10.3.6.1.7

EE24BTECH11027 - satwikagv

Question:
Solve the given pair of equations by reducing into a pair of linear equations.

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \tag{0.1}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \tag{0.2}$$

Solution:
Let us assume

$$\frac{1}{x+y} = u \tag{0.3}$$

$$\frac{1}{x-y} = v \tag{0.4}$$

we get the equations as

$$10u + 2v = 4 \tag{0.5}$$

$$15u - 5v = -2 \tag{0.6}$$

The matrix form of the above two equations is

$$\begin{pmatrix} 10 & 2 \\ 15 & -5 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{0.7}$$

Any non singular matrix can be represented as a product of lower triangular matrix L and a upper triangular matrix U and this is called LU decomposition.
LU decomposition:
For the system of equations $Ax = b$ we decompose A into a lower triangular matrix L and an upper triangular matrix U

$$A = LU \tag{0.8}$$

Thus the equation becomes,

$$LUx = b \tag{0.9}$$

Take

$$y = Ux \tag{0.10}$$

and the equation becomes

$$Ly = b \tag{0.11}$$

To find L and U we use gaussian elimination.
The upper triangular matrix U is formed by applying elimination on A as

$$U_{ij} = A_{ij} - \sum_{k=1}^{i-1} L_{ik} U_{kj},\ i \le j \tag{0.12}$$

The elements of L are computed as:

$$L_{ij} = \frac{1}{U_{ii}} \left(A_{ij} - \sum_{k=1}^{i-1} L_{ik} U_{kj} \right),\ i > j \tag{0.13}$$

For L the diagonal elements are always 1 i.e $L_{ii} = 1$
We find L and U as :

$$L = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix} \text{ and } U = \begin{pmatrix} 10 & 2 \\ 0 & -8 \end{pmatrix} \tag{0.14}$$

First we solve for y in $Ly = b$

$$\begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{0.15}$$

$$y_1 = 4 \tag{0.16}$$

$$y_2 = -8 \tag{0.17}$$

$$y = \begin{pmatrix} 4 \\ -8 \end{pmatrix} \tag{0.18}$$

and then we solve for x in $Ux = y$

$$\begin{pmatrix} 10 & 2 \\ 0 & -8 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} \tag{0.19}$$

$$v = 1 \tag{0.20}$$

$$u = \frac{1}{5} \tag{0.21}$$

$$x = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \tag{0.22}$$

we have

$$\frac{1}{x+y} = \frac{1}{5} \tag{0.23}$$

$$x + y = 5 \tag{0.24}$$

And

$$\frac{1}{x-y} = 2 \tag{0.25}$$

$$x - y = \frac{1}{2} \tag{0.26}$$

Matrix form of the eq(0.24) and eq(0.26) is:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{1}{2} \end{pmatrix} \tag{0.27}$$

Let B=$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
Multiply B on both sides, we have

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 5 \\ \frac{1}{2} \end{pmatrix} \tag{0.28}$$

On solving we have the solution :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{11}{4} \\ \frac{9}{4} \end{pmatrix} \tag{0.29}$$

Plot:

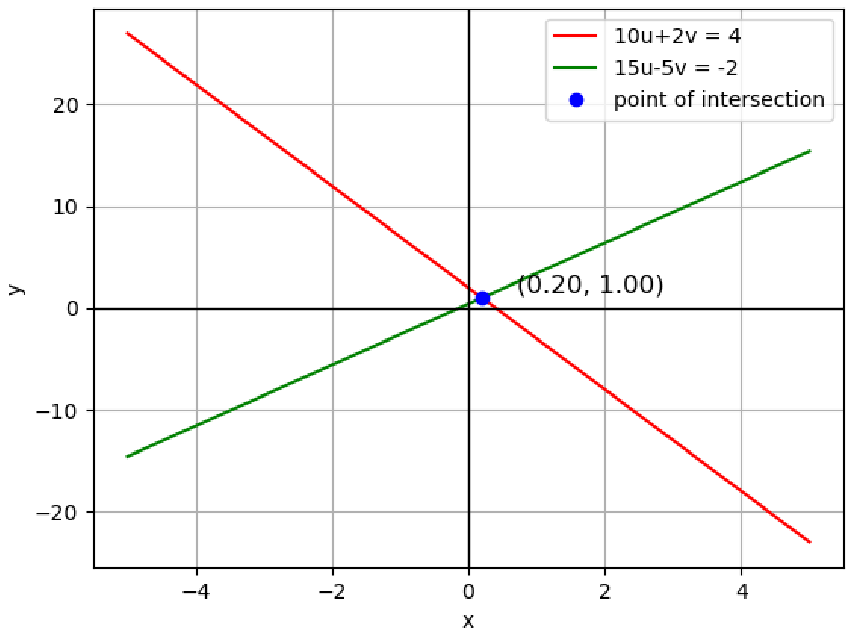

Fig. 0.1: Intersection of two lines in $u - v$ space