Chapter 9: Systems of Equations and Inequalities

9.1 Exercises

Note: The notation E_1 and E_2 refers to the first equation and the second equation.

$\boxed{1}$ Substituting y in E_2 into E_1 yields $2x - 1 = x^2 - 4 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow$

$(x-3)(x+1) = 0 \Rightarrow x = 3, -1; y = 5, -3.$ ★ $(3, 5), (-1, -3)$

$\boxed{2}$ Substituting y in E_1 into E_2 yields $x + x^2 + 1 = 3 \Rightarrow x^2 + x - 2 = 0 \Rightarrow$

$(x+2)(x-1) = 0 \Rightarrow x = -2, 1; y = 5, 2.$ ★ $(-2, 5), (1, 2)$

$\boxed{3}$ Solving E_2 for x and substituting into E_1 yields $y^2 = 1 - (1 - 2y) \Rightarrow$

$y^2 - 2y = 0 \Rightarrow y(y-2) = 0 \Rightarrow y = 0, 2; x = 1, -3.$ ★ $(1, 0), (-3, 2)$

$\boxed{4}$ Solving E_2 for x and substituting into E_1 yields $y^2 = -2y - 3 \Rightarrow$

$y^2 + 2y + 3 = 0 \Rightarrow y = -1 \pm \sqrt{2}\,i.$ There are *no real solutions.*

$\boxed{5}$ Substituting y in E_2 into E_1 yields $8x^3 = x^2 \Rightarrow x^2(8x - 1) = 0 \Rightarrow$

$x = 0, \frac{1}{8}; y = 0, \frac{1}{128}.$ ★ $(0, 0), (\frac{1}{8}, \frac{1}{128})$

$\boxed{6}$ Solving E_1 for x and substituting into E_2 yields $2(y^3 + 1) = 9y^2 + 2 \Rightarrow$

$2y^3 - 9y^2 = 0 \Rightarrow y^2(2y - 9) = 0 \Rightarrow y = 0, \frac{9}{2}; x = 1, \frac{737}{8}.$ ★ $(1, 0), (\frac{737}{8}, \frac{9}{2})$

$\boxed{7}$ Solving E_1 for x and substituting into E_2 yields $2(-2y - 1) - 3y = 12 \Rightarrow$

$-7y = 14 \Rightarrow y = -2; x = 3.$ ★ $(3, -2)$

$\boxed{8}$ Solving E_2 for y and substituting into E_1 yields

$3x - 4(-4 - \frac{3}{2}x) + 20 = 0 \Rightarrow 9x = -36 \Rightarrow x = -4; y = 2.$ ★ $(-4, 2)$

$\boxed{9}$ Solving E_1 for x and substituting into E_2 yields $-6(\frac{1}{2} + \frac{3}{2}y) + 9y = 4 \Rightarrow -3 = 4.$

There are *no solutions.*

$\boxed{10}$ Solving E_1 for x and substituting into E_2 yields $8(\frac{1}{2} + \frac{5}{4}y) - 10y = -5 \Rightarrow 4 = -5.$

There are *no solutions.*

$\boxed{11}$ Solving E_1 for x and substituting into E_2 yields $(5 - 3y)^2 + y^2 = 25 \Rightarrow$

$10y^2 - 30y = 0 \Rightarrow 10y(y-3) = 0 \Rightarrow y = 0, 3; x = 5, -4.$ ★ $(-4, 3), (5, 0)$

$\boxed{12}$ Solving E_1 for x and substituting into E_2 yields $(\frac{25}{3}+\frac{4}{3}y)^2+y^2=25 \Rightarrow$

$\frac{25}{9}y^2+\frac{200}{9}y+\frac{400}{9}=0 \Rightarrow y^2+8y+16=0 \Rightarrow$

$(y+4)^2=0 \Rightarrow y=-4;\ x=3.$ ★ $(3, -4)$

$\boxed{13}$ Solving E_2 for y and substituting into E_1 yields $x^2+(x+4)^2=8 \Rightarrow$

$2x^2+8x+8=0 \Rightarrow 2(x+2)^2=0 \Rightarrow x=-2;\ y=2.$ ★ $(-2, 2)$

$\boxed{14}$ Solving E_2 for x and substituting into E_1 yields

$(-\frac{4}{3}y-\frac{25}{3})^2+y^2=25 \Rightarrow \frac{25}{9}y^2+\frac{200}{9}y+\frac{400}{9}=0 \Rightarrow$

$y^2+8y+16=0 \Rightarrow (y+4)^2=0 \Rightarrow y=-4;\ x=-3.$ ★ $(-3, -4)$

$\boxed{15}$ Solving E_2 for y and substituting into E_1 yields

$x^2+(3x+2)^2=9 \Rightarrow 10x^2+12x-5=0 \Rightarrow x=\dfrac{-6\pm\sqrt{86}}{10}=-\frac{3}{5}\pm\frac{1}{10}\sqrt{86}.$

$y=3\left(\dfrac{-6\pm\sqrt{86}}{10}\right)+2=\dfrac{-18\pm3\sqrt{86}}{10}+\dfrac{20}{10}=\dfrac{2\pm3\sqrt{86}}{10}=\frac{1}{5}\pm\frac{3}{10}\sqrt{86}.$

★ $(-\frac{3}{5}+\frac{1}{10}\sqrt{86},\ \frac{1}{5}+\frac{3}{10}\sqrt{86}),\ (-\frac{3}{5}-\frac{1}{10}\sqrt{86},\ \frac{1}{5}-\frac{3}{10}\sqrt{86})$

$\boxed{16}$ Solving E_2 for y and substituting into E_1 yields $x^2+(-2x-1)^2=16 \Rightarrow$

$5x^2+4x-15=0 \Rightarrow x=-\frac{2}{5}\pm\frac{1}{5}\sqrt{79};\ y=-2(-\frac{2}{5}\pm\frac{1}{5}\sqrt{79})-1=$

$\frac{4}{5}\mp\frac{2}{5}\sqrt{79}-\frac{5}{5}=-\frac{1}{5}\mp\frac{2}{5}\sqrt{79}.$ ★ $(-\frac{2}{5}\pm\frac{1}{5}\sqrt{79},\ -\frac{1}{5}\mp\frac{2}{5}\sqrt{79})$

$\boxed{17}$ Solving E_2 for x and substituting into E_1 yields $(2y-4)^2+y^2=16 \Rightarrow$

$5y^2-16y=0 \Rightarrow y(5y-16)=0 \Rightarrow y=0,\ \frac{16}{5};\ x=-4,\ \frac{12}{5}.$ ★ $(-4, 0),\ (\frac{12}{5}, \frac{16}{5})$

$\boxed{18}$ Solving E_2 for y and substituting into E_1 yields $x^2+(-2x-3)^2=1 \Rightarrow$

$5x^2+12x+8=0 \Rightarrow x=-\frac{6}{5}\pm\frac{2}{5}i.$ There are *no real solutions.*

$\boxed{19}$ Solving E_2 for x and substituting into E_1 yields

$(1-y-1)^2+(y+2)^2=10 \Rightarrow 2y^2+4y-6=0 \Rightarrow$

$2(y+3)(y-1)=0 \Rightarrow y=-3,\ 1;\ x=4,\ 0.$ ★ $(0, 1),\ (4, -3)$

$\boxed{20}$ Solving E_1 for y and substituting into E_2 yields $3x-\dfrac{2}{x}+5=0 \Rightarrow$

$3x^2+5x-2=0 \Rightarrow (3x-1)(x+2)=0 \Rightarrow x=\frac{1}{3},\ -2;\ y=6,\ -1.$

★ $(\frac{1}{3}, 6),\ (-2, -1)$

$\boxed{21}$ Substituting y in E_1 into E_2 yields $20/x^2=9-x^2 \Rightarrow x^4-9x^2+20=0 \Rightarrow$

$(x^2-4)(x^2-5)=0 \Rightarrow x=\pm2,\ \pm\sqrt{5};\ y=5,\ 4.$ ★ $(\pm2, 5),\ (\pm\sqrt{5}, 4)$

$\boxed{22}$ Solving E_2 for x and substituting into E_1 yields $y+1=y^2-4y+5 \Rightarrow$

$y^2-5y+4=0 \Rightarrow (y-1)(y-4)=0 \Rightarrow y=1,\ 4;\ x=2,\ 5.$ ★ $(2, 1),\ (5, 4)$

[23] Solving E_1 for y^2 and substituting into E_2 yields $9(4x^2+4)+16x^2=140 \Rightarrow$

$52x^2=104 \Rightarrow x=\pm\sqrt{2};\ y=\pm 2\sqrt{3}$. ★ $(\sqrt{2},\ \pm 2\sqrt{3}),\ (-\sqrt{2},\ \pm 2\sqrt{3})$

[24] Solving E_2 for x^2 and substituting into E_1 yields $25y^2-16(\frac{9}{4}y^2-9)=400 \Rightarrow$

$-11y^2=256 \Rightarrow y^2=-\frac{256}{11}$. There are *no real solutions.*

[25] Solving E_1 for x^2 and substituting into E_2 yields $(y^2+4)+y^2=12 \Rightarrow$

$2y^2=8 \Rightarrow y=\pm 2;\ x=\pm 2\sqrt{2}$. ★ $(2\sqrt{2},\ \pm 2),\ (-2\sqrt{2},\ \pm 2)$

[26] Solving E_1 for y^3 and substituting into E_2 yields $3x^3+4(6x^3-1)=5 \Rightarrow$

$27x^3=9 \Rightarrow x^3=\frac{1}{3} \Rightarrow x=\sqrt[3]{\frac{1}{3}}$ or $\frac{1}{3}\sqrt[3]{9};\ y=1$. ★ $(\frac{1}{3}\sqrt[3]{9},\ 1)$

[27] Solving E_2 for y and substituting into E_1 and E_3 yields

$$\begin{cases} x+2(2x+z-9)-z & =-1 \\ x+3(2x+z-9)+3z & =6 \end{cases} \Rightarrow \begin{cases} 5x+z & =17 \quad (E_4) \\ 7x+6z & =33 \quad (E_5) \end{cases}$$

Solving E_4 for z and substituting into E_5 yields

$7x+6(17-5x)=33 \Rightarrow -23x=-69 \Rightarrow x=3$.

Now $z=17-5x=2$ and $y=2x+z-9=-1$. ★ $(3,\ -1,\ 2)$

[28] Solving E_2 for z^2 and substituting into E_1 yields $\begin{cases} x-2y & =1 \quad (E_4) \\ x^2-xy & =0 \quad (E_3) \end{cases}$

Now solve E_4 for x and substitute into E_3 yielding

$(2y+1)^2-(2y+1)y=0 \Rightarrow 2y^2+3y+1=0 \Rightarrow (2y+1)(y+1)=0 \Rightarrow$

$y=-\frac{1}{2},\ -1.\ \ x=2y+1=0,\ -1;\ z^2=x-y+1=\frac{3}{2},\ 1 \Rightarrow$

$z=\pm\sqrt{\frac{3}{2}},\ \pm 1$. ★ $(0,\ -\frac{1}{2},\ \pm\sqrt{\frac{3}{2}}),\ (-1,\ -1,\ \pm 1)$

[29] Solving E_3 for y and substituting into E_2 yields $\begin{cases} x^2+z^2 & =5 \quad (E_1) \\ 2x-z & =0 \quad (E_4) \end{cases}$

Now solve E_4 for z and substitute into E_1 yielding $x^2+(2x)^2=5 \Rightarrow 5x^2=5 \Rightarrow$

$x=\pm 1;\ z=2x=\pm 2;\ y=1-z=-1,\ 3$. ★ $(1,\ -1,\ 2),\ (-1,\ 3,\ -2)$

[30] From E_3, $xyz=0$, we see that $x=0$, $y=0$, or $z=0$.

Substituting 0 for x in E_1 gives us $2z=1 \Rightarrow z=\frac{1}{2}$;

substituting $\frac{1}{2}$ for z in E_2 gives us $2y=\frac{9}{2} \Rightarrow y=\frac{9}{4}$. ★ $(0,\ \frac{9}{4},\ \frac{1}{2})$

Substituting 0 for y in E_2 gives us $-z=4 \Rightarrow z=-4$;

substituting -4 for z in E_1 gives us $x-8=1 \Rightarrow x=9$. ★ $(9,\ 0,\ -4)$

Substituting 0 for z in E_1 gives us $x=1$;

substituting 0 for z in E_2 gives us $2y=4 \Rightarrow y=2$. ★ $(1,\ 2,\ 0)$

31 Substituting $y = 4x - b$ into $y = x^2$ yields $4x - b = x^2 \Rightarrow$

$$x^2 - 4x + b = 0 \Rightarrow x = \frac{4 \pm \sqrt{16 - 4b}}{2} = 2 \pm \sqrt{4 - b}.$$

For x to have 1, 2, or no values, the discriminant $4 - b$ must be equal to zero, greater than zero, or less than zero, respectively. Graphically, the line would be (a) tangent to the parabola, (b) intersect the parabola in two points, and (c) not intersect the parabola.

(a) $4 - b = 0 \Rightarrow b = 4$.

(b) $4 - b > 0 \Rightarrow 4 > b \Rightarrow b < 4$.

(c) $4 - b < 0 \Rightarrow 4 < b \Rightarrow b > 4$.

32 Substituting $y = x + b$ into $x^2 + y^2 = 4$ yields $x^2 + (x + b)^2 = 4 \Rightarrow$

$$2x^2 + 2bx + (b^2 - 4) = 0 \Rightarrow x = \frac{-2b \pm \sqrt{4b^2 - 8(b^2 - 4)}}{4} = \frac{-b \pm \sqrt{8 - b^2}}{2}.$$

For x to have 1, 2, or no values, the discriminant $8 - b^2$ must be equal to zero, greater than zero, or less than zero, respectively. Graphically, the line would be (a) tangent to the circle, (b) intersect the circle in two points, and (c) not intersect the circle.

(a) $8 - b^2 = 0 \Rightarrow b = \pm\sqrt{8} = \pm 2\sqrt{2}$.

(b) $8 - b^2 > 0 \Rightarrow b^2 < 8 \Rightarrow |b| < 2\sqrt{2} \Rightarrow -2\sqrt{2} < b < 2\sqrt{2}$.

(c) $8 - b^2 < 0 \Rightarrow b > 2\sqrt{2}$ or $b < -2\sqrt{2}$ from parts (a) and (b).

33 From the graph, it is clear that there is a point of intersection and therefore a single solution between 0 and 1.

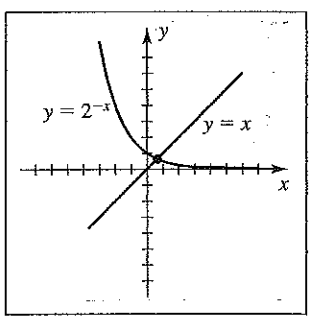

Figure 33

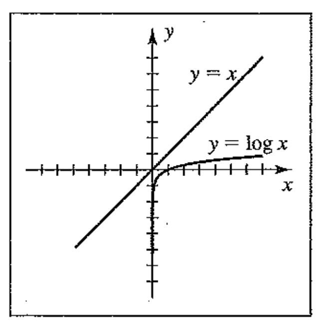

Figure 34

34 From the graph, it is clear that there are no intersection points of the two graphs, and therefore there are no solutions.

35 The equation of the line is $y - 2 = m(x - 4)$. Substituting y^2 for x gives us
$y - 2 = m(y^2 - 4) \Rightarrow 0 = my^2 - y + (2 - 4m) \Rightarrow y = \dfrac{1 \pm \sqrt{1 - 4m(2 - 4m)}}{2m}$.
For there to be only one value of y, the discriminant $1 - 4m(2 - 4m)$ must be equal to zero, i.e., $1 - 4m(2 - 4m) = 0 \Rightarrow 1 - 8m + 16m^2 = 0 \Rightarrow (4m - 1)^2 = 0 \Rightarrow$ $m = \frac{1}{4}$. This line, $y - 2 = \frac{1}{4}(x - 4)$, or $y = \frac{1}{4}x + 1$, is tangent to the parabola.

36 The equation of the line is $y - 1 = m(x - 1)$, or $y = mx + (1 - m)$. Setting this expression equal to x^2 yields $x^2 - mx + m - 1 = 0 \Rightarrow x = \dfrac{m \pm \sqrt{m^2 - 4m + 4}}{2}$.
For there to be only one value of x, the discriminant $m^2 - 4m + 4 = (m - 2)^2$ must be zero, i.e., $m = 2$. This line, $y = 2x - 1$, is tangent to the parabola.

37 Use the fact that $(0, 3)$, $(-1, \frac{5}{3})$, and $(1, 7)$ are on the graph of $f(x) = ba^x + c$.

$$\begin{cases} f(0) = 3 \\ f(-1) = \frac{5}{3} \\ f(1) = 7 \end{cases} \Rightarrow \begin{cases} 3 = ba^0 + c & (\mathrm{E}_1) \\ \frac{5}{3} = ba^{-1} + c & (\mathrm{E}_2) \\ 7 = ba^1 + c & (\mathrm{E}_3) \end{cases}$$

From E_1, $3 = b + c \Rightarrow c = 3 - b$. Substitute $3 - b$ for c in E_2 and E_3.

E_2: $\frac{5}{3} = ba^{-1} + 3 - b \Rightarrow b - ba^{-1} = \frac{4}{3} \Rightarrow b\left(1 - \frac{1}{a}\right) = \frac{4}{3} \Rightarrow b\left(\frac{a-1}{a}\right) = \frac{4}{3}$

E_3: $7 = ba^1 + 3 - b \Rightarrow b - ba = -4 \Rightarrow b(1 - a) = -4 \Rightarrow b(a - 1) = 4$

Solving both equations for b and equating the expressions gives us
$\dfrac{4a}{3(a-1)} = \dfrac{4}{a-1} \Rightarrow \frac{a}{3} = \frac{1}{1} \Rightarrow a = 3$. Substituting 3 for a in $b(a - 1) = 4$ gives us $2b = 4 \Rightarrow b = 2$. Substituting 2 for b in $c = 3 - b$ gives us $c = 1$, so the function is $f(x) = 2(3)^x + 1$.

38 Use the fact that $(0, 3)$, $(-1, 9)$, and $(1, \frac{3}{2})$ are on the graph of $f(x) = ba^x + c$.

$$\begin{cases} f(0) = 3 \\ f(-1) = 9 \\ f(1) = \frac{3}{2} \end{cases} \Rightarrow \begin{cases} 3 = ba^0 + c & (\mathrm{E}_1) \\ 9 = ba^{-1} + c & (\mathrm{E}_2) \\ \frac{3}{2} = ba^1 + c & (\mathrm{E}_3) \end{cases}$$

From E_1, $3 = b + c \Rightarrow c = 3 - b$. Substitute $3 - b$ for c in E_2 and E_3.

E_2: $9 = ba^{-1} + 3 - b \Rightarrow b - ba^{-1} = -6 \Rightarrow b\left(1 - \frac{1}{a}\right) = -6 \Rightarrow b\left(\frac{a-1}{a}\right) = -6$

E_3: $\frac{3}{2} = ba^1 + 3 - b \Rightarrow b - ba = \frac{3}{2} \Rightarrow b(1 - a) = \frac{3}{2} \Rightarrow b(1 - a) = \frac{3}{2}$

Solving both equations for b and equating the expressions gives us
$\dfrac{-6a}{a-1} = \dfrac{3}{2(1-a)} \Rightarrow \dfrac{6a}{1-a} = \dfrac{3}{2(1-a)} \Rightarrow \dfrac{6a}{1} = \dfrac{3}{2} \Rightarrow 12a = 3 \Rightarrow a = \frac{1}{4}$.
Substituting $\frac{1}{4}$ for a in $b(1 - a) = \frac{3}{2}$ gives us $\frac{3}{4}b = \frac{3}{2} \Rightarrow b = 2$.
Substituting 2 for b in $c = 3 - b$ gives us $c = 1$, so the function is $f(x) = 2(\frac{1}{4})^x + 1$.

[39] Using $P = 40 = 2l + 2w$ and $A = 96 = lw$, we have $\begin{cases} 2l + 2w &= 40 \quad (\mathrm{E}_1) \\ lw &= 96 \quad (\mathrm{E}_2) \end{cases}$

Solving E_1 for l and substituting into E_2 yields $(20 - w)w = 96 \Rightarrow$

$w^2 - 20w + 96 = 0 \Rightarrow (w - 8)(w - 12) = 0 \Rightarrow w = 8,\ 12;\ l = 12,\ 8.$

The rectangle is 12 in. × 8 in.

[40] $A = 200 = (2\pi r)h \Rightarrow \pi r h = 100.\ V = 200 = \pi r^2 h \Rightarrow h = 200/(\pi r^2).$

Substituting h into $\pi r h = 100$ yields $200/r = 100 \Rightarrow r = 2$ in.; $h = 50/\pi \approx 15.9$ in.

[41] (a) We have $R = aS/(S + b)$ and for 1998, let $S = 40{,}000$ and $R = 60{,}000$.

Then for 1999, let $S = 60{,}000$ and $R = 72{,}000$.

$$\begin{cases} 60{,}000 = (40{,}000a)/(40{,}000 + b) \\ 72{,}000 = (60{,}000a)/(60{,}000 + b) \end{cases} \Rightarrow \begin{cases} 120{,}000 + 3b = 2a \quad (\mathrm{E}_1) \\ 360{,}000 + 6b = 5a \quad (\mathrm{E}_2) \end{cases}$$

Solving E_1 for a and substituting into E_2 yields

$$360{,}000 + 6b = 5(60{,}000 + \tfrac{3}{2}b) \Rightarrow 60{,}000 = \tfrac{3}{2}b \Rightarrow b = 40{,}000;\ a = 120{,}000.$$

(b) Now let $S = 72{,}000$ and thus $R = \dfrac{(120{,}000)(72{,}000)}{72{,}000 + 40{,}000} = \dfrac{540{,}000}{7} \approx 77{,}143.$

[42] Follow the outline of the solution in Exercise 41 with $R = aSe^{-bS}$.

(a) $$\begin{cases} 60{,}000 = a(40{,}000)e^{(-b)(40{,}000)} \\ 72{,}000 = a(60{,}000)e^{(-b)(60{,}000)} \end{cases} \Rightarrow \begin{cases} 3 = 2ae^{-40{,}000b} \\ 6 = 5ae^{-60{,}000b} \end{cases} \Rightarrow$$

$$6 = 5\left(\frac{3}{2e^{-40{,}000b}}\right)e^{-60{,}000b} \Rightarrow 0.8 = e^{-20{,}000b} \Rightarrow -20{,}000b = \ln 0.8 \Rightarrow$$

$$b = \frac{\ln 0.8}{-20{,}000} \approx 0.00001116;\ a = \frac{3}{2e^{-40{,}000\,[\ln 0.8/(-20{,}000)]}} = \frac{3}{2(e^{\ln 0.8})^2} = \frac{75}{32}$$

(b) $R = \frac{75}{32}(72{,}000)e^{-[\ln 0.8/(-20{,}000)](72{,}000)}$

$= 168{,}750(e^{\ln 0.8})^{3.6} = 168{,}750(0.8)^{3.6} \approx 75{,}573$

[43] Let R_1 and R_2 equal 0. The system is then $\begin{cases} 0 = 0.01x(50 - x - y) \\ 0 = 0.02y(100 - y - 0.5x) \end{cases}$

There are two possibilities for each equation to equal zero.

Case (1): $x = 0$ and $y = 0$ give us the solution $(0,\ 0)$.

Case (2): $x = 0$ and $100 - y - 0.5x = 0\ \{y = 100\}$ give us the solution $(0,\ 100)$.

Case (3): $y = 0$ and $50 - x - y = 0\ \{x = 50\}$ give us the solution $(50,\ 0)$.

Case (4): $50 - x - y = 0$ and $100 - y - 0.5x = 0$. Solve the first equation for y $\{y = 50 - x\}$ and substitute into the second equation:

$100 - (50 - x) - 0.5x = 0 \Rightarrow 50 = -0.5x \Rightarrow x = -100;\ y = 150.$

This solution is meaningless for this problem.

[44] Let l be the length of the side opposite the river and w the length of the other two sides. { 10 acres = 435,600 ft^2 } $\begin{cases} l + 2w &= 2420 & (\mathrm{E}_1) \\ lw &= 435{,}600 & (\mathrm{E}_2) \end{cases}$

Solve E_1 for l and substitute into E_2. $(2420 - 2w)w = 435{,}600 \Rightarrow$
$2w^2 - 2420w + 435{,}600 = 0 \Rightarrow w^2 - 1210w + 217{,}800 = 0 \Rightarrow$
$(w - 990)(w - 220) = 0 \Rightarrow w = 990$ or 220; $l = 2420 - 2w = 440$ or 1980.

[45] $\begin{cases} x^2y = 2 & \textit{Volume} \\ 2x^2 + 3xy = 8 & \textit{Surface Area} \end{cases}$ Solving E_1 for y and substituting into

E_2 yields $2x^2 + 3x(2/x^2) = 8 \Rightarrow 2x^3 - 8x + 6 = 0 \Rightarrow$

$2(x-1)(x^2 + x - 3) = 0 \Rightarrow \{ x > 0 \}\ x = 1, \dfrac{-1 + \sqrt{13}}{2}$.

There are two solutions: 1 ft × 1 ft × 2 ft or

$$\frac{\sqrt{13}-1}{2}\text{ ft} \times \frac{\sqrt{13}-1}{2}\text{ ft} \times \frac{8}{(\sqrt{13}-1)^2}\text{ ft} \approx 1.30\text{ ft} \times 1.30\text{ ft} \times 1.18\text{ ft}.$$

[46] Let r denote the radius of a circle, and l and w the length and width of a rectangle. If the rectangle and circle have equal perimeters and areas, then

$\begin{cases} 2l + 2w = 2\pi r & \textit{Perimeter} \\ lw = \pi r^2 & \textit{Area} \end{cases}$ Solve E_1 for l and substitute into E_2.

$(\pi r - w)w = \pi r^2 \Rightarrow \pi r w - w^2 = \pi r^2 \Rightarrow w^2 - \pi r w + \pi r^2 = 0 \Rightarrow$

$$w = \frac{\pi r \pm \sqrt{\pi^2 r^2 - 4\pi r^2}}{2} = \frac{\pi r \pm r\sqrt{\pi(\pi - 4)}}{2}.$$

Since the discriminant is negative, there are no real roots.

[47] We eliminate n from the equations to determine all intersection points.

(a) $x^2 + y^2 = n^2$ and $y = n - 1 \Rightarrow x^2 + y^2 = (y+1)^2 \Rightarrow x^2 = 2y + 1 \Rightarrow$
$y = \frac{1}{2}x^2 - \frac{1}{2}$. The points are on the parabola $y = \frac{1}{2}x^2 - \frac{1}{2}$.

(b) $x^2 + y^2 = (y+2)^2 \Rightarrow x^2 = 4y + 4 \Rightarrow y = \frac{1}{4}x^2 - 1$

[48] (a) $\begin{cases} \frac{4}{3}\pi(\frac{1}{2})^3 &= \pi r^2 h & \textit{Volume} \\ 2\left[4\pi(\frac{1}{2})^2\right] &= 2\pi r h + 2\pi r^2 & \textit{S.A.} \end{cases} \Rightarrow \begin{cases} 1 = 6r^2h & (\mathrm{E}_1) \\ 1 = r^2 + rh & (\mathrm{E}_2) \end{cases}$

Solve E_1 for h and substitute into E_2. $r^2 + r\left(\dfrac{1}{6r^2}\right) = 1 \Rightarrow 6r^3 - 6r + 1 = 0$.

(b) $r = 0.172 \Rightarrow h \approx 5.63$ (unreasonable) and
$r = 0.903 \Rightarrow h \approx 0.204$ (reasonable).

[49] (a) The slope of the line from $(-4, -3)$ to the origin is $\frac{3}{4}$ so the slope of the tangent line (which is perpendicular to the line to the origin) is $-\frac{4}{3}$. The line through $(-4, -3)$ is $y + 3 = -\frac{4}{3}(x + 4)$, or $4x + 3y = -25$. Letting $y = -50$, $x = 31.25$.

(b) The slope of the line from $(x,\ y)$ to the origin is $\frac{y}{x}$ so the slope of the tangent line is $-\frac{x}{y}$. The line through $(0,\ -50)$ is $y+50=\left(-\frac{x}{y}\right)(x-0)$, so $y^2+50y=-x^2$. Substituting $x^2=25-y^2$ and solving for y, we have $y^2+50y=-25+y^2 \Rightarrow$

$$y=-\tfrac{1}{2} \text{ and } x=-\sqrt{25-(-\tfrac{1}{2})^2}=-\sqrt{\tfrac{99}{4}}=-\tfrac{3}{2}\sqrt{11}\approx -4.975.$$

[50] (a) Since $y=-\frac{3}{4}x$ is a line that describes the slope, and $x^2+y^2=50^2$, the point where the ball hits the ground can be found by solving $(x)^2+(-\frac{3}{4}x)^2=50^2$ for x. The point is $(40,\ -30)$.

c is 0, so substitute $(40,\ -30)$ for $(x,\ y)$ in $y=ax^2+x$ to give $a=-\frac{7}{160}$.

(b) The maximum height *off the ground* is found when the difference d between the parabola and the line is greatest. $d=(-\frac{7}{160}x^2+x)-(-\frac{3}{4}x)=-\frac{7}{160}x^2+\frac{7}{4}x$.

d will be a maximum when $x=\dfrac{-b}{2a}=\dfrac{-7/4}{2(-7/160)}=20$. $x=20 \Rightarrow d=17.5$ ft.

Note that the vertex *of the parabola* is $(\frac{80}{7},\ \frac{40}{7})\approx(11.4,\ 5.7)$.

[51] Graphically: $x^2+y^2=4 \Rightarrow y=\pm\sqrt{4-x^2}$ and $x+y=1 \Rightarrow y=1-x$.
Graph $Y_1=\sqrt{4-x^2}$, $Y_2=-Y_1$, and $Y_3=1-x$. There are two points of intersection at approximately $(-0.82,\ 1.82)$ and $(1.82,\ -0.82)$.
Algebraically: $y=1-x$ and $x^2+y^2=4 \Rightarrow x^2+(1-x)^2=4 \Rightarrow$
$2x^2-2x-3=0$. Using the quadratic formula, $x=\dfrac{2\pm\sqrt{4-4(2)(-3)}}{4}=\dfrac{1\pm\sqrt{7}}{2}$.
$y=1-x \Rightarrow y=1-\dfrac{1\pm\sqrt{7}}{2}=\dfrac{1\mp\sqrt{7}}{2}$.

The points of intersection are $\left(\frac{1}{2}\pm\frac{\sqrt{7}}{2},\ \frac{1}{2}\mp\frac{\sqrt{7}}{2}\right)$.

The graphical solution approximates the algebraic solution.

[−6, 6] by [−4, 4]

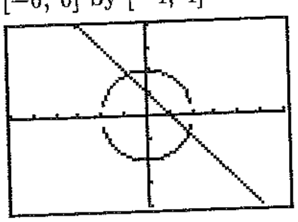

Figure 51

[−9, 9] by [−6, 6]

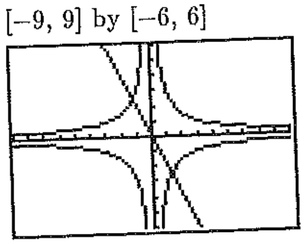

Figure 52

[52] Graphically: $x^2y^2=9 \Rightarrow y=\pm\frac{3}{x}$ and $2x+y=0 \Rightarrow y=-2x$.
Graph $Y_1=\frac{3}{x}$, $Y_2=-Y_1$, and $Y_3=-2x$. There are two points of intersection at approximately $(-1.22,\ 2.45)$ and $(1.22,\ -2.45)$.

Algebraically: $y=-2x$, $x^2y^2=9 \Rightarrow x^2(-2x)^2=9 \Rightarrow 4x^4=9 \Rightarrow$
$x=\pm\sqrt{\frac{3}{2}}=\pm\frac{1}{2}\sqrt{6}$. $y=-2x=\mp\sqrt{6}$. The points of intersection are $(\pm\frac{1}{2}\sqrt{6},\ \mp\sqrt{6})$. The graphical solution approximates the algebraic solution.

$\boxed{53}$ After zooming-in near the region of interest in the second quadrant, we see that the cubic $y = 5x^3 - 5x$ intersects the circle $x^2 + y^2 = 4$ twice. Due to the symmetry, we know there are two more points of intersection in the fourth quadrant. Thus, the six points of intersection are approximately $(\mp 0.56,\ \pm 1.92)$, $(\mp 0.63,\ \pm 1.90)$, and $(\pm 1.14,\ \pm 1.65)$.

$[-6, 6]$ by $[-4, 4]$

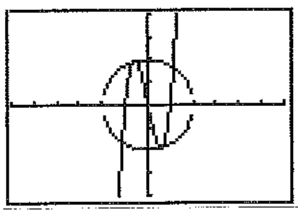

Figure 53

$[-4.5, 4.5]$ by $[-3, 3]$

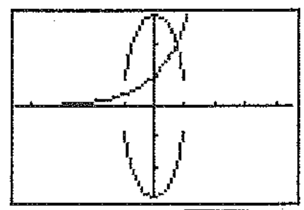

Figure 54

$\boxed{54}$ From the graphs of $Y_1 = \sqrt{9 - 9x^2}$, $Y_2 = -Y_1$, and $Y_3 = e^x$, there are two points of intersection for Y_1 and Y_3.

They are approximately $(-0.992, 0.371)$ and $(0.725, 2.065)$.

$\boxed{55}$ $|\,x + \ln|\,x\,|\,| - y^2 = 0 \;\Rightarrow\; y = \pm\sqrt{|\,x + \ln|\,x\,|\,|}$ and $\dfrac{x^2}{4} + \dfrac{y^2}{2.25} = 1 \;\Rightarrow$

$y = \pm 1.5\sqrt{1 - x^2/4}$. The graph is symmetric with respect to x-axis.

There are 8 points of intersection. Their coordinates are approximately

$(-1.44,\ \pm 1.04)$, $(-0.12,\ \pm 1.50)$, $(0.10,\ \pm 1.50)$, and $(1.22,\ \pm 1.19)$.

$[-3, 3]$ by $[-2, 2]$

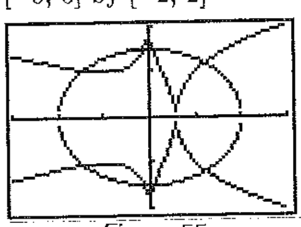

Figure 55

$[-3, 3]$ by $[-1.8, 2.2]$

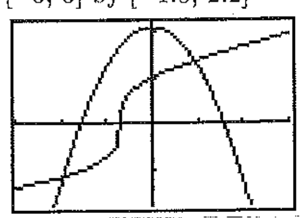

Figure 56

$\boxed{56}$ $y^3 - e^{x/2} = x \;\Rightarrow\; y = (x + e^{x/2})^{1/3}$ and $y + 0.85x^2 = 2.1 \;\Rightarrow\; y = 2.1 - 0.85x^2$.

There are 2 points of intersection.

Their coordinates are approximately $(-1.96, -1.17)$ and $(0.93, 1.36)$.

$\boxed{57}$ If $f(x) = ae^{-bx}$ and $x = 1$, then $f(1) = ae^{-b} = 0.80487 \Rightarrow a = 0.80487e^{b}$. When $x = 2$, $f(2) = ae^{-2b} = 0.53930 \Rightarrow a = 0.53930e^{2b}$. Let $a = y$ and $b = x$ and then graph $Y_1 = 0.80487e^{x}$ and $Y_2 = 0.53930e^{2x}$. The graphs intersect at approximately $(0.4004,\ 1.2012) = (b,\ a)$. Thus, $a \approx 1.2012$, $b \approx 0.4004$, and $f(x) = 1.2012e^{-0.4004x}$. The function f is also accurate at $x = 3,\ 4$.

[0, 3] by [0, 2]

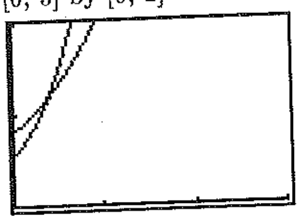

Figure 57

[1, 13] by [−7, 1]

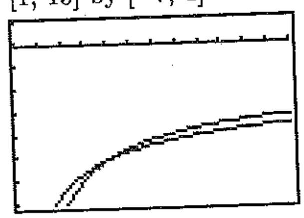

Figure 58

$\boxed{58}$ If $f(x) = a\ln bx$ and $x = 1$, then $f(1) = a\ln b = -8.208 \Rightarrow a = \dfrac{-8.208}{\ln b}$. When $x = 2$, $f(2) = a\ln 2b = -11.74 \Rightarrow a = \dfrac{-11.74}{\ln 2b}$. Graph $Y_1 = \dfrac{-8.208}{\ln x}$ and $Y_2 = \dfrac{-11.74}{\ln 2x}$. The graphs intersect at approximately $(5.0068,\ -5.0956)$. Thus, $a \approx -5.0956$, $b \approx 5.0068$, and $f(x) = -5.0956\ln(5.0068x)$. The function f is also accurate at $x = 3,\ 4$.

$\boxed{59}$ If $f(x) = ax^2 + e^{bx}$ and $x = 2$, then $f(2) = 4a + e^{2b} = 17.2597 \Rightarrow$ $a = (17.2597 - e^{2b})/4$. When $x = 3$, $f(3) = 9a + e^{3b} = 40.1058 \Rightarrow$ $a = (40.1058 - e^{3b})/9$. Graph $Y_1 = (17.2597 - e^{2x})/4$ and $Y_2 = (40.1058 - e^{3x})/9$. The graphs intersect at approximately $(0.9002,\ 2.8019)$. Thus, $a \approx 2.8019$, $b \approx 0.9002$, and $f(x) = 2.8019x^2 + e^{0.9002x}$. The function f is also accurate at $x = 4$.

[0, 4] by [0, 4]

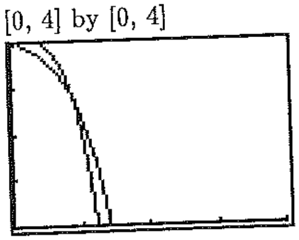

Figure 59

[0, 12] by [0, 8]

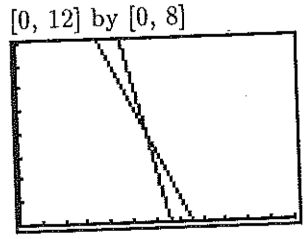

Figure 60

$\boxed{60}$ If $f(x) = \sqrt{ax+b}$ and $x = 2$, then $f(2) = \sqrt{2a+b} = 3.8859 \Rightarrow$ $2a + b = (3.8859)^2 \Rightarrow b = (3.8859)^2 - 2a$. When $x = 4$, $f(4) = \sqrt{4a+b} = 5.1284 \Rightarrow 4a + b = (5.1284)^2 \Rightarrow b = (5.1284)^2 - 4a$. Graph $Y_1 = (3.8859)^2 - 2x$ and $Y_2 = (5.1284)^2 - 4x$. The graphs intersect at approximately $(5.6001,\ 3.9000)$. Thus, $a \approx 5.6001$, $b \approx 3.9000$, and $f(x) = \sqrt{5.6001x + 3.9000}$. The function f is also accurate at $x = 6$.

9.2 Exercises

[1] $-2\,\mathrm{E}_2 + \mathrm{E}_1 \;\Rightarrow\; 7y = -14 \;\Rightarrow\; y = -2;\ x = 4$ ★ $(4,\ -2)$

[2] $-5\,\mathrm{E}_2 + \mathrm{E}_1 \;\Rightarrow\; -11x = 33 \;\Rightarrow\; x = -3;\ y = 5$ ★ $(-3,\ 5)$

[3] $3\,\mathrm{E}_1 - 2\,\mathrm{E}_2 \;\Rightarrow\; 29y = 0 \;\Rightarrow\; y = 0;\ x = 8$ ★ $(8,\ 0)$

[4] $4\,\mathrm{E}_1 - 7\,\mathrm{E}_2 \;\Rightarrow\; -53y = 106 \;\Rightarrow\; y = -2;\ x = -1$ ★ $(-1,\ -2)$

[5] $2\,\mathrm{E}_2 + \mathrm{E}_1 \;\Rightarrow\; 5r = -5 \;\Rightarrow\; r = -1;\ s = \frac{3}{2}$ ★ $(-1,\ \frac{3}{2})$

[6] $-3\,\mathrm{E}_2 + \mathrm{E}_1 \;\Rightarrow\; 17v = -51 \;\Rightarrow\; v = -3;\ u = \frac{2}{3}$ ★ $(\frac{2}{3},\ -3)$

[7] $3\,\mathrm{E}_1 - 5\,\mathrm{E}_2 \;\Rightarrow\; -53y = -28 \;\Rightarrow\; y = \frac{28}{53};$

Instead of substituting into one of the equations to find the value of the other variable, it is usually easier to pick different multipliers and re-solve the system for the other variable. $7\,\mathrm{E}_1 + 6\,\mathrm{E}_2 \;\Rightarrow\; 53x = 76 \;\Rightarrow\; x = \frac{76}{53}$ ★ $(\frac{76}{53},\ \frac{28}{53})$

[8] $3\,\mathrm{E}_1 - 2\,\mathrm{E}_2 \;\Rightarrow\; 34y = 13 \;\Rightarrow\; y = \frac{13}{34};\ 5\,\mathrm{E}_1 + 8\,\mathrm{E}_2 \;\Rightarrow\; 34x = 67 \;\Rightarrow\; x = \frac{67}{34}$ ★ $(\frac{67}{34},\ \frac{13}{34})$

[9] $\begin{cases} 6\,\mathrm{E}_1 \\ 3\,\mathrm{E}_2 \end{cases} \Rightarrow \begin{cases} 2c + 3d = 30 & (\mathrm{E}_3) \\ 3c - 2d = -3 & (\mathrm{E}_4) \end{cases}$ $3\,\mathrm{E}_3 - 2\,\mathrm{E}_4 \;\Rightarrow\; 13d = 96 \;\Rightarrow\; d = \frac{96}{13};$

$2\,\mathrm{E}_3 + 3\,\mathrm{E}_4 \;\Rightarrow\; 13c = 51 \;\Rightarrow\; c = \frac{51}{13}$ ★ $(\frac{51}{13},\ \frac{96}{13})$

[10] $\begin{cases} 10\,\mathrm{E}_1 \\ 12\,\mathrm{E}_2 \end{cases} \Rightarrow \begin{cases} 5t - 2v = 15 & (\mathrm{E}_3) \\ 8t + 3v = 5 & (\mathrm{E}_4) \end{cases}$

$8\,\mathrm{E}_3 - 5\,\mathrm{E}_4 \;\Rightarrow\; -31v = 95 \;\Rightarrow\; v = -\frac{95}{31};\ 3\,\mathrm{E}_3 + 2\,\mathrm{E}_4 \;\Rightarrow\; 31t = 55 \;\Rightarrow\; t = \frac{55}{31}$ ★ $(\frac{55}{31},\ -\frac{95}{31})$

[11] $\sqrt{3}\,\mathrm{E}_1 + \sqrt{2}\,\mathrm{E}_2 \;\Rightarrow\; 7x = 8 \;\Rightarrow\; x = \frac{8}{7};$

$2\sqrt{2}\,\mathrm{E}_1 - \sqrt{3}\,\mathrm{E}_2 \;\Rightarrow\; -7y = 3\sqrt{6} \;\Rightarrow\; y = -\frac{3}{7}\sqrt{6}$ ★ $(\frac{8}{7},\ -\frac{3}{7}\sqrt{6})$

[12] $\begin{cases} 100\,\mathrm{E}_1 \\ 100\,\mathrm{E}_2 \end{cases} \Rightarrow \begin{cases} 11x - 3y = 25 & (\mathrm{E}_3) \\ 12x + 5y = 70 & (\mathrm{E}_4) \end{cases}$

$12\,\mathrm{E}_3 - 11\,\mathrm{E}_4 \;\Rightarrow\; -91y = -470 \;\Rightarrow\; y = \frac{470}{91};$

$5\,\mathrm{E}_3 + 3\,\mathrm{E}_4 \;\Rightarrow\; 91x = 335 \;\Rightarrow\; x = \frac{335}{91}$ ★ $(\frac{335}{91},\ \frac{470}{91})$

[13] $3\,\mathrm{E}_1 + \mathrm{E}_2 \;\Rightarrow\; 0 = 27$; *no solution.* [14] $4\,\mathrm{E}_1 + \mathrm{E}_2 \;\Rightarrow\; 0 = 31$; *no solution.*

[15] $2\,\mathrm{E}_1 + \mathrm{E}_2 \;\Rightarrow\; 0 = 0$; all ordered pairs $(m,\ n)$ such that $3m - 4n = 2$.

[16] $-3\,\mathrm{E}_1 + \mathrm{E}_2 \;\Rightarrow\; 0 = 0$; all ordered pairs $(x,\ y)$ such that $x - 5y = 2$.

[17] $3\,\mathrm{E}_1 - 2\,\mathrm{E}_2 \;\Rightarrow\; -23x = 0 \;\Rightarrow\; x = 0;\ y = 0$ ★ $(0,\ 0)$

[18] $-7\,\mathrm{E}_2 + \mathrm{E}_1 \;\Rightarrow\; 3x = -26 \;\Rightarrow\; x = -\frac{26}{3};\ y = 5$ ★ $(-\frac{26}{3},\ 5)$

[19] Using the hint, the system is $\begin{cases} 2u + 3v = -2 & (\mathrm{E}_3) \\ 4u - 5v = 1 & (\mathrm{E}_4) \end{cases}$

$-2\,\mathrm{E}_3 + \mathrm{E}_4 \;\Rightarrow\; -11v = 5 \;\Rightarrow\; v = -\frac{5}{11};\ 5\,\mathrm{E}_3 + 3\,\mathrm{E}_4 \;\Rightarrow\; 22u = -7 \;\Rightarrow\; u = -\frac{7}{22};$

Resubstituting, we have $x = -\frac{22}{7}$ and $y = -\frac{11}{5}$. ★ $(-\frac{22}{7},\ -\frac{11}{5})$

20 Let $u = \dfrac{1}{x-1}$ and $v = \dfrac{1}{y+2}$. $\begin{cases} 3u + 4v &= 2 & (E_3) \\ 6u - 7v &= -3 & (E_4) \end{cases}$

$-2E_3 + E_4 \Rightarrow -15v = -7 \Rightarrow v = \frac{7}{15}$; $7E_3 + 4E_4 \Rightarrow 45u = 2 \Rightarrow u = \frac{2}{45}$;

$x = \frac{1}{u} + 1 = \frac{47}{2}$; $y = \frac{1}{v} - 2 = \frac{1}{7}$ ★ $(\frac{47}{2}, \frac{1}{7})$

21 Let x denote the number of \$3.00 tickets and y the number of \$4.50 tickets.

$\begin{cases} x + y = 450 & \textit{quantity} \\ 3.00x + 4.50y = 1555.50 & \textit{value} \end{cases}$ $E_2 - 3E_1 \Rightarrow 1.5y = 205.50 \Rightarrow$

$y = 137$; $x = 313$

22 Let x denote the number of passengers that purchased a ticket to Phoenix and y the number of passengers that purchased a ticket to Albuquerque.

$\begin{cases} x + y = 185 & \textit{quantity} \\ 45x + 60y = 10{,}500 & \textit{value} \end{cases}$ $E_2 - 45E_1 \Rightarrow 15y = 2175 \Rightarrow y = 145$; $x = 40$

23 The radius is $\frac{1}{2}$ cm. $\begin{cases} x + y = 8 & \textit{length} \\ \pi(\frac{1}{2})^2 x + \frac{1}{3}\pi(\frac{1}{2})^2 y = 5 & \textit{volume} \end{cases}$

Solving E_1 for y and substituting into E_2 yields $\frac{\pi}{4}x + \frac{\pi}{12}(8 - x) = 5 \Rightarrow$

$\frac{\pi}{6}x = \dfrac{15 - 2\pi}{3} \Rightarrow x = \dfrac{30 - 4\pi}{\pi} = \dfrac{30}{\pi} - 4 \approx 5.55$ cm;

$y = 8 - \left(\dfrac{30}{\pi} - 4\right) = 12 - \dfrac{30}{\pi} \approx 2.45$ cm

24 Let x denote the rate at which he can row in still water and y the speed of the current. $D = RT$ $\begin{cases} 500 = (x - y)10 & \textit{upstream} \\ 300 = (x + y)5 & \textit{downstream} \end{cases} \Rightarrow \begin{cases} 50 = x - y \\ 60 = x + y \end{cases}$

$E_1 + E_2 \Rightarrow 2x = 110 \Rightarrow x = 55$ ft/min; $y = 5$ ft/min

25 $\begin{cases} 2l + 2\pi(\frac{1}{2}w) = 40 & \textit{perimeter} \\ lw = 2\left[\pi(\frac{1}{2}w)^2\right] & \textit{area} \end{cases}$ Solving E_1 for l and substituting into E_2

yields $\left(\dfrac{40 - \pi w}{2}\right)w = \dfrac{\pi w^2}{2} \Rightarrow 40w = 2\pi w^2 \Rightarrow 2w(\pi w - 20) = 0 \Rightarrow w = 0, \dfrac{20}{\pi}$;

$w = 20/\pi \approx 6.37$ ft, $l = 10$ ft

26 Let x denote the amount invested in the 6% fund and y the amount in the 8% fund.

$\begin{cases} x + y = 15{,}000 & \textit{amount} \\ 0.06x + 0.08y = 1000 & \textit{interest} \end{cases}$

$100E_2 - 6E_1 \Rightarrow 2y = 10{,}000 \Rightarrow y = \$5000$; $x = \$10{,}000$

27 Let x denote the number of adults and y the number of kittens. Thus, $(\frac{1}{2}x)$ is the number of adult females. $\begin{cases} x + y = 6000 & \textit{total} \\ y = 3(\frac{1}{2}x) & \textit{3 kittens per adult female} \end{cases}$

Substituting y from E_2 into E_1 yields $x + \frac{3}{2}x = 6000 \Rightarrow x = 2400$; $y = 3600$

$\boxed{28}$ Let x denote the flow rate of the inlet pipe and y the flow rate of one of the outlet pipes. { (inlet rate − outlet rate) (hours) = gallons }

$$\begin{cases} (x-2y)\,5 &= 300 & \textit{both open} \\ (x-y)\,3 &= 300 & \textit{one closed} \end{cases} \Rightarrow \begin{cases} x-2y &= 60 \\ x-y &= 100 \end{cases}$$

$$\mathrm{E}_2-\mathrm{E}_1 \;\Rightarrow\; y=40 \text{ gal/hr};\ x=140 \text{ gal/hr}$$

$\boxed{29}$ Let x denote the number of grams of the 35% alloy and y the number of grams of the 60% alloy.

$$\begin{cases} x+y=100 & \textit{quantity} \\ 0.35x+0.60y=(0.50)(100) & \textit{quality} \end{cases}$$

$$100\,\mathrm{E}_2-35\,\mathrm{E}_1 \;\Rightarrow\; 25y=1500 \;\Rightarrow\; y=60;\ x=40$$

$\boxed{30}$ Let x denote the number of pounds of peanuts used and y the number of pounds of cashews used.

$$\begin{cases} x+\ \ y=60 & \textit{quantity} \\ 3x+8y=5(60) & \textit{quality} \end{cases}$$

$$\mathrm{E}_2-3\,\mathrm{E}_1 \;\Rightarrow\; 5y=120 \;\Rightarrow\; y=24;\ x=36$$

$\boxed{31}$ Let x denote the speed of the plane and y the speed of the wind. D = RT

$$\begin{cases} 1200=(x+y)(2) & \textit{with the wind} \\ 1200=(x-y)(2\tfrac{1}{2}) & \textit{against the wind} \end{cases} \Rightarrow \begin{cases} 600=x+y \\ 480=x-y \end{cases}$$

$$\mathrm{E}_1+\mathrm{E}_2 \;\Rightarrow\; 2x=1080 \;\Rightarrow\; x=540 \text{ mi/hr};\ y=60 \text{ mi/hr}$$

$\boxed{32}$ Let x denote the number of \$0.50 notebooks and y the number of \$0.70 notebooks.

$$\begin{cases} x+y=500 & \textit{quantity} \\ 0.50x+0.70y=286 & \textit{value} \end{cases}$$

$$10\,\mathrm{E}_2-5\,\mathrm{E}_1 \;\Rightarrow\; 2y=360 \;\Rightarrow\; y=180;\ x=320$$

$\boxed{33}$ $v(t)=v_0+at$

$$\begin{cases} v(2)=16 \\ v(5)=25 \end{cases} \Rightarrow \begin{cases} 16=v_0+2a & (\mathrm{E}_1) \\ 25=v_0+5a & (\mathrm{E}_2) \end{cases}$$

$$\mathrm{E}_2-\mathrm{E}_1 \;\Rightarrow\; 9=3a \;\Rightarrow\; a=3;\ v_0=10$$

$\boxed{34}$ $s(t)=-16t^2+v_0t+s_0$

$$\begin{cases} s(1)=84 \\ s(2)=116 \end{cases} \Rightarrow \begin{cases} 84=-16+v_0+s_0 & (\mathrm{E}_1) \\ 116=-64+2v_0+s_0 & (\mathrm{E}_2) \end{cases}$$

$$\mathrm{E}_2-\mathrm{E}_1 \;\Rightarrow\; v_0=80;\ s_0=20$$

$\boxed{35}$ Let x denote the number of sofas produced and y the number of recliners produced.

$$\begin{cases} 8x+\ 6y=340 & \textit{labor hours} \\ 60x+35y=2250 & \textit{cost of materials} \end{cases}$$

$$6\,\mathrm{E}_2-35\,\mathrm{E}_1 \;\Rightarrow\; 80x=1600 \;\Rightarrow\; x=20;\ y=30$$

$\boxed{36}$ Let x denote the number of ounces of oats used and y the number of ounces of cornmeal used. $\begin{cases} 4x + 3y = 200 & \textit{protein} \\ 18x + 24y = 1320 & \textit{carbohydrates} \end{cases}$

$$\mathrm{E}_2 - 8\,\mathrm{E}_1 \;\Rightarrow\; -14x = -280 \;\Rightarrow\; x = 20;\; y = 40$$

$\boxed{37}$ (a) $\underline{6x + 5y}$ represents the total bill for the plumber's business. This should equal the plumber's income, i.e., $(6 + 4)\,x$. $\underline{4x + 6y}$ represents the total bill for the electrician's business. This should equal the electrician's income, i.e., $(5 + 6)\,y$.

$$\begin{cases} 6x + 5y = 10x \\ 4x + 6y = 11y \end{cases} \;\Rightarrow\; \begin{cases} 5y = 4x \\ 4x = 5y \end{cases} \;\Rightarrow\; y = \tfrac{4}{5}x, \text{ or } y = 0.80x$$

(b) The electrician should charge 80% of what the plumber charges.

80% of \$20 per hour is \$16 per hour.

$\boxed{38}$ We want the equations of the 3 lines that are perpendicular to a side and pass through the opposite vertex. $m_{AB} = \frac{1}{4}$; the equation of the line through C with a slope of -4 is $y + 8 = -4(x - 3) \;\Leftrightarrow\; \underline{4x + y = 4}$ (E_1). Similarly, the other two altitudes are:

$$m_{BC} = 6; \quad y - 2 = -\tfrac{1}{6}(x + 3) \;\Leftrightarrow\; \underline{x + 6y = 9} \;\; (\mathrm{E}_2)$$

$$m_{AC} = -\tfrac{5}{3}; \quad y - 4 = \tfrac{3}{5}(x - 5) \;\Leftrightarrow\; \underline{3x - 5y = -5} \;\; (\mathrm{E}_3)$$

Finding the intersection of E_1 and E_2, we have

$$\mathrm{E}_1 - 4\,\mathrm{E}_2 \;\Rightarrow\; -23y = -32 \;\Rightarrow\; y = \tfrac{32}{23};\; x = \tfrac{15}{23}.\; \left(\tfrac{15}{23}, \tfrac{32}{23}\right) \text{ also lies on } \mathrm{E}_3.$$

$\boxed{39}$ Let $t = 0$ correspond to the year 1891. The average daily maximum can then be approximated by the equation $y_1 = 0.011t + 15.1$ and the average daily minimum by the equation $y_2 = 0.019t + 5.8$. We must determine t when y_1 and y_2 differ by 9. $y_1 - y_2 = 9 \;\Rightarrow\; (0.011t + 15.1) - (0.019t + 5.8) = 9 \;\Rightarrow\; -0.008t = -0.3 \;\Rightarrow$ $t = 37.5$. $1891 + 37.5 = 1928.5$ or during 1928.

$$t = 37.5 \;\Rightarrow\; y_1 = 0.011(37.5) + 15.1 = 15.5125 \approx 15.5^\circ\mathrm{C}.$$

$\boxed{40}$ (a) Let x denote the cost of the first minute and y the cost of each additional minute. $\begin{cases} x + 35y = 2.93 & 36 \textit{ minute call} \\ x + 12y = 1.09 & 13 \textit{ minute call} \end{cases}$

$$\mathrm{E}_1 - \mathrm{E}_2 \;\Rightarrow\; 23y = 1.84 \;\Rightarrow\; y = \$0.08;\; x = \$0.13$$

(b) If C denotes the cost of an n minute phone call, then $C = 0.13 + 0.08(n-1)$.

The total cost T of the call is

$T = (\text{cost}) + (\text{federal tax}) + (\text{state tax}) = C + 0.032C + 0.072C = 1.104C$.

$T = 5.00 \Rightarrow 1.104C = 5.00 \Rightarrow C = \frac{5.00}{1.104} \Rightarrow$

$0.13 + 0.08(n-1) = \frac{5.00}{1.104} \Rightarrow 0.08(n-1) = \frac{5.00}{1.104} - 0.13 \Rightarrow$

$$n = \frac{\frac{5.00}{1.104} - 0.13}{0.08} + 1 \approx 55.99 \text{ min, or } 55 \text{ min.}$$

41 Let x and y denote the amount of time in hours at the LP and SLP speeds, respectively. 5 hours and 20 minutes $= 5\frac{1}{3}$ hours, so $\frac{x}{5\frac{1}{3}}$ is the portion of the tape used at the LP speed. $\frac{y}{8}$ is the portion of the tape used at the SLP speed, and together, these fractions add up to 1 whole tape.

$$\begin{cases} x + y = 6 & \textit{total time} \\ \dfrac{x}{5\frac{1}{3}} + \dfrac{y}{8} = 1 & \textit{portions of tape} \end{cases} \qquad 16\,\mathrm{E}_2 - 2\,\mathrm{E}_1 \Rightarrow x = 4;\ y = 2$$

Note: If you convert hours to minutes, the equations can be written as $x + y = 360$ and $x/320 + y/480 = 1$ with the solution $x = 240$ and $y = 120$.

42 (a) Model this as a line through the two points (10, 1,000,000) and (11, 900,000).

$$Q - 1{,}000{,}000 = \frac{900{,}000 - 1{,}000{,}000}{11 - 10}(p - 10) \Rightarrow Q = -100{,}000p + 2{,}000{,}000$$

(b) $K - 2{,}000{,}000 = \frac{150{,}000}{1}(p - 15) \Rightarrow K = 150{,}000p - 250{,}000$

(c) $Q = K \Rightarrow -100{,}000p + 2{,}000{,}000 = 150{,}000p - 250{,}000 \Rightarrow$

$$250{,}000p = 2{,}250{,}000 \Rightarrow p = \$9.00$$

43 $\begin{cases} a\,e^{3x} + b\,e^{-3x} = 0 & (\mathrm{E}_1) \\ a\,(3e^{3x}) + b\,(-3e^{-3x}) = e^{3x} & (\mathrm{E}_2) \end{cases}$

$-3\,\mathrm{E}_1 + \mathrm{E}_2 \Rightarrow -3b\,e^{-3x} - 3b\,e^{-3x} = e^{3x} \Rightarrow -6b\,e^{-3x} = e^{3x} \Rightarrow b = -\frac{1}{6}e^{6x}$.

Substituting back into E_1 yields

$$a\,e^{3x} + (-\tfrac{1}{6}e^{6x})e^{-3x} = 0 \Rightarrow a\,e^{3x} = \tfrac{1}{6}e^{3x} \Rightarrow a = \tfrac{1}{6}.$$

44 $\begin{cases} a\,e^{-x} + b\,e^{4x} = 0 & (\mathrm{E}_1) \\ -a\,e^{-x} + b\,(4e^{4x}) = 2 & (\mathrm{E}_2) \end{cases}$

$\mathrm{E}_1 + \mathrm{E}_2 \Rightarrow b\,e^{4x} + 4b\,e^{4x} = 2 \Rightarrow 5b\,e^{4x} = 2 \Rightarrow b = \frac{2}{5}e^{-4x}$.

Substituting back into E_1 yields

$$a\,e^{-x} + (\tfrac{2}{5}e^{-4x})e^{4x} = 0 \Rightarrow a\,e^{-x} + \tfrac{2}{5} = 0 \Rightarrow a\,e^{-x} = -\tfrac{2}{5} \Rightarrow a = -\tfrac{2}{5}e^{x}.$$

Note: The solutions for Exercises 45 and 46 are on page 526 at the end of this chapter.

9.3 Exercises

[1] $3x - 2y < 6 \Leftrightarrow y > \frac{3}{2}x - 3$; test point $(0, 0) \rightarrow$ True

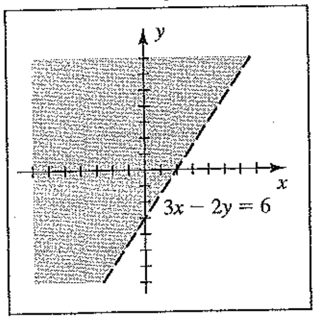

Figure 1

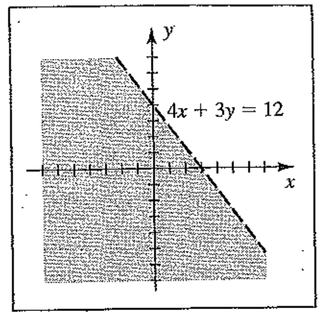

Figure 2

[2] $4x + 3y < 12 \Leftrightarrow y < -\frac{4}{3}x + 4$; test point $(0, 0) \rightarrow$ True

[3] $2x + 3y \geq 2y + 1 \Leftrightarrow y \geq -2x + 1$; test point $(0, 0) \rightarrow$ False

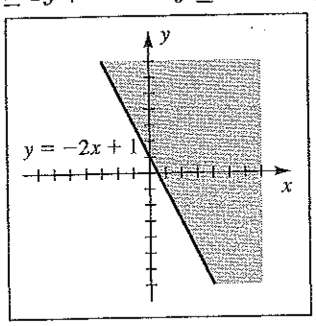

Figure 3

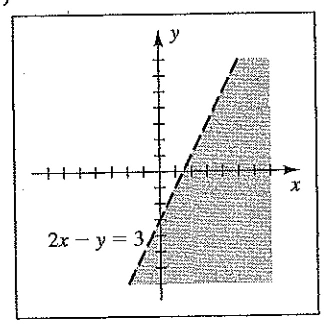

Figure 4

[4] $2x - y > 3 \Leftrightarrow y < 2x - 3$; test point $(0, 0) \rightarrow$ False

[5] $y + 2 < x^2 \Leftrightarrow y < x^2 - 2$; test point $(0, 0) \rightarrow$ False

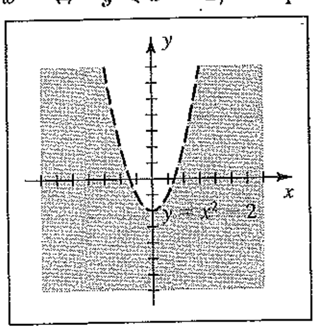

Figure 5

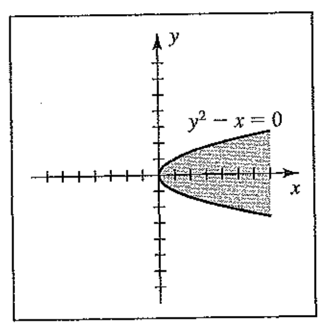

Figure 6

[6] $y^2 - x \leq 0 \Leftrightarrow x \geq y^2$; test point $(1, 0) \rightarrow$ True

[7] $x^2+1 \le y \quad \Leftrightarrow \quad y \ge x^2+1$; test point $(0,\ 0) \rightarrow$ False

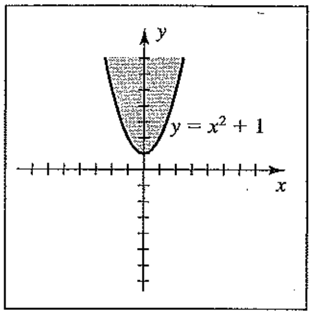

Figure 7

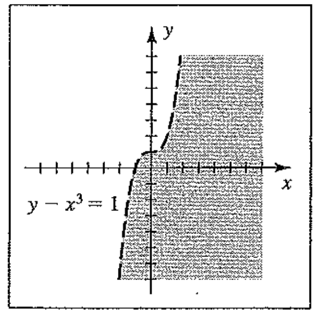

Figure 8

[8] $y - x^3 < 1 \quad \Leftrightarrow \quad y < x^3+1$; test point $(0,\ 0) \rightarrow$ True

[9] $yx^2 \ge 1 \quad \Leftrightarrow \quad y \ge 1/x^2\ \{x \ne 0\}$; test point $(1,\ 0) \rightarrow$ False

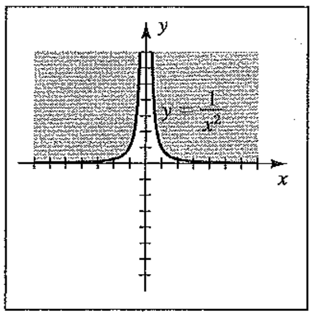

Figure 9

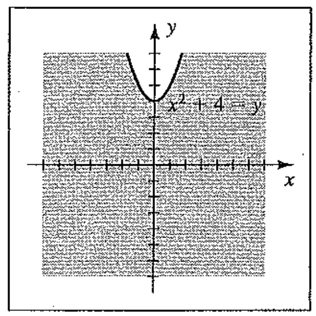

Figure 10

[10] $x^2+4 \ge y \quad \Leftrightarrow \quad y \le x^2+4$; test point $(0,\ 0) \rightarrow$ True

Note: The notation V @ $(a,\ b)$, $(c,\ d)$, ... is used to denote the intersection point(s) of the solution region of the graph.

[11] $\begin{cases} 3x+y<3 \\ 4-y<2x \end{cases} \quad \Leftrightarrow \quad \begin{cases} y<-3x+3 \\ y>-2x+4 \end{cases} \qquad V$ @ $(-1,\ 6)$

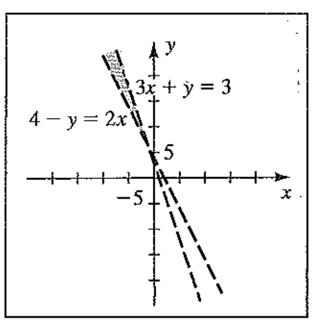

Figure 11

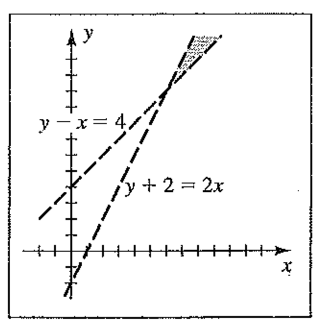

Figure 12

[12] $\begin{cases} y+2<2x \\ y-x>4 \end{cases} \quad \Leftrightarrow \quad \begin{cases} y<2x-2 \\ y>x+4 \end{cases} \qquad V$ @ $(6,\ 10)$

$\boxed{13}$ $\begin{cases} y - x < 0 \\ 2x + 5y < 10 \end{cases} \Leftrightarrow \begin{cases} y < x \\ y < -\frac{2}{5}x + 2 \end{cases}$ V @ $(\frac{10}{7}, \frac{10}{7})$

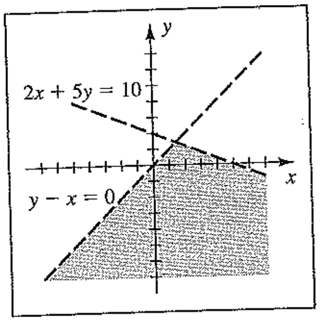

Figure 13

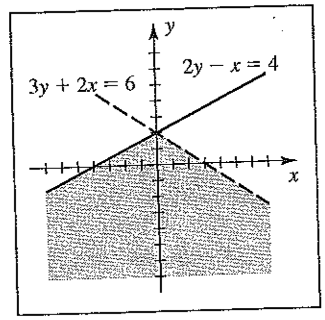

Figure 14

$\boxed{14}$ $\begin{cases} 2y - x \leq 4 \\ 3y + 2x < 6 \end{cases} \Leftrightarrow \begin{cases} y \leq \frac{1}{2}x + 2 \\ y < -\frac{2}{3}x + 2 \end{cases}$ V @ $(0, 2)$

$\boxed{15}$ $\begin{cases} 3x + y \leq 6 \\ y - 2x \geq 1 \\ x \geq -2 \\ y \leq 4 \end{cases} \Leftrightarrow \begin{cases} y \leq -3x + 6 \\ y \geq 2x + 1 \\ x \geq -2 \\ y \leq 4 \end{cases}$ V @ $(-2, -3)$, $(-2, 4)$, $(\frac{2}{3}, 4)$, $(1, 3)$

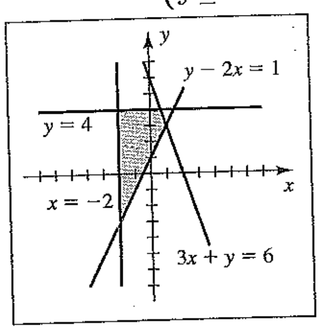

Figure 15

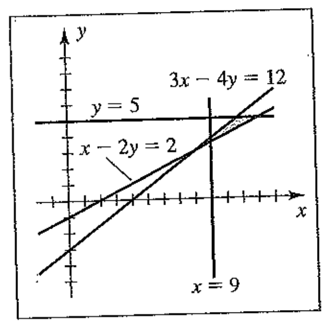

Figure 16

$\boxed{16}$ $\begin{cases} 3x - 4y \geq 12 \\ x - 2y \leq 2 \\ x \geq 9 \\ y \leq 5 \end{cases} \Leftrightarrow \begin{cases} y \leq \frac{3}{4}x - 3 \\ y \geq \frac{1}{2}x - 1 \\ x \geq 9 \\ y \leq 5 \end{cases}$ V @ $(9, \frac{7}{2})$, $(9, \frac{15}{4})$, $(\frac{32}{3}, 5)$, $(12, 5)$

$\boxed{17}$ $\begin{cases} x + 2y \leq 8 \\ 0 \leq x \leq 4 \\ 0 \leq y \leq 3 \end{cases}$

$\boxed{18}$ $\begin{cases} 2x + 3y \geq 6 \\ 0 \leq x \leq 5 \\ 0 \leq y \leq 4 \end{cases}$

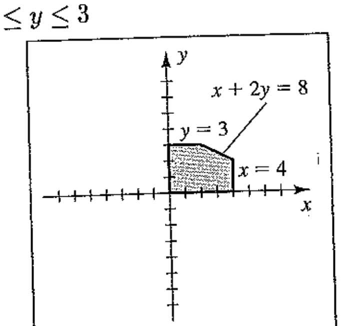

Figure 17

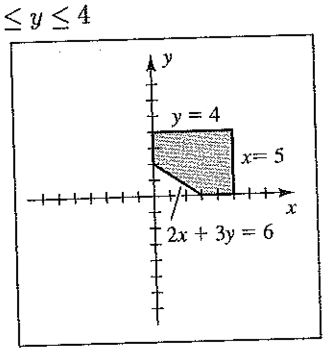

Figure 18

$\boxed{19}$ $|x| \geq 2 \;\Leftrightarrow\; x \geq 2$ or $x \leq -2$; $|y| < 3 \;\Leftrightarrow\; -3 < y < 3$

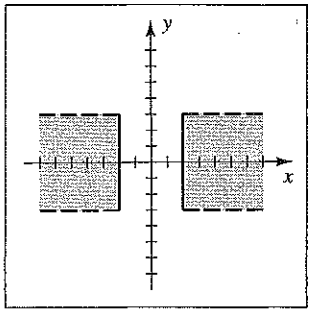

Figure 19

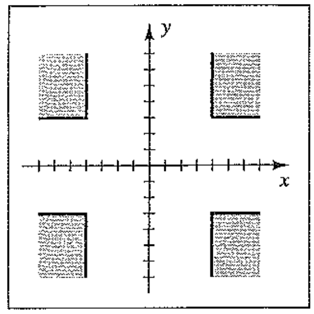

Figure 20

$\boxed{20}$ $|x| \geq 4 \;\Leftrightarrow\; x \geq 4$ or $x \leq -4$; $|y| \geq 3 \;\Leftrightarrow\; y \geq 3$ or $y \leq -3$

$\boxed{21}$ $|x+2| \leq 1 \;\Leftrightarrow\; -1 \leq x+2 \leq 1 \;\Leftrightarrow\; -3 \leq x \leq -1$;

$|y-3| < 5 \;\Leftrightarrow\; -5 < y-3 < 5 \;\Leftrightarrow\; -2 < y < 8$

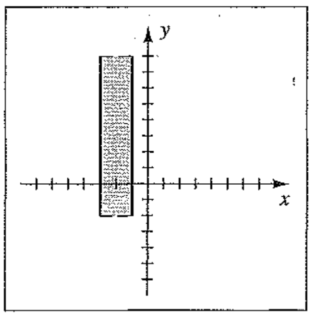

Figure 21

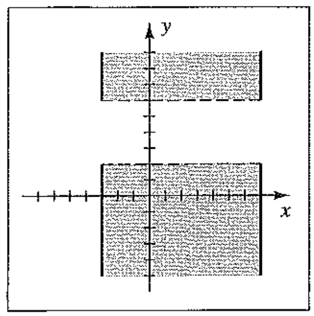

Figure 22

$\boxed{22}$ $|x-2| \leq 5 \;\Leftrightarrow\; -5 \leq x-2 \leq 5 \;\Leftrightarrow\; -3 \leq x \leq 7$;

$|y-4| > 2 \;\Leftrightarrow\; y-4 > 2$ or $y-4 < -2 \;\Leftrightarrow\; y > 6$ or $y < 2$

$\boxed{23}$ $\begin{cases} x^2+y^2 \leq 4 \\ x+y \geq 1 \end{cases} \;\Leftrightarrow\; \begin{cases} x^2+y^2 \leq 2^2 \\ y \geq -x+1 \end{cases}$ $\quad V$ @ $(\frac{1}{2} \mp \frac{1}{2}\sqrt{7}, \frac{1}{2} \pm \frac{1}{2}\sqrt{7})$

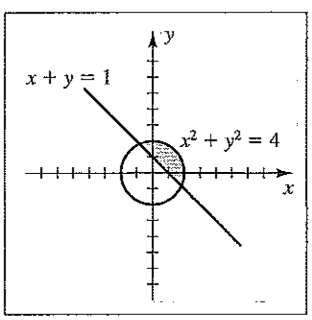

Figure 23

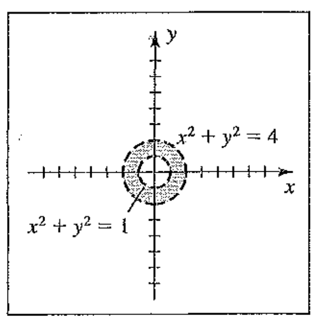

Figure 24

$\boxed{24}$ $\begin{cases} x^2+y^2 > 1 \\ x^2+y^2 < 4 \end{cases}$

[25] $\begin{cases} x^2 \leq 1 - y \\ x \geq 1 + y \end{cases} \quad \Leftrightarrow \quad \begin{cases} y \leq -x^2 + 1 \\ y \leq x - 1 \end{cases}$ $\quad V$ @ $(-2,\ -3)$, $(1,\ 0)$

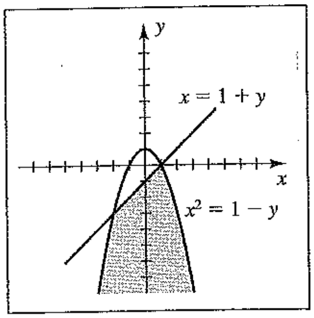

Figure 25

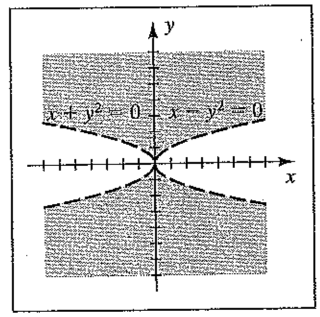

Figure 26

[26] $\begin{cases} x - y^2 < 0 \\ x + y^2 > 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} x < y^2 \\ x > -y^2 \end{cases}$

[27] $\begin{cases} 0 \leq x < 3 \\ y < -x + 4 \\ y \geq x - 4 \end{cases}$
slope between $(0,\ 4)$ and $(4,\ 0)$ is -1, y-intercept is 4
slope between $(0,\ -4)$ and $(4,\ 0)$ is 1, y-intercept is -4

[28] $\begin{cases} x^2 + y^2 \leq 9 \\ (x-1)^2 + y^2 < 9 \end{cases}$
center is $(0,\ 0)$, radius is 3
center is $(1,\ 0)$, radius is 3

[29] $\begin{cases} x^2 + y^2 \leq 9 \\ y > -2x + 4 \end{cases}$
center is $(0,\ 0)$, radius is 3
slope between $(0,\ 4)$ and $(2,\ 0)$ is -2, y-intercept is 4

[30] $\begin{cases} 0 \leq y \leq 3 \\ x \geq 0 \\ y < -x + 4 \\ y \geq \frac{3}{2}x - 3 \end{cases}$
slope between $(0,\ 4)$ and $(4,\ 0)$ is -1, y-intercept is 4
slope between $(0,\ -3)$ and $(2,\ 0)$ is $\frac{3}{2}$, y-intercept is -3

[31] $\begin{cases} y < x \\ y \leq -x + 4 \\ (x-2)^2 + (y-2)^2 \leq 8 \end{cases}$
slope between $(0,\ 4)$ and $(4,\ 0)$ is -1, y-intercept is 4
center is $(2,\ 2)$, radius is $\sqrt{8}$

[32] $\begin{cases} y < x^2 \\ x^2 + y^2 \leq 9 \end{cases}$
center is $(0,\ 0)$, radius is 3

[33] $\begin{cases} y > \frac{1}{8}x + \frac{1}{2} \\ y \leq x + 4 \\ y \leq -\frac{3}{4}x + 4 \end{cases}$
slope between $(-4,\ 0)$ and $(4,\ 1)$ is $\frac{1}{8}$, y-intercept is $\frac{1}{2}$
slope between $(-4,\ 0)$ and $(0,\ 4)$ is 1, y-intercept is 4
slope between $(0,\ 4)$ and $(4,\ 1)$ is $-\frac{3}{4}$, y-intercept is 4

[34] $\begin{cases} y < \frac{4}{3}x + 4 \\ y \leq -x + 2 \end{cases}$
slope between $(-3,\ 0)$ and $(0,\ 4)$ is $\frac{4}{3}$, y-intercept is 4
slope between $(2,\ 0)$ and $(0,\ 2)$ is -1, y-intercept is 2

35 If x and y denote the number of sets of brand A and brand B, respectively, then a system is $x \geq 20$, $y \geq 10$, $x \geq 2y$, $x + y \leq 100$. The graph is the region bounded by the triangle with vertices (20, 10), (90, 10), $(\frac{200}{3}, \frac{100}{3})$.

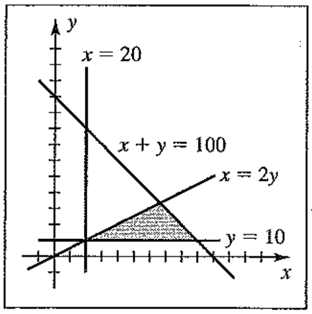

Figure 35

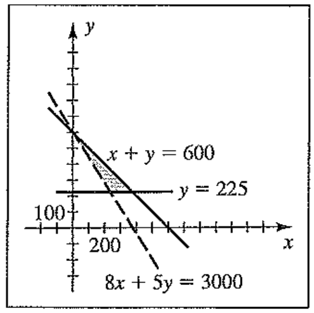

Figure 36

36 If x and y denote the number of \$8.00 and \$5.00 seats, respectively, then a system is $x + y \leq 600$, $y \geq 225$, $8x + 5y \geq 3000$. The graph is the region bounded by the triangle with vertices (0, 600), (375, 225), $(\frac{1875}{8}\ \{= 234.375\}, 225)$.

37 If x and y denote the amount placed in the high-risk and low-risk investment, respectively, then a system is $x \geq 2000$, $y \geq 3x$, $x + y \leq 15{,}000$. The graph is the region bounded by the triangle with vertices (2000, 6000), (2000, 13,000), and (3750, 11,250).

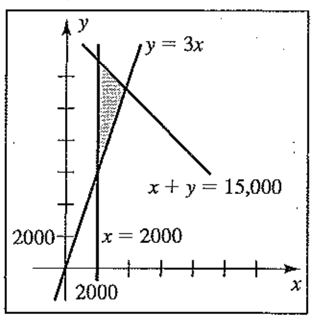

Figure 37

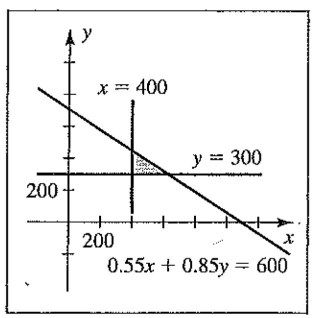

Figure 38

38 If x and y denote the number of 55¢ and 85¢ notebooks, respectively, then a system is $0.55x + 0.85y \leq 600$, $y \geq 300$, $x \geq 400$.
The graph is the region bounded by the triangle with vertices (400, 300), $(400, \frac{7600}{17}\ \{\approx 447.1\})$, $(\frac{6900}{11}\ \{\approx 627.3\}, 300)$.

[39] A system is $x+y\le 9$, $y\ge x$, $x\ge 1$. To justify the condition $y\ge x$, start with

$\dfrac{\text{cylinder volume}}{\text{total volume}}\ge 0.75 \;\Rightarrow\; \dfrac{\pi r^2 y}{\pi r^2 y+\frac{1}{3}\pi r^2 x}\ge\dfrac{3}{4} \;\Rightarrow\; 4\pi r^2 y\ge 3\pi r^2 y+\pi r^2 x \;\Rightarrow$

$\pi r^2 y\ge \pi r^2 x \;\Rightarrow\; y\ge x$. The graph is the region bounded by the triangle with vertices $(1, 1)$, $(1, 8)$, $(\frac{9}{2}, \frac{9}{2})$.

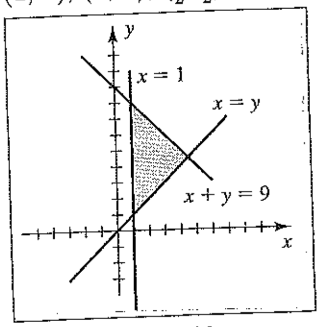

Figure 39

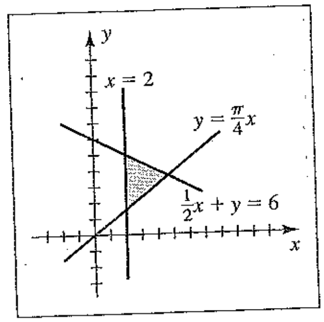

Figure 40

[40] If ℓ denotes the length of the rectangle (the height of the rectangular portion of the window), then a system is $\frac{1}{2}d+\ell\le 6$, $\ell\ge\frac{\pi}{4}d$, $d\ge 2$. The figure has $\ell=y$ and $d=x$. To justify the condition $\ell\ge\frac{\pi}{4}d$, start with

(area of rectangle) ≥ 2 (area of semicircle) $\;\Rightarrow\; d\ell\ge 2\left[\frac{1}{2}\pi(\frac{1}{2}d)^2\right] \;\Rightarrow\; d\ell\ge\frac{\pi}{4}d^2 \;\Rightarrow$

$\ell\ge\frac{\pi}{4}d$. The graph is the region bounded by the triangle with vertices

$(2, 5)$, $(2, \frac{\pi}{2})$, and $\left(\dfrac{24}{\pi+2}, \dfrac{6\pi}{\pi+2}\right)\approx(4.67, 3.67)$.

[41] If the plant is located at (x, y), then a system is $(60)^2\le x^2+y^2\le(100)^2$, $(60)^2\le(x-100)^2+y^2\le(100)^2$, $y\ge 0$. The graph is the region in the first quadrant that lies between the two concentric circles with center $(0, 0)$ and radii 60 and 100, and also between the two concentric circles with center $(100, 0)$ and radii 60 and 100. Equating the different circle equations, we obtain the vertices of the solution region $(50, 50\sqrt{3})$, $(50, 10\sqrt{11})$, $(18, 6\sqrt{91})$, $(82, 6\sqrt{91})$.

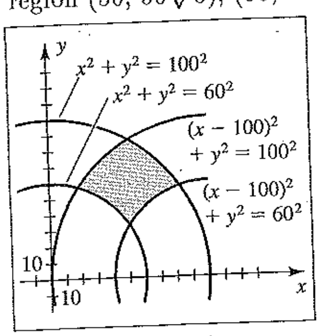

Figure 41

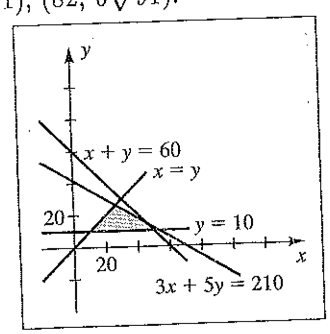

Figure 42

[42] If x and y denote the depths of the patio and pool areas, respectively, then a system is $y\ge 10$, $x+y\le 60$ {sum of depths cannot exceed 60}, $x\ge y$ {patio area $\ge$ pool area},

(continued)

$3x+5y \le 210$ $\{3(50x)+5(50y) \le 10{,}500\}$. The graph is the region bounded by the quadrilateral with vertices (10, 10), $(26\frac{1}{4}, 26\frac{1}{4})$, (45, 15), (50, 10).

$\boxed{43}$ $64y^3 - x^3 \le e^{1-2x} \Rightarrow 64y^3 \le e^{1-2x} + x^3 \Rightarrow y^3 \le \frac{1}{64}(e^{1-2x}+x^3) \Rightarrow$ $y \le \frac{1}{4}(e^{1-2x}+x^3)^{1/3}$. Graph the curve $y = \frac{1}{4}(e^{1-2x}+x^3)^{1/3}$.
The solution includes the curve and the region below it.

[−3.5, 4] by [−1, 4]

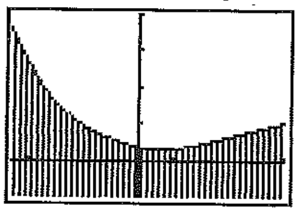

Figure 43

[−3.5, 4] by [−1, 4]

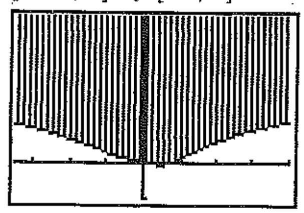

Figure 44

$\boxed{44}$ $e^{5y} - e^{-x} \ge x^4 \Rightarrow e^{5y} \ge x^4 + e^{-x} \Rightarrow 5y \ge \ln(x^4+e^{-x}) \Rightarrow y \ge \frac{1}{5}\ln(x^4+e^{-x})$.
Graph the curve $y = \frac{1}{5}\ln(x^4+e^{-x})$.
The solution includes the curve and the region above it.

$\boxed{45}$ $5^{1-y} \ge x^4+x^2+1 \Rightarrow 1-y \ge \log_5(x^4+x^2+1) \Rightarrow y \le 1-\log_5(x^4+x^2+1)$. $x+3y \ge x^{5/3} \Rightarrow y \ge \frac{1}{3}(x^{5/3}-x)$. Graph $y = 1-\log_5(x^4+x^2+1)$ $\{Y_1\}$ and $y=\frac{1}{3}(x^{5/3}-x)$ $\{Y_2\}$. The curves intersect at approximately (−1.32, −0.09) and (1.21, 0.05). The solution is located between the points of intersection. It includes the curves and the region below the first curve and above the second curve.

[−1.5, 1.5, 0.5] by [−1, 1, 0.5]

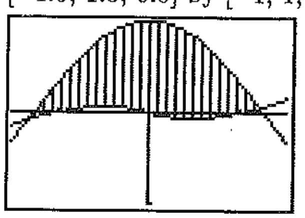

Figure 45

[−3, 3] by [−2, 2]

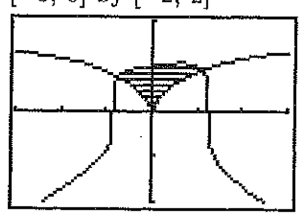

Figure 46

$\boxed{46}$ $x^4+y^5 < 2^x \Rightarrow y < (2^x - x^4)^{1/5}$. $y^3 > \ln(x^2+1) \Rightarrow y > [\ln(x^2+1)]^{1/3}$. Graph $y = (2^x-x^4)^{1/5}$ and $y = [\ln(x^2+1)]^{1/3}$. The curves intersect at approximately (−0.76, 0.77) and (1.10, 0.93). The solution does not include the curves. It includes the region located between the points of intersection that is below the first curve and above the second curve.

$\boxed{47}$ $x^4 - 2x < 3y \Rightarrow y > \frac{1}{3}(x^4 - 2x)$. $x + 2y < x^3 - 5 \Rightarrow y < \frac{1}{2}(x^3 - x - 5)$. Graph $y = \frac{1}{3}(x^4 - 2x)$ and $y = \frac{1}{2}(x^3 - x - 5)$. The curves do not intersect. The solution must be above the first curve and below the second curve. Since the regions do not intersect, there is no solution.

[−4.5, 4.5] by [−3, 3]

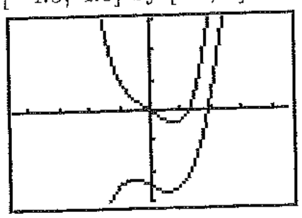

Figure 47

[0, 9] by [−3, 3]

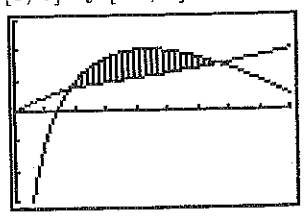

Figure 48

$\boxed{48}$ $e^x + x^2 \le 2^{x+2y} \Rightarrow \log_2(e^x + x^2) \le x + 2y \Rightarrow y \ge \frac{1}{2}\left[\log_2(e^x + x^2) - x\right]$.

$2^{x+2y} \le x^3 2^y \Rightarrow x + 2y \le \log_2(x^3 2^y) \Rightarrow 2y \le \log_2 x^3 + \log_2 2^y - x \Rightarrow$

$y \le 3\log_2 x - x$. Graph $y = \frac{1}{2}\left[\log_2(e^x + x^2) - x\right]$ and $y = 3\log_2 x - x$. The curves intersect at approximately (1.77, 0.70) and (6.72, 1.53). The solution is located between the points of intersection. It includes the curves and the region above the first curve and below the second curve.

$\boxed{49}$ (a) $29T - 39P < 450 \Rightarrow 29(37) - 39(21.2) < 450 \Rightarrow 246.2 < 450$ (true).

Yes, forests can grow.

(b) $29T - 39P < 450 \Rightarrow -39P < -29T + 450 \Rightarrow P > \frac{29}{39}T - \frac{150}{13}$.

Graph $Y_1 = \frac{29}{39}x - \frac{150}{13}$, $Y_3 = 13$, $Y_4 = 45$, and use Shade(Y_1, 45, 33, 76).

(c) Forests can grow whenever the point (T, P) lies in the shaded region with vertices (33, 13), ($\approx$ 76, 45), and (33, 45).

[33, 80, 5] by [0, 50, 5]

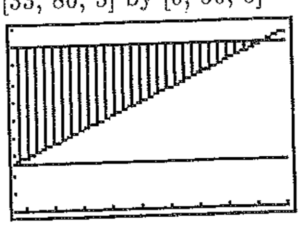

Figure 49

[33, 80, 5] by [0, 50, 5]

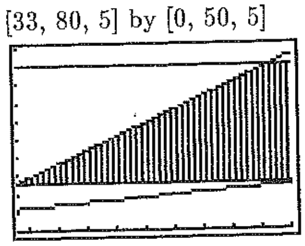

Figure 50

$\boxed{50}$ (a) $22P - 3T > 33 \Rightarrow 22(7.8) - 3(70) > 33 \Rightarrow -38.4 > 33$ (false).

No, grasslands will not grow.

(b) $22P - 3T > 33 \Rightarrow 22P > 3T + 33 \Rightarrow P > \frac{3}{22}T + \frac{3}{2}$.

Graph $Y_2 = \frac{3}{22}x + \frac{3}{2}$ together with Y_1, Y_3, and Y_4 from the previous exercise, and use Shade(13, Y_1, 33, 76) and Shade(13, 45, 76, 80).

(c) Grasslands can grow, but not forests, whenever the point (T, P) lies in the shaded region with vertices (33, 13), ($\approx$ 76, 45), (80, 45), and (80, 13).

9.4 Exercises

1

(x, y)	(0, 2)	(0, 4)	(3, 5)	(6, 2)	(5, 0)	(2, 0)
C	9 ■	13	24	27 ■	20	11

$C = 3x + 2y + 5$; maximum of 27 at (6, 2); minimum of 9 at (0, 2)

2

(x, y)	(1, 3)	(0, 5)	(2, 5)	(6, 2)	(6, 0)	(3, 1)
C	26	38	42 ■	29	15 ■	16

$C = 2x + 7y + 3$; maximum of 42 at (2, 5); minimum of 15 at (6, 0)

3

(x, y)	(0, 0)	(0, 3)	(4, 6)	(6, 3)	(5, 0)
C	0	3	18	21 ■	15

$C = 3x + y$; maximum of 21 at (6, 3)

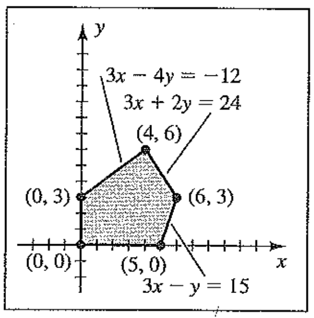

Figure 3

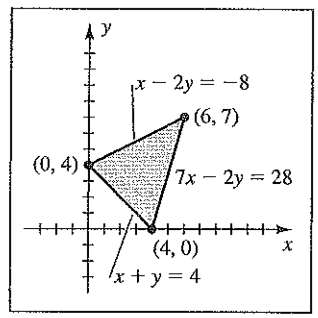

Figure 4

4

(x, y)	(0, 4)	(6, 7)	(4, 0)
C	−8	10	16 ■

$C = 4x - 2y$; maximum of 16 at (4, 0)

5

(x, y)	(8, 0)	(3, 2)	(0, 4)
C	24	21 ■	24

$C = 3x + 6y$; minimum of 21 at (3, 2)

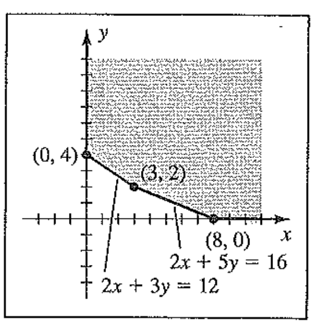

Figure 5

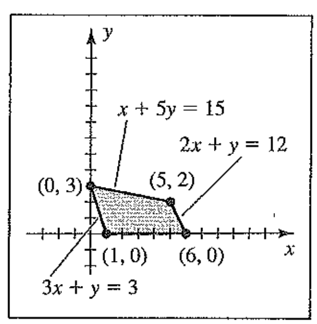

Figure 6

6

(x, y)	(0, 3)	(5, 2)	(6, 0)	(1, 0)
C	3 ■	32	36	6

$C = 6x + y$; minimum of 3 at (0, 3)

[7]

(x, y)	(0, 0)	(0, 4)	(2, 5)	(6, 3)	(8, 0)
C	0	16	24 ■	24 ■	16

$C = 2x + 4y$

C has the maximum value 24 for any point on the line segment from (2, 5) to (6, 3).

$C = 2x + 4y = 2x + 4(6 - \frac{1}{2}x) = 2x + 24 - 2x = 24$. See *Figure 7*.

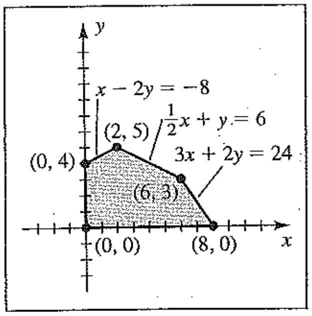

Figure 7

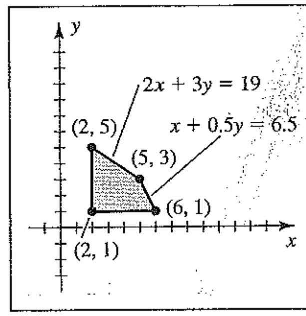

Figure 8

[8]

(x, y)	(2, 1)	(2, 5)	(5, 3)	(6, 1)
C	15	27	39 ■	39 ■

$C = 6x + 3y$

C has the maximum value 39 for any point on the line segment from (6, 1) to (5, 3).

$C = 6x + 3y = 6(6.5 - 0.5y) + 3y = 39 - 3y + 3y = 39$.

[9] Let x and y denote the number of oversized and standard rackets, respectively.

Profit function: $P = 15x + 8y$

$$\begin{cases} 30 \leq y \leq 80 \\ 10 \leq x \leq 30 \\ x + y \leq 80 \end{cases}$$

(x, y)	(10, 30)	(30, 30)	(30, 50)	(10, 70)
P	390	690	850 ■	710

The maximum profit of $850 per day occurs when 30 oversized rackets and 50 standard rackets are manufactured.

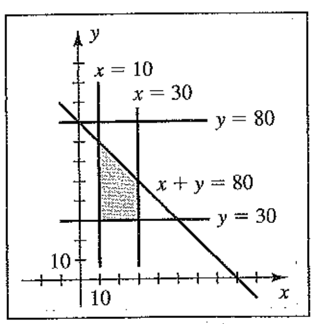

Figure 9

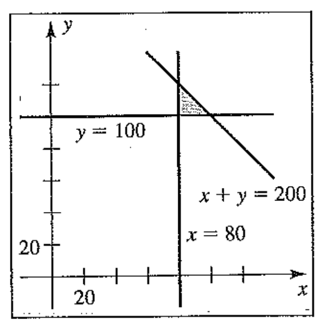

Figure 10

[10] Let x and y denote the number of deluxe and standard model CB radios, respectively.

Profit function: $P = 25x + 30y$

$$\begin{cases} x + y \leq 200 \\ x \geq 80 \\ y \geq 100 \end{cases}$$

(x, y)	(80, 100)	(100, 100)	(80, 120)
P	5000	5500	5600 ■

The maximum profit of $5,600 per day occurs when 80 deluxe models and 120 standard models are produced.

[11] Let x and y denote the number of pounds of S and T, respectively.

Cost function: $C = 3x + 4y$

$$\begin{cases} 2x + 2y \geq 9 & \textit{amount of } \mathrm{I} \\ 4x + 6y \geq 20 & \textit{amount of } \mathrm{G} \\ x \geq 0 \\ y \geq 0 \end{cases}$$

(x, y)	(0, 4.5)	(3.5, 1)	(5, 0)
C	18	14.5 ■	15

The minimum cost of \$14.50 occurs when 3.5 pounds of S and 1 pound of T are used.

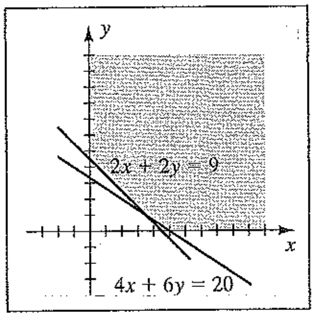

Figure 11

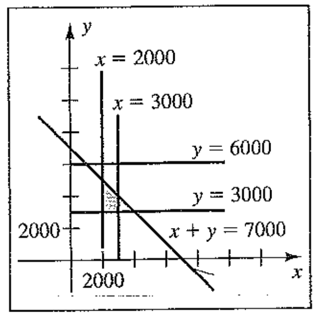

Figure 12

[12] Let x and y denote the number of deluxe and regular notebooks, respectively.

Difference function: $D = 0.25x + 0.15y$

$$\begin{cases} 2000 \leq x \leq 3000 \\ 3000 \leq y \leq 6000 \\ x + y \leq 7000 \end{cases}$$

(x, y)	(2000, 3000)	(3000, 3000)	(3000, 4000)	(2000, 5000)
D	950	1200	1350 ■	1250

The maximum difference of \$1350 occurs when 3000 deluxe notebooks and 4000 regular notebooks are produced.

[13] Let x and y denote the number of units sent to A and B, respectively, from W_1.

Cost function: $C = 12x + 10(35 - x) + 16y + 12(60 - y) = \underline{2x + 4y + 1070}$

The points are the same as those in Example 4.

$$\begin{cases} 0 \leq x \leq 35 \\ 0 \leq y \leq 60 \\ x + y \leq 80 \\ x + y \geq 25 \end{cases}$$

(x, y)	(0, 25)	(0, 60)	(20, 60)	(35, 45)	(35, 0)	(25, 0)
C	1170	1310	1350	1320	1140	1120 ■

To minimize the shipping costs, send 25 units from W_1 to A and 0 from W_1 to B. Send 10 $\{35 - 25\}$ units from W_2 to A and 60 $\{60 - 0\}$ units from W_2 to B. See *Figure 13*.

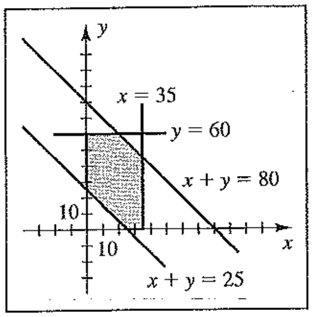

Figure 13

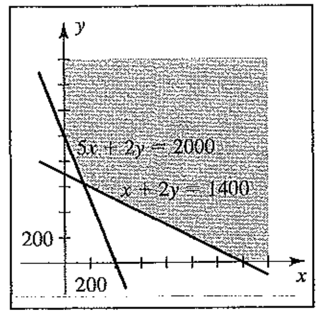

Figure 14

[14] Let x and y denote the number of tons purchased from supplier A and B, respectively. Cost function: $C = 125x + 200y$

$$\left\{\begin{array}{rl} 0.20x + 0.40y \geq 280 & \textit{premium grade} \\ 0.50x + 0.20y \geq 200 & \textit{regular grade} \\ x \geq 0 & \\ y \geq 0 & \end{array}\right. \Rightarrow \left\{\begin{array}{r} x + 2y \geq 1400 \\ 5x + 2y \geq 2000 \\ x \geq 0 \\ y \geq 0 \end{array}\right.$$

(x, y)	(0, 1000)	(150, 625)	(1400, 0)
C	200,000	143,750 ■	175,000

The minimum cost of \$143,750 occurs when 150 tons are purchased from supplier A and 625 tons are purchased from supplier B.

[15] Let x and y denote the number of acres planted with alfalfa and corn, respectively. Profit function: $P = 110x - 4x - 20x + 150y - 6y - 10y = \underline{86x + 134y}$

$$\left\{\begin{array}{rl} 4x + 6y \leq 480 & \textit{seed cost} \\ 20x + 10y \leq 1400 & \textit{labor cost} \\ x + y \leq 90 & \textit{area} \\ x,\ y \geq 0 & \end{array}\right.$$

(x, y)	(70, 0)	(50, 40)	(30, 60)	(0, 80)	(0, 0)
P	6020	9660	10,620	10,720 ■	0

The maximum profit of \$10,720 occurs when

0 acres of alfalfa are planted and 80 acres of corn are planted.

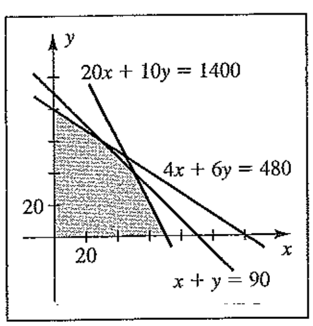

Figure 15

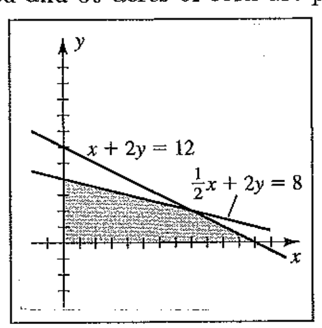

Figure 16

16 Let x and y denote the number of bookshelves and desks manufactured, respectively.

Profit function: $P = 20x + 50y$

$$\begin{cases} 1x + 2y \leq 12 & \textit{router use} \\ \frac{1}{2}x + 2y \leq 8 & \textit{saw use} \\ x,\, y \geq 0 \end{cases}$$

(x, y)	(12, 0)	(8, 2)	(0, 4)	(0, 0)
P	240	260 ■	200	0

The maximum profit of \$260 occurs when 8 bookshelves and 2 desks are manufactured daily. See *Figure 16.*

17 Let x, y, and z denote the number of ounces of X, Y, and Z, respectively.

Cost function: $C = 0.25x + 0.35y + 0.50z$

$= 0.25x + 0.35y + 0.50(20 - x - y) = \underline{10 - 0.25x - 0.15y}$

$$\begin{cases} 0.20x + 0.20y + 0.10z \geq 0.14(20) & \textit{amount of A} \\ 0.10x + 0.40y + 0.20z \geq 0.16(20) & \textit{amount of B} \\ 0.25x + 0.15y + 0.25z \geq 0.20(20) & \textit{amount of C} \end{cases} \Rightarrow \begin{cases} x + y \geq 8 \\ x - 2y \leq 8 \\ y \leq 10 \\ x + y \leq 20 \\ 0 \leq x,\, y \leq 20 \end{cases}$$

The new restrictions are found by substituting $z = 20 - x - y$ into the 3 inequalities, simplifying, and adding the last 2 inequalities.

(x, y)	(8, 0)	(16, 4)	(10, 10)	(0, 10)	(0, 8)
C	8.00	5.40 ■	6.00	8.50	8.80 ■■

The minimum cost of \$5.40 requires 16 oz of X, 4 oz of Y, and 0 oz of Z.

The maximum cost of \$8.80 requires 0 oz of X, 8 oz of Y, and 12 oz of Z.

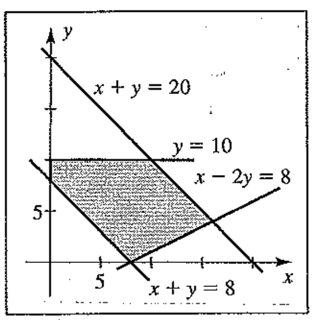

Figure 17

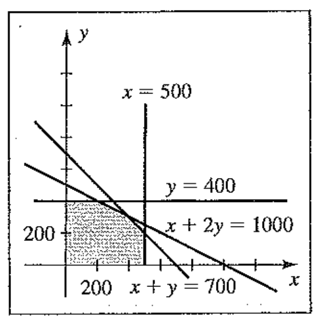

Figure 18

18 Let x and y denote the number of bags of peanuts and candy, respectively.

Profit function: $P = 0.60x + 0.80y$

$$\begin{cases} 0.40x + 0.80y \leq 400 & \textit{purchase} \\ x + y \leq 700 & \textit{sell} \\ 0 \leq x \leq 500 \\ 0 \leq y \leq 400 \end{cases} \Rightarrow \begin{cases} x + 2y \leq 1000 \\ x + y \leq 700 \\ 0 \leq x \leq 500 \\ 0 \leq y \leq 400 \end{cases}$$

(x, y)	(500, 0)	(500, 200)	(400, 300)	(200, 400)	(0, 400)	(0, 0)
P	300	460	480 ■	440	320	0

The maximum profit of \$480 occurs when he sells 400 bags of peanuts and 300 bags of candy.

[19] Let x and y denote the number of vans and buses purchased, respectively.

$$\begin{cases} 10{,}000x + 20{,}000y \le 100{,}000 & \textit{purchase} \\ 100x + 75y \le 500 & \textit{maintenance} \\ x \ge 0 \\ y \ge 0 \end{cases} \Rightarrow \begin{cases} x + 2y \le 10 \\ 4x + 3y \le 20 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

(x, y)	(5, 0)	(2, 4)	(0, 5)	(0, 0)
P	75	130 ■	125	0

Passenger capacity function: $P = 15x + 25y$

The maximum passenger capacity of 130 would occur if the community purchases 2 vans and 4 buses.

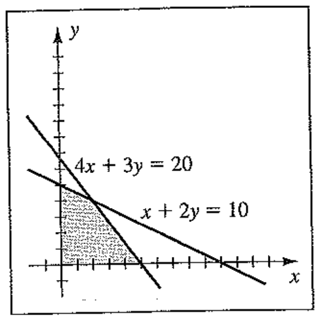

Figure 19

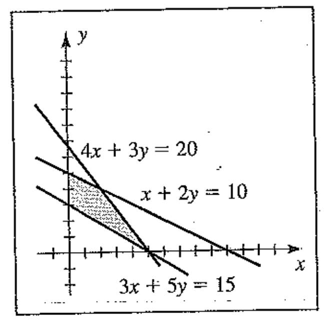

Figure 20

[20] Let x and y denote the number of vans and buses purchased, respectively.

$$\begin{cases} 10{,}000x + 20{,}000y \le 100{,}000 & \textit{purchase} \\ 100x + 75y \le 500 & \textit{maintenance} \\ 15x + 25y \ge 75 & \textit{passengers} \\ x,\ y \ge 0 \end{cases} \Rightarrow \begin{cases} x + 2y \le 10 \\ 4x + 3y \le 20 \\ 3x + 5y \ge 15 \\ x,\ y \ge 0 \end{cases}$$

(x, y)	(5, 0)	(2, 4)	(0, 5)	(0, 3)
C	2750	4500	4250	2550 ■

Cost function: $C = 550x + 850y$

The minimum cost of $2550 per month would occur if the community purchased 3 buses and no vans.

[21] Let x and y denote the number of trout and bass, respectively.

Pound function: $P = 3x + 4y$

$$\begin{cases} x + y \le 5000 & \textit{number of fish} \\ 0.50x + 0.75y \le 3000 & \textit{cost} \\ x \ge 0 \\ y \ge 0 \end{cases} \Rightarrow \begin{cases} x + y \le 5000 \\ 2x + 3y \le 12{,}000 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

(x, y)	(5000, 0)	(3000, 2000)	(0, 4000)	(0, 0)
P	15,000	17,000 ■	16,000	0

The total number of pounds of fish will be a maximum of 17,000 if 3000 trout and 2000 bass are purchased. See *Figure 21*.

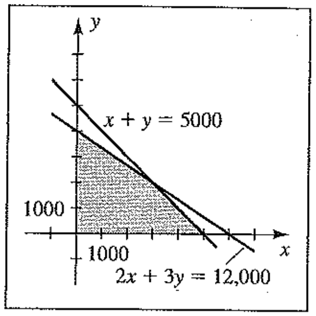

Figure 21

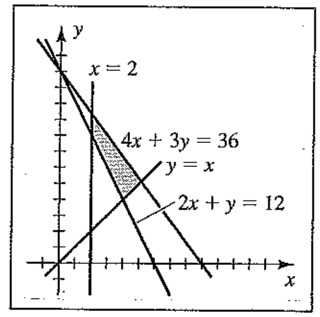

Figure 22

[22] Let x and y denote the number of ounces of corn and squash, respectively.

$$\begin{cases} \frac{1}{2}x + \frac{1}{4}y \geq 3 & \textit{protein} \\ 4x + 3y \leq 36 & \textit{cost} \\ x \geq 2 & \textit{corn} \\ y \geq x & \textit{ratio} \end{cases} \Rightarrow \begin{cases} 2x + y \geq 12 \\ 4x + 3y \leq 36 \\ x \geq 2 \\ y \geq x \end{cases}$$

(x, y)	$(4, 4)$	$(\frac{36}{7}, \frac{36}{7})$	$(2, \frac{28}{3})$	$(2, 8)$
W	8 ■	$10\frac{2}{7}$	$11\frac{1}{3}$	10

Weight function: $W = x + y$

The weight is a minimum of 8 ounces when 4 ounces of each are used.

[23] Let x and y denote the number of basic and deluxe units constructed, respectively.

$$\begin{cases} 300x + 600y \leq 30{,}000 & \textit{cost} \\ x \geq 2y & \textit{ratio} \\ 80x + 120y \leq 7200 & \textit{area} \\ x,\ y \geq 0 \end{cases} \Rightarrow \begin{cases} x + 2y \leq 100 \\ y \leq \frac{1}{2}x \\ 2x + 3y \leq 180 \\ x,\ y \geq 0 \end{cases}$$

(x, y)	$(90, 0)$	$(60, 20)$	$(50, 25)$	$(0, 0)$
R	3600	3900 ■	3875	0

Revenue function: $R = 40x + 75y$

The maximum monthly revenue of \$3900 occurs if 60 basic units and 20 deluxe units are constructed.

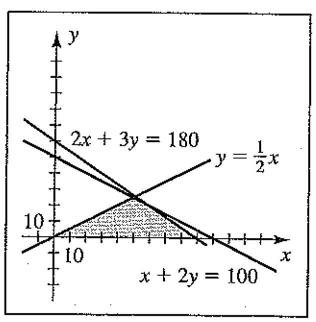

Figure 23

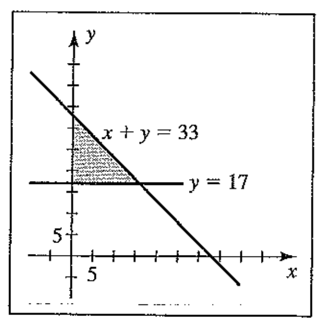

Figure 24

[24] Let x and y denote the number of kg of tree leaves and aquatic plants, respectively.

Energy function: $E = 4x + y$

(x, y)	$(0, 17)$	$(0, 33)$	$(16, 17)$
E	17	33	81 ■

$$\begin{cases} x + y \leq 33 \\ y \geq 17 \\ x \geq 0 \end{cases}$$

(continued)

The maximum daily energy intake of 81 units occurs if the moose eats 16 kg of tree leaves and 17 kg of aquatic plants.

9.5 Exercises

Note: Some equations are interchanged to obtain the matrix in the first step. Most systems are solved using the back substitution method. The solution for Exercise 18 uses the reduced echelon method. To avoid fractions, some solutions include linear combinations of rows.

$\boxed{1}$ $\left[\begin{array}{cccc} 1 & -2 & -3 & -1 \\ 2 & 1 & 1 & 6 \\ 1 & 3 & -2 & 13 \end{array}\right]$ $\begin{array}{l} R_2 - 2R_1 \rightarrow R_2 \\ R_3 - R_1 \rightarrow R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & -2 & -3 & -1 \\ 0 & 5 & 7 & 8 \\ 0 & 5 & 1 & 14 \end{array}\right]$ $R_3 - R_2 \rightarrow R_3$

$\left[\begin{array}{cccc} 1 & -2 & -3 & -1 \\ 0 & 5 & 7 & 8 \\ 0 & 0 & -6 & 6 \end{array}\right]$ $-\frac{1}{6}R_3 \rightarrow R_3$

$\left[\begin{array}{cccc} 1 & -2 & -3 & -1 \\ 0 & 5 & 7 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right]$

R_3: $z = -1$ ★ (2, 3, −1)
R_2: $5y + 7z = 8 \Rightarrow y = 3$
R_1: $x - 2y - 3z = -1 \Rightarrow x = 2$

$\boxed{2}$ $\left[\begin{array}{cccc} 1 & 3 & -1 & -3 \\ 3 & -1 & 2 & 1 \\ 2 & -1 & 1 & -1 \end{array}\right]$ $\begin{array}{l} R_2 - 3R_1 \rightarrow R_2 \\ R_3 - 2R_1 \rightarrow R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & 3 & -1 & -3 \\ 0 & -10 & 5 & 10 \\ 0 & -7 & 3 & 5 \end{array}\right]$ $2R_2 - 3R_3 \rightarrow R_2$

$\left[\begin{array}{cccc} 1 & 3 & -1 & -3 \\ 0 & 1 & 1 & 5 \\ 0 & -7 & 3 & 5 \end{array}\right]$ $R_3 + 7R_2 \rightarrow R_3$

$\left[\begin{array}{cccc} 1 & 3 & -1 & -3 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 10 & 40 \end{array}\right]$ $\frac{1}{10}R_3 \rightarrow R_3$

R_3: $z = 4$ ★ (−2, 1, 4)
R_2: $y + z = 5 \Rightarrow y = 1$
R_1: $x + 3y - z = -3 \Rightarrow x = -2$

[3] $\left[\begin{array}{rrrr|}1 & -2 & 2 & 0\\ 5 & 2 & -1 & -7\\ 0 & 3 & 1 & 17\end{array}\right]$ $R_2 - 5R_1 \rightarrow R_2$

$\left[\begin{array}{rrrr|}1 & -2 & 2 & 0\\ 0 & 12 & -11 & -7\\ 0 & 3 & 1 & 17\end{array}\right]$ $4R_3 - R_2 \rightarrow R_3$

$\left[\begin{array}{rrrr|}1 & -2 & 2 & 0\\ 0 & 12 & -11 & -7\\ 0 & 0 & 15 & 75\end{array}\right]$ $\frac{1}{15}R_3 \rightarrow R_3$

R_3: $z = 5$
R_2: $12y - 11z = -7 \Rightarrow y = 4$ ★ $(-2, 4, 5)$
R_1: $x - 2y + 2z = 0 \Rightarrow x = -2$

[4] $\left[\begin{array}{rrrr|}4 & -1 & 3 & 6\\ -8 & 3 & -5 & -6\\ 5 & -4 & 0 & -9\end{array}\right]$ $R_3 - R_1 \rightarrow R_3$

$\left[\begin{array}{rrrr|}4 & -1 & 3 & 6\\ -8 & 3 & -5 & -6\\ 1 & -3 & -3 & -15\end{array}\right]$ $R_1 - 4R_3 \rightarrow R_1$, $R_2 + 8R_3 \rightarrow R_2$

$\left[\begin{array}{rrrr|}0 & 11 & 15 & 66\\ 0 & -21 & -29 & -126\\ 1 & -3 & -3 & -15\end{array}\right]$ $R_2 + 2R_1 \rightarrow R_2$

$\left[\begin{array}{rrrr|}0 & 11 & 15 & 66\\ 0 & 1 & 1 & 6\\ 1 & -3 & -3 & -15\end{array}\right]$ $R_1 - 11R_2 \rightarrow R_1$

$\left[\begin{array}{rrrr|}0 & 0 & 4 & 0\\ 0 & 1 & 1 & 6\\ 1 & -3 & -3 & -15\end{array}\right]$ $\frac{1}{4}R_1 \rightarrow R_1$

R_1: $z = 0$
R_2: $y + z = 6 \Rightarrow y = 6$ ★ $(3, 6, 0)$
R_3: $x - 3y - 3z = -15 \Rightarrow x = 3$

[5] $\left[\begin{array}{rrrr|}1 & 3 & -2 & 4\\ 2 & 6 & -4 & 1\\ 2 & 1 & -3 & -7\end{array}\right]$ $R_2 - 2R_1 \rightarrow R_2$, $R_3 - 2R_1 \rightarrow R_3$

$\left[\begin{array}{rrrr|}1 & 3 & -2 & 4\\ 0 & 0 & 0 & -7\\ 0 & -5 & 1 & -15\end{array}\right]$

The second row, $0x + 0y + 0z = -7$, has *no solution.*

[6] $\left[\begin{array}{rrrr|}1 & 3 & -3 & -5\\ 2 & -1 & 1 & -3\\ -6 & 3 & -3 & 4\end{array}\right]$ $R_2 - 2R_1 \rightarrow R_2$, $R_3 + 6R_1 \rightarrow R_3$

$\left[\begin{array}{rrrr|}1 & 3 & -3 & -5\\ 0 & -7 & 7 & 7\\ 0 & 21 & -21 & -26\end{array}\right]$ $-\frac{1}{7}R_2 \rightarrow R_2$

(continued)

$$\left[\begin{array}{cccc} 1 & 3 & -3 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 21 & -21 & -26 \end{array}\right] R_3 - 21\,R_2 \to R_3$$

$$\left[\begin{array}{cccc} 1 & 3 & -3 & -5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & -5 \end{array}\right]$$

The third row, $0x + 0y + 0z = -5$, has *no solution.*

[7]
$$\left[\begin{array}{cccc} -3 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 2 & -3 & 2 & -3 \end{array}\right] R_1 + R_2 \to R_1$$

$$\left[\begin{array}{cccc} 1 & 3 & -2 & 5 \\ 4 & 1 & -3 & 4 \\ 2 & -3 & 2 & -3 \end{array}\right] \begin{array}{l} R_2 - 4\,R_1 \to R_2 \\ R_3 - 2\,R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 3 & -2 & 5 \\ 0 & -11 & 5 & -16 \\ 0 & -9 & 6 & -13 \end{array}\right] 4\,R_2 - 5\,R_3 \to R_2$$

$$\left[\begin{array}{cccc} 1 & 3 & -2 & 5 \\ 0 & 1 & -10 & 1 \\ 0 & -9 & 6 & -13 \end{array}\right] R_3 + 9\,R_2 \to R_3$$

$$\left[\begin{array}{cccc} 1 & 3 & -2 & 5 \\ 0 & 1 & -10 & 1 \\ 0 & 0 & -84 & -4 \end{array}\right] -\tfrac{1}{84}R_3 \to R_3$$

R_3: $z = \frac{1}{21}$
R_2: $y - 10z = 1 \;\Rightarrow\; y = \frac{31}{21}$ ★ $\left(\frac{2}{3}, \frac{31}{21}, \frac{1}{21}\right)$
R_1: $x + 3y - 2z = 5 \;\Rightarrow\; x = \frac{14}{21} = \frac{2}{3}$

[8]
$$\left[\begin{array}{cccc} 2 & -3 & 1 & 2 \\ 3 & 2 & -1 & -5 \\ 5 & -2 & 1 & 0 \end{array}\right] -R_1 + R_2 \to R_1$$

$$\left[\begin{array}{cccc} 1 & 5 & -2 & -7 \\ 3 & 2 & -1 & -5 \\ 5 & -2 & 1 & 0 \end{array}\right] \begin{array}{l} R_2 - 3\,R_1 \to R_2 \\ R_3 - 5\,R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 5 & -2 & -7 \\ 0 & -13 & 5 & 16 \\ 0 & -27 & 11 & 35 \end{array}\right] 2\,R_2 - R_3 \to R_2$$

$$\left[\begin{array}{cccc} 1 & 5 & -2 & -7 \\ 0 & 1 & -1 & -3 \\ 0 & -27 & 11 & 35 \end{array}\right] R_3 + 27\,R_2 \to R_3$$

$$\left[\begin{array}{cccc} 1 & 5 & -2 & -7 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & -16 & -46 \end{array}\right] -\tfrac{1}{16}R_3 \to R_3$$

R_3: $z = \frac{23}{8}$
R_2: $y - z = -3 \;\Rightarrow\; y = -\frac{1}{8}$ ★ $\left(-\frac{5}{8}, -\frac{1}{8}, \frac{23}{8}\right)$
R_1: $x + 5y - 2z = -7 \;\Rightarrow\; x = -\frac{5}{8}$

Note: Exer. 9–16: There are other forms for the answers; c is any real number.

9 $\left[\begin{array}{cccc} 1 & 3 & 1 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & -2 & -4 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & 3 & 1 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \begin{array}{l} -\frac{1}{2}R_2 \to R_2 \\ \\ -\frac{1}{5}R_3 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & 3 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 - 3R_2 \to R_1 \\ \\ R_3 - R_2 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

R_1: $x - 2z = 0 \Rightarrow x = 2z$
R_2: $y + z = 0 \Rightarrow y = -z$

★ $(2c, -c, c)$

10 $\left[\begin{array}{cccc} 1 & -1 & -2 & 0 \\ 2 & -1 & 1 & 0 \\ 2 & -3 & -1 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & -1 & 3 & 0 \end{array}\right] \begin{array}{l} \\ \\ R_3 + R_2 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & -1 & -2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \begin{array}{l} \\ \\ \frac{1}{8}R_3 \to R_3 \end{array}$

R_3: $z = 0$
R_2: $y + 5z = 0 \Rightarrow y = 0$
R_1: $x - y - 2z = 0 \Rightarrow x = 0$

★ $(0, 0, 0)$

11 $\left[\begin{array}{cccc} 1 & -2 & -2 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - 2R_1 \to R_2 \\ R_3 - R_1 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & -2 & -2 & 0 \\ 0 & 5 & 5 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right] \begin{array}{l} \frac{1}{5}R_2 \to R_2 \\ \\ \frac{1}{3}R_3 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & -2 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 + 2R_2 \to R_1 \\ \\ R_3 - R_2 \to R_3 \end{array}$

$\left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

R_1: $x = 0$
R_2: $y + z = 0 \Rightarrow y = -z$

★ $(0, -c, c)$

[12]
$$\begin{bmatrix} 1 & 1 & -2 & 0 \\ 1 & -1 & -4 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \; R_2 - R_1 \to R_2$$

$$\begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \; -\tfrac{1}{2}R_2 \to R_2$$

$$\begin{bmatrix} 1 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \; \begin{array}{l} R_1 - R_2 \to R_1 \\ \\ R_3 - R_2 \to R_3 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_1: $x - 3z = 0 \;\Rightarrow\; x = 3z$ ★ $(3c, -c, c)$

R_2: $y + z = 0 \;\Rightarrow\; y = -z$

[13]
$$\begin{bmatrix} 1 & 4 & -1 & -2 \\ 3 & -2 & 5 & 7 \end{bmatrix} \; R_2 - 3R_1 \to R_2$$

$$\begin{bmatrix} 1 & 4 & -1 & -2 \\ 0 & -14 & 8 & 13 \end{bmatrix}$$

R_2: $-14y + 8z = 13 \;\Rightarrow\; y = \frac{4}{7}z - \frac{13}{14}$

R_1: $x + 4y - z = -2 \;\Rightarrow\; x = -4(\frac{4}{7}z - \frac{13}{14}) + z - 2 = -\frac{9}{7}z + \frac{12}{7}$ ★ $(\frac{12}{7} - \frac{9}{7}c, \frac{4}{7}c - \frac{13}{14}, c)$

[14]
$$\begin{bmatrix} 2 & -1 & 4 & 8 \\ -3 & 1 & -2 & 5 \end{bmatrix} \; -R_1 - R_2 \to R_1$$

$$\begin{bmatrix} 1 & 0 & -2 & -13 \\ -3 & 1 & -2 & 5 \end{bmatrix} \; R_2 + 3R_1 \to R_2$$

$$\begin{bmatrix} 1 & 0 & -2 & -13 \\ 0 & 1 & -8 & -34 \end{bmatrix}$$

R_2: $y - 8z = -34 \;\Rightarrow\; y = 8z - 34$ ★ $(2c - 13, 8c - 34, c)$

R_1: $x - 2z = -13 \;\Rightarrow\; x = 2z - 13$

[15]
$$\begin{bmatrix} 4 & -2 & 1 & 5 \\ 3 & 1 & -4 & 0 \end{bmatrix} \; R_1 - R_2 \to R_1$$

$$\begin{bmatrix} 1 & -3 & 5 & 5 \\ 3 & 1 & -4 & 0 \end{bmatrix} \; R_2 - 3R_1 \to R_2$$

$$\begin{bmatrix} 1 & -3 & 5 & 5 \\ 0 & 10 & -19 & -15 \end{bmatrix}$$

R_2: $10y - 19z = -15 \;\Rightarrow\; y = \frac{19}{10}z - \frac{3}{2}$

R_1: $x - 3y + 5z = 5 \;\Rightarrow\; x = 3(\frac{19}{10}z - \frac{3}{2}) - 5z + 5 = \frac{7}{10}z + \frac{1}{2}$ ★ $(\frac{7}{10}c + \frac{1}{2}, \frac{19}{10}c - \frac{3}{2}, c)$

[16]
$$\begin{bmatrix} 5 & 2 & -1 & 10 \\ 0 & 1 & 1 & -3 \end{bmatrix}$$

R_2: $y + z = -3 \;\Rightarrow\; y = -z - 3$

R_1: $5x + 2y - z = 10 \;\Rightarrow\; x = -\frac{2}{5}(-z - 3) + \frac{1}{5}z + 2 = \frac{3}{5}z + \frac{16}{5}$ ★ $(\frac{3}{5}c + \frac{16}{5}, -c - 3, c)$

17
$$\left[\begin{array}{cccc} 5 & 0 & 2 & 1 \\ 0 & 1 & -3 & 2 \\ 2 & 1 & 0 & 3 \end{array}\right] \quad R_1 - 2R_3 \rightarrow R_1$$

$$\left[\begin{array}{cccc} 1 & -2 & 2 & -5 \\ 0 & 1 & -3 & 2 \\ 2 & 1 & 0 & 3 \end{array}\right] \quad R_3 - 2R_1 \rightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & -2 & 2 & -5 \\ 0 & 1 & -3 & 2 \\ 0 & 5 & -4 & 13 \end{array}\right] \quad R_3 - 5R_2 \rightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & -2 & 2 & -5 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 11 & 3 \end{array}\right] \quad \tfrac{1}{11}R_3 \rightarrow R_3$$

R_3: $z = \frac{3}{11}$

R_2: $y - 3z = 2 \;\Rightarrow\; y = \frac{31}{11}$

R_1: $x - 2y + 2z = -5 \;\Rightarrow\; x = \frac{1}{11}$ ★ $(\frac{1}{11}, \frac{31}{11}, \frac{3}{11})$

18
$$\left[\begin{array}{cccc} 2 & -3 & 0 & 12 \\ 0 & 3 & 1 & -2 \\ 5 & 0 & -3 & 3 \end{array}\right] \quad 3R_1 - R_3 \rightarrow R_1$$

$$\left[\begin{array}{cccc} 1 & -9 & 3 & 33 \\ 0 & 3 & 1 & -2 \\ 5 & 0 & -3 & 3 \end{array}\right] \quad R_3 - 5R_1 \rightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & -9 & 3 & 33 \\ 0 & 3 & 1 & -2 \\ 0 & 45 & -18 & -162 \end{array}\right] \quad \tfrac{1}{9}R_3 \rightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & -9 & 3 & 33 \\ 0 & 3 & 1 & -2 \\ 0 & 5 & -2 & -18 \end{array}\right] \quad 2R_2 - R_3 \rightarrow R_2$$

$$\left[\begin{array}{cccc} 1 & -9 & 3 & 33 \\ 0 & 1 & 4 & 14 \\ 0 & 5 & -2 & -18 \end{array}\right] \quad \begin{array}{l} R_1 + 9R_2 \rightarrow R_1 \\ R_3 - 5R_2 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 39 & 159 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & -22 & -88 \end{array}\right] \quad -\tfrac{1}{22}R_3 \rightarrow R_3$$

$$\left[\begin{array}{cccc} 1 & 0 & 39 & 159 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{l} R_1 - 39R_3 \rightarrow R_1 \\ R_2 - 4R_3 \rightarrow R_2 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

★ $(3, -2, 4)$

[19] $\left[\begin{array}{ccc} 4 & -3 & 1 \\ 2 & 1 & -7 \\ -1 & 1 & -1 \end{array}\right]$ $R_1 + 3R_3 \to R_1$

$\left[\begin{array}{ccc} 1 & 0 & -2 \\ 2 & 1 & -7 \\ -1 & 1 & -1 \end{array}\right]$ $R_2 - 2R_1 \to R_2$; $R_3 + R_1 \to R_3$

$\left[\begin{array}{ccc} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{array}\right]$ $R_3 - R_2 \to R_3$

$\left[\begin{array}{ccc} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$ R_2: $y = -3$; R_1: $x = -2$ ★ $(-2, -3)$

[20] $\left[\begin{array}{ccc} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 1 & -2 & 13 \end{array}\right]$ $R_2 - 2R_1 \to R_2$; $R_3 - R_1 \to R_3$

$\left[\begin{array}{ccc} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & -3 & 12 \end{array}\right]$ $R_1 - R_2 \to R_1$; $R_3 + 3R_2 \to R_3$

$\left[\begin{array}{ccc} 1 & 0 & 5 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{array}\right]$ R_2: $y = -4$; R_1: $x = 5$ ★ $(5, -4)$

[21] $\left[\begin{array}{ccc} 1 & -3 & 4 \\ 2 & 3 & 5 \\ 1 & 1 & -2 \end{array}\right]$ $R_2 - 2R_1 \to R_2$; $R_3 - R_1 \to R_3$

$\left[\begin{array}{ccc} 1 & -3 & 4 \\ 0 & 9 & -3 \\ 0 & 4 & -6 \end{array}\right]$ $R_2 - 2R_3 \to R_2$

$\left[\begin{array}{ccc} 1 & -3 & 4 \\ 0 & 1 & 9 \\ 0 & 4 & -6 \end{array}\right]$ $R_1 + 3R_2 \to R_1$; $R_3 - 4R_2 \to R_3$

$\left[\begin{array}{ccc} 1 & 0 & 31 \\ 0 & 1 & 9 \\ 0 & 0 & -42 \end{array}\right]$

There is a contradiction in row 3, therefore there is *no solution.*

[22] $\left[\begin{array}{ccc} 4 & -1 & 2 \\ 2 & 2 & 1 \\ 4 & -5 & 3 \end{array}\right]$ $2R_2 - R_1 \to R_2$; $R_3 - R_1 \to R_3$

$\left[\begin{array}{ccc} 4 & -1 & 2 \\ 0 & 5 & 0 \\ 0 & -4 & 1 \end{array}\right]$

$R_2 \Rightarrow y = 0$ and $R_3 \Rightarrow y = -\frac{1}{4}$, a contradiction, there is *no solution.*

[23] Let x, y, and z denote the number of liters of the 10% acid, 30% acid, and 50% acid.

$$\begin{cases} x+y+z=50 & \textit{quantity} & (E_1) \\ 0.10x+0.30y+0.50z=(0.32)(50) & \textit{quality} & (E_2) \\ z=2y & \textit{constraint} & (E_3) \end{cases}$$

Substitute $z=2y$ into E_1 and $100\,E_2$ to obtain

$$\begin{cases} x+3y = 50 & (E_4) \\ 10x+130y = 1600 & (E_5) \end{cases}$$

$E_5-10\,E_4 \Rightarrow 100y=1100 \Rightarrow y=11;\ x=17;\ z=22$

[24] Let z denote the number of hours it takes for pipe C to fill the pool alone. By adding the hourly rates, that is, how much of the pool each pipe fills in one hour, we have $\frac{1}{8}+\frac{1}{z}=\frac{1}{6} \Rightarrow \frac{1}{z}=\frac{4}{24}-\frac{3}{24} \Rightarrow z=24$. Let y denote the number of hours it takes for pipe B to fill the pool alone. Then $\frac{1}{y}+\frac{1}{24}=\frac{1}{10} \Rightarrow \frac{1}{y}=\frac{12}{120}-\frac{5}{120} \Rightarrow y=\frac{120}{7}$. Let x denote the number of hours it takes for all three pipes to fill the pool. Then

$$\frac{1}{8}+\frac{7}{120}+\frac{1}{24}=\frac{1}{x} \Rightarrow \frac{1}{x}=\frac{15}{120}+\frac{7}{120}+\frac{5}{120} \Rightarrow x=\frac{120}{27}=\frac{40}{9}.$$

[25] Let x, y, and z denote the number of hours needed for A, B, and C, respectively, to produce 1000 items. In one hour, A, B, and C produce $\frac{1000}{x}$, $\frac{1000}{y}$, and $\frac{1000}{z}$ items, respectively. In 6 hours, A and B produce $\frac{6000}{x}$ and $\frac{6000}{y}$ items. From the table, this sum must equal 4500. The system of equations is then:

$$\begin{cases} \dfrac{6000}{x} + \dfrac{6000}{y} = 4500 \\ \dfrac{8000}{x} + \dfrac{8000}{z} = 3600 \\ \dfrac{7000}{y} + \dfrac{7000}{z} = 4900 \end{cases}$$

To simplify, let $a=1/x$, $b=1/y$, $c=1/z$ and divide each equation by its greatest common factor {1500, 400, and 700}.

$$\begin{cases} 4a+4b = 3 & (E_1) \\ 20a + 20c = 9 & (E_2) \\ 10b+10c = 7 & (E_3) \end{cases}$$

$E_2-5\,E_1 \Rightarrow 20c-20b=-6 \quad (E_4)$

$E_4+2\,E_3 \Rightarrow 40c=8 \Rightarrow c=\frac{1}{5};\ b=\frac{1}{2};\ a=\frac{1}{4}$. Resubstituting, $x=4$, $y=2$, $z=5$.

[26] $$\begin{cases} \dfrac{1}{A}+\dfrac{1}{B} = \dfrac{1}{48} \\ \dfrac{1}{B}+\dfrac{1}{C} = \dfrac{1}{80} \\ \dfrac{1}{A} + \dfrac{1}{C} = \dfrac{1}{60} \end{cases} \Rightarrow \begin{cases} 48x+48y = 1 & (E_1) \\ 80y+80z = 1 & (E_2) \\ 60x + 60z = 1 & (E_3) \end{cases}$$

where $x=1/A$, $y=1/B$, and $z=1/C$. $5\,E_1-4\,E_3 \Rightarrow 240y-240z=1 \quad (E_4)$

$E_4+3\,E_2 \Rightarrow 480y=4 \Rightarrow y=\frac{1}{120};\ z=\frac{1}{240};\ x=\frac{1}{80}$.

Resubstituting, A = 80, B = 120, and C = 240.

[27] Let x, y, and z denote the amounts of G_1, G_2, and G_3, respectively.

$$\begin{cases} x+y+z=600 & \textit{quantity} & (E_1) \\ 0.30x+0.20y+0.15z=(0.25)(600) & \textit{quality} & (E_2) \\ z=100+y & \textit{constraint} & (E_3) \end{cases}$$

Substitute $z=100+y$ into E_1 and $100\,E_2$ to obtain

$$\begin{cases} x+\ \ 2y= \ \ \ 500 & (E_4) \\ 30x+35y= 13{,}500 & (E_5) \end{cases}$$

$$E_5-30\,E_4 \ \Rightarrow\ -25y=-1500 \ \Rightarrow\ y=60;\ z=160;\ x=380$$

[28] $s(\frac{1}{2})=7$ $\qquad 7=\frac{1}{8}a+\frac{1}{2}v_0+s_0 \qquad (E_1)$

$s(1)=11 \quad \Rightarrow \quad 11=\frac{1}{2}a+v_0+s_0 \qquad (E_2)$

$s(\frac{3}{2})=17$ $\qquad 17=\frac{9}{8}a+\frac{3}{2}v_0+s_0 \qquad (E_3)$

Solving E_2 for s_0 and substituting into E_1 and E_3 yields

$$\begin{cases} -4=-\frac{3}{8}a-\frac{1}{2}v_0 & (E_4) \\ 6=\frac{5}{8}a+\frac{1}{2}v_0 & (E_5) \end{cases} \qquad E_4+E_5 \ \Rightarrow\ 2=\frac{1}{4}a \ \Rightarrow\ a=8;\ v_0=2;\ s_0=5.$$

[29] (a) Let $R_1=R_2=R_3=3$ in the given system of equations.

$$\begin{cases} I_1-I_2+I_3=0 \\ 3I_1+3I_2=6 \\ 3I_2+3I_3=12 \end{cases} \Rightarrow \begin{cases} I_1-I_2+I_3=0 & (E_1) \\ I_1+I_2=2 & (E_2) \\ I_2+I_3=4 & (E_3) \end{cases}$$

Substitute $I_1=I_2-I_3$ from E_1 into E_2 to obtain $2\,I_2-I_3=2$ (E_4).

Now $E_4+E_3 \ \Rightarrow\ 3\,I_2=6 \ \Rightarrow\ I_2=2;\ I_3=2;\ I_1=0.$

(b) Let $R_1=4$, $R_2=1$, and $R_3=4$ in the given system of equations.

$$\begin{cases} I_1-I_2+I_3=0 & (E_1) \\ 4I_1+1I_2=6 & (E_2) \\ 1I_2+4I_3=12 & (E_3) \end{cases}$$

Substitute $I_1=I_2-I_3$ from E_1 into E_2 to obtain $4(I_2-I_3)+I_2=6 \ \Rightarrow$

$5\,I_2-4\,I_3=6$ (E_4). Now $E_4+E_3 \ \Rightarrow\ 6\,I_2=18 \ \Rightarrow\ I_2=3;\ I_3=\frac{9}{4};\ I_1=\frac{3}{4}.$

[30] Let x, y, and z denote the number of birds on island A, B, and C, respectively.

$$\begin{cases} x+y+z=35{,}000 & \textit{quantity} & (E_1) \\ x-0.10x+0.05z=x & \textit{island A} & (E_2) \\ y-0.20y+0.10x=y & \textit{island B} & (E_3) \\ z-0.05z+0.20y=z & \textit{island C} & (E_4) \end{cases}$$

Solve E_2 for z $\{z=2x\}$, E_3 for y $\{y=\frac{1}{2}x\}$, and substitute both expressions into E_1 to obtain $x+\frac{1}{2}x+2x=35{,}000 \ \Rightarrow\ \frac{7}{2}x=35{,}000 \ \Rightarrow\ x=10{,}000;\ y=5000;$

$z=20{,}000.$

31 Let x, y, and z denote the amount of Colombian, Brazilian, and Kenyan coffee used, respectively.

$$\begin{cases} x+y+z=1 & \textit{quantity} & (\mathrm{E}_1) \\ 10x+6y+8z=(8.50)(1) & \textit{quality} & (\mathrm{E}_2) \\ x=3y & \textit{constraint} & (\mathrm{E}_3) \end{cases}$$

Substitute $x=3y$ into E_1 and E_2 to obtain

$$\begin{cases} 4y+z=1 & (\mathrm{E}_4) \\ 36y+8z=8.5 & (\mathrm{E}_5) \end{cases} \qquad \mathrm{E}_5-8\,\mathrm{E}_4 \Rightarrow 4y=\tfrac{1}{2} \Rightarrow y=\tfrac{1}{8};\ z=\tfrac{1}{2};\ x=\tfrac{3}{8}.$$

32 Let x, y, and z denote the weight of the small, medium, and large links, respectively.

$$\begin{cases} 10x+20y+30z=450 & \textit{450-ounce chain} & (\mathrm{E}_1) \\ 10x+30y+40z=610 & \textit{610-ounce chain} & (\mathrm{E}_2) \\ 10x+40y+70z=950 & \textit{950-ounce chain} & (\mathrm{E}_3) \end{cases}$$

Divide each equation by 10 and write the system in matrix form.

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 45 \\ 1 & 3 & 4 & 61 \\ 1 & 4 & 7 & 95 \end{array}\right] \begin{array}{l} \\ \mathrm{R}_2-\mathrm{R}_1\to\mathrm{R}_2 \\ \mathrm{R}_3-\mathrm{R}_1\to\mathrm{R}_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 45 \\ 0 & 1 & 1 & 16 \\ 0 & 2 & 4 & 50 \end{array}\right] \begin{array}{l} \mathrm{R}_1-2\,\mathrm{R}_2\to\mathrm{R}_1 \\ \\ \mathrm{R}_3-2\,\mathrm{R}_2\to\mathrm{R}_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 13 \\ 0 & 1 & 1 & 16 \\ 0 & 0 & 2 & 18 \end{array}\right] \begin{array}{l} \\ \\ \tfrac{1}{2}\mathrm{R}_3\to\mathrm{R}_3 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 13 \\ 0 & 1 & 1 & 16 \\ 0 & 0 & 1 & 9 \end{array}\right] \begin{array}{l} \mathrm{R}_1-\mathrm{R}_3\to\mathrm{R}_1 \\ \mathrm{R}_2-\mathrm{R}_3\to\mathrm{R}_2 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 9 \end{array}\right]$$

The weights of the small, medium, and large links, are 4, 7, & 9 ounces, respectively.

33 (a) A: $x_1+x_4=75$, B: $x_1+x_2=150$, C: $x_2+x_3=225$, D: $x_3+x_4=150$

(b) From C, $x_3=100 \Rightarrow x_2=125$. From D, $x_3=100 \Rightarrow x_4=50$.

From A, $x_4=50 \Rightarrow x_1=25$.

(c) From D, $x_3=150-x_4 \Rightarrow x_3\le 150$ since $x_4\ge 0$. From C,

$x_3=225-x_2=225-(150-x_1)$ {from B} $=75+x_1 \Rightarrow x_3\ge 75$ since $x_1\ge 0$.

34 $$\begin{cases} f(-3)=-12 \\ f(-1)=22 \\ f(2)=13 \end{cases} \Rightarrow \begin{cases} -27a-3b+c=-12 & (\mathrm{E}_1) \\ -a-b+c=22 & (\mathrm{E}_2) \\ 8a+2b+c=13 & (\mathrm{E}_3) \end{cases}$$

Solving E_2 for c and substituting into E_1 and E_3 yields

$$\begin{cases} -13a-b=-17 & (\mathrm{E}_4) \\ 3a+b=-3 & (\mathrm{E}_5) \end{cases}$$

$\mathrm{E}_4+\mathrm{E}_5 \Rightarrow -10a=-20 \Rightarrow a=2;\ b=-9;\ c=15.$

$\boxed{35}$ $t = 2070 - 1990 = 80$. $rt = (0.025)(80) = 2$ for E_1, $(0.015)(80) = 1.2$ for E_2, and $(0.01)(0) = 0$ for E_3. Summarizing as a system, we have:

$$\begin{cases} a + 80c + e^2 k = 800 & (E_1) \\ a + 80c + e^{1.2} k = 560 & (E_2) \\ a \quad\quad + k = 340 & (E_3) \end{cases}$$

We want to find t when $A = 2(340)$. First we find c and k.

$E_1 - E_2 \;\Rightarrow\; e^2 k - e^{1.2} k = 240 \;\Rightarrow\; k = \dfrac{240}{e^2 - e^{1.2}} \approx 58.98.$

Substituting into E_3 gives $a = 340 - k \approx 281.02$.

Substituting into E_1 gives $c = \dfrac{800 - a - e^2 k}{80} \approx \dfrac{800 - 281.02 - (58.98)e^2}{80} \approx 1.04.$

Thus, $A = 281.02 + 1.04t + 58.98e^{rt}$. If $A = 680$ and $r = 0.01$, then

$680 = 281.02 + 1.04t + 58.98e^{0.01t} \;\Rightarrow\; 1.04t + 58.98e^{0.01t} - 398.98 = 0$. Graphing

$y = 1.04t + 58.98e^{0.01t} - 398.98$, we see there is an x-intercept at $x \approx 144.08$.

$1990 + 144.08 = 2134.08$, or during the year 2134.

[0, 1050, 100] by [−350, 350, 100]

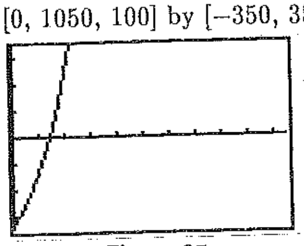

Figure 35

[0, 1050, 100] by [−350, 350, 100]

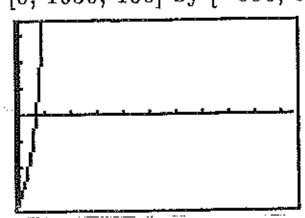

Figure 36

$\boxed{36}$
$$\begin{cases} a + 40c + e^{0.8} k = 455 & (E_1) \\ a + 40c + e^{0.6} k = 430 & (E_2) \\ a \quad\quad + k = 340 & (E_3) \end{cases}$$

$E_1 - E_2 \;\Rightarrow\; e^{0.8} k - e^{0.6} k = 25 \;\Rightarrow\; k = \dfrac{25}{e^{0.8} - e^{0.6}} \approx 61.97.$

Substituting into E_3 gives $a = 340 - k \approx 278.03$.

Substituting into E_1 gives $c = \dfrac{455 - a - ke^{0.8}}{40} \approx \dfrac{455 - 278.03 - (61.97)e^{0.8}}{40} \approx 0.98.$

Thus, $A = 278.03 + 0.98t + 61.97e^{rt}$. If $A = 680$ and $r = 0.025$, then

$680 = 278.03 + 0.98t + 61.97e^{rt} \;\Rightarrow\; 0.98t + 61.97e^{0.025t} - 401.97 = 0$. Graphing

$y = 0.98t + 61.97e^{0.025t} - 401.97$, we see there is an x-intercept at $x \approx 67.59$.

$1990 + 67.59 = 2057.59$, or during the year 2057.

[37] The circle has an equation of the form $x^2+y^2+ax+by+c=0$. Substituting the x and y values of $P(2,\ 1)$, $Q(-1,\ -4)$, and $R(3,\ 0)$ into this equation yields:

$$\begin{cases} 2a + b + c = -5 & P \quad (E_1) \\ -a - 4b + c = -17 & Q \quad (E_2) \\ 3a + c = -9 & R \quad (E_3) \end{cases}$$

Solving E_3 for c $\{c=-9-3a\}$ and substituting into E_1 and E_2 yields:

$$\begin{cases} -a + b = 4 & (E_4) \\ -4a - 4b = -8 & (E_5) \end{cases} \Rightarrow \begin{cases} -a + b = 4 & (E_6) \\ a + b = 2 & (E_7) \end{cases}$$

$E_6 + E_7 \Rightarrow 2b = 6 \Rightarrow b = 3;\ a = -1;\ c = -6.$

The equation is $x^2+y^2-x+3y-6=0$.

[38] The circle has an equation of the form $x^2+y^2+ax+by+c=0$. Substituting the x and y values of $P(-5,\ 5)$, $Q(-2,\ -4)$, and $R(2,\ 4)$ into this equation yields:

$$\begin{cases} -5a + 5b + c = -50 & P \quad (E_1) \\ -2a - 4b + c = -20 & Q \quad (E_2) \\ 2a + 4b + c = -20 & R \quad (E_3) \end{cases}$$

Solving E_2 for c $\{c=2a+4b-20\}$ and substituting into E_1 and E_3 yields:

$$\begin{cases} -3a + 9b = -30 & (E_4) \\ 4a + 8b = 0 & (E_5) \end{cases} \Rightarrow \begin{cases} -a + 3b = -10 & (E_6) \\ a + 2b = 0 & (E_7) \end{cases}$$

$E_6 + E_7 \Rightarrow 5b = -10 \Rightarrow b = -2;\ a = 4;\ c = -20.$

The equation is $x^2+y^2+4x-2y-20=0$.

[39] Using $y=f(x)=ax^3+bx^2+cx+d$ and the point $P(0,\ -6)$, we should recognize that $d=-6$ since substituting 0 for x leaves only the constant term. Now use the points $Q(1,\ -11)$, $R(-1,\ -5)$, and $S(2,\ -14)$ to obtain the system:

$$\begin{cases} a + b + c - 6 = -11 & Q \quad (E_1) \\ -a + b - c - 6 = -5 & R \quad (E_2) \\ 8a + 4b + 2c - 6 = -14 & S \quad (E_3) \end{cases}$$

Add 6 to each equation and divide the resulting third equation by 2 to obtain:

$$\left[\begin{array}{rrrr} 1 & 1 & 1 & -5 \\ -1 & 1 & -1 & 1 \\ 4 & 2 & 1 & -4 \end{array}\right] \begin{array}{l} \\ R_2 + R_1 \to R_2 \\ R_3 - 4R_1 \to R_3 \end{array}$$

$$\left[\begin{array}{rrrr} 1 & 1 & 1 & -5 \\ 0 & 2 & 0 & -4 \\ 0 & -2 & -3 & 16 \end{array}\right] \begin{array}{l} \\ \tfrac{1}{2}R_2 \to R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr} 1 & 1 & 1 & -5 \\ 0 & 1 & 0 & -2 \\ 0 & -2 & -3 & 16 \end{array}\right] \begin{array}{l} R_1 - R_2 \to R_1 \\ \\ R_3 + 2R_2 \to R_3 \end{array}$$

$$\left[\begin{array}{rrrr} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & 12 \end{array}\right] \begin{array}{l} \\ \\ -\tfrac{1}{3}R_3 \to R_3 \end{array}$$

(continued)

$$\text{(repeated)}\left[\begin{array}{cccc} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & 12 \end{array}\right] -\tfrac{1}{3}\mathrm{R}_3 \rightarrow \mathrm{R}_3$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{array}\right] \mathrm{R}_1 - \mathrm{R}_3 \rightarrow \mathrm{R}_1$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -4 \end{array}\right]$$

Thus, $a = 1$, $b = -2$, and $c = -4$, so $f(x) = x^3 - 2x^2 - 4x - 6$.

[40] Using $y = f(x) = ax^3 + bx^2 + cx + d$ and the point $P(0, 4)$, we should recognize that $d = 4$ since substituting 0 for x leaves only the constant term. Now use the points $Q(1, 2)$, $R(-1, 10)$, and $S(2, -2)$ to obtain the system:

$$\left\{\begin{array}{rrrcrll} a + & b + & c + 4 & = & 2 & Q & (\mathrm{E}_1) \\ -a + & b - & c + 4 & = & 10 & R & (\mathrm{E}_2) \\ 8a + & 4b + & 2c + 4 & = & -2 & S & (\mathrm{E}_3) \end{array}\right.$$

Solving the system yields $a = -1$, $b = 2$, and $c = -3$, so $f(x) = -x^3 + 2x^2 - 3x + 4$.

[41] For $y = f(x) = ax^3 + bx^2 + cx + d$ and the points $(-1, 2)$, $(0.5, 2)$, $(1, 3)$, and $(2, 4.5)$, we obtain the system:

$$\left\{\begin{array}{rrrrcrl} -a + & b - & c + & d & = & 2 & (-1, 2) \\ 0.125a + & 0.25b + & 0.5c + & d & = & 2 & (0.5, 2) \\ a + & b + & c + & d & = & 3 & (1, 3) \\ 8a + & 4b + & 2c + & d & = & 4.5 & (2, 4.5) \end{array}\right.$$

Solving the system yields $a = -\frac{4}{9}$, $b = \frac{11}{9}$, $c = \frac{17}{18}$, and $d = \frac{23}{18}$.

[42] For $y = f(x) = ax^4 + bx^3 + cx^2 + dx + e$ and the points $(-2, 1.5)$, $(-1, -2)$, $(1, -3)$, $(2, -3.5)$, and $(3, -4.8)$, we obtain the system:

$$\left\{\begin{array}{rrrrrcrl} 16a - & 8b + & 4c - & 2d + & e & = & 1.5 & (-2, 1.5) \\ a - & b + & c - & d + & e & = & -2 & (-1, -2) \\ a + & b + & c + & d + & e & = & -3 & (1, -3) \\ 16a + & 8b + & 4c + & 2d + & e & = & -3.5 & (2, -3.5) \\ 81a + & 27b + & 9c + & 3d + & e & = & -4.8 & (3, -4.8) \end{array}\right.$$

Solving the system yields $a = 0.03$, $b = -0.25$, $c = 0.35$, $d = -0.25$, and $e = -2.88$.

9.6 Exercises

[1] $A + B = \left[\begin{array}{cc} 5 & -2 \\ 1 & 3 \end{array}\right] + \left[\begin{array}{cc} 4 & 1 \\ -3 & 2 \end{array}\right] = \left[\begin{array}{cc} 9 & -1 \\ -2 & 5 \end{array}\right]$,

$A - B = \left[\begin{array}{cc} 1 & -3 \\ 4 & 1 \end{array}\right]$, $2A = \left[\begin{array}{cc} 10 & -4 \\ 2 & 6 \end{array}\right]$, $-3B = \left[\begin{array}{cc} -12 & -3 \\ 9 & -6 \end{array}\right]$

[2] $A+B=\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}+\begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 6 & -4 \\ 0 & 3 \end{bmatrix}$,

$$A-B=\begin{bmatrix} 0 & 4 \\ -2 & 1 \end{bmatrix}, 2A=\begin{bmatrix} 6 & 0 \\ -2 & 4 \end{bmatrix}, -3B=\begin{bmatrix} -9 & 12 \\ -3 & -3 \end{bmatrix}$$

[3] $A+B=\begin{bmatrix} 6 & -1 \\ 2 & 0 \\ -3 & 4 \end{bmatrix}+\begin{bmatrix} 3 & 1 \\ -1 & 5 \\ 6 & 0 \end{bmatrix}=\begin{bmatrix} 9 & 0 \\ 1 & 5 \\ 3 & 4 \end{bmatrix}$,

$$A-B=\begin{bmatrix} 3 & -2 \\ 3 & -5 \\ -9 & 4 \end{bmatrix}, 2A=\begin{bmatrix} 12 & -2 \\ 4 & 0 \\ -6 & 8 \end{bmatrix}, -3B=\begin{bmatrix} -9 & -3 \\ 3 & -15 \\ -18 & 0 \end{bmatrix}$$

[4] $A+B=\begin{bmatrix} 0 & -2 & 7 \\ 5 & 4 & -3 \end{bmatrix}+\begin{bmatrix} 8 & 4 & 0 \\ 0 & 1 & 4 \end{bmatrix}=\begin{bmatrix} 8 & 2 & 7 \\ 5 & 5 & 1 \end{bmatrix}$,

$$A-B=\begin{bmatrix} -8 & -6 & 7 \\ 5 & 3 & -7 \end{bmatrix}, 2A=\begin{bmatrix} 0 & -4 & 14 \\ 10 & 8 & -6 \end{bmatrix}, -3B=\begin{bmatrix} -24 & -12 & 0 \\ 0 & -3 & -12 \end{bmatrix}$$

[5] $A+B=\begin{bmatrix} 4 & -3 & 2 \end{bmatrix}+\begin{bmatrix} 7 & 0 & -5 \end{bmatrix}=\begin{bmatrix} 11 & -3 & -3 \end{bmatrix}$,

$$A-B=\begin{bmatrix} -3 & -3 & 7 \end{bmatrix}, 2A=\begin{bmatrix} 8 & -6 & 4 \end{bmatrix}, -3B=\begin{bmatrix} -21 & 0 & 15 \end{bmatrix}$$

[6] $A+B=\begin{bmatrix} 7 \\ -16 \end{bmatrix}+\begin{bmatrix} -11 \\ 9 \end{bmatrix}=\begin{bmatrix} -4 \\ -7 \end{bmatrix}, A-B=\begin{bmatrix} 18 \\ -25 \end{bmatrix}, 2A=\begin{bmatrix} 14 \\ -32 \end{bmatrix}, -3B=\begin{bmatrix} 33 \\ -27 \end{bmatrix}$

[7] $A+B$ and $A-B$ are not possible since A and B are different sizes.

$$2A=2\begin{bmatrix} 3 & -2 & 2 \\ 0 & 1 & -4 \\ -3 & 2 & -1 \end{bmatrix}=\begin{bmatrix} 6 & -4 & 4 \\ 0 & 2 & -8 \\ -6 & 4 & -2 \end{bmatrix}, -3B=-3\begin{bmatrix} 4 & 0 \\ 2 & -1 \\ -1 & 3 \end{bmatrix}=\begin{bmatrix} -12 & 0 \\ -6 & 3 \\ 3 & -9 \end{bmatrix}$$

[8] $A+B$ and $A-B$ are not possible since A and B are different sizes.

$$2A=2\begin{bmatrix} 2 & 1 \end{bmatrix}=\begin{bmatrix} 4 & 2 \end{bmatrix}, -3B=-3\begin{bmatrix} 3 & -1 & 5 \end{bmatrix}=\begin{bmatrix} -9 & 3 & -15 \end{bmatrix}$$

[9] To find c_{21} in Exercise 15, use the second row of A and the first column of B.

$$c_{21}=(-5)(2)+(2)(0)+(2)(-4)=-10+0-8=-18.$$

[10] To find c_{23} in Exercise 16, use the second row of A and the third column of B.

$$c_{23}=(3)(1)+(-2)(0)+(0)(4)+(5)(3)=3+0+0+15=18.$$

[11] $AB=\begin{bmatrix} 2 & 6 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 1 & 7 \end{bmatrix}=\begin{bmatrix} 16 & 38 \\ 11 & -34 \end{bmatrix}, BA=\begin{bmatrix} 4 & 38 \\ 23 & -22 \end{bmatrix}$

[12] $AB=\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, BA=\begin{bmatrix} 6 & -3 \\ 12 & -6 \end{bmatrix}$

[13] $AB=\begin{bmatrix} 3 & 0 & -1 \\ 0 & 4 & 2 \\ 5 & -3 & 1 \end{bmatrix}\begin{bmatrix} 1 & -5 & 0 \\ 4 & 1 & -2 \\ 0 & -1 & 3 \end{bmatrix}=\begin{bmatrix} 3 & -14 & -3 \\ 16 & 2 & -2 \\ -7 & -29 & 9 \end{bmatrix}$,

$$BA=\begin{bmatrix} 3 & -20 & -11 \\ 2 & 10 & -4 \\ 15 & -13 & 1 \end{bmatrix}$$

[14] $AB = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$,

$BA = \begin{bmatrix} 15 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & -4 \end{bmatrix}$

[15] $AB = \begin{bmatrix} 4 & -3 & 1 \\ -5 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ -18 & 11 \end{bmatrix}$, $BA = \begin{bmatrix} 3 & -4 & 4 \\ -5 & 2 & 2 \\ -51 & 26 & 10 \end{bmatrix}$

[16] $AB = \begin{bmatrix} 2 & 1 & -1 & 0 \\ 3 & -2 & 0 & 5 \\ -2 & 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 5 & -3 & 1 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \\ 0 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & -2 \\ 13 & -23 & 18 \\ -13 & 4 & 20 \end{bmatrix}$,

$BA = \begin{bmatrix} -1 & 12 & -1 & -13 \\ 8 & -3 & -1 & 10 \\ -10 & 3 & 17 & 8 \\ -12 & 7 & 12 & -4 \end{bmatrix}$

[17] $AB = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $BA = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

[18] $AB = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix}$, $BA = \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 6 & 2 & 4 \end{bmatrix}$

[19] $AB = \begin{bmatrix} -3 & 7 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix} = \begin{bmatrix} 15 \end{bmatrix}$, $BA = \begin{bmatrix} -3 & 7 & 2 \\ -12 & 28 & 8 \\ 15 & -35 & -10 \end{bmatrix}$

[20] $AB = \begin{bmatrix} 4 & 8 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \end{bmatrix}$, $BA = \begin{bmatrix} -12 & -24 \\ 8 & 16 \end{bmatrix}$

[21] $AB = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 5 \\ 5 & 3 & -2 \end{bmatrix}$, BA is not possible since the number of columns of B, 3, and the number of rows of A, 2, are not equal.

[22] AB is not possible, $BA = \begin{bmatrix} -2 \\ 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -6 & 2 & -8 \\ 15 & -5 & 20 \end{bmatrix}$.

[23] $AB = \begin{bmatrix} 4 & -2 \\ 0 & 3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -1 \end{bmatrix}$

[24] $AB = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix} \begin{bmatrix} 5 & 1 \end{bmatrix} = \begin{bmatrix} 20 & 4 \\ -15 & -3 \\ 10 & 2 \end{bmatrix}$

[25] $AB = \begin{bmatrix} 2 & 1 & 0 & -3 \\ -7 & 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 4 & -2 & 0 \\ 1 & 1 & -2 \\ 0 & 0 & 5 \\ -3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 18 & 0 & -2 \\ -40 & 10 & -10 \end{bmatrix}$

[26] $AB = \begin{bmatrix} 1 & 2 & -3 \\ 4 & -5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 2 \\ -2 & 3 & 1 & 0 \\ 0 & 4 & 0 & -3 \end{bmatrix} = \begin{bmatrix} -3 & -7 & 2 & 11 \\ 14 & 5 & -5 & -10 \end{bmatrix}$

$\boxed{27}$ $(A+B)(A-B)=\begin{bmatrix} 3 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ -3 & -4 \end{bmatrix}=\begin{bmatrix} -6 & 5 \\ 3 & 17 \end{bmatrix}$;

$A^2-B^2=\begin{bmatrix} 1 & -4 \\ 0 & 9 \end{bmatrix}-\begin{bmatrix} 1 & -3 \\ 9 & -2 \end{bmatrix}=\begin{bmatrix} 0 & -1 \\ -9 & 11 \end{bmatrix}$; $(A+B)(A-B)\neq A^2-B^2$

$\boxed{28}$ $(A+B)(A+B)=\begin{bmatrix} 3 & 1 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 3 & -2 \end{bmatrix}=\begin{bmatrix} 12 & 1 \\ 3 & 7 \end{bmatrix}$;

$A^2+2AB+B^2=\begin{bmatrix} 1 & -4 \\ 0 & 9 \end{bmatrix}+\begin{bmatrix} 16 & 2 \\ -18 & -6 \end{bmatrix}+\begin{bmatrix} 1 & -3 \\ 9 & -2 \end{bmatrix}=\begin{bmatrix} 18 & -5 \\ -9 & 1 \end{bmatrix}$

$\boxed{29}$ $A(B+C)=\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 5 & 0 \\ 1 & 1 \end{bmatrix}=\begin{bmatrix} 7 & 2 \\ -3 & -3 \end{bmatrix}$;

$AB+AC=\begin{bmatrix} 8 & 1 \\ -9 & -3 \end{bmatrix}+\begin{bmatrix} -1 & 1 \\ 6 & 0 \end{bmatrix}=\begin{bmatrix} 7 & 2 \\ -3 & -3 \end{bmatrix}$

$\boxed{30}$ $A(BC)=\begin{bmatrix} 1 & 2 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} 8 & 2 \\ 7 & 3 \end{bmatrix}=\begin{bmatrix} 22 & 8 \\ -21 & -9 \end{bmatrix}$;

$(AB)C=\begin{bmatrix} 8 & 1 \\ -9 & -3 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 0 \end{bmatrix}=\begin{bmatrix} 22 & 8 \\ -21 & -9 \end{bmatrix}$

$\boxed{31}$
$$\begin{aligned} m(A+B) &= m\begin{bmatrix} a+p & b+q \\ c+r & d+s \end{bmatrix}=\begin{bmatrix} m(a+p) & m(b+q) \\ m(c+r) & m(d+s) \end{bmatrix} \\ &= \begin{bmatrix} ma+mp & mb+mq \\ mc+mr & md+ms \end{bmatrix}=\begin{bmatrix} ma & mb \\ mc & md \end{bmatrix}+\begin{bmatrix} mp & mq \\ mr & ms \end{bmatrix} \\ &= m\begin{bmatrix} a & b \\ c & d \end{bmatrix}+m\begin{bmatrix} p & q \\ r & s \end{bmatrix}=mA+mB \end{aligned}$$

$\boxed{32}$
$$\begin{aligned} (m+n)A &= (m+n)\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} (m+n)a & (m+n)b \\ (m+n)c & (m+n)d \end{bmatrix} \\ &= \begin{bmatrix} ma+na & mb+nb \\ mc+nc & md+nd \end{bmatrix}=\begin{bmatrix} ma & mb \\ mc & md \end{bmatrix}+\begin{bmatrix} na & nb \\ nc & nd \end{bmatrix} \\ &= m\begin{bmatrix} a & b \\ c & d \end{bmatrix}+n\begin{bmatrix} a & b \\ c & d \end{bmatrix}=mA+nA \end{aligned}$$

$\boxed{33}$
$$\begin{aligned} A(B+C) &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p+w & q+x \\ r+y & s+z \end{bmatrix} \\ &= \begin{bmatrix} a(p+w)+b(r+y) & a(q+x)+b(s+z) \\ c(p+w)+d(r+y) & c(q+x)+d(s+z) \end{bmatrix} \\ &= \begin{bmatrix} ap+aw+br+by & aq+ax+bs+bz \\ cp+cw+dr+dy & cq+cx+ds+dz \end{bmatrix} \\ &= \begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix}+\begin{bmatrix} aw+by & ax+bz \\ cw+dy & cx+dz \end{bmatrix} \\ &= \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix}+\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix}=AB+AC \end{aligned}$$

34 $A(BC)=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\left(\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix}\right)=\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} pw+qy & px+qz \\ rw+sy & rx+sz \end{bmatrix}$

$$=\begin{bmatrix} a(pw+qy)+b(rw+sy) & a(px+qz)+b(rx+sz) \\ c(pw+qy)+d(rw+sy) & c(px+qz)+d(rx+sz) \end{bmatrix}$$

$$=\begin{bmatrix} apw+aqy+brw+bsy & apx+aqz+brx+bsz \\ cpw+cqy+drw+dsy & cpx+cqz+drx+dsz \end{bmatrix}$$

$$=\begin{bmatrix} (ap+br)w+(aq+bs)y & (ap+br)x+(aq+bs)z \\ (cp+dr)w+(cq+ds)y & (cp+dr)x+(cq+ds)z \end{bmatrix}$$

$$=\begin{bmatrix} ap+br & aq+bs \\ cp+dr & cq+ds \end{bmatrix}\begin{bmatrix} w & x \\ y & z \end{bmatrix}$$

$$=\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix}\right)\begin{bmatrix} w & x \\ y & z \end{bmatrix}=(AB)C$$

Note: For Exercises 35–38, $A=\begin{bmatrix} 3 & -3 & 7 \\ 2 & 6 & -2 \\ 4 & 2 & 5 \end{bmatrix}$ and $B=\begin{bmatrix} -9 & 5 & -8 \\ 3 & -7 & 1 \\ -1 & 2 & 6 \end{bmatrix}$.

35 $A^2+B^2=\begin{bmatrix} 31 & -13 & 62 \\ 10 & 26 & -8 \\ 36 & 10 & 49 \end{bmatrix}+\begin{bmatrix} 104 & -96 & 29 \\ -49 & 66 & -25 \\ 9 & -7 & 46 \end{bmatrix}=\begin{bmatrix} 135 & -109 & 91 \\ -39 & 92 & -33 \\ 45 & 3 & 95 \end{bmatrix}$

36 $3A-BA=\begin{bmatrix} 9 & -9 & 21 \\ 6 & 18 & -6 \\ 12 & 6 & 15 \end{bmatrix}-\begin{bmatrix} -49 & 41 & -113 \\ -1 & -49 & 40 \\ 25 & 27 & 19 \end{bmatrix}=\begin{bmatrix} 58 & -50 & 134 \\ 7 & 67 & -46 \\ -13 & -21 & -4 \end{bmatrix}$

37 $A^2-5B=\begin{bmatrix} 31 & -13 & 62 \\ 10 & 26 & -8 \\ 36 & 10 & 49 \end{bmatrix}-\begin{bmatrix} -45 & 25 & -40 \\ 15 & -35 & 5 \\ -5 & 10 & 30 \end{bmatrix}=\begin{bmatrix} 76 & -38 & 102 \\ -5 & 61 & -13 \\ 41 & 0 & 19 \end{bmatrix}$

38 $A+A^2+B+B^2=\begin{bmatrix} 3 & -3 & 7 \\ 2 & 6 & -2 \\ 4 & 2 & 5 \end{bmatrix}+\begin{bmatrix} 31 & -13 & 62 \\ 10 & 26 & -8 \\ 36 & 10 & 49 \end{bmatrix}+$

$$\begin{bmatrix} -9 & 5 & -8 \\ 3 & -7 & 1 \\ -1 & 2 & 6 \end{bmatrix}+\begin{bmatrix} 104 & -96 & 29 \\ -49 & 66 & -25 \\ 9 & -7 & 46 \end{bmatrix}$$

$$=\begin{bmatrix} 129 & -107 & 90 \\ -34 & 91 & -34 \\ 48 & 7 & 106 \end{bmatrix}$$

39 (a) For the inventory matrix, we have 5 colors and 3 sizes of towels, and for each size of towel, we have 1 price. We choose a 5×3 matrix A and a 3×1 matrix B.

$$\text{inventory matrix } A = \begin{bmatrix} 400 & 550 & 500 \\ 400 & 450 & 500 \\ 300 & 500 & 600 \\ 250 & 200 & 300 \\ 100 & 100 & 200 \end{bmatrix}, \text{ price matrix } B = \begin{bmatrix} \$1.39 \\ \$2.99 \\ \$4.99 \end{bmatrix}$$

(b) $$C = AB = \begin{bmatrix} 400 & 550 & 500 \\ 400 & 450 & 500 \\ 300 & 500 & 600 \\ 250 & 200 & 300 \\ 100 & 100 & 200 \end{bmatrix} \begin{bmatrix} \$1.39 \\ \$2.99 \\ \$4.99 \end{bmatrix} = \begin{bmatrix} \$4695.50 \\ \$4396.50 \\ \$4906.00 \\ \$2442.50 \\ \$1436.00 \end{bmatrix}$$

(c) The \$1436.00 represents the amount the store would receive if all the yellow towels were sold.

40 (a) For the cost matrix, we have 3 sizes and 2 types of cost, and for the order matrix, we have 3 quantities. We choose a 1×3 matrix A and a 3×2 matrix B.

$$\text{order matrix } A = \begin{bmatrix} 4 & 10 & 6 \end{bmatrix}, \text{ cost matrix } B = \begin{bmatrix} 34 & 50 \\ 40 & 60 \\ 43 & 67 \end{bmatrix}$$

(b) $$C = AB = \begin{bmatrix} 4 & 10 & 6 \end{bmatrix} \begin{bmatrix} 34 & 50 \\ 40 & 60 \\ 43 & 67 \end{bmatrix} = \begin{bmatrix} 794 & 1202 \end{bmatrix}$$

(c) The \$794,000 represents the amount needed for labor and the \$1,202,000 represents the amount needed for materials.

9.7 Exercises

1 $$AB = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \text{ and}$$

$$BA = \begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2.$$

Since $AB = I_2$ and $BA = I_2$, B is the inverse of A.

2 $$AB = \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \text{ and}$$

$$BA = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2.$$

Since $AB = I_2$ and $BA = I_2$, B is the inverse of A.

Note: Exer. 3–12: Let A denote the given matrix.

[3] $\left[\begin{array}{cc|cc} 2 & -4 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right] R_1 - R_2 \to R_1$

$\left[\begin{array}{cc|cc} 1 & -7 & 1 & -1 \\ 1 & 3 & 0 & 1 \end{array}\right] R_2 - R_1 \to R_2$

$\left[\begin{array}{cc|cc} 1 & -7 & 1 & -1 \\ 0 & 10 & -1 & 2 \end{array}\right] \frac{1}{10} R_2 \to R_2$

$\left[\begin{array}{cc|cc} 1 & -7 & 1 & -1 \\ 0 & 1 & -\frac{1}{10} & \frac{2}{10} \end{array}\right] R_1 + 7R_2 \to R_1$

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{3}{10} & \frac{4}{10} \\ 0 & 1 & -\frac{1}{10} & \frac{2}{10} \end{array}\right] \quad \Rightarrow \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}$

[4] $\left[\begin{array}{cc|cc} 3 & 2 & 1 & 0 \\ 4 & 5 & 0 & 1 \end{array}\right] -R_1 + R_2 \to R_1$

$\left[\begin{array}{cc|cc} 1 & 3 & -1 & 1 \\ 4 & 5 & 0 & 1 \end{array}\right] R_2 - 4R_1 \to R_2$

$\left[\begin{array}{cc|cc} 1 & 3 & -1 & 1 \\ 0 & -7 & 4 & -3 \end{array}\right] -\frac{1}{7} R_2 \to R_2$

$\left[\begin{array}{cc|cc} 1 & 3 & -1 & 1 \\ 0 & 1 & -\frac{4}{7} & \frac{3}{7} \end{array}\right] R_1 - 3R_2 \to R_1$

$\left[\begin{array}{cc|cc} 1 & 0 & \frac{5}{7} & -\frac{2}{7} \\ 0 & 1 & -\frac{4}{7} & \frac{3}{7} \end{array}\right] \quad \Rightarrow \quad A^{-1} = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$

[5] $\left[\begin{array}{cc|cc} 2 & 4 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right] \frac{1}{2} R_1 \to R_1$

$\left[\begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 4 & 8 & 0 & 1 \end{array}\right] R_2 - 4R_1 \to R_2$

$\left[\begin{array}{cc|cc} 1 & 2 & \frac{1}{2} & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$

Since the identity matrix cannot be obtained on the left, *no inverse exists.*

[6] $\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 6 & -2 & 0 & 1 \end{array}\right] R_2 - 2R_1 \to R_2$

$\left[\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ 0 & 0 & -2 & 1 \end{array}\right]$

Since the identity matrix cannot be obtained on the left, *no inverse exists.*

[7]
$$\left[\begin{array}{ccc|ccc} 3 & -1 & 0 & 1 & 0 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \mathrm{R}_1 - \mathrm{R}_2 \rightarrow \mathrm{R}_1$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & -1 & 0 \\ 2 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \mathrm{R}_2 - 2\,\mathrm{R}_1 \rightarrow \mathrm{R}_2$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & -1 & 0 \\ 0 & 8 & 0 & -2 & 3 & 0 \\ 0 & 0 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (1/8)\,\mathrm{R}_2 \rightarrow \mathrm{R}_2 \\ (1/4)\,\mathrm{R}_3 \rightarrow \mathrm{R}_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -\frac{2}{8} & \frac{3}{8} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{4} \end{array}\right] \mathrm{R}_1 + 3\,\mathrm{R}_2 \rightarrow \mathrm{R}_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{8} & \frac{1}{8} & 0 \\ 0 & 1 & 0 & -\frac{2}{8} & \frac{3}{8} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{2}{8} \end{array}\right] \qquad A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

[8]
$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -4 & 0 & 2 & 0 & 0 & 1 \end{array}\right] -\mathrm{R}_1 - \mathrm{R}_3 \rightarrow \mathrm{R}_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ -4 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \mathrm{R}_3 + 4\,\mathrm{R}_1 \rightarrow \mathrm{R}_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -14 & -4 & 0 & -3 \end{array}\right] -\tfrac{1}{14}\mathrm{R}_3 \rightarrow \mathrm{R}_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -4 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{14} & 0 & \frac{3}{14} \end{array}\right] \mathrm{R}_1 + 4\,\mathrm{R}_3 \rightarrow \mathrm{R}_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{2}{14} & 0 & -\frac{2}{14} \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{14} & 0 & \frac{3}{14} \end{array}\right] \qquad A^{-1} = \frac{1}{14}\begin{bmatrix} 2 & 0 & -2 \\ 0 & 14 & 0 \\ 4 & 0 & 3 \end{bmatrix}$$

9

$$\left[\begin{array}{ccc|ccc} -2 & 2 & 3 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 + 2\,R_2 \leftrightarrow R_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 + R_3 \rightarrow R_1 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 1 & 1 \\ 0 & 0 & 3 & 1 & 2 & 0 \\ 0 & 1 & 4 & 0 & 0 & 1 \end{array}\right] \tfrac{1}{3}R_2 \leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & 1 & 1 \\ 0 & 1 & 4 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 \end{array}\right] \begin{array}{l} R_1 - 4\,R_3 \rightarrow R_1 \\ R_2 - 4\,R_3 \rightarrow R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{4}{3} & -\frac{5}{3} & 1 \\ 0 & 1 & 0 & -\frac{4}{3} & -\frac{8}{3} & 1 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{2}{3} & 0 \end{array}\right] \qquad A^{-1} = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

10

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ -2 & 1 & 0 & 0 & 1 & 0 \\ 3 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \\ R_2 + 2\,R_1 \rightarrow R_2 \\ R_3 - 3\,R_1 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 5 & 6 & 2 & 1 & 0 \\ 0 & -7 & -8 & -3 & 0 & 1 \end{array}\right] 3\,R_2 + 2\,R_3 \rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 3 & 2 \\ 0 & -7 & -8 & -3 & 0 & 1 \end{array}\right] \begin{array}{l} R_1 - 2\,R_2 \rightarrow R_1 \\ \\ R_3 + 7\,R_2 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -6 & -4 \\ 0 & 1 & 2 & 0 & 3 & 2 \\ 0 & 0 & 6 & -3 & 21 & 15 \end{array}\right] \begin{array}{l} \\ \\ \frac{1}{6}R_3 \rightarrow R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -6 & -4 \\ 0 & 1 & 2 & 0 & 3 & 2 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{7}{2} & \frac{5}{2} \end{array}\right] \begin{array}{l} R_1 + R_3 \rightarrow R_1 \\ R_2 - 2\,R_3 \rightarrow R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & -\frac{5}{2} & -\frac{3}{2} \\ 0 & 1 & 0 & 1 & -4 & -3 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{7}{2} & \frac{5}{2} \end{array}\right] \qquad A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -5 & -3 \\ 2 & -8 & -6 \\ -1 & 7 & 5 \end{bmatrix}$$

11 $\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 & 1 & 0 \\ 0 & 0 & 6 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} (1/2)\,\mathrm{R}_1 \to \mathrm{R}_1 \\ (1/4)\,\mathrm{R}_2 \to \mathrm{R}_2 \\ (1/6)\,\mathrm{R}_3 \to \mathrm{R}_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{6} \end{array}\right]$ $\quad A^{-1} = \frac{1}{12}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

12 $\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 & 0 \\ 3 & 3 & 3 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{R}_2 - 2\,\mathrm{R}_1 \to \mathrm{R}_2 \\ \mathrm{R}_3 - 3\,\mathrm{R}_1 \to \mathrm{R}_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & -3 & 0 & 1 \end{array}\right]$ *No inverse exists.*

13 $\left[\begin{array}{cc|cc} a & 0 & 1 & 0 \\ 0 & b & 0 & 1 \end{array}\right] \begin{array}{l} (1/a)\,\mathrm{R}_1 \to \mathrm{R}_1 \\ (1/b)\,\mathrm{R}_2 \to \mathrm{R}_2 \end{array} \Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 1/a & 0 \\ 0 & 1 & 0 & 1/b \end{array}\right]$

The inverse is the matrix with main diagonal elements $(1/a)$ and $(1/b)$.

The required conditions are that a and b are nonzero to avoid division by zero.

14 Refer to Exercise 13. Generalizing, we have the inverse $\begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$.

15 $AI_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$

$I_3A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = A$

16 Show that the same conditions in Exercise 15 hold using a square matrix of order 4.

17 (a) $X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 13 \\ -1 \end{bmatrix}$; $(\frac{13}{10}, -\frac{1}{10})$

(b) $X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 3 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 5 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 14 \\ 12 \end{bmatrix}$; $(\frac{7}{5}, \frac{6}{5})$

18 (a) $X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} -7 \\ 7 \end{bmatrix}$; $(-1, 1)$

(b) $X = A^{-1}B = \frac{1}{7}\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 14 \\ -7 \end{bmatrix}$; $(2, -1)$

19 (a) $X = A^{-1}B = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -25 \\ -34 \\ 7 \end{bmatrix}$; $(-\frac{25}{3}, -\frac{34}{3}, \frac{7}{3})$

(b) $X = A^{-1}B = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 16 \\ 16 \\ -1 \end{bmatrix}$; $(\frac{16}{3}, \frac{16}{3}, -\frac{1}{3})$

20 (a) $X = A^{-1}B = \frac{1}{2}\begin{bmatrix} 1 & -5 & -3 \\ 2 & -8 & -6 \\ -1 & 7 & 5 \end{bmatrix}\begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -27 \\ -46 \\ 39 \end{bmatrix}$; $(-\frac{27}{2}, -23, \frac{39}{2})$

(b) $X = A^{-1}B = \frac{1}{2}\begin{bmatrix} 1 & -5 & -3 \\ 2 & -8 & -6 \\ -1 & 7 & 5 \end{bmatrix}\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 \\ 4 \\ -6 \end{bmatrix}$; $(2, 2, -3)$

21 $A = \begin{bmatrix} 2 & -5 & 8 \\ 3 & 7 & -1 \\ 0 & 2 & 1 \end{bmatrix} \Rightarrow A^{-1} \approx \begin{bmatrix} 0.11111 & 0.25926 & -0.62963 \\ -0.03704 & 0.02469 & 0.32099 \\ 0.07407 & -0.04938 & 0.35802 \end{bmatrix}$

22 $A = \begin{bmatrix} 0 & 1.2 & 4.1 \\ -1 & 0 & -1 \\ 5.9 & 2 & 0 \end{bmatrix} \Rightarrow A^{-1} \approx \begin{bmatrix} -0.13089 & -0.53665 & 0.07853 \\ 0.38613 & 1.58312 & 0.26832 \\ 0.13089 & -0.46335 & -0.07853 \end{bmatrix}$

23 $A = \begin{bmatrix} 2 & -1 & 1 & 4 \\ 7 & 1.2 & -8 & 0 \\ 2.5 & 0 & 1.9 & 7.9 \\ 1 & -1 & 3 & 1 \end{bmatrix} \Rightarrow$

$$A^{-1} \approx \begin{bmatrix} -0.22278 & 0.12932 & 0.06496 & 0.37796 \\ -1.17767 & 0.09503 & 0.55936 & 0.29171 \\ -0.37159 & 0.00241 & 0.14074 & 0.37447 \\ 0.15987 & -0.04150 & 0.07218 & -0.20967 \end{bmatrix}$$

24 $A = \begin{bmatrix} -3 & -7 & 4 & 0 \\ -7 & 0 & 5.5 & 9 \\ 3 & 1 & 0 & 0 \\ 9 & -11 & 4 & 1 \end{bmatrix} \Rightarrow$

$$A^{-1} \approx \begin{bmatrix} -0.03078 & -0.00404 & 0.18416 & 0.03633 \\ 0.09233 & 0.01211 & 0.44753 & -0.10898 \\ 0.38850 & 0.01816 & 0.92129 & -0.16347 \\ -0.26135 & 0.09687 & -0.41978 & 0.12815 \end{bmatrix}$$

25 (a) $AX = B \quad \Leftrightarrow \quad \begin{bmatrix} 4.0 & 7.1 \\ 2.2 & -4.9 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6.2 \\ 2.9 \end{bmatrix}$

(b) $A^{-1} \approx \begin{bmatrix} 0.1391 & 0.2016 \\ 0.0625 & -0.1136 \end{bmatrix}$

(c) $X = A^{-1}B \approx \begin{bmatrix} 0.1391 & 0.2016 \\ 0.0625 & -0.1136 \end{bmatrix}\begin{bmatrix} 6.2 \\ 2.9 \end{bmatrix} \approx \begin{bmatrix} 1.4472 \\ 0.0579 \end{bmatrix}$

$\boxed{26}$ (a) $AX = B \Leftrightarrow \begin{bmatrix} 5.1 & 8.7 & 2.5 \\ 9.9 & 15 & 12 \\ -4.3 & -2.2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.1 \\ 3.8 \\ -7.1 \end{bmatrix}$

(b) $A^{-1} \approx \begin{bmatrix} -0.0576 & -0.0162 & -0.3381 \\ 0.2108 & -0.0286 & 0.1842 \\ -0.2159 & 0.1324 & 0.0487 \end{bmatrix}$

(c) $X = A^{-1}B \approx \begin{bmatrix} -0.0576 & -0.0162 & -0.3381 \\ 0.2108 & -0.0286 & 0.1842 \\ -0.2159 & 0.1324 & 0.0487 \end{bmatrix} \begin{bmatrix} 1.1 \\ 3.8 \\ -7.1 \end{bmatrix} = \begin{bmatrix} 2.2759 \\ -1.1847 \\ -0.0801 \end{bmatrix}$

$\boxed{27}$ (a) $AX = B \Leftrightarrow \begin{bmatrix} 3.1 & 6.7 & -8.7 \\ 4.1 & -5.1 & 0.2 \\ 0.6 & 1.1 & -7.4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1.5 \\ 2.1 \\ 3.9 \end{bmatrix}$

(b) $A^{-1} \approx \begin{bmatrix} 0.1474 & 0.1572 & -0.1691 \\ 0.1197 & -0.0696 & -0.1426 \\ 0.0297 & 0.0024 & -0.1700 \end{bmatrix}$

(c) $X = A^{-1}B \approx \begin{bmatrix} 0.1474 & 0.1572 & -0.1691 \\ 0.1197 & -0.0696 & -0.1426 \\ 0.0297 & 0.0024 & -0.1700 \end{bmatrix} \begin{bmatrix} 1.5 \\ 2.1 \\ 3.9 \end{bmatrix} \approx \begin{bmatrix} -0.1081 \\ -0.5227 \\ -0.6135 \end{bmatrix}$

$\boxed{28}$ (a) $AX = B \Leftrightarrow \begin{bmatrix} 5.6 & 8.4 & -7.2 & 4.2 \\ 8.4 & 9.2 & -6.1 & -6.2 \\ -7.2 & -6.1 & 9.2 & 4.5 \\ 4.2 & -6.2 & -4.5 & 5.8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 8.1 \\ 5.3 \\ 0.4 \\ 2.7 \end{bmatrix}$

(b) $A^{-1} \approx \begin{bmatrix} -0.0295 & 0.2513 & 0.2069 & 0.1294 \\ 0.0784 & 0.0139 & 0.0363 & -0.0701 \\ -0.0163 & 0.2085 & 0.2530 & 0.0384 \\ 0.0925 & -0.0054 & 0.0852 & 0.0335 \end{bmatrix}$

(c) $X = A^{-1}B \approx \begin{bmatrix} -0.0295 & 0.2513 & 0.2069 & 0.1294 \\ 0.0784 & 0.0139 & 0.0363 & -0.0701 \\ -0.0163 & 0.2085 & 0.2530 & 0.0384 \\ 0.0925 & -0.0054 & 0.0852 & 0.0335 \end{bmatrix} \begin{bmatrix} 8.1 \\ 5.3 \\ 0.4 \\ 2.7 \end{bmatrix} \approx \begin{bmatrix} 1.5255 \\ 0.5341 \\ 1.1778 \\ 0.8456 \end{bmatrix}$

[29] (a) $f(2) = 4a + 2b + c = 19$; $f(8) = 64a + 8b + c = 59$; $f(11) = 121a + 11b + c = 26$

Solve the 3×3 linear system using the inverse method.

$$\begin{bmatrix} 4 & 2 & 1 \\ 64 & 8 & 1 \\ 121 & 11 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 19 \\ 59 \\ 26 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} \approx \begin{bmatrix} -1.9630 \\ 26.2963 \\ -25.7407 \end{bmatrix}$$

Thus, let $f(x) = -1.9630x^2 + 26.2963x - 25.7407$.

(c) For June, $f(6) \approx 61$°F and for October, $f(10) \approx 41$°F.

[1, 12] by [−15, 70, 5]

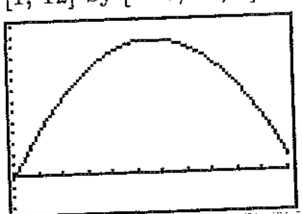

Figure 29

[1, 12] by [−15, 70, 5]

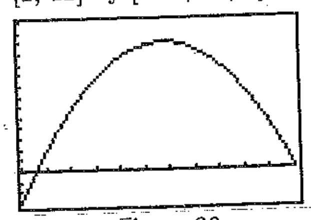

Figure 30

[30] (a) $f(2) = 4a + 2b + c = 9$; $f(7) = 49a + 7b + c = 60$; $f(11) = 121a + 11b + c = 21$

Solve the 3×3 linear system using the inverse method.

$$\begin{bmatrix} 4 & 2 & 1 \\ 49 & 7 & 1 \\ 121 & 11 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9 \\ 60 \\ 21 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} \approx \begin{bmatrix} -2.2167 \\ 30.1500 \\ -42.4333 \end{bmatrix}$$

Thus, let $f(x) = -2.2167x^2 + 30.15x - 42.4333$.

(c) For June, $f(6) \approx 59$°F and for October, $f(10) \approx 37$°F.

9.8 Exercises

[1] $M_{11} = 0 = A_{11}$; $M_{12} = 5$ and $A_{12} = -5$; $M_{21} = -1$ and $A_{21} = 1$; $M_{22} = 7 = A_{22}$

[2] $M_{11} = 2 = A_{11}$; $M_{12} = 3$ and $A_{12} = -3$; $M_{21} = 4$ and $A_{21} = -4$; $M_{22} = -6 = A_{22}$

[3] $M_{11} = \begin{vmatrix} 3 & 2 \\ 7 & 0 \end{vmatrix} = -14 = A_{11}$; $M_{12} = \begin{vmatrix} 0 & 2 \\ -5 & 0 \end{vmatrix} = 10$; $A_{12} = -10$;

$M_{13} = \begin{vmatrix} 0 & 3 \\ -5 & 7 \end{vmatrix} = 15 = A_{13}$; $M_{21} = \begin{vmatrix} 4 & -1 \\ 7 & 0 \end{vmatrix} = 7$; $A_{21} = -7$;

$M_{22} = \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} = -5 = A_{22}$; $M_{23} = \begin{vmatrix} 2 & 4 \\ -5 & 7 \end{vmatrix} = 34$; $A_{23} = -34$;

$M_{31} = \begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix} = 11 = A_{31}$; $M_{32} = \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = 4$; $A_{32} = -4$;

$M_{33} = \begin{vmatrix} 2 & 4 \\ 0 & 3 \end{vmatrix} = 6 = A_{33}$

[4] $M_{11} = \begin{vmatrix} 7 & 0 \\ 4 & -1 \end{vmatrix} = -7 = A_{11}$; $M_{12} = \begin{vmatrix} 4 & 0 \\ -3 & -1 \end{vmatrix} = -4$; $A_{12} = 4$;

$M_{13} = \begin{vmatrix} 4 & 7 \\ -3 & 4 \end{vmatrix} = 37 = A_{13}$; $M_{21} = \begin{vmatrix} -2 & 1 \\ 4 & -1 \end{vmatrix} = -2$; $A_{21} = 2$;

$M_{22} = \begin{vmatrix} 5 & 1 \\ -3 & -1 \end{vmatrix} = -2 = A_{22}$; $M_{23} = \begin{vmatrix} 5 & -2 \\ -3 & 4 \end{vmatrix} = 14$; $A_{23} = -14$;

$M_{31} = \begin{vmatrix} -2 & 1 \\ 7 & 0 \end{vmatrix} = -7 = A_{31}$; $M_{32} = \begin{vmatrix} 5 & 1 \\ 4 & 0 \end{vmatrix} = -4$; $A_{32} = 4$;

$M_{33} = \begin{vmatrix} 5 & -2 \\ 4 & 7 \end{vmatrix} = 43 = A_{33}$

Note: Exer. 5–20: Let A denote the given matrix.

[5] $\begin{vmatrix} 7 & -1 \\ 5 & 0 \end{vmatrix} = (7)(0) - (-1)(5) = 0 + 5 = 5$

[6] $\begin{vmatrix} -6 & 4 \\ 3 & 2 \end{vmatrix} = (-6)(2) - (4)(3) = -12 - 12 = -24$

[7] Expand by the first column.

$|A| = a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} = 2(-14) + 0(A_{21}) - 5(11) = -83$

[8] Expand by the third column.

$|A| = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33} = 1(37) + 0(A_{23}) - 1(43) = -6$

[9] $|A| = (-5)(2) - (4)(-3) = -10 + 12 = 2$

[10] $|A| = (6)(2) - (4)(-3) = 12 + 12 = 24$

[11] $|A| = (a)(-b) - (-a)(b) = -ab + ab = 0$

[12] $|A| = (c)(c) - (d)(-d) = c^2 + d^2$

[13] Expand by the first row.

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 3(-17) + 1(-26) - 2(24) = -125$

[14] Expand by the first row.

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = 2(15) - 5(33) + 1(2) = -133$

[15] Expand by the third row.

$|A| = a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = 2(30) + 0(A_{32}) + 6(-2) = 48$

[16] Expand by the second row.

$|A| = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} = 1(17) + 0(A_{22}) + 4(30) = 137$

[17] Expand $|A|$ by the third row. $|A| = 6\,A_{32} = -6\,M_{32} = -6 \begin{vmatrix} 3 & 2 & 0 \\ 4 & -3 & 5 \\ 1 & -4 & 2 \end{vmatrix}$.

Expand M_{32} by the first row.

$M_{32} = 3(14) + 2(-3) + 0(-13) = 36 \Rightarrow |A| = -6(36) = -216.$

18 Expand $|A|$ by the fourth column. $|A| = 6\,A_{34} = -6\,M_{34} = -6\begin{vmatrix} 2 & 5 & 1 \\ -4 & 0 & -3 \\ -1 & 4 & 2 \end{vmatrix}$.

Expand M_{34} by the second column.

$5(11) + 0(5) + 4(2) = 63$, $\therefore$ $|A| = -6(63) = -378$

19 Expand by the first row. $|A| = -b\begin{vmatrix} 0 & c & 0 \\ a & 0 & 0 \\ 0 & 0 & d \end{vmatrix}$.

Expand again by the first row. $|A| = (-b)(-c)\begin{vmatrix} a & 0 \\ 0 & d \end{vmatrix} = abcd.$

20 Expand by the first column. $|A| = a\begin{vmatrix} b & x & y \\ 0 & c & z \\ 0 & 0 & d \end{vmatrix}$.

Expand again by the first column. $|A| = (ab)\begin{vmatrix} c & z \\ 0 & d \end{vmatrix} = abcd.$

21 LS $= ad - bc$; RS $= -(bc - ad) = ad - bc$

22 LS $= ad - bc$; RS $= -(bc - ad) = ad - bc$

23 LS $= adk - bck$; RS $= k(ad - bc) = adk - bck$

24 LS $= adk - bck$; RS $= k(ad - bc) = adk - bck$

25 LS $= ad - bc$; RS $= abk + ad - abk - bc = ad - bc$

26 LS $= ad - bc$; RS $= ack + ad - ack - bc = ad - bc$

27 LS $= ad - bc + af - ce$; RS $= ad + af - bc - ce$

28 LS $= ad - bc + af - be$; RS $= ad + af - bc - be$

29 All elements in A above the main diagonal are zero. Similar to Exercise 20, we can evaluate the determinant using n expansions by the first row.

30 $\left[\begin{array}{cc|cc} a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}\right] \frac{1}{a_{11}}\mathrm{R}_1 \to \mathrm{R}_1$ {a_{11} or a_{21} must be nonzero since $|A| \neq 0$.} {The rows could be swapped.}

$\left[\begin{array}{cc|cc} 1 & a_{12}/a_{11} & 1/a_{11} & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}\right] \mathrm{R}_2 - a_{21}\,\mathrm{R}_1 \to \mathrm{R}_2$

$\left[\begin{array}{cc|cc} 1 & a_{12}/a_{11} & 1/a_{11} & 0 \\ 0 & (a_{11}a_{22} - a_{12}a_{21})/a_{11} & -a_{21}/a_{11} & 1 \end{array}\right] (a_{11}/|A|)\,\mathrm{R}_2 \to \mathrm{R}_2,$

where $|A| = a_{11}a_{22} - a_{12}a_{21}$

$\left[\begin{array}{cc|cc} 1 & a_{12}/a_{11} & 1/a_{11} & 0 \\ 0 & 1 & -a_{21}/|A| & a_{11}/|A| \end{array}\right] \mathrm{R}_1 - (a_{12}/a_{11})\,\mathrm{R}_2 \to \mathrm{R}_1$

(continued)

$$\left[\begin{array}{cc|cc} 1 & 0 & 1/a_{11} + (a_{12}a_{21})/(a_{11}\,|\,A\,|) & -a_{12}/\,|\,A\,| \\ 0 & 1 & -a_{21}/\,|\,A\,| & a_{11}/\,|\,A\,| \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & (\,|\,A\,| + a_{12}a_{21})/(a_{11}\,|\,A\,|) & -a_{12}/\,|\,A\,| \\ 0 & 1 & -a_{21}/\,|\,A\,| & a_{11}/\,|\,A\,| \end{array}\right]$$

{ Use the definition of $|\,A\,|$ above. }

$$= \left[\begin{array}{cc|cc} 1 & 0 & a_{22}/\,|\,A\,| & -a_{12}/\,|\,A\,| \\ 0 & 1 & -a_{21}/\,|\,A\,| & a_{11}/\,|\,A\,| \end{array}\right] \qquad A^{-1} = \frac{1}{|\,A\,|}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$\boxed{31}$ (a) $A - xI = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} - x\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1-x & 2 \\ 3 & 2-x \end{bmatrix}$.

$$f(x) = |\,A - xI\,| = \begin{vmatrix} 1-x & 2 \\ 3 & 2-x \end{vmatrix} = x^2 - 3x - 4.$$

(b) $(x-4)(x+1) = 0 \;\Rightarrow\; x = -1,\ 4$

$\boxed{32}$ (a) $f(x) = |\,A - xI\,| = \begin{vmatrix} 3-x & 1 \\ 2 & 2-x \end{vmatrix} = x^2 - 5x + 4$

(b) $(x-1)(x-4) = 0 \;\Rightarrow\; x = 1,\ 4$

$\boxed{33}$ (a) $f(x) = |\,A - xI\,| = \begin{vmatrix} -3-x & -2 \\ 2 & 2-x \end{vmatrix} = x^2 + x - 2$

(b) $(x+2)(x-1) = 0 \;\Rightarrow\; x = -2,\ 1$

$\boxed{34}$ (a) $f(x) = |\,A - xI\,| = \begin{vmatrix} 2-x & -4 \\ -3 & 5-x \end{vmatrix} = x^2 - 7x - 2$

(b) $x^2 - 7x - 2 = 0 \;\Rightarrow\; x = \dfrac{7 \pm \sqrt{49+8}}{2} = \dfrac{7 \pm \sqrt{57}}{2} \approx 7.27,\ -0.27$

$\boxed{35}$ (a) $f(x) = \begin{vmatrix} 1-x & 0 & 0 \\ 1 & 0-x & -2 \\ -1 & 1 & -3-x \end{vmatrix}$ { Expand by the first row. }

$$\begin{aligned} &= (1-x)\big[(-x)(-3-x) - (-2)\big] \\ &= (1-x)(x^2 + 3x + 2) \\ &= (1-x)(x+1)(x+2) \text{ or } \\ &\qquad (-x^3 - 2x^2 + x + 2) \end{aligned}$$

(b) $x = -2,\ -1,\ 1$

$\boxed{36}$ (a) $f(x) = \begin{vmatrix} 2-x & 1 & 0 \\ -1 & 0-x & 0 \\ 1 & 3 & 2-x \end{vmatrix}$ { Expand by the third column. }

$$\begin{aligned} &= (2-x)\big[(2-x)(-x) - (-1)\big] \\ &= (2-x)(x^2 - 2x + 1) \\ &= (2-x)(x-1)^2 \text{ or } (-x^3 + 4x^2 - 5x + 2) \end{aligned}$$

(b) $x = 1,\ 2$

37 (a) $f(x) = \begin{vmatrix} 0-x & 2 & -2 \\ -1 & 3-x & 1 \\ -3 & 3 & 1-x \end{vmatrix}$ {Expand by the first row.}

$= (-x)[(3-x)(1-x)-3] - 2[(x-1)+3] - 2[-3+3(3-x)]$

$= -x^3 + 4x^2 + 4x - 16 = (x-2)(-x^2+2x+8) = (x+2)(x-2)(-x+4)$

(b) $x = -2, 2, 4$

38 (a) $f(x) = \begin{vmatrix} 3-x & 2 & 2 \\ 1 & 0-x & 2 \\ -1 & -1 & 0-x \end{vmatrix}$ {Expand by the first row.}

$= (3-x)(x^2+2) - 2(-x+2) + 2(-1-x)$

$= -x^3 + 3x^2 - 2x = (-x)(x^2-3x+2) = (-x)(x-1)(x-2)$

(b) $x = 0, 1, 2$

Note: Exer. 39–42: Expand by the first row.

39 $\begin{vmatrix} i & j & k \\ 2 & -1 & 6 \\ -3 & 5 & 1 \end{vmatrix} = i\begin{vmatrix} -1 & 6 \\ 5 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & 6 \\ -3 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & -1 \\ -3 & 5 \end{vmatrix} = -31i - 20j + 7k$

40 $\begin{vmatrix} i & j & k \\ 1 & -2 & 3 \\ 2 & 1 & -4 \end{vmatrix} = i\begin{vmatrix} -2 & 3 \\ 1 & -4 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} + k\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 5i + 10j + 5k$

41 $\begin{vmatrix} i & j & k \\ 5 & -6 & -1 \\ 3 & 0 & 1 \end{vmatrix} = i\begin{vmatrix} -6 & -1 \\ 0 & 1 \end{vmatrix} - j\begin{vmatrix} 5 & -1 \\ 3 & 1 \end{vmatrix} + k\begin{vmatrix} 5 & -6 \\ 3 & 0 \end{vmatrix} = -6i - 8j + 18k$

42 $\begin{vmatrix} i & j & k \\ 4 & -6 & 2 \\ -2 & 3 & -1 \end{vmatrix} = i\begin{vmatrix} -6 & 2 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 4 & 2 \\ -2 & -1 \end{vmatrix} + k\begin{vmatrix} 4 & -6 \\ -2 & 3 \end{vmatrix} = 0i + 0j + 0k$

43 $A = \begin{bmatrix} 29 & -17 & 90 \\ -34 & 91 & -34 \\ 48 & 7 & 10 \end{bmatrix} \Rightarrow |A| = -359{,}284.$

44 $A = \begin{bmatrix} -2 & 5.5 & 8 \\ -0.3 & 8.5 & 7 \\ 4.9 & 6.7 & 11 \end{bmatrix} \Rightarrow |A| = -235.68.$

45 $A = \begin{bmatrix} 4 & -7 & -3 & 13 \\ -17 & -0.8 & 5 & 0.9 \\ 1.1 & 0.2 & 10 & -4 \\ 3 & -6 & 2 & 1 \end{bmatrix} \Rightarrow |A| = 10{,}739.92.$

46 $A = \begin{bmatrix} 4.2 & 1.7 & -2 & -4 \\ -7 & 0.1 & 4.6 & 2.7 \\ 4.1 & -7 & 12 & 6.8 \\ 4.6 & 2 & 3.2 & 1.2 \end{bmatrix} \Rightarrow |A| = 1323.1608.$

47 (a) $f(x) = |A - xI| = \begin{vmatrix} 1-x & 0 & 1 \\ 0 & 2-x & 1 \\ 1 & 1 & -2-x \end{vmatrix} = -x^3 + x^2 + 6x - 7$

(b) The characteristic values of A are equal to the zeros of f. From the graph, we see that the zeros are approximately -2.51, 1.22, and 2.29.

[−10, 11] by [−12, 2]

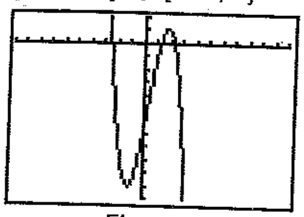

Figure 47

[−15, 15, 2] by [−10, 10, 2]

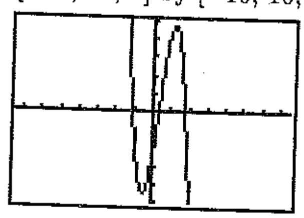

Figure 48

48 (a) $f(x) = |A - xI| = \begin{vmatrix} 3-x & -1 & -1 \\ -1 & 1-x & 0 \\ -1 & 0 & -2-x \end{vmatrix} = -x^3 + 2x^2 + 7x - 5$

(b) The characteristic values of A are equal to the zeros of f. From the graph, we see that the zeros are approximately -2.20, 0.64, and 3.57.

9.9 Exercises

1 $R_2 \leftrightarrow R_3$

2 $C_2 \leftrightarrow C_3$

3 $-R_1 + R_3 \rightarrow R_3$

4 $-R_2 + R_1 \rightarrow R_1$

5 2 is a common factor of R_1 and R_3

6 2 is a common factor of C_1 and 3 is a common factor of C_3

7 R_1 and R_3 are identical

8 C_1 and C_3 are identical

9 -1 is a common factor of R_2

10 -1 is a common factor of R_1

11 Every number in C_2 is 0

12 Every number in R_2 is 0

13 $2C_1 + C_3 \rightarrow C_3$

14 $C_1 \leftrightarrow C_3$

Note: The notation $\{R_i\ (C_i)\}$ means expand the determinant by the ith row (column).

15 $\begin{vmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \\ 1 & 3 & -1 \end{vmatrix} R_3 - 3R_1 \rightarrow R_3 = \begin{vmatrix} 3 & 1 & 0 \\ -2 & 0 & 1 \\ -8 & 0 & -1 \end{vmatrix} \{C_2\} = (-1)\begin{vmatrix} -2 & 1 \\ -8 & -1 \end{vmatrix} =$

$(-1)(2 + 8) = -10$

16 $\begin{vmatrix} -3 & 0 & 4 \\ 1 & 2 & 0 \\ 4 & 1 & -1 \end{vmatrix} R_2 - 2R_3 \rightarrow R_2 = \begin{vmatrix} -3 & 0 & 4 \\ -7 & 0 & 2 \\ 4 & 1 & -1 \end{vmatrix} \{C_2\} = (-1)\begin{vmatrix} -3 & 4 \\ -7 & 2 \end{vmatrix} =$

$(-1)(-6 + 28) = -22$

17 $\begin{vmatrix} 5 & 4 & 3 \\ -3 & 2 & 1 \\ 0 & 7 & -2 \end{vmatrix} \begin{matrix} R_1 - 3R_2 \to R_1 \\ \\ R_3 + 2R_2 \to R_3 \end{matrix} = \begin{vmatrix} 14 & -2 & 0 \\ -3 & 2 & 1 \\ -6 & 11 & 0 \end{vmatrix} \{C_3\} = (-1)\begin{vmatrix} 14 & -2 \\ -6 & 11 \end{vmatrix} =$

$(-1)(154 - 12) = -142$.

18 $\begin{vmatrix} 0 & 2 & -6 \\ 5 & 1 & -3 \\ 6 & -2 & 5 \end{vmatrix} \begin{matrix} R_1 - 2R_2 \to R_1 \\ \\ R_3 + 2R_2 \to R_3 \end{matrix} = \begin{vmatrix} -10 & 0 & 0 \\ 5 & 1 & -3 \\ 16 & 0 & -1 \end{vmatrix} \{C_2\} = (1)\begin{vmatrix} -10 & 0 \\ 16 & -1 \end{vmatrix} =$

$(1)(10 - 0) = 10$

19 $\begin{vmatrix} 2 & 2 & -3 \\ 3 & 6 & 9 \\ -2 & 5 & 4 \end{vmatrix}$ {3 is a common factor of R_2} $= (3)\begin{vmatrix} 2 & 2 & -3 \\ 1 & 2 & 3 \\ -2 & 5 & 4 \end{vmatrix} \begin{matrix} R_1 - 2R_2 \to R_1 \\ \\ R_3 + 2R_2 \to R_3 \end{matrix}$

$= (3)\begin{vmatrix} 0 & -2 & -9 \\ 1 & 2 & 3 \\ 0 & 9 & 10 \end{vmatrix} \{C_1\} = (3)(-1)\begin{vmatrix} -2 & -9 \\ 9 & 10 \end{vmatrix} = (-3)(-20 + 81) = -183$

20 $\begin{vmatrix} 3 & 8 & 5 \\ 5 & 3 & -6 \\ 2 & 4 & -2 \end{vmatrix}$ {2 is a common factor of R_3} $= (2)\begin{vmatrix} 3 & 8 & 5 \\ 5 & 3 & -6 \\ 1 & 2 & -1 \end{vmatrix} \begin{matrix} R_1 - 3R_3 \to R_1 \\ R_2 - 5R_3 \to R_2 \\ \\ \end{matrix}$

$= (2)\begin{vmatrix} 0 & 2 & 8 \\ 0 & -7 & -1 \\ 1 & 2 & -1 \end{vmatrix} \{C_1\} = (2)(1)\begin{vmatrix} 2 & 8 \\ -7 & -1 \end{vmatrix} = (2)(-2 + 56) = 108$

21 $\begin{vmatrix} 3 & 1 & -2 & 2 \\ 2 & 0 & 1 & 4 \\ 0 & 1 & 3 & 5 \\ -1 & 2 & 0 & -3 \end{vmatrix} \begin{matrix} \\ \\ R_3 - R_1 \to R_3 \\ R_4 - 2R_1 \to R_4 \end{matrix} = \begin{vmatrix} 3 & 1 & -2 & 2 \\ 2 & 0 & 1 & 4 \\ -3 & 0 & 5 & 3 \\ -7 & 0 & 4 & -7 \end{vmatrix} \{C_2\}$

$= (-1)\begin{vmatrix} 2 & 1 & 4 \\ -3 & 5 & 3 \\ -7 & 4 & -7 \end{vmatrix} \begin{matrix} \\ R_2 - 5R_1 \to R_2 \\ R_3 - 4R_1 \to R_3 \end{matrix} = (-1)\begin{vmatrix} 2 & 1 & 4 \\ -13 & 0 & -17 \\ -15 & 0 & -23 \end{vmatrix} \{C_2\}$

$= (-1)(-1)\begin{vmatrix} -13 & -17 \\ -15 & -23 \end{vmatrix} = (1)(299 - 255) = 44$

22 $\begin{vmatrix} 3 & 2 & 0 & 4 \\ -2 & 0 & 5 & 0 \\ 4 & -3 & 1 & 6 \\ 2 & -1 & 2 & 0 \end{vmatrix} \begin{matrix} \\ R_2 - 5R_3 \to R_2 \\ \\ R_4 - 2R_3 \to R_4 \end{matrix} = \begin{vmatrix} 3 & 2 & 0 & 4 \\ -22 & 15 & 0 & -30 \\ 4 & -3 & 1 & 6 \\ -6 & 5 & 0 & -12 \end{vmatrix} \{C_3\}$

$= (1)\begin{vmatrix} 3 & 2 & 4 \\ -22 & 15 & -30 \\ -6 & 5 & -12 \end{vmatrix} \begin{matrix} C_1 - C_2 \to C_1 \\ \\ \\ \end{matrix} = \begin{vmatrix} 1 & 2 & 4 \\ -37 & 15 & -30 \\ -11 & 5 & -12 \end{vmatrix} \begin{matrix} \\ R_2 + 37R_1 \to R_2 \\ R_3 + 11R_1 \to R_3 \end{matrix}$

$= \begin{vmatrix} 1 & 2 & 4 \\ 0 & 89 & 118 \\ 0 & 27 & 32 \end{vmatrix} \{C_1\} = (1)\begin{vmatrix} 89 & 118 \\ 27 & 32 \end{vmatrix} = (1)(2848 - 3186) = -338$

$\boxed{23}$ $\begin{vmatrix} 2 & -2 & 0 & 0 & -3 \\ 3 & 0 & 3 & 2 & -1 \\ 0 & 1 & -2 & 0 & 2 \\ -1 & 2 & 0 & 3 & 0 \\ 0 & 4 & 1 & 0 & 0 \end{vmatrix} \begin{matrix} C_2 - 4C_3 \to C_2 \end{matrix} = \begin{vmatrix} 2 & -2 & 0 & 0 & -3 \\ 3 & -12 & 3 & 2 & -1 \\ 0 & 9 & -2 & 0 & 2 \\ -1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{vmatrix} \{R_5\}$

$= (1)\begin{vmatrix} 2 & -2 & 0 & -3 \\ 3 & -12 & 2 & -1 \\ 0 & 9 & 0 & 2 \\ -1 & 2 & 3 & 0 \end{vmatrix} \begin{matrix} R_1 + 2R_4 \to R_1 \\ R_2 + 3R_4 \to R_2 \end{matrix} = \begin{vmatrix} 0 & 2 & 6 & -3 \\ 0 & -6 & 11 & -1 \\ 0 & 9 & 0 & 2 \\ -1 & 2 & 3 & 0 \end{vmatrix} \{C_1\}$

$= (1)\begin{vmatrix} 2 & 6 & -3 \\ -6 & 11 & -1 \\ 9 & 0 & 2 \end{vmatrix} \begin{matrix} R_1 - 3R_2 \to R_1 \\ \\ R_3 + 2R_2 \to R_3 \end{matrix} = \begin{vmatrix} 20 & -27 & 0 \\ -6 & 11 & -1 \\ -3 & 22 & 0 \end{vmatrix} \{C_3\}$

$= (1)\begin{vmatrix} 20 & -27 \\ -3 & 22 \end{vmatrix} = (1)(440 - 81) = 359$

$\boxed{24}$ $\begin{vmatrix} 2 & 0 & -1 & 0 & 2 \\ 1 & 3 & 0 & 0 & 1 \\ 0 & 4 & 3 & 0 & -1 \\ -1 & 2 & 0 & -2 & 0 \\ 0 & 1 & 5 & 0 & -4 \end{vmatrix} \{C_4\} = (-2)\begin{vmatrix} 2 & 0 & -1 & 2 \\ 1 & 3 & 0 & 1 \\ 0 & 4 & 3 & -1 \\ 0 & 1 & 5 & -4 \end{vmatrix} \begin{matrix} R_1 - 2R_2 \to R_1 \end{matrix}$

$= (-2)\begin{vmatrix} 0 & -6 & -1 & 0 \\ 1 & 3 & 0 & 1 \\ 0 & 4 & 3 & -1 \\ 0 & 1 & 5 & -4 \end{vmatrix} \{C_1\} = (-2)(-1)\begin{vmatrix} -6 & -1 & 0 \\ 4 & 3 & -1 \\ 1 & 5 & -4 \end{vmatrix} \begin{matrix} R_1 + 6R_3 \to R_1 \\ R_2 - 4R_3 \to R_2 \end{matrix}$

$= (2)\begin{vmatrix} 0 & 29 & -24 \\ 0 & -17 & 15 \\ 1 & 5 & -4 \end{vmatrix} \{C_1\} = (2)(1)\begin{vmatrix} 29 & -24 \\ -17 & 15 \end{vmatrix} = (2)(435 - 408) = 54$

$\boxed{25}$ $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} \begin{matrix} C_1 - C_2 \to C_1 \\ \\ C_3 - C_2 \to C_3 \end{matrix}$

$= \begin{vmatrix} 0 & 1 & 0 \\ a-b & b & c-b \\ a^2 - b^2 & b^2 & c^2 - b^2 \end{vmatrix} \begin{matrix} a-b \text{ is a common factor of } C_1 \\ c-b \text{ is a common factor of } C_3 \end{matrix}$

$= (a-b)(c-b)\begin{vmatrix} 0 & 1 & 0 \\ 1 & b & 1 \\ a+b & b^2 & c+b \end{vmatrix} \{R_1\}$

$= (a-b)(c-b)(-1)\begin{vmatrix} 1 & 1 \\ a+b & c+b \end{vmatrix}$

$= (a-b)(b-c)(c+b-a-b) = \underline{(a-b)(b-c)(c-a)}$

26 $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} \begin{matrix} \\ R_2 - a\,R_1 \to R_2 \\ R_3 - a^3\,R_1 \to R_3 \end{matrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & b-a & c-a \\ 0 & b^3-a^3 & c^3-a^3 \end{vmatrix} \{C_1\}$

$$= (1)\begin{vmatrix} b-a & c-a \\ b^3-a^3 & c^3-a^3 \end{vmatrix}$$
$$= (b-a)(c^3-a^3) - (c-a)(b^3-a^3)$$
$$= (b-a)(c-a)(c^2+ac+a^2) - (c-a)(b-a)(b^2+ab+a^2)$$
$$= (b-a)(c-a)(c^2+ac+a^2-b^2-ab-a^2)$$
$$= (b-a)(c-a)(c^2+ac-b^2-ab)$$
$$= (b-a)(c-a)\left[(c-b)(c+b)+a(c-b)\right]$$
$$= (b-a)(c-a)\left[(c-b)(c+b+a)\right]$$
$$= (a-b)(b-c)(c-a)(a+b+c)$$

27 $\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{vmatrix} \{C_1\} = (a_{11})\begin{vmatrix} a_{22} & a_{23} & a_{24} \\ 0 & a_{33} & a_{34} \\ 0 & 0 & a_{44} \end{vmatrix} \{C_1\}$

$$= (a_{11})(a_{22})\begin{vmatrix} a_{33} & a_{34} \\ 0 & a_{44} \end{vmatrix} = (a_{11}\,a_{22})(a_{33}\,a_{44} - 0) = a_{11}\,a_{22}\,a_{33}\,a_{44}$$

28 $|\,A\,| = \begin{vmatrix} a & b & 0 & 0 \\ c & d & 0 & 0 \\ 0 & 0 & e & f \\ 0 & 0 & g & h \end{vmatrix} \{C_1\}$

$$= a\begin{vmatrix} d & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{vmatrix} - c\begin{vmatrix} b & 0 & 0 \\ 0 & e & f \\ 0 & g & h \end{vmatrix} \{R_1 \text{ for both determinants}\}$$
$$= ad\begin{vmatrix} e & f \\ g & h \end{vmatrix} - cb\begin{vmatrix} e & f \\ g & h \end{vmatrix} = (ad-bc)\begin{vmatrix} e & f \\ g & h \end{vmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}\begin{vmatrix} e & f \\ g & h \end{vmatrix}$$

29 $|\,AB\,| = \begin{vmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{vmatrix}$

$$= (a_{11}b_{11}+a_{12}b_{21})(a_{21}b_{12}+a_{22}b_{22}) - (a_{11}b_{12}+a_{12}b_{22})(a_{21}b_{11}+a_{22}b_{21})$$
$$= a_{11}b_{11}a_{21}b_{12} + a_{11}b_{11}a_{22}b_{22} + a_{12}b_{21}a_{21}b_{12} + a_{12}b_{21}a_{22}b_{22}$$
$$- a_{11}b_{12}a_{21}b_{11} - a_{11}b_{12}a_{22}b_{21} - a_{12}b_{22}a_{21}b_{11} - a_{12}b_{22}a_{22}b_{21}$$
$$= a_{11}a_{22}b_{11}b_{22} - a_{11}a_{22}b_{21}b_{12} - a_{21}a_{12}b_{11}b_{22} + a_{21}a_{12}b_{21}b_{12}$$
$$= (a_{11}a_{22} - a_{21}a_{12})(b_{11}b_{22} - b_{21}b_{12}) = |\,A\,|\,|\,B\,|$$

30 Let $B = kA$. Since k is a factor of each of the n rows of A, n repeated applications of property (2) of the theorem on row and column transformations of a determinant yields $|\,B\,| = k^n\,|\,A\,|$.

$\boxed{31}$ Expanding by the first row yields $Ax + By + C = 0$ {an equation of a line} where A, B, and C are constants. To show that the line contains $(x_1,\ y_1)$ and $(x_2,\ y_2)$, we must show that these points are solutions of the equation. Substituting x_1 for x and y_1 for y, we obtain two identical rows and the determinant is zero. Hence, $(x_1,\ y_1)$ is a solution of the equation and a similar argument can be made for $(x_2,\ y_2)$.

$\boxed{32}$ Expanding by the first row yields $A(x^2+y^2)+Bx+Cy+D=0$ {an equation of a circle} where A, B, C, and D are constants. To show that the circle contains $(x_1,\ y_1)$, $(x_2,\ y_2)$, and $(x_3,\ y_3)$ we must show that these points are solutions of the equation. Substituting x_1 for x and y_1 for y, we obtain two identical rows and the determinant is zero. Hence, $(x_1,\ y_1)$ is a solution of the equation and similar arguments can be made for $(x_2,\ y_2)$ and $(x_3,\ y_3)$.

$\boxed{33}$ For the system $\left\{\begin{array}{rcr} 2x + 3y &=& 2 \\ x - 2y &=& 8 \end{array}\right.$, $|D| = \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4 - 3 = -7.$

Since $|D| = -7 \neq 0$, we may solve the system using Cramer's rule.

$|D_x| = \begin{vmatrix} 2 & 3 \\ 8 & -2 \end{vmatrix} = -4 - 24 = -28.$ $|D_y| = \begin{vmatrix} 2 & 2 \\ 1 & 8 \end{vmatrix} = 16 - 2 = 14.$

$x = \dfrac{|D_x|}{|D|} = \dfrac{-28}{-7} = 4.$ $y = \dfrac{|D_y|}{|D|} = \dfrac{14}{-7} = -2.$ ★ $(4,\ -2)$

$\boxed{34}$ $|D_x| = 33,$ $|D_y| = -55,$ $|D| = -11;$ $x = -3,$ $y = 5$

$\boxed{35}$ $|D_x| = -232,$ $|D_y| = 0,$ $|D| = -29;$ $x = 8,$ $y = 0$

$\boxed{36}$ $|D_x| = -53,$ $|D_y| = -106,$ $|D| = 53;$ $x = -1,$ $y = -2$

$\boxed{37}$ $|D| = 18 - 18 = 0$, so Cramer's rule cannot be used.

$\boxed{38}$ $|D| = 12 - 12 = 0$, so Cramer's rule cannot be used.

$\boxed{39}$ $|D_x| = -60,$ $|D_y| = -90,$ $|D_z| = 30,$ $|D| = -30;$
$x = 2,$ $y = 3,$ $z = -1$

$\boxed{40}$ $|D_x| = -10,$ $|D_y| = 5,$ $|D_z| = 20,$ $|D| = 5;$
$x = -2,$ $y = 1,$ $z = 4$

$\boxed{41}$ $|D_x| = 90,$ $|D_y| = -180,$ $|D_z| = -225,$ $|D| = -45;$
$x = -2,$ $y = 4,$ $z = 5$

$\boxed{42}$ $|D_x| = -12,$ $|D_y| = -24,$ $|D_z| = 0,$ $|D| = -4;$
$x = 3,$ $y = 6,$ $z = 0$

$\boxed{43}$ To solve for x, find $|D|$ and $|D_x|$, and then use Cramer's Rule.

$$|D| = \begin{vmatrix} a & b & c \\ e & 0 & f \\ h & i & 0 \end{vmatrix} (1/c)\mathrm{R}_1 \to \mathrm{R}_1$$

$$= \begin{vmatrix} a/c & b/c & 1 \\ e & 0 & f \\ h & i & 0 \end{vmatrix} \mathrm{R}_2 - f\,\mathrm{R}_1 \to \mathrm{R}_2$$

$$= \begin{vmatrix} a/c & b/c & 1 \\ e - af/c & -bf/c & 0 \\ h & i & 0 \end{vmatrix} \{\mathrm{C}_3\}$$

$$= (1)\begin{vmatrix} e - af/c & -bf/c \\ h & i \end{vmatrix}$$

$$= \frac{ce - af}{c} \cdot i - \left(-\frac{bf}{c} \cdot h\right) = \frac{cei - afi + bfh}{c}$$

$$|D_x| = \begin{vmatrix} d & b & c \\ g & 0 & f \\ j & i & 0 \end{vmatrix} (1/c)\mathrm{R}_1 \to \mathrm{R}_1$$

$$= \begin{vmatrix} d/c & b/c & 1 \\ g & 0 & f \\ j & i & 0 \end{vmatrix} \mathrm{R}_2 - f\,\mathrm{R}_1 \to \mathrm{R}_2$$

$$= \begin{vmatrix} d/c & b/c & 1 \\ g - df/c & -bf/c & 0 \\ j & i & 0 \end{vmatrix} \{\mathrm{C}_3\}$$

$$= (1)\begin{vmatrix} g - df/c & -bf/c \\ j & i \end{vmatrix}$$

$$= \frac{cg - df}{c} \cdot i - \left(-\frac{bf}{c} \cdot j\right) = \frac{cgi - dfi + bfj}{c}$$

Thus, $x = \dfrac{|D_x|}{|D|} = \dfrac{cgi - dfi + bfj}{cei - afi + bfh}$.

9.10 Exercises

Note: The general outline for the solutions in this section is as follows:

1st line) The expression is shown on the left side of the equation and its decomposition is on the right side.

2nd line) The equation in the first line is multiplied by its least common denominator and left in factored form.

3rd line and beyond) Values are substituted into the equation in the second line and the coefficients are found by solving the resulting equations. It will be stated when the method of equating coefficients is used.

$\boxed{1}$ $\dfrac{8x-1}{(x-2)(x+3)} = \dfrac{A}{x-2} + \dfrac{B}{x+3}$

$8x - 1 = A(x+3) + B(x-2)$

$x = -3$: $-25 = -5B \ \Rightarrow\ B = 5$ $\qquad \star\ \dfrac{3}{x-2} + \dfrac{5}{x+3}$

$x = 2$: $15 = 5A \ \Rightarrow\ A = 3$

$\boxed{2}$ $\dfrac{x-29}{(x-4)(x+1)} = \dfrac{A}{x-4} + \dfrac{B}{x+1}$

$x - 29 = A(x+1) + B(x-4)$

$x = -1$: $-30 = -5B \ \Rightarrow\ B = 6$ $\qquad \star\ -\dfrac{5}{x-4} + \dfrac{6}{x+1}$

$x = 4$: $-25 = 5A \ \Rightarrow\ A = -5$

$\boxed{3}$ $\dfrac{x+34}{(x-6)(x+2)} = \dfrac{A}{x-6} + \dfrac{B}{x+2}$

$x + 34 = A(x+2) + B(x-6)$

$x = -2$: $32 = -8B \ \Rightarrow\ B = -4$ $\qquad \star\ \dfrac{5}{x-6} - \dfrac{4}{x+2}$

$x = 6$: $40 = 8A \ \Rightarrow\ A = 5$

$\boxed{4}$ $\dfrac{5x-12}{x(x-4)} = \dfrac{A}{x} + \dfrac{B}{x-4}$

$5x - 12 = A(x-4) + Bx$

$x = 4$: $8 = 4B \ \Rightarrow\ B = 2$ $\qquad \star\ \frac{3}{x} + \dfrac{2}{x-4}$

$x = 0$: $-12 = -4A \ \Rightarrow\ A = 3$

$\boxed{5}$ $\dfrac{4x^2-15x-1}{(x-1)(x+2)(x-3)} = \dfrac{A}{x-1} + \dfrac{B}{x+2} + \dfrac{C}{x-3}$

$4x^2 - 15x - 1 = A(x+2)(x-3) + B(x-1)(x-3) + C(x-1)(x+2)$

$x = -2$: $45 = 15B \ \Rightarrow\ B = 3$

$x = 1$: $-12 = -6A \ \Rightarrow\ A = 2$ $\qquad \star\ \dfrac{2}{x-1} + \dfrac{3}{x+2} - \dfrac{1}{x-3}$

$x = 3$: $-10 = 10C \ \Rightarrow\ C = -1$

$\boxed{6}$ $\dfrac{x^2+19x+20}{x(x+2)(x-5)} = \dfrac{A}{x} + \dfrac{B}{x+2} + \dfrac{C}{x-5}$

$x^2 + 19x + 20 = A(x+2)(x-5) + Bx(x-5) + Cx(x+2)$

$x = -2$: $-14 = 14B \ \Rightarrow\ B = -1$

$x = 5$: $140 = 35C \ \Rightarrow\ C = 4$ $\qquad \star\ -\frac{2}{x} - \dfrac{1}{x+2} + \dfrac{4}{x-5}$

$x = 0$: $20 = -10A \ \Rightarrow\ A = -2$

[7] $\dfrac{4x^2-5x-15}{x(x-5)(x+1)} = \dfrac{A}{x} + \dfrac{B}{x-5} + \dfrac{C}{x+1}$

$4x^2-5x-15 = A(x-5)(x+1) + Bx(x+1) + Cx(x-5)$

$x=-1$: $-6 = 6C \Rightarrow C = -1$

$x=0$: $-15 = -5A \Rightarrow A = 3$

$x=5$: $60 = 30B \Rightarrow B = 2$

★ $\frac{3}{x} + \dfrac{2}{x-5} - \dfrac{1}{x+1}$

[8] $\dfrac{37-11x}{(x+1)(x-2)(x-3)} = \dfrac{A}{x+1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

$37-11x = A(x-2)(x-3) + B(x+1)(x-3) + C(x+1)(x-2)$

$x=-1$: $48 = 12A \Rightarrow A = 4$

$x=2$: $15 = -3B \Rightarrow B = -5$

$x=3$: $4 = 4C \Rightarrow C = 1$

★ $\dfrac{4}{x+1} - \dfrac{5}{x-2} + \dfrac{1}{x-3}$

[9] $\dfrac{2x+3}{(x-1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2}$

$2x+3 = A(x-1) + B$

$x=1$: $5 = B$

$x=0$: $3 = -A + B \Rightarrow A = 2$

★ $\dfrac{2}{x-1} + \dfrac{5}{(x-1)^2}$

[10] $\dfrac{5x^2-4}{x^2(x+2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x+2}$

$5x^2-4 = Ax(x+2) + B(x+2) + Cx^2$

$x=-2$: $16 = 4C \Rightarrow C = 4$

$x=0$: $-4 = 2B \Rightarrow B = -2$

$x=1$: $1 = 3A + 3B + C \Rightarrow A = 1$

★ $\frac{1}{x} - \dfrac{2}{x^2} + \dfrac{4}{x+2}$

[11] $\dfrac{19x^2+50x-25}{x^2(3x-5)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{3x-5}$

$19x^2+50x-25 = Ax(3x-5) + B(3x-5) + Cx^2$

$x=\frac{5}{3}$: $\frac{1000}{9} = \frac{25}{9}C \Rightarrow C = 40$

$x=0$: $-25 = -5B \Rightarrow B = 5$

$x=1$: $44 = -2A - 2B + C \Rightarrow A = -7$

★ $-\frac{7}{x} + \dfrac{5}{x^2} + \dfrac{40}{3x-5}$

[12] $\dfrac{10-x}{(x+5)^2} = \dfrac{A}{x+5} + \dfrac{B}{(x+5)^2}$

$10-x = A(x+5) + B$

$x=-5$: $15 = B$

$x=0$: $10 = 5A + B \Rightarrow A = -1$

★ $-\dfrac{1}{x+5} + \dfrac{15}{(x+5)^2}$

$\boxed{13}$ $\dfrac{x^2-6}{(x+2)^2(2x-1)} = \dfrac{A}{x+2} + \dfrac{B}{(x+2)^2} + \dfrac{C}{2x-1}$

$x^2 - 6 = A(x+2)(2x-1) + B(2x-1) + C(x+2)^2$

$x = -2$: $-2 = -5B \;\Rightarrow\; B = \frac{2}{5}$

$x = \frac{1}{2}$: $-\frac{23}{4} = \frac{25}{4}C \;\Rightarrow\; C = -\frac{23}{25}$

$x = 1$: $-5 = 3A + B + 9C \;\Rightarrow\; A = \frac{24}{25}$

★ $\dfrac{\frac{24}{25}}{x+2} + \dfrac{\frac{2}{5}}{(x+2)^2} - \dfrac{\frac{23}{25}}{2x-1}$

$\boxed{14}$ $\dfrac{2x^2+x}{(x-1)^2(x+1)^2} = \dfrac{A}{x-1} + \dfrac{B}{(x-1)^2} + \dfrac{C}{x+1} + \dfrac{D}{(x+1)^2}$

$2x^2 + x = A(x-1)(x+1)^2 + B(x+1)^2 + C(x+1)(x-1)^2 + D(x-1)^2$

$x = -1$: $1 = 4D \;\Rightarrow\; D = \frac{1}{4}$

$x = 1$: $3 = 4B \;\Rightarrow\; B = \frac{3}{4}$

$x = 0$: $0 = -A + B + C + D \quad (\mathrm{E}_1)$

$x = 2$: $10 = 9A + 9B + 3C + D \quad (\mathrm{E}_2)$

Substituting the values for B and D into E_1 and E_2 yields

$$\begin{cases} -A + C &= -1 \\ 9A + 3C &= 3 \end{cases} \quad\Rightarrow\quad \begin{cases} A - C &= 1 \quad (\mathrm{E}_3) \\ 3A + C &= 1 \quad (\mathrm{E}_4) \end{cases}$$

★ $\dfrac{\frac{1}{2}}{x-1} + \dfrac{\frac{3}{4}}{(x-1)^2} - \dfrac{\frac{1}{2}}{x+1} + \dfrac{\frac{1}{4}}{(x+1)^2}$

$\mathrm{E}_3 + \mathrm{E}_4 \;\Rightarrow\; 4A = 2 \;\Rightarrow\; A = \frac{1}{2};\; C = -\frac{1}{2}$

$\boxed{15}$ $\dfrac{3x^3+11x^2+16x+5}{x(x+1)^3} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{(x+1)^3}$

$3x^3 + 11x^2 + 16x + 5 = A(x+1)^3 + Bx(x+1)^2 + Cx(x+1) + Dx$

$x = -1$: $-3 = -D \;\Rightarrow\; D = 3$

$x = 0$: $5 = A$

$x = 1$: $35 = 8A + 4B + 2C + D \quad (\mathrm{E}_1)$

$x = -2$: $-7 = -A - 2B + 2C - 2D \quad (\mathrm{E}_2)$

Substituting the values for A and D into E_1 and E_2 yields

$$\begin{cases} 4B + 2C &= -8 \\ -2B + 2C &= 4 \end{cases} \quad\Rightarrow\quad \begin{cases} 2B + C &= -4 \quad (\mathrm{E}_3) \\ -B + C &= 2 \quad (\mathrm{E}_4) \end{cases}$$

★ $\dfrac{5}{x} - \dfrac{2}{x+1} + \dfrac{3}{(x+1)^3}$

$\mathrm{E}_3 + 2\,\mathrm{E}_4 \;\Rightarrow\; 3C = 0 \;\Rightarrow\; C = 0;\; B = -2$

$\boxed{16}$ $\dfrac{4x^3+3x^2+5x-2}{x^3(x+2)} = \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x^3} + \dfrac{D}{x+2}$

$4x^3+3x^2+5x-2 = Ax^2(x+2) + Bx(x+2) + C(x+2) + Dx^3$

$x=-2$: $-32 = -8D \ \Rightarrow\ D = 4$

$x=0$: $-2 = 2C \ \Rightarrow\ C = -1$

$x=-1$: $-8 = A - B + C - D \quad (\mathrm{E}_1)$

$x=1$: $10 = 3A + 3B + 3C + D \quad (\mathrm{E}_2)$

Substituting the values for C and D into E_1 and E_2 yields

$\begin{cases} A - B &= -3 \\ 3A + 3B &= 9 \end{cases} \Rightarrow \begin{cases} A - B &= -3 \quad (\mathrm{E}_3) \\ A + B &= 3 \quad (\mathrm{E}_4) \end{cases}$ ★ $\dfrac{3}{x^2} - \dfrac{1}{x^3} + \dfrac{4}{x+2}$

$\mathrm{E}_3 + \mathrm{E}_4 \ \Rightarrow\ 2A = 0 \ \Rightarrow\ A = 0;\ B = 3$

$\boxed{17}$ $\dfrac{x^2+x-6}{(x^2+1)(x-1)} = \dfrac{Ax+B}{x^2+1} + \dfrac{C}{x-1}$

$x^2+x-6 = (Ax+B)(x-1) + C(x^2+1)$

$x=1$: $-4 = 2C \ \Rightarrow\ C = -2$

$x=0$: $-6 = -B + C \ \Rightarrow\ B = 4$ ★ $-\dfrac{2}{x-1} + \dfrac{3x+4}{x^2+1}$

$x=2$: $0 = 2A + B + 5C \ \Rightarrow\ A = 3$

$\boxed{18}$ $\dfrac{x^2-x-21}{(x^2+4)(2x-1)} = \dfrac{Ax+B}{x^2+4} + \dfrac{C}{2x-1}$

$x^2-x-21 = (Ax+B)(2x-1) + C(x^2+4)$

$x=\frac{1}{2}$: $-\frac{85}{4} = \frac{17}{4}C \ \Rightarrow\ C = -5$

$x=0$: $-21 = -B + 4C \ \Rightarrow\ B = 1$ ★ $-\dfrac{5}{2x-1} + \dfrac{3x+1}{x^2+4}$

$x=1$: $-21 = A + B + 5C \ \Rightarrow\ A = 3$

$\boxed{19}$ $\dfrac{9x^2-3x+8}{x(x^2+2)} = \dfrac{A}{x} + \dfrac{Bx+C}{x^2+2}$

$9x^2-3x+8 = A(x^2+2) + (Bx+C)x$

$x=0$: $8 = 2A \ \Rightarrow\ A = 4$

$x=1$: $14 = 3A + B + C \quad (\mathrm{E}_1)$

$x=-1$: $20 = 3A + B - C \quad (\mathrm{E}_2)$ ★ $\frac{4}{x} + \dfrac{5x-3}{x^2+2}$

$\mathrm{E}_1 - \mathrm{E}_2 \ \Rightarrow\ -6 = 2C \ \Rightarrow\ C = -3;\ B = 5$

20 $\dfrac{2x^3+2x^2+4x-3}{x^2(x^2+1)}=\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+1}$

$$\begin{aligned}2x^3+2x^2+4x-3 &= Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2\\ &= (A+C)x^3+(B+D)x^2+Ax+B\end{aligned}$$

Equating coefficients, we have the following:

constant : $B=-3$

x : $A=4$

x^2 : $B+D=2 \Rightarrow D=5$

x^3 : $A+C=2 \Rightarrow C=-2$

★ $\dfrac{4}{x}-\dfrac{3}{x^2}+\dfrac{-2x+5}{x^2+1}$

21 $\dfrac{4x^3-x^2+4x+2}{(x^2+1)^2}=\dfrac{Ax+B}{x^2+1}+\dfrac{Cx+D}{(x^2+1)^2}$

$$\begin{aligned}4x^3-x^2+4x+2 &= (Ax+B)(x^2+1)+Cx+D\\ &= Ax^3+Bx^2+(A+C)x+(B+D)\end{aligned}$$

Equating coefficients, we have the following:

x^3 : $A=4$

x^2 : $B=-1$

x : $A+C=4 \Rightarrow C=0$

constant : $B+D=2 \Rightarrow D=3$

★ $\dfrac{4x-1}{x^2+1}+\dfrac{3}{(x^2+1)^2}$

22 $\dfrac{3x^3+13x-1}{(x^2+4)^2}=\dfrac{Ax+B}{x^2+4}+\dfrac{Cx+D}{(x^2+4)^2}$

$$\begin{aligned}3x^3+13x-1 &= (Ax+B)(x^2+4)+Cx+D\\ &= Ax^3+Bx^2+(4A+C)x+(4B+D)\end{aligned}$$

Equating coefficients, we have the following:

x^3 : $A=3$

x^2 : $B=0$

x : $4A+C=13 \Rightarrow C=1$

constant : $4B+D=-1 \Rightarrow D=-1$

★ $\dfrac{3x}{x^2+4}+\dfrac{x-1}{(x^2+4)^2}$

23 By first dividing and then factoring, we have the following:

$$2x+\frac{4x^2-3x+1}{(x^2+1)(x-1)}=2x+\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$$

$4x^2-3x+1=(Ax+B)(x-1)+C(x^2+1)$

$x=1$: $2=2C \Rightarrow C=1$

$x=0$: $1=-B+C \Rightarrow B=0$

$x=-1$: $8=2A-2B+2C \Rightarrow A=3$

★ $2x+\dfrac{1}{x-1}+\dfrac{3x}{x^2+1}$

[24] By first dividing and then factoring, we have the following:

$$1+\frac{3x^2-9x+27}{(x^2+9)(x-3)}=1+\frac{Ax+B}{x^2+9}+\frac{C}{x-3}$$

$3x^2-9x+27=(Ax+B)(x-3)+C(x^2+9)$

$x=3$: $27=18C \Rightarrow C=\frac{3}{2}$

$x=0$: $27=-3B+9C \Rightarrow B=-\frac{9}{2}$

$x=2$: $21=-2A-B+13C \Rightarrow A=\frac{3}{2}$

★ $1+\dfrac{\frac{3}{2}}{x-3}+\dfrac{\frac{3}{2}x-\frac{9}{2}}{x^2+9}$

[25] By first dividing and then factoring, we have the following:

$$3+\frac{12x-16}{x(x-4)}=3+\frac{A}{x}+\frac{B}{x-4}$$

$12x-16=A(x-4)+Bx$

$x=0$: $-16=-4A \Rightarrow A=4$

$x=4$: $32=4B \Rightarrow B=8$

★ $3+\frac{4}{x}+\dfrac{8}{x-4}$

[26] By first dividing and then factoring, we have the following:

$$2+\frac{-5x-18}{(x+3)^2}=2+\frac{A}{x+3}+\frac{B}{(x+3)^2}$$

$-5x-18=A(x+3)+B$

$x=-3$: $-3=B \Rightarrow B=-3$

Equating coefficients of x, we see that $A=-5$.

★ $2-\dfrac{5}{x+3}-\dfrac{3}{(x+3)^2}$

[27] By first dividing and then factoring, we have the following:

$$2x+3+\frac{x+5}{(2x+1)(x-1)}=2x+3+\frac{A}{2x+1}+\frac{B}{x-1}$$

$x+5=A(x-1)+B(2x+1)$

$x=1$: $6=3B \Rightarrow B=2$

$x=-\frac{1}{2}$: $\frac{9}{2}=-\frac{3}{2}A \Rightarrow A=-3$

★ $2x+3+\dfrac{2}{x-1}-\dfrac{3}{2x+1}$

[28] By first dividing and then factoring, we have the following:

$$x^2-2x+1+\frac{2x^2-4x+12}{x^2(x-3)}=x^2-2x+1+\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-3}$$

$2x^2-4x+12=Ax(x-3)+B(x-3)+Cx^2$

$x=3$: $18=9C \Rightarrow C=2$

$x=0$: $12=-3B \Rightarrow B=-4$

$x=1$: $10=-2A-2B+C \Rightarrow A=0$

★ $x^2-2x+1-\frac{4}{x^2}+\dfrac{2}{x-3}$

Chapter 9 Review Exercises

$\boxed{1}$ $4\,E_1 + 3\,E_2 \;\Rightarrow\; 23x = 19 \;\Rightarrow\; x = \frac{19}{23}$; $-5\,E_1 + 2\,E_2 \;\Rightarrow\; 23y = -18 \;\Rightarrow\; y = -\frac{18}{23}$

★ $(\frac{19}{23}, -\frac{18}{23})$

$\boxed{2}$ $2\,E_1 + E_2 \;\Rightarrow\; 0 = 10$; *no solution.*

$\boxed{3}$ Solve E_2 for y $\{y = -2x - 1\}$ and substitute into E_1 to yield $x^2 + 2x - 3 = 0 \;\Rightarrow$ $(x+3)(x-1) = 0 \;\Rightarrow\; x = -3, 1$ and $y = 5, -3$. ★ $(-3, 5), (1, -3)$

$\boxed{4}$ Solve E_2 for x $\{x = y + 7\}$ and substitute into E_1 to yield $y^2 + 7y + 12 = 0 \;\Rightarrow$ $(y+3)(y+4) = 0 \;\Rightarrow\; y = -3, -4$ and $x = 4, 3$. ★ $(4, -3), (3, -4)$

$\boxed{5}$ $4\,E_2 + E_1 \;\Rightarrow\; 13x^2 = 156 \;\Rightarrow\; x = \pm\, 2\sqrt{3}$ and $y = \pm\sqrt{2}$.

There are four solutions: $(2\sqrt{3}, \pm\sqrt{2}), (-2\sqrt{3}, \pm\sqrt{2})$.

$\boxed{6}$ From E_3, $x^2 - xz = 0 \;\Rightarrow\; x(x - z) = 0 \;\Rightarrow\; x = 0$ or $x = z$.

If $x = 0$, E_1 is $0 = y^2 + 3z$ and E_2 is $z = 1 - y^2$. Substituting z into E_1 yields

$2y^2 = 3$ or $y = \pm\frac{1}{2}\sqrt{6}$ and z is $-\frac{1}{2}$ for both values of y.

If $x = z$, E_1 is $0 = y^2 + z$ and E_2 is $y^2 = 1$.

Thus, $y = \pm 1$ and in either case, $x = z = -1$.

There are four solutions: $(-1, \pm 1, -1), (0, \pm\frac{1}{2}\sqrt{6}, -\frac{1}{2})$.

$\boxed{7}$ $-4\,E_1 + E_2 \;\Rightarrow\; -14/y = -27 \;\Rightarrow\; y = \frac{14}{27}$; $2\,E_1 + 3\,E_2 \;\Rightarrow\; 14/x = 17 \;\Rightarrow\; x = \frac{14}{17}$;

★ $(\frac{14}{17}, \frac{14}{27})$

$\boxed{8}$ Treat 3^{y+1} as $3 \cdot 3^y$ and 2^{x+1} as $2 \cdot 2^x$. Now $E_1 + 3\,E_2 \;\Rightarrow\; 7 \cdot 2^x = 25 \;\Rightarrow$ $2^x = \frac{25}{7} \;\Rightarrow\; x = \log_2 \frac{25}{7}$. Resolving the original system for y;

$-2\,E_1 + E_2 \;\Rightarrow\; -7 \cdot 3^y = -15 \;\Rightarrow\; 3^y = \frac{15}{7} \;\Rightarrow\; y = \log_3 \frac{15}{7}$. ★ $(\log_2 \frac{25}{7}, \log_3 \frac{15}{7})$

$\boxed{9}$ Solve E_3 for z and substitute into E_1 and E_2 to yield

$$\begin{cases} 3x + y - 2(4x + 5y + 2) = -1 \\ 2x - 3y + (4x + 5y + 2) = 4 \end{cases} \;\Rightarrow\; \begin{cases} -5x - 9y = 3 & (E_4) \\ 6x + 2y = 2 & (E_5) \end{cases}$$

$6\,E_4 + 5\,E_5 \;\Rightarrow\; -44y = 28 \;\Rightarrow\; y = -\frac{7}{11}$;

$2\,E_4 + 9\,E_5 \;\Rightarrow\; 44x = 24 \;\Rightarrow\; x = \frac{6}{11}$; $z = 1$ ★ $(\frac{6}{11}, -\frac{7}{11}, 1)$

$\boxed{10}$ Solve E_1 for x $\{x = -3y\}$ and substitute into E_3 to yield

$$\begin{cases} y - 5z = 3 & (E_2) \\ -6y + z = -1 & (E_4) \end{cases}$$

$6\,E_2 + E_4 \;\Rightarrow\; -29z = 17 \;\Rightarrow\; z = -\frac{17}{29}$;

$E_2 + 5\,E_4 \;\Rightarrow\; -29y = -2 \;\Rightarrow\; y = \frac{2}{29}$; $x = -\frac{6}{29}$ ★ $(-\frac{6}{29}, \frac{2}{29}, -\frac{17}{29})$

[11] Solve E_2 for x and substitute into E_1 and E_3 to yield

$$\begin{cases} 4(y+z) - 3y - z = 0 \\ 3(y+z) - y + 3z = 0 \end{cases} \Rightarrow \begin{cases} y + 3z = 0 & (E_4) \\ 2y + 6z = 0 & (E_5) \end{cases}$$

Now E_5 is $2\,E_4$, hence $y = -3z$ and $x = y + z = -3z + z = -2z$.

The general solution is $(-2c, -3c, c)$ for any real number c.

[12] Solve E_2 for x and substitute into E_1 and E_3 to yield

$$\begin{cases} 2(2y-z) + y - z = 0 \\ 3(2y-z) + 3y + 2z = 0 \end{cases} \Rightarrow \begin{cases} 5y - 3z = 0 & (E_4) \\ 9y - z = 0 & (E_5) \end{cases}$$

$E_4 - 3\,E_5 \;\Rightarrow\; -22y = 0 \;\Rightarrow\; y = 0,\ z = 0,\ x = 0;$ ★ $(0, 0, 0)$

[13] $E_1 - E_2 \;\Rightarrow\; x - 5z = -1 \;\Rightarrow\; x = 5z - 1;$

Substitute this value into E_1 to obtain $y = \dfrac{-19z+5}{2}$; $\left(5c - 1, \dfrac{-19c+5}{2}, c\right)$ is

the general solution, where c is any real number.

[14] $3\,E_1 + E_2 \;\Rightarrow\; 7x = 35 \;\Rightarrow\; x = 5$ and $y = -4$. This solution also satisfies E_3.

★ $(5, -4)$

[15] Let $a = 1/x$, $b = 1/y$, and $c = 1/z$ to obtain the system

$$\begin{cases} 4a + b + 2c = 4 & (E_1) \\ 2a + 3b - c = 1 & (E_2) \\ a + b + c = 4 & (E_3) \end{cases}$$

Solving E_2 for c and substituting into E_1 and E_3 yields

$$\begin{cases} 4a + b + 2(2a + 3b - 1) = 4 \\ a + b + (2a + 3b - 1) = 4 \end{cases} \Rightarrow \begin{cases} 8a + 7b = 6 & (E_4) \\ 3a + 4b = 5 & (E_5) \end{cases}$$

$3\,E_4 - 8\,E_5 \;\Rightarrow\; -11b = -22 \;\Rightarrow\; b = 2$

$4\,E_4 - 7\,E_5 \;\Rightarrow\; 11a = -11 \;\Rightarrow\; a = -1,\ c = 3;\ (x, y, z) = (-1, \tfrac{1}{2}, \tfrac{1}{3})$

[16] Solve E_1 for w and substitute into E_2, E_3, and E_4 to obtain

$$\begin{cases} 5x + y + 2z = 10 & (E_5) \\ -3x - y - 5z = 2 & (E_6) \\ 5x - 2y + 13z = -9 & (E_7) \end{cases}$$

Solve E_6 for y and substitute into E_5 and E_7 to obtain

$$\begin{cases} 2x - 3z = 12 & (E_8) \\ 11x + 23z = -13 & (E_9) \end{cases}$$

$11\,E_8 - 2\,E_9 \;\Rightarrow\; -79z = 158 \;\Rightarrow\; z = -2$

$23\,E_8 + 3\,E_9 \;\Rightarrow\; 79x = 237 \;\Rightarrow\; x = 3,\ y = -1,\ w = 4$ ★ $(3, -1, -2, 4)$

[17] $\begin{cases} x^2+y^2<16 \\ y-x^2>0 \end{cases} \Leftrightarrow \begin{cases} x^2+y^2<4^2 \\ y>x^2 \end{cases}$

V @ $\left(\pm\sqrt{-\frac{1}{2}+\frac{1}{2}\sqrt{65}},\ -\frac{1}{2}+\frac{1}{2}\sqrt{65}\right) \approx (\pm 1.88,\ 3.53)$

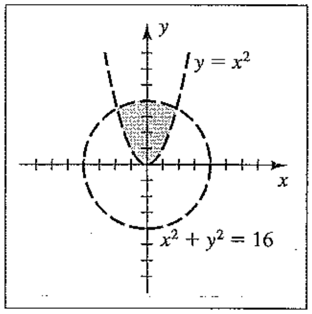

Figure 17

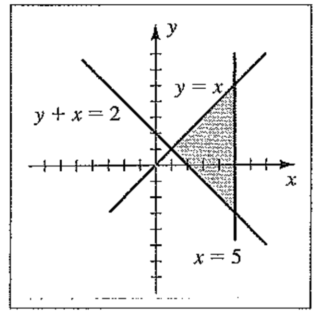

Figure 18

[18] $\begin{cases} y-x\le 0 \\ y+x\ge 2 \\ x\le 5 \end{cases} \Leftrightarrow \begin{cases} y\le x \\ y\ge -x+2 \\ x\le 5 \end{cases}$ V @ $(1,\ 1),\ (5,\ 5),\ (5,\ -3)$

[19] $\begin{cases} x-2y\le 2 \\ y-3x\le 4 \\ 2x+y\le 4 \end{cases} \Leftrightarrow \begin{cases} y\ge \frac{1}{2}x-1 \\ y\le 3x+4 \\ y\le -2x+4 \end{cases}$ V @ $(-2,\ -2),\ (0,\ 4),\ (2,\ 0)$

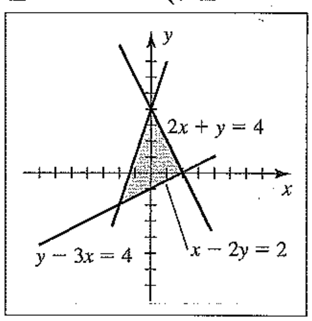

Figure 19

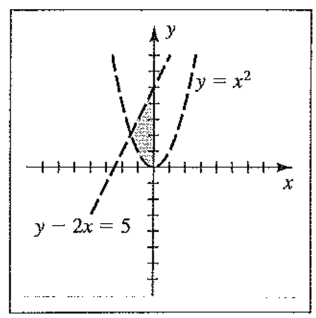

Figure 20

[20] $\begin{cases} x^2-y<0 \\ y-2x<5 \\ xy<0 \end{cases} \Leftrightarrow \begin{cases} y>x^2 \\ y<2x+5 \\ xy<0 \end{cases}$

V @ $(1-\sqrt{6},\ 7-2\sqrt{6}) \approx (-1.45,\ 2.10),\ (0,\ 5),\ (0,\ 0)$

[21] $\begin{bmatrix} 2 & -1 & 0 \\ 3 & 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ 0 & 3 & 0 \\ 1 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -5 & 6 \\ 4 & -11 & 5 \end{bmatrix}$

[22] $\begin{bmatrix} 4 & 2 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 26 \\ -6 \end{bmatrix}$

[23] $\begin{bmatrix} 2 & 0 \\ 1 & 4 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 & -3 \\ 4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 & -6 \\ 16 & 22 & 1 \\ 12 & 11 & 9 \end{bmatrix}$

24 $\begin{bmatrix} 0 & -2 & 3 \\ 4 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 3 & 8 \\ 2 & -7 \end{bmatrix} = \begin{bmatrix} 0 & -37 \\ 15 & -6 \end{bmatrix}$

25 $2\begin{bmatrix} 0 & -1 & -4 \\ 3 & 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 4 & -2 & 1 \\ 0 & 5 & -1 \end{bmatrix} =$

$$\begin{bmatrix} 0 & -2 & -8 \\ 6 & 4 & 2 \end{bmatrix} + \begin{bmatrix} -12 & 6 & -3 \\ 0 & -15 & 3 \end{bmatrix} = \begin{bmatrix} -12 & 4 & -11 \\ 6 & -11 & 5 \end{bmatrix}$$

26 $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix} = \begin{bmatrix} a & 3a \\ 2a & 4a \end{bmatrix}$

27 $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} a & 3a \\ 2b & 4b \end{bmatrix}$

28 $\begin{bmatrix} 3 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

29 $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\left\{\begin{bmatrix} 2 & -4 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 5 \\ -2 & -3 \end{bmatrix}\right\} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 9 \\ 13 & 19 \end{bmatrix}$

30 $AA^{-1} = I_3$

Note: Let A denote each of the matrices in Exercises 31–46.

31 $\left[\begin{array}{cc|cc} 5 & -4 & 1 & 0 \\ -3 & 2 & 0 & 1 \end{array}\right] 2\mathrm{R}_1 + 3\mathrm{R}_2 \rightarrow \mathrm{R}_1$

$\left[\begin{array}{cc|cc} 1 & -2 & 2 & 3 \\ -3 & 2 & 0 & 1 \end{array}\right] \mathrm{R}_2 + 3\mathrm{R}_1 \rightarrow \mathrm{R}_2$

$\left[\begin{array}{cc|cc} 1 & -2 & 2 & 3 \\ 0 & -4 & 6 & 10 \end{array}\right] -\tfrac{1}{4}\mathrm{R}_2 \rightarrow \mathrm{R}_2$

$\left[\begin{array}{cc|cc} 1 & -2 & 2 & 3 \\ 0 & 1 & -\tfrac{3}{2} & -\tfrac{5}{2} \end{array}\right] \mathrm{R}_1 + 2\mathrm{R}_2 \rightarrow \mathrm{R}_1$

$\left[\begin{array}{cc|cc} 1 & 0 & -1 & -2 \\ 0 & 1 & -\tfrac{3}{2} & -\tfrac{5}{2} \end{array}\right]$ $\qquad A^{-1} = -\tfrac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

32 $\left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 4 & 2 & 0 & 1 & 0 \\ 3 & -2 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \mathrm{R}_1 - 2\mathrm{R}_2 \leftrightarrow \mathrm{R}_2 \\ \\ \mathrm{R}_3 - 3\mathrm{R}_2 \rightarrow \mathrm{R}_3 \end{array}$

$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 0 & 1 & 0 \\ 0 & -9 & -4 & 1 & -2 & 0 \\ 0 & -14 & -5 & 0 & -3 & 1 \end{array}\right] 3\mathrm{R}_2 - 2\mathrm{R}_3 \rightarrow \mathrm{R}_2$

$\left[\begin{array}{ccc|ccc} 1 & 4 & 2 & 0 & 1 & 0 \\ 0 & 1 & -2 & 3 & 0 & -2 \\ 0 & -14 & -5 & 0 & -3 & 1 \end{array}\right] \begin{array}{l} \mathrm{R}_1 - 4\mathrm{R}_2 \rightarrow \mathrm{R}_1 \\ \\ \mathrm{R}_3 + 14\mathrm{R}_2 \rightarrow \mathrm{R}_3 \end{array}$

(continued)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 10 & -12 & 1 & 8 \\ 0 & 1 & -2 & 3 & 0 & -2 \\ 0 & 0 & -33 & 42 & -3 & -27 \end{array}\right] -\tfrac{1}{33}R_3 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 10 & -12 & 1 & 8 \\ 0 & 1 & -2 & 3 & 0 & -2 \\ 0 & 0 & 1 & -\frac{14}{11} & \frac{1}{11} & \frac{9}{11} \end{array}\right] \begin{array}{l} R_1 - 10R_3 \to R_1 \\ R_2 + 2R_3 \to R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{8}{11} & \frac{1}{11} & -\frac{2}{11} \\ 0 & 1 & 0 & \frac{5}{11} & \frac{2}{11} & -\frac{4}{11} \\ 0 & 0 & 1 & -\frac{14}{11} & \frac{1}{11} & \frac{9}{11} \end{array}\right] \qquad A^{-1} = \frac{1}{11}\begin{bmatrix} 8 & 1 & -2 \\ 5 & 2 & -4 \\ -14 & 1 & 9 \end{bmatrix}$$

33 $$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 4 & 7 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] R_3 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 4 & 7 & 0 & 1 & 0 \end{array}\right] R_3 - 4R_2 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & -4 \end{array}\right] -R_3 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 4 \end{array}\right] R_2 - 2R_3 \to R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & -7 \\ 0 & 0 & 1 & 0 & -1 & 4 \end{array}\right] \qquad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & -1 & 4 \end{bmatrix}$$

34 $$\left[\begin{array}{ccc|ccc} 2 & 0 & 5 & 1 & 0 & 0 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 & 0 & 1 \end{array}\right] R_3 - R_1 \leftrightarrow R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & -5 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 2 & 0 & 5 & 1 & 0 & 0 \end{array}\right] R_3 - 2R_1 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 4 & -5 & -1 & 0 & 1 \\ 0 & 3 & -1 & 0 & 1 & 0 \\ 0 & -8 & 15 & 3 & 0 & -2 \end{array}\right] 3R_2 + R_3 \to R_2$$

(continued)

$$\left[\begin{array}{ccc|ccc} 1 & 4 & -5 & -1 & 0 & 1 \\ 0 & 1 & 12 & 3 & 3 & -2 \\ 0 & -8 & 15 & 3 & 0 & -2 \end{array}\right] \begin{array}{l} R_1 - 4R_2 \to R_1 \\ \\ R_3 + 8R_2 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -53 & -13 & -12 & 9 \\ 0 & 1 & 12 & 3 & 3 & -2 \\ 0 & 0 & 111 & 27 & 24 & -18 \end{array}\right] \tfrac{1}{111}R_3 \to R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -53 & -13 & -12 & 9 \\ 0 & 1 & 12 & 3 & 3 & -2 \\ 0 & 0 & 1 & \frac{9}{37} & \frac{8}{37} & -\frac{6}{37} \end{array}\right] \begin{array}{l} R_1 + 53R_3 \to R_1 \\ R_2 - 12R_3 \to R_2 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{4}{37} & -\frac{20}{37} & \frac{15}{37} \\ 0 & 1 & 0 & \frac{3}{37} & \frac{15}{37} & -\frac{2}{37} \\ 0 & 0 & 1 & \frac{9}{37} & \frac{8}{37} & -\frac{6}{37} \end{array}\right] \qquad A^{-1} = \frac{1}{37}\begin{bmatrix} -4 & -20 & 15 \\ 3 & 15 & -2 \\ 9 & 8 & -6 \end{bmatrix}$$

35 $X = A^{-1}B = -\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 30 \\ -16 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -4 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$; $(x, y) = (2, -5)$

36 $X = A^{-1}B = \frac{1}{11}\begin{bmatrix} 8 & 1 & -2 \\ 5 & 2 & -4 \\ -14 & 1 & 9 \end{bmatrix}\begin{bmatrix} -5 \\ 15 \\ -7 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -11 \\ 33 \\ 22 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$;

$(x, y, z) = (-1, 3, 2)$

37 $A = [\,-6\,] \Rightarrow |A| = -6.$

38 $|A| = -15 - (-24) = 9$

39 $|A| = 24 - (-24) = 48$

40 $\{R_1\}$ $|A| = 0(A_{11}) - 4(20) - 3(2) = -86$

41 $\{R_1\}$ $|A| = 2(-7) + 3(-5) + 5(-11) = -84$

42 $\{R_1\}$ $|A| = 3(0) - 1(58) - 2(-29) = 0$

43 From Exercise 29 of §9.8, $|A| = (5)(-3)(-4)(2) = 120$

44 $$\begin{vmatrix} 1 & 2 & 0 & 3 & 1 \\ -2 & -1 & 4 & 1 & 2 \\ 3 & 0 & -1 & 0 & -1 \\ 2 & -3 & 2 & -4 & 2 \\ -1 & 1 & 0 & 1 & 3 \end{vmatrix} \begin{array}{l} R_2 + 4R_3 \to R_2 \\ \\ R_4 + 2R_3 \to R_4 \end{array} = \begin{vmatrix} 1 & 2 & 0 & 3 & 1 \\ 10 & -1 & 0 & 1 & -2 \\ 3 & 0 & -1 & 0 & -1 \\ 8 & -3 & 0 & -4 & 0 \\ -1 & 1 & 0 & 1 & 3 \end{vmatrix} \{C_3\}$$

$$= (-1)\begin{vmatrix} 1 & 2 & 3 & 1 \\ 10 & -1 & 1 & -2 \\ 8 & -3 & -4 & 0 \\ -1 & 1 & 1 & 3 \end{vmatrix} \begin{array}{l} R_2 + 2R_1 \to R_2 \\ \\ R_4 - 3R_1 \to R_4 \end{array} = (-1)\begin{vmatrix} 1 & 2 & 3 & 1 \\ 12 & 3 & 7 & 0 \\ 8 & -3 & -4 & 0 \\ -4 & -5 & -8 & 0 \end{vmatrix} \{C_4\}$$

$$= (-1)(-1)\begin{vmatrix} 12 & 3 & 7 \\ 8 & -3 & -4 \\ -4 & -5 & -8 \end{vmatrix} \{4 \text{ is a common factor of } C_1 \text{ and } -1 \text{ of } R_3\}$$ (cont.)

$$= (-4)\begin{vmatrix} 3 & 3 & 7 \\ 2 & -3 & -4 \\ 1 & 5 & 8 \end{vmatrix} \begin{matrix} \mathrm{R}_1 - 3\,\mathrm{R}_3 \to \mathrm{R}_1 \\ \mathrm{R}_2 - 2\,\mathrm{R}_3 \to \mathrm{R}_2 \\ \end{matrix} = (-4)\begin{vmatrix} 0 & -12 & -17 \\ 0 & -13 & -20 \\ 1 & 5 & 8 \end{vmatrix} \{\mathrm{C}_1\}$$

$$= (-4)\begin{vmatrix} -12 & -17 \\ -13 & -20 \end{vmatrix} = (-4)(240 - 221) = -76$$

45 C_2 and C_4 are equal, so $|A| = 0$.

46 As in Exercise 28 of §9.9, $|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}\begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix}$.

Expand the 3×3 by R_1.

$|A| = (-2)\big[1(-2) - 2(-3) + 3(7)\big] = -50$

47 $\begin{vmatrix} 2-x & 3 \\ 1 & -4-x \end{vmatrix} = 0 \Rightarrow (2-x)(-4-x) - 3 = 0 \Rightarrow$

$$x^2 + 2x - 11 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4+44}}{2} = -1 \pm 2\sqrt{3}$$

48 $\begin{vmatrix} 2-x & -1 & 3 \\ 0 & 4-x & 0 \\ 1 & 0 & -2-x \end{vmatrix} = 0 \Rightarrow \{\mathrm{C}_1\}$

$(2-x)(4-x)(-2-x) - 3(4-x) = 0 \Rightarrow$

$$(4-x)\big[(2-x)(-2-x) - 3\big] = 0 \Rightarrow (4-x)(x^2 - 7) = 0 \Rightarrow x = 4, \pm\sqrt{7}$$

49 2 is a common factor of R_1, 2 is a common factor of C_2,

and 3 is a common factor of C_3.

50 Interchange R_1 with R_2 and then R_2 with R_3 to obtain the determinant on the right.

The effect is to multiply by -1 twice.

51 This is an extension of Exercise 29 of §9.8. Expanding by C_1, only a_{11} is not 0.

Expanding by the new C_1 again, only a_{22} is not 0. Repeating this process yields

$|A| = a_{11}a_{22}a_{33}\cdots a_{nn}$, the product of the main diagonal elements.

52 $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} \mathrm{C}_3 + \mathrm{C}_2 \to \mathrm{C}_2$

$= \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix} \mathrm{C}_3 - (a+b+c)\,\mathrm{C}_1 \to \mathrm{C}_3$

$= \begin{vmatrix} 1 & a & 0 \\ 1 & b & 0 \\ 1 & c & 0 \end{vmatrix} = 0$, since C_3 consists of all zeros.

53 $|D_x| = 76$, $|D_y| = 28$, $|D| = 53$; $x = \frac{76}{53}$, $y = \frac{28}{53}$

54 $|D_x| = -14$, $|D_y| = -31$, $|D_z| = -1$, $|D| = -21$;

$x = \frac{2}{3}$, $y = \frac{31}{21}$, $z = \frac{1}{21}$

[55] $\dfrac{4x^2+54x+134}{(x+3)(x^2+4x-5)} = \dfrac{A}{x+3} + \dfrac{B}{x+5} + \dfrac{C}{x-1}$

$4x^2+54x+134 = A(x+5)(x-1) + B(x+3)(x-1) + C(x+3)(x+5)$

$x = 1$: $192 = 24C \;\Rightarrow\; C = 8$

$x = -3$: $8 = -8A \;\Rightarrow\; A = -1$

$x = -5$: $-36 = 12B \;\Rightarrow\; B = -3$

★ $\dfrac{8}{x-1} - \dfrac{3}{x+5} - \dfrac{1}{x+3}$

[56] By first dividing and then factoring, we have the following:

$\dfrac{2x^2+7x+9}{x^2+2x+1} = 2 + \dfrac{3x+7}{(x+1)^2} = 2 + \dfrac{A}{x+1} + \dfrac{B}{(x+1)^2}$

$3x+7 = A(x+1) + B$

$x = -1$: $4 = B \;\Rightarrow\; B = 4$

$x = 0$: $7 = A + B \;\Rightarrow\; A = 3$

★ $2 + \dfrac{3}{x+1} + \dfrac{4}{(x+1)^2}$

[57] $\dfrac{x^2+14x-13}{x^3+5x^2+4x+20} = \dfrac{A}{x+5} + \dfrac{Bx+C}{x^2+4}$

$x^2+14x-13 = A(x^2+4) + (Bx+C)(x+5)$

$x = -5$: $-58 = 29A \;\Rightarrow\; A = -2$

$x = 0$: $-13 = 4A + 5C \;\Rightarrow\; C = -1$

$x = 1$: $2 = 5A + 6B + 6C \;\Rightarrow\; B = 3$

★ $-\dfrac{2}{x+5} + \dfrac{3x-1}{x^2+4}$

[58] $\dfrac{x^3+2x^2+2x+16}{x^4+7x^2+10} = \dfrac{Ax+B}{x^2+2} + \dfrac{Cx+D}{x^2+5}$

$$\begin{aligned} x^3+2x^2+2x+16 &= (Ax+B)(x^2+5) + (Cx+D)(x^2+2) \\ &= (A+C)x^3 + (B+D)x^2 + (5A+2C)x + (5B+2D) \end{aligned}$$

Equating coefficients, we have the following:

x^3 : $A + C = 1$

x^2 : $B + D = 2$

x : $5A + 2C = 2$ $\{A + C = 1\}$ $\Rightarrow$ $A = 0$ and $C = 1$

constant : $5B + 2D = 16$ $\{B + D = 2\}$ $\Rightarrow$ $3B = 12$ $\Rightarrow$ $B = 4$ and $D = -2$

★ $\dfrac{4}{x^2+2} + \dfrac{x-2}{x^2+5}$

[59] Let x and y denote the length and width, respectively, of the rectangle.

$$\begin{cases} xy = 4000 & \textit{area} \quad (\mathrm{E}_1) \\ x^2 + y^2 = 100^2 & \textit{diagonal} \quad (\mathrm{E}_2) \end{cases}$$

Solve E_1 for y $\{y = 4000/x\}$ and substitute into E_2.

$x^2 + \dfrac{4000^2}{x^2} = 100^2 \;\Rightarrow\; x^4 - 10{,}000x^2 + 16{,}000{,}000 = 0 \;\Rightarrow$

$(x^2 - 2000)(x^2 - 8000) = 0 \;\Rightarrow\; x = 20\sqrt{5},\ 40\sqrt{5}$ and $y = 40\sqrt{5},\ 20\sqrt{5}$.

The dimensions are $20\sqrt{5}$ ft $\times$ $40\sqrt{5}$ ft.

[60] Following the hint, we have $x^2+(mx+3)^2=1 \Rightarrow$

$(m^2+1)x^2+(6m)x+(8)=0 \Rightarrow x=\dfrac{-6m\pm\sqrt{36m^2-32m^2-32}}{2(m^2+1)}.$

If there is to be only one solution to the system, i.e., one point of intersection between the circle and the line, then the discriminant must equal 0.

$4m^2-32=0 \Rightarrow m=\pm 2\sqrt{2}$ and the equations of the lines are $y=\pm 2\sqrt{2}\,x+3$.

[61] Let x and y denote the total amount of taxes paid and bonus money, respectively.

$$\begin{cases} x=0.40(50{,}000-y) & \textit{taxes} \\ y=0.10(50{,}000-x) & \textit{bonuses} \end{cases} \Rightarrow \begin{cases} 10x+4y=200{,}000 & (\mathrm{E}_1) \\ x+10y=50{,}000 & (\mathrm{E}_2) \end{cases}$$

$\mathrm{E}_1-10\,\mathrm{E}_2 \Rightarrow -96y=-300{,}000 \Rightarrow y=\$3{,}125;\ x=\$18{,}750$

[62] Let r_1 and r_2 denote the inside radius and the outside radius, respectively.

Inside distance $=90\%$ (outside distance) $\Rightarrow 2\pi r_1=0.90\,(2\pi r_2) \Rightarrow$

$r_1=0.90\,(r_1+10)$ $\{$since $r_2=r_1+10\}$ $\Rightarrow 0.1r_1=9 \Rightarrow$

$r_1=90$ ft and $r_2=100$ ft

[63] Let x, y, and z denote the number of $\mathrm{ft}^3/\mathrm{hr}$ flowing through pipes A, B, and C, respectively.

$$\begin{cases} 10x+10y+10z=1000 & \textit{all 3 working} \\ 20x+20y=1000 & \text{A } \textit{and} \text{ B } \textit{only} \\ 12.5x+12.5z=1000 & \text{A } \textit{and} \text{ C } \textit{only} \end{cases} \Rightarrow \begin{cases} x+y+z=100 & (\mathrm{E}_1) \\ x+y=50 & (\mathrm{E}_2) \\ x+z=80 & (\mathrm{E}_3) \end{cases}$$

$\mathrm{E}_1-\mathrm{E}_2 \Rightarrow z=50;\ \mathrm{E}_1-\mathrm{E}_3 \Rightarrow y=20$; from E_2, $x=30$.

[64] Let x and y denote the number of desks shipped from the western warehouse and the eastern warehouse, respectively.

$$\begin{cases} x+y=150 & \textit{quantity} \\ 24x+35y=4205 & \textit{price} \end{cases}$$

$\mathrm{E}_2-24\,\mathrm{E}_1 \Rightarrow 11y=605 \Rightarrow y=55;\ x=95$

[65] If x and y denote the length and the width, respectively, then a system is $x\le 12$, $y\le 8$, $y\ge \frac{1}{2}x$. The graph is the region bounded by the quadrilateral with vertices (0, 0), (0, 8), (12, 8), (12, 6).

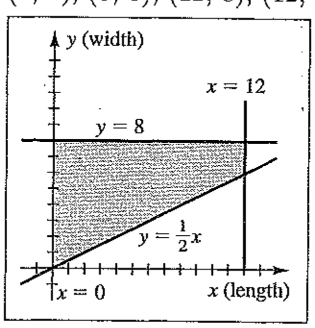

Figure 65

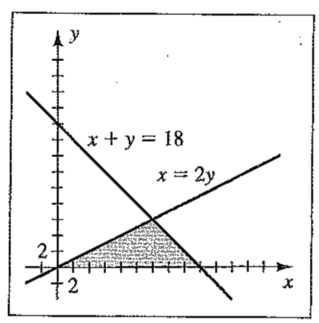

Figure 66

[66] The number of hours spent resting is $24 - x - y$.

A system is $24 - x - y \geq 6$ {hours resting}, $x \geq 2y$, $x \geq 0$, $y \geq 0$.

The first inequality is $x + y \leq 18$. See *Figure 66*.

The graph is the region bounded by the triangle with vertices (0, 0), (18, 0), (12, 6).

[67] Let x and y denote the number of lawn mowers and edgers produced, respectively.

Profit function: $P = 100x + 80y$

$$\begin{cases} 6x + 4y \leq 600 & \textit{machining} \\ 2x + 3y \leq 300 & \textit{welding} \\ 5x + 5y \leq 550 & \textit{assembly} \\ x,\ y \geq 0 & \end{cases}$$

(x, y)	(100, 0)	(80, 30)	(30, 80)	(0, 100)	(0, 0)
P	10,000	10,400 ■	9400	8000	0

The maximum weekly profit of $10,400 occurs when 80 lawn mowers and 30 edgers are produced.

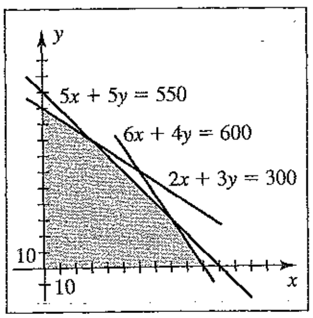

Figure 67

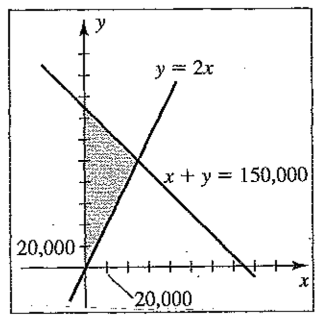

Figure 68

[68] Let x and y denote the amount in the high- and low-risk investments, respectively.

The amount in bonds is $150{,}000 - x - y$.

Profit function: $P = 0.15x + 0.10y + 0.08(150{,}000 - x - y) = \underline{12{,}000 + 0.07x + 0.02y}$

(x, y)	(0, 0)	(0, 150,000)	(50,000, 100,000)
P	12,000	15,000	17,500 ■

$$\begin{cases} x + y \leq 150{,}000 \\ y \geq 2x \\ x,\ y \geq 0 \end{cases}$$

The maximum return of $17,500 occurs when

$50,000 is invested in the high-risk investment,

$100,000 is invested in the low-risk investment, and $0 is invested in bonds.

Chapter 9 Discussion Exercises

[1] (a) For $b = 1.99$, we get $x = 204$ and $y = -100$. For $b = 1.999$, we get $x = 2004$ and $y = -1000$.

(b) Solving $\begin{cases} x + \ 2y = \ 4 \\ x + \ by = \ 5 \end{cases}$ for y gives us $y = \frac{1}{b-2}$ and then solving for x we obtain $x = \frac{4b-10}{b-2}$. Both x and y are rational functions of b and as $b \to 2^-$, $x \to \infty$ and $y \to -\infty$.

(c) If b gets very large, (x, y) gets close to $(4, 0)$.

[2] (a) $D = [\,12{,}000 \quad 9000 \quad 14{,}000\,]$; $E = \begin{bmatrix} 0.90 & 0.10 & 0.00 \\ 0.00 & 0.80 & 0.20 \\ 0.05 & 0.00 & 0.95 \end{bmatrix}$

(b) $F = DE = [\,12{,}000 \quad 9000 \quad 14{,}000\,]\begin{bmatrix} 0.90 & 0.10 & 0.00 \\ 0.00 & 0.80 & 0.20 \\ 0.05 & 0.00 & 0.95 \end{bmatrix} = [\,11{,}500 \quad 8400 \quad 15{,}100\,]$.

The elements of F represent the populations on islands A, B, and C, respectively, after one year.

(c) After repeated multiplications of the population matrix by the proportion matrix, we obtain the matrix $G = [\,10{,}000 \quad 5000 \quad 20{,}000\,]$. Our conclusion is that the population stabilizes with 10,000 birds on A, 5000 birds on B, and 20,000 birds on C.

(d) If we begin with $D = [\,34{,}000 \quad 500 \quad 500\,]$, multiply by E, and continue to multiply the result by E, we eventually get the values in G. Our conclusion is that regardless of the initial population distribution of the 35,000 birds, the populations tend toward the distribution described in part (c).

[3] If we let A be an $m \times n$ matrix $(m \neq n)$, then B would have to be an $n \times m$ matrix so that AB and BA are both defined. But then $AB = I_m$ and $BA = I_n$, different orders of the identity matrix, which we can't have (they must be the same).

[4] $$AX = B \Leftrightarrow \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.2 & 0.5 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.34 \\ 0.33 \\ 0.33 \end{bmatrix} \Rightarrow X = A^{-1}B = \begin{bmatrix} 0.35 \\ 0.3\overline{3} \\ 0.31\overline{6} \end{bmatrix}.$$

Give 35% to the AD, $33\frac{1}{3}$% to the DS, and $31\frac{2}{3}$% to the SP.

[5] Synthetically dividing x^4+ax^2+bx+c by $x+1$, $x-2$, and $x-3$ yields the remainders $a-b+c+1$, $4a+2b+c+16$, and $9a+3b+c+81$, respectively. Setting each of the remainders equal to 0 gives us the following system of equations in matrix form.

$$AX=B \quad\Leftrightarrow\quad \begin{bmatrix} 1 & -1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}=\begin{bmatrix} -1 \\ -16 \\ -81 \end{bmatrix} \quad\Rightarrow\quad X=A^{-1}B=\begin{bmatrix} -15 \\ 10 \\ 24 \end{bmatrix}.$$

Hence, $a=-15$, $b=10$, $c=24$, and we graph $Y_1=x^4-15x^2+10x+24$.

The roots of Y_1 are -1, 2, 3, and -4.

[6] With $y=ax^3+bx^2+cx+d$ and the points $(-6,-6)$, $(-4,3)$, $(2,2)$, and $(6,6)$, we have

$$AX=B \quad\Leftrightarrow\quad \begin{bmatrix} -216 & 36 & -6 & 1 \\ 8 & 4 & 2 & 1 \\ 216 & 36 & 6 & 1 \\ -64 & 16 & -4 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}=\begin{bmatrix} -6 \\ 2 \\ 6 \\ 3 \end{bmatrix} \quad\Rightarrow\quad X=A^{-1}B=\begin{bmatrix} 0.058\overline{3} \\ -0.11\overline{6} \\ -1.1 \\ 4.2 \end{bmatrix}$$

So the equation is $y=0.058\overline{3}x^3-0.11\overline{6}x^2-1.1x+4.2$. As y gets large positive, the values of a and d get larger positively and the values of b and c get larger negatively. As y gets large negative, the values of a and d get larger negatively and the values of b and c get larger positively. In either case, changing the y-value for $(-4, y)$ makes the graph appear nearly vertical through the other three points.

[7] Using $a_{11}x+a_{12}y=k_1$ and the value of y that was found, we substitute for y and solve for x. Let $|A|=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ and $|K|=\begin{vmatrix} a_{11} & k_1 \\ a_{21} & k_2 \end{vmatrix}$.

$$a_{11}x+a_{12}y=k_1 \quad\Rightarrow\quad a_{11}x+a_{12}\cdot\frac{|K|}{|A|}=k_1 \quad\Rightarrow$$

$$x=\frac{k_1-a_{12}\cdot\dfrac{|K|}{|A|}}{a_{11}}$$

$$=\frac{|A|\cdot k_1-a_{12}\cdot|K|}{a_{11}\cdot|A|}$$

$$=\frac{(a_{11}a_{22}-a_{12}a_{21})k_1-a_{12}(a_{11}k_2-k_1a_{21})}{a_{11}\cdot|A|}$$

$$=\frac{a_{11}a_{22}k_1-a_{12}a_{21}k_1-a_{12}a_{11}k_2-a_{12}a_{21}k_1}{a_{11}\cdot|A|}$$

(continued)

$$= \frac{a_{11}a_{22}k_1 - a_{12}a_{11}k_2}{a_{11}\cdot|A|}$$

$$= \frac{a_{11}(a_{22}k_1 - a_{12}k_2)}{a_{11}\cdot|A|}$$

$$= \frac{\begin{vmatrix} k_1 & a_{12} \\ k_2 & a_{22} \end{vmatrix}}{|A|}$$, which is what we wanted to show.

$\boxed{8}$ The points are $P(-1,\ 3)$, $Q(0,\ 4)$, and $R(3,\ 2)$.

(a) $m_{PQ} = 1$ and $m_{QR} = -\frac{2}{3}$, so we cannot have a line through P, Q, and R.

(b) The circle has an equation of the form $x^2 + y^2 + ax + by + c = 0$. Substituting the x and y values of $P(-1,\ 3)$, $Q(0,\ 4)$, and $R(3,\ 2)$ into this equation yields:

$$\begin{cases} -a + 3b + c = -10 & P\ (\mathrm{E}_1) \\ \phantom{-a+{}} 4b + c = -16 & Q\ (\mathrm{E}_2) \\ 3a + 2b + c = -13 & R\ (\mathrm{E}_3) \end{cases}$$

Solving the system gives us $a = -1.8$, $b = -4.2$, and $c = 0.8$.

(c) The parabola with vertical axis has an equation of the form $f(x) = ax^2 + bx + c$. Substituting the x and y values of $P(-1,\ 3)$, $Q(0,\ 4)$, and $R(3,\ 2)$ into this equation yields:

$$\begin{cases} a - b + c = 3 & P\ (\mathrm{E}_1) \\ \phantom{a-b+{}} c = 4 & Q\ (\mathrm{E}_2) \\ 9a + 3b + c = 2 & R\ (\mathrm{E}_3) \end{cases}$$

Solving the system gives us $a = -\frac{5}{12}$, $b = \frac{7}{12}$, and $c = 4$.

(d) For a cubic, $f(x) = ax^3 + bx^2 + cx + d$, we see that $d = 4$ from $Q(0,\ 4)$. Substituting the x and y values of $P(-1,\ 3)$ and $R(3,\ 2)$ gives us

$$\begin{cases} -a + b - c = -1 & P\ (\mathrm{E}_1) \\ 27a + 9b + 3c = -2 & Q\ (\mathrm{E}_2) \end{cases}$$

$3\,\mathrm{E}_1 + \mathrm{E}_2 \ \Rightarrow\ 24a + 12b = -5 \ \Rightarrow\ b = -2a - \frac{5}{12}$.

Using E_1, $-a + (-2a - \frac{5}{12}) - c = -1 \ \Rightarrow\ c = -3a + \frac{7}{12}$.

Thus, we can use any cubic of the form

$$f(x) = ax^3 + (-2a - \tfrac{5}{12})x^2 + (-3a + \tfrac{7}{12})x + 4,\ a \neq 0.$$

(e) Use $f(x) = ba^x + c$ with the points $P(-1, 3)$, $Q(0, 4)$, and $R(3, 2)$.

$$\begin{cases} f(0) = 4 \\ f(-1) = 3 \\ f(3) = 2 \end{cases} \Rightarrow \begin{cases} 4 = ba^0 + c & Q & (\mathrm{E}_1) \\ 3 = ba^{-1} + c & P & (\mathrm{E}_2) \\ 2 = ba^3 + c & R & (\mathrm{E}_3) \end{cases}$$

From E_1, $4 = b + c \;\Rightarrow\; c = 4 - b$. Substitute $4 - b$ for c in E_2 and E_3.

E_2: $3 = ba^{-1} + 4 - b \;\Rightarrow\; b - ba^{-1} = 1 \;\Rightarrow\; b\left(1 - \frac{1}{a}\right) = 1 \;\Rightarrow\; b\left(\frac{a-1}{a}\right) = 1$

E_3: $2 = ba^3 + 4 - b \;\Rightarrow\; b - ba^3 = 2 \;\Rightarrow\; b(1 - a^3) = 2 \;\Rightarrow\; b(a^3 - 1) = -2$

Solving both equations for b and equating the expressions gives us

$\dfrac{a}{a-1} = \dfrac{-2}{a^3 - 1} \;\Rightarrow\; \dfrac{a}{a-1} = \dfrac{-2}{(a-1)(a^2+a+1)} \;\Rightarrow\; a(a^2 + a + 1) = -2 \;\Rightarrow$

$a^3 + a^2 + a + 2 = 0$. The last equation is true only for a value of a that is negative, but a is the base and it must be positive, so we cannot have an exponential of the form $f(x) = ba^x + c$ through the three points.

Note: The next two solutions are for Exercises 45 and 46 in Section 9.2.

$\boxed{45}$

$$\begin{cases} a\cos x + b\sin x = 0 & (\mathrm{E}_1) \\ -a\sin x + b\cos x = \tan x & (\mathrm{E}_2) \end{cases}$$

$\sin x$ (E_1) and $\cos x$ (E_2) yield $\begin{cases} a\sin x\cos x + b\sin^2 x = 0 & (\mathrm{E}_3) \\ -a\sin x\cos x + b\cos^2 x = \sin x & (\mathrm{E}_4) \end{cases}$

$\mathrm{E}_3 + \mathrm{E}_4 \;\Rightarrow\; b\sin^2 x + b\cos^2 x = \sin x \;\Rightarrow\; b(\sin^2 x + \cos^2 x) = \sin x \;\Rightarrow$

$b(1) = \sin x \;\Rightarrow\; b = \sin x$. Substituting back into E_1 yields $a\cos x + \sin^2 x = 0 \;\Rightarrow$

$$a = -\frac{\sin^2 x}{\cos x} = -\frac{1 - \cos^2 x}{\cos x} = -\frac{1}{\cos x} + \frac{\cos^2 x}{\cos x} = -\sec x + \cos x = \cos x - \sec x.$$

$\boxed{46}$

$$\begin{cases} a\cos x + b\sin x = 0 & (\mathrm{E}_1) \\ -a\sin x + b\cos x = \sin x & (\mathrm{E}_2) \end{cases}$$

$\sin x$ (E_1) and $\cos x$ (E_2) yield $\begin{cases} a\sin x\cos x + b\sin^2 x = 0 & (\mathrm{E}_3) \\ -a\sin x\cos x + b\cos^2 x = \sin x\cos x & (\mathrm{E}_4) \end{cases}$

$\mathrm{E}_3 + \mathrm{E}_4 \;\Rightarrow\; b\sin^2 x + b\cos^2 x = \sin x\cos x \;\Rightarrow\; b = \sin x\cos x$.

Substituting back into E_1 yields $a\cos x = -\sin^2 x\cos x \;\Rightarrow\; a = -\sin^2 x$.