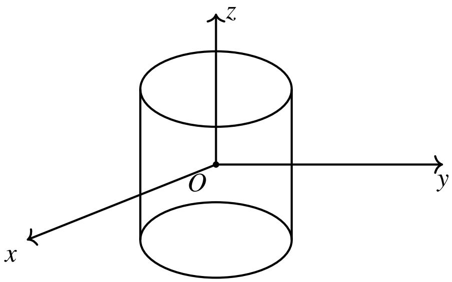
2011-PH

EE24BTECH11027 - satwikagv

1) Consider a cylinder of height h and radius a, closed at both ends, centered at the origin. Let $\mathbf{r} = \hat{i}x + \hat{j}y + \hat{k}z$ be the position vector and $\hat{n}$ a vector normal to the surface. The surface integral $\int_S \mathbf{r} \cdot \hat{n}\, ds$ over the closed surface of the cylinder is



   a) $2\pi a^2 (a + h)$
   b) $3\pi a^2 h$
   c) $2\pi a^2 h$
   d) zero

2) The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

   are a family of
   a) circles with different radii
   b) circles with different centres
   c) straight lines with different slopes
   d) straight lines with different intercepts on the y-axis

3) A particle is moving under the action of a generalized potential

$$V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$$

   The magnitude of the generalized force is

   a) $\frac{2(1+\dot{q})}{q^3}$
   b) $\frac{2(1-\dot{q})}{q^3}$
   c) $\frac{2}{q^3}$
   d) $\frac{\dot{q}}{q^3}$

4) Two bodies mass m and $2m$ are connected by a spring of spring constant k. The frequency of the normal mode is

a) $\sqrt{\frac{3k}{2m}}$ b) $\sqrt{\frac{k}{m}}$ c) $\sqrt{\frac{2k}{3m}}$ d) $\sqrt{\frac{k}{2m}}$

5) Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$$Q = q^{\alpha} \cos \beta p$$
$$P = q^{\alpha} \sin \beta p$$

is canonical for

a) $\alpha = 2, \beta = \frac{1}{2}$ b) $\alpha = 2, \beta = 2$ c) $\alpha = 1, \beta = 1$ d) $\alpha = \frac{1}{2}, \beta = 2$

6) Two particles, each of rest mass m collide head-on and stick together. Before collision, the speed of each, mass was 0.6 times the speed of light in free space. The mass of the final entity is

a) $\frac{5m}{4}$ b) $2m$ c) $\frac{5m}{2}$ d) $\frac{25m}{8}$

7) The normalized eigenstate of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq a \\ \infty & \text{otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right), \qquad \text{where } n = 1, 2, 3 \ldots$$

The particle is subjected to a perturbation

$$V'(x) = \begin{cases} V_0 \cos \frac{\pi x}{a} & \text{for } 0 \leq x \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is

a) $\frac{2V_0}{3\pi}$ b) $\frac{V_0}{3\pi}$ c) $-\frac{V_0}{3\pi}$ d) $-\frac{2V_0}{3\pi}$

8) If the isothermal compressibility of a solid is $K_T = 10^{-10}\,(\text{Pa})^{-1}$, the pressure required to increase its density by 1% is approximately

a) 10^4 Pa b) 10^6 Pa c) 10^8 Pa d) 10^{10} Pa

9) A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is

a) $2k_B \ln N$ b) $3k_B \ln N$ c) $Nk_B \ln 2$ d) $Nk_B \ln 3$

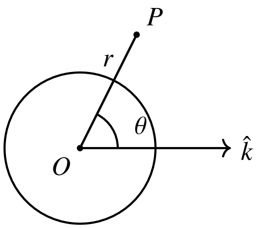
10) A system has two energy levels with energies ϵ and 2ϵ. The lower level is 4-fold-degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which is in thermodynamic equilibrium at a temperature T, the fraction of particles in the upper level is

a) $\frac{1}{1+e^{-\frac{\epsilon}{k_BT}}}$
b) $\frac{1}{1+2e^{\frac{\epsilon}{k_BT}}}$
c) $\frac{1}{2e^{\frac{\epsilon}{k_BT}}+e^{\frac{2\epsilon}{k_BT}}}$
d) $\frac{1}{2e^{\frac{\epsilon}{k_BT}}-e^{\frac{2\epsilon}{k_BT}}}$

11) A spherical conductor of radius a is placed in a uniform electric field $\mathbf{E} = E_0\hat{k}$. The potential at a point $\mathbf{P}(r, \theta)$ for $r > a$, is given by

$$\pi(r, \theta) = constant - E_0 r \cos\theta + \frac{E_0 a^3}{r^2} \cos\theta$$

where r is the distance of $\mathbf{P}$ from the centre $\mathbf{O}$ of the sphere and θ is the angle OP



with the z-axis.
The charge density of the sphere at $\theta = 30°$ is

a) $\frac{3\sqrt{3}\epsilon_0 E_0}{2}$ b) $\frac{3\epsilon_0 E_0}{2}$ c) $\frac{\sqrt{3}\epsilon_0 E_0}{2}$ d) $\frac{\epsilon_0 E_0}{2}$

12) According to the single particle nuclear shell model, the spin-parity of the ground state of ${}^{17}_{8}O$ is

a) $\frac{1}{2}^-$ b) $\frac{3}{2}^-$ c) $\frac{3}{2}^+$ d) $\frac{5}{2}^+$

13) In the β-decay of neutron $n \rightarrow p + e^- + V_e$, the anti-neutrino V_e escapes detection. Its existence is inferred from the measurement of

a) energy distribution of electrons
b) angular distribution of electrons
c) helicity distribution of electrons
d) forward-backward asymmetry of electrons