Discrete Mathematics
Assignment 1

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Note: No late submussions allowed.

1. Let A = {a, b, {a, b}, {{a, b}}}
   Identify each of the following statements as true or false. Justify your answers.
   (a) a ∈ A (b) {a} ∈ A (c) {a, b} ∈ A (d) {{a, b}} ⊆ A (e) {a, b} ⊆ A (f) {a, {b}} ⊆ A

2. If $U = \{n|n \in N, n \leq 15\}$,
   $A = \{n|n \in N, 4 < n < 12\}$,
   $B = \{n|n \in N, 8 < n < 15\}$
   $C = \{n|n \in N, 5 < n < 10\}$
   Find $\overline{A} - \overline{B}$ & $\overline{C} - \overline{A}$

3. Show that by using venn diagram
   (a) $A \cup (\overline{B} \cap C) = (A \cup \overline{B}) \cap (A \cup C)$
   (b) $A \cap B \cap C = A - [(A - B) \cup (A - C)]$

4. State and prove the distributive law(both the laws).

5. State and prove De Morgan's law(both the laws).

6. Let A, B and C be subsets of universal set U. Given that $A \cap B = A \cap C$ and $\overline{A} \cap B = \overline{A} \cap C$. Is it necessary that B = C?

7. (a) Given that $A \cap B = A \cap C$. Is it necessary that B = C?
   (b) Given that $A \cup B = A \cup C$. Is it necessary that B = C?

8. Determine whether the following statements are true or false. Justify your answers.
   i. $\{a, \phi\} \in \{a, \{a, \phi\}\}$
   ii. $\{a, b\} \subseteq \{a, b, \{a, b\}\}$
   iii. $\{a, b\} \in \{a, b, \{a, b\}\}$
   iv. $\{a, c\} \in \{a, b, c, \{a, b, c\}\}$

9. How many integers between 1-1000 are divisible by 2,3,5,7,9 or 11?

10. An investigator interviewed 100 students to determine their preferences for the three drinks-milk (M), coffee (C), and tea (T). He reported the following:
Ten students had all three drinks, 20 had M and C, 30 had C and T, 25 had M and T, 12 had M only, 5 had C only, and 8 had T only.

    (a) How many did not take any of the three drinks?
    (b) How many take milk but not coffee?
    (c) How many take tea and coffee but not milk?

11. It was found that in the first year of computer science of 80 students, 50 know COBOL, 55 know C language, and 46 know Pascal. It was also known that 37 know C and COBOL, 28 know C and Pascal, 25 know Pascal and COBOL. However, 7 students do not know any of the languages. Find:

    (a) How many know all the three languages?
    (b) How many know exactly two languages?
    (c) How many know exactly one language?

12. Find the number of positive integers not exceeding 100 that are either odd or the square of an integer?

13. Let A = $\{\phi, b\}$, construct the following sets:

    (a) $A - \phi$
    (b) $\{\phi\} - A$
    (c) $A \cup P(A)$
    (d) $A \cap P(A)$

14. Using the following statements:
p: Rajni is tall
q: Rajni is beautiful
Write the following statements in symbolic forms.

    (a) Rajni is tall and beautiful.
    (b) Rajni is tall but not beautiful.
    (c) It is false that Rajni is short or beautiful.
    (d) Rajni is tall or Rajni is short and beautiful.

15. To describe the various restaurants in the city, let p denote the statement "the food is good", q denote the statement "the service is good" and r denote the statement "the rating is three star". Write the following statements in symbolic form:

(a) Either the food is good or service is good, or both.
(b) Either the food is good or service is good, but not both.
(c) The food is good while is service is poor.
(d) It is not the case that both the food is good and the rating is three star.
(e) If both the food and service is good, then the rating is three star.
(f) It is not true that a three star rating always means good food and good service.

16. Write the logical negation of the following statements in the symbolic froms:

    (a) Gopal is intelligent and rich.
    (b) Gopal is intelligent but not rich.
    (c) Gopal is either intelligent or rich.

17. Construct truth tables to determine whether each of the following is a tautology, contradiction or continguency:

    (a) $p \rightarrow (q \rightarrow p)$
    (b) $(p \wedge q) \wedge \neg(p \vee q)$
    (c) $(p \wedge q) \rightarrow p$
    (d) $(p \rightarrow q) \leftrightarrow (q \vee \neg p)$
    (e) $(p \wedge (\neg p \vee q)) \wedge \neg q$

18. Show that the following statements is tautological:
    $(p \wedge (p \rightarrow q)) \rightarrow q$

19. Show that

    (a) $(p \wedge (\neg p \vee q)) \vee (q \wedge \neg(p \wedge q))$ is equivalent to q.
    (b) $((p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee q$ is a tautology.

20. Prove that $(p \rightarrow (q \rightarrow r))$ is logical equivalent to $(p \rightarrow q) \rightarrow (p \rightarrow r)$.

21. Suppose the universe of discourse is the set of integers. Let P(x,y) be predicate $x - y = 0$. Indicate which of the propositions are true and which are false.

    (a) P (2, 3)
    (b) P (3, 3)
    (c) $\forall x \; \exists y \; P(x, y)$
    (d) $\exists x \; \forall y \; P(y, x)$

22. Transcribe the following into logical notation. Let the universe of discourse the real numbers.

    (a) For any value of x, x^2 is non-negative.
    (b) For every value of x, there is some value of y such that x.y = 1.
    (c) There are positive values of x and y such that $x.y > 0$.
    (d) There is a value of x such that if y is positive, then x + y is negative.
    (e) For every value of x, there is some value of y such that x - y = 1.

23. Negate the following in such a way that the symbol $\neg$ does not appear outside the square brackets:

    (a) $\forall x\ [x^2 \geq 0]$
    (b) $\exists x\ [x.2 = 1]$
    (c) $\forall x\ \exists y\ [x + y = 1]$
    (d) $\forall x\ \forall y\ [(x > y) \rightarrow (x^2 > y^2)]$

24. Write the following statements in symbolic form, using quantifiers.

    (a) All students have taken a course in communication skills.
    (b) There is a girl student in the class who is also a sports person.
    (c) Some students are intelligent, but not hardworking.

25. For the universe of all integers, let P(x), Q(x), R(x), S(x) and T(x) be the following statements:
    P(x): $x > 0$
    Q(x): x is even
    R(x): x is a perfect square
    S(x): x is divisible by 4
    T(x): x is divisible by 5
    Write the following statements in symbolic forms.

    (a) At least one integer is even.
    (b) There exists a positive integer that is even.
    (c) If x is even, then x is not divisible by 5.
    (d) No even integer is divisible by 5.
    (e) There exists an even integer divisible by 5.
    (f) If x is even and x is a perfect square, then x is divisible by 4.

26. Rewrite the following statements using quantifier variables and predicate symbols:

    (a) All birds can fly.
    (b) Not all birds can fly.

(c) Some men are genius.

(d) Some numbers are not rational.

(e) There is a student who likes Mathematics but not Geography.

(f) Each integer is either odd or even.

27. Negate each of the following statements:

    (a) $\forall x,\ |x| = x$

    (b) $\exists x,\ x^2 = x$

28. Determine the truth value of each of the statement and negate every statement.

    (a) $\exists x,\ x + 2 = x$

    (b) $\forall x,\ x + 1 > x$