Chapter 5: Inverse, Exponential, and Logarithmic Functions

5.1 Exercises

$\boxed{1}$ (a) f is one-to-one and $f(4) = 5$, so $f^{-1}(5) = 4$.

(b) g is *not* one-to-one since $g(1) = g(5) = 6$, so we cannot find $g^{-1}(6)$. Not possible

$\boxed{2}$ (a) f is one-to-one and $f(3) = 5$, so $f^{-1}(5) = 3$.

(b) g is *not* one-to-one since $g(2) = g(4) = 6$, so we cannot find $g^{-1}(6)$. Not possible

$\boxed{3}$ (a) The graph is the graph of a function since it passes the Vertical Line Test. Since it passes the Horizontal Line Test, it is also one-to-one.

(b) The graph is the graph of a function, but since it doesn't pass the Horizontal Line Test, it is not one-to-one.

(c) The graph is not the graph of a function since it doesn't pass the Vertical Line Test.

$\boxed{4}$ (a) The graph is the graph of a function, but since it doesn't pass the Horizontal Line Test, it is not one-to-one.

(b) The graph is the graph of a function since it passes the Vertical Line Test. Since it passes the Horizontal Line Test, it is also one-to-one.

(c) The graph is not the graph of a function since it doesn't pass the Vertical Line Test.

Note: For Exer. 5–16, we use the method illustrated in Example 1; however, the Horizontal Line Test or the theorem on increasing and decreasing functions could also be used.

$\boxed{5}$ Suppose $f(a) = f(b)$. $3a - 7 = 3b - 7 \Rightarrow 3a = 3b \Rightarrow a = b$.

Since $f(a) = f(b)$ implies that $a = b$, we conclude that f is one-to-one.

$\boxed{6}$ Suppose $f(a) = f(b)$. $\dfrac{1}{a-2} = \dfrac{1}{b-2} \Rightarrow a - 2 = b - 2 \Rightarrow a = b$. f is one-to-one.

$\boxed{7}$ Since $f(3) = f(-3) = 0$, but $3 \neq -3$, we conclude that f is *not* one-to-one.

$\boxed{8}$ For $f(x) = x^2 + 4$, $f(1) = 5 = f(-1)$. f is *not* one-to-one.

$\boxed{9}$ Suppose $f(a) = f(b)$ with $a, b \geq 0$. $\sqrt{a} = \sqrt{b} \Rightarrow (\sqrt{a})^2 = (\sqrt{b})^2 \Rightarrow a = b$.

f is one-to-one.

$\boxed{10}$ Suppose $f(a) = f(b)$. $\sqrt[3]{a} = \sqrt[3]{b} \Rightarrow (\sqrt[3]{a})^3 = (\sqrt[3]{b})^3 \Rightarrow a = b$. f is one-to-one.

$\boxed{11}$ For $f(x) = |\,x\,|$, $f(-1) = 1 = f(1)$. f is *not* one-to-one.

$\boxed{12}$ For $f(x) = 3$, $f(2) = 3 = f(4)$. f is *not* one-to-one.

$\boxed{13}$ For $f(x) = \sqrt{4 - x^2}$, $f(-1) = \sqrt{3} = f(1)$. f is *not* one-to-one.

$\boxed{14}$ Suppose $f(a) = f(b)$. $2a^3 - 4 = 2b^3 - 4 \Rightarrow 2a^3 = 2b^3 \Rightarrow a^3 = b^3 \Rightarrow a = b$.

f is one-to-one.

[15] Suppose $f(a) = f(b)$. $\frac{1}{a} = \frac{1}{b} \Rightarrow a = b$. f is one-to-one.

[16] For $f(x) = 1/x^2$, $f(-1) = 1 = f(1)$. f is *not* one-to-one.

[17] $f(g(x)) = 3\left(\frac{x+2}{3}\right) - 2 = x + 2 - 2 = x$. $g(f(x)) = \frac{(3x-2)+2}{3} = \frac{3x}{3} = x$.

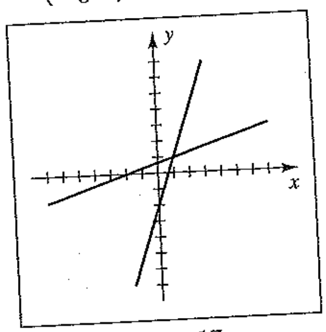

Figure 17

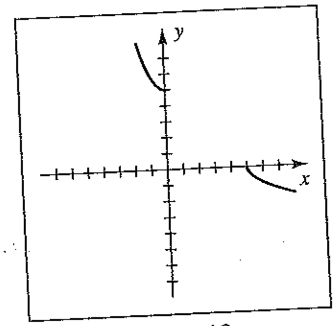

Figure 18

[18] $f(g(x)) = (-\sqrt{x-5})^2 + 5 = x - 5 + 5 = x$.

$g(f(x)) = -\sqrt{(x^2+5)-5} = -\sqrt{x^2} = -\mid x \mid = -(-x)$ { since $x \leq 0$ } $= x$.

[19] $f(g(x)) = -(\sqrt{3-x})^2 + 3 = -(3-x) + 3 = x$.

$g(f(x)) = \sqrt{3-(-x^2+3)} = \sqrt{x^2} = \mid x \mid = x$ { since $x \geq 0$ }.

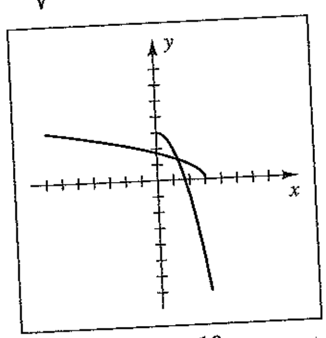

Figure 19

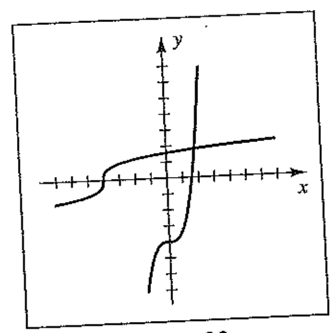

Figure 20

[20] $f(g(x)) = (\sqrt[3]{x+4})^3 - 4 = x + 4 - 4 = x$. $g(f(x)) = \sqrt[3]{(x^3-4)+4} = \sqrt[3]{x^3} = x$.

[21] $f(x) = -\frac{2}{x-1}$ • The domain of f is $\mathbb{R} - \{1\}$. The horizontal asymptote of f is $y = 0$ and the range of f is $\mathbb{R} - \{0\}$. The domain of f^{-1} is equal to the range of f; that is, $\mathbb{R} - \{0\}$. The range of f^{-1} is equal to the domain of f; that is, $\mathbb{R} - \{1\}$.

★ $(-\infty, 0) \cup (0, \infty)$; $(-\infty, 1) \cup (1, \infty)$

[22] $f(x) = \frac{5}{x+3}$ • Domain of f^{-1} = range of $f = \mathbb{R} - \{0\}$.

Range of f^{-1} = domain of $f = \mathbb{R} - \{-3\}$.

★ $(-\infty, 0) \cup (0, \infty)$; $(-\infty, -3) \cup (-3, \infty)$

$\boxed{23}$ $f(x)=\dfrac{4x+5}{3x-8}$ • The domain of f is $\mathbb{R}-\{\frac{8}{3}\}$. The horizontal asymptote of f is $y=\frac{4}{3}$ and the range of f is $\mathbb{R}-\{\frac{4}{3}\}$. The domain of f^{-1} is equal to the range of f; that is, $\mathbb{R}-\{\frac{4}{3}\}$. The range of f^{-1} is equal to the domain of f; that is, $\mathbb{R}-\{\frac{8}{3}\}$.

★ $(-\infty, \frac{4}{3})\cup(\frac{4}{3}, \infty)$; $(-\infty, \frac{8}{3})\cup(\frac{8}{3}, \infty)$

$\boxed{24}$ $f(x)=\dfrac{2x-7}{9x+1}$ • Domain of f^{-1} = range of $f=\mathbb{R}-\{\frac{2}{9}\}$.
Range of f^{-1} = domain of $f=\mathbb{R}-\{-\frac{1}{9}\}$.

★ $(-\infty, \frac{2}{9})\cup(\frac{2}{9}, \infty)$; $(-\infty, -\frac{1}{9})\cup(-\frac{1}{9}, \infty)$

$\boxed{25}$ $f(x)=3x+5 \Rightarrow y-5=3x \Rightarrow x=\dfrac{y-5}{3} \Rightarrow f^{-1}(x)=\dfrac{x-5}{3}$

$\boxed{26}$ $f(x)=7-2x \Rightarrow 2x=7-y \Rightarrow x=\dfrac{7-y}{2} \Rightarrow f^{-1}(x)=\dfrac{7-x}{2}$

$\boxed{27}$ $f(x)=\dfrac{1}{3x-2} \Rightarrow 3xy-2y=1 \Rightarrow 3xy=2y+1 \Rightarrow x=\dfrac{2y+1}{3y} \Rightarrow$
$f^{-1}(x)=\dfrac{2x+1}{3x}$

$\boxed{28}$ $f(x)=\dfrac{1}{x+3} \Rightarrow xy+3y=1 \Rightarrow xy=1-3y \Rightarrow x=\dfrac{1-3y}{y} \Rightarrow$
$f^{-1}(x)=\dfrac{1-3x}{x}$

$\boxed{29}$ $f(x)=\dfrac{3x+2}{2x-5} \Rightarrow 2xy-5y=3x+2 \Rightarrow 2xy-3x=5y+2 \Rightarrow$
$x(2y-3)=5y+2 \Rightarrow x=\dfrac{5y+2}{2y-3} \Rightarrow f^{-1}(x)=\dfrac{5x+2}{2x-3}$

$\boxed{30}$ $f(x)=\dfrac{4x}{x-2} \Rightarrow xy-2y=4x \Rightarrow xy-4x=2y \Rightarrow x=\dfrac{2y}{y-4} \Rightarrow$
$f^{-1}(x)=\dfrac{2x}{x-4}$

$\boxed{31}$ $f(x)=2-3x^2,\ x\leq 0 \Rightarrow y+3x^2=2 \Rightarrow$
$x^2=\dfrac{2-y}{3} \Rightarrow x=\pm\sqrt{\dfrac{2-y}{3}}$ { choose minus since $x\leq 0$ } $\Rightarrow f^{-1}(x)=-\sqrt{\dfrac{2-x}{3}}$

$\boxed{32}$ $f(x)=5x^2+2,\ x\geq 0 \Rightarrow y-2=5x^2 \Rightarrow$
$x^2=\dfrac{y-2}{5} \Rightarrow x=\pm\sqrt{\dfrac{y-2}{5}}$ { choose plus since $x\geq 0$ } $\Rightarrow f^{-1}(x)=\sqrt{\dfrac{x-2}{5}}$

$\boxed{33}$ $f(x)=2x^3-5 \Rightarrow \dfrac{y+5}{2}=x^3 \Rightarrow x=\sqrt[3]{\dfrac{y+5}{2}} \Rightarrow f^{-1}(x)=\sqrt[3]{\dfrac{x+5}{2}}$

$\boxed{34}$ $f(x)=-x^3+2 \Rightarrow x^3=2-y \Rightarrow x=\sqrt[3]{2-y} \Rightarrow f^{-1}(x)=\sqrt[3]{2-x}$

$\boxed{35}$ $f(x)=\sqrt{3-x} \Rightarrow y^2=3-x \Rightarrow$
$x=3-y^2$ { Since $y\geq 0$ for f, $x\geq 0$ for f^{-1}. } $\Rightarrow f^{-1}(x)=3-x^2,\ x\geq 0$

$\boxed{36}$ $f(x)=\sqrt{4-x^2},\ 0\leq x\leq 2 \Rightarrow y^2=4-x^2 \Rightarrow x^2=4-y^2 \Rightarrow$
$x=\pm\sqrt{4-y^2} \Rightarrow$ { choose plus since $0\leq x\leq 2$ } $\Rightarrow$
$f^{-1}(x)=\sqrt{4-x^2},\ 0\leq x\leq 2$

$\boxed{37}$ $f(x)=\sqrt[3]{x}+1 \Rightarrow y-1=\sqrt[3]{x} \Rightarrow x=(y-1)^3 \Rightarrow f^{-1}(x)=(x-1)^3$

[38] $f(x) = (x^3+1)^5 \;\Rightarrow\; \sqrt[5]{y} = x^3+1 \;\Rightarrow\; x^3 = \sqrt[5]{y}-1 \;\Rightarrow$
$x = \sqrt[3]{\sqrt[5]{y}-1} \;\Rightarrow\; f^{-1}(x) = \sqrt[3]{\sqrt[5]{x}-1}$

[39] $f(x) = x \;\Rightarrow\; y = x \;\Rightarrow\; x = y \;\Rightarrow\; f^{-1}(x) = x$

[40] $f(x) = -x \;\Rightarrow\; y = -x \;\Rightarrow\; x = -y \;\Rightarrow\; f^{-1}(x) = -x$

[41] $f(x) = x^2 - 6x,\ x \geq 3 \;\Rightarrow\; y = x^2 - 6x \;\Rightarrow\; x^2 - 6x - y = 0$. This is a quadratic equation in x with $a = 1$, $b = -6$, and $c = -y$. Solving with the quadratic formula gives us $x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-y)}}{2(1)} = \dfrac{6 \pm \sqrt{36+4y}}{2} = \dfrac{6 \pm 2\sqrt{9+y}}{2} =$ $3 \pm \sqrt{9+y}$. Since $x \geq 3$, we choose the "+" and obtain $f^{-1}(x) = 3 + \sqrt{x+9}$.

[42] $f(x) = x^2 - 4x + 3,\ x \leq 2 \;\Rightarrow\; y = x^2 - 4x + 3 \;\Rightarrow\; x^2 - 4x + (3-y) = 0 \;\Rightarrow$
$x = \dfrac{4 \pm \sqrt{16 - 4(1)(3-y)}}{2} = \dfrac{4 \pm \sqrt{4+4y}}{2} = \dfrac{4 \pm 4\sqrt{1+y}}{2} = 2 \pm \sqrt{y+1}.$

Since $x \leq 2$, we choose the "–" and obtain $f^{-1}(x) = 2 - \sqrt{x+1}$.

[43] (a) $(g^{-1} \circ f^{-1})(2) = g^{-1}(f^{-1}(2))$
$= g^{-1}(5)$ $\{\text{since } f(5) = 2\}$
$= 3$ $\{\text{since } g(3) = 5\}$

(b) $(g^{-1} \circ h)(3) = g^{-1}(h(3)) = g^{-1}(1) = -1$ $\{\text{since } g(-1) = 1\}$

(c) $(h^{-1} \circ f \circ g^{-1})(3) = h^{-1}(f(g^{-1}(3)))$
$= h^{-1}(f(2))$ $\{\text{since } g(2) = 3\}$
$= h^{-1}(-1)$
$= 5$ $\{\text{since } -1 = 4 - x \;\Rightarrow\; x = 5\}$

[44] (a) $(g \circ f^{-1})(-1) = g(f^{-1}(-1)) = g(2) = 3$

(b) $(f^{-1} \circ g^{-1})(3) = f^{-1}(g^{-1}(3)) = f^{-1}(2) = 5$

(c) $(h^{-1} \circ g^{-1} \circ f)(6) = h^{-1}(g^{-1}(f(6))) = h^{-1}(g^{-1}(3)) = h^{-1}(2) =$
2 $\{\text{since } 2 = 4 - x \;\Rightarrow\; x = 2\}$

[45] (b) $D = [-1, 2]$; $R = [\frac{1}{2}, 4]$

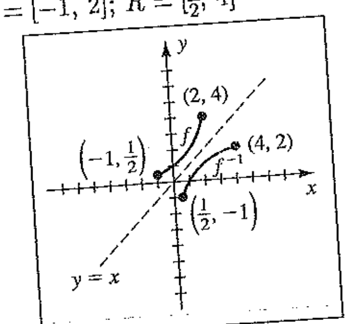

Figure 45

(c) $D_1 = R = [\frac{1}{2}, 4]$; $R_1 = D = [-1, 2]$

[46] (b) $D = [1, 10]$; $R = [0, 9]$

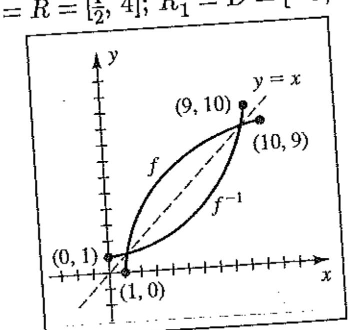

Figure 46

(c) $D_1 = R = [0, 9]$; $R_1 = D = [1, 10]$

[47] (b) $D = [-3, 3]$; $R = [-2, 2]$ (c) $D_1 = R = [-2, 2]$; $R_1 = D = [-3, 3]$

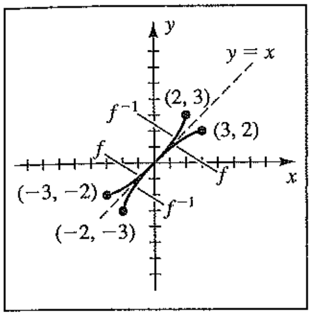

Figure 47

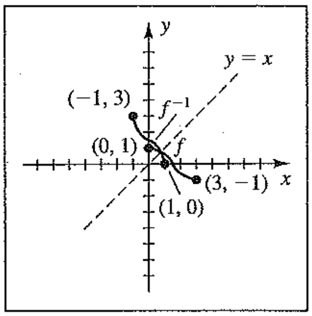

Figure 48

[48] (b) $D = [0, 3]$; $R = [-1, 1]$ (c) $D_1 = R = [-1, 1]$; $R_1 = D = [0, 3]$

[49] (a) Since f is one-to-one, an inverse exists.

If $f(x) = ax + b$, then $f^{-1}(x) = \frac{x-b}{a}$ for $a \neq 0$.

(b) No, because a constant function is not one-to-one.

[50] (1) If $P(a, b)$ is on the graph of f, then $f(a) = b$.

Also, $f^{-1}(f(a)) = f^{-1}(b)$ or $a = f^{-1}(b)$. So $Q(b, a)$ is on the graph of f^{-1}.

(2) $M_{PQ} = \left(\frac{a+b}{2}, \frac{b+a}{2}\right)$.

Since the x and y coordinates are equal, the point is on the line $y = x$.

(3) $m_{PQ} = \frac{a-b}{b-a} = -1$ and the slope of the line $y = x$ is 1. Since the slopes of the lines are negative reciprocals of each other, the lines are perpendicular.

[51] (a) $f(x) = -x + b \Rightarrow y = -x + b \Rightarrow x = -y + b$, or $f^{-1}(x) = -x + b$.

(b) $f(x) = \frac{ax+b}{cx-a}$ for $c \neq 0 \Rightarrow y = \frac{ax+b}{cx-a} \Rightarrow cyx - ya = ax + b \Rightarrow$

$cyx - ax = ay + b \Rightarrow x(cy - a) = ay + b \Rightarrow x = \frac{ay+b}{cy-a}$, or $f^{-1}(x) = \frac{ax+b}{cx-a}$.

(c) The graph of f is symmetric through the line $y = x$. Thus, $f(x) = f^{-1}(x)$.

[52] (a) $f(x) = x^n$ for $x \geq 0$. • If n is odd, then $y = x^n \Rightarrow x = y^{1/n}$. If n is even, then $y = x^n \Rightarrow x = \pm y^{1/n}$ {choose plus since $x \geq 0$}, or $f^{-1}(x) = x^{1/n}$.

(b) $f(x) = x^{m/n}$ for $x \geq 0$ and m any positive integer. •

$y = x^{m/n} \Rightarrow y^n = x^m \Rightarrow x = y^{n/m}$ {for the same reasons as in part (a)}.

[53] From a graph of f, we see that f is always increasing. Thus, f is one-to-one.

$[-6, 6]$ by $[-4, 4]$

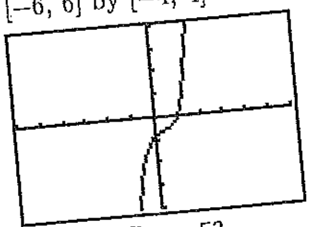

Figure 53

$[-2, 2]$ by $[-2.5, 0.5]$

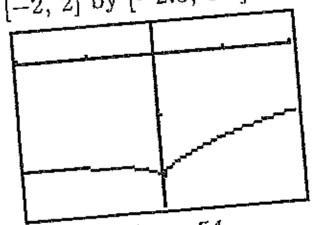

Figure 54

[54] From a graph of f, we see that f is both decreasing and increasing near the point $(0, -2)$. The horizontal line $y = -1.9$ will intersect the graph of f more than once. Thus, f is not one-to-one.

[55] (a) f decreases on $[-0.27, 1.22]$.

(b) Domain of g^{-1} is $[-0.20, 3.31]$; range of g^{-1} is $[-0.27, 1.22]$.

$[-1, 2]$ by $[-1, 4]$

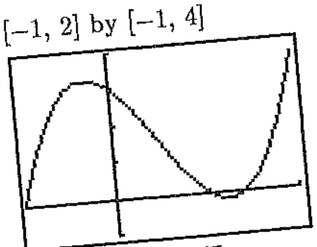

Figure 55

$[-2, 2]$ by $[-1.33, 1.33]$

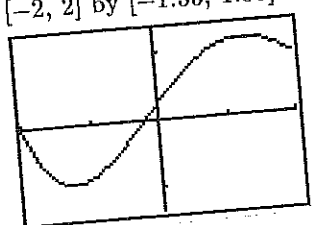

Figure 56

[56] (a) $[-1.27, 1.31]$

(b) Domain of g^{-1} is $[-0.88, 1.14]$; range of g^{-1} is $[-1.27, 1.31]$.

[57] The graph of f will be reflected through the line $y = x$.

$y = \sqrt[3]{x-1} \;\Rightarrow\; y^3 = x - 1 \;\Rightarrow\; x = y^3 + 1 \;\Rightarrow\; f^{-1}(x) = x^3 + 1.$

Graph $Y_1 = \sqrt[3]{x-1}$, $Y_2 = x^3 + 1$, and $Y_3 = x$.

$[-12, 12]$ by $[-8, 8]$

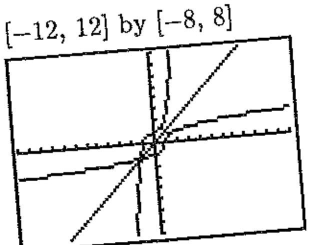

Figure 57

$[0, 12]$ by $[0, 8]$

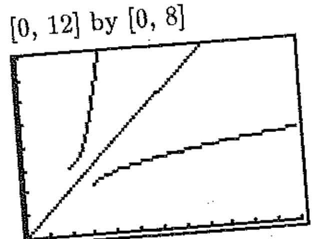

Figure 58

[58] The graph of f will be reflected through the line $y = x$. $y = 2(x-2)^2 + 3 \;\Rightarrow$

$\dfrac{y-3}{2} = (x-2)^2 \;\Rightarrow\; x - 2 = \pm\sqrt{\dfrac{y-3}{2}} \;\Rightarrow\; x = 2 \pm \sqrt{\dfrac{y-3}{2}}.$

Let $f^{-1}(x) = 2 + \sqrt{\dfrac{x-3}{2}}$ since we must have $f^{-1}(x) \geq 2$.

Graph $Y_1 = (2(x-2)^2 + 3)/(x \geq 2)$, $Y_2 = 2 + \sqrt{\dfrac{x-3}{2}}$, and $Y_3 = x$.

$\boxed{59}$ (a) $V(23) = 35(23) = 805 \text{ ft}^3/\text{min}$

(b) $V^{-1}(x) = \frac{1}{35}x$. Given an air circulation of x cubic feet per minute, $V^{-1}(x)$ computes the maximum number of people that should be in the restaurant at one time.

(c) $V^{-1}(2350) = \frac{1}{35}(2350) \approx 67.1 \;\Rightarrow\;$ the maximum number of people is 67.

$\boxed{60}$ (a) [1940, 2000, 10] by [0, 13E3, 1E3]

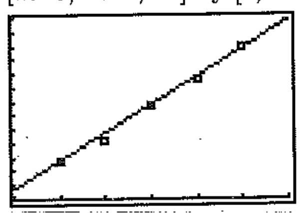

Figure 60

(b) Use the two arbitrary points (1950, 2773) and (1990, 10,819) to determine f.

$$m = \frac{10{,}819 - 2773}{1990 - 1950} = 201.15 \;\Rightarrow$$

$$y - 2773 = 201.15(x - 1950) = 201.15x - 392{,}242.5 \;\Rightarrow$$

$$f(x) = 201.15x - 389{,}469.5$$

(c) $y = 201.15x - 389{,}469.5 \;\Rightarrow\; \dfrac{y + 389{,}469.5}{201.15} = x \;\Rightarrow\; f^{-1}(x) = \dfrac{x + 389{,}469.5}{201.15}$.

$f^{-1}(x)$ computes the year when x radio stations were on the air.

(d) $f^{-1}(7744) = \dfrac{7744 + 389{,}469.5}{201.15} \approx 1974.7 \approx 1975$

5.2 Exercises

$\boxed{1}$ $7^{x+6} = 7^{3x-4} \;\Rightarrow\; x + 6 = 3x - 4 \;\Rightarrow\; 10 = 2x \;\Rightarrow\; x = 5$

$\boxed{2}$ $6^{7-x} = 6^{2x+1} \;\Rightarrow\; 7 - x = 2x + 1 \;\Rightarrow\; 6 = 3x \;\Rightarrow\; x = 2$

$\boxed{3}$ $3^{2x+3} = 3^{(x^2)} \;\Rightarrow\; 2x + 3 = x^2 \;\Rightarrow\; x^2 - 2x - 3 = 0 \;\Rightarrow\; (x-3)(x+1) = 0 \;\Rightarrow$

$$x = -1,\ 3$$

$\boxed{4}$ $9^{(x^2)} = 3^{3x+2} \;\Rightarrow\; (3^2)^{(x^2)} = 3^{3x+2} \;\Rightarrow\; 3^{(2x^2)} = 3^{3x+2} \;\Rightarrow$

$$2x^2 = 3x + 2 \;\Rightarrow\; 2x^2 - 3x - 2 = 0 \;\Rightarrow\; (2x+1)(x-2) = 0 \;\Rightarrow\; x = -\tfrac{1}{2},\ 2$$

$\boxed{5}$ $2^{-100x} = (0.5)^{x-4} \;\Rightarrow\; (2^{-1})^{100x} = (\frac{1}{2})^{x-4} \;\Rightarrow\; (\frac{1}{2})^{100x} = (\frac{1}{2})^{x-4} \;\Rightarrow$

$$100x = x - 4 \;\Rightarrow\; 99x = -4 \;\Rightarrow\; x = -\tfrac{4}{99}$$

$\boxed{6}$ $(\frac{1}{2})^{6-x} = 2 \;\Rightarrow\; (\frac{1}{2})^{6-x} = (\frac{1}{2})^{-1} \;\Rightarrow\; 6 - x = -1 \;\Rightarrow\; x = 7$

$\boxed{7}$ $4^{x-3} = 8^{4-x} \;\Rightarrow\; (2^2)^{x-3} = (2^3)^{4-x} \;\Rightarrow\; 2^{2x-6} = 2^{12-3x} \;\Rightarrow$

$$2x - 6 = 12 - 3x \;\Rightarrow\; 5x = 18 \;\Rightarrow\; x = \tfrac{18}{5}$$

$\boxed{8}$ $27^{x-1} = 9^{2x-3} \Rightarrow (3^3)^{x-1} = (3^2)^{2x-3} \Rightarrow 3^{3x-3} = 3^{4x-6} \Rightarrow$
$3x - 3 = 4x - 6 \Rightarrow x = 3$

$\boxed{9}$ $4^x \cdot (\frac{1}{2})^{3-2x} = 8 \cdot (2^x)^2 \Rightarrow 2^{2x} \cdot (2^{-1})^{3-2x} = 2^3 \cdot 2^{2x} \Rightarrow$
$2^{2x-3+2x} = 2^{3+2x} \Rightarrow 2^{4x-3} = 2^{2x+3} \Rightarrow 4x - 3 = 2x + 3 \Rightarrow$
$2x = 6 \Rightarrow x = 3$

$\boxed{10}$ $9^{2x} \cdot (\frac{1}{3})^{x+2} = 27 \cdot (3^x)^{-2} \Rightarrow 3^{4x} \cdot (3^{-1})^{x+2} = 3^3 \cdot 3^{-2x} \Rightarrow$
$3^{4x-x-2} = 3^{3-2x} \Rightarrow 3^{3x-2} = 3^{3-2x} \Rightarrow 3x - 2 = 3 - 2x \Rightarrow$
$5x = 5 \Rightarrow x = 1$

$\boxed{11}$ (a) Let $F = f(x) = 2^x$. This graph goes through $(-1, \frac{1}{2})$, $(0, 1)$, and $(1, 2)$.

(b) $f(x) = -2^x$ • reflect F through the x-axis

(c) $f(x) = 3 \cdot 2^x$ • vertically stretch F by a factor of 3

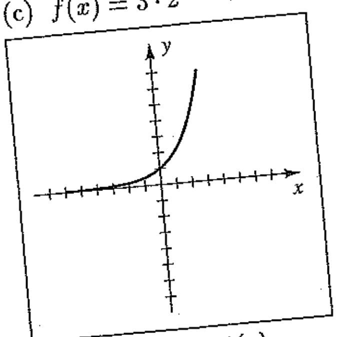

Figure 11(a)

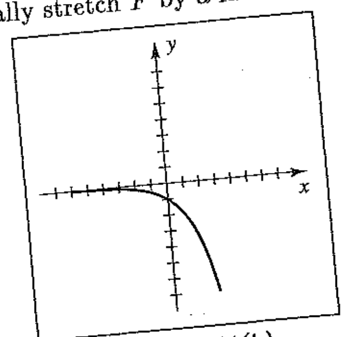

Figure 11(b)

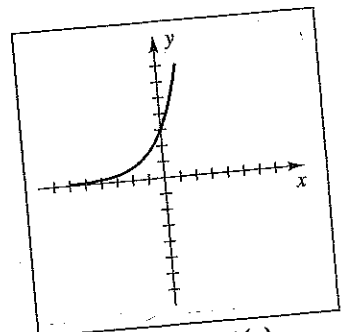

Figure 11(c)

(d) $f(x) = 2^{x+3}$ • shift F left 3 units

(e) $f(x) = 2^x + 3$ • shift F up 3 units

(f) $f(x) = 2^{x-3}$ • shift F right 3 units

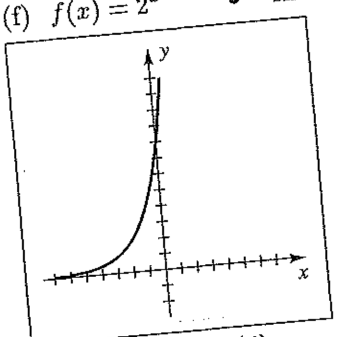

Figure 11(d)

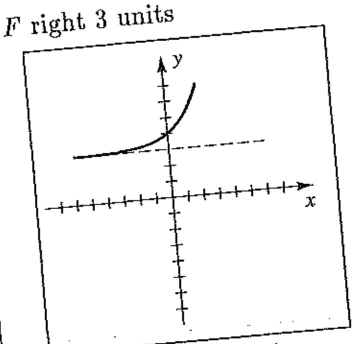

Figure 11(e)

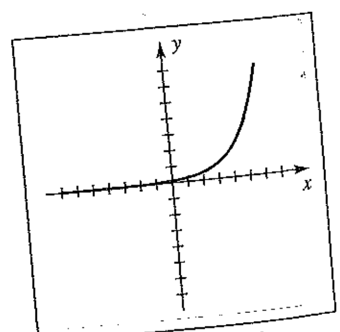

Figure 11(f)

(g) $f(x) = 2^x - 3$ • shift F down 3 units

(h) $f(x) = 2^{-x}$ • reflect F through the y-axis

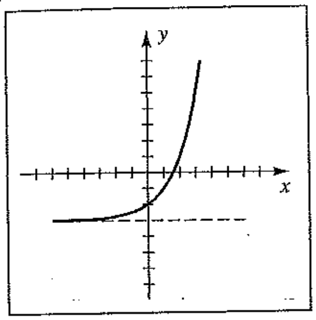

Figure 11(g)

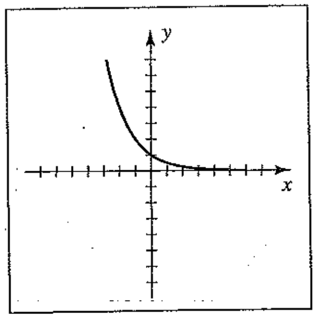

Figure 11(h)

(i) $f(x) = (\frac{1}{2})^x$ • $(\frac{1}{2})^x = (2^{-1})^x = 2^{-x}$, same graph as in part (h)

(j) $f(x) = 2^{3-x}$ • $2^{3-x} = 2^{-(x-3)}$, shift F right 3 units and reflect through the line $x = 3$. Or, $2^{3-x} = 2^3 2^{-x} = 8(\frac{1}{2})^x$, vertically stretch $y = (\frac{1}{2})^x$ by a factor of 8.

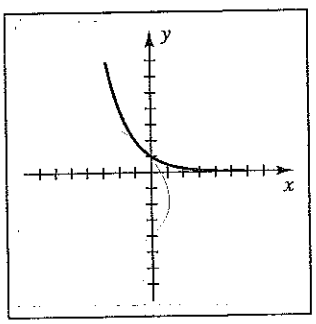

Figure 11(i)

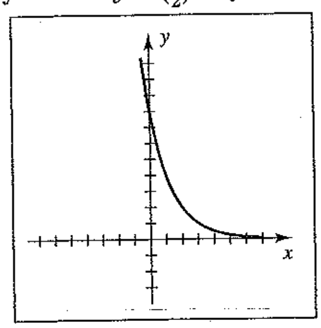

Figure 11(j)

$\boxed{12}$ (a) Let $F = f(x) = (\frac{1}{2})^x$. This graph goes through $(-1, 2)$, $(0, 1)$, and $(1, \frac{1}{2})$.

(b) $f(x) = -(\frac{1}{2})^x$ • reflect F through the x-axis

(c) $f(x) = 3 \cdot (\frac{1}{2})^x$ • vertically stretch F by a factor of 3

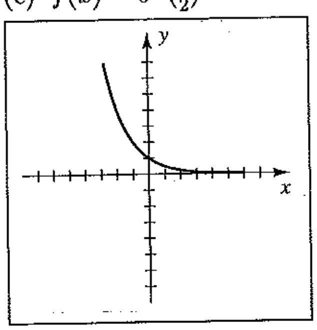

Figure 12(a)

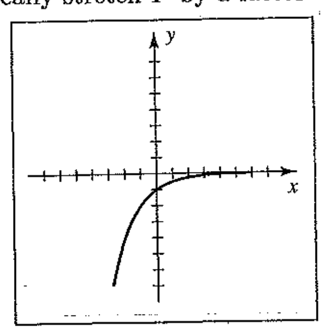

Figure 12(b)

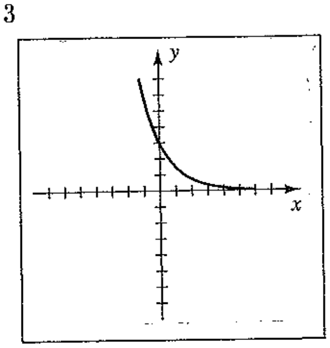

Figure 12(c)

(d) $f(x) = (\frac{1}{2})^{x+3}$ • shift F left 3 units

(e) $f(x) = (\frac{1}{2})^{x} + 3$ • shift F up 3 units

(f) $f(x) = (\frac{1}{2})^{x-3}$ • shift F right 3 units

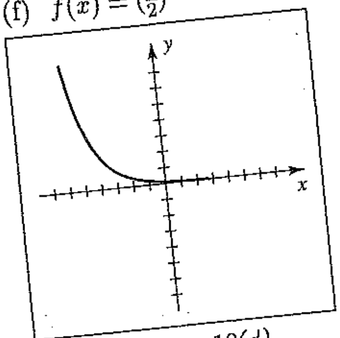

Figure 12(d)

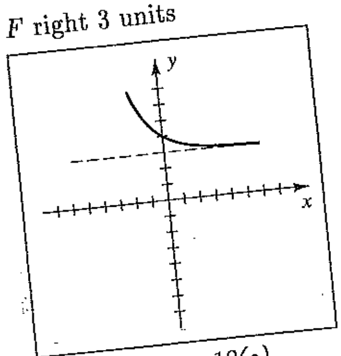

Figure 12(e)

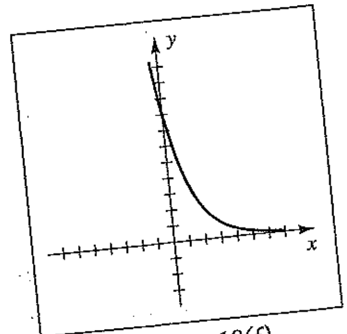

Figure 12(f)

(g) $f(x) = (\frac{1}{2})^{x} - 3$ • shift F down 3 units

(h) $f(x) = (\frac{1}{2})^{-x}$ • reflect F through the y-axis

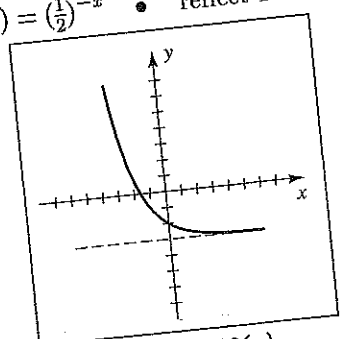

Figure 12(g)

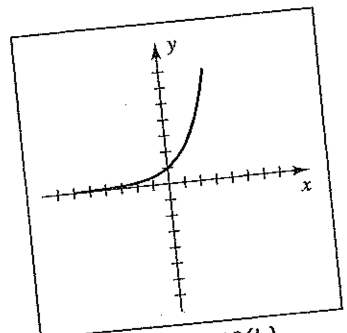

Figure 12(h)

(i) $f(x) = (\frac{1}{1/2})^{x}$ • $(\frac{1}{1/2})^{x} = (2)^{x}$, same graph as in part (h) or #11(a)

(j) $f(x) = (\frac{1}{2})^{3-x}$ • Since $(\frac{1}{2})^{3-x} = (\frac{1}{2})^{-(x-3)}$, we can shift F right 3 units and reflect through the line $x = 3$. Or, since $(\frac{1}{2})^{3-x} = (\frac{1}{2})^{3}(\frac{1}{2})^{-x} = \frac{1}{8} \cdot 2^{x}$, we can vertically compress $y = 2^{x}$ by a factor of 8.

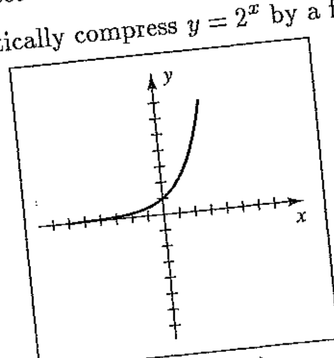

Figure 12(i)

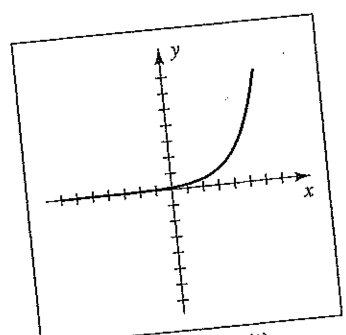

Figure 12(j)

$\boxed{13}$ $f(x) = (\frac{2}{5})^{-x} = \left[(\frac{2}{5})^{-1}\right]^{x} = (\frac{5}{2})^{x}$ • goes through $(-1, \frac{2}{5})$, $(0, 1)$, and $(1, \frac{5}{2})$

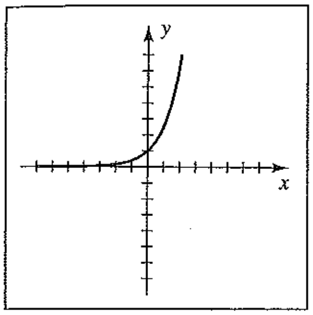

Figure 13

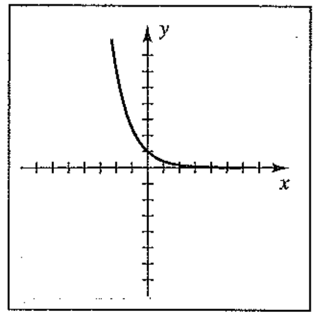

Figure 14

$\boxed{14}$ $f(x) = (\frac{2}{5})^{x}$ • goes through $(-1, \frac{5}{2})$, $(0, 1)$, and $(1, \frac{2}{5})$

$\boxed{15}$ $f(x) = 5(\frac{1}{2})^{x} + 3$ • vertically stretch $y = (\frac{1}{2})^{x}$ by a factor of 5 and shift up 3 units

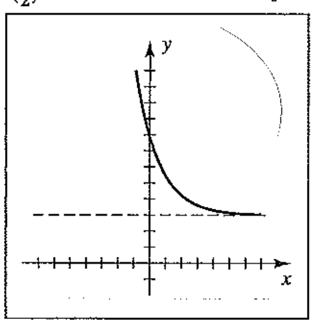

Figure 15

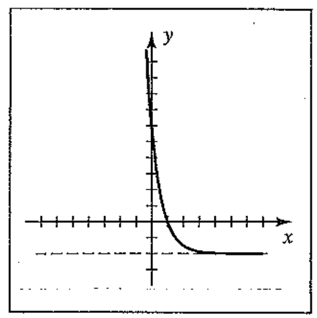

Figure 16

$\boxed{16}$ $f(x) = 8(4)^{-x} - 2$ •

vertically stretch $y = (\frac{1}{4})^{x}$ by a factor of 8 and shift down 2 units

$\boxed{17}$ $f(x) = -(\frac{1}{2})^{x} + 4$ • reflect $y = (\frac{1}{2})^{x}$ through the x-axis and shift up 4 units

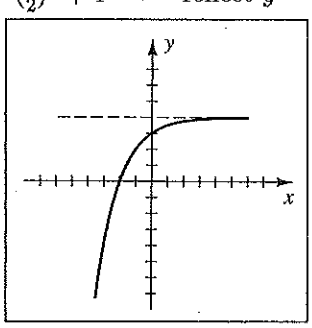

Figure 17

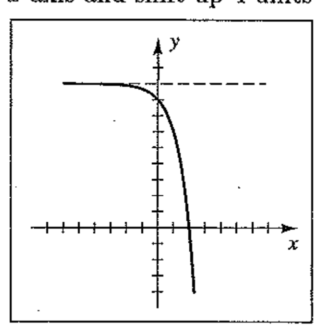

Figure 18

$\boxed{18}$ $f(x) = -3^{x} + 9$ • reflect $y = 3^{x}$ through the x-axis and shift up 9 units

$\boxed{19}$ $f(x) = 2^{|x|}$ • use the portion of $y = 2^x$ with $x \geq 0$ and reflect it through the y-axis since f is even

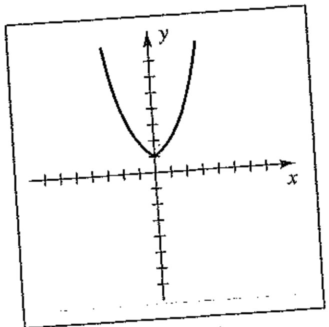
Figure 19

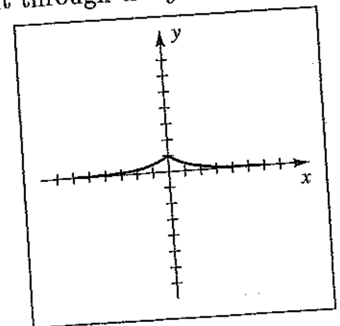
Figure 20

$\boxed{20}$ $f(x) = 2^{-|x|} = (\frac{1}{2})^{|x|}$ • use the portion of $y = (\frac{1}{2})^x$ with $x \geq 0$ and reflect it through the y-axis since f is even

Note: For Exercises 21, 22, and 5 of the review exercises, refer to Example 6 in the text for the basic graph of $y = a^{-x^2} = \left(\frac{1}{a}\right)^{x^2}$, where $a > 1$.

$\boxed{21}$ $f(x) = 3^{1-x^2} = 3^1 3^{-x^2} = 3(\frac{1}{3})^{x^2}$ • vertically stretch $y = (\frac{1}{3})^{x^2}$ by a factor of 3

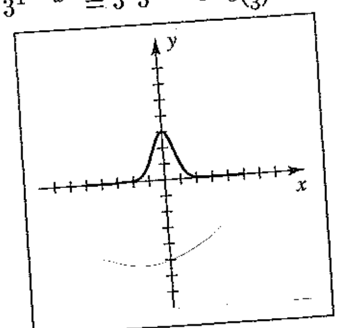
Figure 21

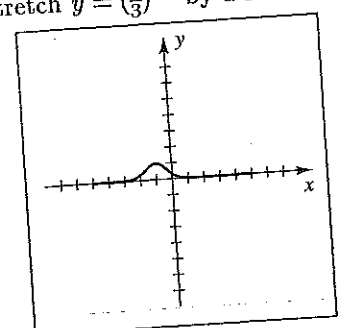
Figure 22

$\boxed{22}$ $f(x) = 2^{-(x+1)^2} = (\frac{1}{2})^{(x+1)^2}$ • shift $y = (\frac{1}{2})^{x^2}$ left 1 unit

$\boxed{23}$ $f(x) = 3^x + 3^{-x}$ • if $x > 0$, f looks like $y = 3^x$ since 3^x dominates 3^{-x}; if $x < 0$, f looks like $y = 3^{-x}$ since 3^{-x} dominates 3^x

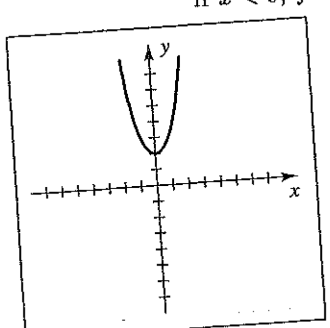
Figure 23

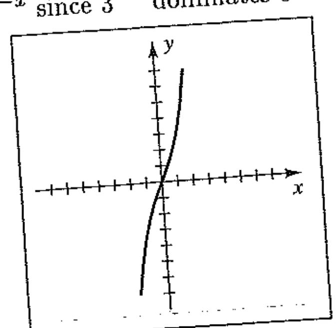
Figure 24

$\boxed{24}$ $f(x) = 3^x - 3^{-x}$ • if $x > 0$, f looks like $y = 3^x$ since $0 < 3^{-x} < 1$; if $x < 0$, f looks like $y = -3^{-x}$ since $0 < 3^x < 1$

[25] The graph has a horizontal asymptote of $y = 0$, so $c = 0$ and the equation has the form $f(x) = ba^x$. The point (0, 2) is on the graph, so $f(0) = 2 \Rightarrow 2 = ba^0 \Rightarrow$ $2 = b(1) \Rightarrow 2 = b$, and hence, $f(x) = 2a^x$. Since the point $P(1, 5)$ is on the graph, $f(1) = 5$. Thus, $5 = 2a^1 \Rightarrow a = \frac{5}{2}$, and the function is $f(x) = 2(\frac{5}{2})^x$.

[26] The graph has a horizontal asymptote of $y = 0$, so $c = 0$ and the equation has the form $f(x) = ba^x$. The point $(0, \frac{1}{2})$ is on the graph, so $f(0) = \frac{1}{2} \Rightarrow \frac{1}{2} = ba^0 \Rightarrow$ $\frac{1}{2} = b(1) \Rightarrow \frac{1}{2} = b$, and hence, $f(x) = \frac{1}{2}a^x$. Since the point $P(-2, 8)$ is on the graph, $f(-2) = 8$. Thus, $8 = \frac{1}{2}a^{-2} \Rightarrow \dfrac{1}{a^2} = 16 \Rightarrow a^2 = \frac{1}{16} \Rightarrow a = \pm\frac{1}{4}$, but a must be positive, and the function is $f(x) = \frac{1}{2}(\frac{1}{4})^x$.

[27] The graph has a horizontal asymptote of $y = -3$, so $c = -3$ and the equation has the form $f(x) = ba^x - 3$. The point $(0, -1)$ is on the graph, so $f(0) = -1 \Rightarrow$ $-1 = ba^0 - 3 \Rightarrow -1 = b(1) - 3 \Rightarrow 2 = b$, and hence, $f(x) = 2a^x - 3$. Since the point $P(-1, 0)$ is on the graph, $f(-1) = 0$. Thus, $0 = 2a^{-1} - 3 \Rightarrow 3 = \dfrac{2}{a} \Rightarrow$ $3a = 2 \Rightarrow a = \frac{2}{3}$, and the function is $f(x) = 2(\frac{2}{3})^x - 3$.

[28] The graph has a horizontal asymptote of $y = 1$, so $c = 1$ and the equation has the form $f(x) = ba^x + 1$. The point (0, 5) is on the graph, so $f(0) = 5 \Rightarrow$ $5 = ba^0 + 1 \Rightarrow 4 = b$, and hence, $f(x) = 4a^x + 1$. Since the point $P(1, 7)$ is on the graph, $f(1) = 7$. Thus, $7 = 4a^1 + 1 \Rightarrow 6 = 4a \Rightarrow a = \frac{3}{2}$, and the function is $f(x) = 4(\frac{3}{2})^x + 1$.

[29] Since the y-intercept is 8, we must have $f(0) = 8$. $f(x) = ba^x$, so $f(0) = 8 \Rightarrow$ $8 = ba^0 \Rightarrow 8 = b(1) \Rightarrow 8 = b$, and hence, $f(x) = 8a^x$. Since the point $P(3, 1)$ is on the graph, $f(3) = 1$. Thus, $1 = 8a^3 \Rightarrow a^3 = \frac{1}{8} \Rightarrow a = \frac{1}{2}$, and the function is $f(x) = 8(\frac{1}{2})^x$.

[30] $f(x) = ba^x$ and $f(0) = 6 \Rightarrow 6 = ba^0 \Rightarrow 6 = b \Rightarrow f(x) = 6a^x$.
$f(2) = \frac{3}{32} \Rightarrow \frac{3}{32} = 6a^2 \Rightarrow a^2 = \frac{1}{64} \Rightarrow a = \frac{1}{8}\ \{a > 0\}$. Thus, $f(x) = 6(\frac{1}{8})^x$.

[31] The horizontal asymptote is $y = 32$, so $f(x) = ba^{-x} + c$ has the form $f(x) = ba^{-x} + 32$. The y-intercept is 212, so $f(0) = 212 \Rightarrow 212 = ba^{-0} + 32 \Rightarrow$ $212 = b(1) + 32 \Rightarrow 180 = b \Rightarrow f(x) = 180a^{-x} + 32$. The function passes through the point $P(2, 112)$, so $f(2) = 112 \Rightarrow 112 = 180a^{-2} + 32 \Rightarrow 80 = 180a^{-2} \Rightarrow$ $80 = \dfrac{180}{a^2} \Rightarrow a^2 = \dfrac{180}{80} \Rightarrow a^2 = \dfrac{9}{4} \Rightarrow a = \dfrac{3}{2}$ {since a must be positive, $a = -\dfrac{3}{2}$ is not allowed} $\Rightarrow f(x) = 180(1.5)^{-x} + 32$.

$\boxed{32}$ The horizontal asymptote is $y = 72$, so $f(x) = ba^{-x} + c$ has the form $f(x) = ba^{-x} + 72$. $f(0) = 425 \Rightarrow 425 = ba^{-0} + 72 \Rightarrow 353 = b$, so $f(x) = 353a^{-x} + 72$. $f(1) = 248.5 \Rightarrow 248.5 = 353a^{-1} + 72 \Rightarrow$

$$176.5 = \frac{353}{a} \Rightarrow a = \frac{353}{176.5} = 2 \Rightarrow f(x) = 353(2)^{-x} + 72.$$

$\boxed{33}$ (a) $N(t) = 100(0.9)^t \Rightarrow N(1) = 100(0.9)^1 = 90$

(b) $N(5) = 100(0.9)^5 \approx 59$ (c) $N(10) = 100(0.9)^{10} \approx 35$

$\boxed{34}$ (a) $A(t) = 10(0.8)^t \Rightarrow A(8) = 10(0.8)^8 \approx 1.68$ mg

(b) In general, $A(t+1)$ is the amount in the body one hour after $A(t)$.

So the percentage of the drug still in the body after one hour is

$$\frac{A(t+1)}{A(t)} = \frac{10(0.8)^{t+1}}{10(0.8)^t} = 0.8;$$ that is, 80% remains, or 20% is eliminated.

$\boxed{35}$ (a) 8:00 A.M. corresponds to $t = 1$ and $f(1) = 600\sqrt{3} \approx 1039$.

10:00 A.M. corresponds to $t = 3$ and $f(3) = 600(3\sqrt{3}) = 1800\sqrt{3} \approx 3118$.

11:00 A.M. corresponds to $t = 4$ and $f(4) = 600(9) = 5400$.

(b) The graph of f is an increasing exponential that passes through (0, 600) and the points in part (a).

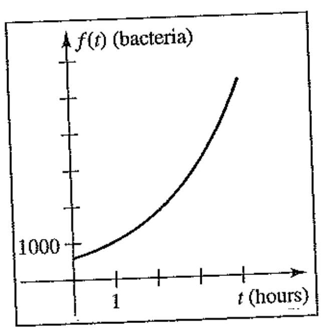

Figure 35

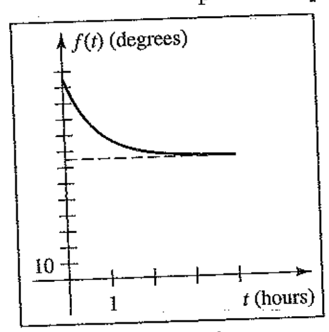

Figure 36

$\boxed{36}$ (a) 2:00 P.M. corresponds to $t = 1$ and $f(1) = 87.5^\circ$.

3:30 P.M. corresponds to $t = \frac{5}{2}$ and $f(\frac{5}{2}) = 76.5625 \approx 76.6^\circ$.

4:00 P.M. corresponds to $t = 3$ and $f(3) = 75.78125 \approx 75.8^\circ$.

(b) The endpoints are (0, 125) and (4, ≈ 75.2).

$\boxed{37}$ (a) $f(t) = 100(2)^{-t/5}$, so $f(5) = 100(2)^{-1} = 50$ mg, $f(10) = 100(2)^{-2} = 25$ mg, and
$$f(12.5) = 100(2)^{-2.5} = \frac{100}{4\sqrt{2}} = \frac{25}{2}\sqrt{2} \approx 17.7 \text{ mg}.$$

(b) The endpoints are (0, 100) and (30, 1.5625).

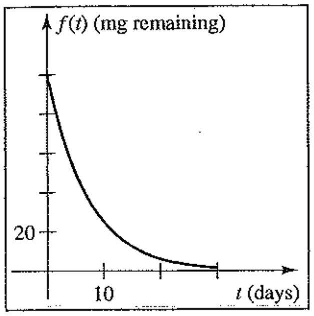

Figure 37

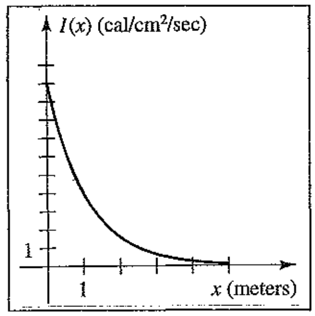

Figure 38

$\boxed{38}$ (a) $I(x) = 10(0.4)^x \Rightarrow I(2) = 10(0.4)^2 = 1.6$ calories/cm^2/sec

(b) The endpoints are (0, 10) and (5, 0.1024).

$\boxed{39}$ $q(t) = \frac{1}{2}q_0$ when $t = 1600 \Rightarrow \frac{1}{2}q_0 = q_0 2^{k(1600)} \Rightarrow 2^{-1} = 2^{1600k} \Rightarrow$
$$1600k = -1 \Rightarrow k = -\tfrac{1}{1600}.$$

$\boxed{40}$ The endpoints of the graph of $q(t) = 10(\frac{4}{5})^t$ are (0, 10) and $(10, \approx 1.07)$.

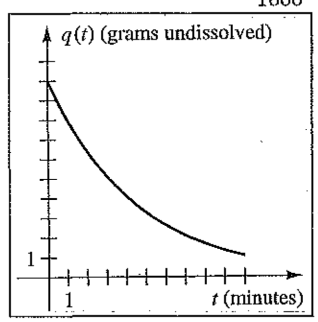

Figure 40

$\boxed{41}$ Using $A = P(1 + \frac{r}{n})^{nt}$, we have $P = 1000$, $r = 0.12$, and $n = 12$.

Consider A to be a function of t; that is, $A(t) = 1000\left(1 + \frac{0.12}{12}\right)^{12t} = 1000(1.01)^{12t}$.

(a) $A(\frac{1}{12}) = \$1010.00$ (b) $A(\frac{6}{12}) \approx \$1061.52$

(c) $A(1) \approx \$1126.83$ (d) $A(20) \approx \$10{,}892.55$

$\boxed{42}$ $A = 5000 \Rightarrow 5000 = P\left(1 + \frac{0.10}{2}\right)^{2 \cdot 1} \Rightarrow P = \dfrac{5000}{(1.05)^2} \approx \$4535.15$

$\boxed{43}$ $C = 10{,}000 \Rightarrow V(t) = 0.78(10{,}000)(0.85)^{t-1} = 7800(0.85)^{t-1}$

(a) $V(1) = \$7800$ (b) $V(4) \approx \$4790.18$, or \$4790 (c) $V(7) \approx \$2941.77$, or \$2942

$\boxed{44}$ The year 2010 corresponds to $t = 2010 - 1986 = 24$.
$$P = 80{,}000 \Rightarrow V = 80{,}000(1.05)^{24} = \$258{,}008.00.$$

$\boxed{45}$ $t = 2006 - 1626 = 380$; $A = \$24(1 + 0.06/4)^{4 \cdot 380} = \$161{,}657{,}351{,}965.80$.

$\boxed{46}$ $P = 500$, $r = 0.18$, and $n = 12 \Rightarrow A = 500\left(1 + \frac{0.18}{12}\right)^{12 \cdot 1} = 500(1.015)^{12} \approx \$597.81$

[47] (a) Examine the pattern formed by the value y in the year n.

year (n)	value (y)
0	y_0
1	$(1-a)y_0 = y_1$
2	$(1-a)y_1 = (1-a)\left[(1-a)y_0\right] = (1-a)^2 y_0 = y_2$
3	$(1-a)y_2 = (1-a)\left[(1-a)^2 y_0\right] = (1-a)^3 y_0 = y_3$

(b) $s = (1-a)^T y_0 \;\Rightarrow\; (1-a)^T = s/y_0 \;\Rightarrow\; 1-a = \sqrt[T]{s/y_0} \;\Rightarrow\; a = 1 - \sqrt[T]{s/y_0}$

[48] (a) $t = \frac{100}{1000} = \frac{1}{10}$ millennium and

$N(\frac{1}{10}) = N_0(0.805)^{1/10} \approx 0.9785\, N_0$.

This is a 97.85% retention, or a 2.15% loss.

(b) The endpoints are (0, 200) and (5, ≈ 67.61).

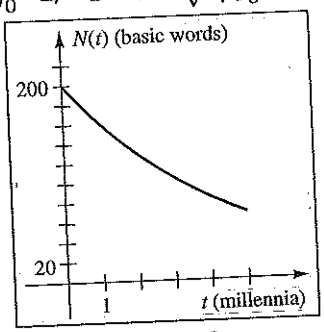

Figure 48

[49] (a) $r = 0.12,\ t = 30,\ L = 90{,}000 \;\Rightarrow\; k \approx 35.95,\ M \approx 925.75$

(b) (360 payments) × \$925.75 − \$90,000 = \$243,270

[50] $r = 0.10,\ t = 25,\ M = 800 \;\Rightarrow\; k \approx 12.06,\ L \approx \$88{,}037.78$

[51] $r = 0.15,\ t = 3,\ M = 220 \;\Rightarrow\; k \approx 1.56,\ L \approx \$6{,}346.40$

[52] (a) $r = 0.125,\ t = 2,\ L = 3000 \;\Rightarrow\; k \approx 1.28,\ M \approx \$141.92$

(b) (24 payments) × \$141.92 − \$3000 = \$406.08

[53] (a) $f(3) = 13^{\sqrt{3+1.1}} \approx 180.1206$ (b) $g(1.43) = \left(\frac{5}{42}\right)^{-1.43} \approx 20.9758$

(c) $h(1.06) = (2^{1.06} + 2^{-1.06})^{2(1.06)} \approx 7.3639$

[54] (a) $f(2.5) = 2^{\sqrt[3]{1-2.5}} \approx 0.4523$ (b) $g(2.1) = \left(\frac{2}{25} + 2.1\right)^{-3(2.1)} \approx 0.0074$

(c) $h(\sqrt{2}) = \dfrac{3^{-\sqrt{2}} + 5}{3^{\sqrt{2}} - 16} \approx -0.4624$

[55] Part (b) may be interpreted as doubling an investment at 8.5%.

(a) If $y = (1.085)^x$ and $x = 40$, then $y \approx 26.13$. (b) If $y = 2$, then $x \approx 8.50$.

[0, 60, 5] by [0, 40, 5]

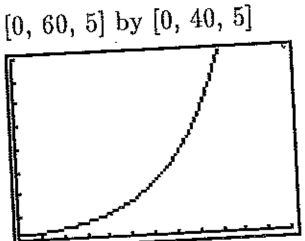
Figure 55

[0, 60, 5] by [0, 40, 5]

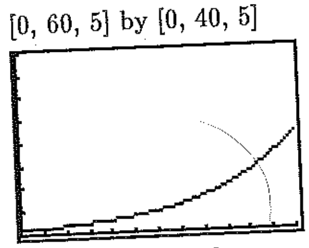
Figure 56

[56] Compare the doubling time in part (b) with the result in Exercise 55(b).

(a) If $y = (1.0525)^x$ and $x = 40$, then $y \approx 7.74$. (b) If $y = 2$, then $x \approx 13.55$.

[57] First graph $y = 1.4x^2 - 2.2^x - 1$. The x-intercepts are the roots of the equation. Using a zero or root feature, the roots are $x \approx -1.02$, 2.14, and 3.62.

[−10.5, 10.5] by [−7, 7]

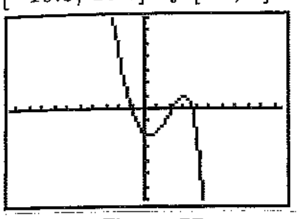

Figure 57

[−10.5, 10.5] by [−7, 7]

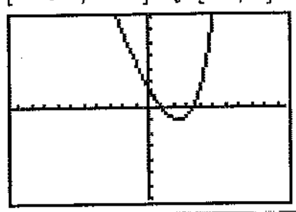

Figure 58

[58] First graph $y = 1.21^{3x} + 1.4^{-1.1x} - 2x - 0.5$. The x-intercepts are the roots of the equation. Using a zero or root feature, the roots are $x \approx 0.97$ and 3.41.

[59] (a) See *Figure 59.* f is not one-to-one since the horizontal line $y = -0.1$ intersects the graph of f more than once.

(b) By inspection, the only zero of f is $x = 0$.

[−3, 3] by [−2, 2]

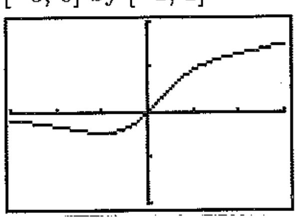

Figure 59

[−4, 4] by [−2.7, 2.7]

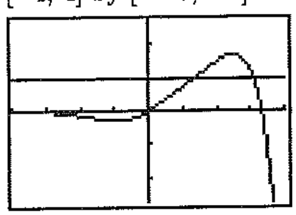

Figure 60

[60] (a) On older calculator models, rewrite $f(x) = \pi^{0.6x} - 1.3^{(x^{1.8})}$ as $f(x) = \pi^{0.6x} - 1.3^{(x^{9/5})}$. f is not one-to-one since the horizontal line $y = 1$ intersects the graph of f more than once.

(b) The zeros of f are $x \approx -3.33$, 0, and 3.33.

[61] (a) f is increasing on $[-3.37, -1.19]$ and $[0.52, 1]$. f is decreasing on $[-4, -3.37]$ and $[-1.19, 0.52]$.

(b) The range of f on $[-4, 1]$ is approximately $[-1.79, 1.94]$.

[−4, 1] by [−2, 3]

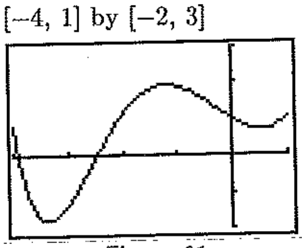

Figure 61

[−3, 3] by [−2, 2]

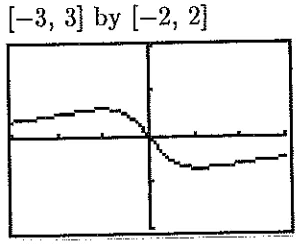

Figure 62

[62] (a) f is increasing on $[-3, -1]$ and $[1.11, 3]$. f is decreasing on $[-1, 1.11]$.

(b) The range of f on $[-3, 3]$ is approximately $[-0.69, 0.62]$. See *Figure 62.*

[63] *Figure 63* is a graph of $N(t) = 1000(0.9)^t$.

Using an intersect feature, we determine that $N = 500$ when $t \approx 6.58$ yr.

[0, 10] by [0, 1000, 100]

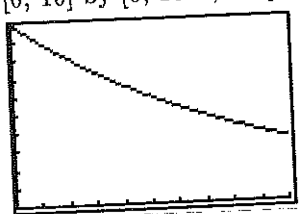

Figure 63

[0, 20] by [0, 1, 0.1]

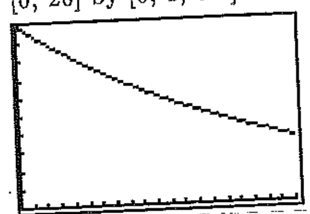

Figure 64

[64] From a graph of $B(t) = (0.95)^t$, we determine that $B = 0.5$ when $t \approx 13.51$ yr.

[65] Graph $y = 4(0.125)^{(0.25^x)}$. The line $y = k = 4$ is a horizontal asymptote for the Gompertz function. The maximum number of sales of the product approaches k.

[0, 7.5] by [0, 5]

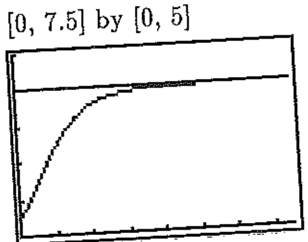

Figure 65

[0, 7.5] by [0, 5]

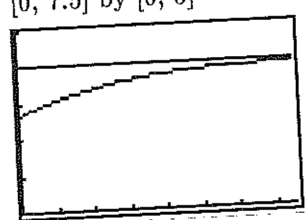

Figure 66

[66] Graph $y = \dfrac{1}{0.25 + 0.125(0.625)^x}$. The line $y = 1/k = 4$ is a horizontal asymptote for the logistic function. The maximum number of sales of the product approaches $1/k$.

[67] Using an intersect feature, we determine that $A = 100{,}000$ when $n \approx 32.8$.

[0, 40, 10] by [0, 2E5, 5E4]

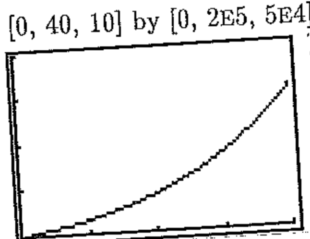

Figure 67

[0, 40, 10] by [0, 2E5, 5E4]

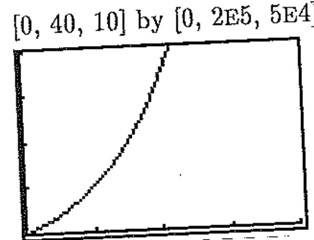

Figure 68

[68] Using an intersect feature, we determine that $A = 100{,}000$ when $n \approx 15.4$.

$\boxed{69}$ (a) Let $x = 0$ correspond to 1910, $x = 20$ to 1930, ..., and $x = 85$ to 1995.

Graph the data together with the functions

(1) $f(x) = 0.809(1.094)^x$ and (2) $g(x) = 0.375x^2 - 18.4x + 88.1$.

[−10, 90, 10] by [−200, 1500, 100]

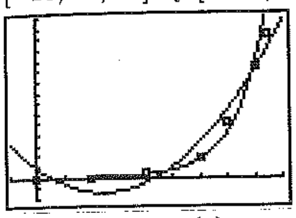

Figure 69(a)

[−10, 90, 10] by [−200, 1500, 100]

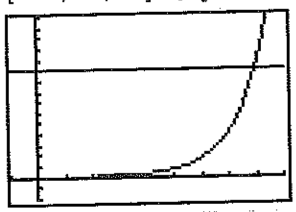

Figure 69(c)

(b) The exponential function f best models the data.

(c) Graph $Y_1 = f(x)$ and $Y_2 = 1000$. The graphs intersect at $x \approx 79$, or in 1989.

$\boxed{70}$ (a) Change the date in the table to the number of days after August 12.

Graph the data together with the functions

$$f(t) = 653(1.028)^t \quad \text{and} \quad g(t) = 54{,}700e^{-(t-200)^2/7500}.$$

t (days)	0	28	56	84	112	140	168
New Cases	506	1289	3487	9597	18,817	33,835	47,191

[0, 400, 100] by [0, 6E4, 1E4]

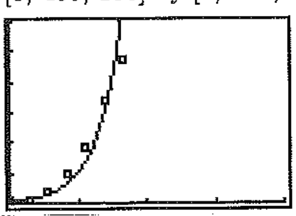

Figure 70(a)

[0, 400, 100] by [0, 6E4, 1E4]

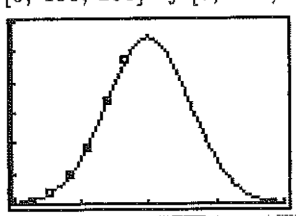

Figure 70(b)

(b) The function g models Farr's prediction better than f since the graph of g attains a maximum and then decreases.

(c) The maximum number of new cases occurred 200 days after August 12, which is February 28, 1841. After this date, the number of new cases decreased.

$\boxed{71}$ The model is $y = 0.03b^t$, with $t = 0$ corresponding to 1958. The year 2002 corresponds to $t = 2002 - 1958 = 44$, so $0.37 = 0.03b^{44} \Rightarrow \frac{37}{3} = b^{44} \Rightarrow$ $b = \sqrt[44]{\frac{37}{3}} \approx 1.05876$. The model is $y = 0.03(1.0588)^t$. Using $t = 2010 - 1958 = 52$, we predict the cost of a first-class stamp in 2010 will be $0.03(1.0588)^{52} \approx 0.5842$, or 58¢.

[72] The model is $y = 37.80b^t$, with $t = 0$ corresponding to 1970.

The year 2000 corresponds to $t = 2000 - 1970 = 30$, so $168.80 = 37.80b^{30}$ $\Rightarrow$

$\frac{1688}{378} = b^{30}$ $\Rightarrow$ $b = \sqrt[30]{\frac{1688}{378}} \approx 1.051145$. The model is $y = 37.80(1.0511)^t$. Using $t = 2010 - 1970 = 40$, we predict the value of the CPI in 2010 will be $37.80(1.0511)^{40} \approx 277.97$, or \$277.97.

[73] (a) $t = 1999 - 1974 = 25$ and r/n is 0.0025, so $A = P\left(1 + \frac{r}{n}\right)^{nt} =$

$\$353{,}022(1 + 0.0025)^{12 \cdot 25} = \$746{,}648.43$; the website gives \$1,192,971.

(b) Let r denote the annual interest rate. $\$6{,}616{,}585 = \$353{,}022\left(1 + \frac{r}{1}\right)^{1 \cdot 25}$ $\Rightarrow$

$$\frac{6{,}616{,}585}{353{,}022} = (1 + r)^{25} \quad\Rightarrow\quad 1 + r = \sqrt[25]{\frac{6{,}616{,}585}{353{,}022}} \quad\Rightarrow$$

$r = \sqrt[25]{\frac{6{,}616{,}585}{353{,}022}} - 1 \approx 0.1244$, so r is about 12.44%.

(c) In part (a), the base is constant and the variable is in the exponent, so this is an *exponential* function. In part (b), the base is variable and the exponent is constant, so this is a *polynomial* function.

5.3 Exercises

[1] (a) $f(x) = e^{-x}$ • reflect $y = e^x$ through the y-axis

(b) $f(x) = -e^x$ • reflect $y = e^x$ through the x-axis

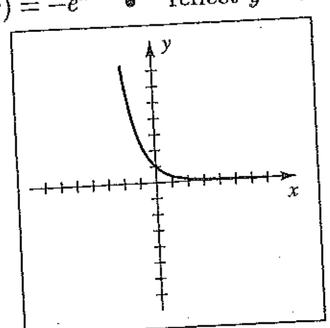

Figure 1(a)

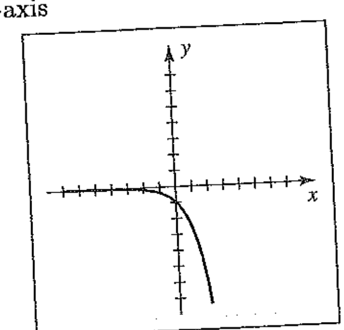

Figure 1(b)

$\boxed{2}$ (a) $f(x) = e^{2x} = (e^x)^2$ • square the y values of $y = e^x$

(b) $f(x) = 2e^x$ • vertically stretch $y = e^x$ by a factor of 2

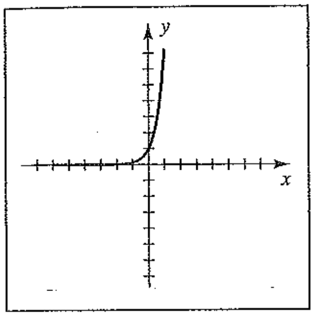

Figure 2(a)

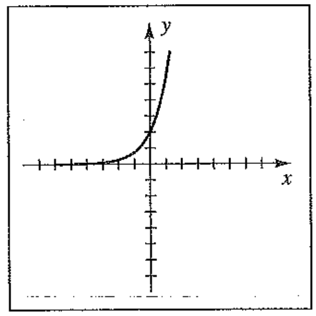

Figure 2(b)

$\boxed{3}$ (a) $f(x) = e^{x+4}$ • shift $y = e^x$ left 4 units

(b) $f(x) = e^x + 4$ • shift $y = e^x$ up 4 units

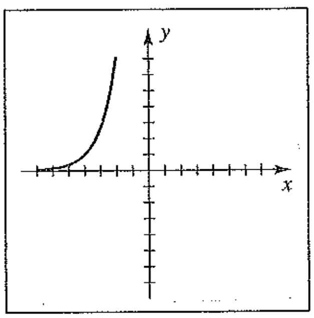

Figure 3(a)

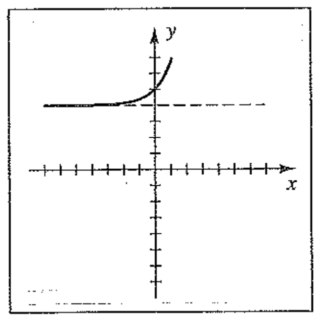

Figure 3(b)

$\boxed{4}$ (a) $f(x) = e^{-2x}$ • reflect $y = e^x$ through the y-axis and square the y values

(b) $f(x) = -2e^x$ • reflect $y = e^x$ through the x-axis and double the y values

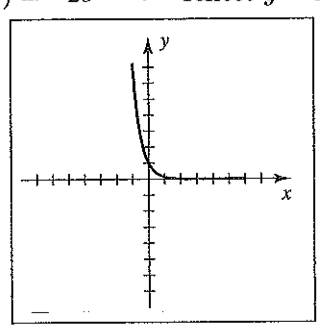

Figure 4(a)

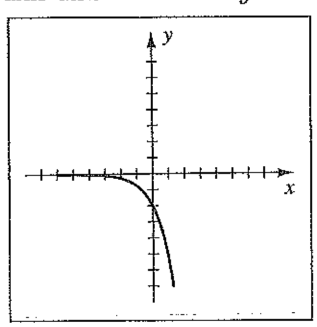

Figure 4(b)

$\boxed{5}$ $A = Pe^{rt} = 1000e^{(0.0825)(5)} \approx \$1510.59$

$\boxed{6}$ $A = Pe^{rt} = 100e^{(0.125)(10)} \approx \$349.03$

$\boxed{7}$ $100{,}000 = Pe^{(0.11)(18)} \Rightarrow P = \dfrac{100{,}000}{e^{1.98}} \approx \$13{,}806.92$

$\boxed{8}$ $15{,}000 = Pe^{(0.095)(4)} \Rightarrow P = \dfrac{15{,}000}{e^{0.38}} \approx \$10{,}257.92$

[9] $13{,}464 = 1000e^{(r)(20)} \Rightarrow e^{20r} = 13.464$. Using a trial and error approach on a scientific calculator or an intersect feature on a graphing calculator, we determine that $e^x = 13.464$ if $x \approx 2.6$. Thus, $20r = 2.6$ and $r = 0.13$ or 13%.

[10] $890.20 = 400e^{(r)(16)} \Rightarrow e^{16r} = 2.2255$. As in Exercise 9, $e^x = 2.2255$ if $x \approx 0.8$.
Thus, $16r = 0.8$ and $r = 0.05$ or 5%.

[11] $e^{(x^2)} = e^{7x-12} \Rightarrow x^2 = 7x - 12 \Rightarrow x^2 - 7x + 12 = 0 \Rightarrow$
$(x-3)(x-4) = 0 \Rightarrow x = 3, 4$

[12] $e^{3x} = e^{2x-1} \Rightarrow 3x = 2x - 1 \Rightarrow x = -1$

[13] $xe^x + e^x = 0 \Rightarrow e^x(x+1) = 0 \Rightarrow x = -1 \; \{e^x \neq 0\}$

[14] $-x^2e^{-x} + 2xe^{-x} = 0 \Rightarrow xe^{-x}(-x+2) = 0 \Rightarrow x = 0, 2 \; \{e^{-x} \neq 0\}$

[15] $x^3(4e^{4x}) + 3x^2e^{4x} = 0 \Rightarrow x^2e^{4x}(4x+3) = 0 \Rightarrow x = -\frac{3}{4}, 0 \; \{e^{4x} \neq 0\}$

[16] $x^2(2e^{2x}) + 2xe^{2x} + e^{2x} + 2xe^{2x} = 0 \Rightarrow e^{2x}(2x^2 + 4x + 1) = 0 \Rightarrow x = -1 \pm \frac{1}{2}\sqrt{2}$

[17] $$\frac{(e^x + e^{-x})(e^x + e^{-x}) - (e^x - e^{-x})(e^x - e^{-x})}{(e^x + e^{-x})^2} =$$
$$\frac{(e^{2x} + 2 + e^{-2x}) - (e^{2x} - 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{4}{(e^x + e^{-x})^2}$$

[18] $$\frac{(e^x - e^{-x})^2 - (e^x + e^{-x})^2}{(e^x + e^{-x})^2} = \frac{(e^{2x} - 2 + e^{-2x}) - (e^{2x} + 2 + e^{-2x})}{(e^x + e^{-x})^2} = \frac{-4}{(e^x + e^{-x})^2}$$

[19] $W(t) = W_0e^{kt}$; $t = 30 \Rightarrow W(30) = 68e^{(0.2)(30)} \approx 27{,}433$ mg, or 27.43 grams

[20] $W(10) = W_0e^{(0.21)(10)} = 575 \Rightarrow W_0 = \frac{575}{e^{2.1}} \approx 70.41$ mg

[21] The year 2010 corresponds to $t = 2010 - 1980 = 30$. Using the law of growth formula with $q_0 = 227$ and $r = 0.007$, we have $N(t) = 227e^{0.007t}$.
Thus, $N(30) = 227e^{(0.007)(30)} = 227e^{0.21} \approx 280.0$ million.

[22] The year 2010 corresponds to $t = 2010 - 1985 = 25$. Using the law of growth formula with $q_0 = 762$ and $r = 0.022$, we have $N(t) = 762e^{0.022t}$.
Thus, $N(25) = 762e^{(0.022)(25)} = 762e^{0.55} \approx 1{,}320.7$ million.

[23] $N(10) = N_0e^{-2}$. The percentage of the original number still alive after 10 years is
$$100 \times \left(\frac{N(10)}{N_0}\right) = 100e^{-2} \approx 13.5\%.$$

[24] (a) $A_0 = 35$, $t = 2 \Rightarrow A(2) = 35e^{-0.0498} \approx 33.3$ units are available

(b) $A(t) = 35$, $t = 2 \Rightarrow 35 = A_0e^{-0.0498} \Rightarrow$
$A_0 = 35e^{0.0498} \approx 36.8$ units should be shipped

[25] 2010 corresponds to $t = 2010 - 1978 = 32$; $N(32) = 5000e^{(0.0036)(32)} \approx 5610$

[26] (a) $f(10) = 200(1 - 0.956e^{-0.18(10)}) \approx 168.4$ cm

(b) $f(0) = 8.8$ cm. As $t \to \infty$, $0.956e^{-0.18t} \to 0$, and $f(t) \to 200$ cm.

[27] $h = 40{,}000 \Rightarrow p = 29e^{-0.000034(40{,}000)} = 29e^{-1.36} \approx 7.44$ in.

[28] Consider A to be a function of t with $c = 50$, so $A(t) = 50e^{-0.00495t}$.

(a) $A(30) = 50e^{-0.00495(30)} = 50e^{-0.1485} \approx 43.10$ mg

(b) $A(180) = 50e^{-0.00495(180)} = 50e^{-0.891} \approx 20.51$ mg

(c) $A(365) = 50e^{-0.00495(365)} = 50e^{-1.80675} \approx 8.21$ mg

[29] $x = 1 \Rightarrow y = 79.041 + 6.39 - e^{2.268} \approx 75.77$ cm.

$x = 1 \Rightarrow R = 6.39 + 0.993e^{2.268} \approx 15.98$ cm/yr.

[30] (a) As t increases, $e^{-at} \to 0$ and $s \approx \frac{v_0}{a}$. (b) $s \approx \frac{v_0}{a} = \frac{10}{8 \times 10^5} = 1.25 \times 10^{-5}$ m.

[31] $2010 - 1971 = 39 \Rightarrow t = 39$ years. $A = 1.60e^{(0.05)(39)} \approx \$11.25$ per hour

[32] Let P denote the cost of one acre in 1867.

$$P = \left(\frac{\$7{,}200{,}000}{586{,}400 \text{ mi}^2}\right)\left(\frac{1 \text{ mi}^2}{640 \text{ acres}}\right)(1 \text{ acre}) \approx \$0.02.$$

$2010 - 1867 = 143 \Rightarrow t = 143$ years. $A = Pe^{(0.03)(143)} = Pe^{4.29} \approx \$1.40$.

[33] (a) $\left(1 + \frac{0.07}{4}\right)^{4 \cdot 1} \approx 1.0719$. $(1.0719 - 1) \times 100\% = 7.19\%$

(b) $e^{(0.07)(1)} \approx 1.0725$. $(1.0725 - 1) \times 100\% = 7.25\%$

[34] (a) $\left(1 + \frac{0.12}{4}\right)^{4 \cdot 1} \approx 1.1255$. $(1.1255 - 1) \times 100\% = 12.55\%$

(b) $e^{(0.12)(1)} \approx 1.1275$. $(1.1275 - 1) \times 100\% = 12.75\%$

[35] The graph of $y = e^{1000x}$ nearly coincides with the negative x-axis and then increases so fast that it nearly coincides with the positive y-axis. To see the graph on a calculator, try "turning off" your axes display.

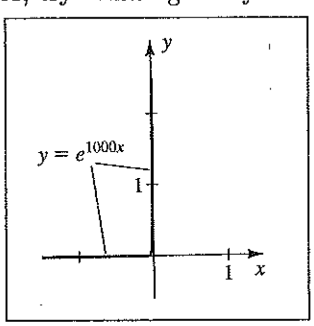

Figure 35

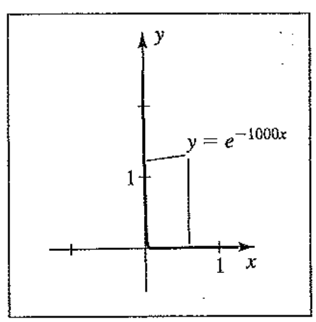

Figure 36

[36] $y = e^{-1000x}$ • Similar to the previous exercise.

[37] It may be of interest to compare this graph with the graph of $y = (1.085)^x$ in Exercise 55 of §5.2. Both are compounding functions with $r = 8.5\%$.
Note that $e^{0.085x} = (e^{0.085})^x \approx (1.0887)^x > (1.085)^x$ for $x > 0$.
(a) If $y = e^{0.085x}$ and $x = 40$, then $y \approx 29.96$. (b) If $y = 2$, then $x \approx 8.15$.

[0, 60, 5] by [0, 40, 5]

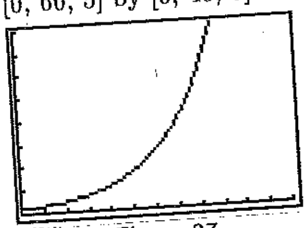

Figure 37

[0, 60, 5] by [0, 40, 5]

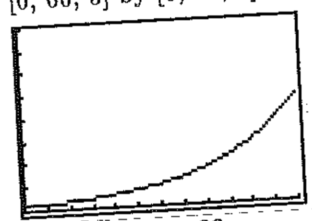

Figure 38

[38] Compare with Exercise 56 of §5.2.
(a) If $y = e^{0.0525x}$ and $x = 40$, then $y \approx 8.17$. (b) If $y = 2$, then $x \approx 13.20$.

[39] (a) As $x \to \infty$, $e^{-x} \to 0$ and f will resemble $\frac{1}{2}e^x$.
As $x \to -\infty$, $e^x \to 0$ and f will resemble $-\frac{1}{2}e^x$.
(b) At $x = 0$, we will have a vertical asymptote. As $x \to \infty$, $f(x) \to \infty$, and $g(x) \to 0$.
As $x \to -\infty$, $f(x) \to -\infty$, and $g(x) \to 0$.

[−7.5, 7.5] by [−5, 5]

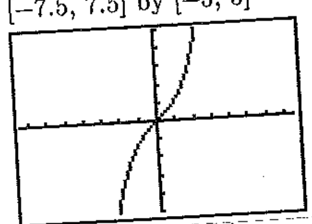

Figure 39(a)

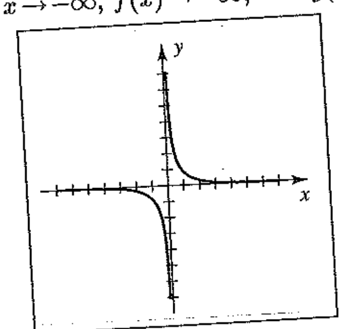

Figure 39(b)

[40] (a) See Example 5 for a graph of $f(x) = \dfrac{e^x + e^{-x}}{2}$.
(b) At $x = 0$, $f(x)$ and $g(x) = 1$. As $x \to \infty$ or $x \to -\infty$, $f(x) \to \infty$ and $g(x) \to 0$.

[−7.5, 7.5] by [−5, 5]

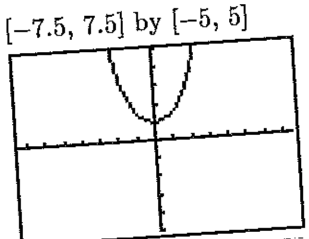

Figure 40(a)

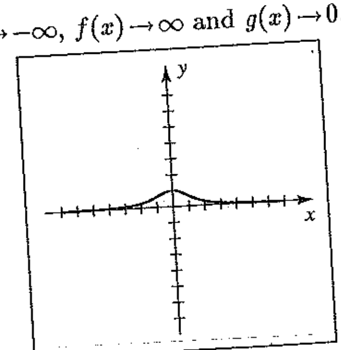

Figure 40(b)

[41] (a) $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \dfrac{e^x - 1/e^x}{e^x + 1/e^x} \cdot \dfrac{e^x}{e^x} = \dfrac{e^{2x} - 1}{e^{2x} + 1}$.

At $x = 0$, $f(x) = 0$. As $x \to \infty$, $f(x) \to 1$. As $x \to -\infty$, $f(x) \to -1$.

(b) At $x = 0$, we will have a vertical asymptote. As $x \to \infty$, $g(x) \to 1$.

As $x \to -\infty$, $g(x) \to -1$.

[−4.5, 4.5] by [−3, 3]

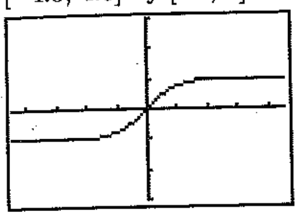

Figure 41(a)

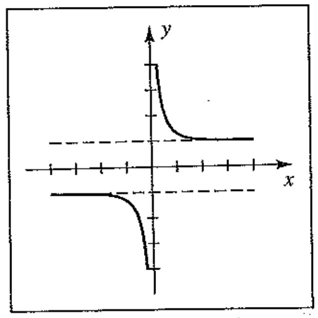

Figure 41(b)

[42] $\sigma = 1$ and $\mu = 0 \Rightarrow z = x$. $f(x) = \dfrac{1}{\sqrt{2\pi}} e^{-x^2/2}$

is an even function with $f(0) = \dfrac{1}{\sqrt{2\pi}} \approx 0.3989$,

$f(\pm 1) = \dfrac{1}{\sqrt{2\pi e}} \approx 0.2420$, and

$f(\pm 2) = \dfrac{1}{\sqrt{2\pi\ e^2}} \approx 0.0540$.

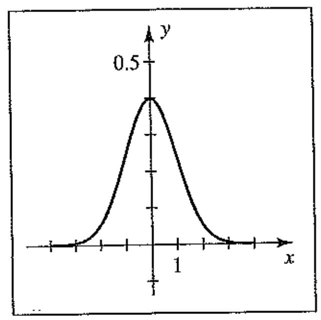

Figure 42

[43] The approximate coordinates of the points where the graphs of f and g intersect are (−1.04, −0.92), (2.11, 2.44), and (8.51, 70.42). The region near the origin in *Figure 43(a)* is enhanced in *Figure 43(b)*. Thus, the solutions of the equation $f(x) = g(x)$ are $x \approx -1.04$, 2.11, and 8.51.

[−3, 11] by [−10, 80, 10]

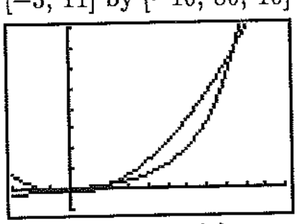

Figure 43(a)

[−2.26, 3.34] by [−7.14, 8.57]

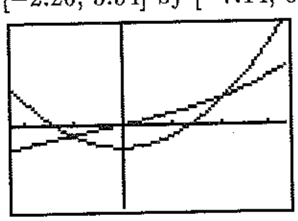

Figure 43(b)

[44] The approximate coordinates of the points where the graphs of f and g intersect are $(-0.93,\ 0.12)$, $(-0.25,\ 0.23)$, $(1.36,\ 1.17)$, and $(7.04,\ 341.46)$. The region near the origin in *Figure 44(a)* is enhanced in *Figure 44(b)*. Thus, the solutions of the equation $f(x) = g(x)$ are $x \approx -0.93,\ -0.25,\ 1.36$, and 7.04.

[−5, 8] by [−50, 400, 50]

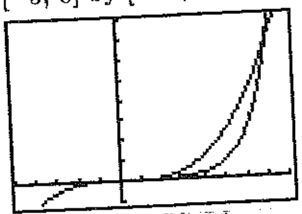

Figure 44(a)

[−1.37, 1.79] by [−1.11, 2.06]

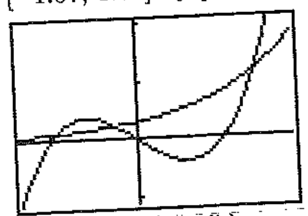

Figure 44(b)

[45] f is a more accurate approximation to e^x near $x = 0$,
whereas g is a more accurate approximation to e^x near $x = 1$.

[0, 4.5] by [0, 3]

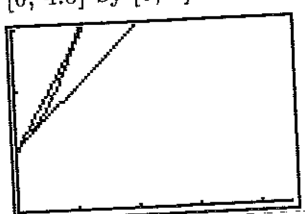

Figure 45

[0, 4.5] by [0, 3]

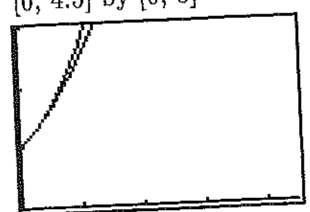

Figure 46

[46] Both approximations are quite accurate for $0 \le x \le \frac{1}{2}$.
For $\frac{1}{2} \le x \le 1$, g is a more accurate approximation.
(Careful inspection will show that f is a more accurate near $x = 0$.)

[47] Using a zero feature, we determine that
$f(x) = x^2e^x - xe^{(x^2)} + 0.1$ has zeros at $x \approx 0.11,\ 0.79$, and 1.13.

[−2, 2.5] by [−1, 2]

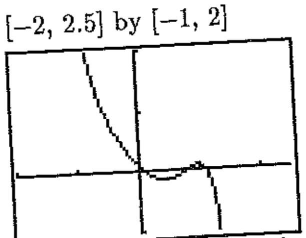

Figure 47

[−6, 3] by [−3, 3]

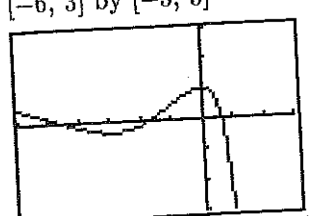

Figure 48

[48] Using a zero feature, we determine that
$f(x) = x^3e^x - x^2e^{2x} + 1$ has zeros at $x \approx -4.54,\ -1.71$, and 0.65.

$\boxed{49}$ From the graph, there is a horizontal asymptote of $y \approx 2.71$.

f is approaching the value of e asymptotically.

[0, 200, 50] by [0, 8]

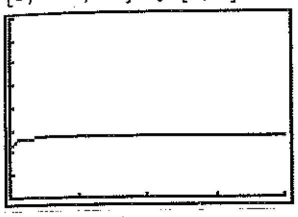

Figure 49

[0, 200, 50] by [0, 8]

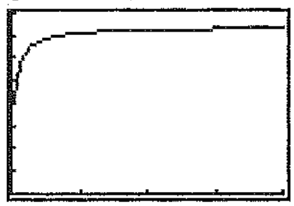

Figure 50

$\boxed{50}$ From the graph, there is a horizontal asymptote of $y \approx 7.32$.

f is approaching the value of e^2 (≈ 7.389) asymptotically.

$\boxed{51}$ Using an intersect feature, $e^{-x} = x$ when $x \approx 0.567$.

[−4.5, 4.5] by [−3, 3]

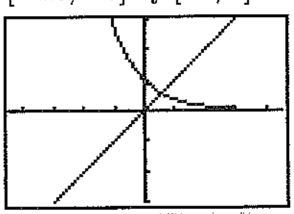

Figure 51

[−4.5, 4.5] by [0, 6]

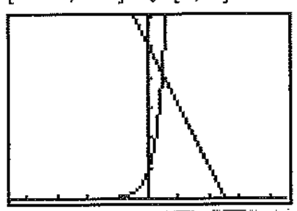

Figure 52

$\boxed{52}$ Using an intersect feature, $e^{3x} = 5 - 2x$ when $x \approx 0.467$.

$\boxed{53}$ $f(x) = xe^x$ is increasing on $[-1, \infty)$ and f is decreasing on $(-\infty, -1]$.

[−5.5, 5] by [−2, 5]

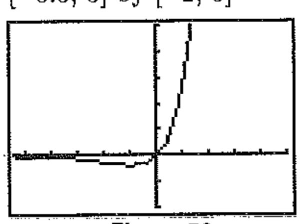

Figure 53

[−2, 2.5] by [−1, 2]

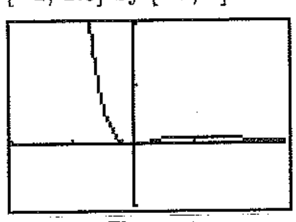

Figure 54

$\boxed{54}$ $f(x) = x^2e^{-2x}$ is increasing on [0, 1] and f is decreasing on $(-\infty, 0]$ and $[1, \infty)$.

$\boxed{55}$ (a) When $y = 0$ and $z = 0$, the equation becomes $C = \dfrac{2Q}{\pi vab}e^{-h^2/(2b^2)}$.

As h increases, the concentration C decreases.

(b) When $z = 0$, the equation becomes $C = \dfrac{2Q}{\pi vab}e^{-y^2/(2a^2)}\,e^{-h^2/(2b^2)}$.

As y increases, the concentration C decreases.

[56] Graph $C = e^{-(z-100)^2/288} + e^{-(z+100)^2/288}$. From the graph, we see that the concentration of the pollution first increases as the height increases and then decreases. The maximum concentration occurs when $z = 100$ m.

[0, 200, 10] by [0, 1, 0.5]

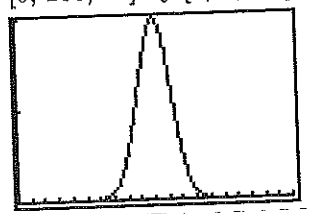

Figure 56

[57] (a) Choose two arbitrary points that appear to lie on the curve such as (0, 1.225) and (10,000, 0.414).

$$f(0) = Ce^0 = C = 1.225 \quad \text{and} \quad f(10{,}000) = 1.225e^{10{,}000k} = 0.414.$$

To solve the last equation, graph $Y_1 = 1.225e^{10{,}000x}$ and $Y_2 = 0.414$. The graph of Y_1 increases so rapidly that it nearly "covers" the positive y-axis, so a viewing rectangle such as [−0.001, 0] by [0, 1] is needed to see the point of intersection. The graphs intersect at $x \approx -0.0001085$. Thus, $f(x) = 1.225e^{-0.0001085x}$.

(b) $f(3000) \approx 0.885$ {actual $= 0.909$} and $f(9000) \approx 0.461$ {actual $= 0.467$}.

[−1000, 10,100, 1000] by [0, 1.5, 0.5]

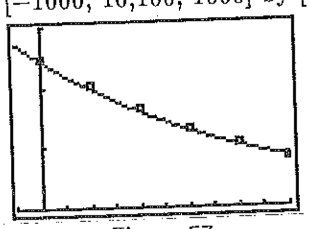

Figure 57

[−10, 90, 10] by [−100, 1600, 100]

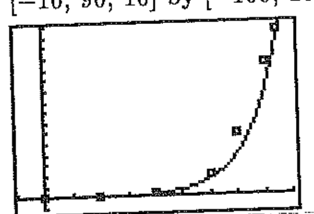

Figure 58

[58] (a) Choose two arbitrary points that appear to lie on the curve such as (0, 0.7) and (85, 1538.9). $A(0) = A_0e^0 = A_0 = 0.7$. $A(85) = 0.7e^{85k} = 1538.9$. To solve the last equation, graph $Y_1 = 0.7e^{85x}$ and $Y_2 = 1538.9$ in the viewing rectangle [0, 0.15, 0.01] by [0, 1600, 100]. The graphs intersect at $x \approx 0.09053527$. Thus, $A(x) = 0.7e^{0.09053527x}$.

(b) Graph $Y_1 = A(x)$ and $Y_2 = 1000$. The graphs intersect at $x \approx 80.2$. The federal government first spent one trillion dollars in 1990.

5.4 Exercises

[1] (a) $4^3 = 64 \Leftrightarrow \log_4 64 = 3$

(b) $4^{-3} = \frac{1}{64} \Leftrightarrow \log_4 \frac{1}{64} = -3$

(c) $t^r = s \Leftrightarrow \log_t s = r$

(d) $3^x = 4 - t \Leftrightarrow \log_3(4 - t) = x$

(e) $5^{7t} = \dfrac{a+b}{a} \Leftrightarrow \log_5 \dfrac{a+b}{a} = 7t$

(f) $(0.7)^t = 5.3 \Leftrightarrow \log_{0.7}(5.3) = t$

2 (a) $3^5 = 243 \Leftrightarrow \log_3 243 = 5$ (b) $3^{-4} = \frac{1}{81} \Leftrightarrow \log_3 \frac{1}{81} = -4$
(c) $c^p = d \Leftrightarrow \log_c d = p$ (d) $7^x = 100p \Leftrightarrow \log_7(100p) = x$
(e) $3^{-2x} = \frac{P}{F} \Leftrightarrow \log_3 \frac{P}{F} = -2x$ (f) $(0.9)^t = \frac{1}{2} \Leftrightarrow \log_{0.9}(\frac{1}{2}) = t$

3 (a) $\log_2 32 = 5 \Leftrightarrow 2^5 = 32$ (b) $\log_3 \frac{1}{243} = -5 \Leftrightarrow 3^{-5} = \frac{1}{243}$
(c) $\log_t r = p \Leftrightarrow t^p = r$ (d) $\log_3(x+2) = 5 \Leftrightarrow 3^5 = (x+2)$
(e) $\log_2 m = 3x+4 \Leftrightarrow 2^{3x+4} = m$ (f) $\log_b 512 = \frac{3}{2} \Leftrightarrow b^{3/2} = 512$

4 (a) $\log_3 81 = 4 \Leftrightarrow 3^4 = 81$ (b) $\log_4 \frac{1}{256} = -4 \Leftrightarrow 4^{-4} = \frac{1}{256}$
(c) $\log_v w = q \Leftrightarrow v^q = w$ (d) $\log_6(2x-1) = 3 \Leftrightarrow 6^3 = 2x-1$
(e) $\log_4 p = 5-x \Leftrightarrow 4^{5-x} = p$ (f) $\log_a 343 = \frac{3}{4} \Leftrightarrow a^{3/4} = 343$

5 $2a^{t/3} = 5 \Leftrightarrow a^{t/3} = \frac{5}{2} \Leftrightarrow t/3 = \log_a \frac{5}{2} \Leftrightarrow t = 3\log_a \frac{5}{2}$

6 $3a^{4t} = 10 \Leftrightarrow a^{4t} = \frac{10}{3} \Leftrightarrow 4t = \log_a \frac{10}{3} \Leftrightarrow t = \frac{1}{4}\log_a \frac{10}{3}$

7 $K = H - Ca^t \Rightarrow Ca^t = H - K \Rightarrow a^t = \frac{H-K}{C} \Rightarrow t = \log_a\left(\frac{H-K}{C}\right)$

8 $F = D + Ba^t \Rightarrow Ba^t = F - D \Rightarrow a^t = \frac{F-D}{B} \Rightarrow t = \log_a\left(\frac{F-D}{B}\right)$

9 $A = Ba^{Ct} + D \Rightarrow A - D = Ba^{Ct} \Rightarrow$

$$\frac{A-D}{B} = a^{Ct} \Rightarrow Ct = \log_a\left(\frac{A-D}{B}\right) \Rightarrow t = \frac{1}{C}\log_a\left(\frac{A-D}{B}\right)$$

10 $L = Ma^{t/N} - P \Rightarrow L + P = Ma^{t/N} \Rightarrow$

$$\frac{L+P}{M} = a^{t/N} \Rightarrow \frac{t}{N} = \log_a\left(\frac{L+P}{M}\right) \Rightarrow t = N\log_a\left(\frac{L+P}{M}\right)$$

11 (a) $10^5 = 100{,}000 \Leftrightarrow \log 100{,}000 = 5$ (b) $10^{-3} = 0.001 \Leftrightarrow \log 0.001 = -3$
(c) $10^x = y+1 \Leftrightarrow \log(y+1) = x$ (d) $e^7 = p \Leftrightarrow \ln p = 7$
(e) $e^{2t} = 3-x \Leftrightarrow \ln(3-x) = 2t$

12 (a) $10^4 = 10{,}000 \Leftrightarrow \log 10{,}000 = 4$ (b) $10^{-2} = 0.01 \Leftrightarrow \log 0.01 = -2$
(c) $10^x = 38z \Leftrightarrow \log(38z) = x$ (d) $e^4 = D \Leftrightarrow \ln D = 4$
(e) $e^{0.1t} = x+2 \Leftrightarrow \ln(x+2) = 0.1t$

13 (a) $\log x = 50 \Leftrightarrow 10^{50} = x$ (b) $\log x = 20t \Leftrightarrow 10^{20t} = x$
(c) $\ln x = 0.1 \Leftrightarrow e^{0.1} = x$ (d) $\ln w = 4+3x \Leftrightarrow e^{4+3x} = w$
(e) $\ln(z-2) = \frac{1}{6} \Leftrightarrow e^{1/6} = z-2$

14 (a) $\log x = -8 \Leftrightarrow 10^{-8} = x$ (b) $\log x = y-2 \Leftrightarrow 10^{y-2} = x$
(c) $\ln x = \frac{1}{2} \Leftrightarrow e^{1/2} = x$ (d) $\ln z = 7+x \Leftrightarrow e^{7+x} = z$
(e) $\ln(t-5) = 1.2 \Leftrightarrow e^{1.2} = t-5$

[15] (a) $\log_5 1 = 0$ (b) $\log_3 3 = 1$ (c) $\log_4(-2)$ is undefined, not possible

(d) $\log_7 7^2 = 2$ (e) $3^{\log_3 8} = 8$ (f) $\log_5 125 = \log_5 5^3 = 3$

(g) $\log_4 \frac{1}{16} = \log_4 4^{-2} = -2$

[16] (a) $\log_8 1 = 0$ (b) $\log_9 9 = 1$ (c) $\log_5 0$ is undefined, not possible

(d) $\log_6 6^7 = 7$ (e) $5^{\log_5 4} = 4$ (f) $\log_3 243 = \log_3 3^5 = 5$

(g) $\log_2 128 = \log_2 2^7 = 7$

[17] (a) $10^{\log 3} = 3$ (b) $\log 10^5 = 5$ (c) $\log 100 = \log 10^2 = 2$

(d) $\log 0.0001 = \log 10^{-4} = -4$

(e) $e^{\ln 2} = 2$ (f) $\ln e^{-3} = -3$ (g) $e^{2+\ln 3} = e^2 e^{\ln 3} = e^2(3) = 3e^2$

[18] (a) $10^{\log 7} = 7$ (b) $\log 10^{-6} = -6$ (c) $\log 100{,}000 = \log 10^5 = 5$

(d) $\log 0.001 = \log 10^{-3} = -3$

(e) $e^{\ln 8} = 8$ (f) $\ln e^{2/3} = \frac{2}{3}$ (g) $e^{1+\ln 5} = e^1 e^{\ln 5} = e(5) = 5e$

[19] $\log_4 x = \log_4(8-x) \;\Rightarrow\; x = 8 - x \;\Rightarrow\; 2x = 8 \;\Rightarrow\; x = 4$

[20] $\log_3(x+4) = \log_3(1-x) \;\Rightarrow\; x + 4 = 1 - x \;\Rightarrow\; 2x = -3 \;\Rightarrow\; x = -\frac{3}{2}$

[21] $\log_5(x-2) = \log_5(3x+7) \;\Rightarrow\; x - 2 = 3x + 7 \;\Rightarrow\; 2x = -9 \;\Rightarrow\; x = -\frac{9}{2}$.

$-\frac{9}{2}$ is extraneous since it makes either of the given logarithm expressions undefined; no solution.

[22] $\log_7(x-5) = \log_7(6x) \;\Rightarrow\; x - 5 = 6x \;\Rightarrow\; 5x = -5 \;\Rightarrow\; x = -1$. -1 is extraneous since it makes either of the given logarithm expressions undefined; no solution.

[23] $\log x^2 = \log(-3x-2) \;\Rightarrow\; x^2 = -3x - 2 \;\Rightarrow\; x^2 + 3x + 2 = 0 \;\Rightarrow\; (x+1)(x+2) = 0 \;\Rightarrow\; x = -1, -2$

[24] $\ln x^2 = \ln(12-x) \;\Rightarrow\; x^2 = 12 - x \;\Rightarrow\; x^2 + x - 12 = 0 \;\Rightarrow\; (x+4)(x-3) = 0 \;\Rightarrow\; x = -4, 3$

[25] $\log_3(x-4) = 2 \;\Rightarrow\; x - 4 = 3^2 \;\Rightarrow\; x = 13$

[26] $\log_2(x-5) = 4 \;\Rightarrow\; x - 5 = 2^4 \;\Rightarrow\; x = 21$

[27] $\log_9 x = \frac{3}{2} \;\Rightarrow\; x = 9^{3/2} = (9^{1/2})^3 = 3^3 = 27$

[28] $\log_4 x = -\frac{3}{2} \;\Rightarrow\; x = 4^{-3/2} = (4^{-1/2})^3 = (\frac{1}{2})^3 = \frac{1}{8}$

[29] $\ln x^2 = -2 \;\Rightarrow\; x^2 = e^{-2} = \frac{1}{e^2} \;\Rightarrow\; x = \pm\frac{1}{e}$

[30] $\log x^2 = -4 \;\Rightarrow\; x^2 = 10^{-4} = \frac{1}{10{,}000} \;\Rightarrow\; x = \pm\frac{1}{100}$

[31] $e^{2\ln x} = 9 \;\Rightarrow\; (e^{\ln x})^2 = 9 \;\Rightarrow\; x^2 = 9 \;\Rightarrow\; x = \pm 3$; -3 is extraneous

[32] $e^{-\ln x} = 0.2 \;\Rightarrow\; (e^{\ln x})^{-1} = 0.2 \;\Rightarrow\; x^{-1} = 0.2 \;\Rightarrow\; 1/x = 1/5 \;\Rightarrow\; x = 5$

[33] $e^{x\ln 3} = 27 \;\Rightarrow\; (e^{\ln 3})^x = 27 \;\Rightarrow\; 3^x = 3^3 \;\Rightarrow\; x = 3$

$\boxed{34}$ $e^{x \ln 2} = 0.25 \;\Rightarrow\; (e^{\ln 2})^x = \frac{1}{4} \;\Rightarrow\; 2^x = 2^{-2} \;\Rightarrow\; x = -2$

$\boxed{35}$ (a) $f(x) = \log_4 x$ • This graph has a vertical asymptote of $x = 0$ and goes through $(\frac{1}{4}, -1)$, $(1, 0)$, and $(4, 1)$. For reference purposes, call this $F(x)$.

(b) $f(x) = -\log_4 x$ • reflect F through the x-axis

(c) $f(x) = 2\log_4 x$ • vertically stretch F by a factor of 2

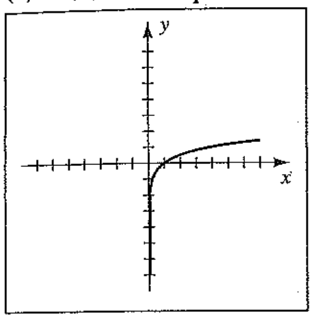

Figure 35(a)

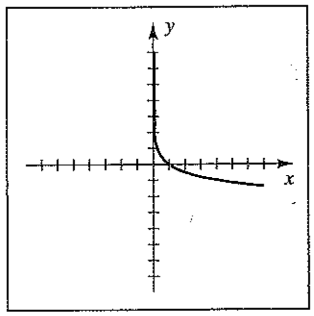

Figure 35(b)

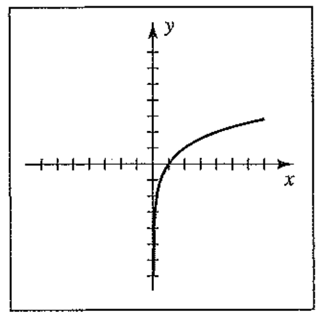

Figure 35(c)

(d) $f(x) = \log_4(x + 2)$ • shift F left 2 units

(e) $f(x) = (\log_4 x) + 2$ • shift F up 2 units

(f) $f(x) = \log_4(x - 2)$ • shift F right 2 units

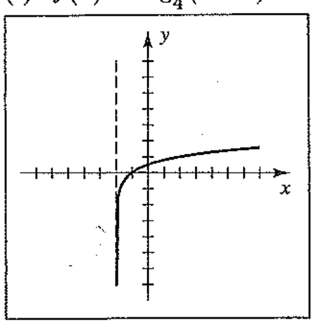

Figure 35(d)

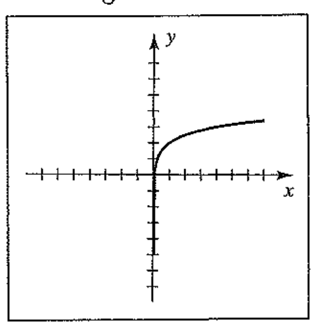

Figure 35(e)

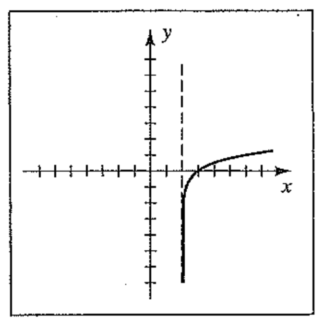

Figure 35(f)

(g) $f(x) = (\log_4 x) - 2$ • shift F down 2 units

(h) $f(x) = \log_4 |x|$ • include the reflection of F through the y-axis

(i) $f(x) = \log_4(-x)$ • reflect F through the y-axis

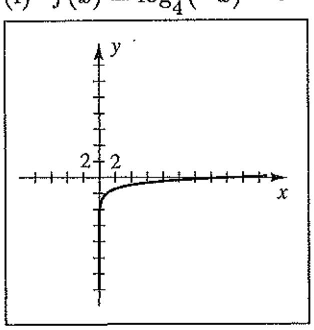

Figure 35(g)

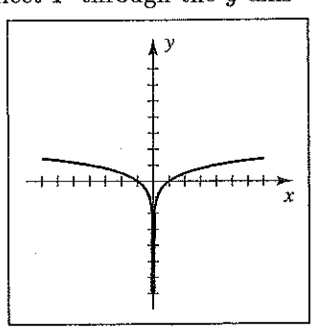

Figure 35(h)

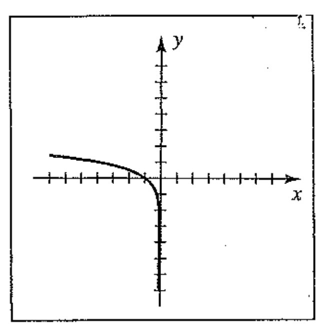

Figure 35(i)

(j) $f(x) = \log_4(3 - x) = \log_4[-(x - 3)]$ • shift F 3 units right and reflect through the line $x = 3$

(k) $f(x) = |\log_4 x|$ • reflect points with negative y-coordinates through the x-axis

(l) $f(x) = \log_{1/4} x$ • reflect the graph of $y = \log_4 x$ about the x-axis

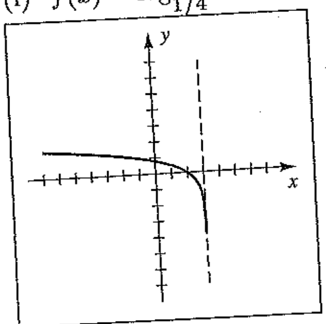

Figure 35(j)

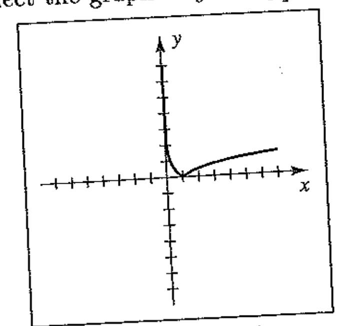

Figure 35(k)

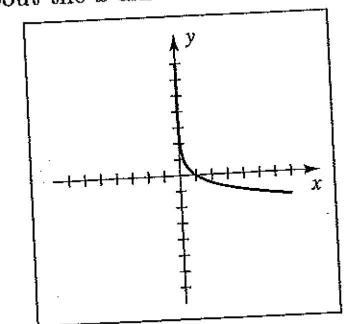

Figure 35(l)

[36] The graphs are found in a manner similar to that of Exercise 35. The graph of $y = \log_5 x$ goes through $(\frac{1}{5}, -1)$, $(1, 0)$, and $(5, 1)$.

[37] $f(x) = \log(x + 10)$ • This is the graph of $g(x) = \log x$ shifted 10 units to the left. There is a vertical asymptote of $x = -10$. g goes through $(\frac{1}{10}, -1)$, $(1, 0)$, and $(10, 1)$, so f goes through $(\frac{1}{10} - 10, -1)$, $(1 - 10, 0)$, and $(10 - 10, 1)$, that is, $(-9.9, -1)$, $(-9, 0)$, and $(0, 1)$.

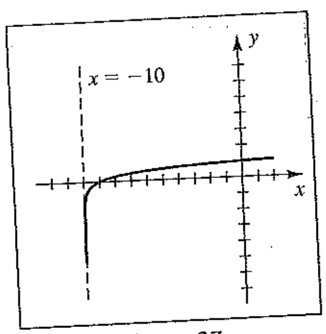

Figure 37

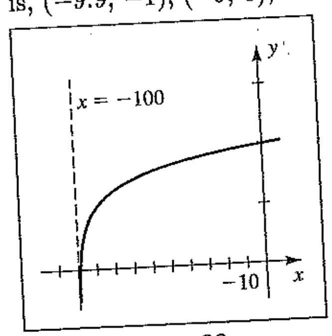

Figure 38

[38] $f(x) = \log(x + 100)$ • This is the graph of $g(x) = \log x$ shifted 100 units to the left. There is a vertical asymptote of $x = -100$.

[39] $f(x) = \ln|x|$ • This is the graph of $g(x) = \ln x$ and its reflection through the y-axis. There is a vertical asymptote of $x = 0$. The graph of g goes through $(\frac{1}{e}, -1)$, $(1, 0)$, and $(e, 1)$, so the graph of f goes through $(\pm\frac{1}{e}, -1)$, $(\pm 1, 0)$, and $(\pm e, 1)$. See *Figure 39*.

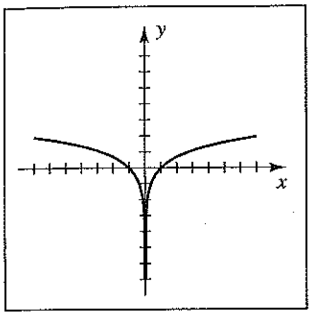

Figure 39

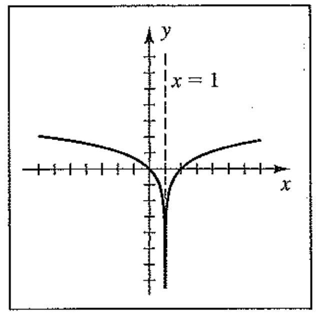

Figure 40

40 $f(x) = \ln|x-1|$ • This is the graph of $g(x) = \ln|x|$ shifted 1 unit to the right. There is a vertical asymptote of $x = 1$.

41 $f(x) = \ln e + x = 1 + x$. This is the graph of a line with slope 1 and y-intercept 1.

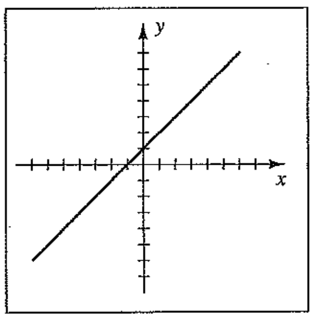

Figure 41

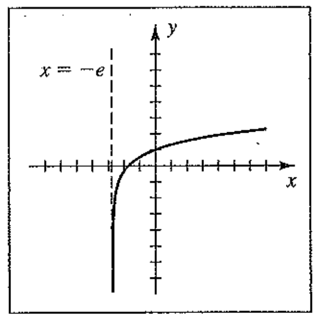

Figure 42

42 $f(x) = \ln(e + x)$ • This is the graph of $g(x) = \ln x$ shifted e units to the left.

There is a vertical asymptote of $x = -e$.

43 The point (9, 2) is on the graph of $f(x) = \log_a x$, so $f(9) = 2 \Rightarrow 2 = \log_a 9 \Rightarrow$

$a^2 = 9 \Rightarrow a = \pm 3$. Since the base must be positive, $a = 3$.

44 The point (8, 3) is on the graph of $f(x) = \log_a x$, so

$f(8) = 3 \Rightarrow 3 = \log_a 8 \Rightarrow a^3 = 8 \Rightarrow a = 2$.

45 This is the graph of F reflected through the x-axis. ★ $f(x) = -F(x)$

46 This is the graph of F reflected through the y-axis. ★ $f(x) = F(-x)$

47 This is the graph of F shifted right 2 units. ★ $f(x) = F(x-2)$

48 This is the graph of F shifted left 3 units. ★ $f(x) = F(x+3)$

49 This is the graph of F shifted up 1 unit. ★ $f(x) = F(x) + 1$

50 The y-coordinates are doubled, so multiply F by 2. ★ $f(x) = 2\,F(x)$

51 (a) $\log x = 3.6274 \Rightarrow x = 10^{3.6274} \approx 4240.333$, or 4240 to three significant figures

(b) $\log x = 0.9469 \Rightarrow x = 10^{0.9469} \approx 8.849$, or 8.85

(c) $\log x = -1.6253 \Rightarrow x = 10^{-1.6253} \approx 0.023697$, or 0.0237

(d) $\ln x = 2.3 \Rightarrow x = e^{2.3} \approx 9.974$, or 9.97

(e) $\ln x = 0.05 \Rightarrow x = e^{0.05} \approx 1.051$, or 1.05

(f) $\ln x = -1.6 \Rightarrow x = e^{-1.6} \approx 0.2019$, or 0.202

[52] (a) $\log x = 1.8965 \Rightarrow x = 10^{1.8965} \approx 78.795$, or 78.8

(b) $\log x = 4.9680 \Rightarrow x = 10^{4.968} \approx 92{,}896.639$, or 92,900

(c) $\log x = -2.2118 \Rightarrow x = 10^{-2.2118} \approx 0.00614$

(d) $\ln x = 3.7 \Rightarrow x = e^{3.7} \approx 40.447$, or 40.4

(e) $\ln x = 0.95 \Rightarrow x = e^{0.95} \approx 2.5857$, or 2.59

(f) $\ln x = -5 \Rightarrow x = e^{-5} \approx 0.00674$

[53] $f(x) = 1000(1.05)^x = 1000e^{x \ln 1.05}$ (see the illustration on page 362).

This is approximately $1000e^{x\,(0.0487901642)} \approx 1000e^{0.0488x}$,

so the growth rate of f is about 4.88%.

[54] $f(x) = 100(\frac{1}{2})^x = 100e^{x \ln (1/2)}$.

Since $\ln \frac{1}{2} \approx -0.6931471806$, the decay rate of f is about 69.31%.

[55] $q = q_0(2)^{-t/1600} \Rightarrow \dfrac{q}{q_0} = 2^{-t/1600} \Rightarrow -\dfrac{t}{1600} = \log_2(\frac{q}{q_0}) \Rightarrow t = -1600\log_2(\frac{q}{q_0})$

[56] $Q = k(2)^{-t/5} \Rightarrow \dfrac{Q}{k} = 2^{-t/5} \Rightarrow -\frac{t}{5} = \log_2(\frac{Q}{k}) \Rightarrow t = -5\log_2(\frac{Q}{k})$

[57] $I = 20e^{-Rt/L} \Rightarrow \dfrac{I}{20} = e^{-Rt/L} \Rightarrow \ln(\frac{I}{20}) = -\dfrac{Rt}{L} \Rightarrow t = -\frac{L}{R}\ln(\frac{I}{20})$

[58] $Q = Q_0e^{kt} \Rightarrow \dfrac{Q}{Q_0} = e^{kt} \Rightarrow \ln(\frac{Q}{Q_0}) = kt \Rightarrow t = \frac{1}{k}\ln(\frac{Q}{Q_0})$

[59] $I = 10^a I_0 \Rightarrow R = \log\left(\dfrac{I}{I_0}\right) = \log\left(\dfrac{10^a I_0}{I_0}\right) = \log 10^a = a.$

Hence, for $10^2 I_0$, $10^4 I_0$, and $10^5 I_0$, the answers are: (a) 2 (b) 4 (c) 5

[60] From the previous solution, these magnitudes are between $10^8 I_0$ and $10^9 I_0$.

[61] $I = 10^a I_0 \Rightarrow \alpha = 10\log\left(\dfrac{I}{I_0}\right) = 10\log\left(\dfrac{10^a I_0}{I_0}\right) = 10\log 10^a = 10a.$

Hence, for $10^1 I_0$, $10^3 I_0$, and $10^4 I_0$, the answers are: (a) 10 (b) 30 (c) 40

[62] $\alpha = 140 \Rightarrow 140 = 10\log\left(\dfrac{I}{I_0}\right) \Rightarrow \log\left(\dfrac{I}{I_0}\right) = 14 \Rightarrow \dfrac{I}{I_0} = 10^{14} \Rightarrow I = 10^{14} I_0.$

[63] 1980 corresponds to $t = 0$ and $N(0) = 227$ million. $2 \cdot 227 = 227e^{0.007t} \Rightarrow$

$2 = e^{0.007t} \Rightarrow \ln 2 = 0.007t \Rightarrow t \approx 99$, which corresponds to the year 2079.

Alternatively, using the doubling time formula, $t = (\ln 2)/r = (\ln 2)/0.007 \approx 99$.

[64] 1985 corresponds to $t = 0$. $N(t) = 1500 \Rightarrow 1500 = 762e^{0.022t} \Rightarrow$

$e^{0.022t} = \frac{1500}{762} \Rightarrow 0.022t = \ln(\frac{1500}{762}) \Rightarrow$

$t \approx 30.79$, which corresponds to the year 2015.

[65] (a) $\ln W = \ln 2.4 + (1.84)h \Rightarrow W = e^{[\ln 2.4 + (1.84)h]} \Rightarrow W = e^{\ln 2.4}e^{1.84h} \Rightarrow$

$W = 2.4e^{1.84h}$

(b) $h = 1.5 \Rightarrow W = 2.4e^{(1.84)(1.5)} = 2.4e^{2.76} \approx 37.92$ kg

[66] $25{,}000 = 6000e^{0.1t} \Rightarrow \frac{25}{6} = e^{0.1t} \Rightarrow 0.1t = \ln(\frac{25}{6}) \Rightarrow t = 10\ln(\frac{25}{6}) \approx 14.27$ yr

19.25

$\boxed{67}$ (a) $p(h) = 10 \Rightarrow 10 = 14.7e^{-0.0000385h} \Rightarrow h = -\frac{1}{0.0000385}\ln\left(\frac{10}{14.7}\right) \approx 10{,}007$ ft.

(b) At sea level, $h = 0$ and $p = 14.7$. Setting $p(h)$ equal to $\frac{1}{2}(14.7)$,

and solving as in part (a), we have $h = -\frac{1}{0.0000385}\ln\left(\frac{1}{2}\right) \approx 18{,}004$ ft.

$\boxed{68}$ $\log P = a + \dfrac{b}{c+T} \Rightarrow P(T) = 10^{a + b/(c+T)} \Rightarrow P(T) = 10^a 10^{b/(c+T)}$

$\boxed{69}$ (a) $t = 0 \Rightarrow W = 2600(1 - 0.51e^{-0.075(0)})^3 = 2600(0.49)^3 \approx 305.9$ kg

(b) (1) From the graph, if $W = 1800$, t appears to be about 20.

(2) Solving the equation for t, we have $1800 = 2600(1 - 0.51e^{-0.075t})^3 \Rightarrow$

$\frac{1800}{2600} = (1 - 0.51e^{-0.075t})^3 \Rightarrow \sqrt[3]{\frac{9}{13}} = 1 - 0.51e^{-0.075t} \Rightarrow$

$e^{-0.075t} = (1 - \sqrt[3]{\frac{9}{13}})(\frac{100}{51})$ {call this A} $\Rightarrow (-0.075)t = \ln A \Rightarrow$

$t \approx 19.8$ yr.

$\boxed{70}$ The formula for R is not needed. $T = 50 \Rightarrow 50 = 325(e^{0.02t} - 1) \Rightarrow$

$e^{0.02t} - 1 = \frac{2}{13} \Rightarrow e^{0.02t} = \frac{15}{13} \Rightarrow 0.02t = \ln\frac{15}{13} \Rightarrow t = 50\ln\frac{15}{13} \approx 7.16$ yr

$\boxed{71}$ $D = 2 \Rightarrow 5.5e^{-0.1x} = 2 \Rightarrow e^{-0.1x} = \frac{4}{11} \Rightarrow -0.1x = \ln\frac{4}{11} \Rightarrow$

$x = -10\ln\frac{4}{11}$ mi ≈ 10.1 mi

$\boxed{72}$ (a) $m = 6 - 2.5\log\left(\dfrac{10^{0.4}L_0}{L_0}\right) = 6 - 2.5\log 10^{0.4} = 6 - 2.5(0.4) = 6 - 1 = 5$

(b) $m = 6 - 2.5\log\left(\dfrac{L}{L_0}\right) \Rightarrow \log\left(\dfrac{L}{L_0}\right) = \dfrac{6-m}{2.5} \Rightarrow \dfrac{L}{L_0} = 10^{(6-m)/2.5} \Rightarrow$

$L = L_0 10^{(6-m)/2.5}$

$\boxed{73}$ $A(t) = \frac{1}{2}A_0$ when $t = 8 \Rightarrow \frac{1}{2}A_0 = A_0a^{-8} \Rightarrow a^{-8} = \frac{1}{2} \Rightarrow a^8 = 2 \Rightarrow$

$A(t) = A_0a^{-t}$

$a = 2^{1/8} \approx 1.09.$

$\boxed{74}$ Since the field is *currently* 2.5 times the safe level S, we let $A_0 = 2.5S$ and $A(t) = S$.

$S = 2.5Se^{-0.0239t} \Rightarrow 0.4 = e^{-0.0239t} \Rightarrow \ln 0.4 = -0.0239t \Rightarrow$

$t = \dfrac{\ln 0.4}{-0.0239} \approx 38.3$ yr.

$\boxed{75}$ (a) Since $\log P$ is an increasing function, increasing the population increases the walking speed. Pedestrians have faster average walking speeds in large cities.

(b) $5 = 0.05 + 0.86\log P \Rightarrow 4.95 = 0.86\log P \Rightarrow P = 10^{4.95/0.86} \approx 570{,}000$

$\boxed{76}$ (a) Larger values of c cause F to decrease more rapidly.

This indicates that the chip will fail sooner and be less reliable.

(b) $c = 0.125$ and $F = 35\% \Rightarrow 0.35 = 1 - e^{-0.125t} \Rightarrow e^{-0.125t} = 0.65 \Rightarrow$

$-0.125t = \ln 0.65 \Rightarrow t = \dfrac{\ln 0.65}{-0.125} \approx 3.45$ yr.

$\boxed{77}$ (a) $f(2) = \ln(2+1) + e^2 \approx 8.4877$

(b) $g(3.97) = \dfrac{(\log 3.97)^2 - \log 3.97}{4} \approx -0.0601$

[78] (a) $f(1.95) = \log(2 \times 1.95^2 + 1) - 10^{-1.95} \approx 0.9235$

(b) $g(0.55) = \dfrac{0.55 - 3.4}{\ln 0.55 + 4} \approx -0.8377$

[79] Using an intersect feature, $x \ln x = 1$ when $x \approx 1.763$.

[0, 4] by [−1, 1.67]

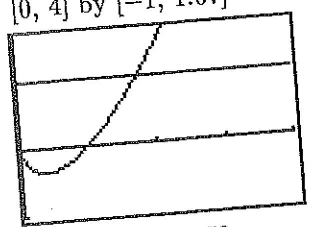

Figure 79

[−3, 4.5] by [−1, 4]

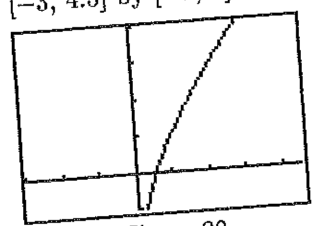

Figure 80

[80] Using a root feature, $\ln x + x = 0$ when $x \approx 0.567$.

[81] The domain of $g(x) = \ln x$ is $x > 0$. From the graph, we determine that $f(x) = 2.2 \log(x + 2)$ intersects g at about 14.90. Thus, $f(x) \geq g(x)$ on (0, 14.90].

[−2, 16] by [−4, 8]

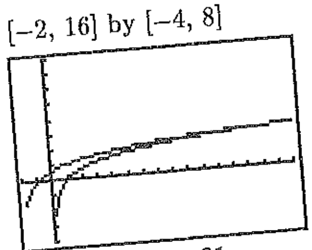

Figure 81

[−2, 4] by [−1, 4]

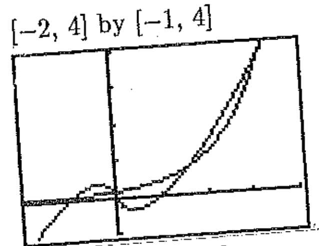

Figure 82

[82] Note that $f(x) = x \ln|x|$ is not defined at $x = 0$. The graph of f intersects the graph of $g(x) = 0.15e^x$ four times: $x \approx -0.94, -0.05, 1.59$, and 3.23.

Thus, $f(x) \geq g(x)$ on $[-0.94, -0.05] \cup [1.59, 3.23]$.

[83] (a) $R = 2.07 \ln x - 2.04 = 2.07 \ln \frac{C}{H} - 2.04 = 2.07 \ln \frac{242}{78} - 2.04 \approx 0.3037 \approx 30\%$.

(b) Graph $Y_1 = 2.07 \ln x - 2.04$ and $Y_2 = 0.75$.

From the graph, $R \approx 0.75$ when $x = \frac{C}{H} \approx 3.85$.

[3, 5, 0.5] by [0, 1, 0.5]

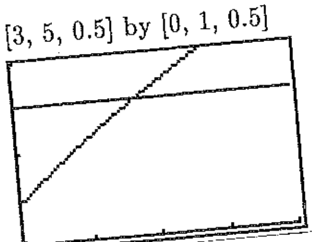

Figure 83

[3, 5, 0.5] by [0, 1, 0.5]

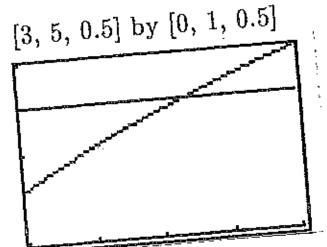

Figure 84

[84] (a) $R = 1.36 \ln x - 1.19 = 1.36 \ln \frac{C}{H} - 1.19 = 1.36 \ln \frac{287}{65} - 1.19 \approx 0.8297 \approx 83\%$.

(b) Graph $Y_1 = 1.36 \ln x - 1.19$ and $Y_2 = 0.75$.

From the graph, $R \approx 0.75$ when $x = \frac{C}{H} \approx 4.16$.

5.5 Exercises

$\boxed{1}$ (a) $\log_4(xz) = \log_4 x + \log_4 z$ (b) $\log_4(y/x) = \log_4 y - \log_4 x$

(c) $\log_4 \sqrt[3]{z} = \log_4 z^{1/3} = \frac{1}{3}\log_4 z$

$\boxed{2}$ (a) $\log_3(xyz) = \log_3(xy) + \log_3 z = \log_3 x + \log_3 y + \log_3 z$

(b) $\log_3(xz/y) = \log_3(xz) - \log_3 y = \log_3 x + \log_3 z - \log_3 y$

(c) $\log_3 \sqrt[5]{y} = \log_3 y^{1/5} = \frac{1}{5}\log_3 y$

$\boxed{3}$ $\log_a \dfrac{x^3 w}{y^2 z^4} = \log_a x^3 w - \log_a y^2 z^4 = \log_a x^3 + \log_a w - (\log_a y^2 + \log_a z^4) =$

$3\log_a x + \log_a w - 2\log_a y - 4\log_a z$

$\boxed{4}$ $\log_a \dfrac{y^5 w^2}{x^4 z^3} = \log_a y^5 w^2 - \log_a x^4 z^3 = \log_a y^5 + \log_a w^2 - (\log_a x^4 + \log_a z^3) =$

$5\log_a y + 2\log_a w - 4\log_a x - 3\log_a z$

$\boxed{5}$ $\log \dfrac{\sqrt[3]{z}}{x\sqrt{y}} = \log \sqrt[3]{z} - \log x\sqrt{y} = \log z^{1/3} - \log x - \log y^{1/2} = \frac{1}{3}\log z - \log x - \frac{1}{2}\log y$

$\boxed{6}$ $\log \dfrac{\sqrt{y}}{x^4 \sqrt[3]{z}} = \log \sqrt{y} - \log x^4 \sqrt[3]{z} = \log y^{1/2} - \log x^4 - \log y^{1/3} = \frac{1}{2}\log y - 4\log x - \frac{1}{3}\log z$

$\boxed{7}$ $\ln \sqrt[4]{\dfrac{x^7}{y^5 z}} = \ln x^{7/4} - \ln y^{5/4} z^{1/4} = \ln x^{7/4} - \ln y^{5/4} - \ln z^{1/4} = \frac{7}{4}\ln x - \frac{5}{4}\ln y - \frac{1}{4}\ln z$

$\boxed{8}$ $\ln x \sqrt[3]{\dfrac{y^4}{z^5}} = \ln x + \ln y^{4/3} - \ln z^{5/3} = \ln x + \frac{4}{3}\ln y - \frac{5}{3}\ln z$

$\boxed{9}$ (a) $\log_3 x + \log_3(5y) = \log_3(x \cdot 5y) = \log_3(5xy)$

(b) $\log_3(2z) - \log_3 x = \log_3(2z/x)$ (c) $5\log_3 y = \log_3 y^5$

$\boxed{10}$ (a) $\log_4(3z) + \log_4 x = \log_4(3xz)$ (b) $\log_4 x - \log_4(7y) = \log_4[x/(7y)]$

(c) $\frac{1}{3}\log_4 w = \log_4 w^{1/3} = \log_4 \sqrt[3]{w}$

$\boxed{11}$ $2\log_a x + \frac{1}{3}\log_a(x-2) - 5\log_a(2x+3) = \log_a x^2 + \log_a(x-2)^{1/3} - \log_a(2x+3)^5 =$

$\log_a x^2 \sqrt[3]{x-2} - \log_a(2x+3)^5 = \log_a \dfrac{x^2 \sqrt[3]{x-2}}{(2x+3)^5}$

$\boxed{12}$ $5\log_a x - \frac{1}{2}\log_a(3x-4) - 3\log_a(5x+1) = \log_a x^5 - \log_a(3x-4)^{1/2} - \log_a(5x+1)^3 =$

$\log_a x^5 - \log_a \sqrt{3x-4}(5x+1)^3 = \log_a \dfrac{x^5}{\sqrt{3x-4}\,(5x+1)^3}$

$\boxed{13}$ $\log(x^3 y^2) - 2\log x\sqrt[3]{y} - 3\log\left(\frac{x}{y}\right) = \log \dfrac{x^3 y^2}{x^2 y^{2/3}(x^3/y^3)} = \log \dfrac{y^{13/3}}{x^2}$

$\boxed{14}$ $2\log \dfrac{y^3}{x} - 3\log y + \frac{1}{2}\log x^4 y^2 = \log \dfrac{(y^6/x^2)(x^2 y)}{y^3} = \log y^4$

$\boxed{15}$ $\ln y^3 + \frac{1}{3}\ln(x^3 y^6) - 5\ln y = \ln y^3 + \ln(xy^2) - \ln y^5 = \ln[(xy^5)/y^5] = \ln x$

$\boxed{16}$ $2\ln x - 4\ln(1/y) - 3\ln(xy) = \ln x^2 - \ln(1/y^4) - \ln(x^3 y^3) =$

$\ln x^2 - [\ln(1/y^4) + \ln(x^3 y^3)] = \ln x^2 - \ln(x^3/y) = \ln \dfrac{x^2}{x^3/y} = \ln(y/x)$

$\boxed{17}$ $\log_6(2x-3) = \log_6 12 - \log_6 3 \Rightarrow \log_6(2x-3) = \log_6 \frac{12}{3} \Rightarrow 2x-3 = 4 \Rightarrow x = \frac{7}{2}$

$\boxed{18}$ $\log_4(3x+2) = \log_4 5 + \log_4 3 \Rightarrow \log_4(3x+2) = \log_4(5 \cdot 3) \Rightarrow 3x+2 = 15 \Rightarrow$
$x = \frac{13}{3}$

$\boxed{19}$ $2\log_3 x = 3\log_3 5 \Rightarrow \log_3 x^2 = \log_3 5^3 \Rightarrow x^2 = 125 \Rightarrow x = \pm 5\sqrt{5}$;
$-5\sqrt{5}$ is extraneous since it would make $\log_3 x$ undefined

$\boxed{20}$ $3\log_2 x = 2\log_2 3 \Rightarrow \log_2 x^3 = \log_2 3^2 \Rightarrow x^3 = 9 \Rightarrow x = \sqrt[3]{9}$

$\boxed{21}$ $\log x - \log(x+1) = 3\log 4 \Rightarrow \log\frac{x}{x+1} = \log 4^3 \Rightarrow$
$\frac{x}{x+1} = 64 \Rightarrow x = 64x + 64 \Rightarrow x = -\frac{64}{63}$; $-\frac{64}{63}$ is extraneous, no solution

$\boxed{22}$ $\log(x+2) - \log x = 2\log 4 \Rightarrow \log\frac{x+2}{x} = \log 16 \Rightarrow x+2 = 16x \Rightarrow x = \frac{2}{15}$

$\boxed{23}$ $\ln(-4-x) + \ln 3 = \ln(2-x) \Rightarrow \ln(-12-3x) = \ln(2-x) \Rightarrow$
$-12-3x = 2-x \Rightarrow 2x = -14 \Rightarrow x = -7$

$\boxed{24}$ $\ln x + \ln(x+6) = \frac{1}{2}\ln 9 \Rightarrow \ln(x^2+6x) = \ln 3 \Rightarrow x^2+6x-3 = 0 \Rightarrow$
$x = \frac{-6 \pm \sqrt{48}}{2} = -3 \pm 2\sqrt{3}$; $-3-2\sqrt{3}$ is extraneous

$\boxed{25}$ $\log_2(x+7) + \log_2 x = 3 \Rightarrow \log_2(x^2+7x) = 3 \Rightarrow x^2+7x = 8 \Rightarrow$
$(x+8)(x-1) = 0 \Rightarrow x = -8, 1$; -8 is extraneous

$\boxed{26}$ $\log_6(x+5) + \log_6 x = 2 \Rightarrow \log_6(x^2+5x) = 2 \Rightarrow x^2+5x = 36 \Rightarrow$
$(x+9)(x-4) = 0 \Rightarrow x = -9, 4$; -9 is extraneous

$\boxed{27}$ $\log_3(x+3) + \log_3(x+5) = 1 \Rightarrow \log_3(x^2+8x+15) = 1 \Rightarrow$
$x^2+8x+15 = 3 \Rightarrow (x+2)(x+6) = 0 \Rightarrow x = -6, -2$; -6 is extraneous

$\boxed{28}$ $\log_3(x-2) + \log_3(x-4) = 2 \Rightarrow \log_3(x^2-6x+8) = 2 \Rightarrow x^2-6x+8 = 9 \Rightarrow$
$x^2-6x-1 = 0 \Rightarrow x = \frac{6 \pm \sqrt{40}}{2} = 3 \pm \sqrt{10}$; $3-\sqrt{10}$ is extraneous

$\boxed{29}$ $\log(x+3) = 1 - \log(x-2) \Rightarrow \log(x+3) + \log(x-2) = 1 \Rightarrow$
$\log[(x+3)(x-2)] = 1 \Rightarrow x^2+x-6 = 10^1 \Rightarrow$
$x^2+x-16 = 0 \Rightarrow x = \frac{-1+\sqrt{65}}{2} \approx 3.53$; $\frac{-1-\sqrt{65}}{2} \approx -4.53$ is extraneous

$\boxed{30}$ $\log(57x) = 2 + \log(x-2) \Rightarrow \log(57x) - \log(x-2) = 2 \Rightarrow$
$\log\frac{57x}{x-2} = 2 \Rightarrow \frac{57x}{x-2} = 10^2 \Rightarrow 57x = 100x - 200 \Rightarrow 200 = 43x \Rightarrow x = \frac{200}{43}$

$\boxed{31}$ $\ln x = 1 - \ln(x+2) \Rightarrow \ln x + \ln(x+2) = 1 \Rightarrow \ln[x(x+2)] = 1 \Rightarrow$
$x^2+2x = e^1 \Rightarrow x^2+2x-e = 0 \Rightarrow$
$x = \frac{-2+\sqrt{4+4e}}{2} = -1+\sqrt{1+e} \approx 0.93$; $-1-\sqrt{1+e} \approx -2.93$ is extraneous

$\boxed{32}$ $\ln x = 1 + \ln(x+1) \Rightarrow \ln x - \ln(x+1) = 1 \Rightarrow \ln\left(\frac{x}{x+1}\right) = 1 \Rightarrow$
$\frac{x}{x+1} = e^1 \Rightarrow x = ex + e \Rightarrow (1-e)x = e \Rightarrow x = \frac{e}{1-e} \approx -1.58$ and is
extraneous, no solution

[33]
$$\log_3(x-2) = \log_3 27 - \log_3(x-4) - 5^{\log_5 1} \quad \text{given}$$
$$\log_3(x-2) + \log_3(x-4) = \log_3 3^3 - 1 \quad 5^{\log_5 1} = 1$$
$$\log_3[(x-2)(x-4)] = 3 - 1 \quad \log_3 3^3 = 3$$
$$\log_3(x^2 - 6x + 8) = 2 \quad \text{simplify}$$
$$x^2 - 6x + 8 = 3^2 \quad \text{exponential form}$$
$$x^2 - 6x - 1 = 0 \quad \text{subtract 9}$$

Now solve $x^2 - 6x - 1 = 0$ using the quadratic formula to get $x = (6 \pm \sqrt{40})/2 = 3 \pm \sqrt{10}$. Since $3 - \sqrt{10}$ makes the arguments of the logarithms negative, the only solution is $3 + \sqrt{10}$.

[34] $\log_2(x+3) = \log_2(x-3) + \log_3 9 + 4^{\log_4 3} \Rightarrow$

$\log_2(x+3) - \log_2(x-3) = 2 + 3 \Rightarrow \log_2 \dfrac{x+3}{x-3} = 5 \Rightarrow \dfrac{x+3}{x-3} = 2^5 \Rightarrow$

$x + 3 = 32(x-3) \Rightarrow x + 3 = 32x - 96 \Rightarrow 99 = 31x \Rightarrow x = \frac{99}{31}$

[35] $f(x) = \log_3(3x) = \log_3 3 + \log_3 x = \log_3 x + 1$ • shift $y = \log_3 x$ up 1 unit

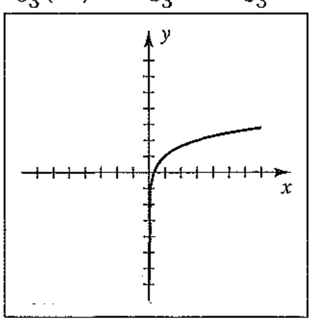

Figure 35

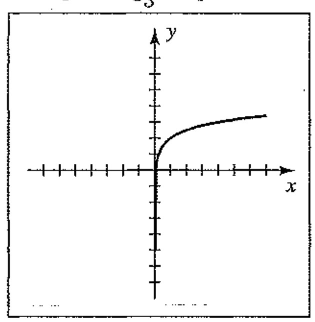

Figure 36

[36] $f(x) = \log_4(16x) = \log_4 16 + \log_4 x = \log_4 x + 2$ • shift $y = \log_4 x$ up 2 units

[37] $f(x) = 3\log_3 x$ • vertically stretch $y = \log_3 x$ by a factor of 3

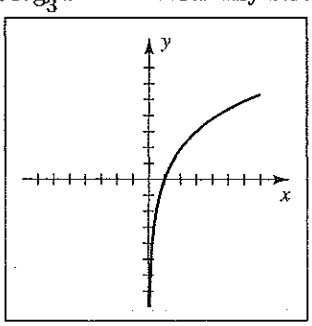

Figure 37

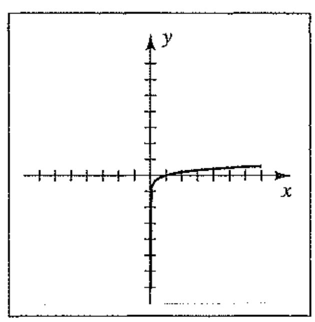

Figure 38

[38] $f(x) = \frac{1}{3}\log_3 x$ • vertically compress $y = \log_3 x$ by a factor of 3

$\boxed{39}$ $f(x) = \log_3(x^2) = 2\log_3 x$ • Vertically stretch $y = \log_3 x$ by a factor of 2 and include its reflection through the y-axis since the domain of the original function, $f(x) = \log_3(x^2)$, is $\mathbb{R} - \{0\}$.

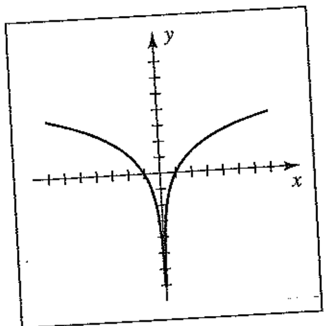

Figure 39

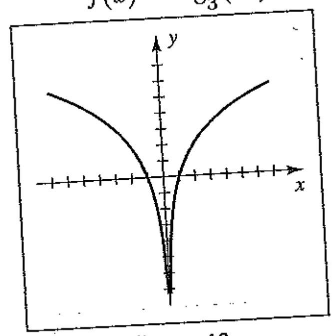

Figure 40

$\boxed{40}$ $f(x) = \log_2(x^2) = 2\log_2 x$ • similar to Exercise 39

$\boxed{41}$ $f(x) = \log_2(x^3) = 3\log_2 x$ • vertically stretch $y = \log_2 x$ by a factor of 3

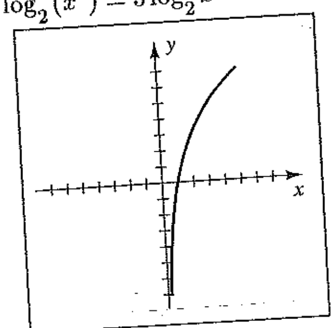

Figure 41

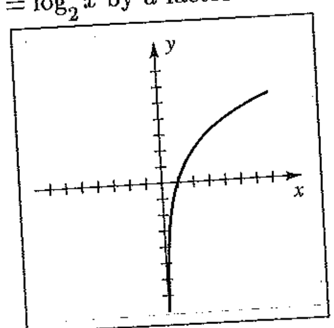

Figure 42

$\boxed{42}$ $f(x) = \log_3(x^3) = 3\log_3 x$ • vertically stretch $y = \log_3 x$ by a factor of 3

$\boxed{43}$ $f(x) = \log_2\sqrt{x} = \frac{1}{2}\log_2 x$ • vertically compress $y = \log_2 x$ by a factor of 2

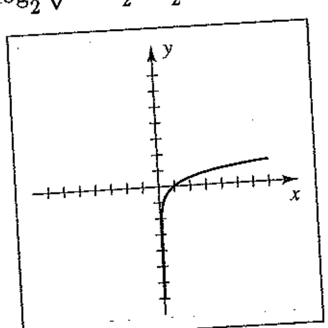

Figure 43

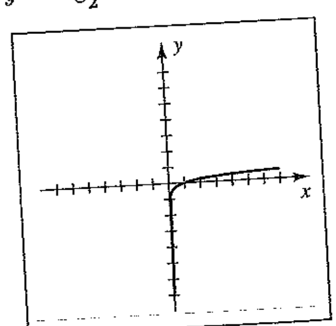

Figure 44

$\boxed{44}$ $f(x) = \log_2\sqrt[3]{x} = \frac{1}{3}\log_2 x$ • vertically compress $y = \log_2 x$ by a factor of 3

[45] $f(x) = \log_3(\frac{1}{x}) = \log_3 x^{-1} = -\log_3 x$ • reflect $y = \log_3 x$ through the x-axis

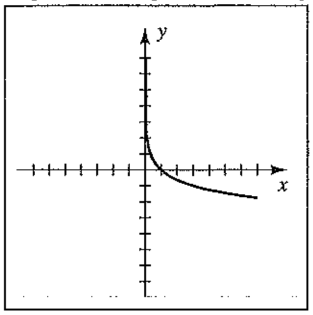

Figure 45

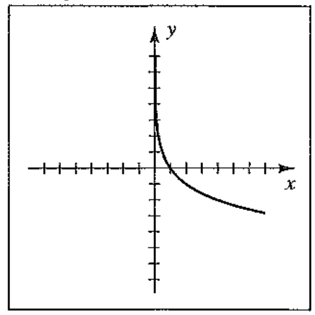

Figure 46

[46] $f(x) = \log_2(\frac{1}{x}) = \log_2 x^{-1} = -\log_2 x$ • reflect $y = \log_2 x$ through the x-axis

[47] The values of $F(x) = \log_2 x$ are doubled and the reflection of F through the y-axis is included. The domain of f is $\mathbb{R} - \{0\}$ and $f(x) = \log_2 x^2$.

[48] The domain of f is $\mathbb{R} - \{0\}$ and f has the same y-values as $F(x) = \log_2 x$.

Hence, $f(x) = \log_2 |x|$. $\{$or $f(x) = \frac{1}{2}\log_2 x^2\}$

[49] $F(x) = \log_2 x$ is shifted up 3 units.

Hence, $f(x) = 3 + \log_2 x = \log_2 2^3 + \log_2 x = \log_2(8x)$.

[50] $F(x) = \log_2 x$ is reflected through the x-axis and shifted up 1 unit.

Hence, $f(x) = -\log_2 x + 1 = \log_2 2 - \log_2 x = \log_2 \frac{2}{x}$.

[51] $V_1 = 2$ and $V_2 = 4.5 \Rightarrow \text{db} = 20\log\frac{V_2}{V_1} = 20\log\frac{4.5}{2} \approx 7.04$, or $+7$.

[52] Let $\frac{V_2}{V_1} = k$. A $+20$ decibel gain $\Rightarrow +20 = 20\log k \Rightarrow 1 = \log k \Rightarrow k = 10$.

A $+40$ decibel gain $\Rightarrow +40 = 20\log k \Rightarrow 2 = \log k \Rightarrow k = 100$.

Note that a ten-fold increase in the voltage ratio is needed to double the decibel gain.

[53] $\log y = \log b - k\log x \Rightarrow \log y = \log b - \log x^k \Rightarrow \log y = \log\frac{b}{x^k} \Rightarrow y = \frac{b}{x^k}$

[54] $p = p_0 e^{-ax} \Rightarrow \frac{p}{p_0} = e^{-ax} \Rightarrow \ln(\frac{p}{p_0}) = -ax \Rightarrow x(p) = -\frac{1}{a}\ln(\frac{p}{p_0}) = \frac{1}{a}\ln(\frac{p_0}{p})$

[55] $c = 0.5$ and $z_0 = 0.1 \Rightarrow$

$v = c\ln(z/z_0)$

$= (0.5)\ln(10z)$

$= \frac{1}{2}(\ln 10 + \ln z) \approx \frac{1}{2}\ln z + 1.15$.

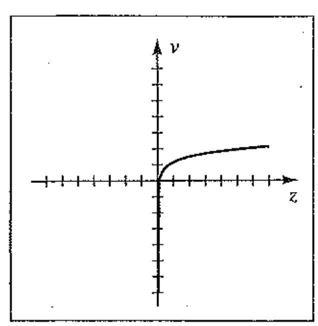

Figure 55

[56] $y = 50\%$ of $y_0 \Rightarrow \frac{1}{2}y_0 = y_0 e^{-0.3821t} \Rightarrow \frac{1}{2} = e^{-0.3821t} \Rightarrow$
$\ln\frac{1}{2} = -0.3821t \Rightarrow t = \frac{-\ln 2}{-0.3821} \approx 1.814$ yr.

[57] (a) $R(x) = a\log\frac{x}{x_0} \Rightarrow R(x_0) = a\log\left(\frac{x_0}{x_0}\right) = a\log 1 = a\cdot 0 = 0$

(b) $R(2x) = a\log\left(\frac{2x}{x_0}\right) = a\left[\log 2 + \log\left(\frac{x}{x_0}\right)\right] = a\log 2 + a\log\left(\frac{x}{x_0}\right) = R(x) + a\log 2$

[58] (a) $E(x) = E_0 e^{-x/x_0} \Rightarrow E(x_0) = E_0 e^{-x_0/x_0} = E_0 e^{-1} = \frac{1}{e}E_0$

(b) $E = E_0 - 99\%\,E_0 \Rightarrow (1-0.99)\,E_0 = E_0 e^{-x/x_0} \Rightarrow 0.01 = e^{-x/x_0} \Rightarrow$
$-\frac{x}{x_0} = \ln(0.01) = -\ln 100 \Rightarrow x = (\ln 100)\,x_0 \approx 4.6\,x_0$

[59] $\ln I_0 - \ln I = kx \Rightarrow \ln\frac{I_0}{I} = kx \Rightarrow x = \frac{1}{k}\ln\frac{I_0}{I} = \frac{1}{0.39}\ln 1.12 \approx 0.29$ cm.

[60] $\ln I_0 - \ln I = kx \Rightarrow \ln\frac{I_0}{I} = kx \Rightarrow \frac{I_0}{I} = e^{kx} = e^{(0.39)(0.24)} \approx 1.10 \Rightarrow \frac{I}{I_0} \approx 0.91.$
The intensity decreases by approximately 9%.

[61] The domain of $g(x) = \log 3x$ is $x > 0$. The graphs intersect at $x \approx 1.02$ and 2.40.
The solution of the inequality $f(x) \geq g(x)$ is $(0,\ 1.02] \cup [2.40,\ \infty)$.

[0, 6] by [−1, 3]

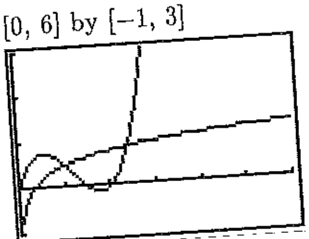

Figure 61

[0, 6] by [−1, 3]

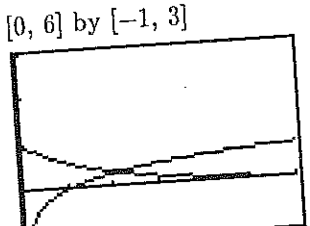

Figure 62

[62] The domain of $g(x) = \log x$ is $x > 0$. The graphs intersect at $x \approx 2.08$.
The solution of the inequality $f(x) \geq g(x)$ is $(0,\ 2.08]$.

[63] Graph $y = e^{-x} - 2\log(1+x^2) + 0.5x$ and estimate any x-intercepts. From the graph, we determine that the roots of the equation are approximately 1.41 and 6.59.

[0, 8] by [−1.67, 3.67]

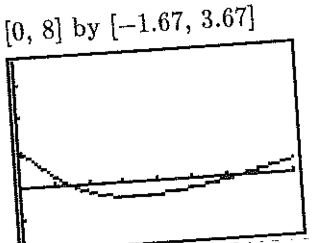

Figure 63

[−1, 5] by [−1, 3]

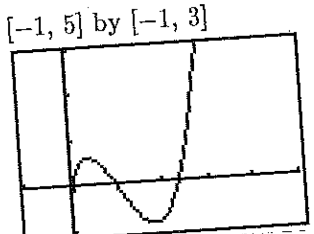

Figure 64

[64] Graph $y = 0.3\ln x + x^3 - 3.1x^2 + 1.3x + 0.8$ and estimate any x-intercepts. From the graph, we determine that the roots of the equation are approximately 0.056 and 2.359 as well as $x = 1$.

65 (a) f is increasing on $[0.2, 0.63]$ and $[6.87, 16]$. f is decreasing on $[0.63, 6.87]$.

(b) The maximum value of f is 4.61 when $x = 16$.

The minimum value of f is approximately -3.31 when $x \approx 6.87$.

$[0.2, 16, 2]$ by $[-4.77, 5.77]$

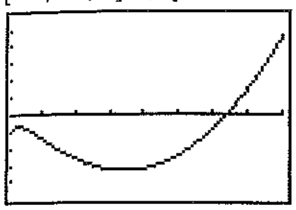

Figure 65

$[0.2, 16, 2]$ by $[-6, 14]$

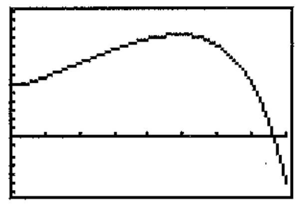

Figure 66

66 (a) f is increasing on $[0.44, 9.76]$. f is decreasing on $[0.2, 0.44]$ and $[9.76, 16]$.

(b) The maximum value of f is approximately 11.35 when $x \approx 9.76$.

The minimum value of f is approximately -4.9 when $x = 16$.

67 Graph $Y_1 = x \log x - \log x$ and $Y_2 = 5$. The graphs intersect at $x \approx 6.94$.

$[-5, 10]$ by $[-2, 8]$

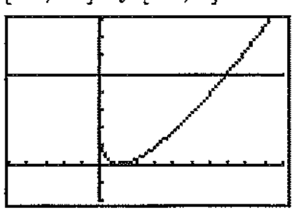

Figure 67

$[-5, 10]$ by $[-2, 8]$

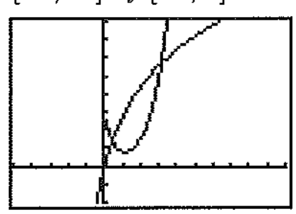

Figure 68

68 Graph $Y_1 = 0.3e^x - \ln x$ and $Y_2 = 4 \ln (x + 1)$.

The graphs intersect at $x \approx 0.40, 3.12$.

69 Let $d = x$. Graph $Y_1 = I_0 - 20 \log x - kx = 70 - 20 \log x - 0.076x$ and $Y_2 = 20$.

At the point of intersection, $x \approx 115.3$. The distance is approximately 115 meters.

$[0, 150, 10]$ by $[0, 100, 10]$

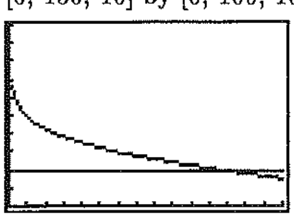

Figure 69

$[0, 150, 10]$ by $[0, 100, 10]$

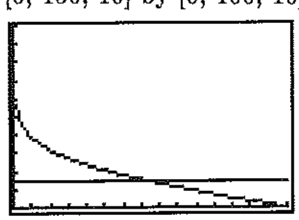

Figure 70

70 Let $d = x$. Graph $Y_1 = I_0 - 20 \log x - kx = 60 - 20 \log x - 0.11x$ and $Y_2 = 15$.

At the point of intersection, $x \approx 71.7$. The distance is approximately 72 meters.

5.6 Exercises

[1] (a) $5^x = 8 \Rightarrow \log 5^x = \log 8 \Rightarrow x \log 5 = \log 8 \Rightarrow x = \frac{\log 8}{\log 5} \approx 1.29$

(b) $5^x = 8 \Rightarrow x = \log_5 8 = \frac{\log 8}{\log 5} \approx 1.29$

[2] (a) $4^x = 3 \Rightarrow \log 4^x = \log 3 \Rightarrow x \log 4 = \log 3 \Rightarrow x = \frac{\log 3}{\log 4} \approx 0.79$

(b) $4^x = 3 \Rightarrow x = \log_4 3 = \frac{\log 3}{\log 4} \approx 0.79$

[3] (a) $3^{4-x} = 5 \Rightarrow \log(3^{4-x}) = \log 5 \Rightarrow (4-x)\log 3 = \log 5 \Rightarrow$

$4 - x = \frac{\log 5}{\log 3} \Rightarrow x = 4 - \frac{\log 5}{\log 3} \approx 2.54.$ *Note:* The answer could also be written as $4 - \frac{\log 5}{\log 3} = \frac{4\log 3 - \log 5}{\log 3} = \frac{\log 81 - \log 5}{\log 3} = \frac{\log \frac{81}{5}}{\log 3}$.

(b) $3^{4-x} = 5 \Rightarrow 4 - x = \log_3 5 \Rightarrow x = 4 - \frac{\log 5}{\log 3} \approx 2.54.$

[4] (a) $(\frac{1}{3})^x = 100 \Rightarrow \log(\frac{1}{3})^x = \log 100 \Rightarrow x \log\frac{1}{3} = 2 \Rightarrow x = \frac{2}{-\log 3} \approx -4.19$

(b) $(\frac{1}{3})^x = 100 \Rightarrow x = \log_{(1/3)} 100 = \frac{\log 100}{\log \frac{1}{3}} = \frac{2}{-\log 3} \approx -4.19$

[5] $\log_5 6 = \frac{\log 6}{\log 5} \left\{ \text{or } \frac{\ln 6}{\ln 5} \right\} \approx 1.1133$

[6] $\log_2 20 = \frac{\log 20}{\log 2} \approx 4.3219$

[7] $\log_9 0.2 = \frac{\log 0.2}{\log 9} \approx -0.7325$

[8] $\log_6 \frac{1}{2} = \frac{\log \frac{1}{2}}{\log 6} \approx -0.3869$

[9] $\frac{\log_5 16}{\log_5 4} = \log_4 16 = \log_4 4^2 = 2$

[10] $\frac{\log_7 243}{\log_7 3} = \log_3 243 = \log_3 3^5 = 5$

[11] $3^{x+4} = 2^{1-3x} \Rightarrow (x+4)\log 3 = (1-3x)\log 2 \Rightarrow$

$x \log 3 + 4\log 3 = \log 2 - 3x \log 2 \Rightarrow x(\log 3 + 3\log 2) = \log 2 - 4\log 3 \Rightarrow$

$x = \frac{\log 2 - \log 81}{\log 3 + \log 8} \Rightarrow x = \frac{\log \frac{2}{81}}{\log 24} \approx -1.16$

[12] $4^{2x+3} = 5^{x-2} \Rightarrow (2x+3)\log 4 = (x-2)\log 5 \Rightarrow$

$2x \log 4 + 3\log 4 = x \log 5 - 2\log 5 \Rightarrow 2\log 5 + 3\log 4 = x(\log 5 - 2\log 4) \Rightarrow$

$x = \frac{\log 25 + \log 64}{\log 5 - \log 16} \Rightarrow x = \frac{\log 1600}{\log \frac{5}{16}} \approx -6.34$

[13] $2^{2x-3} = 5^{x-2} \Rightarrow (2x-3)\log 2 = (x-2)\log 5 \Rightarrow$

$2x \log 2 - 3\log 2 = x \log 5 - 2\log 5 \Rightarrow x(2\log 2 - \log 5) = 3\log 2 - 2\log 5 \Rightarrow$

$x = \frac{\log 8 - \log 25}{\log 4 - \log 5} \Rightarrow x = \frac{\log \frac{8}{25}}{\log \frac{4}{5}} \approx 5.11$

[14] $3^{2-3x} = 4^{2x+1} \Rightarrow (2-3x)\log 3 = (2x+1)\log 4 \Rightarrow$

$2\log 3 - 3x \log 3 = 2x \log 4 + \log 4 \Rightarrow 2\log 3 - \log 4 = x(2\log 4 + 3\log 3) \Rightarrow$

$x = \frac{\log 9 - \log 4}{\log 16 + \log 27} \Rightarrow x = \frac{\log \frac{9}{4}}{\log 432} \approx 0.13$

$\boxed{15}$ $2^{-x} = 8 \Rightarrow 2^{-x} = 2^3 \Rightarrow -x = 3 \Rightarrow x = -3$

$\boxed{16}$ $2^{-x^2} = 5 \Rightarrow -x^2 = \log_2 5 \Rightarrow x^2 = -\dfrac{\log 5}{\log 2}$ $\{x^2 \geq 0\}$, no solution

$\boxed{17}$ $\log x = 1 - \log(x-3) \Rightarrow \log x + \log(x-3) = 1 \Rightarrow \log(x^2 - 3x) = 1 \Rightarrow$
$x^2 - 3x = 10^1 \Rightarrow (x-5)(x+2) = 0 \Rightarrow x = 5, -2$; -2 is extraneous

$\boxed{18}$ $\log(5x+1) = 2 + \log(2x-3) \Rightarrow \log(5x+1) - \log(2x-3) = 2 \Rightarrow$
$\log\left(\dfrac{5x+1}{2x-3}\right) = 2 \Rightarrow \dfrac{5x+1}{2x-3} = 10^2 \Rightarrow 5x + 1 = 200x - 300 \Rightarrow x = \frac{301}{195} \approx 1.54$

$\boxed{19}$ $\log(x^2+4) - \log(x+2) = 2 + \log(x-2) \Rightarrow \log\left(\dfrac{x^2+4}{x+2}\right) - \log(x-2) = 2 \Rightarrow$
$\log\left(\dfrac{x^2+4}{x^2-4}\right) = 2 \Rightarrow \dfrac{x^2+4}{x^2-4} = 10^2 \Rightarrow x^2 + 4 = 100x^2 - 400 \Rightarrow 404 = 99x^2 \Rightarrow$
$x = \pm\sqrt{\frac{404}{99}} = \pm\frac{2}{3}\sqrt{\frac{101}{11}} \approx \pm 2.02$; $-\frac{2}{3}\sqrt{\frac{101}{11}}$ is extraneous

$\boxed{20}$ $\log(x-4) - \log(3x-10) = \log(1/x) \Rightarrow$
$\log\left(\dfrac{x-4}{3x-10}\right) = \log\left(\frac{1}{x}\right) \Rightarrow \dfrac{x-4}{3x-10} = \frac{1}{x} \Rightarrow x^2 - 4x = 3x - 10 \Rightarrow$
$(x-2)(x-5) = 0 \Rightarrow x = 2, 5$; 2 is extraneous

$\boxed{21}$ $5^x + 125(5^{-x}) = 30 \Rightarrow$ {multiply by 5^x} $(5^x)^2 - 30(5^x) + 125 = 0 \Rightarrow$
$(5^x - 5)(5^x - 25) = 0 \Rightarrow 5^x = 5, 25 \Rightarrow 5^x = 5^1, 5^2 \Rightarrow x = 1, 2$

$\boxed{22}$ $3(3^x) + 9(3^{-x}) = 28 \Rightarrow 3(3^x)^2 - 28(3^x) + 9 = 0 \Rightarrow$
$[3(3^x) - 1](3^x - 9) = 0 \Rightarrow 3^x = \frac{1}{3}, 9 \Rightarrow 3^x = 3^{-1}, 3^2 \Rightarrow x = -1, 2$

$\boxed{23}$ $4^x - 3(4^{-x}) = 8 \Rightarrow (4^x)^2 - 8(4^x) - 3 = 0 \Rightarrow$
$4^x = \dfrac{8+\sqrt{76}}{2}$ $\left\{\text{since } 4^x > 0 \text{ and } \dfrac{8-\sqrt{76}}{2} < 0\right\}$ $= 4 + \sqrt{19} \Rightarrow$
$x = \log_4(4+\sqrt{19}) = \dfrac{\log(4+\sqrt{19})}{\log 4} \approx 1.53$

$\boxed{24}$ $2^x - 6(2^{-x}) = 6 \Rightarrow (2^x)^2 - 6(2^x) - 6 = 0 \Rightarrow$
$2^x = \dfrac{6+\sqrt{60}}{2}$ $\left\{\text{since } 2^x > 0 \text{ and } \dfrac{6-\sqrt{60}}{2} < 0\right\}$ $= 3 + \sqrt{15} \Rightarrow$
$x = \log_2(3+\sqrt{15}) = \dfrac{\log(3+\sqrt{15})}{\log 2} \approx 2.78$

$\boxed{25}$ $\log(x^2) = (\log x)^2 \Rightarrow 2\log x = (\log x)^2 \Rightarrow (\log x)^2 - 2\log x = 0 \Rightarrow$
$(\log x)(\log x - 2) = 0 \Rightarrow \log x = 0, 2 \Rightarrow x = 10^0, 10^2 \Rightarrow x = 1$ or 100

$\boxed{26}$ $\log\sqrt{x} = \sqrt{\log x} \Rightarrow \frac{1}{2}\log x = (\log x)^{1/2} \Rightarrow (\log x)^2 = 4\log x \Rightarrow$
$(\log x)(\log x - 4) = 0 \Rightarrow \log x = 0, 4 \Rightarrow x = 10^0, 10^4 \Rightarrow x = 1$ or $10{,}000$

$\boxed{27}$ $\log(\log x) = 2 \Rightarrow \log x = 10^2 = 100 \Rightarrow x = 10^{100}$

$\boxed{28}$ $\log\sqrt{x^3 - 9} = 2 \Rightarrow \sqrt{x^3-9} = 10^2 \Rightarrow x^3 - 9 = 10^4 \Rightarrow x = \sqrt[3]{10{,}009}$

$\boxed{29}$ $x^{\sqrt{\log x}} = 10^8 \Rightarrow \log\left(x^{\sqrt{\log x}}\right) = \log 10^8 \Rightarrow \sqrt{\log x}(\log x) = 8 \Rightarrow$
$(\log x)^{3/2} = 8 \Rightarrow \log x = 8^{2/3} = 4 \Rightarrow x = 10{,}000$

$\boxed{30}$ $\log(x^3) = (\log x)^3 \;\Rightarrow\; 3\log x = (\log x)^3 \;\Rightarrow\; (\log x)[(\log x)^2 - 3] = 0 \;\Rightarrow$

$$\log x = 0,\ \pm\sqrt{3} \;\Rightarrow\; x = 1,\ 10^{\sqrt{3}},\ 10^{-\sqrt{3}}$$

$\boxed{31}$ Since $e^{2x} = (e^x)^2$, we recognize $e^{2x} + 2e^x - 15$ as a quadratic in e^x and factor it.

$e^{2x} + 2e^x - 15 = 0 \;\Rightarrow\; (e^x+5)(e^x-3) = 0 \;\Rightarrow\; e^x = -5,\ 3.$

But $e^x > 0$, so $e^x = 3 \;\Rightarrow\; x = \ln 3$.

$\boxed{32}$ $e^x + 4e^{-x} = 5 \;\Rightarrow\; e^{2x} + 4 = 5e^x$ {multiply by e^x} $\Rightarrow\; e^{2x} - 5e^x + 4 = 0 \;\Rightarrow$

$(e^x-1)(e^x-4) = 0 \;\Rightarrow\; e^x = 1,\ 4 \;\Rightarrow\; x = \ln 1,\ \ln 4 \;\Rightarrow\; x = 0,\ \ln 4$

$\boxed{33}$ $\log_3 x - \log_9(x+42) = 0 \;\Rightarrow\; \dfrac{\ln x}{\ln 3} - \dfrac{\ln(x+42)}{\ln 9} = 0$ {change of base theorem} $\Rightarrow$

$\dfrac{2\ln x}{2\ln 3} - \dfrac{\ln(x+42)}{\ln 9} = 0$ {get a common denominator, $2\ln 3 = \ln 3^2 = \ln 9$} $\Rightarrow$

$\ln x^2 - \ln(x+42) = 0$ {multiply by $\ln 9$ and change $2\ln x$ to $\ln x^2$} $\Rightarrow$

$\ln\left(\dfrac{x^2}{x+42}\right) = 0 \;\Rightarrow\; \dfrac{x^2}{x+42} = e^0 \;\Rightarrow\; \dfrac{x^2}{x+42} = 1 \;\Rightarrow\; x^2 = x + 42 \;\Rightarrow$

$x^2 - x - 42 = 0 \;\Rightarrow\; (x-7)(x+6) = 0 \;\Rightarrow\; x = 7,\ -6.$ -6 is extraneous.

$\boxed{34}$ $\log_4 x + \log_8 x = 1 \;\Rightarrow\; \dfrac{\ln x}{\ln 4} + \dfrac{\ln x}{\ln 8} = 1 \;\Rightarrow\; \dfrac{\ln x}{\ln 2^2} + \dfrac{\ln x}{\ln 2^3} = 1 \;\Rightarrow$

$\dfrac{\ln x}{2\ln 2} + \dfrac{\ln x}{3\ln 2} = 1 \;\Rightarrow\; \dfrac{3\ln x}{6\ln 2} + \dfrac{2\ln x}{6\ln 2} = 1 \;\Rightarrow\; 3\ln x + 2\ln x = 6\ln 2 \;\Rightarrow$

$\ln x^3 + \ln x^2 = \ln 2^6 \;\Rightarrow\; \ln(x^3\cdot x^2) = \ln 2^6 \;\Rightarrow\; x^5 = 64 \;\Rightarrow\; x = \sqrt[5]{64}$

Note: For Exercises 35–38 and 49–50 of the review exercises, let D denote the domain of the function determined by the original equation, and R its range. These are then the range and domain, respectively, of the equation listed in the answer.

$\boxed{35}$ $y = \dfrac{10^x + 10^{-x}}{2}$ $\{D = \mathbb{R},\ R = [1, \infty)\}$ $\Rightarrow$

$2y = 10^x + 10^{-x}$ $\left\{10^{-x} = \dfrac{1}{10^x}\text{, so multiply by } 10^x \text{ to eliminate denominator}\right\}$ $\Rightarrow$

$10^{2x} - 2y\,10^x + 1 = 0$ {treat as a quadratic in 10^x} $\Rightarrow$

$$10^x = \frac{2y \pm \sqrt{4y^2-4}}{2} = y \pm \sqrt{y^2-1} \;\Rightarrow\; x = \log\left(y \pm \sqrt{y^2-1}\right)$$

$\boxed{36}$ The solution is similar to that of Exercise 35.

$y = \dfrac{10^x - 10^{-x}}{2}$ $\{D = R = \mathbb{R}\}$ $\Rightarrow\; 2y = 10^x - 10^{-x} \;\Rightarrow$

$10^{2x} - 2y\,10^x - 1 = 0 \;\Rightarrow\; 10^x = \dfrac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1};$

$\sqrt{y^2+1} > y$, so $y - \sqrt{y^2+1} < 0$, but $10^x > 0$ and thus, $x = \log\left(y + \sqrt{y^2+1}\right)$

$\boxed{37}$ $y = \dfrac{10^x - 10^{-x}}{10^x + 10^{-x}}$ $\{ D = \mathbb{R},\ R = (-1, 1) \}$ $\Rightarrow$ $y\,10^x + y\,10^{-x} = 10^x - 10^{-x}$ $\Rightarrow$

$y\,10^{2x} + y = 10^{2x} - 1$ $\Rightarrow$ $(y-1)\,10^{2x} = -1 - y$ $\Rightarrow$

$$10^{2x} = \frac{-1-y}{y-1} \Rightarrow 2x = \log\left(\frac{1+y}{1-y}\right) \Rightarrow x = \tfrac{1}{2}\log\left(\frac{1+y}{1-y}\right)$$

$\boxed{38}$ $y = \dfrac{10^x + 10^{-x}}{10^x - 10^{-x}}$ $\{ D = \mathbb{R} - \{0\},\ R = (-\infty, -1) \cup (1, \infty) \}$ $\Rightarrow$

$y\,10^x - y\,10^{-x} = 10^x + 10^{-x}$ $\Rightarrow$ $y\,10^{2x} - y = 10^{2x} + 1$ $\Rightarrow$

$$(y-1)\,10^{2x} = y + 1 \Rightarrow 10^{2x} = \frac{y+1}{y-1} \Rightarrow 2x = \log\left(\frac{y+1}{y-1}\right) \Rightarrow x = \tfrac{1}{2}\log\left(\frac{y+1}{y-1}\right)$$

$\boxed{39}$ Use ln instead of log in Exercise 36. $x = \ln(y + \sqrt{y^2 + 1})$

$\boxed{40}$ Use ln instead of log in Exercise 35. $x = \ln(y \pm \sqrt{y^2 - 1})$

$\boxed{41}$ Use ln instead of log in Exercise 38. $x = \frac{1}{2}\ln\left(\dfrac{y+1}{y-1}\right)$

$\boxed{42}$ Use ln instead of log in Exercise 37. $x = \frac{1}{2}\ln\left(\dfrac{1+y}{1-y}\right)$

$\boxed{43}$ $f(x) = \log_2(x+3)$ • $x = 0$ $\Rightarrow$ y-intercept $= \log_2 3 = \dfrac{\log 3}{\log 2} \approx 1.5850$

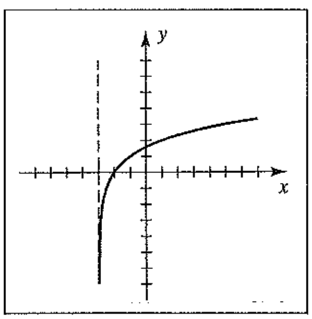

Figure 43

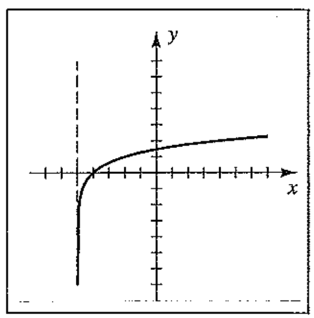

Figure 44

$\boxed{44}$ $f(x) = \log_3(x+5)$ • $x = 0$ $\Rightarrow$ y-intercept $= \log_3 5 = \dfrac{\log 5}{\log 3} \approx 1.4650$

$\boxed{45}$ $f(x) = 4^x - 3$ • $y = 0$ $\Rightarrow$ $4^x = 3$ $\Rightarrow$ x-intercept $= \log_4 3 = \dfrac{\log 3}{\log 4} \approx 0.7925$

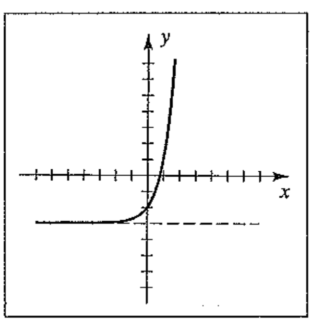

Figure 45

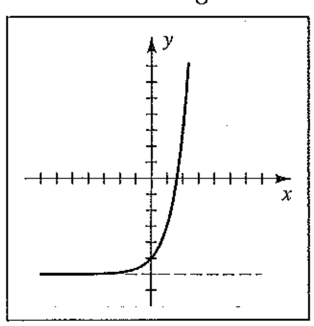

Figure 46

$\boxed{46}$ $f(x) = 3^x - 6$ • $y = 0$ $\Rightarrow$ $3^x = 6$ $\Rightarrow$ x-intercept $= \log_3 6 = \dfrac{\log 6}{\log 3} \approx 1.6309$

[47] (a) vinegar: $\text{pH} \approx -\log(6.3 \times 10^{-3}) = -(\log 6.3 + \log 10^{-3}) = -(\log 6.3 - 3) = 3 - \log 6.3 \approx 2.2$

(b) carrots: $\text{pH} \approx -\log(1.0 \times 10^{-5}) = 5 - \log 1.0 = 5$

(c) sea water: $\text{pH} \approx -\log(5.0 \times 10^{-9}) = 9 - \log 5.0 \approx 8.3$

[48] $\text{pH} = -\log[\text{H}^+] \Rightarrow [\text{H}^+] = 10^{-\text{pH}}$

(a) apples: $[\text{H}^+] = 10^{-3} = 0.001$

(b) beer: $[\text{H}^+] = 10^{-4.2} \approx 0.0000631$

(c) milk: $[\text{H}^+] = 10^{-6.6} \approx 0.00000025$

[49] $[\text{H}^+] < 10^{-7} \Rightarrow \log[\text{H}^+] < \log 10^{-7}$ {since log is an increasing function} $\Rightarrow$

$\log[\text{H}^+] < -7 \Rightarrow -\log[\text{H}^+] > -(-7) \Rightarrow \text{pH} > 7$ for basic solutions;

similarly, $\text{pH} < 7$ for acidic solutions.

[50] $1 < \text{pH} < 14 \Rightarrow 1 < -\log[\text{H}^+] < 14 \Rightarrow -1 > \log[\text{H}^+] > -14 \Rightarrow$

$10^{-1} > 10^{\log[\text{H}^+]} > 10^{-14} \Rightarrow 10^{-14} < [\text{H}^+] < 10^{-1}$

[51] Solving $A = P(1 + \frac{r}{n})^{nt}$ for t with $A = 2P$, $r = 0.06$, and $n = 12$ yields

$2P = P(1 + \frac{0.06}{12})^{12t} \Rightarrow 2 = (1.005)^{12t} \Rightarrow \ln 2 = 12t \ln(1.005) \Rightarrow$

$t = \dfrac{\ln 2}{12 \ln(1.005)} \approx 11.58$ yr, or about 11 years and 7 months.

[52] $A = P(1 + \frac{r}{n})^{nt} \Rightarrow \frac{A}{P} = (1 + \frac{r}{n})^{nt} \Rightarrow \ln\left(\frac{A}{P}\right) = nt \ln(1 + \frac{r}{n}) \Rightarrow t = \dfrac{\ln(A/P)}{n \ln(1 + \frac{r}{n})}$

[53] 50% of the light reaching a depth of 13 meters corresponds to the equation

$\frac{1}{2}I_0 = I_0 c^{13}$. Solving for c, we have $c^{13} = \frac{1}{2} \Rightarrow c = \sqrt[13]{\frac{1}{2}} = 2^{-1/13}$.

Now letting $I = 0.01\, I_0$, $c = 2^{-1/13}$, and using the formula from Example 8(a),

$$x = \frac{\log(I/I_0)}{\log c} = \frac{\log[(0.01\, I_0)/I_0]}{\log 2^{-1/13}} = \frac{\log 10^{-2}}{-\frac{1}{13}\log 2} = \frac{26}{\log 2} \approx 86.4 \text{ m.}$$

[54] Solving as in Exercise 53 with $x = 10$ cm $= 0.1$ m, $\frac{1}{2}I_0 = I_0 c^{0.1} \Rightarrow c^{1/10} = \frac{1}{2} \Rightarrow$

$c = \frac{1}{1024}$. $x = \dfrac{\log[(0.01\, I_0)/I_0]}{\log(\frac{1}{1024})} = \dfrac{-2}{-\log 1024} \approx 0.664$ m.

[55] (a) A is an increasing function that passes through $(0, 0)$, $(5, \approx 41)$, and $(10, \approx 65)$.

(b) $A = 50 \Rightarrow 50 = 100[1 - (0.9)^t] \Rightarrow$

$1 - (0.9)^t = 0.5 \Rightarrow (0.9)^t = 0.5 \Rightarrow$

$t = \log_{0.9}(0.5) = \dfrac{\log 0.5}{\log 0.9} \approx 6.58$ min.

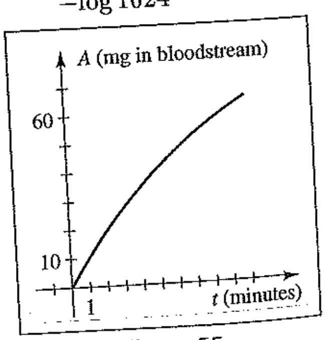

Figure 55

[56] (a) $A(t) = 2 \Rightarrow 2 = 10(0.8)^t \Rightarrow 0.2 = (0.8)^t \Rightarrow$

$t = \log_{0.8}(0.2) = \dfrac{\log 0.2}{\log 0.8} \approx 7.21$ hr

(b) $A(t)=\frac{1}{2}(10) \Rightarrow \frac{1}{2}(10)=10(0.8)^t \Rightarrow t=\log_{0.8}(0.5)=\dfrac{\log 0.5}{\log 0.8}\approx 3.11$ hr

$\boxed{57}$ (a) $F=F_0(1-m)^t \Rightarrow$

$$(1-m)^t=\frac{F}{F_0} \Rightarrow t\log(1-m)=\log\left(\frac{F}{F_0}\right) \Rightarrow t=\frac{\log(F/F_0)}{\log(1-m)}$$

(b) Using part (a) with $F=\frac{1}{2}F_0$ and $m=0.00005$,

$$t=\frac{\log\frac{1}{2}}{\log 0.99995}\approx 13{,}863 \text{ generations.}$$

$\boxed{58}$ (a) $f(5)=3+20\left[1-e^{-0.1(5)}\right]\approx 10.87\approx 11$;

$f(9)\approx 15$; $f(24)\approx 21$; $f(30)\approx 22$.

(b) The graph of $f(n)=3+20(1-e^{-0.1n})$ is shown in *Figure 58*.

(c) As $n\to\infty$, $e^{-0.1n}\to 0$, and $f(n)\to 23$.

Thus, there is a horizontal asymptote of $y=23$.

[0, 36, 2] by [0, 24, 2]

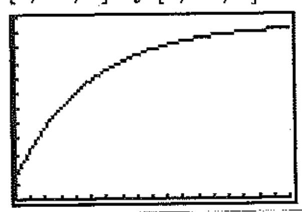

Figure 58

$\boxed{59}$ (a) $h=\dfrac{120}{1+200e^{-0.2t}}$ and $t=10 \Rightarrow h=\dfrac{120}{1+200e^{-2}}\approx 4.28$ ft

(b) $50=\dfrac{120}{1+200e^{-0.2t}} \Rightarrow 1+200e^{-0.2t}=\frac{120}{50} \Rightarrow e^{-0.2t}=(\frac{120}{50}-1)\cdot\frac{1}{200} \Rightarrow$

$$e^{-0.2t}=0.007 \Rightarrow -0.2t=\ln 0.007 \Rightarrow t=\frac{\ln 0.007}{-0.2}\approx 24.8 \text{ yr}$$

$\boxed{60}$ (a) $T(2)=T_1 2^{-k}=(0.80)T_1 \Rightarrow 2^{-k}=0.8 \Rightarrow -k=\log_2 0.8 \Rightarrow$

$$k=-\log_2 0.8\approx 0.32$$

(b) $T(4)=T_1(4)^{-k}=T_1(2^2)^{-k}=T_1(2^{-k})^2=T_1(2^{\log_2 0.8})^2=T_1(0.8)^2=(0.64)T_1$

(c) $T(2n)=T_1(2n)^{-k}=2^{-k}(T_1 n^{-k})=2^{\log_2 0.8}T(n)=(0.80)T(n)$

$\boxed{61}$ $\dfrac{v_0}{v_1}=\left(\dfrac{h_0}{h_1}\right)^P \Rightarrow \ln\dfrac{v_0}{v_1}=P\ln\dfrac{h_0}{h_1} \Rightarrow P=\dfrac{\ln(v_0/v_1)}{\ln(h_0/h_1)}=\dfrac{\ln(25/6)}{\ln(200/35)}\approx 0.82$

$\boxed{62}$ Without loss of generality, let $h_0=200$, $h_1=35$ and $v_0>v_1$. Then $\dfrac{v_0}{v_1}>1$ and $\ln\dfrac{v_0}{v_1}$ is positive. Since $P=\dfrac{\ln(v_0/v_1)}{\ln(h_0/h_1)}\approx 0.57\ln\dfrac{v_0}{v_1}$ and $y=\ln x$ is an increasing function, P increases when the ratio $\dfrac{v_0}{v_1}$ increases. Since $v_0>v_1>1$, if $\dfrac{v_0}{v_1}$ increases, then v_0-v_1 must also increase. Thus, $s=\dfrac{v_1-v_0}{h_1-h_0}=\dfrac{v_0-v_1}{165}$ must increase.

Increasing values for P correspond with larger values for s.

$\boxed{63}$ If $y=c2^{kx}$ and $x=0$, then $y=c2^0=c=4$. Thus, $y=4(2)^{kx}$. Similarly, $x=1 \Rightarrow y=4(2)^k=3.249 \Rightarrow 2^k=\frac{3.249}{4} \Rightarrow k=\log_2(\frac{3.249}{4})\approx -0.300$. Thus, $y=4(2)^{-0.3x}$. Checking the remaining two points, we see that $x=2 \Rightarrow y\approx 2.639$ and $x=3 \Rightarrow y\approx 2.144$. The four points lie on the graph of $y=4(2)^{-0.3x}$ to within three-decimal-place accuracy.

[64] If $y = c2^{kx}$ and $x = 0$, then $y = c2^0 = c = -0.3$. Thus, $y = -0.3(2)^{kx}$. Similarly, $x = 1 \Rightarrow y = -0.3(2)^k = -0.397 \Rightarrow 2^k = \frac{-0.397}{-0.3} \Rightarrow k = \log_2\left(\frac{-0.397}{-0.3}\right) \approx 0.404$. Thus, $y = -0.3(2)^{0.404x}$. Checking $x = 2$ gives us $y \approx -0.525$, not -0.727. The points do not lie on the graph of $y = c2^{kx}$ to within three-decimal-place accuracy.

[65] If $y = c\log(kx + 10)$ and $x = 0$, then $y = c\log 10 = c = 1.5$. Thus, $y = 1.5\log(kx + 10)$. Similarly, $x = 1 \Rightarrow y = 1.5\log(k + 10) = 1.619 \Rightarrow \frac{1.619}{1.5} = \log(k + 10) \Rightarrow k + 10 = 10^{1.619/1.5} \Rightarrow k = 10^{1.619/1.5} - 10 \approx 2.004$. Thus, $y = 1.5\log(2.004x + 10)$. Checking the remaining two points, we see that $x = 2 \Rightarrow y \approx 1.720$, and $x = 3 \Rightarrow y \approx 1.807$, not 1.997. The points do not lie on the graph of $y = c\log(kx + 10)$ to within three-decimal-place accuracy.

[66] If $y = c\log(kx + 10)$ and $x = 0$, then $y = c\log 10 = c = 0.7$. Thus, $y = 0.7\log(kx + 10)$. Similarly, $x = 1 \Rightarrow y = 0.7\log(k + 10) = 0.782 \Rightarrow k + 10 = 10^{0.782/0.7} \Rightarrow k = 10^{0.782/0.7} - 10 \approx 3.096$. Thus, $y = 0.7\log(3.096x + 10)$. Checking the remaining three points, we see that $x = 2 \Rightarrow y \approx 0.847$, $x = 3 \Rightarrow y \approx 0.900$, and $x = 4 \Rightarrow y \approx 0.945$. The points lie on the graph of $y = 0.7\log(3.096x + 10)$ to within three-decimal-place accuracy.

[67] $h(5.3) = \log_4 5.3 - 2\log_8(1.2 \times 5.3) = \frac{\ln 5.3}{\ln 4} - \frac{2\ln 6.36}{\ln 8} \approx -0.5764$

[68] $h(52.6) = 3\log_3(2 \times 52.6 - 1) + 7\log_2(52.6 + 0.2) = \frac{3\ln 104.2}{\ln 3} + \frac{7\ln 52.8}{\ln 2} \approx 52.7450$

[69] The minimum point on the graph of $y = x - \ln(0.3x) - 3\log_3 x$ is about (3.73, 0.023). So there are *no* x-intercepts, and hence, no roots of the equation on (0, 9).

[0, 9] by [−1, 5]

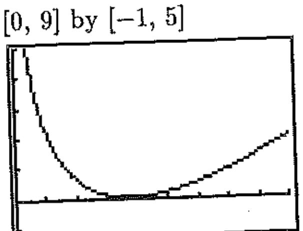

Figure 69

[0, 3] by [−1, 1]

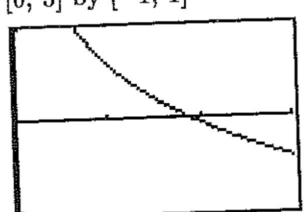

Figure 70

[70] From the graph of $y = 2\log 2x - \log_3 x^2$, we see that the root of the equation on (0, 3) is $x \approx 1.88$.

$\boxed{71}$ The graphs of $f(x) = x$ and $g(x) = 3\log_2 x$ intersect at approximately (1.37, 1.37) and (9.94, 9.94). Hence, the solutions of the equation $f(x) = g(x)$ are 1.37 and 9.94.

[−1, 17] by [−1, 11]

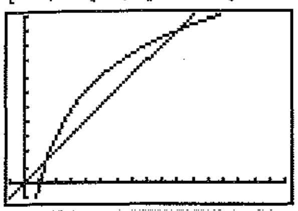

Figure 71

[−1, 2] by [−1, 1]

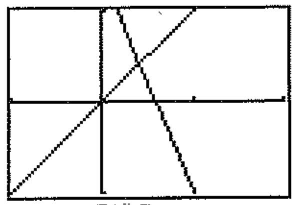

Figure 72

$\boxed{72}$ The graphs of $f(x) = x$ and $g(x) = -x^2 - \log_5 x$ intersect at approximately (0.40, 0.40). Hence, the solution of the equation $f(x) = g(x)$ is 0.40.

$\boxed{73}$ From the graph, we see that the graphs of $f(x) = 3^{-x} - 4^{0.2x}$ and $g(x) = \ln(1.2) - x$ intersect at three points. Their coordinates are approximately (−0.32, 0.50), (1.52, −1.33), and (6.84, −6.65). The region near the origin in *Figure 73(a)* is enhanced in *Figure 73(b)*. Thus, $f(x) > g(x)$ on $(-\infty, -0.32)$ and (1.52, 6.84).

[−5, 10] by [−8, 2]

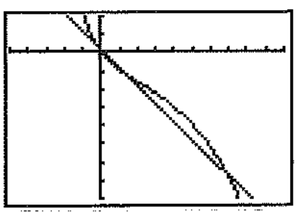

Figure 73(a)

[−1.53, 2.26] by [−2.92, 1.05]

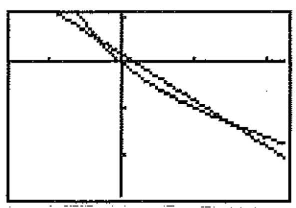

Figure 73(b)

$\boxed{74}$ From the figure, we see that the graphs of $f(x) = 3\log_4 x - \log x$ and $g(x) = e^x - 0.25x^4$ intersect at two points. Their coordinates are approximately (2.68, 1.70) and (5.30, 2.88). Thus, $f(x) > g(x)$ on (2.68, 5.30).

[−8, 19, 2] by [−13, 5, 2]

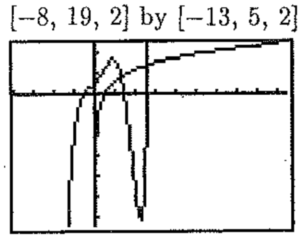

Figure 74

$\boxed{75}$ (1) The graph of $n(t) = 85e^{t/3}$ is increasing rapidly and soon becomes greater than 100. It is doubtful that the average score would improve dramatically without any review.

[0, 5] by [0, 200, 20]

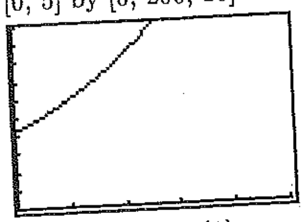

Figure 75(1)

[0, 5] by [0, 100, 10]

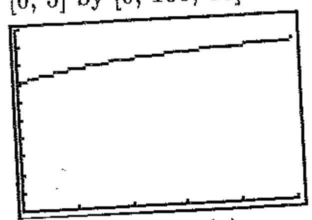

Figure 75(2)

(2) The graph of $n(t) = 70 + 10\ln(t+1)$ is also increasing. It is incorrect because $n(0) = 70 \neq 85$. Also, one would not expect the average score to improve without any review.

(3) The graph of $n(t) = 86 - e^t$ decreases rapidly to zero. The *average* score probably would not be zero after 5 weeks.

[0, 5] by [0, 100, 10]

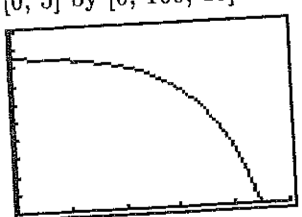

Figure 75(3)

[0, 5] by [0, 100, 10]

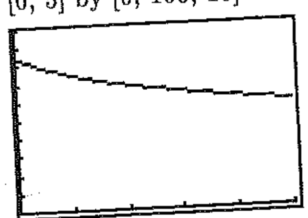

Figure 75(4)

(4) The graph of $n(t) = 85 - 15\ln(t+1)$ is decreasing. During the first weeks it decreases most rapidly and then starts to level off. Of the four functions, **this function seems to best model the situation.**

$\boxed{76}$ (1) $T(t) = 212 - 50t$ decreases at a constant rate of 50°F each hour. Initially the temperature is 212°F, but after 3 hours the temperature of the water is 62°F, which is cooler than the room.

[0, 5] by [0, 250, 50]

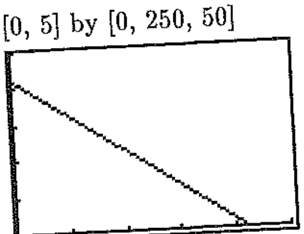

Figure 76(1)

[0, 5] by [0, 250, 50]

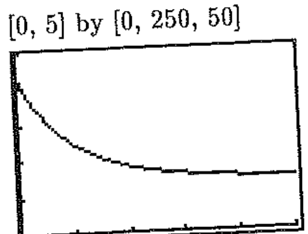

Figure 76(2)

(2) $T(t) = 140e^{-t} + 72$ is initially 212°F, cools rapidly at first and then gradually approaches a temperature of 72°F. This equation best models the situation.

(3) $T(t) = 212e^{-t}$ is initially 212°F, but decreases rapidly to temperatures cooler than 72°F.

[0, 5] by [0, 250, 50]

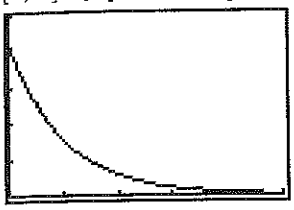

Figure 76(3)

[0, 5] by [0, 250, 50]

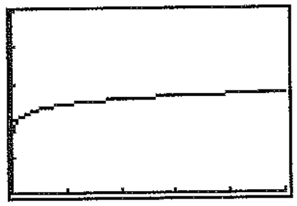

Figure 76(4)

(4) $T(t) = 72 + 10\ln(140t + 1)$ is initially 72°F and is an *increasing* function.

77 (a) The ozone level is decreasing by 11% per year. The fraction of ozone remaining x years after April 1992 is given by the function $f(x) = (0.89)^x$. We must approximate x when $f(x) = 0.5$. Using a table, $f(x) = 0.5$ when $x \approx 6$. Thus, in 1998 the ozone level would be 50% of its normal level.

x	1	2	3	4	5	6	7
$f(x)$	0.89	0.79	0.70	0.63	0.56	0.50	0.44

(b) $(0.89)^t = 0.5 \Rightarrow \ln(0.89)^t = \ln 0.5 \Rightarrow t\ln 0.89 = \ln 0.5 \Rightarrow$

$$t = \frac{\ln 0.5}{\ln 0.89} \approx 5.948, \text{ or in 1998.}$$

78 (a) Let $f(t)$ be the increased likelihood of skin cancer. f is increasing by a factor of 1.07 each year. Thus, we must solve $f(t) = (1.07)^t = 2$. Graph $Y_1 = (1.07)^t$ and $Y_2 = 2$. The point of intersection occurs at $x \approx 10.2448 \approx 10.2$ years.

[0, 15, 5] by [0, 3]

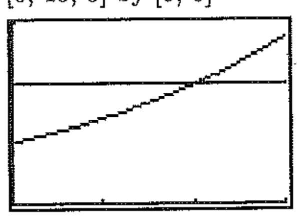

Figure 78

(b) $(1.07)^t = 2 \Rightarrow \ln(1.07)^t = \ln 2 \Rightarrow t\ln 1.07 = \ln 2 \Rightarrow$

$$t = \frac{\ln 2}{\ln 1.07} \approx 10.2448 \approx 10.2 \text{ yr.}$$

Chapter 5 Review Exercises

1 Suppose $f(a) = f(b)$. $2a^3 - 5 = 2b^3 - 5 \Rightarrow 2a^3 = 2b^3 \Rightarrow a^3 = b^3 \Rightarrow a = b$.

Thus, f is a one-to-one function.

2 Reflect the graph of $y = f(x)$ through the line $y = x$ to obtain the graph of $y = f^{-1}(x)$. See *Figure 2* on the next page.

3 $f(x) = 10 - 15x \Rightarrow 15x = 10 - y \Rightarrow x = \dfrac{10-y}{15} \Rightarrow f^{-1}(x) = \dfrac{10-x}{15}$

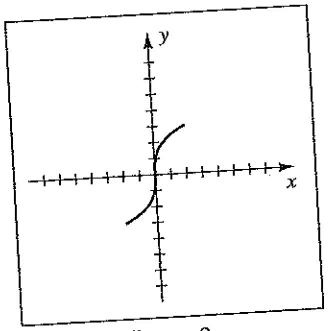

Figure 2

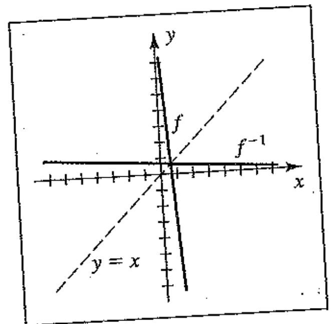

Figure 3

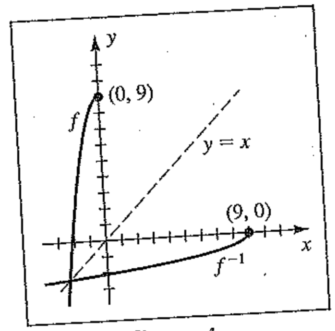

Figure 4

4 $f(x) = 9 - 2x^2,\ x \leq 0 \Rightarrow y + 2x^2 = 9 \Rightarrow$

$x^2 = \dfrac{9-y}{2} \Rightarrow x = \pm\sqrt{\dfrac{9-y}{2}}$ {choose minus since $x \leq 0$} $\Rightarrow f^{-1}(x) = -\sqrt{\dfrac{9-x}{2}}$

5 (a) $f(1) = 2$ (b) $(f \circ f)(1) = f(f(1)) = f(2) = 4$

(c) $f(2) = 4$ and f is one-to-one $\Rightarrow f^{-1}(4) = 2$

(d) $f(x) = 4 \Rightarrow x = 2$ (e) $f(x) > 4 \Rightarrow x > 2$

6 Since f and g are one-to-one functions, we know that $f(2) = 7$, $f(4) = 2$, and $g(2) = 5$ imply that $f^{-1}(7) = 2$, $f^{-1}(2) = 4$, and $g^{-1}(5) = 2$, respectively.

(a) $(g \circ f^{-1})(7) = g(f^{-1}(7)) = g(2) = 5$ (b) $(f \circ g^{-1})(5) = f(g^{-1}(5)) = f(2) = 7$

(c) $(f^{-1} \circ g^{-1})(5) = f^{-1}(g^{-1}(5)) = f^{-1}(2) = 4$

(d) $(g^{-1} \circ f^{-1})(2) = g^{-1}(f^{-1}(2)) = g^{-1}(4)$, which is not known.

7 $f(x) = 3^{x+2}$ • shift $y = 3^x$ left 2 units

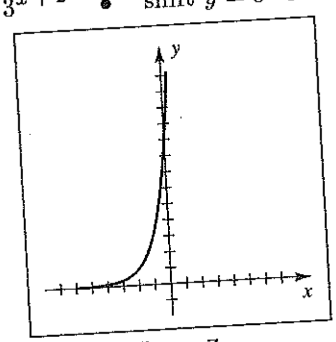

Figure 7

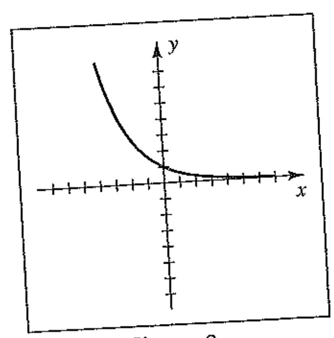

Figure 8

8 $f(x) = (\frac{3}{5})^x$ • goes through $(-1, \frac{5}{3})$, $(0, 1)$, and $(1, \frac{3}{5})$

[9] $f(x) = (\frac{3}{2})^{-x} = (\frac{2}{3})^x$ • goes through $(-1, \frac{3}{2})$, $(0, 1)$, and $(1, \frac{2}{3})$

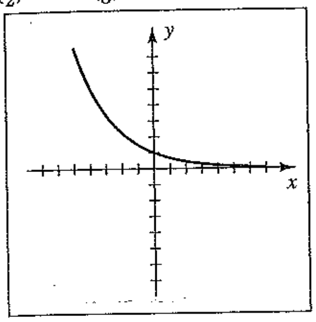

Figure 9

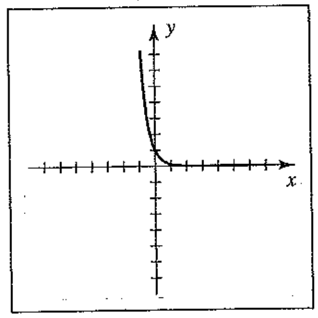

Figure 10

[10] $f(x) = 3^{-2x} = (3^{-2})^x = (\frac{1}{9})^x$ • goes through $(-1, 9)$, $(0, 1)$, and $(1, \frac{1}{9})$

Or: reflect $y = 3^x$ through the y-axis and horizontally compress it by a factor of 2

[11] $f(x) = 3^{-x^2} = (\frac{1}{3})^{x^2}$ • see the note in §5.2 before Exercise 21

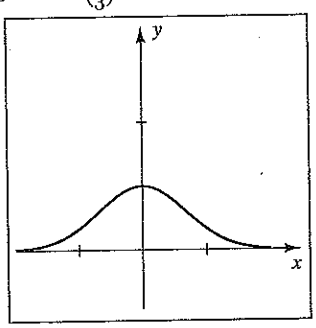

Figure 11

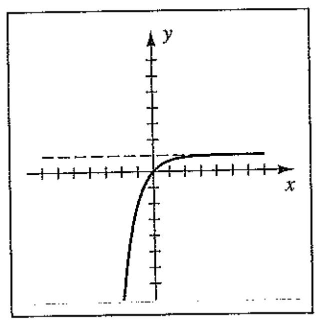

Figure 12

[12] $f(x) = 1 - 3^{-x} = -(\frac{1}{3})^x + 1$ •

reflect $y = (\frac{1}{3})^x$ through the x-axis and shift it up 1 unit

[13] $f(x) = e^{x/2} = (e^{1/2})^x \approx (1.65)^x$ • goes through $(-1, 1/\sqrt{e})$, $(0, 1)$, and $(1, \sqrt{e})$;

or approximately $(-1, 0.61)$, $(0, 1)$, and $(1, 1.65)$

Or: horizontally stretch $y = e^x$ by a factor of 2

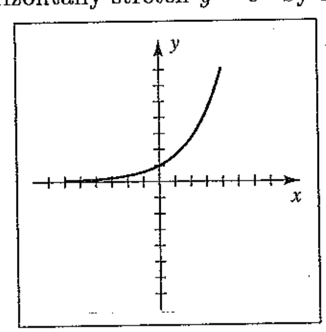

Figure 13

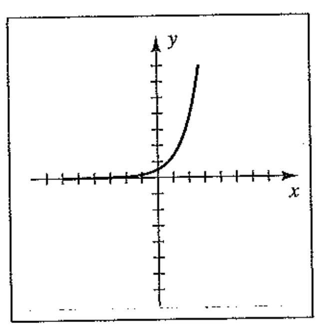

Figure 14

[14] $f(x) = \frac{1}{2}e^x$ • vertically compress $y = e^x$ by a factor of 2

[15] $f(x) = e^{x-2}$ • shift $y = e^x$ right 2 units

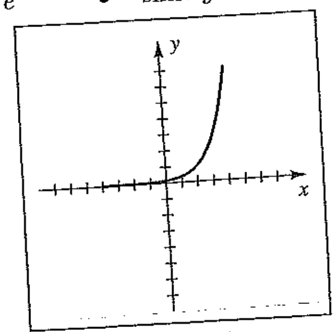

Figure 15

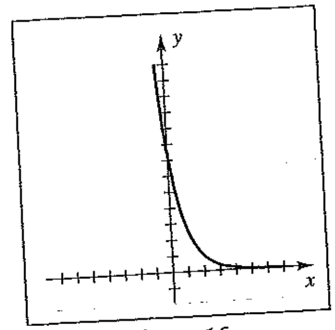

Figure 16

[16] $f(x) = e^{2-x} = e^{-(x-2)} = (\frac{1}{e})^{x-2}$ • the graph of $y = (\frac{1}{e})^x$ is the graph of $y = e^x$ reflected through the y-axis; shift it right 2 units

[17] $f(x) = \log_6 x$ • goes through $(\frac{1}{6}, -1)$, $(1, 0)$, and $(6, 1)$

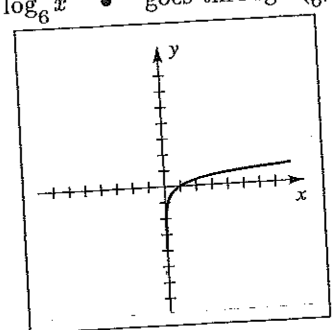

Figure 17

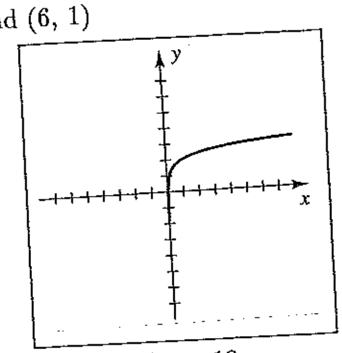

Figure 18

[18] $f(x) = \log_6(36x) = \log_6 36 + \log_6 x = \log_6 6^2 + \log_6 x = \log_6 x + 2$ • shift $y = \log_6 x$ up 2 units

[19] $f(x) = \log_4(x^2) = 2\log_4 x$ • stretch $y = \log_4 x$ by a factor of 2 and include its reflection through the y-axis since the domain of the original function is $\mathbb{R} - \{0\}$

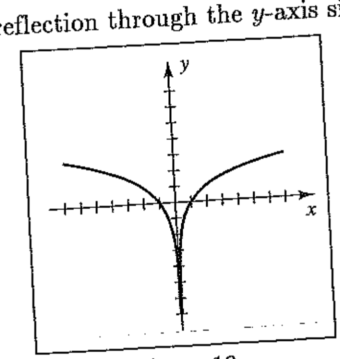

Figure 19

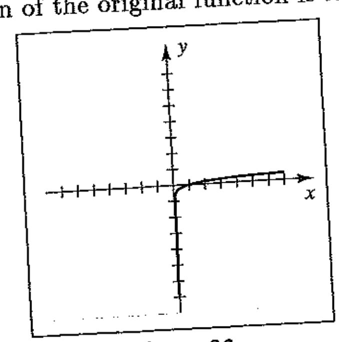

Figure 20

[20] $f(x) = \log_4 \sqrt[3]{x} = \log_4 x^{1/3} = \frac{1}{3}\log_4 x$ • vertically compress $y = \log_4 x$ by a factor of 3

[21] $f(x) = \log_2(x+4)$ • shift $y = \log_2 x$ left 4 units

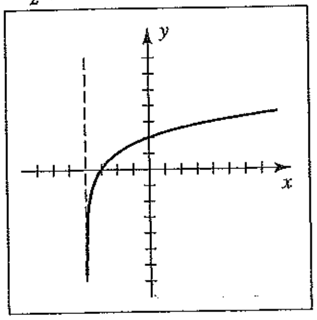

Figure 21

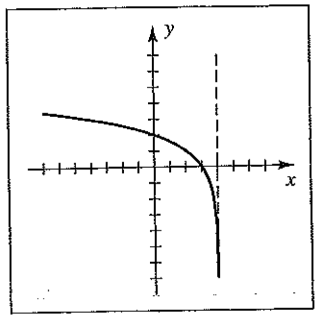

Figure 22

[22] $f(x) = \log_2(4-x) = \log_2[-(x-4)]$ •

shift $y = \log_2 x$ four units right and reflect through the line $x = 4$

[23] (a) $\log_2 \frac{1}{16} = \log_2 2^{-4} = -4$ (b) $\log_\pi 1 = 0$ (c) $\ln e = 1$

(d) $6^{\log_6 4} = 4$ (e) $\log 1{,}000{,}000 = \log 10^6 = 6$

(f) $10^{3\log 2} = 10^{\log 2^3} = 2^3 = 8$ (g) $\log_4 2 = \log_4 4^{1/2} = \frac{1}{2}$

[24] (a) $\log_5 \sqrt[3]{5} = \log_5 5^{1/3} = \frac{1}{3}$ (b) $\log_5 1 = 0$ (c) $\log 10 = 1$

(d) $e^{\ln 5} = 5$ (e) $\log\log 10^{10} = \log(\log 10^{10}) = \log(10) = 1$

(f) $e^{2\ln 5} = e^{\ln 5^2} = 5^2 = 25$ (g) $\log_{27} 3 = \log_{27} 27^{1/3} = \frac{1}{3}$

[25] $2^{3x-1} = \frac{1}{2} \Rightarrow 2^{3x-1} = 2^{-1} \Rightarrow 3x - 1 = -1 \Rightarrow 3x = 0 \Rightarrow x = 0$

[26] $8^{2x} \cdot (\frac{1}{4})^{x-2} = 4^{-x} \cdot (\frac{1}{2})^{2-x} \Rightarrow (2^3)^{2x} \cdot (2^{-2})^{x-2} = (2^2)^{-x} \cdot (2^{-1})^{2-x} \Rightarrow$

$2^{6x} \cdot 2^{-2x+4} = 2^{-2x} \cdot 2^{-2+x} \Rightarrow 2^{4x+4} = 2^{-2-x} \Rightarrow 4x + 4 = -2 - x \Rightarrow$

$5x = -6 \Rightarrow x = -\frac{6}{5}$

[27] $\log\sqrt{x} = \log(x-6) \Rightarrow \sqrt{x} = x - 6 \Rightarrow x = x^2 - 12x + 36 \Rightarrow$

$x^2 - 13x + 36 = 0 \Rightarrow (x-4)(x-9) = 0 \Rightarrow x = 4, 9$; 4 is extraneous

[28] $\log_8(x-5) = \frac{2}{3} \Rightarrow x - 5 = 8^{2/3} = 4 \Rightarrow x = 9$

[29] $\log_4(x+1) = 2 + \log_4(3x-2) \Rightarrow \log_4(x+1) - \log_4(3x-2) = 2 \Rightarrow$

$$\log_4\left(\frac{x+1}{3x-2}\right) = 2 \Rightarrow \frac{x+1}{3x-2} = 16 \Rightarrow x + 1 = 48x - 32 \Rightarrow x = \frac{33}{47}$$

[30] $2\ln(x+3) - \ln(x+1) = 3\ln 2 \Rightarrow \ln\frac{(x+3)^2}{x+1} = \ln 2^3 \Rightarrow (x+3)^2 = 8(x+1) \Rightarrow$

$x^2 + 6x + 9 = 8x + 8 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1$

[31] $\ln(x+2) = \ln e^{\ln 2} - \ln x \Rightarrow \ln(x+2) + \ln x = \ln 2 \Rightarrow \ln(x^2 + 2x) = \ln 2 \Rightarrow$

$x^2 + 2x = 2 \Rightarrow x^2 + 2x - 2 = 0 \Rightarrow x = (-2 \pm \sqrt{12})/2 = -1 + \sqrt{3}\ \{x > 0\}$.

[32] $\log\sqrt[4]{x+1} = \frac{1}{2} \Rightarrow (x+1)^{1/4} = 10^{1/2} \Rightarrow x + 1 = 10^2 \Rightarrow x = 99$

[33] $2^{5-x} = 6 \Rightarrow 5 - x = \log_2 6 \Rightarrow x = 5 - \frac{\log 6}{\log 2}$, or $\frac{\log\frac{16}{3}}{\log 2}$

$\boxed{34}$ $3^{(x^2)} = 7 \Rightarrow x^2 = \log_3 7 \Rightarrow x^2 = \dfrac{\log 7}{\log 3} \Rightarrow x = \pm\sqrt{\dfrac{\log 7}{\log 3}}$

$\boxed{35}$ $2^{5x+3} = 3^{2x+1} \Rightarrow (5x+3)\log 2 = (2x+1)\log 3 \Rightarrow$

$5x \log 2 + 3\log 2 = 2x \log 3 + \log 3 \Rightarrow x(5\log 2 - 2\log 3) = \log 3 - 3\log 2 \Rightarrow$

$x = \dfrac{\log 3 - \log 8}{\log 32 - \log 9} = \dfrac{\log\frac{3}{8}}{\log\frac{32}{9}}$

$\boxed{36}$ $\log_3(3x) = \log_3 x + \log_3(4-x) \Rightarrow \log_3(3x) = \log_3[x(4-x)] \Rightarrow$

$3x = 4x - x^2 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0,\ 1;$ 0 is extraneous

$\boxed{37}$ $\log_4 x = \sqrt[3]{\log_4 x} \Rightarrow \log_4 x = (\log_4 x)^{1/3} \Rightarrow (\log_4 x)^3 = \log_4 x \Rightarrow$

$\log_4 x\left[(\log_4 x)^2 - 1\right] = 0 \Rightarrow \log_4 x = 0$ or $\log_4 x = \pm 1 \Rightarrow x = 1$ or $x = 4, \frac{1}{4}$

$\boxed{38}$ $e^{x+\ln 4} = 3e^x \Rightarrow e^x e^{\ln 4} = 3e^x \Rightarrow 4e^x = 3e^x \Rightarrow e^x = 0$, which is never true.

There is no real solution.

$\boxed{39}$ $10^{2\log x} = 5 \Rightarrow 10^{\log x^2} = 5 \Rightarrow x^2 = 5 \Rightarrow x = \pm\sqrt{5};\ -\sqrt{5}$ is extraneous

$\boxed{40}$ $e^{\ln(x+1)} = 3 \Rightarrow x + 1 = 3 \Rightarrow x = 2$

$\boxed{41}$ $x^2(-2xe^{-x^2}) + 2xe^{-x^2} = 0 \Rightarrow 2xe^{-x^2}(-x^2+1) = 0 \Rightarrow x = 0,\ \pm 1\ \left\{e^{-x^2} \neq 0\right\}$

$\boxed{42}$ $e^x + 2 = 8e^{-x} \Rightarrow e^{2x} + 2e^x - 8 = 0 \Rightarrow (e^x+4)(e^x-2) = 0 \Rightarrow e^x = -4,\ 2 \Rightarrow$

$x = \ln 2$ since $e^x \neq -4$

$\boxed{43}$ (a) $\log x^2 = \log(6-x) \Rightarrow x^2 = 6 - x \Rightarrow x^2 + x - 6 = 0 \Rightarrow$

$(x+3)(x-2) = 0 \Rightarrow x = -3,\ 2$

(b) $2\log x = \log(6-x) \Rightarrow \log x^2 = \log(6-x)$, which is the equation in part (a).

This equation has the same solutions provided they are in the domain.

But -3 is extraneous, so 2 is the only solution.

$\boxed{44}$ (a) $\ln(e^x)^2 = 16 \Rightarrow 2\ln e^x = 16 \Rightarrow 2x = 16 \Rightarrow x = 8$

(b) $\ln e^{(x^2)} = 16 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\boxed{45}$ $\log x^4 \sqrt[3]{y^2/z} = \log(x^4 y^{2/3} z^{-1/3}) = \log x^4 + \log y^{2/3} + \log z^{-1/3} =$

$4\log x + \frac{2}{3}\log y - \frac{1}{3}\log z$

$\boxed{46}$ $\log(x^2/y^3) + 4\log y - 6\log\sqrt{xy} = \log\left(\dfrac{x^2}{y^3}\right) + \log y^4 - \log(x^3y^3)$

$= \log\left(\dfrac{x^2 y}{x^3 y^3}\right) = \log\left(\dfrac{1}{xy^2}\right) = -\log(xy^2)$

$\boxed{47}$ We can assume that the graph has a horizontal asymptote of $y = 0$, and use the form $f(x) = ba^x$. The point $(0, 6)$ is on the graph, so $f(0) = 6 \Rightarrow 6 = ba^0 \Rightarrow$ $6 = b(1) \Rightarrow 6 = b$, and hence, $f(x) = 6a^x$. Since the point $P(1, 8)$ is on the graph, $f(1) = 8$. Thus, $8 = 6a^1 \Rightarrow a = \frac{4}{3}$, and the function is $f(x) = 6(\frac{4}{3})^x$.

[48] $f(x) = \log_3(x+2)$ • $x = 0 \Rightarrow$

y-intercept $= \log_3 2 = \dfrac{\log 2}{\log 3} \approx 0.6309$

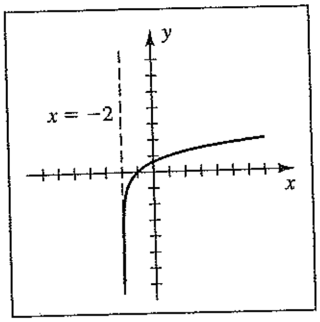

Figure 48

[49] $y = \dfrac{1}{10^x + 10^{-x}} \left\{ D = \mathbb{R},\ R = (0, \tfrac{1}{2}] \right\} \Rightarrow y\ 10^x + y\ 10^{-x} = 1 \Rightarrow$

$y\ 10^{2x} + y = 10^x \Rightarrow y\ 10^{2x} - 10^x + y = 0 \Rightarrow 10^x = \dfrac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow$

$$x = \log\left(\frac{1 \pm \sqrt{1-4y^2}}{2y}\right)$$

[50] $y = \dfrac{1}{10^x - 10^{-x}} \{ D = R = \mathbb{R} - \{0\} \} \Rightarrow y\ 10^x - y\ 10^{-x} = 1 \Rightarrow$

$y\ 10^{2x} - y = 10^x \Rightarrow y\ 10^{2x} - 10^x - y = 0 \Rightarrow 10^x = \dfrac{1 \pm \sqrt{1+4y^2}}{2y}$.

There are 2 cases to consider.

(1) If $y < 0$, then $\dfrac{1 - \sqrt{1+4y^2}}{2y} > 0$, so $x = \log\left(\dfrac{1 - \sqrt{1+4y^2}}{2y}\right)$.

(2) If $y > 0$, then $\dfrac{1 + \sqrt{1+4y^2}}{2y} > 0$, so $x = \log\left(\dfrac{1 + \sqrt{1+4y^2}}{2y}\right)$.

[51] (a) $x = \ln 6.6 \approx 1.89$ (b) $\log x = 1.8938 \Rightarrow x = 10^{1.8938} \approx 78.3$

(c) $\ln x = -0.75 \Rightarrow x = e^{-0.75} \approx 0.472$

[52] (a) $x = \log 8.4 \approx 0.924$ (b) $\log x = -2.4260 \Rightarrow x = 10^{-2.4260} \approx 0.00375$

(c) $\ln x = 1.8 \Rightarrow x = e^{1.8} \approx 6.05$

[53] (a) For $y = \log_2(x+1)$, $D = (-1, \infty)$ and $R = \mathbb{R}$.

(b) $y = \log_2(x+1) \Rightarrow x = \log_2(y+1) \Rightarrow 2^x = y+1 \Rightarrow y = 2^x - 1$,

$D = \mathbb{R},\ R = (-1, \infty)$

[54] (a) For $y = 2^{3-x} - 2$, $D = \mathbb{R}$ and $R = (-2, \infty)$.

(b) $y = 2^{3-x} - 2 \Rightarrow x = 2^{3-y} - 2 \Rightarrow x + 2 = 2^{3-y} \Rightarrow$

$\log_2(x+2) = 3 - y \Rightarrow y = 3 - \log_2(x+2),\ D = (-2, \infty),\ R = \mathbb{R}$

[55] (a) $Q(t) = 2(3^t)$, so $Q(0) = 2(3^0) = 2$ {in thousands}, or 2000.

(b) $Q(\frac{10}{60}) = 2000(3^{1/6}) \approx 2401$; $Q(\frac{30}{60}) = 2000(3^{1/2}) \approx 3464$; $Q(1) = 2(3) = 6000$

[56] $A = P\left(1+\frac{r}{n}\right)^{nt} = 1000\left(1+\frac{0.12}{4}\right)^{4\cdot 1} \approx \$1125.51$

[57] (a) $N = 64(0.5)^{t/8} = 64\left[(0.5)^{1/8}\right]^t \approx 64(0.917)^t$

(b) $N = \frac{1}{2}N_0 \Rightarrow \frac{1}{2}N_0 = N_0(0.5)^{t/8} \Rightarrow$

$\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^{t/8} \Rightarrow 1 = t/8 \Rightarrow t = 8$ days

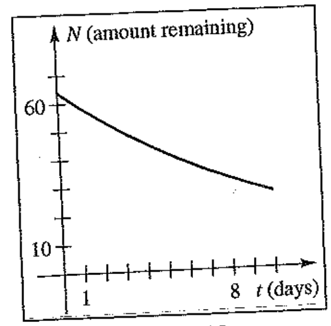

Figure 57

[58] $t = 0$ and $N = 1000 \Rightarrow N_0 = 1000$. $t = 3$ and $N = 600 \Rightarrow 600 = 1000(a^c)^3 \Rightarrow$

$(a^c)^3 = \frac{600}{1000} \Rightarrow a^c = \left(\frac{3}{5}\right)^{1/3} \Rightarrow N = 1000\left(\frac{3}{5}\right)^{t/3}$, with $a = \frac{3}{5}$ and $c = \frac{1}{3}$.

[59] (a) $A = Pe^{rt} \Rightarrow 35{,}000 = 10{,}000e^{0.11t} \Rightarrow e^{0.11t} = 3.5 \Rightarrow 0.11t = \ln 3.5 \Rightarrow$

$t \approx 11.39$ yr

(b) $2\cdot 10{,}000 = 10{,}000e^{0.11t} \Rightarrow e^{0.11t} = 2 \Rightarrow 0.11t = \ln 2 \Rightarrow t \approx 6.30$ yr.

Alternatively, using the doubling time formula, $t = (\ln 2)/r = (\ln 2)/0.11 \approx 6.30$.

[60] The model is $y = 4000b^t$, with $t = 0$ corresponding to 1790. Use $t = 200$ and

$y = 2{,}000{,}000$: $2{,}000{,}000 = 4000b^{200} \Rightarrow 500 = b^{200} \Rightarrow b = \sqrt[200]{500} \approx 1.03156$,

so the annual interest rate is about 3.16%.

[61] $I(t) = 1\%$ of $I_0 \Rightarrow \frac{1}{100}I_0 = I_0e^{-Rt/L} \Rightarrow \frac{1}{100} = e^{-Rt/L} \Rightarrow \ln\frac{1}{100} = -\frac{Rt}{L} \Rightarrow$

$t = (-\ln 100)\left(-\frac{L}{R}\right) = (\ln 100)\frac{L}{R} \approx 4.6\frac{L}{R}$

[62] (a) $\alpha = 10\log\left(\frac{I}{I_0}\right) \Rightarrow \frac{\alpha}{10} = \log\left(\frac{I}{I_0}\right) \Rightarrow 10^{\alpha/10} = \frac{I}{I_0} \Rightarrow I = I_0 10^{\alpha/10}$

(b) Let $I(\alpha)$ be the intensity corresponding to α decibels.

$I(\alpha+1) = I_0 10^{(\alpha+1)/10} = I_0 10^{\alpha/10}10^{1/10} = I(\alpha)\,10^{1/10} \approx 1.26\,I(\alpha)$,

which represents a 26% increase in $I(\alpha)$.

[63] $L = a\left(1-be^{-kt}\right) \Rightarrow \frac{L}{a} = 1-be^{-kt} \Rightarrow be^{-kt} = 1-\frac{L}{a} \Rightarrow be^{-kt} = \frac{a-L}{a} \Rightarrow$

$e^{-kt} = \frac{a-L}{ab} \Rightarrow -kt = \ln\left(\frac{a-L}{ab}\right) \Rightarrow t = -\frac{1}{k}\ln\left(\frac{a-L}{ab}\right)$

[64] $R = 2.3\log(A+3000) - 5.1 \Rightarrow \frac{R+5.1}{2.3} = \log(A+3000) \Rightarrow$

$A + 3000 = 10^{(R+5.1)/2.3} \Rightarrow A = 10^{(R+5.1)/2.3} - 3000$

[65] As in Exercise 64, $A = 10^{(R+7.5)/2.3} - 34{,}000$. Thus, $\frac{A_1}{A_2} = \frac{10^{(R+5.1)/2.3} - 3000}{10^{(R+7.5)/2.3} - 34{,}000}$.

[66] $R = 4 \Rightarrow 2.3\log(A+14{,}000) - 6.6 = 4 \Rightarrow \log(A+14{,}000) = \frac{10.6}{2.3} \Rightarrow$

$A = 10^{106/23} - 14{,}000 \approx 26{,}615.9$ mi^2.

$\boxed{67}$ $p = 29e^{-0.000034h} \Rightarrow \frac{p}{29} = e^{-0.000034h} \Rightarrow \ln\left(\frac{p}{29}\right) = -0.000034h \Rightarrow$

$$h = \frac{\ln(p/29)}{-0.000034} \Rightarrow h = \frac{\ln(29/p)}{0.000034}$$

$\boxed{68}$ Substituting $v = 0$ and $m = m_1 + m_2$ in $v = -a \ln m + b$ yields

$0 = -a \ln(m_1 + m_2) + b$. Thus, $b = a \ln(m_1 + m_2)$. At burnout, $m = m_1$,

and hence, $v = -a \ln m_1 + b = -a \ln m_1 + a \ln(m_1 + m_2)$

$$= a\left[\ln(m_1 + m_2) - \ln m_1\right] \Rightarrow v = a \ln\left(\frac{m_1 + m_2}{m_1}\right)$$

$\boxed{69}$ (a) $\log n = 7.7 - (0.9)R \Rightarrow n = 10^{7.7 - 0.9R}$

(b) $R = 4$, 5, and 6 $\Rightarrow n \approx 12{,}589$; 1585; and 200

$\boxed{70}$ (a) $\log E = 11.4 + (1.5)R \Rightarrow E = 10^{11.4 + 1.5R}$

(b) $R = 8.4 \Rightarrow E = 10^{24}$ ergs

$\boxed{71}$ Let $q(t) = \frac{1}{2}q_0$. $\frac{1}{2}q_0 = q_0e^{-0.0063t} \Rightarrow \frac{1}{2} = e^{-0.0063t} \Rightarrow \ln\frac{1}{2} = -0.0063t \Rightarrow$

$$-\ln 2 = -0.0063t \Rightarrow t = \frac{\ln 2}{0.0063} \approx 110 \text{ days}$$

$\boxed{72}$ $h = 70.228 + 5.104t + 9.222 \ln t$ and $R = 5.104 + (9.222/t)$.

$$t = 2 \Rightarrow h \approx 86.8 \text{ cm and } R = 9.715 \text{ cm/yr.}$$

$\boxed{73}$ $I = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \Rightarrow \frac{RI}{V} = 1 - e^{-Rt/L} \Rightarrow e^{-Rt/L} = 1 - \frac{RI}{V} \Rightarrow$

$$e^{-Rt/L} = \left(\frac{V - RI}{V}\right) \Rightarrow -\frac{Rt}{L} = \ln\left(\frac{V - RI}{V}\right) \Rightarrow t = -\frac{L}{R}\ln\left(\frac{V - RI}{V}\right)$$

$\boxed{74}$ (a) $x = 4\% \Rightarrow T = -8310 \ln(0.04) \approx 26{,}749$ yr.

(b) $T = 10{,}000 \Rightarrow 10{,}000 = -8310 \ln x \Rightarrow x = e^{-10{,}000/8310} \approx 0.30$, or 30%.

$\boxed{75}$ Using the doubling time formula, $t = (\ln 2)/r = (\ln 2)/0.033 \approx 21.00$ yr.

$\boxed{76}$ $N(t) = \frac{1}{2}N_0 \Rightarrow \frac{1}{2}N_0 = N_0(0.805)^t \Rightarrow 2^{-1} = (0.805)^t \Rightarrow$

$\ln(2^{-1}) = \ln(0.805)^t \Rightarrow -\ln 2 = t \ln(0.805) \Rightarrow$

$$t = -\frac{\ln 2}{\ln(0.805)} \approx 3.196 \text{ millennia, or 3196 yr}$$

Chapter 5 Discussion Exercises

$\boxed{1}$ (a) $f(x) = -(x-1)^3 + 1 \Rightarrow$

$y = -(x-1)^3 + 1 \Rightarrow$

$(x-1)^3 = 1 - y \Rightarrow$

$x - 1 = \sqrt[3]{1-y} \Rightarrow$

$x = \sqrt[3]{1-y} + 1 \Rightarrow$

$f^{-1}(x) = \sqrt[3]{1-x} + 1$

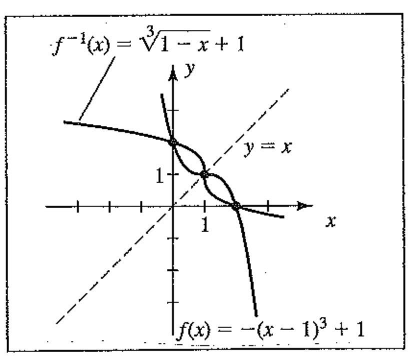

Figure 1

(b) If f is increasing, then f^{-1} moves from left to right and increases.
If f is decreasing, then f^{-1} moves from right to left and decreases.

(c) If $P(a,\ b)$ is a point on f and P is an intersection point of f and f^{-1}, then $Q(b,\ a)$ is a point on f^{-1} (since P is a point on f) and Q is a point on f (since P is also a point on f^{-1}). If $a = b$, then P lies on the line $y = x$. If $a \neq b$, then $m_{PQ} = -1$, and P lies on a line perpendicular to $y = x$; that is, on a line of the form $y = -x + c$.

[2] The values for which $y = (-3)^x$ is defined correspond to points that are either on the graph of $y = 3^x$ or $y = -3^x$. There is no point on $y = (-3)^x$ for $x = 0.1$, but there is for $x = 0.2$ since $y = (-3)^{0.2} = (-3)^{1/5} = -\sqrt[5]{3}$. The base a must be positive so that the function $f(x) = a^x$ is defined for all values of x.

[−4.7, 4.7] by [−3.1, 3.1]

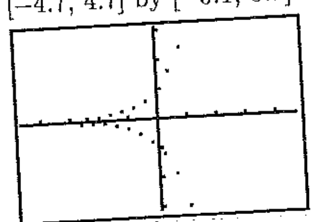

Figure 2

[3] (a) The y-intercept is a, so it increases as a does. The graph flattens out as the value of a increases.

(b) Graph $Y_1 = \frac{a}{2}(e^{x/a} + e^{-x/a}) + (30 - a)$ on $[-20,\ 20]$ by $[30,\ 32]$ and check for $Y_1 < 32$ at $x = 20$. In this case it turns out that $a = 101$ is the smallest integer value that satisfies the conditions, so an equation is

$$y = \tfrac{101}{2}(e^{x/101} + e^{-x/101}) - 71.$$

[4] Sum the values of R for $t = 1$ to $t = x$ so that the sum is 50. Summing R for $t = 1$ to $t = 7$ years gives 49.329 tons. The answer using the formula is about 7.16 years.

[5] (a) $x^y = y^x \;\Rightarrow\; \ln(x^y) = \ln(y^x) \;\Rightarrow\; y\ln x = x\ln y \;\Rightarrow\; \dfrac{\ln x}{x} = \dfrac{\ln y}{y}$

(b) Once you find two values of $(\ln x)/x$ that are the same (such as 0.36652), you know that the corresponding x-values, x_1 and x_2, satisfy the relationship $(x_1)^{x_2} \approx (x_2)^{x_1}$. In particular, when $(\ln x)/x \approx 0.36652$, we find that $x_1 \approx 2.50$ and $x_2 \approx 2.97$. Note that $2.50^{2.97} \approx 2.97^{2.50} \approx 15.20$.

(c) Note that $f(e) = \frac{1}{e}$. Any horizontal line $y = k$, with $0 < k < \frac{1}{e}$, will intersect the graph at the two points $\left(x_1,\ \dfrac{\ln x_1}{x_1}\right)$ and $\left(x_2,\ \dfrac{\ln x_2}{x_2}\right)$, where $1 < x_1 < e$ and $x_2 > e$.

6 (a) $y_1 = (1.085)^x$ could be considered to be the value of an investment growing at 8.5% per year with no compounding. $y_2 = e^{0.085x}$ could be considered to be the value of an investment growing at 8.5% per year compounded continuously. In part (a) of the exercises, $x = 40 \Rightarrow y_1 \approx 26.13$ and $y_2 \approx 29.96$. In part (b) of the exercises, $y = 2 \Rightarrow x_1 \approx 8.50$ and $x_2 \approx 8.15$. The answers in part (b) could be considered to be the doubling time for the investments.

(b) $y_3 = (1 + 0.085/12)^{12x}$ would be greater than y_1, but less than y_2 (closer to y_2).

(c) An estimate of 29 and 8.2 would be reasonable. You would believe that these are correct because they are much closer to the continuously compounded interest values than the simple interest values. Actual values are 29.61 and 8.18.

7 Logarithm law 3 states that it is valid only for positive real numbers, so $y = \log_3(x^2)$ is equivalent to $y = 2\log_3 x$ only for $x > 0$. The domain of $y = \log_3(x^2)$ is $\mathbb{R} - \{0\}$, whereas the domain of $y = 2\log_3 x$ is $x > 0$.

8 (a) $U =$ compound interest amount $-$ accumulated amount $=$

$$P\left(1 + \frac{r}{12}\right)^{12t} - \frac{12M\left[(1 + r/12)^{12t} - 1\right]}{r}.$$

Note that P is the loan amount L and we use 12 for n because there are monthly payments.

(b) $U = 90{,}000\left(1 + \frac{0.12}{12}\right)^{12x} - \frac{12(925.75)\left[(1 + 0.12/12)^{12x} - 1\right]}{0.12}.$

[0, 35, 5] by [0, 1E5, 1E4]

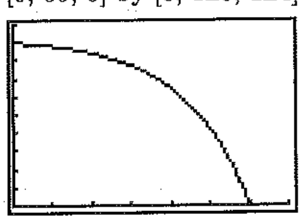

Figure 8

(c) $x = 10$ years $\Rightarrow U = \$84{,}076.50$. $U = \$45{,}000 \Rightarrow x \approx 24.425$ years.

(d) The points (0, 90,000) and (30, 0) must be on the graph. The graph must be decreasing—more near $x = 30$ than in the beginning.

(e) It is an excellent approximation, but we must remember that payments are only made every month—not for every value of x. *Note:* You may want to loosely discuss discrete vs. continuous models at this time.

[9] There are 4 points of intersection. Listed in order of difficulty to find we have: $(-0.9999011,\ 0.00999001)$, $(-0.0001,\ 0.01)$, $(100,\ 0.01105111)$, and $(36{,}102.844,\ 4.6928\times 10^{13})$. Exponential function values (with base > 1) are greater than polynomial function values (with leading term positive) for very large values of x.

[10] There are 3 points of intersection. Listed in order of difficulty to find we have $(x,\ x)$ with $x \approx 0.44239443$, 4.1770774, and 5,503.6647. The y-values for $y = x$ eventually will be larger than the y-values for $y = (\ln x)^n$.

[11] $60{,}000 = 40{,}000b^5 \Rightarrow b = \sqrt[5]{1.5} \approx 1.0844717712$, or 8.447717712%. Mentally—it would take $70/8.5 \approx 8^+$ years to double and there would be about $40/8 = 5$ doubling periods, $2^5 = 32$ and $32 \cdot \$40{,}000 = \$1{,}280{,}000$.

Actual $= \$40{,}000(1 + 0.0844717712)^{40} = \$1{,}025{,}156.25$.

[12] (a) The formula in Exercise 70, $\log E = 11.4 + 1.5\,R$, gives E in terms of ergs and 1 joule $= 10^7$ ergs. The earthquake released about 1.122×10^{15} joules of energy, so about 3.5 earthquakes are equivalent to one 1-megaton bomb. About 425 1-megaton bombs are equivalent to the Mount St. Helens eruption.

(b) $E = 1.7\times 10^{18} \Rightarrow R = (\log E - 11.4)/1.5 = 9.22$. No, the highest recorded reading is about 8.9 in the 1933 earthquake in Japan. The worst death toll by an earthquake was in 1201 in the Near East—it killed an estimated 1.1 million people.

[13] On the TI-83 Plus, enter the sum of the days, {0, 5168, 6728, 8136, 8407, 8735, 8857, 9010, 9274, 9631, 9666}, and the averages, {1003.16, 2002.25, 3004.46, 4003.33, 5023.55, 6010.00, 7022.44, 8038.88, 9033.23, 10,006.78, 11,014.69}, in L_1 and L_2, respectively. Use ExpReg under STAT CALC to obtain $y = ab^x$, where $a = 753.76507865478$ and $b = 1.0002468662739$. Plot the data along with the exponential function and the line $y = 20{,}000$. The functions intersect at approximately 13,282. This value corresponds to March 27, 2009. *Note:* The TI-83 has a convenient "day between dates" function for problems of this nature.

Using the first and last milestone figures and the continuously compounded interest formula gives us $A = Pe^{rt} \Leftrightarrow 11{,}014.69 = 1003.16e^{r(9666)} \Rightarrow$ $r \approx 0.0002478869045$ (daily) or about 9.05% annually. *Note:* The Dow was 100.25 on 1/12/1906. You may want to include this information and examine the differences it makes in any type of prediction.

(continued)

A discussion of practical considerations should lead to mention of crashes, corrections, and the validity of any model over too long of a period of time.

[14] On the TI-83 Plus, enter the sum of the days, {0, 3566, 7368, 8925, 10,020, 10,495, 10,551, 10,615}, and the averages, {100, 200.25, 501.62, 1005.89, 2000.56, 3028.51, 4041.46, 5046.86}, in L_1 and L_2, respectively. Use ExpReg under STAT CALC to obtain $y = ab^x$, where $a = 68.206216283022$ and $b = 1.0003527403467$.

A graph of the data and the model shows a poor fit. The phenomenal growth spurt makes it impossible to obtain a good fit over this long of a period of time.

[15] (a) [−10, 110, 10] by [0, 1E10, 1E9]

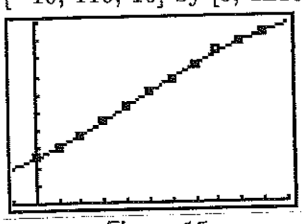

Figure 15

(b) Based on the shape of the curve in part (a), a logistic model is definitely more appropriate.

(c) Using the logistic model feature, we obtain $y = c/(1 + ae^{-bx})$ with

$$a = 3.567015872,\ b = 0.0281317818, \text{ and } c = 1.1355368 \times 10^{10}.$$

(d) As x gets large, the term ae^{-bx} gets closer to zero, so y approaches $c/1 = c$.

[16] $\log_5 x + \log_7 x = 11$ • The left side is the sum of two simple increasing logarithms, so we would expect it to be an increasing function that would be equal to 11 exactly once, and there should be one solution. $\log_5 x + \log_7 x = 11 \Rightarrow$

$$\frac{\ln x}{\ln 5} + \frac{\ln x}{\ln 7} = 11 \Rightarrow \frac{\ln x}{\ln 5} + \frac{\frac{\ln 5}{\ln 7} \cdot \ln x}{\ln 5} = 11 \Rightarrow \ln x\left(1 + \frac{\ln 5}{\ln 7}\right) = 11 \ln 5 \Rightarrow$$

$$\ln x\left(\frac{\ln 7 + \ln 5}{\ln 7}\right) = 11 \ln 5 \Rightarrow \ln x = \frac{11 \ln 5 \cdot \ln 7}{\ln 35} \Rightarrow x = e^b, \text{ where } b = \frac{11 \ln 5 \cdot \ln 7}{\ln 35}.$$