

7-7.2-25

EE24BTECH11027 - satwikagv

Question:

Find the equation of a circle passing through the point (7,3) having radius 3 units and whose centre lies on the line $y = x - 1$.

Solution:

Variable	Description	Value
P	point vector	$\begin{pmatrix} 7 \\ 3 \end{pmatrix}$
$-u$	centre of the circle	$\begin{pmatrix} k \\ k - 1 \end{pmatrix}$
r	radius of the circle	3 units

TABLE 0: Variables Used

From the given information, the following equations can be formulated using the circle equation

$$\|x\|^2 + 2u^\top x + f = 0 \quad (0.1)$$

$$\|P\|^2 + 2u^\top P + f = 0 \quad (0.2)$$

$$\|u\|^2 - f = r^2 \quad (0.3)$$

From (0.2) and (0.3)

$$\|P\|^2 + 2u^\top P + \|u\|^2 = r^2 \quad (0.4)$$

Substituting the values of u, P and r ,

$$k^2 + (1 - k)^2 + 2 \begin{pmatrix} -k & 1 - k \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix} + \|P\|^2 - r^2 = 0 \quad (0.5)$$

$$2k^2 - 2k + 1 + 6 - 20k + 7^2 + 3^2 - 3^2 = 0 \quad (0.6)$$

$$2k^2 - 22k + 56 = 0 \quad (0.7)$$

$$k = 7, 4 \quad (0.8)$$

resulting in circles with centres

$$-u = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \text{ or } \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad (0.9)$$

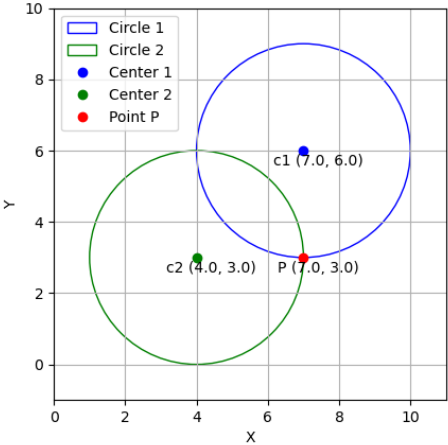


Fig. 0.1: circles passing through point $\mathbf{P}(7, 3)$