

# Digital Design and Computer Architecture: Solutions to the Exercises

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## 1 Chapter One

### 1.1 Exercise

Explain in one paragraph at least three levels of abstraction that are used by

- a) biologists studying the operation of cells
- b) chemists studying the composition of matter

#### Solution

- a) Cells consist of *atoms*. Atoms form special molecules known as *amino acids*. These are used by the cell to create *enzymes*.
- b) There are a lot of *quarks*, which (in threes) form basic particles such as neutrons, electrons, protons, anti-protons, positrons. If you then take a number of neutrons, electrons and protons, you get an atom.

### 1.2 Exercise

Explain in one paragraph how the techniques of hierarchy, modularity, and regularity may be used by

- a) automobile designers
- b) businesses to manage their operations

#### Solution

1.2

### 1.3 Exercise

Ben Bitdiddle is building a house. Explain how he can use the principle of *hierarchy*, *modularity*, and *regularity* to save time and money during construction

#### Solution

First, let's define the terms used here:

**Hierarchy** dividing a system into modules

**Modularity** modules can be used without having to understand how they work inside

**Regularity** modules can be reused easily

1.3

## 1.4 Exercise

An analog voltage is in the range of 0-5V. If it can be measured with an accuracy of  $\pm 50mV$ , at most how many bits of information does it convey?

### Solution

If the analog voltage can be measured with an accuracy of  $\pm 50mV$ , then it can only accurately be measured to the nearest  $100mV$ . This means that between  $0mV$  and  $5,000mV$ , it can take on 50 differentiable values:

$$\frac{5,000mV}{100mV} = 50$$

Since you need  $\log_2(n)$  bits to be able to differentiate between  $n$  values, the analog voltage has  $\log_2(50) \approx 5.64$  bits of information, so it conveys at most 6 bits of information.

## 1.5 Exercise

A classroom has an old clock on the wall whose minute hand broke off.

- If you can read the hour hand to the nearest 15 minutes, how many bits of information does the clock convey about time?
- If you know whether it is before or after noon, how many additional bits of information do you know about the time?

### Solution

- An hour has 60 minutes, if you know the time to the nearest minute that makes 4 different states. Then there are 12 hours per day, which makes a total of  $12 * 4 = 48$  different states. Using  $\log_2(n)$ , that makes  $\log_2(48) \approx 5.58$  bits of information.
- If you know whether it is before or after noon, that makes two states, and that is exactly one bit. So by knowing that, you know one additional bit about the time.

## 1.6 Exercise

The Babylonians developed the *sexagesimal* (base 60) number system about 4000 years ago. How many bits of information is conveyed with one sexagesimal digit? How do you write the number  $4000_{10}$  in sexagesimal?

### Solution

Just like we have ten digits (0..9) in base 10, they had sixty digits in their number system. Since there are sixty different digits, one single one of them conveys  $\log_2(60) \approx 5.91$  bits of information. Converting numbers can be done just like in any other number system:

$$4000/60 = 66 \text{ rem } 40$$

$$66/60 = 1 \text{ rem } 6$$

$$1/60 = 0 \text{ rem } 1$$

By reading the remainders off, the number is:  $1 \cdot 60^2 + 6 \cdot 60^1 + 40 \cdot 60^0$ .

## 1.7 Exercise

How many different numbers can be represented with 16 bits?

### Solution

$2^{16} = 65536$  numbers can be represented with 16 bits.

## 1.8 Exercise

What is the largest unsigned 32-bit number?

### Solution

$2^{32} - 1 = 4,294,967,295$ , because there are  $2^{32} = 4,294,967,295$  unsigned 32-bit numbers and the first one is 0.

## 1.9 Exercise

What is the largest 16-bit binary number that can be represented with

- a) unsigned numbers?                      b) two's complement numbers?                      c) sign/magnitude numbers?

### Solution

- a) There are  $2^{16} = 65536$  unsigned 16-bit numbers, the largest being  $2^{16} - 1 = \mathbf{65535}$ .  
b) There are  $2^{16} = 65536$  16-bit numbers in two's complement form, the biggest is  $2^{15} - 1 = \mathbf{32767}$ .  
c) There are  $2^{16} = 65536$  16-bit numbers in sign/magnitude form, the biggest being  $2^{15} - 1 = \mathbf{32767}$ .

## 1.10 Exercise

What is the largest 32-bit number that can be represented with

- a) unsigned numbers?                      b) two's complement numbers?                      c) sign/magnitude numbers?

### Solution

- a) Largest 32-bit unsigned number is  $2^{32} - 1 = \mathbf{4\,294\,967\,295}$ .  
b) Largest 32-bit two's complement number is  $2^{31} - 1 = \mathbf{2\,147\,483\,648}$ .  
c) Largest 32-bit sign/magnitude number is  $2^{31} - 1 = \mathbf{2\,147\,483\,648}$ .

## 1.11 Exercise

What is the smallest (most negative) 16-bit binary number that can be represented with

- a) unsigned numbers?                      b) two's complement numbers?                      c) sign/magnitude numbers?

### Solution

- a) Smallest 16-bit unsigned number is **0**.  
b) Smallest 16-bit two's complement number is  $-2^{15} = \mathbf{-32768}$ .  
c) Smallest 16-bit sign/magnitude number is  $-2^{15} + 1 = \mathbf{-32767}$ .

### 1.12 Exercise

What is the smallest (most negative) 32-bit binary number that can be represented with

- a) unsigned numbers?                      b) two's complement numbers?                      c) sign/magnitude numbers?

#### Solution

- a) Smallest 32-bit unsigned number is **0**.  
b) Smallest 32-bit two's complement number is  $-2^{31} = -2\,147\,483\,648$ .  
c) Smallest 32-bit sign/magnitude number is  $-2^{31} + 1 = 2\,147\,483\,647$ .

### 1.13 Exercise

Convert the following unsigned binary numbers to decimal. Show your work.

- a)  $1010_2$                       b)  $110110_2$                       c)  $11110000_2$                       d)  $000100010100111_2$

#### Solution

- a)  $1010_2 = 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 8 + 2 = \mathbf{10}$   
b)  $110110_2 = 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = 32 + 16 + 4 + 2 = \mathbf{54}$   
c)  $11110000_2 = 1 \cdot 2^7 + 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 = 128 + 64 + 32 + 16 = \mathbf{240}$   
d)  $000100010100111_2 = 2^1 + 2^7 + 2^5 + 2^2 + 2^1 + 2^0 = 2048 + 256 + 32 + 4 + 2 + 1 = \mathbf{2\,343}$

### 1.14 Exercise

Convert the following unsigned binary numbers to decimal. Show your work.

- a)  $1110_2$                       b)  $100100_2$                       c)  $11010111_2$                       d)  $011101010100100_2$

#### Solution

- a)  $1110_2 = 2^3 + 2^2 + 2^1 = 8 + 4 + 2 = \mathbf{14}$   
b)  $100100_2 = 2^5 + 2^2 = 32 + 4 = \mathbf{36}$   
c)  $11010111_2 = 2^7 + 2^6 + 2^4 + 2^2 + 2^1 + 2^0 = 128 + 64 + 16 + 4 + 2 + 1 = \mathbf{215}$   
d)  $011101010100100_2 = 2^1 + 2^3 + 2^1 + 2^1 + 2^9 + 2^7 + 2^5 + 2^2 = 8192 + 4096 + 2048 + 512 + 128 + 32 + 4 = \mathbf{15\,012}$

### 1.15 Exercise

Repeat Exercise 1.13, but convert to hexadecimal.

#### Solution

- a)  $1010_2 = 1010_2 \cdot 16^0 = \mathbf{A_{16}}$   
b)  $110110_2 = 11_2 \cdot 16^1 + 0110_2 \cdot 16^0 = \mathbf{36_{16}}$   
c)  $11110000_2 = 1111_2 \cdot 16^1 + 0000_2 \cdot 16^0 = \mathbf{F0_{16}}$   
d)  $000100010100111_2 = 000_2 \cdot 16^3 + 1000_2 \cdot 16^2 + 1010_2 \cdot 16^1 + 0111_2 \cdot 16^0 = \mathbf{8A7_{16}}$

### 1.16 Exercise

Repeat Exercise 1.14, but convert to hexadecimal.

#### Solution

- a)  $1110_2 = \mathbf{E}_{16}$
- b)  $100100_2 = \mathbf{24}_{16}$
- c)  $11010111_2 = \mathbf{D7}_{16}$
- d)  $011101010100100_2 = \mathbf{3AA4}_{16}$

### 1.17 Exercise

Convert the following hexadecimal numbers to decimal. Show your work.

- a)  $A5_{16}$
- b)  $3B_{16}$
- c)  $FFFF_{16}$
- d)  $D0000000_{16}$

#### Solution

- a)  $A5_{16} = 10 \cdot 16^1 + 5 \cdot 16^0 = 160 + 5 = \mathbf{165}$
- b)  $3B_{16} = 3 \cdot 16^1 + 11 \cdot 16^0 = 48 + 11 = \mathbf{59}$
- c)  $FFFF_{16} = 15 \cdot 16^3 + 15 \cdot 16^2 + 15 \cdot 16^1 + 15 \cdot 16^0 = 61440 + 3840 + 240 + 15 = \mathbf{65535}$
- d)  $D0000000_{16} = 13 \cdot 16^7 = \mathbf{3489660928}$

### 1.18 Exercise

Convert the following hexadecimal numbers to decimal. Show your work.

- a)  $4E_{16}$
- b)  $7C_{16}$
- c)  $ED3A_{16}$
- d)  $403FB001_{16}$

#### Solution

- a)  $4E_{16} = 4 \cdot 16^1 + 14 \cdot 16^0 = 64 + 14 = \mathbf{78}$
- b)  $7C_{16} = 7 \cdot 16^1 + 12 \cdot 16^0 = 112 + 12 = \mathbf{124}$
- c)  $ED3A_{16} = 14 \cdot 16^3 + 13 \cdot 16^2 + 3 \cdot 16^1 + 10 \cdot 16^0 = 57355 + 3328 + 48 + 10 = \mathbf{60741}$
- d)  $403FB001_{16} = 4 \cdot 16^7 + 0 \cdot 16^6 + 3 \cdot 16^5 + 15 \cdot 16^4 + 11 \cdot 16^3 + 0 \cdot 16^2 + 0 \cdot 16^1 + 1 \cdot 16^0$   
 $= 1073741824 + 3145728 + 983040 + 45056 + 1$   
 $= \mathbf{1077915649}$

### 1.19 Exercise

Repeat Exercise 1.17, but convert to unsigned binary.

### Solution

$$\text{a) } A5_{16} = 10 \cdot 16^1 + 5 \cdot 16^0 = 1010_2 \cdot 16^1 + 0101_2 \cdot 16^0 = \mathbf{1010\,0101_2}$$

$$\text{b) } 3B_{16} = 3 \cdot 16^1 + 11 \cdot 16^0 = 0011_2 \cdot 16^1 + 1011_2 \cdot 16^0 = \mathbf{0011\,1011_2}$$

$$\begin{aligned}\text{c) } FFFF_{16} &= 15 \cdot 16^3 + 15 \cdot 16^2 + 15 \cdot 16^1 + 15 \cdot 16^0 \\ &= 1111_2 \cdot 16^3 + 1111_2 \cdot 16^2 + 1111_2 \cdot 16^1 + 1111_2 \cdot 16^0 \\ &= \mathbf{1111\,1111\,1111\,1111_2}\end{aligned}$$

$$\text{d) } D0000000_{16} = 13 \cdot 16^7 = 1101_2 \cdot 16^7 = \mathbf{1101\,0000\,0000\,0000\,0000\,0000\,0000_2}$$

### 1.20 Exercise

Repeat Exercise 1.18, but convert to unsigned binary.

### Solution

$$\text{a) } 4E_{16} = 4 \cdot 16^1 + 14 \cdot 16^0 = 0100_2 \cdot 16^1 + 1110_2 \cdot 16^0 = \mathbf{0100\,1110_2}$$

$$\text{b) } 7C_{16} = 7 \cdot 16^1 + 12 \cdot 16^0 = 0111_2 \cdot 16^1 + 1100_2 \cdot 16^0 = \mathbf{0111\,1100_2}$$

$$\begin{aligned}\text{c) } ED3A_{16} &= 14 \cdot 16^3 + 13 \cdot 16^2 + 3 \cdot 16^1 + 10 \cdot 16^0 \\ &= 1101_2 \cdot 16^3 + 1100_2 \cdot 16^2 + 0011_2 \cdot 16^1 + 1010_2 \cdot 16^0 \\ &= \mathbf{1101\,1100\,0011\,1010_2}\end{aligned}$$

$$\begin{aligned}\text{d) } 403FB001_{16} &= 4 \cdot 16^7 + 0 \cdot 16^6 + 3 \cdot 16^5 + 15 \cdot 16^4 + 11 \cdot 16^3 + 0 \cdot 16^2 + 0 \cdot 16^1 + 1 \cdot 16^0 \\ &= 0100_2 \cdot 16^7 + 0000_2 \cdot 16^6 + 0011_2 \cdot 16^5 + 1111_2 \cdot 16^4 + 1011_2 \cdot 16^3 + \\ &\quad 0000_2 \cdot 16^2 + 0000_2 \cdot 16^1 + 0001_2 \cdot 16^0 \\ &= \mathbf{0100\,0000\,0011\,1111\,1011\,0000\,0000\,0001_2}\end{aligned}$$

### 1.21 Exercise

Convert the following two's complement binary numbers to decimal.

$$\text{a) } 1010_2$$

$$\text{b) } 110110_2$$

$$\text{c) } 01110000_2$$

$$\text{d) } 10011111_2$$

### Solution

$$\text{a) } 1010_2 = 1 \cdot (-2^3) + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 = -8 + 2 = \mathbf{-6}$$

$$\text{b) } 110110_2 = 1 \cdot (-2^5) + 1 \cdot 2^4 + 1 \cdot 2^2 + 1 \cdot 2^1 = -32 + 16 + 4 + 2 = \mathbf{-10}$$

$$\text{c) } 01110000_2 = 1 \cdot 2^6 + 1 \cdot 2^5 + 1 \cdot 2^4 = 64 + 32 + 16 = \mathbf{112}$$

$$\text{d) } 10011111_2 = 1 \cdot (-2^7) + 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = -128 + 16 + 8 + 4 + 2 + 1 = \mathbf{-97}$$

### 1.22 Exercise

Convert the following two's complement binary numbers to decimal.

$$\text{a) } 1110_2$$

$$\text{b) } 100011_2$$

$$\text{c) } 01001110_2$$

$$\text{d) } 10110101_2$$

### Solution

a)  $1110_2 = -2^3 + 2^2 + 2^1 = -8 + 4 + 2 = -2$

b)  $100011_2 = -2^5 + 2^1 + 2^0 = -32 + 2 + 1 = -29$

c)  $01001110_2 = 2^6 + 2^3 + 2^2 + 2^1 = 64 + 8 + 4 + 2 = 78$

d)  $10110101_2 = -2^7 + 2^5 + 2^4 + 2^2 + 2^0 = -128 + 32 + 16 + 4 + 1 = -75$

### 1.23 Exercise

Repeat Exercise 1.21, assuming the binary numbers are in sign/magnitude form rather than two's complement representation.

### Solution

1.23

### 1.24 Exercise

Repeat Exercise 1.22, assuming the binary numbers are in sign/magnitude form rather than two's complement representation.

### Solution

1.24

### 1.25 Exercise

Convert the following decimal numbers to unsigned binary numbers.

a)  $42_{10}$

b)  $63_{10}$

c)  $229_{10}$

d)  $845_{10}$

### Solution

a)  $42 = 32 + 8 + 2 = 2^5 + 2^3 + 2^1 = 10\ 1010_2$

b)  $63 = 32 + 16 + 8 + 4 + 2 + 1 = 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 11\ 1111_2$

c)  $229 = 128 + 64 + 32 + 4 + 1 = 2^7 + 2^6 + 2^5 + 2^2 + 2^0 = 1110\ 0101_2$

d)  $845 = 512 - 256 - 64 - 8 - 4 - 1 = 2^9 + 2^8 + 2^6 + 2^3 + 2^2 + 2^0 = 11\ 0100\ 1101_2$

1.26

1.27

1.28

1.29

1.30

1.30

1.31

## 2 Chapter Two

### 2.1 Exercise

Write a Boolean equation in sum-of-products canonical form for each of the truth tables.

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#### Solution

(a)  $Y = \overline{A}\overline{B} + \overline{A}B + AB$

(b)  $Y = \overline{A}\overline{B}\overline{C} + ABC$

(c)  $Y = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + ABC$

(d)  $Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D}$

(e)  $Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D} + \overline{A}BC\overline{D} + \overline{A}BC\overline{D}$



## 2.2 Exercise

Write a Boolean equation in sum-of-products canonical form for each of the truth tables.

a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

b)

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

c)

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

d)

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

e)

A	B	C	D	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

### Solution

(a)  $Y = \overline{A}B + A\overline{B} + AB$

(b)  $Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + ABC$

(c)  $Y = \overline{A}\overline{B}C + \overline{A}B\overline{C} + ABC$

(d)  $Y = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}D + \overline{A}BC\overline{D} + \overline{A}BCD + \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}C\overline{D}$

(e)  $Y = \overline{A}\overline{B}CD + \overline{A}B\overline{C}\overline{D} + \overline{A}BCD + A\overline{B}\overline{C}\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D} + A\overline{B}C\overline{D}$

## 2.3 Exercise

Write a Boolean equation in product-of-sums canonical form for the truth tables from Exercise 2.1.

2.3

**Solution**

(a)  $Y = A + \overline{B}$

(b)  $Y = (A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + B + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$

(c)  $Y = (A + B + \overline{C})(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$

(d)  $Y =$

(e)  $Y =$

## 2.4 Exercise

Write a Boolean equation in product-of-sums canonical form for the truth tables from Exercise 2.2.

2.4

**Solution**

(a)  $Y = A + \overline{B}$

(b)  $Y = \dots$

## 2.5 Exercise

Minimize each of the Boolean equations from Exercise 2.1.

**Solution**

2.5

## 2.6 Exercise

Minimize each of the Boolean equations from Exercise 2.2.

**Solution**

2.5

## 2.7 Exercise

Sketch a reasonably simple combinatorial circuit implementing each of the functions from Exercise 2.5.

**Solution**

2.7

## 2.8 Exercise

Sketch a reasonably simple combinatorial circuit implementing each of the functions from Exercise 2.6.

**Solution**

2.8

## 2.9 Exercise

Repeat Exercise 2.7, using only NOT, AND and OR gates.

Solution

2.9

## 2.10 Exercise

Repeat Exercise 2.8, using only NOT, AND and OR gates.

Solution

2.10

## 2.11 Exercise

Repeat Exercise 2.7, using only NOT, NAND and NOR gates.

Solution

2.11

## 2.12 Exercise

Repeat Exercise 2.8, using only NOT, NAND and NOT gates.

Solution

2.12