

2018-ma

EE24BTECH11027 - satwikagv

1) The image of the half plane $\operatorname{Re}(z) + \operatorname{Im}(z) > 0$ under the map $\omega = \frac{z-1}{z+i}$ is given by

- a) $\operatorname{Re}(\omega) > 0$ b) $\operatorname{Im}(\omega) > 0$ c) $|\omega| > 1$ d) $|\omega| < 1$

2) Let $D \subset \mathbb{R}^2$ denote the closed disc with center at the origin and radius 2. Then

$$\iint_D e^{-(x^2+y^2)} dx dy =$$

- a) $\pi(1 - e^{-4})$ b) $\frac{\pi}{2}(1 - e^{-4})$ c) $\pi(1 - e^{-2})$ d) $\frac{\pi}{2}(1 - e^{-2})$

3) Consider the polynomial $p(X) = X^4 + 4$ in the ring $\mathbb{Q}[X]$ of polynomials in the variable X with coefficients in the field \mathbb{Q} of rational numbers. Then

- a) the set of zeroes of $p(X)$ in \mathbb{C} forms a group under multiplication
 b) $p(X)$ is reducible in the ring $\mathbb{Q}[X]$
 c) the splitting field of $p(X)$ has degree 3 over \mathbb{Q}
 d) the splitting field of $p(X)$ has degree 4 over \mathbb{Q}

4) Which one of the following statements is true?

- a) Every group of order 12 has a non trivial proper normal subgroup
 b) Some group of order 12 does not have a non-trivial proper normal subgroup
 c) Every group of order 12 has a subgroup of order 6
 d) Every group of order 12 has an element of order 12

5) For an odd prime p , consider the ring $\mathbb{Z}[\sqrt{-p}] = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}[\sqrt{-p}]$ is

- a) a unit b) a square c) a prime d) irreducible

6) Consider the following two statements:

P : The matrix $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$ has infinitely many LU factorizations, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Q: The matrix $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$ has no LU factorization, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Then which one of the following options is correct?

- a) P is TRUE and Q is FALSE
 b) Both P and Q are TRUE
 c) P is FALSE and Q is TRUE

d) Both P and Q are FALSE

7) If the characteristic curves of the partial differential equation $xu_{xx} + 2x^2u_{xy} = u_x - 1$ are $\mu(x, y) = c_1$ and $\nu(x, y) = c_2$, where c_1 and c_2 are constants, then

- a) $\mu(x, y) = x^2 - y, \nu(x, y) = y$
- b) $\mu(x, y) = x^2 + y, \nu(x, y) = y$
- c) $\mu(x, y) = x^2 + y, \nu(x, y) = x^2$
- d) $\mu(x, y) = x^2 - y, \nu(x, y) = x^2$

8) Let $f : X \rightarrow Y$ be a continuous map from a Hausdorff topological space X to a metric space Y . Consider the following two statements:

P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact for each $y \in Y$.

Q: For every compact subset $K \subset Y$, the inverse image $f^{-1}(K)$ is a compact subset of X .

which one of the following is true?

- a) Q implies P but P does NOT imply Q
- b) P implies Q but Q does NOT imply P
- c) P and Q are equivalent
- d) neither P implies Q nor Q implies P

9) Let X denote \mathbb{R}^2 endowed with the usual topology. Let Y denote \mathbb{R} endowed with the co-finite topology. If Z is the product topological space $Y \times Y$, then

- a) the topology of X is the same as the topology of Z
- b) the topology of X is strictly coarser (weaker) than that of Z
- c) the topology of Z is strictly coarser (weaker) than that of X
- d) the topology of X cannot be compared with that of Z

10) Consider \mathbb{R}^n with the usual topology for $n = 1, 2, 3$. Each of the following options gives topological spaces X and Y with respective induced topologies. In which option is X homeomorphic to Y ?

- a) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, Y = \{(x, y, z) \in \mathbb{R}^3 : z = 0, x^2 + y^2 \neq 0\}$
- b) $X = \{(x, y) \in \mathbb{R}^2 : y = \sin\left(\frac{1}{2}\right), 0 < x \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0, -1 \leq y \leq 1\}, Y = [0, 1] \subseteq \mathbb{R}$
- c) $X = \{(x, y) \in \mathbb{R}^2 : y = x \sin\left(\frac{1}{x}\right), 0 < x \leq 1\}, Y = [0, 1] \subseteq \mathbb{R}$
- d) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, Y = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\}$

11) Let $\{X_i\}$ be a sequence of independent Poisson(λ) and let $W_n = \frac{1}{n} \sum_{i=1}^n X_i$. Then the limiting distribution of $\sqrt{n}(W_n - \lambda)$ is the normal distribution with zero mean and variance given by

- a) 1
- b) $\sqrt{\lambda}$
- c) λ
- d) λ^2

12) Let $X_1, X_2, X_3, \dots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f_X(x, \theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0 & \text{otherwise} \end{cases}$$

Also, let $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$. Then the maximum likelihood estimator of θ is

- a) $\frac{1}{\bar{X}}$ b) $\frac{1}{\bar{X}} - 1$ c) $\frac{1}{(\bar{X}-1)}$ d) \bar{X}

13) Consider the Linear Programming Problem (LPP):

Maximize : $\alpha X_1 + X_2$

Subject to: $2X_1 + X_2 \leq 6$,

$-X_1 + X_2 \leq 1$,

$X_1 + X_2 \leq 4$,

$X_1 \geq 0, X_2 \geq 0$,

where α is a constant. If the $(3, 0)$ is the only optimal solution, then

- a) $\alpha < -2$ b) $-2 < \alpha < 1$ c) $1 < \alpha < 2$ d) $\alpha > 2$