

# 2018-ma

EE24BTECH11027 - satwikagv

1) The image of the half plane  $\operatorname{Re}(z) + \operatorname{Im}(z) > 0$  under the map  $\omega = \frac{z-1}{z+i}$  is given by

- a)  $\operatorname{Re}(\omega) > 0$       b)  $\operatorname{Im}(\omega) > 0$       c)  $|\omega| > 1$       d)  $|\omega| < 1$

2) Let  $D \subset \mathbb{R}^2$  denote the closed disc with center at the origin and radius 2. Then

$$\iint_D e^{-(x^2+y^2)} dx dy =$$

- a)  $\pi(1 - e^{-4})$       b)  $\frac{\pi}{2}(1 - e^{-4})$       c)  $\pi(1 - e^{-2})$       d)  $\frac{\pi}{2}(1 - e^{-2})$

3) Consider the polynomial  $p(X) = X^4 + 4$  in the ring  $\mathbb{Q}[X]$  of polynomials in the variable  $X$  with coefficients in the field  $\mathbb{Q}$  of rational numbers. Then

- a) the set of zeroes of  $p(X)$  in  $\mathbb{C}$  forms a group under multiplication  
 b)  $p(X)$  is reducible in the ring  $\mathbb{Q}[X]$   
 c) the splitting field of  $p(X)$  has degree 3 over  $\mathbb{Q}$   
 d) the splitting field of  $p(X)$  has degree 4 over  $\mathbb{Q}$

4) Which one of the following statements is true?

- a) Every group of order 12 has a non trivial proper normal subgroup  
 b) Some group of order 12 does not have a non-trivial proper normal subgroup  
 c) Every group of order 12 has a subgroup of order 6  
 d) Every group of order 12 has an element of order 12

5) For an odd prime  $p$ , consider the ring  $\mathbb{Z}[\sqrt{-p}] = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$ . Then the element 2 in  $\mathbb{Z}[\sqrt{-p}]$  is

- a) a unit      b) a square      c) a prime      d) irreducible

6) Consider the following two statements:

P : The matrix  $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$  has infinitely many LU factorizations, where  $L$  is lower triangular with each diagonal entry 1 and  $U$  is upper triangular.

Q: The matrix  $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$  has no LU factorization, where  $L$  is lower triangular with each diagonal entry 1 and  $U$  is upper triangular.

Then which one of the following options is correct?

- a) P is TRUE and Q is FALSE  
 b) Both P and Q are TRUE  
 c) P is FALSE and Q is TRUE

d) Both P and Q are FALSE

7) If the characteristic curves of the partial differential equation  $xu_{xx} + 2x^2u_{xy} = u_x - 1$  are  $\mu(x, y) = c_1$  and  $\nu(x, y) = c_2$ , where  $c_1$  and  $c_2$  are constants, then

- a)  $\mu(x, y) = x^2 - y, \nu(x, y) = y$
- b)  $\mu(x, y) = x^2 + y, \nu(x, y) = y$
- c)  $\mu(x, y) = x^2 + y, \nu(x, y) = x^2$
- d)  $\mu(x, y) = x^2 - y, \nu(x, y) = x^2$

8) Let  $f : X \rightarrow Y$  be a continuous map from a Hausdorff topological space  $X$  to a metric space  $Y$ . Consider the following two statements:

P:  $f$  is a closed map and the inverse image  $f^{-1}(y) = \{x \in X : f(x) = y\}$  is compact for each  $y \in Y$ .

Q: For every compact subset  $K \subset Y$ , the inverse image  $f^{-1}(K)$  is a compact subset of  $X$ .

which one of the following is true?

- a) Q implies P but P does NOT imply Q
- b) P implies Q but Q does NOT imply P
- c) P and Q are equivalent
- d) neither P implies Q nor Q implies P

9) Let  $X$  denote  $\mathbb{R}^2$  endowed with the usual topology. Let  $Y$  denote  $\mathbb{R}$  endowed with the co-finite topology. If  $Z$  is the product topological space  $Y \times Y$ , then

- a) the topology of  $X$  is the same as the topology of  $Z$
- b) the topology of  $X$  is strictly coarser (weaker) than that of  $Z$
- c) the topology of  $Z$  is strictly coarser (weaker) than that of  $X$
- d) the topology of  $X$  cannot be compared with that of  $Z$

10) Consider  $\mathbb{R}^n$  with the usual topology for  $n = 1, 2, 3$ . Each of the following options gives topological spaces  $X$  and  $Y$  with respective induced topologies. In which option is  $X$  homeomorphic to  $Y$ ?

- a)  $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, Y = \{(x, y, z) \in \mathbb{R}^3 : z = 0, x^2 + y^2 \neq 0\}$
- b)  $X = \{(x, y) \in \mathbb{R}^2 : y = \sin\left(\frac{1}{2}\right), 0 < x \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0, -1 \leq y \leq 1\}, Y = [0, 1] \subseteq \mathbb{R}$
- c)  $X = \{(x, y) \in \mathbb{R}^2 : y = x \sin\left(\frac{1}{x}\right), 0 < x \leq 1\}, Y = [0, 1] \subseteq \mathbb{R}$
- d)  $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}, Y = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\}$

11) Let  $\{X_i\}$  be a sequence of independent Poisson( $\lambda$ ) and let  $W_n = \frac{1}{n} \sum_{i=1}^n X_i$ . Then the limiting distribution of  $\sqrt{n}(W_n - \lambda)$  is the normal distribution with zero mean and variance given by

- a) 1
- b)  $\sqrt{\lambda}$
- c)  $\lambda$
- d)  $\lambda^2$

12) Let  $X_1, X_2, X_3, \dots, X_n$  be independent and identically distributed random variables with probability density function given by

$$f_X(x, \theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0 & \text{otherwise} \end{cases}$$

Also, let  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ . Then the maximum likelihood estimator of  $\theta$  is

- a)  $\frac{1}{\bar{X}}$                       b)  $\frac{1}{\bar{X}} - 1$                       c)  $\frac{1}{(\bar{X}-1)}$                       d)  $\bar{X}$

13) Consider the Linear Programming Problem (LPP):

Maximize  $\alpha X_1 + X_2$

Subject to  $2X_1 + X_2 \leq 6,$

$-X_1 + X_2 \leq 1,$

$X_1 + X_2 \leq 4,$

$X_1 \geq 0, X_2 \geq 0,$

where  $\alpha$  is a constant. If the  $(3, 0)$  is the only optimal solution, then

- a)  $\alpha < -2$                       b)  $-2 < \alpha < 1$                       c)  $1 < \alpha < 2$                       d)  $\alpha > 2$