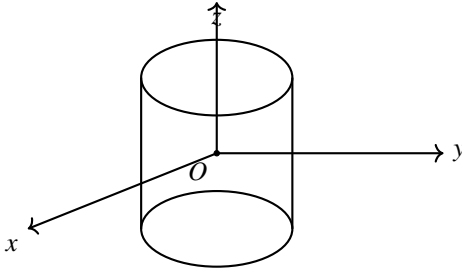


- 1) Consider a cylinder of height h and radius a , closed at both ends, centered at the origin. Let $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and \hat{n} a vector normal to the surface. The surface integral $\int_S \vec{r} \cdot \hat{n} \, ds$ over the closed surface of the cylinder is



- a) $2\pi a^2(a+h)$ c) $2\pi a^2h$
b) $3\pi a^2h$ d) zero

- 2) The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

- a) circles with different radii
 - b) circles with different centres
 - c) straight lines with different slopes
 - d) straight lines with different intercepts on the y-axis
- 3) A particle is moving under the action of a generalized potential

$$V(q, \dot{q}) = \frac{1 + \dot{q}}{q^2}$$

The magnitude of the generalized force is

- a) $\frac{2(1+\dot{q})}{q^3}$ b) $\frac{2(1-\dot{q})}{q^3}$ c) $\frac{2}{q^3}$ d) $\frac{\dot{q}}{q^3}$

- 4) Two bodies mass m and $2m$ are connected by a spring of spring constant k . The frequency of the normal mode is

- a) $\sqrt{\frac{3k}{2m}}$ b) $\sqrt{\frac{k}{m}}$ c) $\sqrt{\frac{2k}{3m}}$ d) $\sqrt{\frac{k}{2m}}$

5) Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$\begin{aligned} Q &= q^\alpha \cos \beta p \\ P &= q^\alpha \sin \beta p \end{aligned}$$

is canonical for

- a) $\alpha = 2, \beta = \frac{1}{2}$ b) $\alpha = 2, \beta = 2$ c) $\alpha = 1, \beta = 1$ d) $\alpha = \frac{1}{2}, \beta = 2$
- 6) Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each, mass was 0.6 times the speed of light in free space. The mass of the final entity is

- a) $\frac{5m}{4}$ b) $2m$ c) $\frac{5m}{2}$ d) $\frac{25m}{8}$

7) The normalized eigenstate of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \quad \text{where } n = 1, 2, 3, \dots$$

The particle is subjected to a perturbation

$$V'(x) = \begin{cases} V_0 \cos \frac{\pi x}{a} & \text{for } 0 \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is

- a) $\frac{2V_0}{3\pi}$ b) $\frac{V_0}{3\pi}$ c) $-\frac{V_0}{3\pi}$ d) $-\frac{2V_0}{3\pi}$

8) If the isothermal compressibility of a solid is $K_T = 10^{-10} (\text{Pa})^{-1}$, the pressure required to increase its density by 1% is approximately

- a) 10^4 Pa b) 10^6 Pa c) 10^8 Pa d) 10^{10} Pa

9) A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

- a) $2k_B \ln N$ b) $3k_B \ln N$ c) $Nk_B \ln 2$ d) $Nk_B \ln 3$

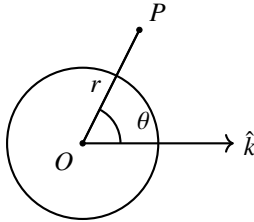
10) A system has two energy levels with energies ϵ and 2ϵ . The lower level is 4-fold-degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T , the fraction of particles in the upper level is

- a) $\frac{1}{1+e^{-\frac{\epsilon}{k_B T}}}$ c) $\frac{1}{2e^{\frac{\epsilon}{k_B T}} + e^{\frac{2\epsilon}{k_B T}}}$
b) $\frac{1}{1+2e^{\frac{\epsilon}{k_B T}}}$ d) $\frac{1}{2e^{\frac{\epsilon}{k_B T}} - e^{\frac{2\epsilon}{k_B T}}}$

11) A spherical conductor of radius a is placed in a uniform electric field $\mathbf{E} = E_0 \hat{k}$. The potential at a point $\mathbf{P}(r, \theta)$ for $r > a$, is given by

$$\pi(r, \theta) = \text{constant} - E_0 r \cos \theta + \frac{E_0 a^3}{r^2} \cos \theta$$

where r is the distance of \mathbf{P} from the centre \mathbf{O} of the sphere and θ is the angle OP



with the z -axis.

The charge density of the sphere at $\theta = 30^\circ$ is

- a) $\frac{3\sqrt{3}\epsilon_0 E_0}{2}$ b) $\frac{3\epsilon_0 E_0}{2}$ c) $\frac{\sqrt{3}\epsilon_0 E_0}{2}$ d) $\frac{\epsilon_0 E_0}{2}$

12) According to the single particle nuclear shell model, the spin-parity of the ground state of $^{17}_8\text{O}$ is

- a) $\frac{1}{2}^-$ b) $\frac{3}{2}^-$ c) $\frac{3}{2}^+$ d) $\frac{5}{2}^+$

13) In the β -decay of neutron $n \rightarrow p + e^- + \bar{\nu}_e$, the anti-neutrino $\bar{\nu}_e$ escapes detection. Its existence is inferred from the measurement of

- a) energy distribution of electrons
b) angular distribution of electrons
c) helicity distribution of electrons
d) forward-backward asymmetry of electrons