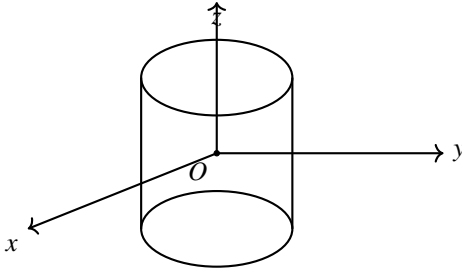


- 1) Consider a cylinder of height  $h$  and radius  $a$ , closed at both ends, centered at the origin. Let  $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$  be the position vector and  $\hat{n}$  a vector normal to the surface. The surface integral  $\int_S \vec{r} \cdot \hat{n} \, ds$  over the closed surface of the cylinder is



- a)  $2\pi a^2(a+h)$                       c)  $2\pi a^2h$   
b)  $3\pi a^2h$                           d) zero

- 2) The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

- a) circles with different radii
  - b) circles with different centres
  - c) straight lines with different slopes
  - d) straight lines with different intercepts on the y-axis
- 3) A particle is moving under the action of a generalized potential

$$V(q, \dot{q}) = \frac{1 + \dot{q}}{q^2}$$

The magnitude of the generalized force is

- a)  $\frac{2(1+\dot{q})}{q^3}$       b)  $\frac{2(1-\dot{q})}{q^3}$       c)  $\frac{2}{q^3}$       d)  $\frac{\dot{q}}{q^3}$

- 4) Two bodies mass  $m$  and  $2m$  are connected by a spring of spring constant  $k$ . The frequency of the normal mode is

- a)  $\sqrt{\frac{3k}{2m}}$       b)  $\sqrt{\frac{k}{m}}$       c)  $\sqrt{\frac{2k}{3m}}$       d)  $\sqrt{\frac{k}{2m}}$

5) Let  $(p, q)$  and  $(P, Q)$  be two pairs of canonical variables. The transformation

$$\begin{aligned} Q &= q^\alpha \cos \beta p \\ P &= q^\alpha \sin \beta p \end{aligned}$$

is canonical for

- a)  $\alpha = 2, \beta = \frac{1}{2}$       b)  $\alpha = 2, \beta = 2$       c)  $\alpha = 1, \beta = 1$       d)  $\alpha = \frac{1}{2}, \beta = 2$

6) Two particles, each of rest mass  $m$  collide head-on and stick together. Before collision, the speed of each, mass was 0.6 times the speed of light in free space. The mass of the final entity is

- a)  $\frac{5m}{4}$       b)  $2m$       c)  $\frac{5m}{2}$       d)  $\frac{25m}{8}$

7) The normalized eigenstate of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad \text{where } n = 1, 2, 3 \dots$$

The particle is subjected to a perturbation

$$V'(x) = \begin{cases} V_0 \cos \frac{\pi x}{a} & \text{for } 0 \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is

- a)  $\frac{2V_0}{3\pi}$       b)  $\frac{V_0}{3\pi}$       c)  $-\frac{V_0}{3\pi}$       d)  $-\frac{2V_0}{3\pi}$

8) If the isothermal compressibility of a solid is  $\kappa_T = 10^{-10} (\text{Pa})^{-1}$ , the pressure required to increase its density by 1% is approximately

- a)  $10^4 \text{ Pa}$       b)  $10^6 \text{ Pa}$       c)  $10^8 \text{ Pa}$       d)  $10^{10} \text{ Pa}$

9) A system of  $N$  non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

- a)  $2k_B \ln N$       b)  $3k_B \ln N$       c)  $Nk_B \ln 2$       d)  $Nk_B \ln 3$

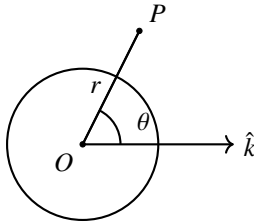
10) A system has two energy levels with energies  $\epsilon$  and  $2\epsilon$ . The lower level is 4-fold-degenerate while the upper level is doubly degenerate. If there are  $N$  non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature  $T$ , the fraction of particles in the upper level is

- a)  $\frac{1}{1+e^{-\frac{\epsilon}{k_B T}}}$       c)  $\frac{1}{2e^{\frac{\epsilon}{k_B T}} + e^{\frac{2\epsilon}{k_B T}}}$   
b)  $\frac{1}{1+2e^{\frac{\epsilon}{k_B T}}}$       d)  $\frac{1}{2e^{\frac{\epsilon}{k_B T}} - e^{\frac{2\epsilon}{k_B T}}}$

11) A spherical conductor of radius  $a$  is placed in a uniform electric field  $\mathbf{E} = E_0 \hat{k}$ . The potential at a point  $\mathbf{P}(r, \theta)$  for  $r > a$ , is given by

$$\pi(r, \theta) = \text{constant} - E_0 r \cos \theta + \frac{E_0 a^3}{r^2} \cos \theta$$

where  $r$  is the distance of  $\mathbf{P}$  from the centre  $\mathbf{O}$  of the sphere and  $\theta$  is the angle  $OP$



with the  $z$ -axis.

The charge density of the sphere at  $\theta = 30^\circ$  is

- a)  $\frac{3\sqrt{3}\epsilon_0 E_0}{2}$       b)  $\frac{3\epsilon_0 E_0}{2}$       c)  $\frac{\sqrt{3}\epsilon_0 E_0}{2}$       d)  $\frac{\epsilon_0 E_0}{2}$

12) According to the single particle nuclear shell model, the spin-parity of the ground state of  $^{17}_8\text{O}$  is

- a)  $\frac{1}{2}^-$       b)  $\frac{3}{2}^-$       c)  $\frac{3}{2}^+$       d)  $\frac{5}{2}^+$

13) In the  $\beta$ -decay of neutron  $n \rightarrow p + e^- + \bar{\nu}_e$ , the anti-neutrino  $\bar{\nu}_e$  escapes detection. Its existence is inferred from the measurement of

- a) energy distribution of electrons  
b) angular distribution of electrons  
c) helicity distribution of electrons  
d) forward-backward asymmetry of electrons