2018-ma

EE24BTECH11027 - satwikagv

1) The image of the half plane $\text{Re}(z) + \text{Im}(z) > 0$ under the map $\omega = \frac{z-1}{z+i}$ is given by

   a) $\text{Re}(\omega) > 0$ b) $\text{Im}(\omega) > 0$ c) $|\omega| > 1$ d) $|\omega| < 1$

2) Let $D \subset \mathbb{R}^2$ denote the closed disc with center at the origin and radius 2. Then

$$\iint_D e^{-\left(x^2+y^2\right)} dxdy =$$

   a) $\pi\left(1-e^{-4}\right)$ b) $\frac{\pi}{2}\left(1-e^{-4}\right)$ c) $\pi\left(1-e^{-2}\right)$ d) $\frac{\pi}{2}\left(1-e^{-2}\right)$

3) Consider the polynomial $p(X) = X^4 + 4$ in the ring $\mathbb{Q}[X]$ of polynomials in the variable X with coefficients in the field $\mathbb{Q}$ of rational numbers. Then
   a) the set of zeroes of $p(X)$ in $\mathbb{C}$ forms a group under multiplication
   b) $p(X)$ is reducible in the ring $\mathbb{Q}[X]$
   c) the splitting field of $p(X)$ has degree 3 over $\mathbb{Q}$
   d) the splitting field of $p(X)$ has degree 4 over $\mathbb{Q}$
4) Which one of the following statements is true?
   a) Every group of order 12 has a non trivial proper normal subgroup
   b) Some group of order 12 does not have a non-trivial proper normal subgroup
   c) Every group of order 12 has a subgroup of order 6
   d) Every group of order 12 has an element of order 12
5) For an odd prime p, consider the ring $\mathbb{Z}\left[\sqrt{-p}\right] = \left\{a + b\sqrt{-p} : a, b \in \mathbb{Z}\right\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}\left[\sqrt{-p}\right]$ is

   a) a unit b) a square c) a prime d) irreducible

6) Consider the following two statements:
   P : The matrix $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$ has infinitely many LU factorizations, where L is lower triangular with each diagonal entry 1 and U is upper triangular.
   Q: The matrix $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$ has no LU factorization, where L is lower triangular with each diagonal entry 1 and U is upper triangular.
   Then which one of the following options is correct?
   a) P is TRUE and Q is FALSE
   b) Both P and Q are TRUE
   c) P is FALSE and Q is TRUE

d) Both P and Q are FALSE

7) If the characteristic curves of the partial differential equation $xu_{xx} + 2x^2u_{xy} = u_x - 1$ are $\mu(x, y) = c_1$ and $\nu(x, y) = c_2$, where c_1 and c_2 are constants, then
   a) $\mu(x, y) = x^2 - y, \nu(x, y) = y$
   b) $\mu(x, y) = x^2 + y, \nu(x, y) = y$
   c) $\mu(x, y) = x^2 + y, \nu(x, y) = x^2$
   d) $\mu(x, y) = x^2 - y, \nu(x, y) = x^2$

8) Let $f : X \to Y$ be a continuous map from a Hausdorff topological space X to a metric space Y. Consider the following two statements:
   P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact for each $y \in Y$.
   Q: For every compact subset $K \subset Y$, the inverse image $f^{-1}(K)$ is a compact subset of X.
   which one of the following is true?
   a) Q implies P but P does NOT imply Q
   b) P implies Q but Q does NOT imply P
   c) P and Q are equivalent
   d) neither P implies Q nor Q implies P

9) Let X denote $\mathbb{R}^2$ endowed with the usual topology. Let Y denote $\mathbb{R}$ endowed with the co-finite topology. If Z is the product topological space $Y \times Y$, then
   a) the topology of X is the same as the topology of Z
   b) the topology of X is strictly coarser (weaker) than that of Z
   c) the topology of Z is strictly coarser (weaker) than that of X
   d) the topology of X cannot be compared with that of Z

10) Consider $\mathbb{R}^n$ with the usual topology for $n = 1, 2, 3$. Each of the following options gives topological spaces X and Y with respective induced topologies. In which option is X homeomorphic to Y?
   a) $X = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\right\}, Y = \left\{(x, y, z) \in \mathbb{R}^3 : z = 0, x^2 + y^2 \neq 0\right\}$
   b) $X = \left\{(x, y) \in \mathbb{R}^2 : y = \sin\left(\frac{1}{2}\right), 0 < x \leq 1\right\} \cup \left\{(x, y) \in \mathbb{R}^2 : x - 0, -1 \leq y \leq 1\right\}, Y = [0, 1] \subseteq \mathbb{R}$
   c) $X = \left\{(x, y) \in \mathbb{R}^2 : y = x\sin\left(\frac{1}{x}\right), 0 < x \leq 1\right\}, Y = [0, 1] \subseteq \mathbb{R}$
   d) $X = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\right\}, Y = \left\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\right\}$

11) Let $\{X_i\}$ be a sequence of independent Poisson(λ) and let $W_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the limiting distribution of $\sqrt{n}(W_n - \lambda)$ is the normal distribution with zero mean and variance given by

   a) 1 b) $\sqrt{\lambda}$ c) λ d) λ^2

12) Let $X_1, X_2, X_3, \ldots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f_X(x, \theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0 & \text{otherwise} \end{cases}$$

Also, let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the maximum likelihood estimator of θ is

a) $\frac{1}{\overline{X}}$ b) $\frac{1}{\overline{X}} - 1$ c) $\frac{1}{(\overline{X}-1)}$ d) $\overline{X}$

13) Consider the Linear Programming Problem (LPP):
Maximize : $\alpha X_1 + X_2$
Subject to: $2X_1 + X2 \leq 6,$

$$-X_1 + X_2 \leq 1,$$
$$X_1 + X_2 \leq 4,$$
$$X_1 \geq 0, X_2 \geq 0,$$

where α is a constant. If the $(3, 0)$ is the only optimal solution, then

a) $\alpha < -2$ b) $-2 < \alpha < 1$ c) $1 < \alpha < 2$ d) $\alpha > 2$