9.3.12B

EE24BTECH11027 - satwikagv

Question:
Plot the solution of the differential equation:

$$y'' + xy' + xy = x. \tag{0.1}$$

Solution:
To plot the curve of the given differential equation (0.1) we can do it using the method of finite differences which is a numerical technique for solving complex differential equations by approximating derivatives with differences.
The approximated forward derivative of $y(x)$ is given as:

$$y'_n \approx \frac{y_{n+1} - y_n}{h} \tag{0.2}$$

On rearranging we get,

$$y_{n+1} = y_n + y'_n(h) \tag{0.3}$$

And also

$$x_{n+1} = x_n + h \tag{0.4}$$

The approximated forward derivative of second order of $y(x)$ is given as:

$$y''_n \approx \frac{y'_{n+1} - y'_n}{h} \tag{0.5}$$

Substitute eq (0.2) in eq (0.5) we get,

$$y''_n \approx \frac{y_{n+2} - 2y_{n+1} - y_n}{h^2} \tag{0.6}$$

Substitute eq (0.2) and eq (0.6) in eq (0.1) and on reaaranging we get,

$$y_{n+2} = y_{n+1}(2 - hx_n) + y_n\left(1 + hx_n - h^2x_n\right) + h^2x_n \tag{0.7}$$

We need to assume two initial conditions as it is a second order differential equation.
So here we assume the initial conditions as

$$x_0 = 0 \tag{0.8}$$
$$y_0 = 0 \tag{0.9}$$
$$y'_0 = 1 \tag{0.10}$$
$$h = 0.1 \tag{0.11}$$

substitute eq (0.8), eq (0.9) and eq (0.10) in eq (0.1)
we get

$$y''(0) = 0 \tag{0.12}$$

Substitute eq (0.10) in eq (0.3)

$$y_1 = y_0 + y_0'(0.1) \tag{0.13}$$

$$y_1 = 0.1 \tag{0.14}$$

For the rest of the points use eq (0.7) we get the other points.