Sahubrata Mohanty 7

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Q1 - Find the rank of the matrix by reducing in row reduced echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

apply elementary row operation to get the matrix into row echelon form then reduce it to RREF.

First column

1) $R_2 = R_2 - 2R_1$

2) $R_3 = R_3 - 3R_1$

3) $R_4 = R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -5 & 5 \end{bmatrix}$$

Second column

1) $R_3 = R_3 - 4/3 R_2$

2) $R_4 = R_4 - 4/3 R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & -4 & 0 & 7/3 \end{bmatrix}$$

Third column

1) $R_4 = R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

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This is now row echelon form.

To convert it to RREF

1> $R_3 = R_3 + 4/3\, R_2$

2> $R_1 = R_1 - 2R_2$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 0 & -1 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

divide third row by -4

$R_3 = \frac{1}{4} R_3$

$$\begin{bmatrix} 1 & 2 & 0 & -4 \\ 0 & 0 & -3 & 2 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 0 & 13/3 \end{bmatrix}$$

We have the matrix in RREF

So rank of matrix A is 3

Q2> Let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

find the rank.

$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ by rank-nullity theorem

rank(T) + nullity(T) = dim (W).

1> Find the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$:

$T\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = (1-0) + (0-0)x + (0-1)x^2 = 1 - x^2$

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2) For the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (0-1) + (1-1)x + (1-0)x^2 = -1+x$$

3) for the matrix $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$:

$$T\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = (0-0) + (0-0)x + (0-0)x^2 = 0$$

The images of the basis vector $\{1-x^2, -1+x, 0\}$ are ~~identity~~ linearly independent of T, so the rank T is 3.

using the rank - nullity theorem

$$\text{rank}(T) + \text{nullity}(T) = \dim(W)$$

$$3 + \text{nullity}(T) = 3$$

$$\text{nullity}(T) = 0.$$

rank of T is 3 and nullity of T is 0