

Chapter 4: Polynomial and Rational Functions

4.1 Exercises

1 $f(x) = 2x^3 + c$ (a) $c = 3$

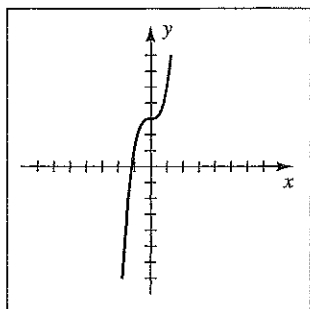


Figure 1(a)

(b) $c = -3$ •

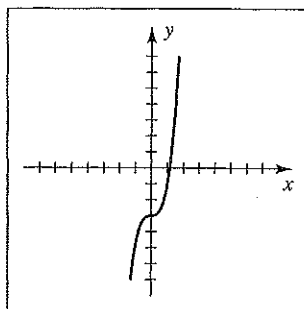


Figure 1(b)

2 $f(x) = -2x^3 + c$ (a) $c = -2$

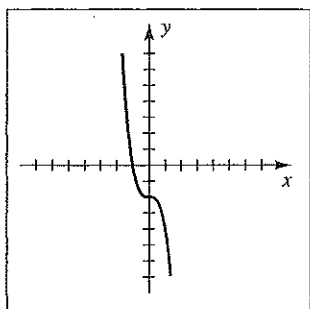


Figure 2(a)

(b) $c = 2$ •

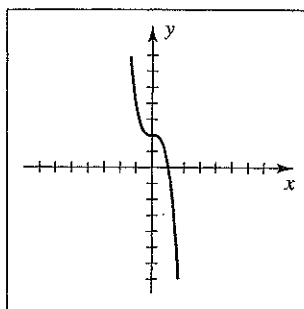


Figure 2(b)

3 $f(x) = ax^3 + 2$ (a) $a = 2$

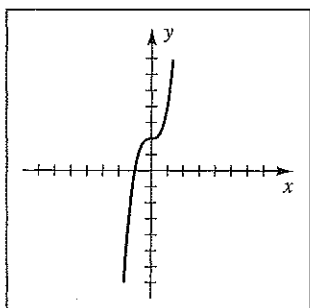


Figure 3(a)

(b) $a = -\frac{1}{3}$ •

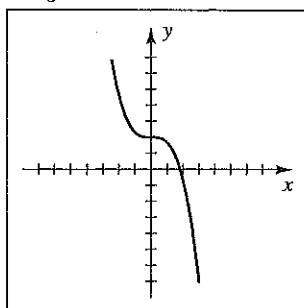


Figure 3(b)

4 $f(x) = ax^3 - 3$ (a) $a = -2$

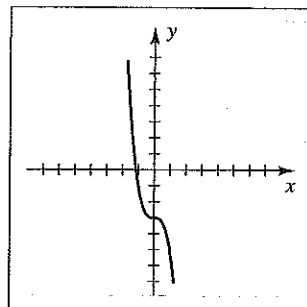


Figure 4(a)

(b) $a = \frac{1}{4}$ •

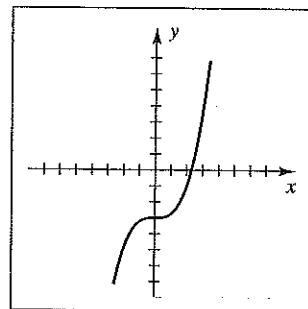


Figure 4(b)

5 $f(x) = x^3 - 4x^2 + 3x - 2$; $a = 3$, $b = 4$; $f(3) = -2 < 0$, $f(4) = 10 > 0$

By the intermediate value theorem for polynomial functions,

f takes on every value between -2 and 10 in the interval $[3, 4]$, namely, 0 .

6 $f(x) = 2x^3 + 5x^2 - 3$; $a = -3$, $b = -2$; $f(-3) = -12 < 0$, $f(-2) = 1 > 0$

7 $f(x) = -x^4 + 3x^3 - 2x + 1$; $a = 2$, $b = 3$; $f(2) = 5 > 0$, $f(3) = -5 < 0$

8 $f(x) = 2x^4 + 3x - 2$; $a = \frac{1}{2}$, $b = \frac{3}{4}$; $f(\frac{1}{2}) = -\frac{3}{8} < 0$, $f(\frac{3}{4}) = \frac{113}{128} > 0$

9 $f(x) = x^5 + x^3 + x^2 + x + 1$; $a = -\frac{1}{2}$, $b = -1$; $f(-\frac{1}{2}) = \frac{19}{32} > 0$, $f(-1) = -1 < 0$

10 $f(x) = x^5 - 3x^4 - 2x^3 + 3x^2 - 9x - 6$; $a = 3$, $b = 4$; $f(3) = -60 < 0$, $f(4) = 134 > 0$

- 11 (a) The graph has x -intercepts at -1 , 1 , and 2 —so $(x+1)$, $(x-1)$, and $(x-2)$ are factors in its equation. It is the graph of a cubic. C

- (b) The graph has x -intercepts at -1 , 1 , and 2 . It is the graph of a quartic. D

- (c) The graph has x -intercepts at 0 and 2 . It is the graph of a cubic with a negative leading coefficient. B

- (d) The graph has x -intercepts at 0 and 2 . It is the graph of a cubic with a positive leading coefficient. A

- 12 (a) The graph has x -intercepts at -2 , -1 , and 1 —so $(x+2)$, $(x+1)$, and $(x-1)$ are factors in its equation. It is the graph of a quartic. D

- (b) The graph has x -intercepts at 0 and 1 . It is the graph of a cubic. A

- (c) The graph has x -intercepts at -2 and 0 . It is the graph of a cubic with a negative leading coefficient. B

- (d) The graph has x -intercepts at -2 , -1 , and 3 . It is the graph of a cubic with a positive leading coefficient. C

[13] $f(x) = \frac{1}{4}x^3 - 2 = \frac{1}{4}(x^3 - 8) = \frac{1}{4}(x - 2)(x^2 + 2x + 4)$; $f(x) > 0$ if $x > 2$, $f(x) < 0$ if $x < 2$

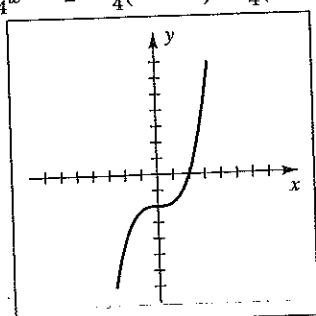


Figure 13

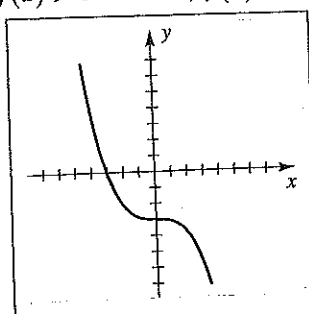


Figure 14

[14] $f(x) = -\frac{1}{9}x^3 - 3 = -\frac{1}{9}(x^3 + 27) = -\frac{1}{9}(x + 3)(x^2 - 3x + 9)$;
 $f(x) > 0$ if $x < -3$, $f(x) < 0$ if $x > -3$

[15] $f(x) = -\frac{1}{16}x^4 + 1 = -\frac{1}{16}(x^4 - 16) = -\frac{1}{16}(x^2 + 4)(x + 2)(x - 2)$;
 $f(x) > 0$ if $|x| < 2$, $f(x) < 0$ if $|x| > 2$

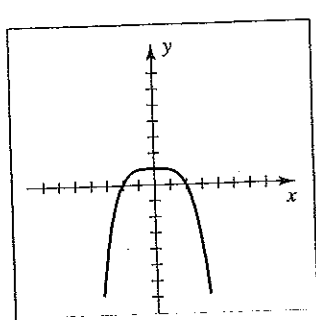


Figure 15

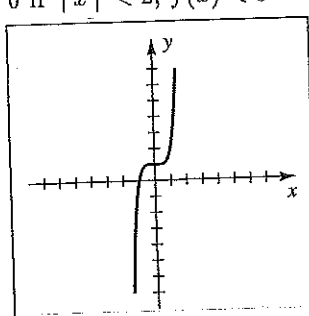


Figure 16

[16] $f(x) = x^5 + 1 = (x + 1)(x^4 - x^3 + x^2 - x + 1)$; $f(x) > 0$ if $x > -1$, $f(x) < 0$ if $x < -1$

[17] $f(x) = x^4 - 4x^2 = x^2(x + 2)(x - 2)$; $f(x) > 0$ if $|x| > 2$, $f(x) < 0$ if $0 < |x| < 2$

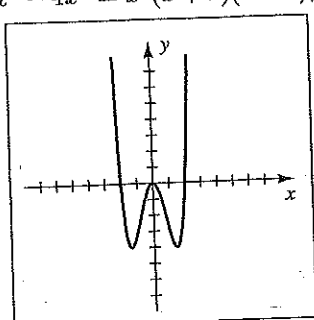


Figure 17

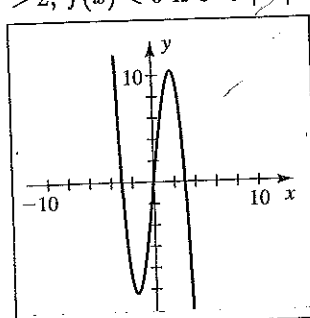


Figure 18

[18] $f(x) = 9x - x^3 = x(3 + x)(3 - x)$;
 $f(x) > 0$ if $x < -3$ or $0 < x < 3$, $f(x) < 0$ if $-3 < x < 0$ or $x > 3$

$$[19] f(x) = -x^3 + 3x^2 + 10x = -x(x^2 - 3x - 10) = -x(x+2)(x-5);$$

$$f(x) > 0 \text{ if } x < -2 \text{ or } 0 < x < 5, f(x) < 0 \text{ if } -2 < x < 0 \text{ or } x > 5$$

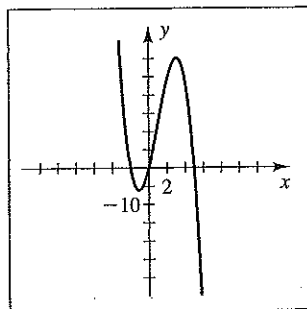


Figure 19

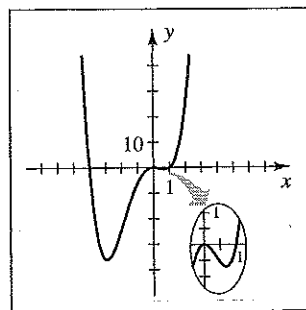


Figure 20

$$[20] f(x) = x^4 + 3x^3 - 4x^2 = x^2(x^2 + 3x - 4) = x^2(x+4)(x-1);$$

$$f(x) > 0 \text{ if } x < -4 \text{ or } x > 1, f(x) < 0 \text{ if } -4 < x < 0 \text{ or } 0 < x < 1$$

$$[21] f(x) = \frac{1}{6}(x+2)(x-3)(x-4);$$

$$f(x) > 0 \text{ if } -2 < x < 3 \text{ or } x > 4, f(x) < 0 \text{ if } x < -2 \text{ or } 3 < x < 4$$

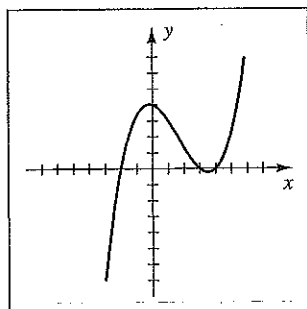


Figure 21

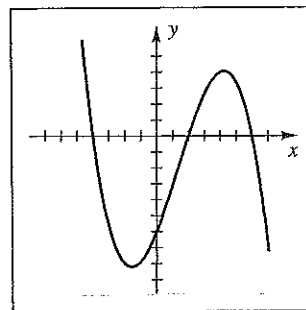


Figure 22

$$[22] f(x) = -\frac{1}{8}(x+4)(x-2)(x-6);$$

$$f(x) > 0 \text{ if } x < -4 \text{ or } 2 < x < 6, f(x) < 0 \text{ if } -4 < x < 2 \text{ or } x > 6$$

$$[23] f(x) = x^3 + 2x^2 - 4x - 8 = x^2(x+2) - 4(x+2) = (x+2)^2(x-2);$$

$$f(x) > 0 \text{ if } x > 2, f(x) < 0 \text{ if } x < -2 \text{ or } |x| < 2$$

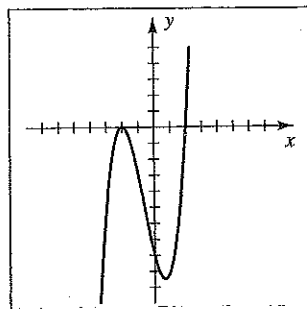


Figure 23

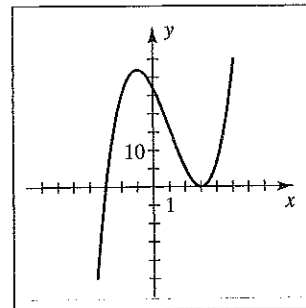


Figure 24

$$[24] f(x) = x^3 - 3x^2 - 9x + 27 = x^2(x-3) - 9(x-3) = (x-3)^2(x+3);$$

$$f(x) > 0 \text{ if } |x| < 3 \text{ or } x > 3, f(x) < 0 \text{ if } x < -3$$

[25] $f(x) = x^4 - 6x^2 + 8 = (x^2 - 2)(x + 2)(x - 2);$

$$f(x) > 0 \text{ if } |x| > 2 \text{ or } |x| < \sqrt{2}, f(x) < 0 \text{ if } \sqrt{2} < |x| < 2$$

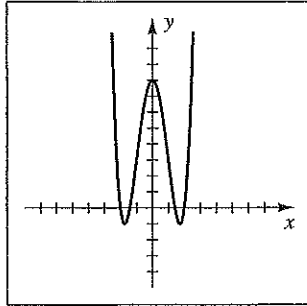


Figure 25

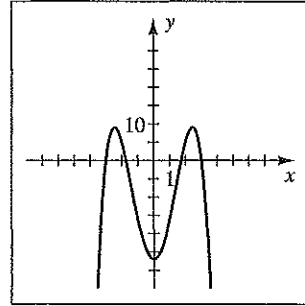


Figure 26

[26] $f(x) = -x^4 + 12x^2 - 27 = (-x^2 + 3)(x^2 - 9) = -(x^2 - 3)(x + 3)(x - 3);$

$$f(x) > 0 \text{ if } \sqrt{3} < |x| < 3, f(x) < 0 \text{ if } |x| > 3 \text{ or } |x| < \sqrt{3}$$

[27] $f(x) = x^2(x + 2)(x - 1)^2(x - 2);$

$$f(x) > 0 \text{ if } |x| > 2, f(x) < 0 \text{ if } |x| < 2, x \neq 0, x \neq 1$$

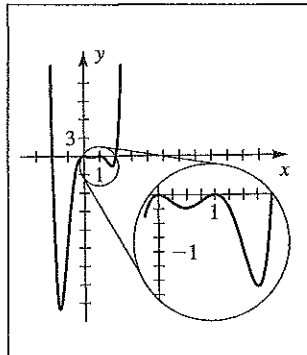


Figure 27

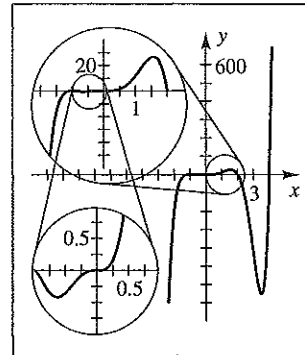


Figure 28

[28] $f(x) = x^3(x + 1)^2(x - 2)(x - 4);$

$$f(x) > 0 \text{ if } 0 < x < 2 \text{ or } x > 4, f(x) < 0 \text{ if } x < 0, x \neq -1 \text{ or } 2 < x < 4$$

- [29] The sign of $f(x)$ is positive on $(-\infty, -4)$, so the graph of f must be above the x -axis on that interval. The sign of $f(x)$ is negative on $(-4, 0)$, so the graph of f must cross the x -axis at $x = -4$ and be below the x -axis on the interval $(-4, 0)$. The sign of $f(x)$ is negative on $(0, 1)$, so the graph of f must touch the x -axis at $x = 0$ and then fall below the x -axis on $(0, 1)$. The sign of $f(x)$ is positive on $(1, 3)$, so the graph of f must cross the x -axis at $x = 1$ and be above the x -axis on that interval. The sign of $f(x)$ is negative on $(3, \infty)$, so the graph of f must cross the x -axis at $x = 3$ and be below the x -axis on the interval $(3, \infty)$. See Figure 29.

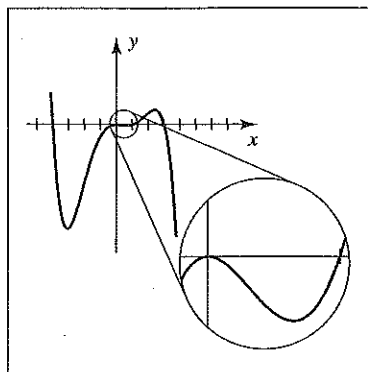


Figure 29

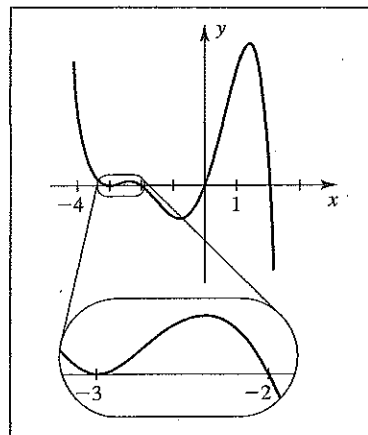


Figure 30

- [30] The graph of f is above the x -axis on $(-\infty, -2)$, touching the axis at $x = -3$. The graph is below the x -axis on $(-2, 0)$, above the x -axis on $(0, 2)$, and below the x -axis on $(2, \infty)$.
- [31] (a) The graph of $f(x) = (x-a)(x-b)(x-c)$, where $a < 0 < b < c$, must have one negative zero, a , and two positive zeros, b and c . The general shape is that of a cubic polynomial.
- (b) The y -intercept is $f(0) = (-a)(-b)(-c) = -abc$.
- (c) The solution to $f(x) < 0$ is $(-\infty, a) \cup (b, c)$; that is, where the graph of f is below the x -axis.
- (d) The solution to $f(x) \geq 0$ is $[a, b] \cup [c, \infty)$; that is, where the graph of f is above or on the x -axis.

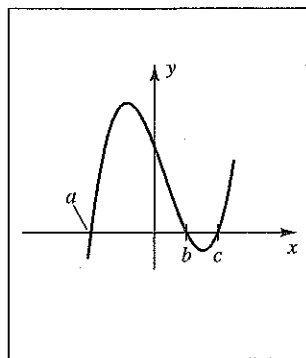


Figure 31

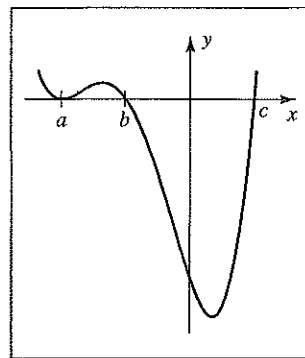


Figure 32

- [32] (a) The graph of $f(x) = (x-a)^2(x-b)(x-c)$, where $a < b < 0 < c$, must have two negative zeros, a and b , and one positive zero, c . The general shape is that of a quartic polynomial.
- (b) The y -intercept is $f(0) = (-a)^2(-b)(-c) = a^2bc$.

(c) The solution to $f(x) > 0$ is $(-\infty, a) \cup (a, b) \cup (c, \infty)$; that is, where the graph of f is above the x -axis.

(d) The solution to $f(x) \leq 0$ is $\{a\} \cup [b, c]$; that is, where the graph of f is below or on the x -axis.

[33] If n is even, then $(-x)^n = x^n$ and hence $f(-x) = f(x)$. Thus, f is an even function.

[34] If n is odd, then $(-x)^n = -x^n$ and hence $f(-x) = -f(x)$. Thus, f is an odd function.

[35] $f(-1) = -4 - 6k$ and $f(-1) = 4 \Rightarrow -4 - 6k = 4 \Rightarrow 6k = -8 \Rightarrow k = -\frac{4}{3}$.

[36] $f(2) = 8k + 4 - 2k + 2$ and $f(2) = 12 \Rightarrow 6k + 6 = 12 \Rightarrow 6k = 6 \Rightarrow k = 1$.

[37] $f(2) = 16k - 32$ and $f(2) = 0 \Rightarrow 16k - 32 = 0 \Rightarrow k = 2$.

$$f(x) = x^3 - 2x^2 - 16x + 32 = x^2(x - 2) - 16(x - 2) = (x + 4)(x - 4)(x - 2).$$

The other two zeros are ± 4 .

[38] $f(-2) = 2k - 8$ and $f(-2) = 0 \Rightarrow 2k - 8 = 0 \Rightarrow 2k = 8 \Rightarrow k = 4$.

$$f(x) = x^3 - 3x^2 - 4x + 12 = x^2(x - 3) - 4(x - 3) = (x - 3)(x^2 - 4) =$$

$(x - 3)(x + 2)(x - 2)$. The other two zeros are 2 and 3.

[39] $P(x) = \frac{1}{2}(5x^3 - 3x) = \frac{1}{2}x(5x^2 - 3)$.

$P(x) > 0$ on $(-\frac{1}{5}\sqrt{15}, 0)$ and $(\frac{1}{5}\sqrt{15}, \infty)$.

$P(x) < 0$ on $(-\infty, -\frac{1}{5}\sqrt{15})$ and $(0, \frac{1}{5}\sqrt{15})$.

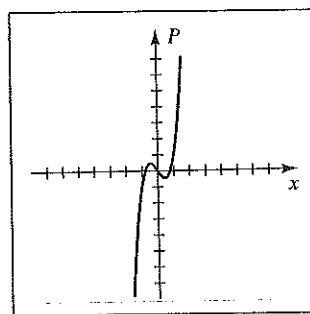


Figure 39

[40] $x^2 = \frac{8 \pm 4\sqrt{2}}{16} = \frac{2 \pm \sqrt{2}}{4} \Rightarrow x = \pm \frac{1}{2}\sqrt{2 \pm \sqrt{2}} \approx \pm 0.92, \pm 0.38;$

$f(x) > 0$ if $|x| < \frac{1}{2}\sqrt{2 - \sqrt{2}}$ or $|x| > \frac{1}{2}\sqrt{2 + \sqrt{2}}$

[41] (a) $V(x) = lwh = (30 - x - x)(20 - x - x)x = x(20 - 2x)(30 - 2x) =$

$$4x(10 - x)(15 - x) = 4x(x - 10)(x - 15).$$

(b) $V(x) > 0$ on $(0, 10)$ and $(15, \infty)$. Allowable values for x are in $(0, 10)$.

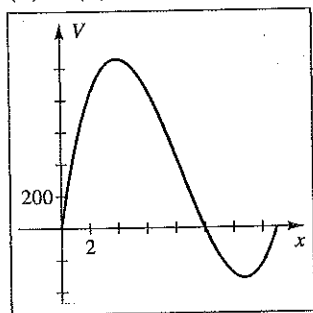


Figure 41

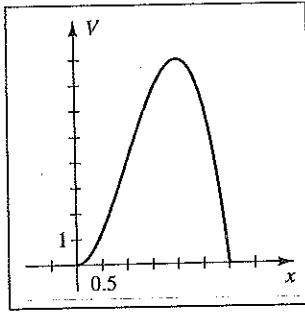


Figure 42

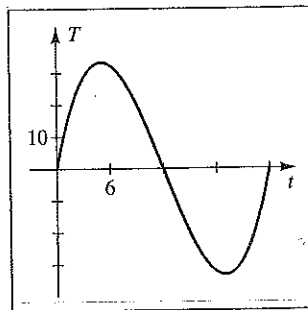


Figure 43

- [42] (a) Since we are disregarding the thickness of the lumber, 4 boards are y feet long and 8 boards are x feet long. Total length = 24 $\Rightarrow 4y + 8x = 24 \Rightarrow$
 $y = 6 - 2x$. $V = x^2y = x^2(6 - 2x) = 6x^2 - 2x^3$.

(b) From part (a), $V = 6x^2 - 2x^3$, or, equivalently, $V = -2x^2(x - 3)$. See Figure 42.

- [43] (a) $T = \frac{1}{20}t(t - 12)(t - 24) = 0 \Rightarrow t = 0, 12, 24$. $T > 0$ for
 $0 < t < 12$ { 6 A.M. to 6 P.M. }; $T < 0$ for $12 < t < 24$ { 6 P.M. to 6 A.M. }.

(b) See Figure 43.

(c) 12 noon corresponds to $t = 6$, $T(6) = 32.4 > 32^\circ\text{F}$ and $T(7) = 29.75 < 32^\circ\text{F}$

- [44] (a) At the end of the board, $s = 10$.

Letting $d = 1$ and $L = 10$ yields $1 = 100c(20) \Rightarrow c = \frac{1}{2000}$.

- (b) $s = 6.5 \Rightarrow d = (\frac{1}{2000})(6.5)^2[3(10) - 6.5] \approx 0.4964 < \frac{1}{2}$.
 $s = 6.6 \Rightarrow d = (\frac{1}{2000})(6.6)^2[3(10) - 6.6] \approx 0.5097 > \frac{1}{2}$.

- [45] (a) $N(t) = -t^4 + 21t^2 + 100$
 $= -(t^4 - 21t^2 - 100)$
 $= -(t^2 - 25)(t^2 + 4)$
 $= -(t + 5)(t - 5)(t^2 + 4)$

If $t > 0$, then $N(t) > 0$ for $0 < t < 5$.

- (b) The population becomes extinct when $N = 0$.

This occurs after 5 years.

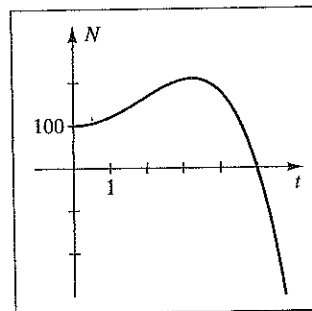


Figure 45

- [46] (a) $R = 0 \Rightarrow -4t^3 + 42t = 0 \Rightarrow -2t(2t^2 - 21) = 0 \Rightarrow t = \frac{1}{2}\sqrt{42}$ { for $t > 0$ }.

The population ceases to grow after $\frac{1}{2}\sqrt{42} \approx 3.24$ years.

- (b) $R > 0$ if t is in the interval $(0, \frac{1}{2}\sqrt{42})$.

- [47] (a) $f(x) = 2x^4$, $g(x) = 2x^4 - 5x^2 + 1$, $h(x) = 2x^4 + 5x^2 - 1$, $k(x) = 2x^4 - x^3 + 2x$

x	$f(x)$	$g(x)$	$h(x)$	$k(x)$
-60	25,920,000	25,902,001	25,937,999	26,135,880
-40	5,120,000	5,112,001	5,127,999	5,183,920
-20	320,000	318,001	321,999	327,960
20	320,000	318,001	321,999	312,040
40	5,120,000	5,112,001	5,127,999	5,056,080
60	25,920,000	25,902,001	25,937,999	25,704,120

(b) As $|x|$ becomes large, the function values become similar.

(c) The term with the highest power of x : $2x^4$.

- [48] (a) $f(x) = -3x^3$, $g(x) = -3x^3 - x^2 + 1$, $h(x) = -3x^3 + x^2 - 1$, $k(x) = -3x^3 - 2x^2 + 2x$

$[-2, 2]$ by $[-2, 2]$

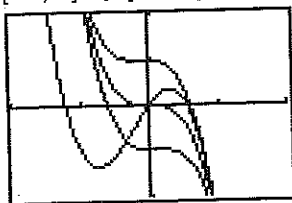


Figure 48(1)

$[-10, 10]$ by $[-10, 10]$

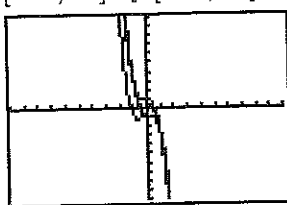


Figure 48(2)

$[-50, 50, 10]$ by $[-5E3, 5E3, 1E3]$

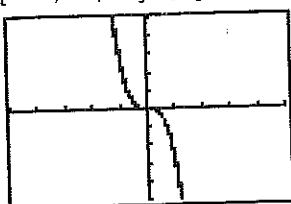


Figure 48(3)

$[-100, 100, 10]$ by $[-5E5, 5E5, 1E5]$

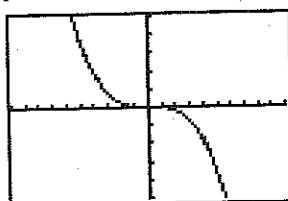


Figure 48(4)

- (b) As the viewing rectangle increases in size, the graphs look alike.
 (c) Their end behavior is similar because their highest degree term is $-3x^3$. This term determines the shape of the graph when $|x|$ is large.

- [49] (a) $[-9, 9]$ by $[-6, 6]$

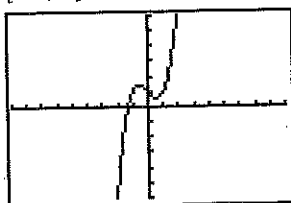


Figure 49(1)

$[-9, 9]$ by $[-6, 6]$

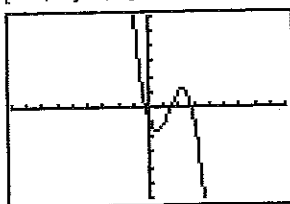


Figure 49(2)

$[-9, 9]$ by $[-6, 6]$

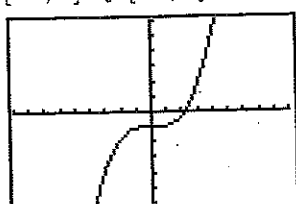


Figure 49(3)

$[-9, 9]$ by $[-6, 6]$

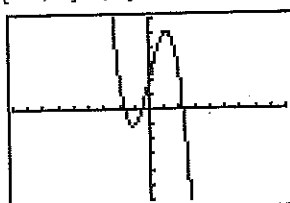


Figure 49(4)

- (b) (1) $f(x) = x^3 - x + 1$ • As x approaches ∞ , $f(x)$ approaches ∞ ;
 as x approaches $-\infty$, $f(x)$ approaches $-\infty$
 (2) $f(x) = -x^3 + 4x^2 - 3x - 1$ • As x approaches ∞ , $f(x)$ approaches $-\infty$;
 as x approaches $-\infty$, $f(x)$ approaches ∞
 (3) $f(x) = 0.1x^3 - 1$ • As x approaches ∞ , $f(x)$ approaches ∞ ;
 as x approaches $-\infty$, $f(x)$ approaches $-\infty$

- (4) $f(x) = -x^3 + 4x + 2$ • As x approaches ∞ , $f(x)$ approaches $-\infty$;
as x approaches $-\infty$, $f(x)$ approaches ∞
- (c) For the cubic function $f(x) = ax^3 + bx^2 + cx + d$ with $a > 0$, $f(x)$ approaches ∞ as x approaches ∞ and $f(x)$ approaches $-\infty$ as x approaches $-\infty$. With $a < 0$, $f(x)$ approaches $-\infty$ as x approaches ∞ and $f(x)$ approaches ∞ as x approaches $-\infty$.

[50] (a) $[-9, 9]$ by $[-6, 6]$

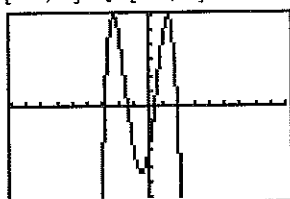


Figure 50(1)

$[-9, 9]$ by $[-6, 6]$

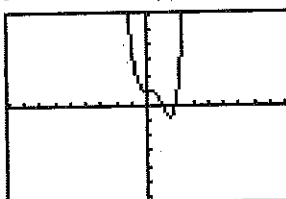


Figure 50(2)

$[-9, 9]$ by $[-6, 6]$

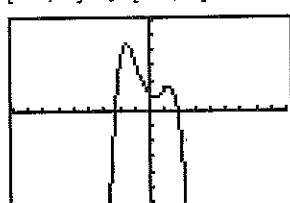


Figure 50(3)

$[-9, 9]$ by $[-6, 6]$

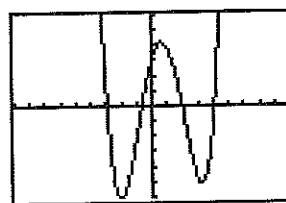


Figure 50(4)

- (b) (1) $f(x) = -x^4 - 2x^3 + 5x^2 + 6x - 3$ • As $x \rightarrow \pm\infty$, $f(x) \rightarrow -\infty$
- (2) $f(x) = x^4 - 2x^3 + 1$ • As $x \rightarrow \pm\infty$, $f(x) \rightarrow \infty$
- (3) $f(x) = -\frac{1}{2}x^4 + 2x^2 - x + 1$ • As $x \rightarrow \pm\infty$, $f(x) \rightarrow -\infty$
- (4) $f(x) = \frac{1}{5}x^4 - \frac{1}{2}x^3 - \frac{7}{3}x^2 + \frac{7}{2}x + 3$ • As $x \rightarrow \pm\infty$, $f(x) \rightarrow \infty$
- (c) For the fourth-degree polynomial $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ with $a > 0$, $f(x)$ approaches ∞ as $|x|$ approaches ∞ and with $a < 0$, $f(x)$ approaches $-\infty$ as $|x|$ approaches ∞ .

[51] From the graph, f has three zeros. They are approximately -1.89 , 0.49 , and 1.20 .

$[-4.5, 4.5]$ by $[-3, 3]$

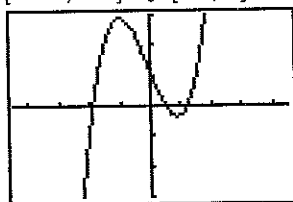


Figure 51

$[-4.5, 4.5]$ by $[-3, 3]$

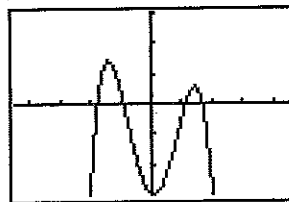


Figure 52

[52] From the graph, f has four zeros.

They are approximately -1.78 , -0.91 , 1.11 , and 1.67 .

- [53] From the graph, f has three zeros. They are approximately -1.88 , 0.35 , and 1.53 .

$[-4.5, 4.5]$ by $[-3, 3]$

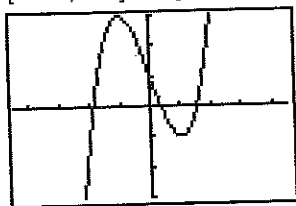


Figure 53

$[-4.5, 4.5]$ by $[-3, 3]$

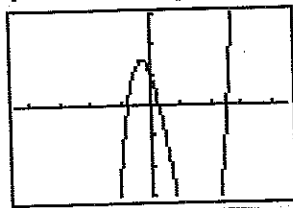


Figure 54

- [54] From the graph, f has three zeros. They are approximately -0.77 , 0.26 , and 2.52 .

- [55] If $f(x) = x^3 + 5x - 2$ and $k = 1$, then $f(x) > k$ on $(0.56, \infty)$.

$[-4.5, 4.5]$ by $[-3, 3]$

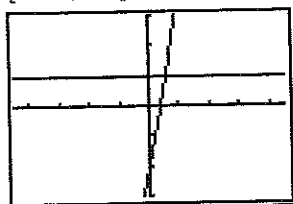


Figure 55

$[-4.5, 4.5]$ by $[-2, 4]$

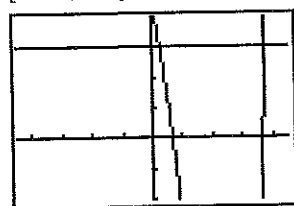


Figure 56

- [56] If $f(x) = x^4 - 4x^3 + 3x^2 - 8x + 5$ and $k = 3$, then $f(x) > k$ on $(-\infty, 0.27) \cup (3.73, \infty)$.

- [57] If $f(x) = x^5 - 2x^2 + 2$ and $k = -2$, then $f(x) > k$ on $(-1.10, \infty)$.

$[-4.5, 4.5]$ by $[-3, 3]$

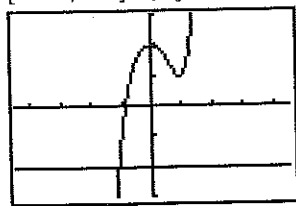


Figure 57

$[-4.5, 4.5]$ by $[-3, 3]$

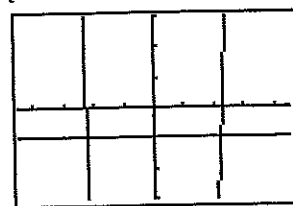


Figure 58

- [58] If $f(x) = x^4 - 2x^3 + 10x - 26$ and $k = -1$, then $f(x) > k$ on $(-\infty, -2.24) \cup (2.24, \infty)$.

- [59] From the graph, there are three points of intersection.

Their coordinates are approximately $(-1.29, -0.77)$, $(0.085, 2.66)$, and $(1.36, -0.42)$.

$[-4.5, 4.5]$ by $[-2, 4]$

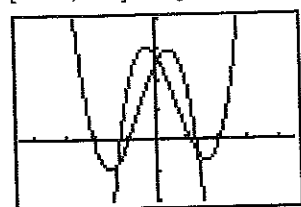


Figure 59

$[-5.5, 5]$ by $[-3, 4]$

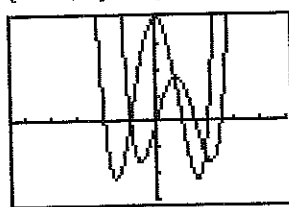


Figure 60

- [60] From the graph, there are three points of intersection. One is $(-1, 0)$.

The others are approximately $(0.71, 1.72)$ and $(1.87, -1.25)$.

[61] (a)

[1975, 1995] by [20, 40]

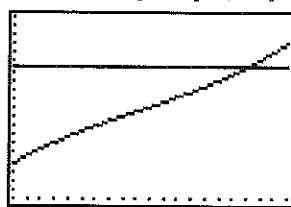


Figure 61

- (b) Graph $Y_1 = 0.0014(x - 1975)^3 - 0.0388(x - 1975)^2 + 0.8783(x - 1975) + 23.82$ and $Y_2 = 34.4$. Their graphs intersect when $x \approx 1991.9792 \approx 1992$. There were 34.4 million recipients in 1992.

[62] (a) The number of preschool children decreased during 1970 to 1976, but since then, it has increased.

[0, 25, 5] by [3E5, 8E5, 1E5]

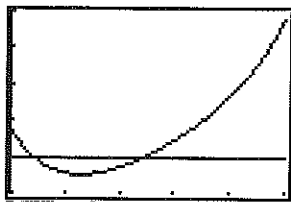


Figure 62

- (b) $f(x) = 4.363x^4 - 236.3x^3 + 5527x^2 - 46,519x + 475,913 \Rightarrow f(6) \approx 350,385$

(c) Graph $Y_1 = f(x)$ and $Y_2 = 400,000$. There are two points of intersection at $x \approx 2.12, 12.30$. Thus, there were approximately 400,000 participants during the years 1972 and 1982.

4.2 Exercises

- | | | | |
|---|-------------------------|---|--|
| [1-8] $f(x) = \text{dividend};$ | $p(x) = \text{divisor}$ | • | ★ quotient; remainder |
| [1] $f(x) = 2x^4 - x^3 - 3x^2 + 7x - 12;$ | $p(x) = x^2 - 3$ | • | ★ $2x^2 - x + 3; 4x - 3$ |
| [2] $f(x) = 3x^4 + 2x^3 - x^2 - x - 6;$ | $p(x) = x^2 + 1$ | • | ★ $3x^2 + 2x - 4; -3x - 2$ |
| [3] $f(x) = 3x^3 + 2x - 4;$ | $p(x) = 2x^2 + 1$ | • | ★ $\frac{3}{2}x; \frac{1}{2}x - 4$ |
| [4] $f(x) = 3x^3 - 5x^2 - 4x - 8;$ | $p(x) = 2x^2 + x$ | • | ★ $\frac{3}{2}x - \frac{13}{4}; -\frac{3}{4}x - 8$ |
| [5] $f(x) = 7x + 2;$ | $p(x) = 2x^2 - x - 4$ | • | ★ 0; $7x + 2$ |
| [6] $f(x) = -5x^2 + 3;$ | $p(x) = x^3 - 3x + 9$ | • | ★ 0; $-5x^2 + 3$ |
| [7] $f(x) = 9x + 4;$ | $p(x) = 2x - 5$ | • | ★ $\frac{9}{2}; \frac{53}{2}$ |
| [8] $f(x) = 7x^2 + 3x - 10;$ | $p(x) = x^2 - x + 10$ | • | ★ 7; $10x - 80$ |
- [9] Dividing $f(x) = 3x^3 - x^2 + 5x - 4$ by $x - 2$ using either long division or

synthetic division yields a remainder of 26.

[10] Divide $f(x) = 2x^3 + 4x^2 - 3x - 1$ by $x - 3$ to obtain a remainder of 80.

[11] Divide $f(x) = x^4 - 6x^2 + 4x - 8$ by $x + 3$ to obtain a remainder of 7.

[12] Divide $f(x) = x^4 + 3x^2 - 12$ by $x + 2$ to obtain a remainder of 16.

[13] Since $f(-3) = 0$, $x + 3$ is a factor of $f(x) = x^3 + x^2 - 2x + 12$.

[14] Since $f(2) = 0$, $x - 2$ is a factor of $f(x) = x^3 + x^2 - 11x + 10$.

[15] Since $f(-2) = 0$, $x + 2$ is a factor of $f(x) = x^{12} - 4096$.

[16] Since $f(2) = 0$, $x - 2$ is a factor of $f(x) = x^4 - 3x^3 - 2x^2 + 5x + 6$.

Note: In Exercises 17–20, let $a = 1$.

[17] f has degree 3 with zeros $-2, 0, 5 \Rightarrow$

$$f(x) = a[x - (-2)](x - 0)(x - 5) = x(x + 2)(x - 5) = x(x^2 - 3x - 10) = x^3 - 3x^2 - 10x$$

[18] f has degree 3 with zeros $\pm 2, 3 \Rightarrow$

$$f(x) = a(x + 2)(x - 2)(x - 3) = (x^2 - 4)(x - 3) = x^3 - 3x^2 - 4x + 12$$

[19] f has degree 4 with zeros $-2, \pm 1, 4 \Rightarrow$

$$f(x) = a(x + 2)(x + 1)(x - 1)(x - 4) = (x^2 - 1)(x^2 - 2x - 8) = x^4 - 2x^3 - 9x^2 + 2x + 8$$

[20] f has degree 4 with zeros $-3, 0, 1, 5 \Rightarrow$

$$f(x) = a(x + 3)(x)(x - 1)(x - 5) = x(x^2 + 2x - 3)(x - 5) = x^4 - 3x^3 - 13x^2 + 15x$$

[21]
$$\begin{array}{r|rrrr} 2 & 2 & -3 & 4 & -5 \\ & & 4 & 2 & 12 \\ \hline & 2 & 1 & 6 & 7 \end{array}$$
 The synthetic division indicates that the quotient is $2x^2 + x + 6$ and the remainder is 7.

[22] $3x^3 - 4x^2 - x + 8;$ $x + 4 \bullet$ $\star 3x^2 - 16x + 63; -244$

[23] $x^3 - 8x - 5;$ $x + 3 \bullet$ $\star x^2 - 3x + 1; -8$

[24] $5x^3 - 6x^2 + 15;$ $x - 4 \bullet$ $\star 5x^2 + 14x + 56; 239$

[25] $3x^5 + 6x^2 + 7;$ $x + 2 \bullet$ $\star 3x^4 - 6x^3 + 12x^2 - 18x + 36; -65$

[26] $-2x^4 + 10x - 3;$ $x - 3 \bullet$ $\star -2x^3 - 6x^2 - 18x - 44; -135$

[27] $4x^4 - 5x^2 + 1;$ $x - \frac{1}{2} \bullet$ $\star 4x^3 + 2x^2 - 4x - 2; 0$

[28] $9x^3 - 6x^2 + 3x - 4;$ $x - \frac{1}{3} \bullet$ $\star 9x^2 - 3x + 2; -\frac{10}{3}$

[29]
$$\begin{array}{r|rrrr} 3 & 2 & 3 & -4 & 4 \\ & & 6 & 27 & 69 \\ \hline & 2 & 9 & 23 & 73 \end{array}$$
 The synthetic division indicates that $f(3) = 73$.

[30] $f(x) = -x^3 + 4x^2 + x;$ $c = -2 \bullet$ $f(-2) = 22$

[31] $f(x) = 0.3x^3 + 0.04x - 0.034;$ $c = -0.2 \bullet$ $f(-0.2) = -0.0444$

[32] $f(x) = 8x^5 - 3x^2 + 7;$ $c = \frac{1}{2} \bullet$ $f(\frac{1}{2}) = \frac{13}{2}$

[33] $f(x) = x^2 + 3x - 5;$ $c = 2 + \sqrt{3} \bullet$ $f(2 + \sqrt{3}) = 8 + 7\sqrt{3}$

[34] $f(x) = x^3 - 3x^2 - 8;$ $c = 1 + \sqrt{2} \bullet$ $f(1 + \sqrt{2}) = -10 - \sqrt{2}$

$$\boxed{35} \quad f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4; \quad c = -2 \quad \bullet \quad \star f(-2) = 0$$

$$\boxed{36} \quad f(x) = 4x^3 - 9x^2 - 8x - 3; \quad c = 3 \quad \bullet \quad \star f(3) = 0$$

$$\boxed{37} \quad f(x) = 4x^3 - 6x^2 + 8x - 3; \quad c = \frac{1}{2} \quad \bullet \quad \star f\left(\frac{1}{2}\right) = 0$$

$$\boxed{38} \quad f(x) = 27x^4 - 9x^3 + 3x^2 + 6x + 1; \quad c = -\frac{1}{3} \quad \bullet \quad \star f\left(-\frac{1}{3}\right) = 0$$

$$\boxed{39} \quad f(-2) = k^2 - 8k + 15. \text{ This remainder must be zero if } f(x) \text{ is to be divisible by } x + 2.$$

$$k^2 - 8k + 15 = 0 \Rightarrow k = 3, 5.$$

$$\boxed{40} \quad f(1) = k^2 - 4k + 3. \text{ As in the previous exercise, } k^2 - 4k + 3 = 0 \Rightarrow k = 1, 3.$$

$$\boxed{41} \quad f(c) = 3c^4 + c^2 + 5 \geq 5 \quad \forall c \in \mathbb{R}; \text{ that is, the remainder cannot be zero.}$$

$$\boxed{42} \quad f(c) = -c^4 - 3c^2 - 2 \leq -2 \quad \forall c \in \mathbb{R}; \text{ that is, the remainder cannot be zero.}$$

$$\boxed{43} \quad f(x) = 3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6 \Rightarrow f(-1) = 3 - 5 - 4 - 2 - 6 = -14.$$

{ change sign on coefficients of odd powers of x }

$$\boxed{44} \quad \text{If } f(x) = x^n - y^n, \text{ then } f(y) = y^n - y^n = 0. \text{ Hence, } x - y \text{ is a factor of } f.$$

$$\boxed{45} \quad \text{If } f(x) = x^n - y^n \text{ and } n \text{ is even, then } f(-y) = (-y)^n - (y)^n = y^n - y^n = 0.$$

Hence, $x + y$ is a factor of $x^n - y^n$.

$$\boxed{46} \quad \text{If } f(x) = x^n + y^n \text{ and } n \text{ is odd, then } f(-y) = (-y)^n + y^n = -y^n + y^n = 0.$$

Hence, $x + y$ is a factor of $x^n + y^n$.

$$\boxed{47} \quad (a) \quad V = \pi r^2 h = \pi x^2(6 - x)$$

$$(b) \quad \text{The volume of the cylinder of radius 1 and altitude 5 is } \pi(1)^2 5 = 5\pi.$$

$$5\pi = \pi x^2(6 - x) \Rightarrow x^3 - 6x^2 + 5 = 0 \Rightarrow (x - 1)(x^2 - 5x - 5) = 0 \Rightarrow$$

$$x = 1, \frac{5 \pm \sqrt{45}}{2}. \quad \frac{5 + \sqrt{45}}{2} \approx 5.85 \text{ would be an allowable value of } x.$$

$$\text{The point } P \text{ is } P(x, y) = \left(\frac{1}{2}(5 + \sqrt{45}), \frac{1}{2}(7 - \sqrt{45})\right).$$

$$\boxed{48} \quad \text{The width, depth, and diameter form a right triangle.}$$

$$w^2 + d^2 = 2^2 \Rightarrow d^2 = 4 - w^2 = \frac{7}{4} \text{ for } w = \frac{3}{2}. \quad S = kwd^2 = k\left(\frac{3}{2}\right)\left(\frac{7}{4}\right) = \frac{21}{8}k. \text{ Now}$$

$$\frac{21}{8}k = kw(4 - w^2) \Rightarrow 8w^3 - 32w + 21 = 0 \Rightarrow \left(w - \frac{3}{2}\right)(8w^2 + 12w - 14) = 0 \Rightarrow$$

$$w = \frac{3}{2}, \frac{-3 \pm \sqrt{37}}{4}. \quad \frac{\sqrt{37} - 3}{4} \approx 0.77 \text{ would be an allowable value of } w.$$

$$\boxed{49} \quad (a) \quad A = lw = (2x)(y) = 2x(4 - x^2) = 8x - 2x^3$$

$$(b) \quad A = 6 \Rightarrow 6 = 8x - 2x^3 \Rightarrow x^3 - 4x + 3 = 0 \Rightarrow (x - 1)(x^2 + x - 3) = 0 \Rightarrow$$

$$x = 1, \frac{-1 \pm \sqrt{13}}{2}. \quad \frac{\sqrt{13} - 1}{2} \text{ would be an allowable value of } x.$$

$$\text{The base would then be } \sqrt{13} - 1 \approx 2.61.$$

$$\boxed{50} \quad (a) \quad \text{Let } \frac{3}{2} - 2r \text{ denote the length of the right circular cylinder portion of the capsule.}$$

$$V = \frac{4}{3}\pi r^3 + \pi r^2\left(\frac{3}{2} - 2r\right) = \pi r^2\left(\frac{4}{3}r + \frac{3}{2} - 2r\right) = \pi r^2\left(\frac{3}{2} - \frac{2}{3}r\right).$$

(b) The tablet has volume $\pi(\frac{1}{2})^2 \cdot \frac{1}{3} = \frac{\pi}{12}$. $\frac{\pi}{12} = \pi r^2(\frac{3}{2} - \frac{2}{3}r) \Rightarrow$

$$8r^3 - 18r^2 + 1 = 0 \Rightarrow (4r - 1)(2r^2 - 4r - 1) = 0 \Rightarrow r = \frac{1}{4}, 1 \pm \frac{1}{2}\sqrt{6}.$$

$\frac{1}{4}$ cm is the only allowable solution.

51 If $f(x) = x^8 - 7.9x^5 - 0.8x^4 + x^3 + 1.2x - 9.81$,

then the remainder is $f(0.21) \approx -9.55$.

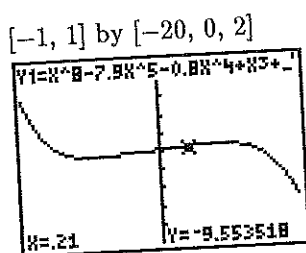


Figure 51

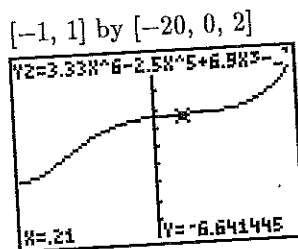


Figure 52

52 If $f(x) = 3.33x^6 - 2.5x^5 + 6.9x^3 - 4.1x^2 + 1.22x - 6.78$,

then the remainder is $f(0.21) \approx -6.64$.

53 $f(1.6) = -2k^4 + 2.56k^3 + 3.2k + 4.096$. Graph $y = -2k^4 + 2.56k^3 + 3.2k + 4.096$ (that is, $y = -2x^4 + 2.56x^3 + 3.2x + 4.096$). From the graph, we see that $y = 0$ when $k \approx -0.75, 1.96$. Thus, if k assumes either of these values, $f(1.6) = 0$ and f will be divisible by $x - 1.6$ by the factor theorem.

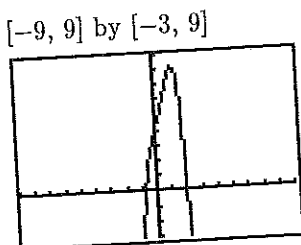


Figure 53

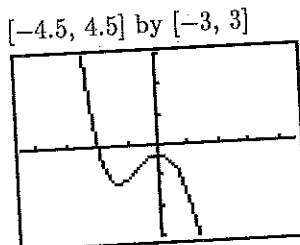


Figure 54

54 $f(-0.4) = -0.064k^5 - 0.4k^3 - 1.2k^2 - 0.336$.

Graph $y = -0.064k^5 - 0.4k^3 - 1.2k^2 - 0.336$. From the graph, we see that $y = 0$ when $k \approx -1.98$. Thus, if k assumes this value, $f(-0.4) = 0$ and f will be divisible by $x + 0.4$ by the factor theorem.

4.3 Exercises

1 $f(x) = a(x+1)(x-2)(x-3);$

$f(-2) = a(-1)(-4)(-5) = 80 \Rightarrow -20a = 80 \Rightarrow a = -4 \quad \star -4x^3 + 16x^2 - 4x - 24$

2 $f(x) = a(x+5)(x-2)(x-4);$

$f(3) = a(8)(1)(-1) = -24 \Rightarrow -8a = -24 \Rightarrow a = 3 \quad \star 3x^3 - 3x^2 - 66x + 120$

$$\boxed{3} \quad f(x) = a(x+4)(x-3)(x);$$

$$f(2) = a(6)(-1)(2) = -36 \Rightarrow -12a = -36 \Rightarrow a = 3$$

$$\star 3x^3 + 3x^2 - 36x$$

$$\boxed{4} \quad f(x) = a(x+3)(x+2)(x);$$

$$f(-4) = a(-1)(-2)(-4) = 16 \Rightarrow -8a = 16 \Rightarrow a = -2$$

$$\star -2x^3 - 10x^2 - 12x$$

$$\boxed{5} \quad f(x) = a(x+2i)(x-2i)(x-3); f(1) = a(1+2i)(1-2i)(-2) = 20 \Rightarrow$$

$$-10a = 20 \Rightarrow a = -2$$

$$\star -2x^3 + 6x^2 - 8x + 24$$

$$\boxed{6} \quad f(x) = a(x+3i)(x-3i)(x-4); f(-1) = a(-1+3i)(-1-3i)(-5) = 50 \Rightarrow$$

$$-50a = 50 \Rightarrow a = -1$$

$$\star -x^3 + 4x^2 - 9x + 36$$

$$\boxed{7} \quad f(x) = a(x+4)^2(x-3)^2 = (x^2+x-12)^2 \{a=1\} = x^4 + 2x^3 - 23x^2 - 24x + 144$$

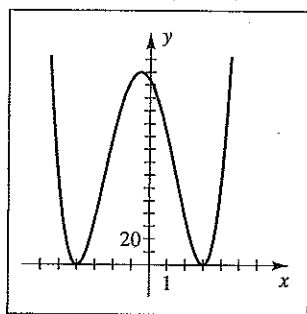


Figure 7

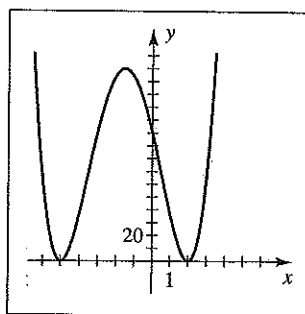


Figure 8

$$\boxed{8} \quad f(x) = a(x+5)^2(x-2)^2 = (x^2+3x-10)^2 \{a=1\} = x^4 + 6x^3 - 11x^2 - 60x + 100.$$

Note that the figure can be obtained by shifting Figure 7 left 1 unit.

$$\boxed{9} \quad f(x) = a(x)^3(x-3)^3 \text{ so } f(2) = a(8)(-1) = -8a. \text{ But } f(2) = -24, \text{ so } -8a = -24, \text{ or,}$$

$$a = 3. \quad f(x) = 3(x)^3(x^3 - 9x^2 + 27x - 27) = 3x^6 - 27x^5 + 81x^4 - 81x^3.$$

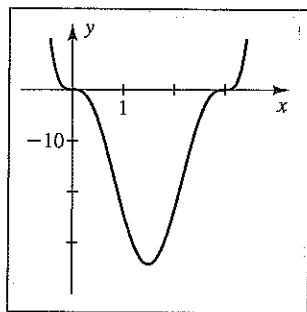


Figure 9

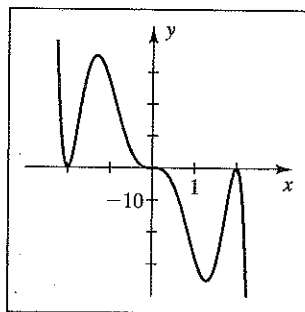


Figure 10

$$\boxed{10} \quad f(x) = a(x)^3(x+2)^2(x-2)^2 \text{ so } f(-1) = a(-1)(1)(9) = -9a.$$

$$\text{But } f(-1) = 27, \text{ so } -9a = 27, \text{ or, } a = -3.$$

$$f(x) = -3x^3(x^2-4)^2 = -3x^3(x^4-8x^2+16) = -3x^7+24x^5-48x^3.$$

$$\boxed{11} \quad \text{The graph has } x\text{-intercepts at } -1, \frac{3}{2}, 3, \text{ and } f(0) = \frac{7}{2}.$$

$$f(x) = a(x+1)(x-\frac{3}{2})(x-3); f(0) = a(1)(-\frac{3}{2})(-3) = \frac{7}{2} \Rightarrow \frac{9}{2}a = \frac{7}{2} \Rightarrow a = \frac{7}{9}.$$

$$f(x) = \frac{7}{9}(x+1)(x-\frac{3}{2})(x-3)$$

- [12] The graph has x -intercepts at 0, 1, 3, 5, and $f(-1) = 4$.

$$f(x) = a(x)(x-1)(x-3)(x-5); f(-1) = a(-1)(-2)(-4)(-6) = 4 \Rightarrow$$

$$48a = 4 \Rightarrow a = \frac{1}{12}.$$

$$f(x) = \frac{1}{12}x(x-1)(x-3)(x-5)$$

- [13] 3 is a zero of multiplicity one, 1 is a zero of multiplicity two, and $f(0) = 3$.

$$f(x) = a(x-1)^2(x-3); f(0) = a(1)(-3) = 3 \Rightarrow a = -1. f(x) = -1(x-1)^2(x-3).$$

- [14] 4 is a zero of multiplicity one, 2 is a zero of multiplicity two, and $f(1) = -3$.

$$f(x) = a(x-2)^2(x-4); f(1) = a(1)(-3) = -3 \Rightarrow a = 1. f(x) = 1(x-2)^2(x-4).$$

- [15] $\star -\frac{2}{3}$ (multiplicity 1); 0 (multiplicity 2); $\frac{5}{2}$ (multiplicity 3)

- [16] $\star -1$ (multiplicity 4); 0 (multiplicity 1); $\frac{7}{3}$ (multiplicity 2)

- [17] $f(x) = 4x^5 + 12x^4 + 9x^3 = x^3(2x+3)^2 \quad \star -\frac{3}{2}$ (multiplicity 2); 0 (multiplicity 3)

- [18] $\star \pm \frac{1}{2}\sqrt{5}$ (each of multiplicity 2)

- [19] $f(x) = (x^2 + x - 12)^3(x^2 - 9)^2 = (x+4)^3(x-3)^3(x+3)^2(x-3)^2$

$$\star -4 \text{ (multiplicity 3); } -3 \text{ (multiplicity 2); } 3 \text{ (multiplicity 5)}$$

- [20] $f(x) = (6x^2 + 7x - 5)^4(4x^2 - 1)^2 = (3x+5)^4(2x-1)^4(2x+1)^2(2x-1)^2$

$$\star -\frac{5}{3} \text{ (multiplicity 4); } -\frac{1}{2} \text{ (multiplicity 2); } \frac{1}{2} \text{ (multiplicity 6)}$$

- [21] $f(x) = x^4 + 7x^2 - 144 = (x^2 + 16)(x+3)(x-3)$

$$\star \pm 4i, \pm 3 \text{ (each of multiplicity 1)}$$

- [22] $f(x) = x^4 + 21x^2 - 100 = (x^2 + 25)(x+2)(x-2)$

$$\star \pm 5i, \pm 2 \text{ (each of multiplicity 1)}$$

- [23] Using synthetic division, $f(x) = x^4 + 7x^3 + 13x^2 - 3x - 18 =$

$$(x+3)(x^3 + 4x^2 + x - 6) = (x+3)^2(x^2 + x - 2) = (x+3)^2(x+2)(x-1).$$

- [24] $f(x) = x^4 - 9x^3 + 22x^2 - 32 = (x-4)^2(x-2)(x+1)$

- [25] $f(x) = x^6 - 4x^5 + 5x^4 - 5x^2 + 4x - 1 = (x-1)^5(x+1)$

- [26] $f(x) = x^5 + x^4 - 6x^3 - 14x^2 - 11x - 3 = (x+1)^4(x-3)$

Note: For the following exercises, let $f(x)$ denote the polynomial, P , the number of sign changes in $f(x)$, and N , the number of sign changes in $f(-x)$. The types of possible solutions are listed in the order positive, negative, nonreal complex.

- [27] There are 3 sign changes in $f(x) = 4x^3 - 6x^2 + x - 3$, so $P = 3$. There are no sign changes in $f(-x) = -4x^3 - 6x^2 - x - 3$, so $N = 0$, and there are no negative solutions of the given equation. The equation has either 3 positive solutions or 1 positive solution along with 2 nonreal complex solutions. $\star 3, 0, 0$ or $1, 0, 2$

- [28] $5x^3 - 6x - 4 = 0 \quad \bullet \quad P = 1, N = 2 \quad \star 1, 2, 0$ or $1, 0, 2$

- [29] $4x^3 + 2x^2 + 1 = 0 \quad \bullet \quad P = 0, N = 1 \quad \star 0, 1, 2$

- [30] $3x^3 - 4x^2 + 3x + 7 = 0 \quad \bullet \quad P = 2, N = 1 \quad \star 2, 1, 0$ or $0, 1, 2$

- [31] $3x^4 + 2x^3 - 4x + 2 = 0$ • $P = 2, N = 2$ ★ 2, 2, 0; 2, 0, 2; 0, 2, 2; 0, 0, 4
- [32] $2x^4 - x^3 + x^2 - 3x + 4 = 0$ • $P = 4, N = 0$ ★ 4, 0, 0; 2, 0, 2; 0, 0, 4
- [33] $x^5 + 4x^4 + 3x^3 - 4x + 2 = 0$ • $P = 2, N = 3$ ★ 2, 3, 0; 2, 1, 2; 0, 3, 2; 0, 1, 4
- [34] $2x^6 + 5x^5 + 2x^2 - 3x + 4 = 0$ • $P = 2, N = 2$ ★ 2, 2, 2; 2, 0, 4; 0, 2, 4; 0, 0, 6
- [35] From the graph of $f(x) = x^3 - 4x^2 - 5x + 7$, we see that the bounds given by the first theorem (5 and -2) are indeed the smallest and largest integers that are upper and lower bounds.
- [36] From the graph of $f(x) = 2x^3 - 5x^2 + 4x - 8$, we see that the least integer upper bound is 3 and the greatest integer lower bound is 2. According to the first theorem, the greatest *negative* integer lower bound is -1 (the first theorem gives no information about a greatest *positive* integer lower bound).
- [37] From the graph of $f(x) = x^4 - x^3 - 2x^2 + 3x + 6$, we see that there are no real zeros. The first theorem gives us upper and lower bounds of 2 and -2 , respectively.
- [38] Similar to Exercise 35 with
 $f(x) = 2x^4 - 9x^3 - 8x - 10$, upper bound 5, and lower bound -1 .
- [39] Similar to Exercise 35 with
 $f(x) = 2x^5 - 13x^3 + 2x - 5$, upper bound 3, and lower bound -3 .
- [40] Similar to Exercise 36 with $f(x) = 3x^5 + 2x^4 - x^3 - 8x^2 - 7$, upper bound 2, and lower bound -1 . From the graph of f , the lower bound is 1.
- [41] $f(x) = a(x+1)^2(x-1)(x-2)^3$.
 $f(0) = a(1)(-1)(-8) = 8a$ and $f(0) = -2 \Rightarrow a = -\frac{1}{4}$.
- [42] $f(x) = a(x+2)(x+1)(x-2)^2$. $f(0) = a(2)(1)(4) = 8a$ and $f(0) = 1 \Rightarrow a = \frac{1}{8}$.
- [43] (a) $f(x) = a(x+3)^3(x+1)(x-2)^2$.
 (b) $a = 1$ and $x = 0 \Rightarrow f(0) = 1(3)^3(1)(-2)^2 = 108$.
- [44] (a) $f(x) = a(x+2)^3(x-3)^2$.
 (b) $a = -1$ and $x = 0 \Rightarrow f(0) = -1(2)^3(-3)^2 = -72$.
- [45] From the graph, we see that $f(x) = x^5 - 16.75x^3 + 12.75x^2 + 49.5x - 54$ has zeros of $-4, -2, 1.5$, and 3 . There is a double root at 1.5 . Since the leading coefficient of f is 1, we have $f(x) = 1(x+4)(x+2)(x-1.5)^2(x-3)$.

$[-5, 5]$ by $[-150, 150, 25]$

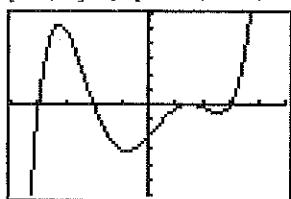


Figure 45

$[-5, 5]$ by $[-100, 100, 25]$

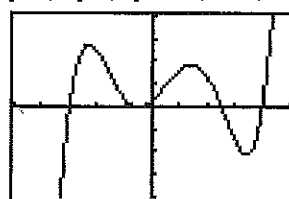


Figure 46

- [46] From the graph, we see that $f(x) = x^5 - 2.5x^4 - 12.75x^3 + 19.625x^2 + 27.625x + 7.5$ has zeros of -3 , -0.5 , 2.5 , and 4 . There is a double root at -0.5 . Since the leading coefficient of f is 1, we have $f(x) = (x+3)(x+0.5)^2(x-2.5)(x-4)$.

- [47] Since the zeros are -2 , 1 , 2 , and 3 , the polynomial must have the form

$$f(x) = a(x+2)(x-1)(x-2)(x-3).$$

Now, $f(0) = a(2)(-1)(-2)(-3) = -12a$ and $f(0) = -24 \Rightarrow -12a = -24 \Rightarrow a = 2$.

Let $f(x) = 2(x+2)(x-1)(x-2)(x-3)$. Since f has been completely determined, we must check the remaining data point(s). $f(-1) = 2(1)(-2)(-3)(-4) = -48 \neq -52$.

Thus, a fourth-degree polynomial *does not* fit the data points.

- [48] $f(x) = a(x)(x+3)(x+1)(x-2)(x-3)$. $f(-2) = a(-2)(1)(-1)(-4)(-5) = 40a$ and $f(-2) = 5 \Rightarrow 40a = 5 \Rightarrow a = \frac{1}{8}$. Let $f(x) = \frac{1}{8}x(x+3)(x+1)(x-2)(x-3)$.
 $f(1) = \frac{1}{8}(1)(4)(2)(-1)(-2) = 2$.

Thus, a fifth-degree polynomial *does* fit the data points.

- [49] $f(x) = a(x-2)(x-5.2)(x-10.1)$. $f(1.1) = a(-33.21) = -49.815 \Rightarrow a = 1.5$.
 Let $f(x) = 1.5(x-2)(x-5.2)(x-10.1)$. $f(3.5) = 25.245$ and $f(6.4) = -29.304$.

Thus, a third-degree polynomial *does* fit the data points.

- [50] $f(x) = a(x-1.25)(x-2)(x-6.5)(x-10)$. $f(2.5) = a(18.75) = 56.25 \Rightarrow a = 3$.
 Let $f(x) = 3(x-1.25)(x-2)(x-6.5)(x-10)$. $f(2.5) = 56.25$ and $f(3) = 128.625$.
 However, $f(9) = -406.875 \neq -307.75$.

Thus, a fourth-degree polynomial *does not* fit the data points.

- [51] The zeros are 0 , 5 , 19 , 24 , and $f(12) = 10$. $f(t) = a(t)(t-5)(t-19)(t-24)$;

$$f(12) = a(12)(7)(-7)(-12) = 10 \Rightarrow 7056a = 10 \Rightarrow a = \frac{5}{3528}.$$

$$f(t) = \frac{5}{3528}t(t-5)(t-19)(t-24)$$

- [52] $f(x) = a(x-c_1)(x-c_2)(x-c_3)$;

$$f(c) = a(c-c_1)(c-c_2)(c-c_3) = 1 \Rightarrow a = \frac{1}{(c-c_1)(c-c_2)(c-c_3)}$$

$$f(x) = \frac{1}{(c-c_1)(c-c_2)(c-c_3)}(x-c_1)(x-c_2)(x-c_3)$$

- [53] The graph of f does not cross the x -axis at a zero of even multiplicity, but does cross the x -axis at a zero of odd multiplicity. The higher the multiplicity of a zero, the more horizontal the graph of f is near that zero. See Figure 53.

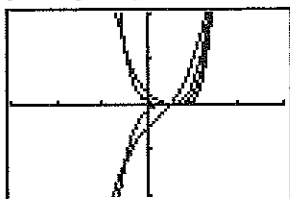
$[-3, 3]$ by $[-2, 2]$


Figure 53

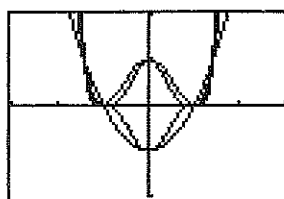
 $[-3, 3]$ by $[-2, 2]$


Figure 54

[54] The conclusions from Exercise 53 do not change when there is more than one zero.

[55] From the graph of f there are two zeros. They are -1.2 and 1.1 . The zero at -1.2 has even multiplicity and the zero at 1.1 has odd multiplicity. Since f has degree 3, the zero at -1.2 must have multiplicity 2 and the zero at 1.1 has multiplicity 1.

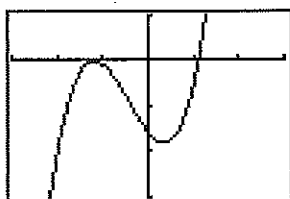
 $[-3, 3]$ by $[-3, 1]$


Figure 55

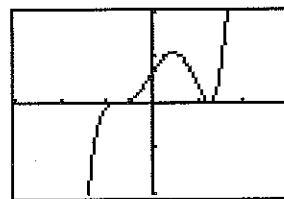
 $[-3, 3]$ by $[-2, 2]$


Figure 56

[56] From the graph of f there are two zeros. They are -0.75 and 1.25 . The zero at -0.75 has odd multiplicity and the zero at 1.25 has even multiplicity. Since f has degree 5, there are two possibilities for the multiplicity: -0.75 has multiplicity 1 and 1.25 has multiplicity 4, or -0.75 has multiplicity 3 and 1.25 has multiplicity 2. By careful inspection we can see that the graph of f levels off as it crosses the x -axis at -0.75 . This means that the multiplicity of this zero is greater than 1. (See Exercise 53.) Thus, -0.75 has multiplicity 3 and 1.25 has multiplicity 2.

[57] From the graph of $A(t) = -\frac{1}{2400}t^3 + \frac{1}{20}t^2 + \frac{7}{6}t + 340$, we see that $A = 400$ when $t \approx 27.1$. Thus, the carbon dioxide concentration will be 400 in $1980 + 27.1 = 2007.1$, or, during the year 2007.

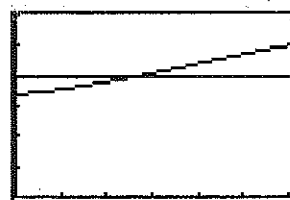
 $[0, 60, 10]$ by $[0, 600, 100]$


Figure 57

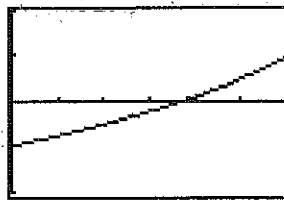
 $[0, 60, 10]$ by $[-2, 2]$


Figure 58

[58] From the graph of $T(t) = \frac{21}{5,000,000}t^3 - \frac{127}{1,000,000}t^2 + \frac{1293}{50,000}t - 1$, we see that $T = 0$ when $t \approx 37.1$. Thus, the average temperature will have increased by 1°C in $1980 + 37.1 = 2017.1$, or, during the year 2017.

[59] (a) Graphing the data and the functions show that the best fit is $h(x)$.

(b) Since the temperature changes sign between April and May and between October and November, an average temperature of 0°F occurs when $4 \leq x \leq 5$ and $10 \leq x \leq 11$.

(c) Finding the zeros of h between 1 and 12 gives us $x \approx 4.02, 10.53$.

[0.5, 12.5] by $[-30, 50, 10]$

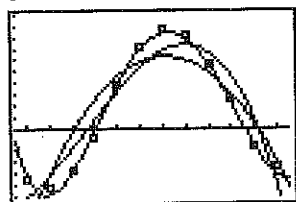


Figure 59

[0.5, 12.5] by $[-20, 70, 10]$

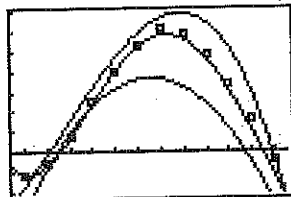


Figure 60

[60] (a) Graphing the data and the functions show that the best fit is $h(x)$.

(b) Since the temperature changes sign between February and March and between November and December, an average temperature of 0°F occurs when $2 \leq x \leq 3$ and $11 \leq x \leq 12$.

(c) Finding the zeros of h between 1 and 12 gives us $x \approx 2.54, 11.42$.

[61] Let $r = 6$ and $k = 0.7$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = \frac{604.8\pi}{3} - 6\pi x^2 + \frac{1}{3}\pi x^3$ and

determine the positive zeros. There are two zeros located at $x \approx 7.64, 15.47$. Since the sphere floats, it will not sink deeper than twice the radius, which is 12 centimeters. Thus, the pine sphere will sink approximately 7.64 centimeters into the water.

$[-20, 20, 5]$ by $[-800, 800, 100]$

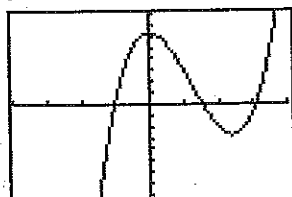


Figure 61

$[-20, 20, 5]$ by $[-1000, 1000, 100]$

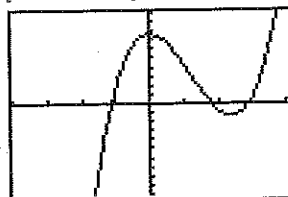


Figure 62

[62] Let $r = 6$ and $k = 0.85$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = \frac{734.4\pi}{3} - 6\pi x^2 + \frac{1}{3}\pi x^3$

and determine the positive zeros. There are two zeros located at $x \approx 9.07, 14.51$. Since the sphere floats, it will not sink deeper than twice the radius, which is 12. Thus, the oak sphere will sink approximately 9.07 centimeters into the water. This is slightly deeper than the pine sphere because the density of oak wood is greater than that of pine.

- [63] Let $r = 6$ and $k = 1$. Graph $Y_1 = \frac{4k}{3}\pi r^3 - \pi x^2 r + \frac{1}{3}\pi x^3 = 288\pi - 6\pi x^2 + \frac{1}{3}\pi x^3$ and determine the positive zero. There is a zero at $x = 12$. This means that the entire sphere is just submerged. The sphere has the same density as water and neither sinks nor floats, much like a balloon filled with water.

$[-20, 20, 5]$ by $[-1000, 1000, 100]$

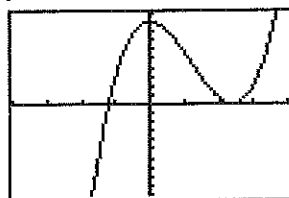


Figure 63

4.4 Exercises

- [1] Since $3 + 2i$ is a root, so is $3 - 2i$. Thus, the polynomial is of the form
 $[x - (3 + 2i)][x - (3 - 2i)] = x^2 - 6x + 13$ { from the discussion on the top of p. 282 }.
- [2] $[x - (-4 + 3i)][x - (-4 - 3i)] = x^2 + 8x + 25$
- [3] $(x - 2)[x - (-2 - 5i)][x - (-2 + 5i)] = (x - 2)(x^2 + 4x + 29)$
- [4] $(x + 3)[x - (1 - 7i)][x - (1 + 7i)] = (x + 3)(x^2 - 2x + 50)$
- [5] $x(x + 1)[x - (3 + i)][x - (3 - i)] = x(x + 1)(x^2 - 6x + 10)$
- [6] $x(x - 2)[x - (-2 - i)][x - (-2 + i)] = x(x - 2)(x^2 + 4x + 5)$
- [7] $[x - (4 + 3i)][x - (4 - 3i)][x - (-2 + i)][x - (-2 - i)] = (x^2 - 8x + 25)(x^2 + 4x + 5)$
- [8] $[x - (3 + 5i)][x - (3 - 5i)][x - (-1 - i)][x - (-1 + i)] = (x^2 - 6x + 34)(x^2 + 2x + 2)$
- [9] $[x - (-2i)][x - (2i)][x - (1 - i)][x - (1 + i)] = x(x^2 + 4)(x^2 - 2x + 2)$
- [10] $[x - (3i)][x - (-3i)][x - (4 + i)][x - (4 - i)] = x(x^2 + 9)(x^2 - 8x + 17)$

Note: Show that none of the possible rational roots listed satisfy the equation in 11–14.

- [11] $x^3 + 3x^2 - 4x + 6 = 0$ • ★ $\pm 1, \pm 2, \pm 3, \pm 6$
- [12] $3x^3 - 4x^2 + 7x + 5 = 0$ • ★ $\pm 1, \pm \frac{1}{3}, \pm 5, \pm \frac{5}{3}$
- [13] $x^5 - 3x^3 + 4x^2 + x - 2 = 0$ • ★ $\pm 1, \pm 2$
- [14] $2x^5 + 3x^3 + 7 = 0$ • ★ $\pm 1, \pm \frac{1}{2}, \pm 7, \pm \frac{7}{2}$
- [15] $x^3 - x^2 - 10x - 8 = 0$ • ★ $-2, -1, 4$
- [16] $x^3 + x^2 - 14x - 24 = 0$ • ★ $-3, -2, 4$
- [17] $2x^3 - 3x^2 - 17x + 30 = 0$ • ★ $-3, 2, \frac{5}{2}$
- [18] $12x^3 + 8x^2 - 3x - 2 = 0$ • ★ $-\frac{2}{3}, \pm \frac{1}{2}$
- [19] $x^4 + 3x^3 - 30x^2 - 6x + 56 = 0$ • ★ $-7, \pm \sqrt{2}, 4$
- [20] $3x^5 - 10x^4 - 6x^3 + 24x^2 + 11x - 6 = 0$ • ★ -1 (multiplicity 2), $\frac{1}{3}, 2, 3$

[21] $6x^5 + 19x^4 + x^3 - 6x^2 = 0$ •

★ $-3, -\frac{2}{3}, 0$ (multiplicity 2), $\frac{1}{2}$

[22] $6x^4 + 5x^3 - 17x^2 - 6x = 0$ •

★ $-2, -\frac{1}{3}, 0, \frac{3}{2}$

[23] $8x^3 + 18x^2 + 45x + 27 = 0$ •

★ $-\frac{3}{4}, -\frac{3}{4} \pm \frac{3}{4}\sqrt{7}i$

[24] $3x^3 - x^2 + 11x - 20 = 0$ •

★ $\frac{4}{3}, -\frac{1}{2} \pm \frac{1}{2}\sqrt{19}i$

[25] $f(x) = 6x^5 - 23x^4 + 24x^3 + x^2 - 12x + 4$ has zeros at $-\frac{2}{3}, \frac{1}{2}, 1$ (mult. 2), and 2.

Thus, $f(x) = 6(x + \frac{2}{3})(x - \frac{1}{2})(x - 1)^2(x - 2) = (3x + 2)(2x - 1)(x - 1)^2(x - 2)$.

[26] $f(x) = -6x^5 + 5x^4 + 14x^3 - 8x^2 - 8x + 3$ has zeros at -1 (mult. 2), $\frac{1}{3}, 1$, and $\frac{3}{2}$.

Thus, $f(x) = -6(x + 1)^2(x - \frac{1}{3})(x - 1)(x - \frac{3}{2}) = (x + 1)^2(3x - 1)(1 - x)(2x - 3)$.

[27] From the graph, we see that $f(x) = 2x^3 - 25.4x^2 + 3.02x + 24.75$ has zeros of approximately $-0.9, 1.1$, and 12.5 . Since the leading coefficient of f is 2, we have

$$f(x) = 2(x + 0.9)(x - 1.1)(x - 12.5).$$

$[-5, 15, 5]$ by $[-600, 100, 100]$

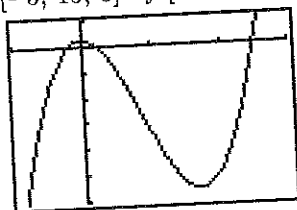


Figure 27

$[-5, 5]$ by $[-15, 15, 5]$

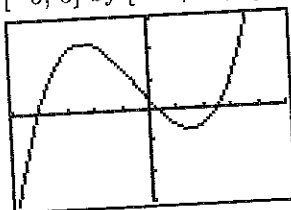


Figure 28

[28] From the graph, we see that $f(x) = 0.5x^3 + 0.65x^2 - 5.365x + 1.5375$ has zeros of approximately $-4.1, 0.3$, and 2.5 . Since the leading coefficient of f is 0.5, we have

$$f(x) = 0.5(x + 4.1)(x - 0.3)(x - 2.5).$$

[29] No. If i is a root, then $-i$ is also a root. Hence, the polynomial would have factors

$x - 1, x + 1, x - i, x + i$ and therefore would be of degree greater than 3.

[30] The theorem applies only to polynomials with real coefficients whereas the polynomial in question has nonreal complex coefficients.

[31] Since n is odd and nonreal complex zeros occur in conjugate pairs for polynomials with real coefficients, there must be at least one real zero.

[32] By the theorem on rational zeros, r is of the form c/d , where c is a factor of a_0 and d is a factor of a_n . Since a_n is 1, its divisors are ± 1 .

Thus, r will be an integer and a factor of a_0 .

[33] (a) $V(x) = x(20 - 2x)(30 - 2x) = 1000 \Rightarrow 4x^3 - 100x^2 + 600x - 1000 = 0 \Rightarrow$

$$4(x - 5)\left[x - (10 - 5\sqrt{2})\right]\left[x - (10 + 5\sqrt{2})\right] = 0. \text{ The allowable range from}$$

Exercise 41 of Section 4.1 was $(0, 10)$, so discard $10 + 5\sqrt{2}$.

The two boxes having volume 1000 in³ have dimensions

$$\text{[A]} 5 \times 10 \times 20 \text{ and [B]} (10 - 5\sqrt{2}) \times (10\sqrt{2}) \times (10 + 10\sqrt{2}).$$

(b) The surface area function is

$$S(x) = (20 - 2x)(30 - 2x) + 2(x)(20 - 2x) + 2(x)(30 - 2x) = -4x^2 + 600.$$

$$S(5) = 500 \text{ and } S(10 - 5\sqrt{2}) = 400\sqrt{2} \approx 565.7 \text{ so box [A] has less surface area.}$$

[34] From Exercise 42 in §4.1, $V(x) = x^2(6 - 2x)$. $V(x) = 4 \Rightarrow x^3 - 3x^2 + 2 = 0 \Rightarrow$

$$(x - 1)(x^2 - 2x - 2) = 0 \Rightarrow \{x > 0\} \ x = 1, 1 + \sqrt{3}.$$

[35] (a) The sides of the triangle are x , $x + 1$, and $\sqrt{2x + 1}$.

$$A = \frac{1}{2}bh \Rightarrow 30 = \frac{1}{2}x\sqrt{2x + 1} \Rightarrow 2x^3 + x^2 - 3600 = 0.$$

(b) There is one sign change in $f(x) = 2x^3 + x^2 - 3600$.

By Descartes' rule of signs there is one positive real root.

Synthetically dividing 13 into f , we obtain a third row of 2, 27, 351, and 963.

These are all positive so 13 is an upper bound for the zeros of f .

(c) $f(x) = 0 \Rightarrow (x - 12)(2x^2 + 25x + 300) = 0.$

The legs of the triangle are 12 and 5, and the hypotenuse is 13.

[36] $V(x) = 27\pi \Rightarrow 10\pi x^2 + \frac{4}{3}\pi x^3 = 27\pi \Rightarrow 4x^3 + 30x^2 - 81 = 0 \Rightarrow$

$$(2x - 3)(2x^2 + 18x + 27) = 0.$$

$$x = \frac{3}{2} \text{ is the only positive solution, so the radius is 1.5 ft.}$$

[37] (a) $\text{Volume}_{\text{total}} = \text{Volume}_{\text{cube}} + \text{Volume}_{\text{roof}}$

$$= x^3 + \frac{1}{2}bhx = x^3 + \frac{1}{2}(x)(6 - x)(x) = x^3 + \frac{1}{2}x^2(6 - x).$$

(b) $\text{Volume} = 80 \Rightarrow x^3 + 6x^2 - 160 = 0 \Rightarrow (x - 4)(x^2 + 10x + 40) = 0.$

The length of the side is 4 ft.

[38] Form a triangle with the center pole and the midpoint of any side.

The hypotenuse (call it y) is the height of one of the triangular sides with base x .

$$\left(\frac{1}{2}x\right)^2 + (8)^2 = y^2 \Rightarrow y = \sqrt{64 + \frac{1}{4}x^2}. \text{ Area}_{\text{total}} = \text{Area}_{\text{base}} + \text{Area}_{4\text{sides}} =$$

$$(\text{side})(\text{side}) + 4\left(\frac{1}{2}\right)(\text{base})(\text{height}) = x^2 + 4\left(\frac{1}{2}\right)(x)\sqrt{64 + \frac{1}{4}x^2} = 384 \Rightarrow$$

$$147,456 - 768x^2 + x^4 = 4x^2(64 + \frac{1}{4}x^2) \Rightarrow x^2 = 144 \Rightarrow x = 12.$$

[39] $x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 0.62 = -1 \Leftrightarrow$

$$x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 1.62 = 0.$$

The graph of $y = x^5 + 1.1x^4 - 3.21x^3 - 2.835x^2 + 2.7x + 1.62$ intersects the x -axis three times. The zeros at -1.5 and 1.2 have even multiplicity (since the graph is tangent to the x -axis at these points) and the zero at -0.5 has odd multiplicity (since the graph crosses the x -axis at this point). Since the equation has degree 5, the only possibility is that the zeros at -1.5 and 1.2 have multiplicity 2 and the zero at -0.5 has multiplicity 1. Thus, the equation has no nonreal solutions. See Figure 39.

[-4.5, 4.5] by [-3, 3]

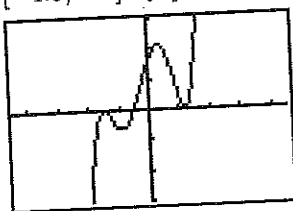


Figure 39

[-4.5, 4.5] by [-3, 3]

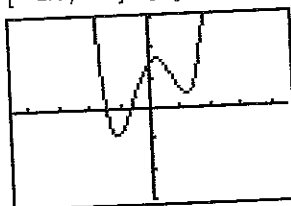


Figure 40

- [40] $x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 3.5 = 2 \Leftrightarrow x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 1.5 = 0$. The graph of $y = x^4 - 0.4x^3 - 2.6x^2 + 1.1x + 1.5$ crosses the x -axis twice. There are two real zeros and they have odd multiplicity. By careful inspection we can see that the graph does not level off at either zero. Therefore, the zeros must have multiplicity 1. (See Exercise 53 in §4.3.) Since the equation has degree 4, there are two nonreal solutions.

- [41] Graph $y = x^4 + 1.4x^3 + 0.44x^2 - 0.56x - 0.96$.

From the graph, zeros are located at -1.2 and 0.8 . Using synthetic division,

$$\frac{x^4 + 1.4x^3 + 0.44x^2 - 0.56x - 0.96}{x + 1.2} = x^3 + 0.2x^2 + 0.2x - 0.8 \text{ and}$$

$$\frac{x^3 + 0.2x^2 + 0.2x - 0.8}{x - 0.8} = x^2 + x + 1. \text{ The zeros of } x^2 + x + 1 \text{ are } -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i.$$

Thus, the solutions to the equation are $-1.2, 0.8, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$.

[-4.5, 4.5] by [-3, 3]

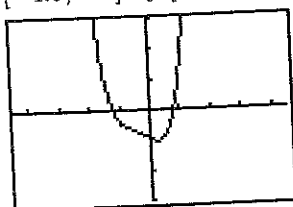


Figure 41

[-6.5, 7] by [-3, 6]

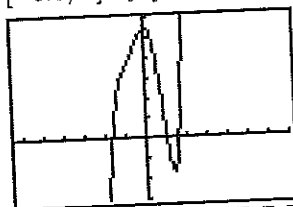


Figure 42

- [42] Graph $y = x^5 + 1.1x^4 - 2.62x^3 - 4.72x^2 - 0.2x + 5.44$.

From the graph, zeros are located at $-1.7, 1$, and 1.6 . Using synthetic division,

$$\frac{x^5 + 1.1x^4 - 2.62x^3 - 4.72x^2 - 0.2x + 5.44}{x + 1.7} = x^4 - 0.6x^3 - 1.6x^2 - 2x + 3.2,$$

$$\frac{x^4 - 0.6x^3 - 1.6x^2 - 2x + 3.2}{x - 1} = x^3 + 0.4x^2 - 1.2x - 3.2, \text{ and}$$

$$\frac{x^3 + 0.4x^2 - 1.2x - 3.2}{x - 1.6} = x^2 + 2x + 2. \text{ The zeros of } x^2 + 2x + 2 \text{ are } -1 \pm i.$$

Thus, the solutions to the equation are $-1.7, 1, 1.6, -1 \pm i$.

- [43] From the graph, we see that $D(h) = 0.4$ when $h \approx 10,200$.

Thus, the density of the atmosphere is 0.4 kg/m^3 at 10,200 m.

$[0, 3E4, 2E3]$ by $[0, 1.2, 0.2]$

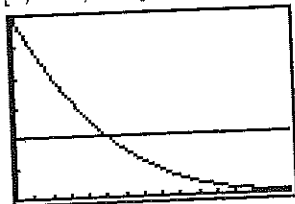


Figure 43

$[0, 1000, 100]$ by $[0, 5]$

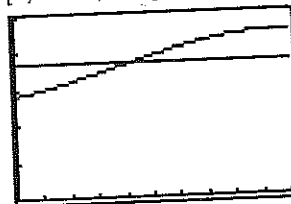


Figure 44

- [44] From the graph, we see that $D(h) = 3.7$ when $h \approx 418$. Thus, the density of the earth is 3.7 g/cm^3 at 418 m. (The graphs also intersect at $h \approx -674$ and $h \approx 1394$. However, these values are not in the domain of D .)

4.5 Exercises

- [1] (b) D = all nonzero real numbers; $R = D$ (c) Decreasing on $(-\infty, 0)$ and on $(0, \infty)$

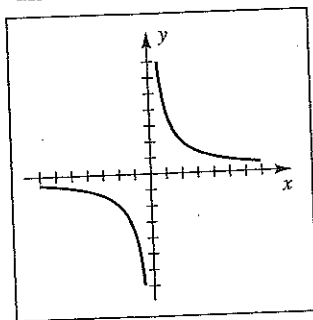


Figure 1

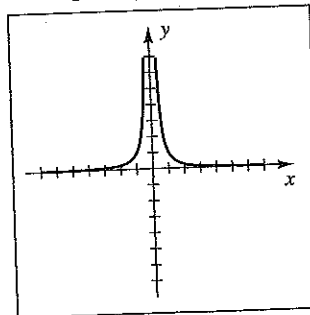


Figure 2

- [2] (b) D = all nonzero real numbers; $R = (0, \infty)$
 (c) Increasing on $(-\infty, 0)$, decreasing on $(0, \infty)$

- [3] $f(x) = \frac{-2(x+5)(x-6)}{(x-3)(x-6)} = \frac{-2(x+5)}{(x-3)}$ if $x \neq 6$ • Note that $x-6$ appears in both the numerator and denominator, so there is a hole at $x = 6$. The y -value for the hole can be found by substituting 6 for x in the rest of the function. $\frac{-2(6+5)}{(6-3)} = \frac{-2(11)}{3} = -\frac{22}{3}$, so the hole is at $(6, -\frac{22}{3})$. There is one other zero of the denominator, namely 3, so there is a vertical asymptote of $x = 3$. The degrees of the numerator and denominator are the same, namely 2, so the ratio of leading coefficients gives us the value of the horizontal asymptote. In this case, it is $y = \frac{-2}{1} = -2$.

[4] $f(x) = \frac{2(x+4)(x+2)}{5(x+2)(x-1)} = \frac{2(x+4)}{5(x-1)}$ if $x \neq -2$ • There is a hole at $x = -2$. Its y -

value is $\frac{2(2)}{5(-3)} = -\frac{4}{15}$. The vertical asymptote is $x = 1$ and the horizontal asymptote is $y = \frac{2}{5}$.

- [5] There is a hole at $x = -2$, so $(x+2)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x = 1$, so $(x-1)$ must be a factor in the denominator. There is an x -intercept at -3 , so $(x+3)$ must be a factor in the numerator. The horizontal asymptote is $y = 2$, so the ratio of leading coefficients must be 2. Combining this information gives us this possibility:

$$f(x) = \frac{2(x+3)(x+2)}{(x-1)(x+2)}$$

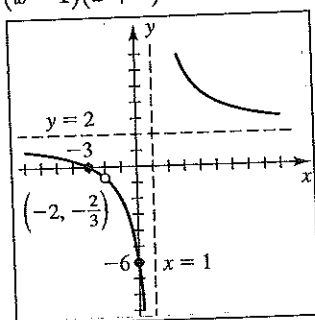


Figure 5

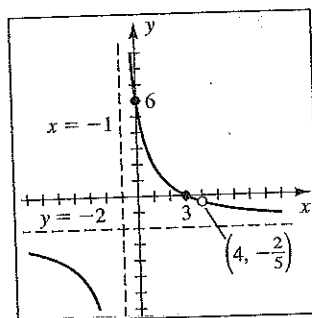


Figure 6

- [6] There is a hole at $x = 4$, so $(x-4)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x = -1$, so $(x+1)$ must be a factor in the denominator. There is an x -intercept at 3, so $(x-3)$ must be a factor in the numerator. The horizontal asymptote is $y = -2$, so the ratio of leading coefficients must be -2 . Combining this information gives us this possibility:

$$f(x) = \frac{-2(x-3)(x-4)}{(x+1)(x-4)}$$

[7] $f(x) = \frac{3}{x-4}$ • VA of $x = 4$

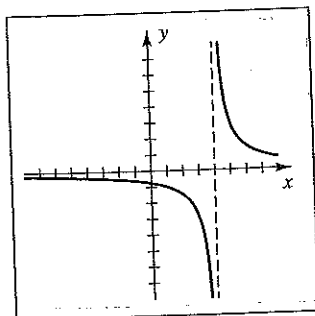


Figure 7

[8] $f(x) = \frac{-3}{x+3}$ • VA of $x = -3$

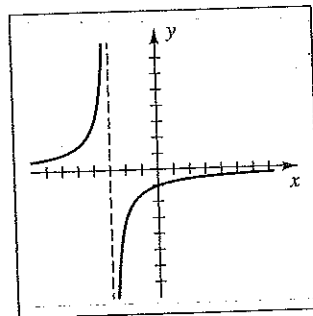


Figure 8

9 $f(x) = \frac{-3x}{x+2}$ • VA of $x = -2$

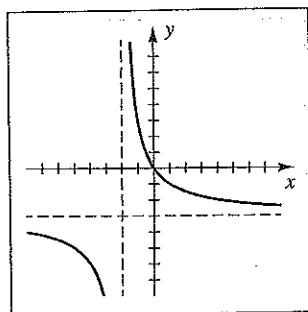


Figure 9

10 $f(x) = \frac{4x}{2x-5}$ • VA of $x = \frac{5}{2}$

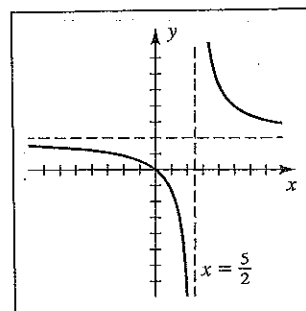


Figure 10

11 $f(x) = \frac{4x-1}{2x+3}$ • VA of $x = -\frac{3}{2}$

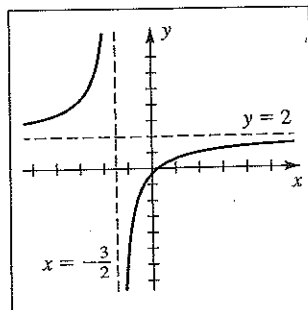


Figure 11

12 $f(x) = \frac{5x+3}{3x-7}$ • VA of $x = \frac{7}{3}$

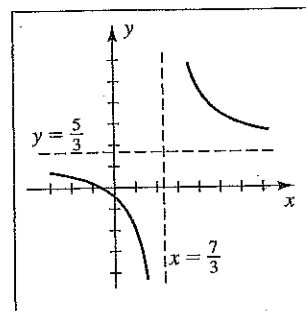


Figure 12

13 $f(x) = \frac{(4x-1)(x-2)}{(2x+3)(x-2)}$ •

VA of $x = -\frac{3}{2}$; hole at $x = 2$

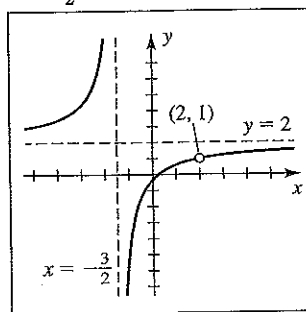


Figure 13

14 $f(x) = \frac{(5x+3)(x+1)}{(3x-7)(x+1)}$ •

VA of $x = \frac{7}{3}$; hole at $x = -1$

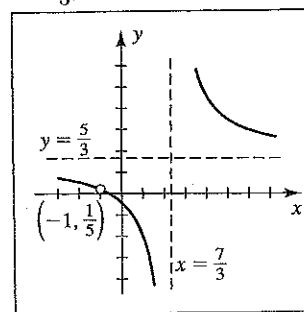


Figure 14

Note: Let I denote the point of intersection between the function and its horizontal or oblique asymptote. See Exercises 22 and 36 for more detailed work on finding I .

$$[15] f(x) = \frac{x-2}{x^2-x-6} = \frac{x-2}{(x+2)(x-3)}; I = (2, 0)$$

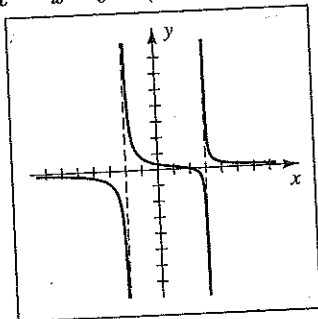


Figure 15

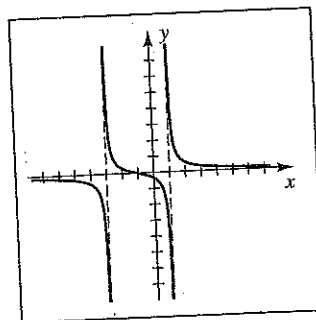


Figure 16

$$[16] f(x) = \frac{x+1}{x^2+2x-3} = \frac{x+1}{(x+3)(x-1)}; I = (-1, 0)$$

$$[17] f(x) = \frac{-4}{(x-2)^2}$$

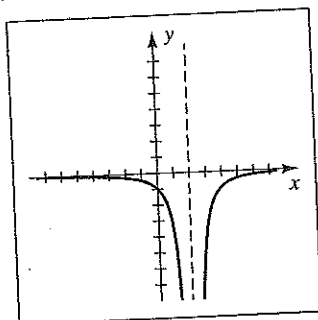


Figure 17

$$[18] f(x) = \frac{2}{(x+1)^2}$$

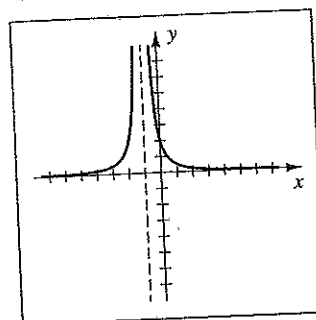


Figure 18

$$[19] f(x) = \frac{x-3}{x^2-1} = \frac{x-3}{(x+1)(x-1)}; I = (3, 0)$$

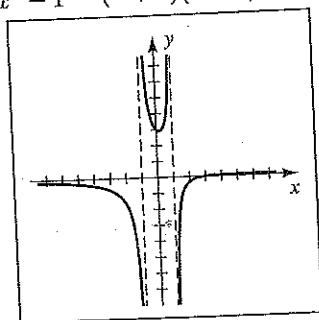


Figure 19

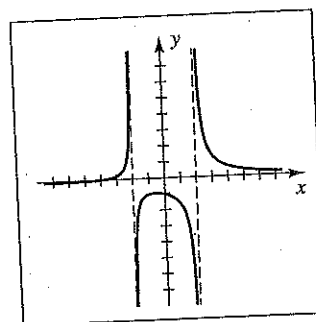


Figure 20

$$[20] f(x) = \frac{x+4}{x^2-4} = \frac{x+4}{(x+2)(x-2)}; I = (-4, 0)$$

$$\boxed{21} \quad f(x) = \frac{2x^2 - 2x - 4}{x^2 + x - 12} = \frac{2(x+1)(x-2)}{(x+4)(x-3)}; I = (5, 2)$$

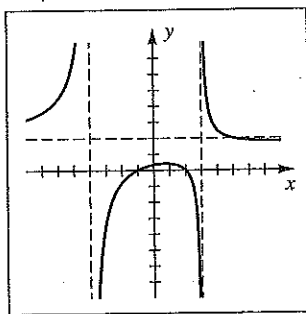


Figure 21

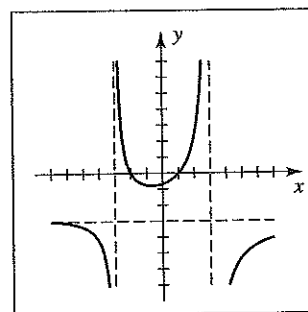


Figure 22

$$\boxed{22} \quad f(x) = \frac{-3x^2 - 3x + 6}{x^2 - 9} = \frac{-3(x+2)(x-1)}{(x+3)(x-3)}$$

$$f(x) = -3 \Rightarrow \frac{-3x^2 - 3x + 6}{x^2 - 9} = -3 \Rightarrow -3x^2 - 3x + 6 = -3x^2 + 27 \Rightarrow -3x = 21 \Rightarrow x = -7, I = (-7, -3).$$

$$\boxed{23} \quad f(x) = \frac{-x^2 - x + 6}{x^2 + 3x - 4} = \frac{-1(x+3)(x-2)}{(x+4)(x-1)}; I = (-1, -1)$$

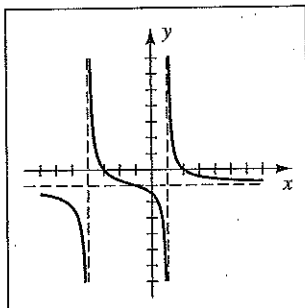


Figure 23

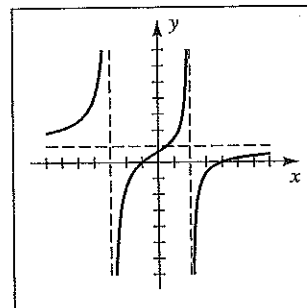


Figure 24

$$\boxed{24} \quad f(x) = \frac{x^2 - 3x - 4}{x^2 + x - 6} = \frac{(x+1)(x-4)}{(x+3)(x-2)}; I = (\frac{1}{2}, 1)$$

$$\boxed{25} \quad f(x) = \frac{3x^2 - 3x - 36}{x^2 + x - 2} = \frac{3(x+3)(x-4)}{(x+2)(x-1)}; I = (-5, 3)$$

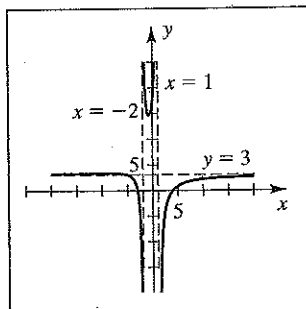


Figure 25

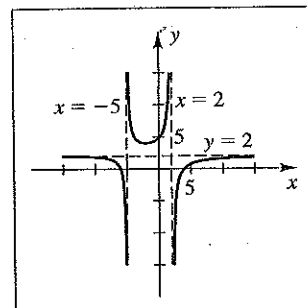


Figure 26

$$\boxed{26} \quad f(x) = \frac{2x^2 + 4x - 48}{x^2 + 3x - 10} = \frac{2(x+6)(x-4)}{(x+5)(x-2)}; I = (-14, 2)$$

$$[27] f(x) = \frac{-2x^2 + 10x - 12}{x^2 + x} = \frac{-2(x-2)(x-3)}{(x+1)(x)}; I = (1, -2)$$

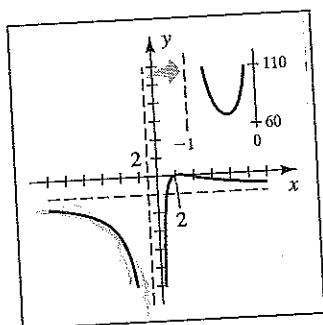


Figure 27

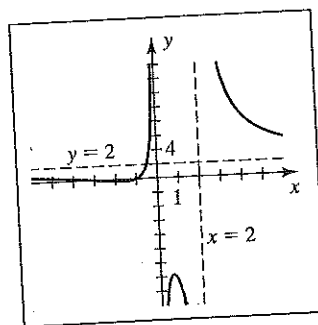


Figure 28

$$[28] f(x) = \frac{2x^2 + 8x + 6}{x^2 - 2x} = \frac{2(x+3)(x+1)}{(x)(x-2)}; I = (-\frac{1}{2}, 2)$$

$$[29] f(x) = \frac{x-1}{x^3-4x} = \frac{x-1}{(x+2)(x)(x-2)}; I = (1, 0)$$

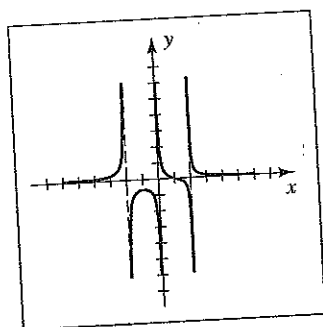


Figure 29

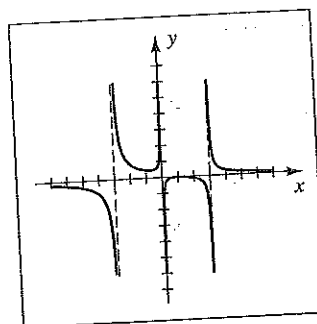


Figure 30

$$[30] f(x) = \frac{x^2 - 2x + 1}{x^3 - 9x} = \frac{(x-1)^2}{(x+3)(x)(x-3)}; I = (1, 0)$$

$$[31] f(x) = \frac{-3x^2}{x^2 + 1}; f \text{ is an even function}$$

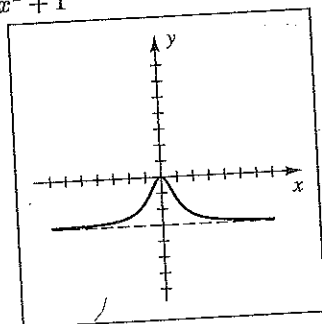


Figure 31

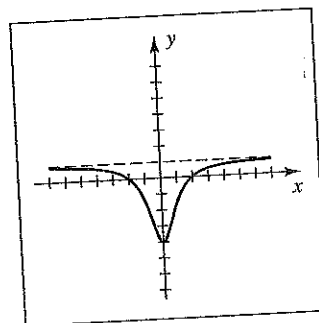


Figure 32

$$[32] f(x) = \frac{x^2 - 4}{x^2 + 1} = \frac{(x+2)(x-2)}{x^2 + 1}; f \text{ is an even function}$$

$$\boxed{33} \quad f(x) = \frac{x^2 - x - 6}{x + 1} = \frac{(x + 2)(x - 3)}{x + 1} = x - 2 - \frac{4}{x + 1}.$$

The expression $\frac{4}{x + 1} \rightarrow 0$ as $x \rightarrow \pm \infty$, so $y = x - 2$ is an oblique asymptote for f .

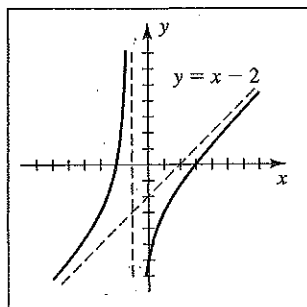


Figure 33

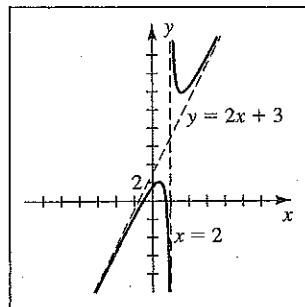


Figure 34

$$\boxed{34} \quad f(x) = \frac{2x^2 - x - 3}{x - 2} = \frac{(x + 1)(2x - 3)}{x - 2} = 2x + 3 + \frac{3}{x - 2}$$

$$\boxed{35} \quad f(x) = \frac{8 - x^3}{2x^2} = \frac{(2 - x)(4 + 2x + x^2)}{2x^2} = -\frac{1}{2}x + \frac{8}{2x^2}$$

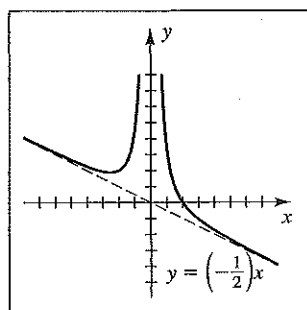


Figure 35

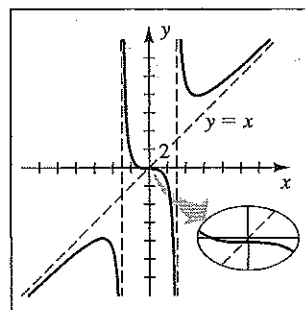


Figure 36

$$\boxed{36} \quad f(x) = \frac{x^3 + 1}{x^2 - 9} = \frac{(x + 1)(x^2 - x + 1)}{(x + 3)(x - 3)} = x + \frac{9x + 1}{x^2 - 9}$$

$$f(x) = x \Rightarrow \frac{x^3 + 1}{x^2 - 9} = x \Rightarrow x^3 + 1 = x^3 - 9x \Rightarrow x = -\frac{1}{9}, I = (-\frac{1}{9}, -\frac{1}{9}).$$

$$\boxed{37} \quad f(x) = \frac{2x^2 + x - 6}{x^2 + 3x + 2} = \frac{(x+2)(2x-3)}{(x+2)(x+1)} = \frac{2x-3}{x+1} \text{ for } x \neq -2;$$

To determine the value of y when $x = -2$, substitute -2 into $\frac{2x-3}{x+1}$ to get 7 .
There is a hole in the graph at $(-2, 7)$.

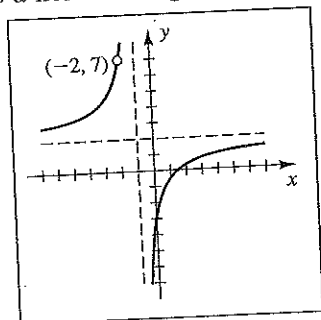


Figure 37

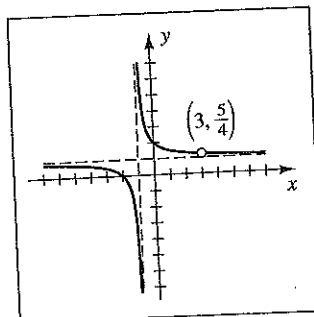


Figure 38

$$\boxed{38} \quad f(x) = \frac{x^2 - x - 6}{x^2 - 2x - 3} = \frac{(x+2)(x-3)}{(x+1)(x-3)} = \frac{x+2}{x+1} \text{ for } x \neq 3; \text{ hole at } (3, \frac{5}{4}).$$

$$\boxed{39} \quad f(x) = \frac{x-1}{1-x^2} = \frac{x-1}{(1+x)(1-x)} = \frac{-1}{x+1} \text{ for } x \neq 1; \text{ hole at } (1, -\frac{1}{2})$$

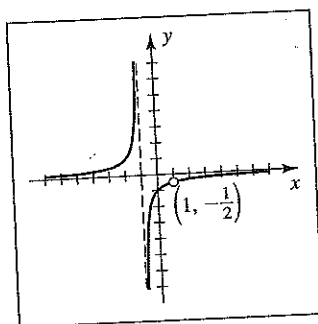


Figure 39

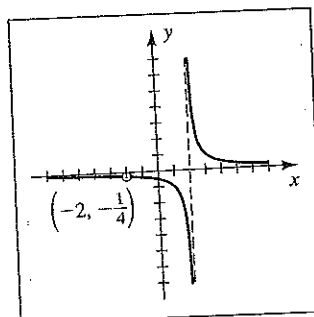


Figure 40

$$\boxed{40} \quad f(x) = \frac{x+2}{x^2-4} = \frac{x+2}{(x+2)(x-2)} = \frac{1}{x-2} \text{ for } x \neq -2; \text{ hole at } (-2, -\frac{1}{4})$$

$$\boxed{41} \quad f(x) = \frac{x^2 + x - 2}{x + 2} = \frac{(x+2)(x-1)}{x+2} = x-1 \text{ for } x \neq -2; \text{ hole at } (-2, -3)$$

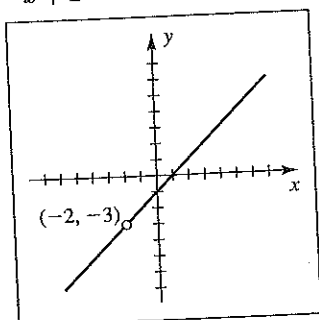


Figure 41

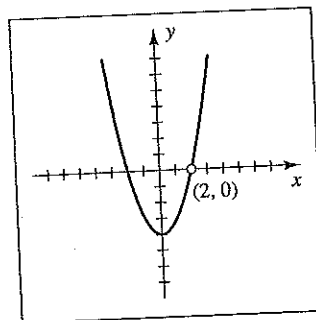


Figure 42

$$\boxed{42} \quad f(x) = \frac{x^3 - 2x^2 - 4x + 8}{x - 2} = \frac{(x^2 - 4)(x - 2)}{x - 2} = x^2 - 4 \text{ for } x \neq 2; \text{ hole at } (2, 0)$$

$$[43] f(x) = \frac{x^2 + 4x + 4}{x^2 + 3x + 2} = \frac{(x+2)^2}{(x+1)(x+2)} = \frac{x+2}{x+1} \text{ for } x \neq -2.$$

Note that the hole is on the x -axis at $(-2, 0)$.

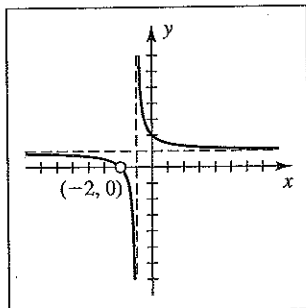


Figure 43

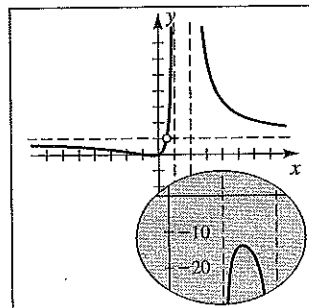


Figure 44

$$[44] f(x) = \frac{(x^2 + x)(2x - 1)}{(x^2 - 3x + 2)(2x - 1)} = \frac{x(x+1)(2x-1)}{(x-1)(x-2)(2x-1)} = \frac{x(x+1)}{(x-1)(x-2)} \text{ for } x \neq \frac{1}{2}.$$

Note that the hole is on the horizontal asymptote at $(\frac{1}{2}, 1)$.

$$[45] f(x) = \frac{-1(x-3)}{x-4} = \frac{3-x}{x-4}$$

$$[46] f(x) = \frac{a(x-2)}{x(x+2)}; f(3) = 1 \text{ and } f(3) = \frac{a(1)}{3(5)} = \frac{a}{15} \Rightarrow a = 15; f(x) = \frac{15x-30}{x^2+2x}$$

$$[47] f(x) = \frac{a(x+1)(x-2)}{(x-1)(x+3)(x-2)}; f(0) = -2 \text{ and } f(0) = \frac{a(1)}{(-1)(3)} = \frac{a}{-3} \Rightarrow a = 6;$$

$$f(x) = \frac{6(x+1)(x-2)}{(x-1)(x+3)(x-2)} = \frac{6x^2-6x-12}{x^3-7x+6}$$

$$[48] f(x) = \frac{2(x+2)(x-1)x}{(x+1)(x-3)x} = \frac{2x^3+2x^2-4x}{x^3-2x^2-3x}$$

$$[49] \text{ (a) The radius of the outside cylinder is } (r+0.5) \text{ ft and its height is } (h+1) \text{ ft.}$$

$$\text{Since the volume is } 16\pi \text{ ft}^3, \text{ we have } 16\pi = \pi(r+0.5)^2(h+1) \Rightarrow$$

$$h = \frac{16}{(r+0.5)^2} - 1.$$

$$\text{(b) } V(r) = \pi r^2 h = \pi r^2 \left[\frac{16}{(r+0.5)^2} - 1 \right]$$

$$\text{(c) } r \text{ and } h \text{ must both be positive. } h > 0 \Rightarrow \frac{16}{(r+0.5)^2} - 1 > 0 \Rightarrow$$

$$16 > (r+0.5)^2 \Rightarrow |r+0.5| < 4 \Rightarrow -4.5 < r < 3.5. \text{ The last inequality}$$

combined with $r > 0$ means that the excluded values are $r \leq 0$ and $r \geq 3.5$.

[50] $a = 100$ and $y = ta/(t+12) \Rightarrow y = 100t/(t+12)$.

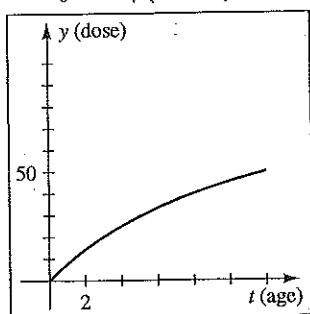


Figure 50

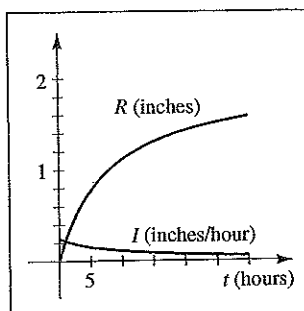


Figure 52

- [51] (a) Since 5 gallons of water flow into the tank each minute, $V(t) = 50 + 5t$. Since each additional gallon of water contains 0.1 lb of salt, $A(t) = 5(0.1)t = 0.5t$.

(b) $c(t) = \frac{A(t)}{V(t)} = \frac{0.5t}{50 + 5t} = \frac{t}{10t + 100}$ lb/gal

- (c) As $t \rightarrow \infty$, $c(t) \rightarrow 0.1$ lb. of salt per gal.

- [52] (a) As t increases, the total number of inches of rain approaches the constant a .

- (b) Note that at the start of the storm, $I = 0.25$ and $R = 0$, i.e.,

the intensity is greater than the accumulation. See Figure 52.

[53] (a) $R > S \Rightarrow \frac{4500S}{S+500} > S \Rightarrow \frac{S(S-4000)}{S+500} < 0 \{S > 0\} \Rightarrow 0 < S < 4000$

- (b) The greatest possible number of offspring that survive to maturity is 4500, the horizontal asymptote value. 90% of 4500 is 4050.

$$R = 4050 \Rightarrow 4050 = \frac{4500S}{S+500} \Rightarrow 4050S + 2,025,000 = 4500S \Rightarrow 2,025,000 = 450S \Rightarrow S = 4500.$$

- (c) 80% of 4500 is 3600. $R = 3600 \Rightarrow S = 2000$.

- (d) A 125% increase $\left\{ \frac{4500 - 2000}{2000} \times 100 \right\}$ in the number S of spawners produces only a 12.5% increase $\left\{ \frac{4050 - 3600}{3600} \times 100 \right\}$ in the number R of offspring surviving to maturity.

- [54] (a) $x = 20 \Rightarrow D \approx 229.4$ and $x = 25 \Rightarrow D = 189.1$; the density decreases

- (b) As $x \rightarrow \infty$, $D \rightarrow 0$.

The density gets closer to 0 as the distance from the center increases.

(c) $D > 400 \Rightarrow \frac{5000x}{x^2 + 36} > 400 \Rightarrow$

$$25x > 2(x^2 + 36) \left\{ \text{multiply by } \frac{1}{200}(x^2 + 36), \text{ which is positive} \right\} \Rightarrow 2x^2 - 25x + 72 < 0 \Rightarrow (2x - 9)(x - 8) < 0 \Rightarrow 4.5 < x < 8$$

- [55] Assign $20x^2 + 80x + 72$ to Y_1 , $10x^2 + 40x + 41$ to Y_2 , and Y_1/Y_2 to Y_3 . Zoom-in around $(-2, -8)$ to confirm that this is a low point and that there is not a vertical asymptote at $x = -2$. To determine the vertical asymptotes, graph Y_2 only {turn off Y_3 }, and look for its zeros. If these values are not zeros of the numerator, then they are the values of the vertical asymptotes. No vertical asymptotes in this case.

$[-9, 3]$ by $[-9, 3]$

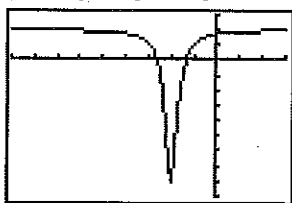


Figure 55

$[-5, 9]$ by $[-1, 13]$

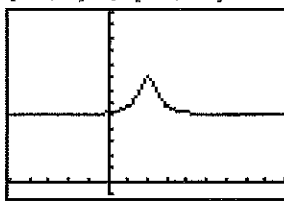


Figure 56

- [56] Similar to Exercise 55, there is a high point at $(2, 8)$. No vertical asymptotes.

- [57] $f(x) = \frac{(x-1)^2}{(x-0.999)^2}$ • Note that a standard viewing rectangle gives the horizontal line $y = 1$. Figure 57 was obtained by using Dot mode. An equation of the vertical asymptote is $x = 0.999$.

$[0.7, 1.3, 0.1]$ by $[0.8, 1.2, 0.1]$

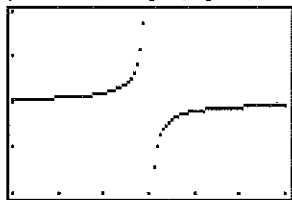


Figure 57

$[-4, 8]$ by $[-1, 7]$

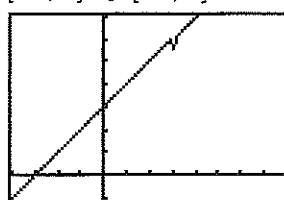


Figure 58

- [58] $f(x) = \frac{x^2 - 9.01}{x - 3}$ • Note that a standard viewing rectangle gives a line, but we recognize that there is a vertical asymptote at $x = 3$ since $x - 3$ is a factor of the denominator but not of the numerator.

- [59] (a) The graph of g is the horizontal line $y = 1$ with holes at $x = 0, \pm 1, \pm 2, \pm 3$.

The TI-85/86 shows a small break in the line to indicate a hole.

- (b) The graph of h is the graph of p with holes at $x = 0, \pm 1, \pm 2, \pm 3$.

- [60] (a) The graph of f is that of a seventh-degree polynomial with zeros at $x = 0, \pm 1, \pm 2, \pm 3$. Its sign changes at each zero.
- (b) The graph of k has vertical asymptotes at $x = 0, \pm 1, \pm 2, \pm 3$. Its sign changes at each asymptote. The values of k are reciprocals of the values of f —so as f gets larger, k gets smaller, and vice versa. Try graphing k with a viewing rectangle of $[-4, 4]$ by $[-0.5, 0.5]$.

[61] (a) GPA with additional credits = desired GPA $\Rightarrow \frac{48(2.75) + y(4.0)}{48 + y} = x \Rightarrow$
 $132 + 4y = 48x + xy \Rightarrow 132 - 48x = xy - 4y \Rightarrow 132 - 48x = y(x - 4) \Rightarrow$
 $y = \frac{132 - 48x}{x - 4}$

(b)

X	Y1
2	12
3	27
4	52
5	102
6	208
7	408
8	808
9	1608
10	3208
11	6408
12	12808
13	25608
14	51208
15	102408
16	204808
17	409608
18	819208
19	1638408
20	3276808
21	6553608
22	13107208
23	26214408
24	52428808
25	104857608
26	209715208
27	419430408
28	838860808
29	1677721608
30	3355443208
31	6710886408
32	13421772808
33	26843545608
34	53687091208
35	107374182408
36	214748364808
37	429496729608
38	858993459208
39	1717986918408
40	3435973836808
41	6871947673608
42	13743895347208
43	27487790694408
44	54975581388808
45	109951162777608
46	219902325555208
47	439804651110408
48	879609302220808
49	1759218604441608
50	3518437208883208
51	7036874417766408
52	14073748835532808
53	28147497671065608
54	56294995342131208
55	112589990684262408
56	225179981368524808
57	450359962737049608
58	900719925474099208
59	1801439850948198408
60	3602879701896396808
61	7205759403792793608
62	14411518807585587208
63	28823037615171174408
64	57646075230342348808
65	115292150460684697608
66	230584300921369395208
67	461168601842738790408
68	922337203685477580808
69	1844674407370955161608
70	3689348814741910323208
71	7378697629483820646408
72	14757395258967641292808
73	29514790517935282585608
74	59029581035870565171208
75	118059162071741130342408
76	236118324143482260684808
77	472236648286964521369608
78	944473296573929042739208
79	1888946593147858085478408
80	3777893186295716170956808
81	7555786372591432341913608
82	15111572745182864683827208
83	30223145490365729367654408
84	60446290980731458735308808
85	120892581961462917470617608
86	241785163922925834941235208
87	483570327845851669882470408
88	967140655691703339764940808
89	1934281311383406679529881608
90	3868562622766813359059763208
91	7737125245533626718119526408
92	15474250491067253436239052808
93	30948500982134506872478105608
94	61897001964269013744956211208
95	123794003928538027489912422408
96	247588007857076054979824844808
97	495176015714152109959649689608
98	990352031428304219919299379208
99	1980704062856608439838598758408
100	3961408125713216879677197516808
101	7922816251426433759354395033608
102	15845632502852867518708790067208
103	31691265005705735037417580134408
104	63382530011411470074835160268808
105	126765060022822940149670320537608
106	253530120045645880299340641075208
107	507060240091291760598681282150408
108	1014120480182583521197362564300808
109	2028240960365167042394725128601608
110	4056481920730334084789450257203208
111	8112963841460668169578900514406408
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113	32451855365842672678315602057625608
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115	129807421463370690713262408230502408
116	259614842926741381426524816461004808
117	519229685853482762853049632922009608
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122	16615349947311448411297588253504307208
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125	132922799578491587290380706028034457608
126	265845599156983174580761412056068915208
127	531691198313966349161522824112137830408
128	1063382396627932698323045648224275660808
129	2126764793255865396646091296448551321608
130	4253529586511730793292182592897102643208
131	8507059173023461586584365185794205286408
132	17014118346046923173168730371588410572808
133	34028236692093846346337460743176821145608
134	68056473384187692692674921486353642291208
135	136112946768375385385349842972707284582408
136	272225893536750770770699685945414569164808
137	544451787073501541541399371890829138329608
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144	69689828745408197317299119602026129706188808
145	139379657490816394634598239204052259412377608
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148	1115037259926531157076785913632418075299020808
149	2230074519853062314153571827264836150598041608
150	4460149039706124628307143654529672301196083208
151	8920298079412249256614287309059344602392166408
152	17840596158824498513228574618118689204784332808
153	35681192317648997026457149236237378409568665608
154	71362384635297994052914298472474756819137331208
155	142724769270595988105828596944949513638274662408
156	285449538541191976211657193889899027276549324808
157	570899077082383952423314387779798054553098649608
158	1141798154164767904846628775559596109106197299208
159	2283596308329535809693257551119192218212394598408
160	4567192616659071619386515102238384436424789196808
161	9134385233318143238773030204476768872849578393608
162	18268770466636286477546060408953537745699156787208
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164	73075081866545145910184241635814150982796627148808
165	146150163733090291820368483271628301965593254297608
166	292300327466180583640736966543256603931186508595208
167	584600654932361167281473933086513207862373017190408
168	1169201309864722334562947866173026415724746034380808
169	2338402619729444669125895732346052831449492068760808
170	4676805239458889338251791464692105662898984137521608
171	9353610478917778676503582929384211325797968275043208
172	18707220957835557353007165858768422651595936550086408
173	37414441915671114706014331717536845303191873100172808
174	74828883831342229412028663435073690606383746200345608
175	149657767662684458824057326870147381212767492400691208
176	299315535325368917648114653740294762425534984801382408
177	598631070650737835296229307480589524851069969602764808
178	1197262141301475670592458614961179049702139939205529608
179	2394524282602951341184917229922358099404279878411059208
180	4789048565205902682369834459844716198808559756822118408
181	9578097130411805364739668919689432397617119513644236808
182	19156194260823610729479337839378864795234239027288473608
183	38312388521647221458958675678757729590468478054576947208
184	76624777043294442917917351357515459180936956109153894408
185	153249554086588885835834702715030918361873912218307788808
186	306499108173177771671669405430061836723747824436615577608
187	612998216346355543343338810860123673447495648873231155208
188	1225996432692711086686677621720247346894991297746462310408
189	2451992865385422173373355243440494693789982595492924620808
190	4903985730770844346746710486880989387579965190985849240808
191	9807971461541688693493420973761978775159930381971698480808
192	19615942923083377386986841947523957550319860763943396960808
193	39231885846166754773973683895047915100639721527886793920808
194	78463771692333509547947367790095830201279443055773587840808
195	156927543384667019095894735580191660402558886111547175680808
196	313855086769334038191789471160383320805117772223094351360808
197	627710173538668076383578942320766641610235544446188702720808
198	1255420347077336152767157884641533283220471088892377405440808
199	2510840694154672305534315769283066566440942177784754810880808
200	5021681388309344611068631538566133132881884355569509621760808
201	10043362776618689222137263077132266265763768711139019243520808
202	20086725553237378444274526154264532531527537422278038487040808
203	40173451106474756888549052308529065063055074844556076974080808
204	80346902212949513777098104617058130126110149689112153948160808
205	160693804425899027554196209234116260252220299378224307896320808
206	321387608851798055108392418468232520504440598756448615792640808
207	642775217703596110216784836936465041008881197512897231585280808
208	1285550435407192220433569673872930082017762395025794463170560808
209	2571100870814384440867139347745860164035524790051588926341120808
210	5142201741628768881734278695491720328071049580103177852682240808
211	10284403483257537763468557390983440656142099160206355705364480808
212	20568806966515075526937114781966881312284198320412711410728960808
213	41137613933030151053874229563933762624568396640825422821457920808
214	82275227866060302107748459127867525249136793281650845642915840808
215	164550455732120604215496918255735050498273586563301691285831680808
216	329100911464241208430993836511470100996547173126603382571663360808
217	658201822928482416861987673022940201993094346253206765143326720808
218	1316403645856964833723975346045880403986188692506413530286653440808
219	2632807291713929667447950692091760807972377385012827060573306880808
220	5265614583427859334895901384183521615944754770025654121146613760808
221	10531229166855718669791802768367043231889509540051308242293227520808
222	21062458333711437339583605536734086463779019080102616484586455040808
223	42124916667422874679167211073468172927558038160205232969172910080808
224	84249833334845749358334422146936345855116076320410465938345820160808
225	168499666669691498716668844293872691710232152640820931876691640320808
226	33699933333938299743

- 14 (a) $F = kx$
 (b) $4 = k(0.3) \Rightarrow k = \frac{40}{3}$
 (c) $F = \frac{40}{3}(1.5) = 20$ lb
 (d) $F = \frac{40}{3}x$ is a simple linear relationship.
 The graph is a line with slope $\frac{40}{3}$
 and y -intercept 0.

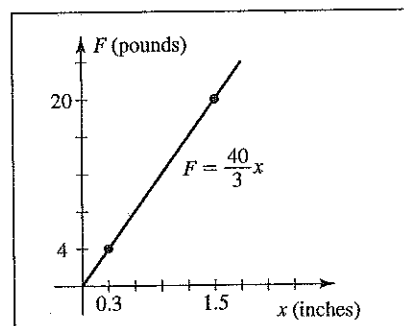


Figure 14(d)

- 15 (a) $R = k \frac{l}{d^2} = \frac{kl}{d^2}$
 (b) $25 = \frac{k(100)}{(0.01)^2} \Rightarrow k = \frac{1}{40,000}$
 (c) $R = \frac{50}{(40,000)(0.015)^2} = \frac{50}{9}$ ohms
 (d) $R = \frac{kl}{d^2} = \frac{(1/40,000)(100)}{d^2} = \frac{1}{400d^2}$

The graph of R for $d > 0$ has
 a vertical asymptote of $R = 0$ and
 a horizontal asymptote of $d = 0$.

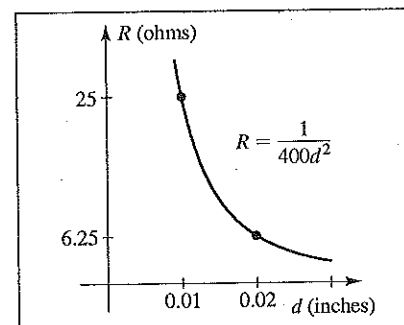


Figure 15(d)

- 16 (a) $I = \frac{k}{d^2}$
 (b) $1,000,000 = \frac{k}{(50)^2} \Rightarrow k = 2.5 \times 10^9$
 (c) $I = \frac{2.5 \times 10^9}{(5280)^2} \approx 89.7$ candlepower
 (d) $I = \frac{2.5 \times 10^9}{d^2}$ has a VA of $I = 0$ and

a HA of $d = 0$.

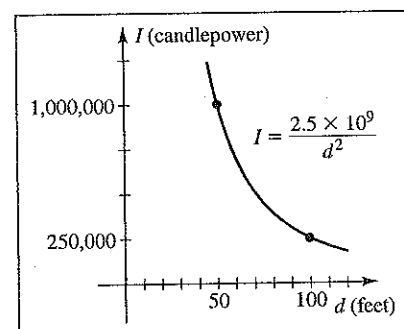


Figure 16(d)

- 17 (a) $P = k\sqrt{l}$
 (c) $P = \frac{3}{4}\sqrt{2}(\sqrt{6}) = \frac{3}{2}\sqrt{3}$ sec

(b) $1.5 = k\sqrt{2} \Rightarrow k = \frac{3}{4}\sqrt{2}$

- 18 (a) $C = 2\pi r \Leftrightarrow r = \frac{C}{2\pi}$. $V = \pi r^2 L = \pi \left(\frac{C}{2\pi}\right)^2 L = \frac{1}{4\pi} C^2 L$.
 (b) $V = \frac{1}{4\pi}(22)^2(27) = \frac{3267}{\pi} \approx 1039.9$ cm³, or 1040 cm³

- 19 (a) $T = kd^{3/2}$
 (c) $T = \frac{365}{(93)^{3/2}} \cdot (67)^{3/2} \approx 223.2$ days

(b) $365 = k(93)^{3/2} \Rightarrow k = \frac{365}{(93)^{3/2}}$

- [20] (a) $R = kv^2$ (b) $150 = k(70)^2 \Rightarrow k = \frac{3}{98}$
 (c) $R = \frac{3}{98}(80)^2 = \frac{9600}{49} \approx 195.9$ ft
- [21] (a) $V = k\sqrt{L}$ (b) $35 = k\sqrt{50} \Rightarrow k = \frac{7}{2}\sqrt{2}$
 (c) $V = \frac{7}{2}\sqrt{2}(\sqrt{150}) = 35\sqrt{3} \approx 60.6$ mi/hr
- [22] (a) $F = k \frac{Q_1 Q_2}{d^2} = \frac{kQ_1 Q_2}{d^2}$
 (b) $F = \frac{kQ_1 Q_2}{(\frac{1}{4}d)^2} = 16 \left(\frac{kQ_1 Q_2}{d^2} \right)$, the force F is multiplied by 16
- [23] (a) $W = kh^3$ (b) $200 = k(6)^3 \Rightarrow k = \frac{25}{27}$
 (c) $W = \frac{25}{27}(\frac{11}{2})^3 \approx 154.1$ lb, or 154 lb
- [24] (a) $V = k \frac{nT}{P} = \frac{knT}{P}$ (b) $V = \frac{k(2n)(\frac{1}{2}T)}{(\frac{1}{2}P)} = 2 \left(\frac{knT}{P} \right)$, i.e., V is doubled.
- [25] (a) $F = kPr^4 \Rightarrow P = \frac{F}{kr^4}$ under normal conditions
 (b) $3F = kP(1.1r)^4 \Rightarrow P = \frac{3F}{(1.1)^4 kr^4} \approx 2.05 \left(\frac{F}{kr^4} \right)$, or about 2.05 times as hard
- [26] $T = kn$; $10 = k(300) \Rightarrow k = \frac{1}{30}$; $200 = \frac{1}{30}n \Rightarrow n = 6000$
- [27] Let k be the constant of variation. Then, $C = \frac{kDE}{Vt} \Rightarrow D = \left(\frac{Ct}{k} \right) \frac{V}{E}$, where $\frac{Ct}{k}$ is constant. If V is twice its original value and E is $\frac{4}{5}$ of its original value, then
 D becomes $\frac{2}{4/5} = \frac{5}{2} = 250\%$ of its original value. Thus, D increases by 250%.
- [28] $C = \frac{kDE}{Vt}$. D increases to $\frac{13}{10}$ of its original value, V decreases to $\frac{9}{10}$ of its original value, and t increases to $\frac{16}{10}$ of its original value. C becomes $\frac{(13/10)}{(9/10)(16/10)} = \frac{65}{72} \approx 90.3\%$ of its original value. Thus, C decreases by approximately 9.7%.
- [29] The square of the distance from the origin to the point (x, y) is $x^2 + y^2$. $d = \frac{k}{x^2 + y^2}$.
 If (x_1, y_1) is the new point that has density d_1 , then
 $d_1 = \frac{k}{x_1^2 + y_1^2} = \frac{k}{(\frac{1}{3}x)^2 + (\frac{1}{3}y)^2} = \frac{k}{\frac{1}{9}x^2 + \frac{1}{9}y^2} = \frac{k}{\frac{1}{9}(x^2 + y^2)} = 9 \cdot \frac{k}{x^2 + y^2} = 9d$.
 The density d is multiplied by 9.
- [30] The distance from the origin to the point (x, y) is $\sqrt{x^2 + y^2}$. $T = k/\sqrt{x^2 + y^2}$.
 $T = 20$ and $P(3, 4) \Rightarrow 20 = k/5 \Rightarrow k = 100$.
 $T = 100/\sqrt{24^2 + 7^2} = 100/25 = 4^\circ\text{C}$.
- [31] $\frac{y}{x} = \frac{0.72}{0.6} = \frac{1.44}{1.2} = \frac{5.04}{4.2} = \frac{8.52}{7.1} = \frac{11.16}{9.3} = 1.2$.
 Thus, y varies directly as x with constant of variation $k = 1.2$, and $y = 1.2x$.
- [32] $xy = -5.3$ for each data point.
 Thus, y varies inversely as x with constant of variation $k = -5.3$, and $y = -\frac{5.3}{x}$.

- [33] $x^2y = -10.1$ for each data point.

Thus, y varies inversely as x^2 with constant of variation $k = -10.1$, and $y = -\frac{10.1}{x^2}$.

- [34] $y/x^3 = 2.67$ for each data point. Thus,

y varies directly as the cube of x with constant of variation $k = 2.67$, and $y = 2.67x^3$.

- [35] (a) $D = kS^{2.3} \Rightarrow k = \frac{D}{S^{2.3}}$. Using the 6 data points: $\frac{33}{20^{2.3}} \approx 0.0336$;

$$\frac{86}{30^{2.3}} \approx 0.0344; \frac{167}{40^{2.3}} \approx 0.0345; \frac{278}{50^{2.3}} \approx 0.0344; \frac{414}{60^{2.3}} \approx 0.0337; \frac{593}{70^{2.3}} \approx 0.0338.$$

Let $k = 0.034$. Thus, $D = 0.034S^{2.3}$.

- (b) Graph the data [0, 75, 10] by [0, 600, 100]
together with
 $Y_1 = 0.034x^{2.3}$.

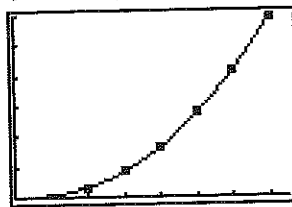


Figure 35

Chapter 4 Review Exercises

- [1] Shift $y = x^3$ left 2 units. $f(x) > 0$ if $x > -2$, $f(x) < 0$ if $x < -2$.

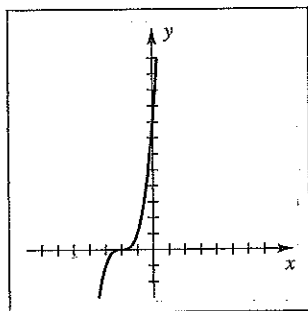


Figure 1

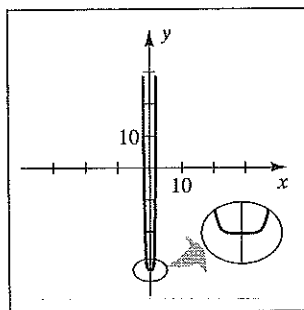


Figure 2

- [2] Shift $y = x^6$ down 32 units.

$$f(x) > 0 \text{ if } x < -\sqrt[6]{32} \text{ or } x > \sqrt[6]{32}, f(x) < 0 \text{ if } -\sqrt[6]{32} < x < \sqrt[6]{32}.$$

- [3] $f(x) = -\frac{1}{4}(x+2)(x-1)^2(x-3)$ has zeros at -2 , 1 (multiplicity 2), and 3 .

$f(x) > 0$ if $-2 < x < 1$ or $1 < x < 3$, $f(x) < 0$ if $x < -2$ or $x > 3$.

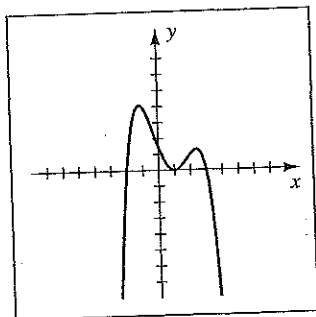


Figure 3

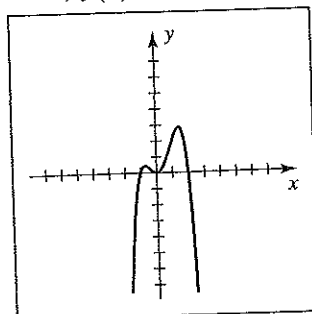


Figure 4

- [4] $f(x) = 2x^2 + x^3 - x^4 = x^2(2 + x - x^2) = -x^2(x-2)(x+1)$.

$f(x) > 0$ if $-1 < x < 0$ or $0 < x < 2$, $f(x) < 0$ if $x < -1$ or $x > 2$.

- [5] $f(x) = x^3 + 2x^2 - 8x = x(x^2 + 2x - 8) = x(x+4)(x-2)$.

$f(x) > 0$ if $-4 < x < 0$ or $x > 2$, $f(x) < 0$ if $x < -4$ or $0 < x < 2$.

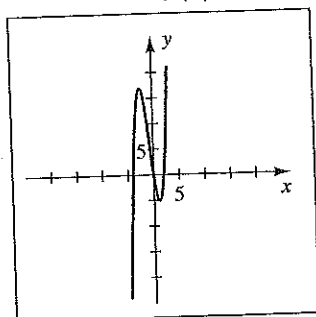


Figure 5

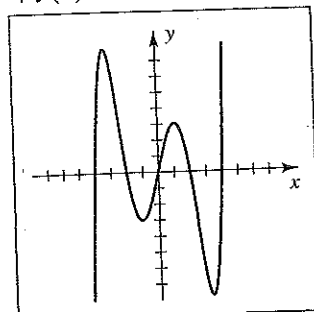


Figure 6

- [6] $f(x) = \frac{1}{15}(x^5 - 20x^3 + 64x) = \frac{1}{15}x(x^4 - 20x^2 + 64) = \frac{1}{15}x(x^2 - 4)(x^2 - 16) = \frac{1}{15}x(x+2)(x-2)(x+4)(x-4)$. $f(x) > 0$ if $-4 < x < -2$, $0 < x < 2$, or $x > 4$,
 $f(x) < 0$ if $x < -4$, $-2 < x < 0$, or $2 < x < 4$.

- [7] $f(0) = -9 < 100$ and $f(10) = 561 > 100$. By the intermediate value theorem for polynomial functions, f takes on every value between -9 and 561 . Hence, there is at least one real number a in $[0, 10]$ such that $f(a) = 100$.

- [8] Let $f(x) = x^5 - 3x^4 - 2x^3 - x + 1$. $f(0) = 1 > 0$ and $f(1) = -4 < 0$. By the intermediate value theorem for polynomial functions, f takes on every value between -4 and 1 . Hence, there is at least one real number a in $[0, 1]$ such that $f(a) = 0$.

- [9] $f(x) = 3x^5 - 4x^3 + x + 5$; $p(x) = x^3 - 2x + 7$ • $\star 3x^2 + 2$; $-21x^2 + 5x - 9$

- [10] $f(x) = 4x^3 - x^2 + 2x - 1$; $p(x) = x^2$ • $\star 4x - 1$; $2x - 1$

- [11] Dividing $f(x) = -4x^4 + 3x^3 - 5x^2 + 7x - 10$ by $x + 2$ yields -132 .

- [12] If $f(x) = 2x^4 - 5x^3 - 4x^2 + 9$, then $f(3) = 0$ and $x - 3$ is a factor of f .

[13] $\star 6x^4 - 12x^3 + 24x^2 - 52x + 104; -200$

[14] $\star 2x^2 + (5 + 2\sqrt{2})x + (2 + 5\sqrt{2}); 11 + 2\sqrt{2}$

[15] $f(x) = a[x - (-3 + 5i)][x - (-3 - 5i)](x + 1) = a(x^2 + 6x + 34)(x + 1).$

$f(1) = a(41)(2)$ and $f(1) = 4 \Rightarrow 82a = 4 \Rightarrow a = \frac{2}{41}.$

Hence, $f(x) = \frac{2}{41}(x^2 + 6x + 34)(x + 1).$

[16] $f(x) = a[x - (1 - i)][x - (1 + i)](x - 3)(x) = ax(x^2 - 2x + 2)(x - 3).$

$f(2) = a(2)(2)(-1)$ and $f(2) = -1 \Rightarrow -4a = -1 \Rightarrow a = \frac{1}{4}.$

Hence, $f(x) = \frac{1}{4}x(x^2 - 2x + 2)(x - 3).$

[17] $f(x) = x^5(x + 3)^2$
 $= x^5(x^2 + 6x + 9)$
 $= x^7 + 6x^6 + 9x^5$

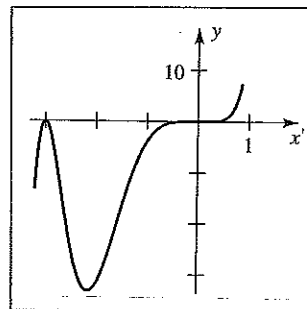


Figure 17

[18] Synthetically dividing $f(x) = x^5 - 4x^4 - 3x^3 + 34x^2 - 52x + 24$ by $x - 2$ three times gives us $(x - 2)^3(x^2 + 2x - 3)$. Hence, $f(x) = (x - 2)^3(x + 3)(x - 1).$

[19] $f(x) = (x^2 - 2x + 1)^2(x^2 + 2x - 3) = [(x - 1)^2]^2(x + 3)(x - 1) = (x + 3)(x - 1)^5$

$\star 1$ (multiplicity 5); -3 (multiplicity 1)

[20] $f(x) = x^6 + 2x^4 + x^2 = x^2(x^4 + 2x^2 + 1) = x^2(x^2 + 1)^2$

$\star 0, \pm i$ (all have multiplicity 2)

[21] (a) Let $f(x) = 2x^4 - 4x^3 + 2x^2 - 5x - 7$. Since there are 3 sign changes in $f(x)$ and 1 sign change in $f(-x)$, there are either 3 positive and 1 negative solution or

1 positive, 1 negative, and 2 nonreal complex solutions.

(b) Upper bound is 3, lower bound is -1

[22] (a) Let $f(x) = x^5 - 4x^3 + 6x^2 + x + 4$. Since there are 2 sign changes in $f(x)$ and 3 sign changes in $f(-x)$, there are either 2 positive and 3 negative solutions;

2 positive, 1 negative, and 2 nonreal complex;

3 negative and 2 nonreal complex;

or 1 negative and 4 nonreal complex solutions.

(b) Upper bound is 2, lower bound is -3

[23] Since there are only even powers, $7x^6 + 2x^4 + 3x^2 + 10 \geq 10$ for every real number x .

[24] $x^4 + 9x^3 + 31x^2 + 49x + 30 = 0 \quad \bullet \quad \star -3, -2, -2 \pm i$

[25] $16x^3 - 20x^2 - 8x + 3 = 0 \quad \bullet \quad \star -\frac{1}{2}, \frac{1}{4}, \frac{3}{2}$

[26] $x^4 - 7x^2 + 6 = 0$ •

★ $\pm\sqrt{6}, \pm 1$

- [27] The graph has x -intercepts at -2 , 1 , and 3 . The equation must have the form $f(x) = a(x+2)^m(x-1)^n(x-3)^p$, where $m+n+p=6$ since f is a sixth-degree polynomial. The graph goes through the x -axis at m and p , so they must be odd, and it doesn't change sign at n , so it must be even. Since the graph flattens out at $x = -2$, m must be greater than p . The only possibility is $m=3$, $n=2$, and $p=1$, so f has the form $f(x) = a(x+2)^3(x-1)^2(x-3)$. The y -intercept is 4 , so $f(0) = 4 \Rightarrow 4 = a(2)^3(-1)^2(-3) \Rightarrow -24a = 4 \Rightarrow a = -\frac{1}{6}$. Thus,

$$f(x) = -\frac{1}{6}(x+2)^3(x-1)^2(x-3).$$

- [28] The graph has x -intercepts at -3 , 0 , and 3 . The equation must have the form

$$f(x) = a(x+3)^2(x)^2(x-3)^2 \text{ since the graph doesn't change sign at any of the intercepts. } f(1) = 4 \Rightarrow 4 = a(4)^2(1)^2(-2)^2 \Rightarrow 64a = 4 \Rightarrow a = \frac{1}{16}.$$

[29] $f(x) = \frac{4(x+2)(x-1)}{3(x+2)(x-5)} = \frac{4(x-1)}{3(x-5)}$ if $x \neq -2$ • Note that $x+2$ appears in both the

numerator and denominator, so there is a hole at $x = -2$. The y -value for the hole can be found by substituting -2 for x in the rest of the function.

$$\frac{4(-2-1)}{3(-2-5)} = \frac{4(-3)}{3(-7)} = \frac{4}{7}, \text{ so the hole is at } (-2, \frac{4}{7}).$$

There is one other zero of the denominator, namely 5 , so there is a vertical asymptote of $x = 5$. The degrees of the numerator and denominator are the same, namely 2 , so the ratio of leading coefficients gives us the value of the horizontal asymptote. In this case, it is $y = \frac{4}{3}$.

There is one other zero of the numerator, namely 1 , so there is an x -intercept of 1 . If we substitute 0 for x , we get $\frac{4(2)(-1)}{3(2)(-5)} = \frac{4}{15}$, the y -intercept.

[30] $f(x) = \frac{-2}{(x+1)^2}$ •

[31] $f(x) = \frac{1}{(x-1)^3}$ •

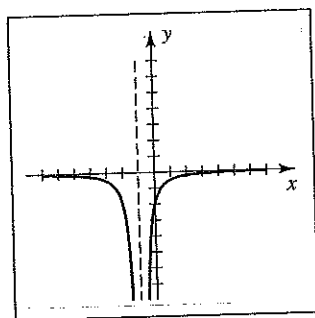


Figure 30

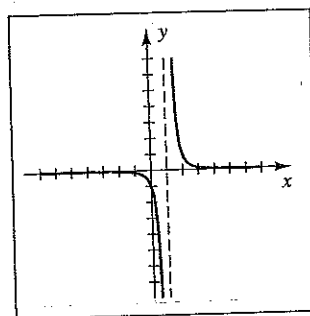


Figure 31

$$\boxed{32} \quad f(x) = \frac{3x^2}{16-x^2} = \frac{3x^2}{(4+x)(4-x)}$$

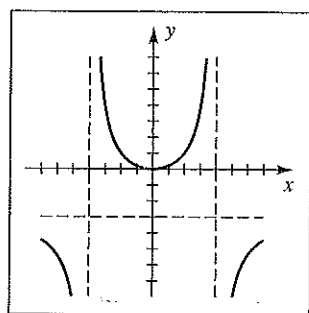


Figure 32

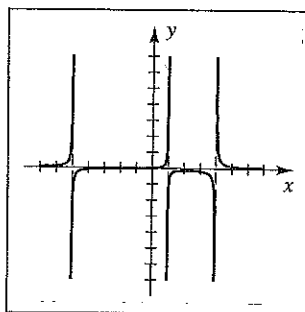


Figure 33

$$\boxed{33} \quad f(x) = \frac{x}{(x+5)(x^2-5x+4)} = \frac{x}{(x+5)(x-1)(x-4)}$$

$$\boxed{34} \quad f(x) = \frac{x^3-2x^2-8x}{-x^2+2x} = \frac{x(x^2-2x-8)}{x(2-x)} = \frac{(x-4)(x+2)}{2-x} \text{ for } x \neq 0; \text{ hole at } (0, -4)$$

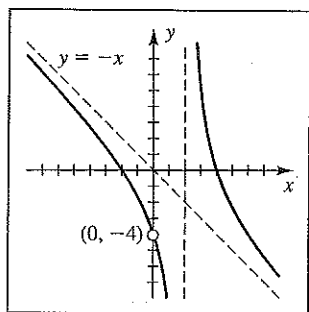


Figure 34

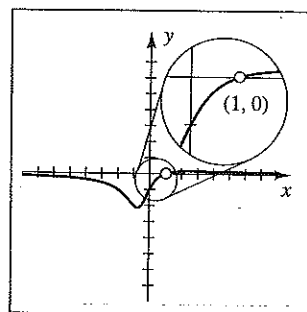


Figure 35

$$\boxed{35} \quad f(x) = \frac{x^2-2x+1}{x^3-x^2+x-1} = \frac{(x-1)(x-1)}{x^2(x-1)+1(x-1)} = \frac{(x-1)^2}{(x^2+1)(x-1)} = \frac{x-1}{x^2+1} \text{ for } x \neq 1;$$

hole at (1, 0)

$$\boxed{36} \quad f(x) = \frac{3x^2+x-10}{x^2+2x} = \frac{(3x-5)(x+2)}{x(x+2)} = \frac{3x-5}{x} \text{ for } x \neq -2$$

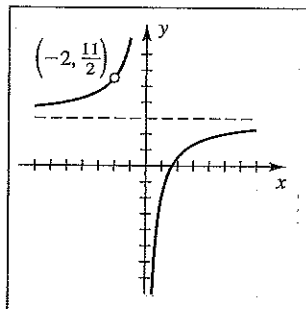


Figure 36

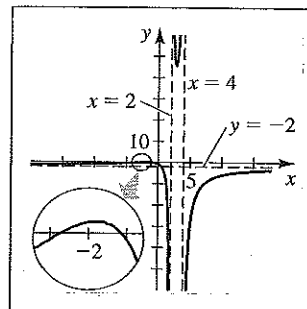


Figure 37

$$\boxed{37} \quad f(x) = \frac{-2x^2-8x-6}{x^2-6x+8} = \frac{-2(x^2+4x+3)}{(x-2)(x-4)} = \frac{-2(x+1)(x+3)}{(x-2)(x-4)}$$

$$[38] f(x) = \frac{x^2 + 2x - 8}{x + 3} = \frac{(x + 4)(x - 2)}{x + 3} = x - 1 - \frac{5}{x + 3}$$

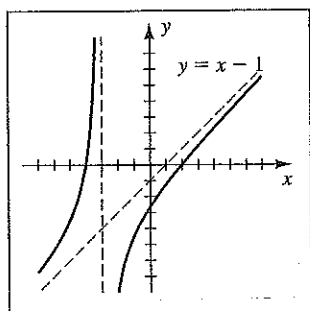


Figure 38

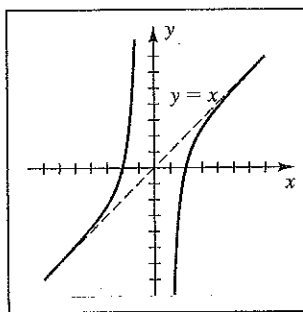


Figure 39

$$[39] f(x) = \frac{x^4 - 16}{x^3} = \frac{(x^2 + 4)(x + 2)(x - 2)}{x^3} = x - \frac{16}{x^3}$$

- [40] There is a hole at $x = 2$, so $(x - 2)$ must be a factor in both the numerator and denominator. There is a vertical asymptote of $x = -3$, so $(x + 3)$ must be a factor in the denominator. There is an x -intercept at 5, so $(x - 5)$ must be a factor in the numerator. The horizontal asymptote is $y = \frac{3}{2}$, so the ratio of leading coefficients must be $\frac{3}{2}$. Combining this information gives us this possibility:

$$f(x) = \frac{3(x - 5)(x - 2)}{2(x + 3)(x - 2)} \quad \text{or} \quad f(x) = \frac{3x^2 - 21x + 30}{2x^2 + 2x - 12}$$

$$[41] y = \frac{k \sqrt[3]{x}}{x^2} \Rightarrow 6 = \frac{k \sqrt[3]{8}}{3^2} \Rightarrow 6 = \frac{2k}{9} \Rightarrow k = \frac{54}{2} = 27$$

$$[42] y = \frac{k}{x^2} \Rightarrow 18 = \frac{k}{4^2} \Rightarrow$$

$$k = 18 \cdot 16 = 288,$$

$$\text{so graph } y = \frac{288}{x^2} \text{ for } x > 0.$$

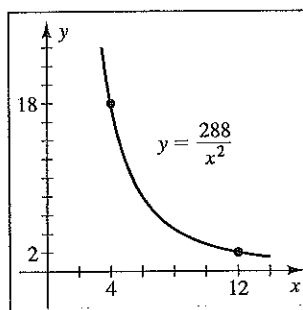


Figure 42

$$[43] \text{ (a) } l = 10, x = 10, \text{ and } y = 2 \Rightarrow 2 = 30,000c \Rightarrow c = \frac{1}{15,000}$$

$$\text{ (b) } y \approx 0.9754 < 1 \text{ if } x = 6.1, \text{ and } y \approx 1.0006 > 1 \text{ if } x = 6.2$$

- [44] (a) Edge AB has length $2\pi r$ where r is the radius of the cylinder.

$$2\pi r = \sqrt{l^2 - x^2} \Rightarrow r^2 = \frac{1}{4\pi^2}(l^2 - x^2).$$

$$\text{Now } V = \pi r^2 x = \pi \left[\frac{1}{4\pi^2}(l^2 - x^2) \right] (x) = \frac{1}{4\pi} x(l^2 - x^2).$$

- (b) If $x > 0$, $V > 0$ if $l^2 - x^2 > 0$ or $l > x$. Thus, when $0 < x < l$, $V > 0$.

[45] $T = \frac{1}{20}t(t-12)(t-24) = 32 \Rightarrow t^3 - 36t^2 + 288t - 640 = 0$. Solving for t yields
 $t = 4$ and $16 \pm 4\sqrt{6}$. Since $0 \leq t \leq 24$, $t = 4$ (10:00 A.M.) and
 $t = 16 - 4\sqrt{6} \approx 6.2020$ (12:12 P.M.) are the times when the temperature was 32°F.

[46] $N(t) = -t^4 + 21t^2 + 100$ and $N(t) > 180 \Rightarrow$
 $t^4 - 21t^2 + 80 < 0 \Rightarrow (t^2 - 5)(t^2 - 16) < 0$.

The positive values of t satisfying this inequality are in the interval $(\sqrt{5}, 4)$.

[47] (a) $R = \frac{kS^n}{S^n + a^n} \cdot \frac{1/S^n}{1/S^n} = \frac{k}{1 + (a/S)^n}$. As S gets large, R approaches k .

(b) k is the maximum rate at which the liver can remove alcohol from the bloodstream.

[48] (a) $C(100) = \$2,000,000.00$ and
 $C(90) \approx \$163,636.36$

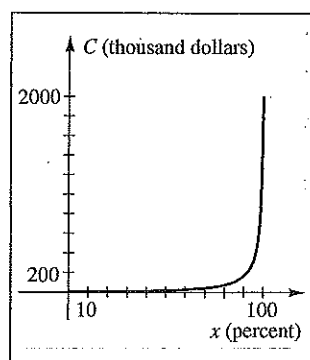


Figure 48

[49] $C = \frac{kP_1P_2}{d^2}$; $2000 = \frac{k(10,000)(5000)}{(25)^2} \Rightarrow k = \frac{1}{40}$; $C = \frac{(10,000)(15,000)}{40(100)^2} = 375$

[50] $P = kA^2v^3$; $3000 = k[\pi(5)^2]^2(20)^3 \Rightarrow k = \frac{3}{5000\pi^2}$

$$P = \frac{3}{5000\pi^2}[\pi(5)^2]^2(30)^3 = 10,125 \text{ watts}$$

Chapter 4 Discussion Exercises

- [1] **For even-degreed polynomials:** the domain is \mathbb{R} and the number of x -intercepts ranges from zero to the degree of the polynomial; if the leading coefficient is positive, the range is of the form $[c, \infty)$ and the general shape has $y \rightarrow \infty$ as $|x| \rightarrow \infty$; if the leading coefficient is negative, the range is of the form $(-\infty, c]$ and the general shape has $y \rightarrow -\infty$ as $|x| \rightarrow \infty$.

For odd-degreed polynomials: the domain is \mathbb{R} , the range is \mathbb{R} , and the number of x -intercepts ranges from one to the degree of the polynomial; if the leading coefficient is positive, then $y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow -\infty$ as $x \rightarrow -\infty$, if the leading coefficient is negative, then $y \rightarrow -\infty$ as $x \rightarrow \infty$ and $y \rightarrow \infty$ as $x \rightarrow -\infty$.

- [2] After trying a few examples, students should come up with the conclusion that complex numbers can be used in the synthetic division process.
- [3] By long division, we obtain the quotient $2x^2 - 7x + 5$ with remainder -6 . By synthetic division with $k = -3/2$, we obtain a bottom row of 4 -14 10 -6. The first three numbers are twice the coefficients of the quotient and the last number is the remainder. For the factor $ax + b$, we can use synthetic division with $k = -b/a$, and obtain a times the quotient and the remainder in the bottom row.
- [4] $f(x) = a(x-1)(x-2)(x-3)$. $f(0) = a(-6) = 6 \Rightarrow a = -1$.
 $f(-1) = 24 \neq 25 \Rightarrow$ the point cannot be on the polynomial.
- [5] After working Discussion Exercise 4, students should guess that 4 points specify a third-degree polynomial. They know that 2 points specify a first-degree polynomial, so a logical conclusion is that $n + 1$ points specify an n -degree polynomial.
- [6] Let us consider the polynomial

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0,$$

where each coefficient a_k is a real number and $a_n \neq 0$. If $f(z) = 0$, then

$$a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 = 0.$$

If two complex numbers are equal, then so are their conjugates. Hence, the conjugate of the left-hand side of the last equation equals the conjugate of the right-hand side; that is,

$$\overline{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0} = \overline{0} = 0.$$

The fact that $\overline{0} = 0$ follows from $\overline{0} = \overline{0 + 0i} = 0 - 0i = 0$.

If z and w are complex numbers, then it can be shown that $\overline{z + w} = \overline{z} + \overline{w}$. More generally, the conjugate of any sum of complex numbers is the sum of the conjugates. Consequently,

$$\overline{a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0} = 0.$$

It can also be shown that $\overline{z \cdot w} = \overline{z} \cdot \overline{w}$, $\overline{z^n} = \overline{z}^n$ for every positive integer n , and $\overline{z} = z$ if and only if z is real. Thus, for every k ,

$$\overline{a_k z^k} = \overline{a_k} \cdot \overline{z^k} = \overline{a_k} \cdot \overline{z}^k = a_k \overline{z}^k,$$

and therefore

$$a_n \overline{z}^n + a_{n-1} \overline{z}^{n-1} + \cdots + a_1 \overline{z} + a_0 = 0.$$

The last equation states that $f(\overline{z}) = 0$, which completes the proof.

- [7] If the common factor is never equal to zero for any real number, then it can be canceled and has no effect on its graph. Such a factor is $x^2 + 1$, and an example of a

$$\text{function is } f(x) = \frac{(x^2 + 1)(x - 1)}{(x^2 + 1)(x - 2)}.$$

- [8] (a) The horizontal asymptote is $y = a/c$. Solving $f(x) = a/c$ gives us

$$\frac{ax + b}{cx + d} = \frac{a}{c} \Rightarrow acx + bc = acx + ad \Rightarrow bc = ad \Rightarrow \text{no solution,}$$

and f doesn't cross its horizontal asymptote.

- (b) The horizontal asymptote is $y = a/d$. Solving $f(x) = a/d$ gives us

$$\frac{ax^2 + bx + c}{dx^2 + ex + f} = \frac{a}{d} \Rightarrow adx^2 + bdx + cd = adx^2 + aex + af \Rightarrow$$

$$aex - bdx = cd - af \Rightarrow x = \frac{cd - af}{ae - bd}, \text{ provided the denominator is not zero.}$$

$$[9] \quad y = \frac{9x}{\sqrt{x^2 + 1}} \Rightarrow y\sqrt{x^2 + 1} = 9x \Rightarrow y^2(x^2 + 1) = 81x^2 \Rightarrow$$

$$y^2x^2 + y^2 = 81x^2 \Rightarrow y^2x^2 - 81x^2 = -y^2 \Rightarrow x^2(y^2 - 81) = -y^2 \Rightarrow$$

$$x^2 = \frac{y^2}{81 - y^2} \Rightarrow x = \pm \sqrt{\frac{y^2}{81 - y^2}} \Rightarrow x = \pm \frac{|y|}{\sqrt{81 - y^2}} \Rightarrow x = \pm \frac{y}{\sqrt{81 - y^2}}.$$

From the original equation, we see that x and y are always both positive or both negative, so the last equation can be simplified to $x = y/\sqrt{81 - y^2}$. Hence,

$f^{-1}(y) = y/\sqrt{81 - y^2}$ or, equivalently, $f^{-1}(x) = x/\sqrt{81 - x^2}$. The denominator of f^{-1} is zero for $x = \pm 9$, which are the vertical asymptotes. They are related to the horizontal asymptotes of f , which are $y = \pm 9$.

- [10] Let x , $x + 1$, and $x + 2$ denote three consecutive integers. Their product is $x(x + 1)(x + 2) = x^3 + 3x^2 + 2x$. Now add the second integer to get

$$(x^3 + 3x^2 + 2x) + (x + 1) = x^3 + 3x^2 + 3x + 1.$$

Only $x + 1$ or $x - 1$ could be factors, and it turns out that $x + 1$ is a factor three times. Thus, if you multiply three consecutive integers together and then add the second integer to that product, you obtain the cube of the second integer.

$$\boxed{11} \text{ (a) } B = \frac{GW}{29.3 + 53.1E - 22.7C} \quad \bullet$$

$G = 3 \cdot 500 = 1500$, $W = 5$, $E = -0.05$, and $C = 0.95$, so

$B = \frac{1500(5)}{29.3 + 53.1(-0.05) - 22.7(0.95)} \approx 1476$. This tells us that a bankroll of about \$1476 would allow us to play 1500 games and be 95% confident that we will survive the 3-hour gambling session.

- (b) If we use the same values for G , W , and E , we get $B = \frac{7500}{26.645 - 22.7C}$. A graph of B has a vertical asymptote at $C \approx 1.17$, but that is 117% confident of surviving the 3-hour gambling session. The highest confidence value for any model like this should be 100, so the formula should have a restricted domain with it.