1-1.2-11

EE24BTECH11027 - satwikagv

Question:
Find the equation of a circle passing through the point $(7,3)$ having radius 3 units adn whose centre lies on the line $y = x - 1$.
solution:
From the given information, the following equations can be formulated using the circle equation

$$\|x\|^2 + 2u^\top x + f = 0 \tag{0.1}$$
$$\|P\|^2 + 2u^\top P + f = 0 \tag{0.2}$$
$$u = \begin{pmatrix} -k \\ 1-k \end{pmatrix} \tag{0.3}$$
$$\|u\|^2 - f = r^2 \tag{0.4}$$
$$\tag{0.5}$$

where

$$p = \begin{pmatrix} 7 \\ 3 \end{pmatrix}, r = 3 \tag{0.6}$$
$$\tag{0.7}$$

From (0.2) and (0.4)

$$\|P\|^2 + 2u^\top P + \|u\|^2 = r^2 \tag{0.8}$$
$$\tag{0.9}$$

Substituting from (0.3) and numerical values,

$$k^2 + (1-k)^2 + 2\begin{pmatrix} -k & 1-k \end{pmatrix}\begin{pmatrix} 7 \\ 3 \end{pmatrix} + \|P\|^2 - r^2 = 0 \tag{0.10}$$
$$2k^2 - 2k + 1 + 6 - 20k + 7^2 + 3^2 - 3^2 = 0 \tag{0.11}$$
$$k^2 - 11k + 28 = 0 \tag{0.12}$$
$$k = 7, 4 \tag{0.13}$$
$$\tag{0.14}$$

resulting in circles with centres

$$-u = \begin{pmatrix} 7 \\ 6 \end{pmatrix} or \begin{pmatrix} 4 \\ 3 \end{pmatrix} \tag{0.15}$$
$$\tag{0.16}$$