 4x^3 + x + 5$; $p(x) = x^3 - 2x + 7$ • ★ $3x^2 + 2$; $-21x^2 + 5x - 9$

[10] $f(x) = 4x^3 - x^2 + 2x - 1$; $p(x) = x^2$ • ★ $4x - 1$; $2x - 1$

[11] Dividing $f(x) = -4x^4 + 3x^3 - 5x^2 + 7x - 10$ by $x + 2$ yields -132.

[12] If $f(x) = 2x^4 - 5x^3 - 4x^2 + 9$, then $f(3) = 0$ and $x - 3$ is a factor of f.

[13] ★ $6x^4 - 12x^3 + 24x^2 - 52x + 104;\ -200$

[14] ★ $2x^2 + (5 + 2\sqrt{2})x + (2 + 5\sqrt{2});\ 11 + 2\sqrt{2}$

[15] $f(x) = a[x - (-3 + 5i)][x - (-3 - 5i)](x + 1) = a(x^2 + 6x + 34)(x + 1)$.

$f(1) = a(41)(2)$ and $f(1) = 4 \Rightarrow 82a = 4 \Rightarrow a = \frac{2}{41}$.

Hence, $f(x) = \frac{2}{41}(x^2 + 6x + 34)(x + 1)$.

[16] $f(x) = a[x - (1 - i)][x - (1 + i)](x - 3)(x) = ax(x^2 - 2x + 2)(x - 3)$.

$f(2) = a(2)(2)(-1)$ and $f(2) = -1 \Rightarrow -4a = -1 \Rightarrow a = \frac{1}{4}$.

Hence, $f(x) = \frac{1}{4}x(x^2 - 2x + 2)(x - 3)$.

[17] $f(x) = x^5(x + 3)^2$
$= x^5(x^2 + 6x + 9)$
$= x^7 + 6x^6 + 9x^5$

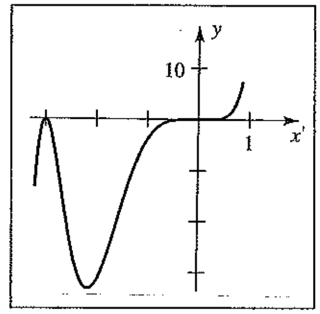

Figure 17

[18] Synthetically dividing $f(x) = x^5 - 4x^4 - 3x^3 + 34x^2 - 52x + 24$ by $x - 2$ three times gives us $(x - 2)^3(x^2 + 2x - 3)$. Hence, $f(x) = (x - 2)^3(x + 3)(x - 1)$.

[19] $f(x) = (x^2 - 2x + 1)^2(x^2 + 2x - 3) = [(x - 1)^2]^2(x + 3)(x - 1) = (x + 3)(x - 1)^5$

★ 1 (multiplicity 5); -3 (multiplicity 1)

[20] $f(x) = x^6 + 2x^4 + x^2 = x^2(x^4 + 2x^2 + 1) = x^2(x^2 + 1)^2$

★ $0,\ \pm i$ (all have multiplicity 2)

[21] (a) Let $f(x) = 2x^4 - 4x^3 + 2x^2 - 5x - 7$. Since there are 3 sign changes in $f(x)$ and 1 sign change in $f(-x)$, there are either 3 positive and 1 negative solution or 1 positive, 1 negative, and 2 nonreal complex solutions.

(b) Upper bound is 3, lower bound is -1

[22] (a) Let $f(x) = x^5 - 4x^3 + 6x^2 + x + 4$. Since there are 2 sign changes in $f(x)$ and 3 sign changes in $f(-x)$, there are either 2 positive and 3 negative solutions;

2 positive, 1 negative, and 2 nonreal complex;

3 negative and 2 nonreal complex;

or 1 negative and 4 nonreal complex solutions.

(b) Upper bound is 2, lower bound is -3

[23] Since there are only even powers, $7x^6 + 2x^4 + 3x^2 + 10 \geq 10$ for every real number x.

[24] $x^4 + 9x^3 + 31x^2 + 49x + 30 = 0$ • ★ $-3,\ -2,\ -2 \pm i$

[25] $16x^3 - 20x^2 - 8x + 3 = 0$ • ★ $-\frac{1}{2},\ \frac{1}{4},\ \frac{3}{2}$

[26] $x^4 - 7x^2 + 6 = 0$ • ★ $\pm\sqrt{6}$, ± 1

[27] The graph has x-intercepts at -2, 1, and 3. The equation must have the form $f(x) = a(x+2)^m(x-1)^n(x-3)^p$, where $m+n+p=6$ since f is a sixth-degree polynomial. The graph goes through the x-axis at m and p, so they must be odd, and it doesn't change sign at n, so it must be even. Since the graph flattens out at $x=-2$, m must be greater than p. The only possibility is $m=3$, $n=2$, and $p=1$, so f has the form $f(x) = a(x+2)^3(x-1)^2(x-3)$. The y-intercept is 4, so $f(0) = 4 \Rightarrow 4 = a(2)^3(-1)^2(-3) \Rightarrow -24a = 4 \Rightarrow a = -\frac{1}{6}$. Thus,

$$f(x) = -\tfrac{1}{6}(x+2)^3(x-1)^2(x-3).$$

[28] The graph has x-intercepts at -3, 0, and 3. The equation must have the form $f(x) = a(x+3)^2(x)^2(x-3)^2$ since the graph doesn't change sign at any of the intercepts. $f(1) = 4 \Rightarrow 4 = a(4)^2(1)^2(-2)^2 \Rightarrow 64a = 4 \Rightarrow a = \frac{1}{16}$.

[29] $f(x) = \dfrac{4(x+2)(x-1)}{3(x+2)(x-5)} = \dfrac{4(x-1)}{3(x-5)}$ if $x \neq -2$ • Note that $x+2$ appears in both the numerator and denominator, so there is a hole at $x=-2$. The y-value for the hole can be found by substituting -2 for x in the rest of the function. $\frac{4(-2-1)}{3(-2-5)} = \frac{4(-3)}{3(-7)} = \frac{4}{7}$, so the hole is at $(-2, \frac{4}{7})$. There is one other zero of the denominator, namely 5, so there is a vertical asymptote of $x=5$. The degrees of the numerator and denominator are the same, namely 2, so the ratio of leading coefficients gives us the value of the horizontal asymptote. In this case, it is $y = \frac{4}{3}$. There is one other zero of the numerator, namely 1, so there is an x-intercept of 1. If we substitute 0 for x, we get $\frac{4(2)(-1)}{3(2)(-5)} = \frac{4}{15}$, the y-intercept.

[30] $f(x) = \dfrac{-2}{(x+1)^2}$ •

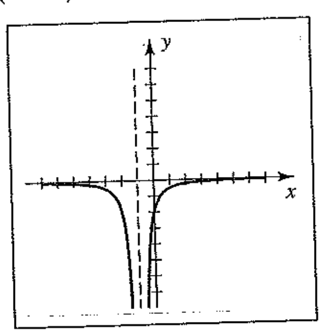

Figure 30

[31] $f(x) = \dfrac{1}{(x-1)^3}$ •

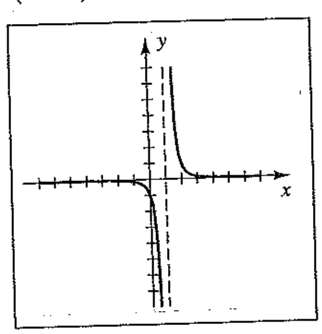

Figure 31

$\boxed{32}$ $f(x) = \dfrac{3x^2}{16 - x^2} = \dfrac{3x^2}{(4+x)(4-x)}$

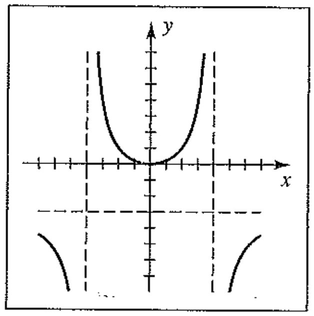

Figure 32

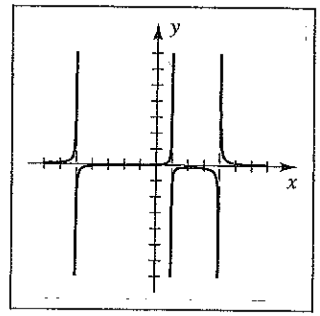

Figure 33

$\boxed{33}$ $f(x) = \dfrac{x}{(x+5)(x^2 - 5x + 4)} = \dfrac{x}{(x+5)(x-1)(x-4)}$

$\boxed{34}$ $f(x) = \dfrac{x^3 - 2x^2 - 8x}{-x^2 + 2x} = \dfrac{x(x^2 - 2x - 8)}{x(2-x)} = \dfrac{(x-4)(x+2)}{2-x}$ for $x \neq 0$; hole at $(0, -4)$

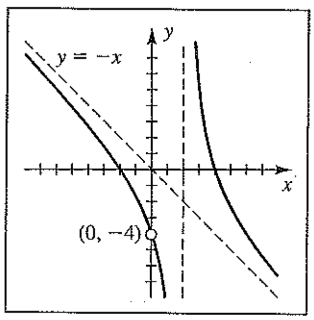

Figure 34

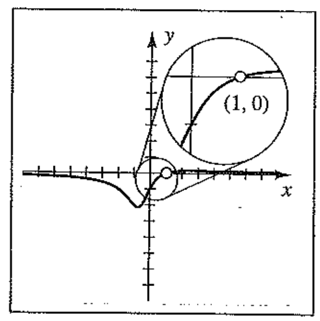

Figure 35

$\boxed{35}$ $f(x) = \dfrac{x^2 - 2x + 1}{x^3 - x^2 + x - 1} = \dfrac{(x-1)(x-1)}{x^2(x-1) + 1(x-1)} = \dfrac{(x-1)^2}{(x^2+1)(x-1)} = \dfrac{x-1}{x^2+1}$ for $x \neq 1$;

hole at $(1, 0)$

$\boxed{36}$ $f(x) = \dfrac{3x^2 + x - 10}{x^2 + 2x} = \dfrac{(3x-5)(x+2)}{x(x+2)} = \dfrac{3x-5}{x}$ for $x \neq -2$

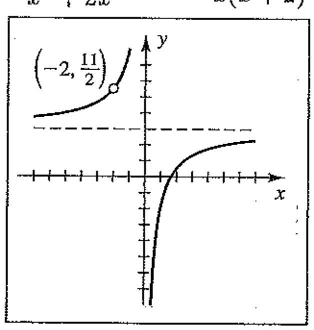

Figure 36

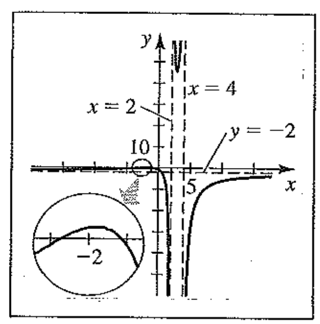

Figure 37

$\boxed{37}$ $f(x) = \dfrac{-2x^2 - 8x - 6}{x^2 - 6x + 8} = \dfrac{-2(x^2 + 4x + 3)}{(x-2)(x-4)} = \dfrac{-2(x+1)(x+3)}{(x-2)(x-4)}$

38 $f(x) = \dfrac{x^2+2x-8}{x+3} = \dfrac{(x+4)(x-2)}{x+3} = x - 1 - \dfrac{5}{x+3}$

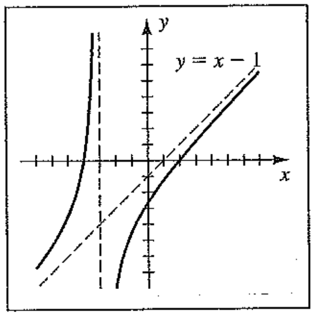

Figure 38

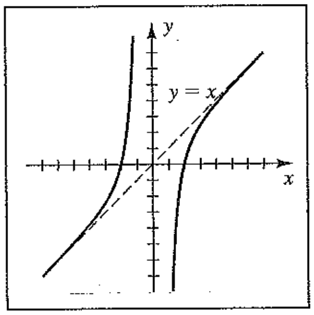

Figure 39

39 $f(x) = \dfrac{x^4-16}{x^3} = \dfrac{(x^2+4)(x+2)(x-2)}{x^3} = x - \dfrac{16}{x^3}$

40 There is a hole at $x=2$, so $(x-2)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x=-3$, so $(x+3)$ must be a factor in the denominator. There is an x-intercept at 5, so $(x-5)$ must be a factor in the numerator. The horizontal asymptote is $y=\frac{3}{2}$, so the ratio of leading coefficients must be $\frac{3}{2}$. Combining this information gives us this possibility:

$$f(x) = \frac{3(x-5)(x-2)}{2(x+3)(x-2)} \quad \text{or} \quad f(x) = \frac{3x^2-21x+30}{2x^2+2x-12}$$

41 $y = \dfrac{k\sqrt[3]{x}}{z^2} \;\Rightarrow\; 6 = \dfrac{k\sqrt[3]{8}}{3^2} \;\Rightarrow\; 6 = \dfrac{2k}{9} \;\Rightarrow\; k = \dfrac{54}{2} = 27$

42 $y = \dfrac{k}{x^2} \;\Rightarrow\; 18 = \dfrac{k}{4^2} \;\Rightarrow$

$k = 18 \cdot 16 = 288,$

so graph $y = \dfrac{288}{x^2}$ for $x > 0$.

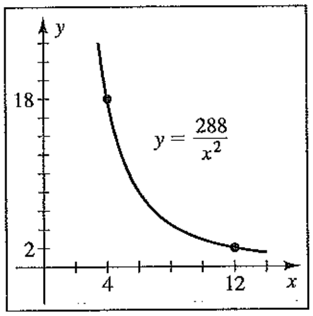

Figure 42

43 (a) $l=10$, $x=10$, and $y=2 \;\Rightarrow\; 2 = 30{,}000c \;\Rightarrow\; c = \frac{1}{15{,}000}$

(b) $y \approx 0.9754 < 1$ if $x = 6.1$, and $y \approx 1.0006 > 1$ if $x = 6.2$

44 (a) Edge AB has length $2\pi r$ where r is the radius of the cylinder.

$2\pi r = \sqrt{l^2-x^2} \;\Rightarrow\; r^2 = \frac{1}{4\pi^2}(l^2-x^2)$.

Now $V = \pi r^2 x = \pi\left[\frac{1}{4\pi^2}(l^2-x^2)\right](x) = \frac{1}{4\pi}x(l^2-x^2)$.

(b) If $x>0$, $V>0$ if $l^2-x^2>0$ or $l>x$. Thus, when $0<x<l$, $V>0$.

$\boxed{45}$ $T = \frac{1}{20}t(t-12)(t-24) = 32 \;\Rightarrow\; t^3 - 36t^2 + 288t - 640 = 0$. Solving for t yields $t = 4$ and $16 \pm 4\sqrt{6}$. Since $0 \le t \le 24$, $t = 4$ (10:00 A.M.) and $t = 16 - 4\sqrt{6} \approx 6.2020$ (12:12 P.M.) are the times when the temperature was 32°F.

$\boxed{46}$ $N(t) = -t^4 + 21t^2 + 100$ and $N(t) > 180 \;\Rightarrow$
$t^4 - 21t^2 + 80 < 0 \;\Rightarrow\; (t^2 - 5)(t^2 - 16) < 0$.

The positive values of t satisfying this inequality are in the interval $(\sqrt{5}, 4)$.

$\boxed{47}$ (a) $R = \dfrac{kS^n}{S^n + a^n} \cdot \dfrac{1/S^n}{1/S^n} = \dfrac{k}{1 + (a/S)^n}$. As S gets large, R approaches k.

(b) k is the maximum rate at which the liver can remove alcohol from the bloodstream.

$\boxed{48}$ (a) $C(100) = \$2{,}000{,}000.00$ and $C(90) \approx \$163{,}636.36$

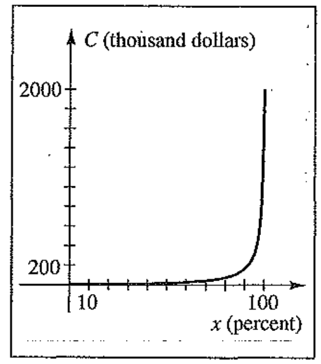

Figure 48

$\boxed{49}$ $C = \dfrac{kP_1P_2}{d^2}$; $2000 = \dfrac{k(10{,}000)(5000)}{(25)^2} \;\Rightarrow\; k = \dfrac{1}{40}$; $C = \dfrac{(10{,}000)(15{,}000)}{40(100)^2} = 375$

$\boxed{50}$ $P = kA^2v^3$; $3000 = k\left[\pi(5)^2\right]^2(20)^3 \;\Rightarrow\; k = \dfrac{3}{5000\pi^2}$;

$$P = \frac{3}{5000\pi^2}\left[\pi(5)^2\right]^2(30)^3 = 10{,}125 \text{ watts}$$

Chapter 4 Discussion Exercises

$\boxed{1}$ **For even-degreed polynomials:** the domain is $\mathbb{R}$ and the number of x-intercepts ranges from *zero* to the degree of the polynomial; if the leading coefficient is positive, the range is of the form $[c, \infty)$ and the general shape has $y \to \infty$ as $|x| \to \infty$; if the leading coefficient is negative, the range is of the form $(-\infty, c]$ and the general shape has $y \to -\infty$ as $|x| \to \infty$.

For odd-degreed polynomials: the domain is $\mathbb{R}$, the range is $\mathbb{R}$, and the number of x-intercepts ranges from *one* to the degree of the polynomial; if the leading coefficient is positive, then $y \to \infty$ as $x \to \infty$ and $y \to -\infty$ as $x \to -\infty$, if the leading coefficient is negative, then $y \to -\infty$ as $x \to \infty$ and $y \to \infty$ as $x \to -\infty$.

[2] After trying a few examples, students should come up with the conclusion that complex numbers can be used in the synthetic division process.

[3] By long division, we obtain the quotient $2x^2 - 7x + 5$ with remainder -6. By synthetic division with $k = -3/2$, we obtain a bottom row of 4 -14 10 -6. The first three numbers are twice the coefficients of the quotient and the last number is the remainder. For the factor $ax + b$, we can use synthetic division with $k = -b/a$, and obtain a times the quotient and the remainder in the bottom row.

[4] $f(x) = a(x-1)(x-2)(x-3)$. $\quad f(0) = a(-6) = 6 \Rightarrow a = -1$.

$f(-1) = 24 \neq 25 \Rightarrow$ the point cannot be on the polynomial.

[5] After working Discussion Exercise 4, students should guess that 4 points specify a third-degree polynomial. They know that 2 points specify a first-degree polynomial, so a logical conclusion is that $n+1$ points specify an n-degree polynomial.

[6] Let us consider the polynomial

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0,$$

where each coefficient a_k is a real number and $a_n \neq 0$. If $f(z) = 0$, then

$$a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0 = 0.$$

If two complex numbers are equal, then so are their conjugates. Hence, the conjugate of the left-hand side of the last equation equals the conjugate of the right-hand side; that is,

$$\overline{a_n z^n + a_{n-1}z^{n-1} + \cdots + a_1 z + a_0} = \overline{0} = 0.$$

The fact that $\overline{0} = 0$ follows from $\overline{0} = \overline{0 + 0i} = 0 - 0i = 0$.

If z and w are complex numbers, then it can be shown that $\overline{z + w} = \overline{z} + \overline{w}$. More generally, the conjugate of *any* sum of complex numbers is the sum of the conjugates. Consequently,

$$\overline{a_n z^n} + \overline{a_{n-1}z^{n-1}} + \cdots + \overline{a_1 z} + \overline{a_0} = 0.$$

It can also be shown that $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$, $\overline{z^n} = \overline{z}^n$ for every positive integer n, and $\overline{z} = z$ if and only if z is real. Thus, for every k,

$$\overline{a_k z^k} = \overline{a_k} \cdot \overline{z^k} = \overline{a_k} \cdot \overline{z}^k = a_k \overline{z}^k,$$

and therefore

$$a_n \overline{z}^n + a_{n-1}\overline{z}^{n-1} + \cdots + a_1 \overline{z} + a_0 = 0.$$

The last equation states that $f(\overline{z}) = 0$, which completes the proof.

[7] If the common factor is never equal to zero for any real number, then it can be canceled and has no effect on its graph. Such a factor is x^2+1, and an example of a function is $f(x)=\dfrac{(x^2+1)(x-1)}{(x^2+1)(x-2)}$.

[8] (a) The horizontal asymptote is $y=a/c$. Solving $f(x)=a/c$ gives us

$$\frac{ax+b}{cx+d}=\frac{a}{c} \;\Rightarrow\; acx+bc=acx+ad \;\Rightarrow\; bc=ad \;\Rightarrow\; \text{no solution,}$$

and f doesn't cross its horizontal asymptote.

(b) The horizontal asymptote is $y=a/d$. Solving $f(x)=a/d$ gives us

$$\frac{ax^2+bx+c}{dx^2+ex+f}=\frac{a}{d} \;\Rightarrow\; adx^2+bdx+cd=adx^2+aex+af \;\Rightarrow$$

$$aex-bdx=cd-af \;\Rightarrow\; x=\frac{cd-af}{ae-bd},$$ provided the denominator is not zero.

[9] $$y=\frac{9x}{\sqrt{x^2+1}} \;\Rightarrow\; y\sqrt{x^2+1}=9x \;\Rightarrow\; y^2(x^2+1)=81x^2 \;\Rightarrow$$

$$y^2x^2+y^2=81x^2 \;\Rightarrow\; y^2x^2-81x^2=-y^2 \;\Rightarrow\; x^2(y^2-81)=-y^2 \;\Rightarrow$$

$$x^2=\frac{y^2}{81-y^2} \;\Rightarrow\; x=\pm\sqrt{\frac{y^2}{81-y^2}} \;\Rightarrow\; x=\pm\frac{|y|}{\sqrt{81-y^2}} \;\Rightarrow\; x=\pm\frac{y}{\sqrt{81-y^2}}.$$

From the original equation, we see that x and y are always both positive or both negative, so the last equation can be simplified to $x=y/\sqrt{81-y^2}$. Hence, $f^{-1}(y)=y/\sqrt{81-y^2}$ or, equivalently, $f^{-1}(x)=x/\sqrt{81-x^2}$. The denominator of f^{-1} is zero for $x=\pm 9$, which are the vertical asymptotes. They are related to the horizontal asymptotes of f, which are $y=\pm 9$.

[10] Let x, $x+1$, and $x+2$ denote three consecutive integers. Their product is $x(x+1)(x+2)=x^3+3x^2+2x$. Now add the second integer to get

$$(x^3+3x^2+2x)+(x+1)=x^3+3x^2+3x+1.$$

Only $x+1$ or $x-1$ could be factors, and it turns out that $x+1$ is a factor three times. Thus, if you multiply three consecutive integers together and then add the second integer to that product, you obtain the cube of the second integer.

$\boxed{11}$ (a) $B = \dfrac{GW}{29.3 + 53.1E - 22.7C}$ •

$G = 3 \cdot 500 = 1500$, $W = 5$, $E = -0.05$, and $C = 0.95$, so

$B = \dfrac{1500(5)}{29.3 + 53.1(-0.05) - 22.7(0.95)} \approx 1476$. This tells us that a bankroll of about \$1476 would allow us to play 1500 games and be 95% confident that we will survive the 3-hour gambling session.

(b) If we use the same values for G, W, and E, we get $B = \dfrac{7500}{26.645 - 22.7C}$. A graph of B has a vertical asymptote at $C \approx 1.17$, but that is 117% confident of surviving the 3-hour gambling session. The highest confidence value for any model like this should be 100, so the formula should have a restricted domain with it.