Chapter 8 Electromagnetism.

Maxwell's equations

(I) Gauss law in electrostatics :

$$\oint_S \vec{E} \cdot d\vec{s} = q/\epsilon_0 \quad \text{(Integral form)}$$

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \text{(Differential form)}$$

Significance:
The electric field passing through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the total charge enclosed by that surface.

(II) Gauss law in magnetism :

$$\oint_S \vec{B} \cdot d\vec{s} = 0 \quad \text{(Integral form)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{(Differential form)}$$

Significance:
Isolated magnetic monopole do not exist. The magnetic flux passing passing through any closed surface enclosing a magnet is zero.

(III) Faraday law of electromagnetic induction :

$$\oint_L \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t} \quad \text{(Integral forms)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Differential form)}$$

Significance:
The rate of change of magnetic flux is equal to the induced emf. Thus changing magnetic field produced electric field.

(iv) Ampere - Maxwell law :

$$\oint_l \vec{B} \cdot \vec{dl} = \mu_0 I + \mu_0 \epsilon_0 \frac{\partial \phi_E}{\partial t} \quad \text{(Integral form)}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{(Differential form)}$$

Significance:

This equation states that there are two ways of producing magnetic field (a) by steady current (b) by changing ~~m~~ electric field.

For steady current: $\oint_l \vec{B} \cdot \vec{dl} = \mu_0 I$

For varying electric field: $\oint_l \vec{B} \cdot \vec{dl} = \mu_0 \epsilon_0 \frac{\partial \phi_E}{\partial t}$

$= \mu_0 I_d$ where $I_d = \epsilon_0 \frac{\partial \phi_E}{\partial t}$ is called displacement current.

Imp.

(*) Integral forms to differential forms of Maxwell's equation

i) Gauss law in Electrostatics :

$$\oint_s \vec{E} \cdot \vec{ds} = q/\epsilon_0 \quad \text{— (1)}$$

According to eqⁿ (1), total electric flux passing through closed surface is equal to $1/\epsilon_0$ times the total charge enclosed.

$$\text{or} \quad \oint_s \vec{E} \cdot \vec{ds} = \frac{1}{\epsilon_0} \oint_v \rho \, dv \quad [\because q = \oint_v \rho \, dv]$$

Using Gauss' divergance theorem, we have

$$\oint_s \vec{E} \cdot \vec{ds} = \oint_v (\nabla \cdot \vec{E}) \, dv$$

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or $\oint_V (\nabla\cdot\vec{E})\,dV = \frac{1}{\epsilon_0}\oint_V \rho\, dV$

or $\oint_V (\nabla\cdot\vec{E} - \rho/\epsilon_0)\,dV = 0$

since, $dV \neq 0$ so $\nabla\cdot\vec{E} - \rho/\epsilon_0 = 0$

Thus $\nabla\cdot\vec{E} = \rho/\epsilon_0$ which is differential form of Maxwell's 1st equation.

II) <u>Gauss law in magnetism</u>:

$$\oint_S \vec{B}\cdot d\vec{s} = 0 \quad \text{—} (1)$$

According to equation (1) flux of magnetic induction $\vec{B}$ across any closed surface is zero.

Using Gauss' divergance theorem, we have

$$\oint \vec{B}\cdot d\vec{s} = \oint_V (\nabla\cdot\vec{B})\,dV$$

or $\oint_V (\nabla\cdot\vec{B})\,dV = 0$

since, $dV \neq 0$, so $\nabla\cdot\vec{B} = 0$ which is differential form of Maxwell's second equation.

(III) Faraday's law of electromagnetic induction:

$$\oint_l \vec{E}\cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t} \quad \text{—} (1)$$

when magnetic field varies with time, an electric field will produced.

using stoke's theorem, we have

$$\oint_l \vec{E}\cdot d\vec{l} = \oint_s (\nabla\times\vec{E})\cdot d\vec{s}$$

Thus $\oint_s (\nabla\times\vec{E})\cdot d\vec{s} = -\frac{\partial}{\partial t}\oint_s \vec{B}\cdot d\vec{s}$ $\quad \left[\because \phi = \oint \vec{B}\cdot d\vec{s}\right]$

i.e $\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ which is differential form of maxwell 3rd equation.

(iv) Ampere-Maxwell law:

$$\oint_l \vec{B}\cdot d\vec{l} = \mu_0\left(i + \epsilon_0 \frac{\partial \phi_E}{\partial t}\right)$$

As soon as electric field is applied between plates of capacitor, electrons of dielectric undergoes displacement within atom. This electronic movement constitutes displacement current. Hence, the total current through the circuit will be sum of displacement current and conduction current.

Using stoke's theorem

$$\oint_l \vec{B}\cdot d\vec{l} = \oint_s (\nabla\times\vec{B})\cdot d\vec{s}$$

or $$\oint_s (\nabla\times\vec{B})\cdot d\vec{s} = \mu_0 \oint_s \vec{J}\cdot d\vec{s} + \mu_0\epsilon_0 \frac{\partial}{\partial t}\oint_s \vec{E}\cdot d\vec{s}$$

where $I = \oint_s \vec{J}\cdot d\vec{s}$ and $\phi_E = \oint_s \vec{E}\cdot d\vec{s}$

$$\oint(\nabla\times\vec{B})\cdot d\vec{s} = \mu_0 \oint_s \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right]\cdot d\vec{s}$$

$$\therefore \nabla\times\vec{B} = \mu_0\left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right]$$

This is maxwell's fourth equation in differential form.

Imp. ✱ Differential form to integral form of maxwell's equations

(I) Gauss law in electrostatics

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \quad \text{— (1)}$$

Integrating over volume v we get

$$\oint_V (\nabla \cdot \vec{E})\, dv = \frac{1}{\epsilon_0} \oint_V \rho\, dV \quad \text{— (2)}$$

From Gauss divergance theorem

$$\oint_V (\nabla \cdot \vec{E})\, dv = \oint_S \vec{E} \cdot d\vec{s} \quad \text{— (3)}$$

From eq^n (2) and (3)

$$\oint_S \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \oint_V \rho\, dv$$

$$\oint_S \vec{E} \cdot d\vec{s} = q/\epsilon_0 \quad \text{where } \oint_V \rho\, dv = q$$

which means that flux through a closed surface is equal to $\frac{1}{\epsilon_0}$ times total charge enclosed.

(II) Gauss law in magnetism:

$$\nabla \cdot \vec{B} = 0 \quad \text{— (1)}$$

~~Integrating~~ where $\vec{B}$ is magnetic induction vector.

The eq^n (1) represents that magnetic poles exist only in pairs i.e isolated magnetic poles can not be found.

Integrating over volume v we get

$$\oint_V (\nabla \cdot \vec{B})\, dv = 0 \quad \text{— (2)}$$

From Gauss divergance theorem

$$\oint_V (\nabla \cdot \vec{B})\, dv = \oint_S \vec{B} \cdot d\vec{s} \quad \text{— (3)}$$

From equation (2) and (3)

$$\oint \vec{B} \cdot d\vec{s} = 0$$

which means magnetic flux through a closed surface is zero.

(iii) Faraday law of electromagnetic induction

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{——} (1)$$

This equation shows that time variation of magnetic induction $\vec{B}$ generates the electric field $\vec{E}$.

Integrating eq^n (1) over a bounded surface

$$\oint_s (\nabla \times \vec{E}) \cdot d\vec{s} = -\oint_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad \text{——} (2)$$

From stoke's theorem

$$\oint_s (\nabla \times \vec{E}) \cdot d\vec{s} = \oint_l \vec{E} \cdot d\vec{l} \quad \text{——} (3)$$

From eq^n (2) and (3)

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint_s \vec{B} \cdot d\vec{s}$$

$$\oint_l \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t} \quad \text{where } \phi_B = \oint_s \vec{B} \cdot d\vec{s}$$

where $\oint \vec{E} \cdot d\vec{l}$ represents as enduced emf in a closed circuit and $-\frac{\partial \phi_B}{\partial t}$ represents negative rate of change of magnetic flux through the circuit.

(iv) Ampere - Maxwell law:

$$\nabla \times \vec{B} = \mu_0 \left[\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right]$$

$$\text{or} \quad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{——} (1)$$

There are two ways of producing magnetic field by steady current $(\nabla \times \vec{B} = \mu_0 \vec{J})$ and by changing electric field $\left(\nabla\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$.

Integrating equation (1) over a bounded surface.

$$\oint_s (\nabla\times\vec{B})\cdot\vec{ds} = \mu_0\oint_s \vec{J}\cdot\vec{ds} + \mu_0\epsilon_0\frac{\partial}{\partial t}\oint_s \vec{E}\cdot\vec{ds} \quad \text{--- (2)}$$

From stoke's theorem

$$\oint_s (\nabla\times\vec{B})\cdot\vec{ds} = \oint_l \vec{B}\cdot\vec{dl} \quad \text{--- (3)}$$

From equation (2) and (3)

$$\oint_l \vec{B}\cdot\vec{dl} = \mu_0 i + \mu_0\epsilon_0\frac{\partial\phi_E}{\partial t}$$

where $i = \oint \vec{J}\cdot\vec{ds}$ and $\phi_E = \oint \vec{E}\cdot\vec{ds}$

$I_d = \epsilon_0\frac{\partial\phi_E}{\partial t}$ is called displacement current.

This equation is the restatement of Amperes law combined together for displacement and conduction currents and states that a changing electric field produces a changing magnetic field.

Imp.

Displacement current :

From Ampere maxwell law, we can write

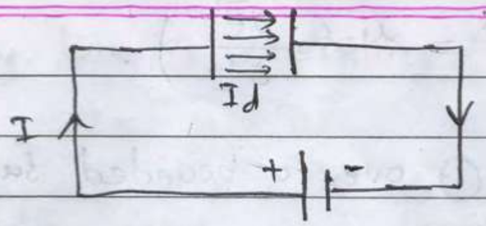
$$\oint_l \vec{B}\cdot\vec{dl} = \mu_0 I + \mu_0\epsilon_0\frac{\partial\phi_E}{\partial t}$$

$$= \mu_0(I + I_d)$$

where $I_d = \epsilon_0\frac{\partial\phi_E}{\partial t}$ is called displacement current.

To understand the displacement current concepts, let us consider a simple circuit of capacitor joined by a conducting wire.

Capacitor



Current in the circuit is equal to the rate of flow of charge i.e $I = \frac{dq}{dt}$ where q is the charge on the plate of a capacitor and is related to electric field as $E = \frac{q}{\epsilon_0 A}$; A = Area of plate of capacitor.

$$\text{so } q = \epsilon_0 E A$$

$$\text{So, } I = \frac{dq}{dt} = \frac{d}{dt}(\epsilon_0 E A) = \epsilon_0 A \frac{dE}{dt} = \epsilon_0 \frac{d(EA)}{dt}$$

$$= \epsilon_0 \frac{d\phi_E}{dt}$$

$$\boxed{\text{i.e } I = I_d = \epsilon_0 \frac{d\phi_E}{dt}}$$

This displacement current arises due to the change in electric field in the circuit.

Extra $\Rightarrow$ In the case of parallel plate capacitor

$$C = \frac{q}{V} \Rightarrow q = CV$$

$$\frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

$$\boxed{I = I_d = C\frac{dV}{dt}}$$

I_d (displacement current) $\xrightarrow{\text{is produced}}$ due to change in

- $q \rightarrow I_d = \frac{dq}{dt}$
- $V \rightarrow I_d = C\frac{dV}{dt}$
- $E \rightarrow I_d = \epsilon_0 A \frac{d\vec{E}}{dt}$
- $\phi_E \rightarrow I_d = \epsilon_0 \frac{d\phi_E}{dt}$

Imp.

Equation of continuity $\left(\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0\right)$

The rate of flow of electric charge is called electric current. The rate of loss of charge density is equivalent to the divergence of current density.

By defination $I = -\frac{dq}{dt}$ — ① where -ve sign indicats current loss.

We know, $I = \oint_s \vec{J}\cdot d\vec{s}$ and $q = \oint_v \rho\, dv$

Now, eq^n ① becomes

$$\oint_s \vec{J}\cdot d\vec{s} = -\frac{d}{dt}\oint_v \rho\, dv$$

$$\oint_s \vec{J}\cdot d\vec{s} = -\oint_v \left(\frac{\partial \rho}{\partial t}\right) dv \quad — ②$$

From Gauss divergence theorem, we have

$$\oint_s \vec{J}\cdot d\vec{s} = \oint_v (\nabla\cdot\vec{J})\, dv \quad — ③$$

From eq^n ② and ③

$$\oint_v (\nabla\cdot\vec{J})\, dv = -\oint_v \left(\frac{\partial \rho}{\partial t}\right) dv$$

Since $dv \neq 0$ $\quad \nabla\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$

i.e $\nabla\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0$ — ④

This is required equation of continuity which explains the law of conservation of electric charge.

For steady state condition, $\frac{\partial \rho}{\partial t} = 0$

Thus, $\nabla\cdot\vec{J} = 0$

For steady state condition, there should be both sources and sink.

Electromagnetic wave equations:

Maxwell's equations predict the existance of em waves propagating in free space with the speed of light.

I) Electromagnetic waves are propagated in the form of electric and magnetic fields such that both the fields are at right angles to each other and also to the direction of propagation of waves. It means that e.m waves are transverse in nature.

II) In vacuum, the e.m waves move with velocity given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ which is the velocity of light in free space.

III) Electric and magnetic fields in steady state are related by $c = \frac{\vec{E}}{\vec{B}}$.

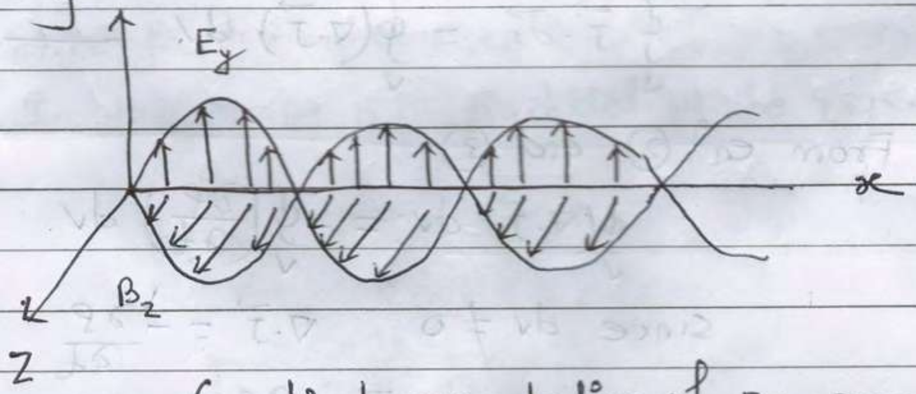


fig. Graphical presentation of EM wave.

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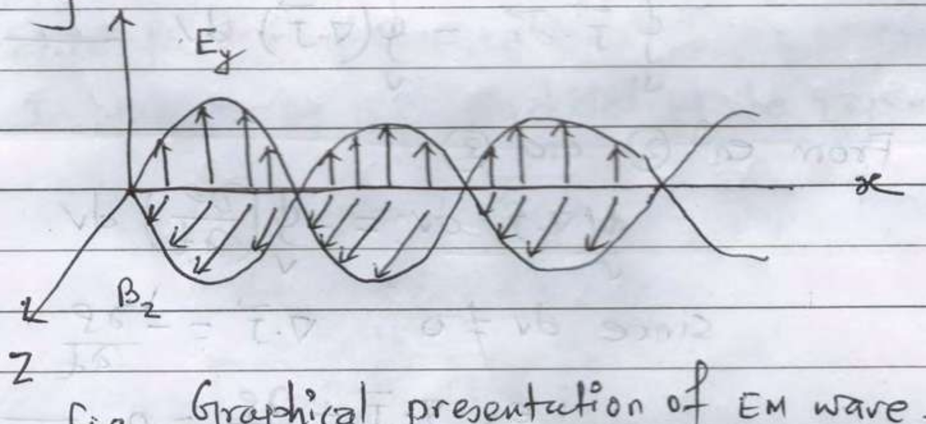


fig: Graphical presentation of EM wave.

Imp

Wave equation in free space:

In free space, charge density $\rho = 0$ and current density $J = 0$

Maxwell's equations in free space are

$$\nabla\cdot\vec{E} = 0 \quad \text{—— (1)}$$
$$\nabla\cdot\vec{B} = 0 \quad \text{—— (2)}$$
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{—— (3)}$$
$$\nabla\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{—— (4)}$$

Taking curl in both sides of equation (3), we get

$$\nabla\times(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$$

$$\nabla(\nabla\cdot\vec{E}) - (\nabla\cdot\nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$-\nabla^2\vec{E} = -\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \qquad \left[\because \nabla\cdot\vec{E} = 0\right]$$

$$\nabla^2\vec{E} = \mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2} \quad \text{—— (5)}$$

This is required electric wave equation in free space.

Also, Taking curl in both side of equation (4), we get

$$\nabla\times(\nabla\times\vec{B}) = \mu_0\epsilon_0\frac{\partial}{\partial t}(\nabla\times\vec{E})$$

$$\nabla(\nabla\cdot\vec{B}) - (\nabla\cdot\nabla)\vec{B} = \mu_0\epsilon_0\frac{\partial}{\partial t}\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$-\nabla^2\vec{B} = -\mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \qquad \left[\because \nabla\cdot\vec{B} = 0\right]$$

$$\nabla^2\vec{B} = \mu_0\epsilon_0\frac{\partial^2\vec{B}}{\partial t^2} \quad \text{—— (6)}$$

This is required magnetic wave eq^n in free space.

Also we have general wave equation

$$\cancel{\frac{\partial^2\psi}{\partial x^2}} \quad \nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{——} \quad (7)$$

where v is the speed of wave and ψ is wave amplitude

Comparing eq^n 5, 6 or 7 we get

$$\frac{1}{v^2} = \mu_0\epsilon_0$$

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$\mu_0 = 4\pi \times 10^{7} \ Wb\, A^{-1} m^{-1}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \ C^2 N^{-1} m^{-2}$$

The speed of ~~light v~~ = wave $v = 3\times10^8$ m/s.

which is the speed of light. so we can replace v by c and write the speed of em wave as

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8 \ m/s.$$

Electromagnetic wave in Dielectric medium (Non-conducting medium)

In case of insulator, there is no charge so the charge density is zero i.e $\rho = 0$. Also there is no current and current density is also zero i.e $J = 0$.

The maxwell's equations in dielectric medium are given by

$$\nabla \cdot \vec{E} = 0 \quad \text{--- (1)}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{--- (2)}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\nabla \times \vec{B} = \mu \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

Taking curl on both side of equation (3)

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\nabla(\nabla \cdot \vec{E}) - (\nabla \cdot \nabla)\vec{E} = -\frac{\partial}{\partial t}\left(\mu \epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{--- (5)}$$

This is the required electric wave equation in dielectric medium

Also, taking curl on both side of equation (4)

$$\nabla \times (\nabla \times \vec{B}) = \mu \epsilon \frac{\partial}{\partial t}(\nabla \times \vec{E})$$

$$\nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B} = \mu \epsilon \cdot \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$-\nabla^2 \vec{B} = -\mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{--- (6)}$$

This is required magnetic wave equation in dielectric medium.

Also we have general wave equation

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{--- (7)}$$

where v is the speed of wave and ψ is wave amplitude.

Comparing eq^n 5, 6 and (7)

we have $v = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}$

where $\mu = \mu_r \mu_0$ and $\epsilon = \epsilon_r \epsilon_0$ and $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

Here $\mu_r, \epsilon_r > 1$ and $v < c$ i.e in dielectric medium, the wave propagates with the velocity lesser than the velocity of light ($v < c$).

Electromagnetic waves in conducting media :

The Maxwell's electromagnetic equations are

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{---} (1)$$

$$\nabla \cdot \vec{B} = 0 \quad \text{---} (2)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{---} (3)$$

$$\nabla \times \vec{B} = \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t} \quad \text{---} (4)$$

Here ϵ and μ are permittivity and permeability of medium

Taking curl on both side of eq^n (3), we get

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t}(\nabla \times \vec{B})$$

$$\nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$-\nabla^2 \vec{E} = -\mu \frac{\partial \vec{J}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu \frac{\partial}{\partial t}(\sigma \vec{E}) + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{E} = \mu \sigma \frac{\partial \vec{E}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad \text{---} (5)$$

Similarly for magnetic field

$$\nabla^2 \vec{B} = \mu \sigma \frac{\partial \vec{B}}{\partial t} + \mu \epsilon \frac{\partial^2 \vec{B}}{\partial t^2} \quad \text{---} (6)$$

The equation (5) and (6) are required wave equation in conducting medium. The wave propagate with the velocity v and it also depends on the conductivity of the medium.

Imp.

Ratio of electric and magnetic field $\left(\frac{E}{B} = \frac{E_0}{B_0} = c\right)$

The sinusoidally varying electric and magnetic fields of a plane electromagnetic wave are given as

$$\vec{E} = \vec{E}_0 \cos(kx - \omega t) \quad \text{—— (1)}$$
$$\vec{B} = \vec{B}_0 \cos(kx - \omega t) \quad \text{—— (2)}$$

where E_0 and B_0 are the maximum values of the electric and magnetic fields associated with the electromagnetic waves.

Also K the propagation constant $= \frac{2\pi}{\lambda}$, where λ is the wavelength of electromagnetic wave and $\omega (2\pi f)$ is angular frequency of the wave.

$$\frac{\omega}{k} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c \quad \text{—— (3)}$$

Taking partial differentiation of eqn (1) and (2), we have

$$\frac{\partial E}{\partial x} = -k\vec{E}_0 \sin(kx - \omega t)$$

and $$\frac{\partial B}{\partial t} = \omega \vec{B}_0 \sin(kx - \omega t)$$

Now from Maxwell's third equation (Faraday's law of electromagnetic induction)

$$\frac{\partial \vec{E}}{\partial x} = -\frac{\partial \vec{B}}{\partial t}$$

$$\therefore -k\vec{E}_0 \sin(kx - \omega t) = -\omega \vec{B}_0 \sin(kx - \omega t)$$

$$k\vec{E}_0 = \omega \vec{B}_0$$

$$\frac{\vec{E}_0}{B_0} = c \quad \text{From eq}^n \text{ (3)}$$

From eq^n ① and ②

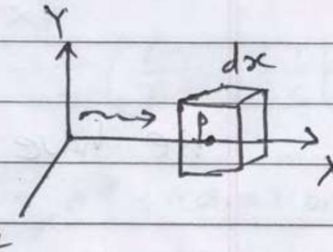
$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

so at every instant the ratio of the electric field to the magnetic field of an electromagnetic wave is equal to the speed of light.

Imp. <u>Poynting vector :</u>

The rate of energy transport per unit area in a plane electromagnetic wave is called magnitude of poynting vector which is denoted by S.

$$S = \frac{1}{A}\frac{dU}{dt} \quad \text{—①}$$



Consider propagation of wave in a box of area A and thickness dx.
At any instant energy stored in the box A is given by

$$dU = dU_E + dU_B = (u_E + u_B)\,A\,dx$$

where $u_E = \frac{1}{2}\epsilon_0 E^2$ and $u_B = \frac{1}{2\mu_0}B^2$ are energy densities in electric and magnetic fields.

Energy stored in the box A is

$$dU = \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2\right)A\,dx \quad \text{—②}$$

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From eq^n ① and ②

$$S = \frac{1}{\cancel{A}}\left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}\frac{E^2}{c^2}\right)\frac{dx}{dt}\cancel{A}$$

$$= \left(\frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2\right) c. \quad \left[\because \frac{E}{B} = c\right]$$

$$= \epsilon_0 E^2 c$$

$$= \epsilon_0 E (Bc) c$$

$$= \epsilon_0 E B c^2$$

$$= \epsilon_0 E B \frac{1}{\mu_0 \epsilon_0} = \frac{1}{\mu_0}(EB)$$

Thus the magnitude of poynting vector is

$$S = \frac{1}{\mu_0} EB$$

In vector notation

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}.$$

We have equation for electric and magnetic field as

$$\vec{E} = \vec{E_0}\cos(kx - \omega t)$$

$$\vec{B} = \vec{B_0}\cos(kx - \omega t)$$

$$\text{so} \quad S = \frac{E_0 B_0}{\mu_0}\cos^2(kx - \omega t)$$

If S_{av} is the average value of S taken over a complete cycle

$$S_{av} = \frac{E_0 B_0}{2\mu_0} \quad \text{where } \left[\cos^2(kx - \omega t)\right]_{av} = \frac{1}{2}.$$

Thus average value of poynting vector (S_{av}) is also called intensity of the wave.

Radiation pressure:

An electromagnetic wave transport energy as well as linear momentum. As this momentum is absorbed by the surface of medium pressure is exerted on it.

Let a parallel beam of radiation fall on an object for a time Δt and it is totally absorbed by the object. Maxwell assume that if the surface absorbs ΔW amount of energy than the magnitude of change in momentum Δp is

$$\Delta p = \frac{\Delta W}{c} \quad \text{--- (1)}$$

This represents the total absorption of the radiation where c is velocity of light.

From Newton's law of motion, the change in pressure is

$$P_a = \frac{\Delta F}{A} = \frac{1}{A}\left(\frac{\Delta p}{\Delta t}\right) = \frac{1}{A}\left(\frac{\Delta W}{c\Delta t}\right) = \frac{1}{Ac}\left(\frac{\Delta W}{\Delta t}\right) = \frac{1}{c}\left(\frac{\Delta W}{A\Delta t}\right)$$

$$P_a = \frac{I}{c} \quad \text{where } I = \text{intensity of radiation}$$

This is also equal to magnitude of poynting vector (S_{av}).

For direct sun light, the value of I before it passes through the earth's atmosphere is approximately $1.4\ kW/m^2$. The corresponding average pressure on completely absorbing surface is

$$P_a = \frac{1.4 \times 10^3}{3 \times 10^8} = 4.7 \times 10^{-6}\ Pa.$$

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Imp.

Why did Maxwell modify Ampere's law. Explain with mathematical details (9 marks)

Ampere's law states that, "The line integral of magnetic field around a closed loop is equal to μ_0 times the current enclosed by that loop.

i.e $\oint_l \vec{B} \cdot d\vec{l} = \mu_0 I$

Using the stoke's theorem

$\oint_s (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \oint_s \vec{J} \cdot ds$

Thus $\nabla \times \vec{B} = \mu_0 J$ —— (1)

Now, taking divergence of above equation

$\nabla \cdot (\nabla \times \vec{B}) = \mu_0 \nabla \cdot \vec{J}$

$0 = \mu_0 \nabla \cdot \vec{J}$

Since $\mu_0 \neq 0$ so $\nabla \cdot \vec{J} = 0$

But the equation of continuity is $\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$

This shows that Ampere's law doesnot satisfy the continuity equation. So, maxwell modified Ampere's law for the changing electric field. Hence he added a term J_d in eq^n (1)

Hence, the modified Ampere's law becomes

$\nabla \times \vec{B} = \mu_0 (J + J_d)$

Again taking the divergence

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0\,\nabla\cdot(\vec{J}+\vec{J}_d)$$

$$0 = \mu_0\,\nabla\cdot(\vec{J}+\vec{J}_d)$$

since $\mu_0 \neq 0$

$$\nabla\cdot(\vec{J}+\vec{J}_d) = 0$$

$$\nabla\cdot\vec{J} + \nabla\cdot\vec{J}_d = 0$$

$$\nabla\cdot\vec{J}_d = -\nabla\cdot\vec{J} \quad \text{---} (2)$$

Again from continuity equation

$$\nabla\cdot\vec{J} + \frac{d\rho}{dt} = 0 \quad \text{---} (3)$$

Using equation (3) in equation (2)

$$\nabla\cdot\vec{J}_d = \frac{d\rho}{dt}$$

From Maxwell first equation

$$\nabla\cdot\vec{E} = \rho/\epsilon_0$$

$$\rho = \epsilon_0\,\nabla\cdot\vec{E}$$

$$\therefore\ \nabla\cdot\vec{J}_d = \frac{d}{dt}(\epsilon_0\,\nabla\cdot\vec{E})$$

$$= \nabla\cdot\epsilon_0\frac{d\vec{E}}{dt}$$

Thus $\vec{J}_d = \epsilon_0\dfrac{d\vec{E}}{dt}$ which is the added term in the Ampere's law in the differential form.

Hence, the modified Ampere's law becomes

$$\nabla\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{d\vec{E}}{dt}\right) \text{---} (4)$$ which is Maxwell fourth equation in differential form.

Taking the surface integral of equation (4)

$$\oint_s (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \left[\oint_s \vec{J} \cdot \vec{ds} + \epsilon_0 \frac{d}{dt} \oint_s \vec{E} \cdot \vec{ds} \right]$$

Using stoke's theorem in LHS

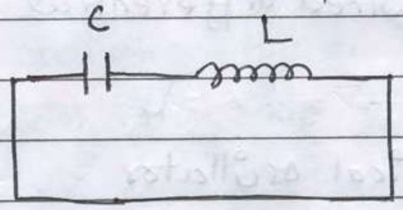
$$\oint_l \vec{B} \cdot d\vec{l} = \mu_0 \left(I + \epsilon_0 \frac{d\phi_E}{dt} \right)$$

This is the required Maxwell ~~eq~~ 4^{th} equation in integral form.

Here $I_d = \epsilon_0 \frac{d\phi_E}{dt}$ is the displacement current which arises due to change in electric flux.

and $\vec{J_d} = \epsilon_0 \frac{d\vec{E}}{dt}$, is the displacement current density, which arises due to change in electric field.

Imp — Free Oscillation / Undamped / LC Oscillation



consider a fully charged capacitor connected in series with a source of emf. After full charged the battery is then disconnected and inductor is conneeted in series with the capacitor.

The capacitor stores the electric energy and the inductor stores magnetic energy.

$$U_E = \frac{1}{2}\frac{q^2}{C}$$

$$U_B = \frac{1}{2} L I^2$$

Since, total energy is conserved / constant such that

$$\frac{dU}{dt} = 0$$

$$\therefore \frac{d}{dt}(U_E + U_B) = 0$$

$$\frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2\right) = 0$$

$$\frac{q}{C}\frac{dq}{dt} + LI\frac{dI}{dt} = 0$$

But $I = \frac{dq}{dt}$ and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

Thus, $LI\frac{d^2q}{dt^2} + \frac{q}{C}I = 0$

$$I\left(L\frac{d^2q}{dt^2} + \frac{q}{C}\right) = 0$$

$$\frac{d^2q}{dt^2} + \left(\frac{1}{LC}\right)q = 0 \quad \text{— (1)} \quad (\text{as } I \neq 0)$$

Equation (1) describes the oscillation of a resistanceless LC circuit. It is required differential equation for EM oscillations.

In case of a mechanical oscillator

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- (2)}$$

The solution of equation (2) is given as

$$x = x_m \cos(\omega t + \phi)$$

By analogy of electromagnetic oscillator with mechanical oscillator, we have the solution of equation (1)

$$q = q_m \cos(\omega t + \phi)$$

The angular frequency $\omega = \frac{1}{\sqrt{LC}}$

since $2\pi f = \frac{1}{\sqrt{LC}}$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

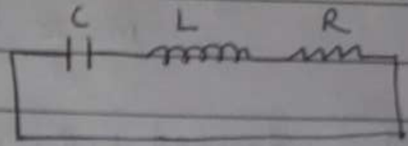
This is frequency of electromagnetic oscillation.

(Undamped frequency).

Damped EM oscillations (LCR oscillation):

In damped oscillation, the amplitude of oscillation decrease exponentially with time due to external resistive force.

consider a fully charged capacitor connected in series with an inductor and a resistor



The total energy stored by capacitor and inductor is given by

$$U = U_E + U_B$$
$$= \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}LI^2 \quad -①$$

U is no longer constant, but

$$\frac{dU}{dt} = -I^2R \quad -②$$

The -ve sign represents that the stored energy U decrease with time.

From eqn ① and ②

$$LI\frac{dI}{dt} + \frac{q}{C}\frac{dq}{dt} = -I^2R$$

$$LI\frac{dI}{dt} + \frac{q}{C}\frac{dq}{dt} + I^2R = 0$$

$$LI\frac{d^2q}{dt^2} + \frac{q}{C}I + I^2R = 0$$

$$I\left(L\frac{d^2q}{dt^2} + R\frac{dI}{dt} + \frac{q}{C}\right) = 0$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \quad ③ \quad (\text{as } I \neq 0)$$

Equation ③ is the required differential equation for damped LCR oscillation.

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Equation (3) can be compared to the mechanical damped oscillation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + Kx = 0 \quad \text{—— (4)}$$

On comparing eq^n (3) and (4)

$$m = L,\ b = R \text{ and } K = \frac{1}{C}.$$

Thus, damped LCR oscillation is equivalent to the mechanical damped oscillation.

The angular frequency $\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$

$$2\pi f' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$f' = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

This is required frequency of damped oscillator.

Now the solution of differential equation (4) is

$$x = x_m e^{-bt/2m} \cos(\omega' t + \phi)$$

The charge in the damped oscillation is given by

$$q = q_m e^{-\frac{Rt}{2L}} \cos(\omega' t + \phi).$$

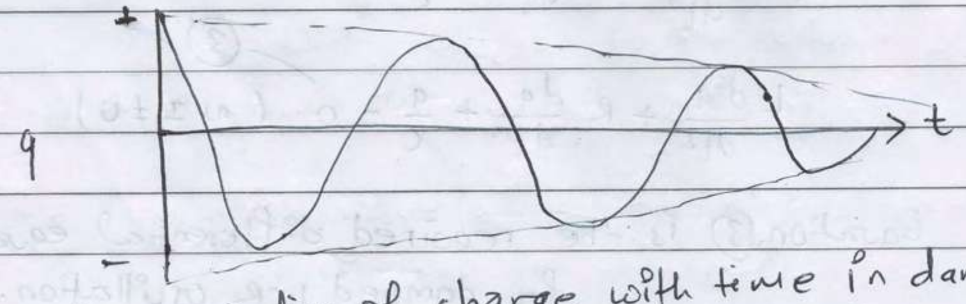
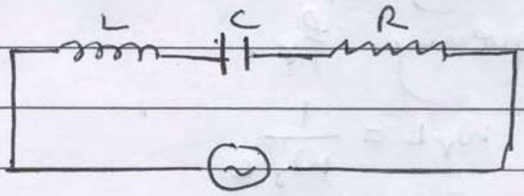


fig. The variation of charge with time in damped oscillation.

Forced or Driven EM oscillation and Resonance:

If we apply external frequency by means of a.c frequency generator to the LCR oscillations, such type of oscillation becomes forced oscillation.



$E = E_m \cos \omega_d t$

By the principle of conservation of energy

$$L\frac{dI}{dt} + \frac{q}{c} + IR = E_m \cos \omega_d t$$

~~Bet~~ we know, $I = \frac{dq}{dt}$ and $\frac{dI}{dt} = \frac{d^2q}{dt^2}$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E_m \cos \omega_d t$$

$$\frac{d^2q}{dt^2} + \left(\frac{R}{L}\right)\frac{dq}{dt} + \left(\frac{1}{LC}\right)q = \left(E_m/L\right)\cos \omega_d t \quad \text{--- (1)}$$

This is the required differential equation for forced EM oscillation, whose solution is given by

$$q = q_m \sin(\omega_d t - \phi)$$

The impedance of LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Inductive reactance $X_L = 2\pi f L$

Capacitive reactance $X_C = \frac{1}{2\pi f C}$

At resonance, $X_L = X_C$ at which current becomes maximum and impedence becomes minimum.

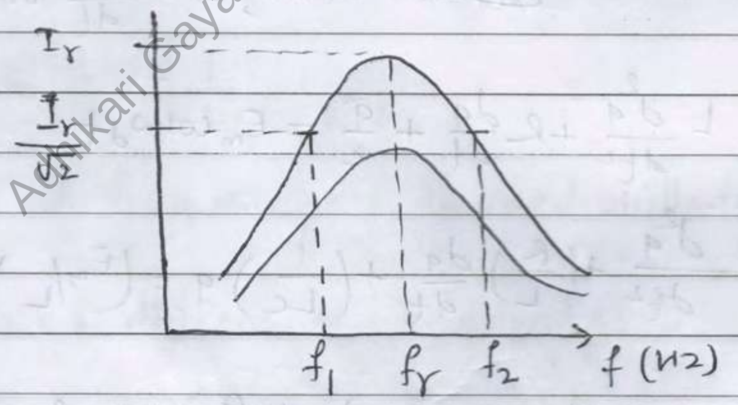
At this condition applied frequency is equal to natural frequency of oscillation which is called resonant frequency f_r.

$$\omega_r L = \frac{1}{\omega_r C}$$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

This is required resonant frequency of LCR ~~circu~~ circuit.



Quality factor $Q = \frac{\omega_r}{f_2 - f_1}$

$= \frac{2\pi f_r}{f_2 - f_1}$ where f_1 and f_2 are frequency at $\frac{I_r}{\sqrt{2}}$.

At resonance, Quality factor for inductance $Q_L = \frac{\omega_r L}{R}$

Quality factor for capacitance $Q_C = \frac{1}{\omega_r C R}$

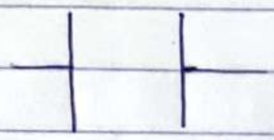
Band width of circuit is given by $\Delta f = f_2 - f_1$

For the smallest value of R, the highest peak of the curve can be obtained.

Capacitors

It is an electric device used for storing charge.

Theory $q \propto v$



Symbol of capacitor   Battery

It is ~~po~~ found that $\boxed{q \propto v}$

where q = charge

v = potential

v.v. Imp → $\boxed{q = CV}$

where C = capacitance

Define capacitance

Since $C = \frac{q}{v}$

Hence the capacitance of the capacitor is defined as the charge store per unit potential applied between the plates of the capacitor.

Unit:

C → Coulomb / volt (CV^{-1})

→ Farad (F)

$1F \rightarrow 1CV^{-1}$

Factors

m = milli : mF $= 10^{-3}$

μ = micro : μf $= 10^{-6}$

n = nano : nF $= 10^{-9}$

p = pico : p = μμf $= 10^{-12}$

Types of capacitors (4)

1. parallel plate capacitor
2. Isolated spherical capacitor
3. Spherical capacitor
4. Cylindrical capacitor

Task:
To find C

Technique
1. First find E, electric field
2. Then find V, potential
3. At last $C = q/V$

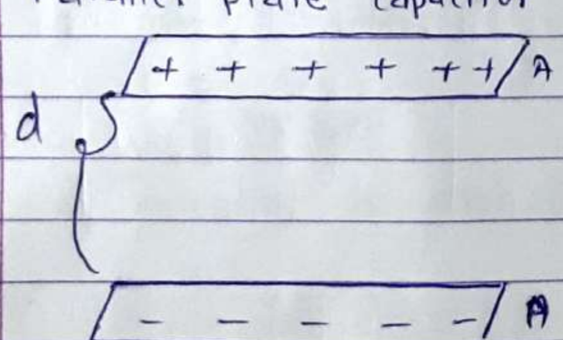
EVC

PVC

formulae

1. Parallel
$$C = \frac{\varepsilon_0 A}{d}$$
2. Isol.
$$C = 4\pi\varepsilon_0 R$$
3. Spherical
$$C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$$
4. Cylindrical
$$C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

1. Parallel plate capacitor



Consider a parallel plate capacitor having '+q' charge on its upper plate such that '-q' charge is developed on the lower plate

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The plate separation is 'd' and plate area is 'A'. Now the electric field is given by,

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

$$= \frac{q}{\varepsilon_0 A}$$

$$V = \int \vec{E} \cdot \vec{dr}$$

$$= \int E\,dr \cos 0^\circ$$

$$= \int_0^d \frac{q}{\varepsilon_0 A}\, dr$$

$$= \frac{q}{\varepsilon_0 A} \int_0^d dr$$

$$= \frac{q}{\varepsilon_0 A} d$$

Then, $C = \frac{q}{V}$ $\left[\because q = CV \right]$

$$\therefore \boxed{C = \frac{\varepsilon_0 A}{d}}$$

In air, $C_{air} = \frac{\varepsilon_0 A}{d}$

If a dielectric (insulator) is inserted,

$C_{med} = \frac{\varepsilon_m A}{d}$

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$$\therefore \frac{C_{med}}{C_{air}} = \frac{\varepsilon_m}{\varepsilon_0}$$

$$= \frac{\cancel{\varepsilon_0} \varepsilon_r}{\cancel{\varepsilon_0}}$$

$$= \varepsilon_r = K > 1$$

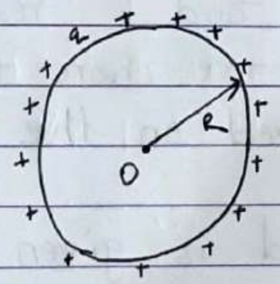
$$\therefore \boxed{C_{med} > C_{air}}$$

Note:

$C \propto A$

$C \propto \frac{1}{d}$

② Isolated spherical capacitor



Consider an isolated spherical capacitor having charge 'q' and radius 'R'. The electric field is given by,

$$E = \frac{q}{4\pi \varepsilon_0 R^2}$$

& potential is given by

$$V = \int \vec{E} \cdot \vec{dr}$$

$$= \frac{q}{4\pi \varepsilon_0 R}$$

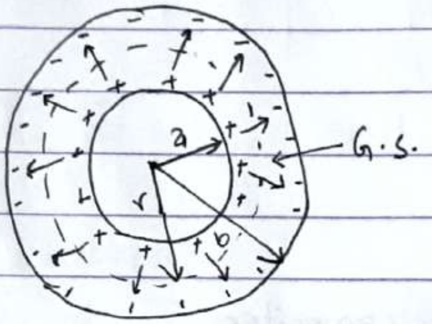
① Parallel:
$C = \frac{\varepsilon_0 A}{d}$
② Isol.
$C = 4\pi\varepsilon_0 R$

Then,

$$C = \frac{q}{V}$$

$$\therefore \boxed{C = 4\pi\varepsilon_0 R}$$

Spherical capacitor:



A spherical capacitor consists of two concentric spheres of radii 'a' and 'b' respectively. The inner sphere is given +ve chare '+q' such that '-q' charge is developed on the outer sphere

Then, the electric field is given by,

$$E = \frac{\sigma}{\varepsilon_0}$$

$$= \frac{q}{A\varepsilon_0}$$

where A = Area of the Gaussian surface of radius 'r' such that $a < r < b$

$$A = 4\pi r^2$$

$$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

③ Spherical

$$C = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)$$

④ Cylindrical

$$C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

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Then the potential is given by

$$V = \int_a^b \vec{E} \cdot \vec{dr}$$

$$V = \int_a^b E\, dr \cos 0^\circ$$

$$V = \int_a^b \frac{q}{4\pi\varepsilon_0 r^2}\, dr$$

$$V = \frac{q}{4\pi\varepsilon_0} \int_a^b r^{-2}\, dr$$

$$V = \frac{q}{4\pi\varepsilon_0} \left[\frac{r^{-2+1}}{-2+1}\right]_a^b$$

$$V = -\frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r}\right]_a^b$$

$$V = -\frac{q}{4\pi\varepsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{b-a}{ab}\right)$$

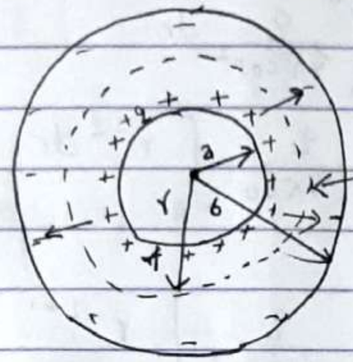
Then, capacitance is given by,

$$C = \frac{q}{V}$$

$$= \frac{\cancel{q}}{\frac{\cancel{q}}{4\pi\varepsilon_0}\left(\frac{b-a}{ab}\right)}$$

$$C = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)$$

Cylindrical capacitor



A cylindrical capacitor consists of two co-axial ~~coaxial~~ of radii 'a' and 'b' $(a<b)$ and length 'l'. The inner cylinder is given '+q' charge such that the outer cylinder has '-q' charge by induction

Now, the electric field is given by,

$$E = \frac{\sigma}{\varepsilon_0}$$

$$= \frac{q}{\varepsilon_0 A}$$

Where A = area of a Gaussian surface of radius of cylinder 'r' through which the electric flux passes. such that $a<r<b$ & length 'l'

$$A = 2\pi r l$$

$$\therefore E = \frac{q}{2\pi\varepsilon_0 r l}$$

Now,

The potential is given by.

$$V = \int \vec{E} \cdot \vec{dr}$$

$$= \int_a^b E\, dr \cdot \cos 0^\circ$$

$$= \frac{q}{2\pi\varepsilon_0 l} \int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi\varepsilon_0 l} \int_a^b \frac{dr}{r}$$

$$= \frac{q}{2\pi\varepsilon_0 l} \left|\ln r\right|_a^b$$

$$= \frac{q}{2\pi\varepsilon_0 l} (\ln b - \ln a)$$

$$= \frac{q}{2\pi\varepsilon_0 l} \ln\left(\frac{b}{a}\right)$$

$$C = \frac{q}{V}$$

$$= \frac{\cancel{q}}{\frac{\cancel{q}}{2\pi\varepsilon_0 l} \ln\left(\frac{b}{a}\right)}$$

$$= \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

$E, R, C \Rightarrow$ const
$q, I, t \Rightarrow$ variable

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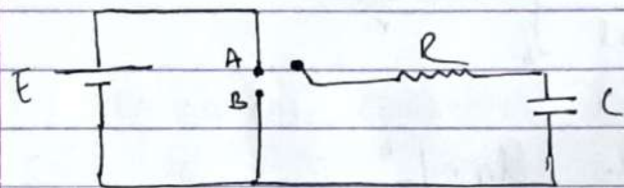
RC circuit → (lab)

Discuss the charging and discharging of capacitor through resistors. (9 marks)

OR

Give the theory of RC circuits with graphs. (9 marks)



Consider a circuit containing a resistor and capacitor connected in series with a source of e.m.f 'E'.

for charging
S is connected to the pointer 'A' then Kirchoff's voltage law (2nd law) gives

Σemf = Σp.d (conservation of energy)

$$E = V_C + V_R$$
$$= IR + \frac{q}{C} \qquad \left[\because \text{cap} = q = CV \right] \quad \Rightarrow V = V_C = q/C$$

$$= R\frac{dq}{dt} + \frac{q}{C}$$

or, $EC = RC\frac{dq}{dt} + q$

Recall:
$q = CV$ maxm
$q_0 = CE$ maxm

Known: $q_0 = CE$, C, E, R

q & t

$q = f(t)$

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or, $q_0 = RC \frac{dq}{dt} + q$

where $q_0 = CE$, maximum or peak or equilibrium or steady state or final equilibrium charge

or, $q_0 - q = RC \frac{dq}{dt}$

or, $\frac{dq}{q_0 - q} = \frac{dt}{RC}$

On Integrating,

$-\ln(q_0 - q) = \frac{t}{RC} + A$ --- (i)

where 'A' is the constant of integration

To find A :

At $t = 0$, $q = 0$

$\therefore$ eq^n (i) becomes

$-\ln q_0 = A$ --- (ii)

from (i) & (ii)

$-\ln(q_0 - q) = \frac{t}{RC} - \ln q_0$

$\cancel{\ln q_0 - \ln(q_0 - q) = \frac{t}{RC}}$

$-[\ln(q_0 - q) - \ln q_0] = \frac{t}{RC}$

or, $\ln\left(\frac{q_0 - q}{q_0}\right) = -\frac{t}{RC}$

Define time constant interms of current

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Taking antilog,

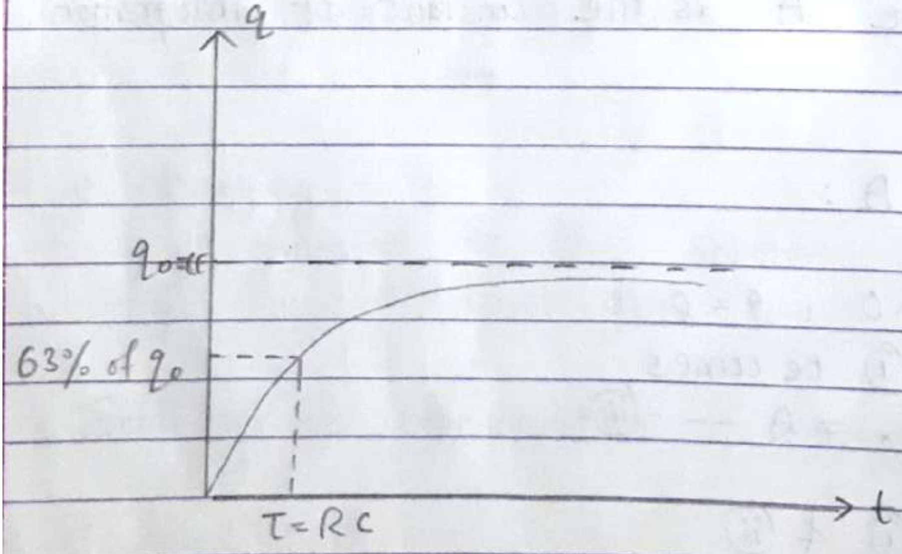
$$\frac{q_0 - q}{q_0} = e^{-t/RC}$$

$$\therefore q_0 - q = q_0 e^{-t/RC}$$

or, $$\boxed{q = q_0 (1 - e^{-t/RC})} \quad \text{--- (iii)}$$

Eq^n (iii) is the required expression for the charge stored at time 't' during charging of the capacitor through resistor.

Graph of q vs t



Capacitive time constant (τ or τ_c):

In eq^n (iii)

$\tau = \tau_c = RC$ is called the capacitive time constant

If $t = \tau = RC$, eq^n (iii) becomes,

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$$q = q_0 (1 - e^{-1})$$
$$q = q_0 \left(1 - \frac{1}{2.718}\right)$$
$$= 0.63\, q_0$$
$$= 63\% \text{ of } q_0$$

Hence the capacitive time constant is defined as that time at which the charge becomes or rises to 63% of the peak charge.

Current during charging (I):-

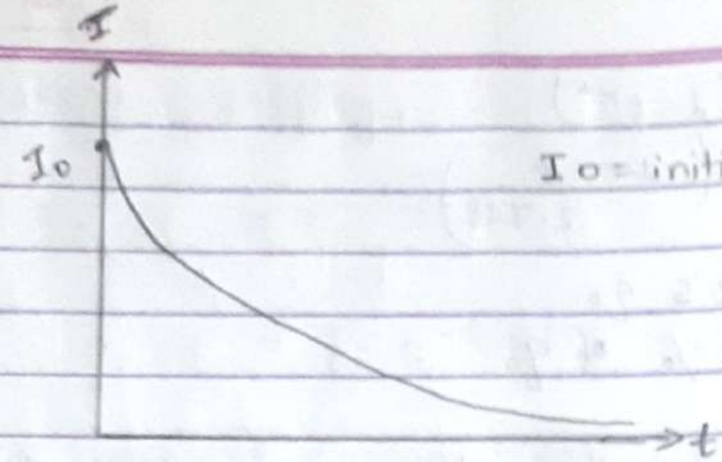
$$I = \frac{dq}{dt}$$
$$= \frac{d}{dt}\left[q_0 (1 - e^{-t/RC})\right]$$
$$= \cancel{\frac{dq_0}{dt} - \frac{d\, q_0 e^{-t/RC}}{dt}}$$
$$= q_0 \frac{d(1 - e^{-t/RC})}{d(-t/RC)} \cdot \frac{d(-t/RC)}{dt}$$
$$= q_0 (0 - e^{-t/RC})\left(-\frac{1}{RC}\right)$$

or, $I = \frac{q_0}{RC} e^{-t/RC}$

Again,

$$q_0 = CE$$
$$\therefore I = \frac{\cancel{C}E}{R\cancel{C}} e^{-t/RC}$$
$$= \frac{E}{R} e^{-t/RC}$$
$$\boxed{I = I_0 e^{-t/RC}}$$

Recall:
$V = IR$
$V/R = I$
$E/R = I_0$, peak current



Io = initial / max / peak current

For Discharging:-

For a discharging ~~series~~ switch 's' is connected to the pointer 'B'.

Then kirchoff's voltage law gives

$$0 = V_R + V_C \qquad [\because \Sigma e.m.f = 0]$$

or, $0 = IR + \frac{q}{c}$

or, $0 = R\frac{dq}{dt} + \frac{q}{c}$

or, $R\frac{dq}{dt} = -\frac{q}{c}$

or, $\frac{dq}{q} = -\frac{dt}{RC}$

On, Integrating

$$\ln q = -\frac{t}{RC} + B \quad \text{--- (1)}$$

Where B is the constant of integration.

To find B:

At $t=0$, $q=q_0$

$\therefore$ eq^n (i) becomes

$$\ln q_0 = -\frac{\overset{\nearrow 0}{t}}{RC} + B$$

$$\ln q_0 = B \quad \text{--- (ii)}$$

So, (i) & (ii) gives

$$\ln q = -\frac{t}{RC} + \ln q_0$$

$$\ln q - \ln q_0 = -t/_{RC}$$

$$\ln\left(\frac{q}{q_0}\right) = -t/_{RC}$$

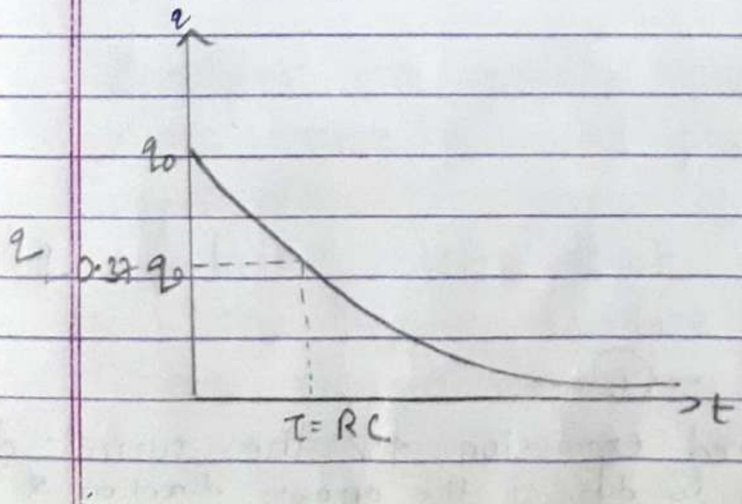
Taking antilog,

$$\frac{q}{q_0} = e^{-t/RC}$$

$$\therefore \boxed{q = q_0 e^{-t/RC}} \quad \text{--- (iii)}$$

Eq^n (iii) is the required expression for the ch at time 't' during discharging.

Graph of q vs t

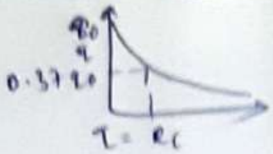


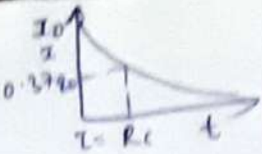
Discharging

$q\downarrow$ $I\downarrow$

$q = q_0 e^{-t/RC}$

$I = -I_0 e^{-t/RC}$





Capacitive Time constant (τ or τ_c)

In eq^n (iii)

$\tau = \tau_c = RC$ called capacitive time constant.

If $t = \tau = RC$, eq^n (iii) becomes

$$q = q_0\left(\frac{1}{e}\right)$$

$$q = 0.37q_0$$
$$= 37\% \text{ of } q_0$$

Hence the capacitive time constant is ~~the~~ defined as that time in which the charge decreases to 37% of its initial value during discharging of capacitor.

Current during discharging (I)

$$I = +\frac{dq}{dt}$$

$$= +\frac{d}{dt}\left[q_0 e^{-t/RC}\right]$$

$$= +q_0 \frac{d(e^{-t/RC})}{d(-t/RC)} \cdot \frac{d(-t/RC)}{dt}$$

$$= +q_0 e^{-t/RC}\left(-\frac{1}{RC}\right)$$

$$I = -\frac{q_0}{RC} e^{-t/RC}$$

Again,

$$q_0 = CE$$
$$I = -\frac{CE}{RC} e^{-t/RC}$$

$$\boxed{I = -I_0 e^{-t/RC}} \quad \text{— (iv)}$$

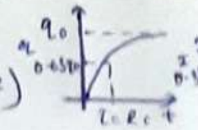
Eq^n (iv) is the required expression for the current during discharging. The -ve sign is due to the opposite direction of flow of charge or current during discharging as compared to that

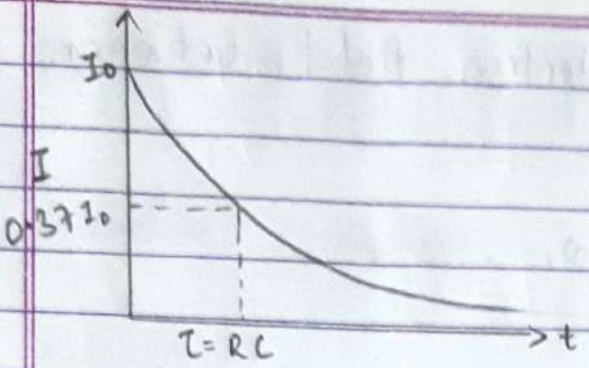
Charging

$q\uparrow \quad I\downarrow$

$q = q_0(1 - e^{-t/RC})$

$I = I_0 e^{-t/RC}$





dielectric → Insulator

Summary:

Tips for numericals:

RC
- Step I (Confirm): Charging
  - Step 2: charge (q) — Step 3: $q = q_0(1-e^{-t/RC})$ → q increases
  - current (I) — $I = I_0 e^{-t/RC}$ → I decrease
- discharging
  - charge (q) — $q = q_0 e^{-t/RC}$ → q decrease
  - current (I) — $I = -I_0 e^{-t/RC}$ → I decrease

Gauss law in dielectric (Theory / Num)

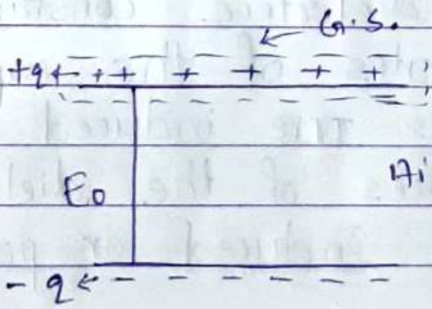


fig (i)

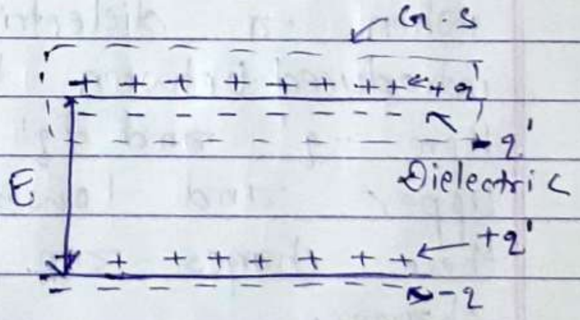


fig (ii)

Consider a parallel plate capacitor having plate area (A) and plate seperation (d). The upper plate is given '+q' charge on the inner surface such that '-q' charge is induced on the upper surface of the lower plate. These charges ~~are~~ are called free charge. Air is in between the plates of the capacitor such

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that E_0 is the electric field between the plates.

Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

or, $\oint_S \vec{E_0} \cdot \vec{dA} = \frac{1}{\varepsilon_0} q$

or, $E_0 \cdot A = \frac{q}{\varepsilon_0}$

or, $\boxed{E_0 = \frac{q}{\varepsilon_0 A}}$ ---- (i)

v.v. Imp

for figure (ii), let E be the electric field when a dielectric of dielectric constant is introduced between the plates of the capacitor. Here $-q'$ and $+q'$ charges are induced on the upper and lower surfaces of the dielectric. Then these charges are called induced or polarized charges.

Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

or, $\oint_S \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} (q - q')$

or, $EA = \frac{q}{\varepsilon_0} - \frac{q'}{\varepsilon_0}$

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V.V.Imp

or, $\boxed{E = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}}$ — (ii)

~~Since~~ Here, ~~$E_0 < E_0$~~

Note:

[Here $E_0 > E$]

Since

$$\boxed{K = \frac{E_0}{E}}$$

or, $E = \frac{E_0}{K}$

Using (i)

$$\boxed{E = \frac{q}{\varepsilon_0 A K}}$$ --- (iii)

from (ii) & (iii)

$$\frac{q}{\varepsilon_0 A K} = \frac{q}{\varepsilon_0 A} - \frac{q'}{\varepsilon_0 A}$$ — (iv)

~~$\frac{q}{k} = (q - q')$~~

Two tasks:-

1st task:

To prove $q' < q$

from (iv)

$$\frac{q'}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 A} - \frac{q}{\varepsilon_0 A K}$$

$$q' = q - \frac{q}{K}$$

$\rightarrow \boxed{q' = q\left(1 - \frac{1}{K}\right)}$ — (v)

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Eq^n (v) is the required relation between q' (induced charge) and q (free charge).

Since $k > 1$

$q' < q$

Hence the induced charge is always less than the free charge.

Note:

For air,

$K = 1$

$\therefore q' = 0$

2nd Task:

To prove : $[D = \varepsilon_0 E + P]$

or. $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

from eq^n (iv),

$$\frac{q}{\varepsilon_0 A} = \frac{q}{\varepsilon_0 AK} + \frac{q'}{\varepsilon_0 A}$$

cancel ε_0

Or, $$\frac{q}{A} = \varepsilon_0\left(\frac{q}{\varepsilon_0 AK}\right) + \frac{q'}{A} \quad \text{-- (vi)}$$

Here, $D = q/A = \frac{\text{free charge}}{\text{Area}}$, magnitude of the displacement vector

$E = \frac{E_0}{K} = \frac{q}{\varepsilon_0 AK}$, magnitude of the electric field in dielectric

w/q

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$P = \frac{q'}{A} = \frac{\text{induced or polarized charged}}{\text{Area}}$, magnitude of the polarization vector

$\therefore$ Eq^n (6) becomes

$$D = \varepsilon_0 E + P$$

In vector form,

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

This is the required relation between the three electric vectors $\vec{D}$, $\vec{E}$ and $\vec{P}$. This is the Gauss law in dielectrics.

Electric or electrostatic Energy (U_E) stored in capacitor

Capacitor is a charge storing device. Since potential is defined as the amount of work done in bringing a charge from infinity to a given point.

$$V = \frac{W}{q}$$

$$\Rightarrow W = qV$$

Amount of workdone in raising the charge through 'dq' is given by

$$dW = dq\,V$$

Since $V = \frac{q}{C}$

$$dW = \frac{q}{C}\,dq$$

Capacitor ~~store~~ store electric enery or electrostatic energy

Then the total workdone in raising the charge from 0 to q.

$$W = \int dW = \int_0^q \frac{q}{C}\, dq$$

$$= \frac{1}{C}\int_0^q q\, dq$$

$$= \frac{1}{C}\,\frac{q^2}{2}$$

or, $W = \frac{1}{2}\,\frac{q^2}{C}$

This amount of work done is stored in the capacitor in the form of electric energy. Therefore energy stored in the capacitor is given by

$$\boxed{U_E = \frac{1}{2}\,\frac{q^2}{C}}$$

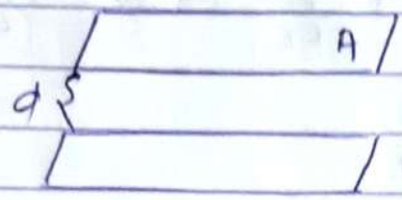
Also $q = CV$

$$\Rightarrow U_E = \frac{1}{2} CV^2$$

$$\& \; U_E = \frac{1}{2} qV$$

Electric energy density (u_E)
It is defined as the electric energy stored per unit volume of the capacitor and is given by

$$u_E = \frac{U_E}{\text{Volume}}$$



For a parallel plate capacitor, volume $= A \cdot d$

$$\& \quad U_E = \frac{1}{2} \frac{q^2}{C}$$

$$\therefore \; u_E = \frac{1}{2} \frac{q^2}{C} \cdot \frac{1}{Ad}$$

$$\text{or,} \; u_E = \frac{1}{2} CV^2 \frac{1}{Ad}$$

where $C = \frac{\varepsilon_0 A}{d}$

$$\therefore \; u_E = \frac{1}{2} \frac{\varepsilon_0 A}{d} \cdot \frac{V^2}{A \cdot d}$$

$$= \frac{1}{2} \varepsilon_0 \left(\frac{V}{d} \right)^2$$

$$\therefore \boxed{u_E = \frac{1}{2} \varepsilon_0 E^2}$$

If a dielectric is placed between the plates then, electric energy density, ~~(u_E)~~

$$u_E = \frac{1}{2} \varepsilon_m E^2$$

where ε_m is the permittivity of the dielectric medium.

Combinations of capacitor (opposite of resistor combination)

i) Capacitors connected in series

p.d across series combination $V = V_1 + V_2 + V_3$

charge 'q' is same in series combination,

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3} \quad \& \quad V = q/C$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3},$$

$$1/C = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Capacitors connected in Parallel

In parallel combination, the p.d. across each capacitor will be same.

$\therefore \quad q = q_1 + q_2 + q_3$

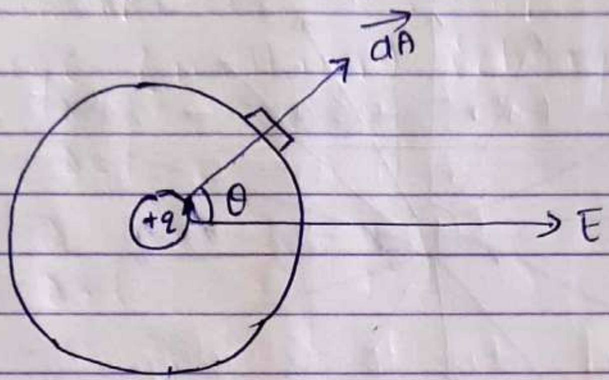
Consider $C_1, C_2 \& C_3$ connected in parallel.

$$\therefore \; q = q_1 + q_2 + q_3$$
$$CV = C_1V + C_2V + C_3V$$
$$CV = V(C_1 + C_2 + C_3)$$
$$C = C_1 + C_2 + C_3$$

Electric flux (ϕ)

The total electric lines of force passing through a surface or area is called the electric flux. It is given by

$$\phi_E = \oint_s \vec{E} \cdot \vec{dA}$$



$$\text{or, } \phi_E = \oint_s E dA \cos\theta$$

Gauss's law

Statement:

It states that the total electric flux passing through a closed surface is equal to $1/\varepsilon_0$ times the charge enclosed by that surface.

$$\text{i.e. } \boxed{\phi_E = \frac{1}{\varepsilon_0} q_{enclosed}}$$

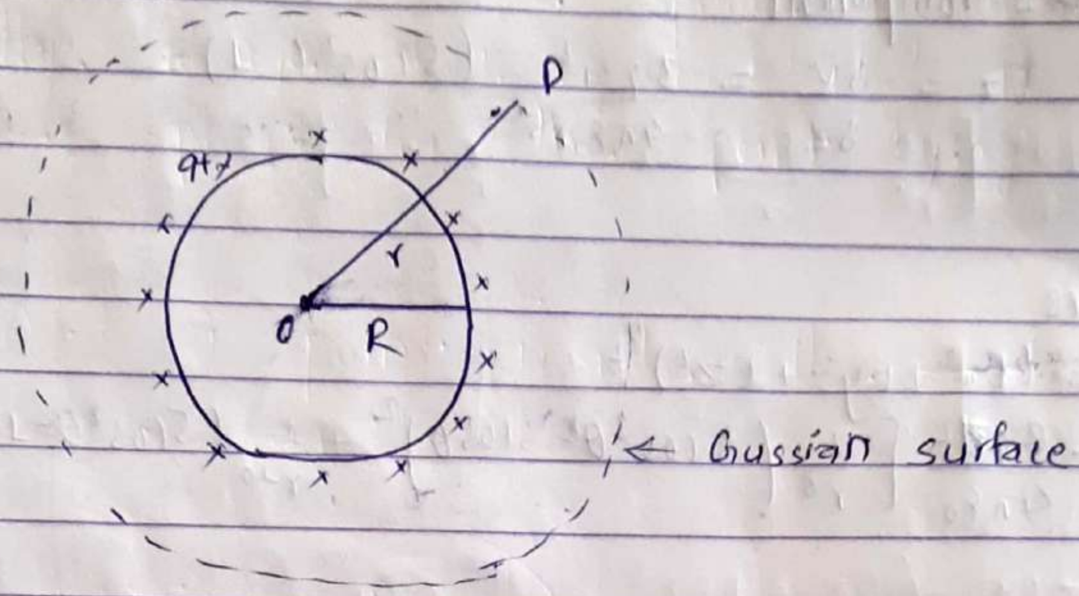
Let ϕ represent the flux through a surface enclosing charge Q. Then Gauss theorem is expressed as

$$\phi = \frac{Q}{\varepsilon_0} \quad \text{--- (1)}$$

where ε_0 = absolute permittivity of free space.

Application of Gauss theorem

Electric field due to a charged sphere



Consider a charged sphere of charge 'q' on the surface and radius 'R'. We have to find the electric field

ⓐ At a point outside the sphere
For this we draw a Gaussian surface passing through the point 'P' such that
$OP = r > R$, which is a sphere of radius 'r'.

Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

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or, $\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} q$

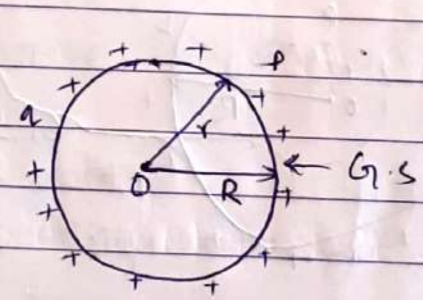
or, $\oint_s E\, dA \cos 0° = \frac{q}{\varepsilon_0}$

or, $E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$

$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$

This shows that the sphere behaves as if all the charge was concentrated at the centre of the sphere for the point lying outside it.

(b) At a point ~~inside~~ on the ~~surface~~ sphere



for the point on the surface
$OP = r = R$,

The Gaussain surface is the sphere itself
Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

$$\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} q$$

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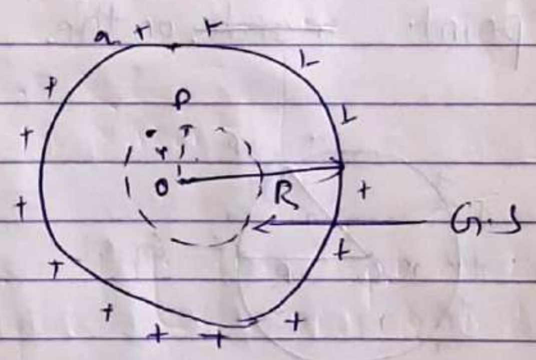
$$\text{or, } \oint_S E \, dA \cos 0^\circ = \frac{q}{\varepsilon_0}$$

$$\text{or, } E \, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}} \quad [\because r = R]$$

This shows that the sphere behaves as if all the charge was concentrated at the centre of the sphere for the point lying on the surface.

(c) At a point inside the ~~surface~~ sphere



For the point inside the sphere, $OP = r < R$, the Gaussian surface is the sphere of the radius 'r'. Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

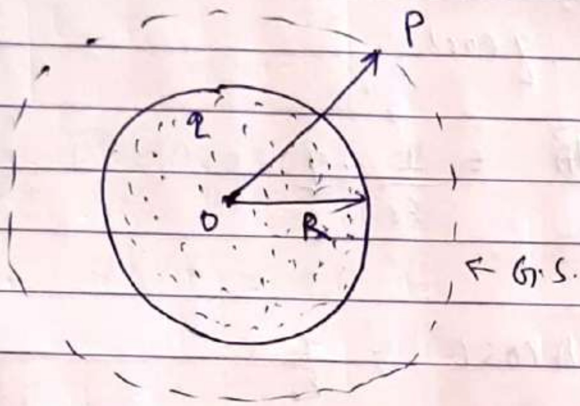
$$\text{or, } \oint_S \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \, 0$$

$$\text{or, } \oint_S E \, dA \cos 0^\circ = 0$$

or, $E \cdot 4\pi r^2 = 0$

$\therefore \boxed{E = 0}$ (zero everywhere inside the charged sphere)

② Electric field due to solid sphere of charge



<u>Note</u>

If charge is distributed uniformly in

① 1-d (length / wire).

λ (linear charge density)

$= \frac{q}{l}$

② 2-d (surface / area)

σ (surface charge density)

$= \frac{q}{A}$

③ 3-d (volume / sphere)

ρ (volume charge density)

$= \frac{q}{v}$

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(a) At the point outside the sphere for point P
For this we draw a Gaussian surface passing through the point 'P' such that $OP = r > R$, which is a sphere of radius 'r'.

Then by Gauss law

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

$$\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \cdot q$$

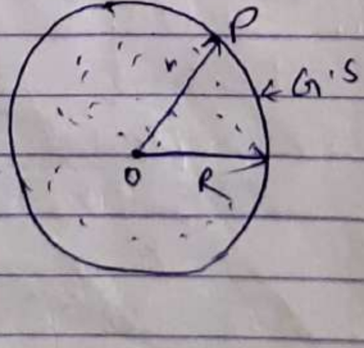
or, $$\oint_s E\, dA \cos 0^\circ = \frac{q}{\varepsilon_0}$$

or, $$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

This shows that the sphere behaves as if all the charge was concentrated at the centre of the sphere for the point lying outside it.

(b) At a point on sphere



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for a point on the surface $OP = r = R$, the Gaussain surface is the sphere itself then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

$$\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} q$$

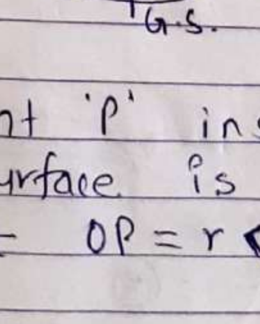
or, $$\oint_s \vec{E}\, dA \cos 0^\circ = \frac{q}{\varepsilon_0}$$

or, $$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}} \quad [\because r = R]$$

This shows that the sphere behaves as if all the charge was concentrated at the centre of the sphere for the point lying on the surface.

(c) At a point ~~ino~~ inside the sphere (or surface) :-



For the point 'P' inside the sphere the Gaussian surface is a sphere of radius 'r' such that $OP = r < R$

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Then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

or, $$\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} q'$$

or, $$E \oint_s dA = \frac{1}{\varepsilon_0} q'$$

or, $$E \cdot 4\pi r^2 = \frac{q'}{\varepsilon_0}$$

or, $$\boxed{E = \frac{q'}{4\pi\varepsilon_0 r^2}} \quad \text{---} (1)$$

Note:
Known: q, R, r
Unknown: q'

To find q' [q' is the charge enclosed by the Gaussian surface

We assume that charge is distributed uniformly ~~through~~ throughout the sphere of radius 'R'. Therefore volume charge density (ρ) is constant througout the sphere of radius 'R'.

i.e $\rho_R = \rho_r$

or, $$\frac{q}{\frac{4}{3}\pi R^3} = \frac{q'}{\frac{4}{3}\pi r^3}$$

$$\therefore \boxed{q' = q\left(\frac{r}{R}\right)^3} \quad \text{---} (2)$$

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[since $r < R$, $q' < q$]

Using ② in ①

$$E = \frac{q \, r^3}{4\pi\varepsilon_0 r^2 \cdot R^3}$$

or, $$\boxed{E = \frac{qr}{4\pi\varepsilon_0 R^3}}$$

Cases :-

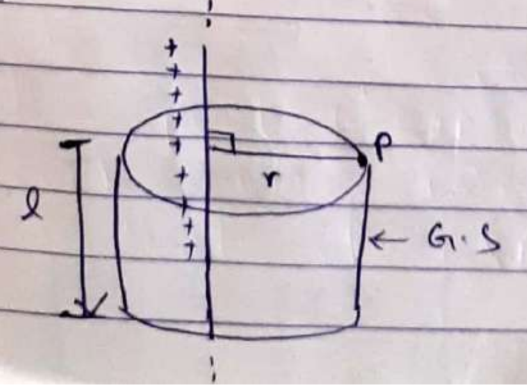
① At a point at the centre,
$r = 0$
$\Rightarrow \boxed{E = 0}$

② At a point on the surface,
$r = R$
$\therefore \boxed{E = \frac{q}{4\pi\varepsilon_0 R^2}}$ (2nd case)

maximum)

[q' is maximum at sphere)

③ Electric field due to an infinite line of charge (wire)



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Consider an infinite line of conductor (charge) having liner charge density 'λ'. We have to find the electric field at a point 'P' such that $OP = r$

For this we draw a Gaussain surface which is a cylinder of radius 'r' and length 'l'.

Then by Gauss law.

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

or, $\oint_s \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \lambda l$

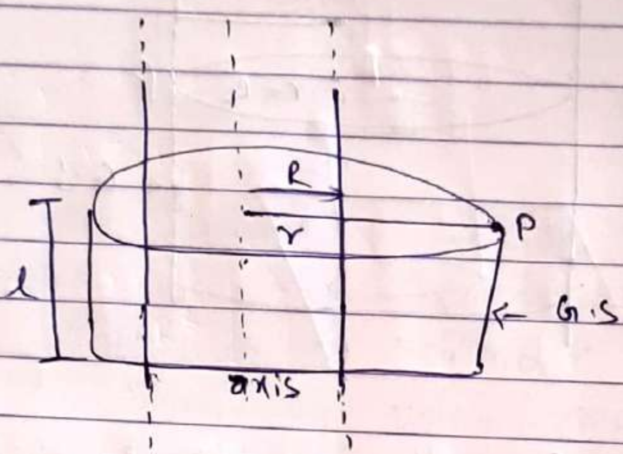
or, $E \oint_s dA = \frac{\lambda l}{\varepsilon_0}$

or, $E \cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0}$

$$\therefore \boxed{E = \frac{\lambda}{2\pi \varepsilon_0 r}}$$

$$E \propto \frac{1}{r}$$

④ Electric field due to an infinitely long cylinder of charge



Consider an infinitely long cylinder of charge having radius 'R' having volume charge density 'ρ'. We have to find the electric field

ⓐ At a point outside the cylinder
For the point 'P' outside the cylinder $OP = r > R$, we draw a Gaussian surface which is a cylinder of radius 'r' and length 'l'. Then by using Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

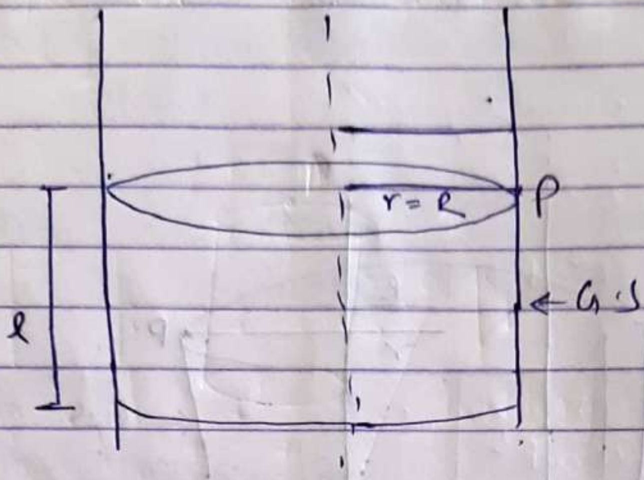
or, $$\oint_S \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \rho \pi R^2 l$$

or, $$E \oint_S \vec{dA} = \frac{\rho \pi R^2 l}{\varepsilon_0}$$

or, $$E \cdot 2\pi r \cancel{l} = \frac{\rho \pi R^2 \cancel{l}}{\varepsilon_0}$$

or, $$\boxed{E = \frac{\rho R^2}{2 \varepsilon_0 r}}$$

(b) At a point on the surface of the cylinder



~~Consider~~ for the point 'P' ~~inside~~ on the surface of the cylinder $OP = r = R$. ~~that~~ We draw a Gaussian surface which is a cylinder of radius 'R' & length 'l'. Then by using Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

or, $\oint_S \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \rho \pi R^2 l$

or, $E \oint \vec{dA} = \frac{1}{\varepsilon_0} \rho \pi R^2 l$

or, $E \cdot 2\cancel{\pi} \cancel{R} \cancel{l} = \frac{1}{\varepsilon_0} \rho \cancel{\pi} R^{\cancel{2}} \cancel{l}$

$\therefore E = \frac{\rho R}{2\varepsilon_0}$ $\qquad \boxed{E = \frac{\rho R}{2\varepsilon_0}}$

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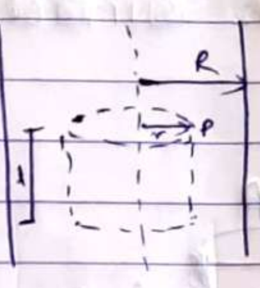
(c) At a point inside the surface of the cylinder



For the point 'P' inside the surface of the cylinder $OP = r < R$ we draw a Gaussian surface which is a cylinder of radius 'r' & length 'l'. Then by using Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

$$\text{or, } \oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \rho \pi r^2 l$$

$$\text{or, } E \cdot 2\pi \cancel{r} \cancel{l} = \frac{\rho \pi r^{\cancel{2}} \cancel{l}}{\varepsilon_0}$$

$$\therefore \boxed{E = \frac{\rho r}{2\varepsilon_0}}$$

Case:

(1) At the axis

$r = 0$

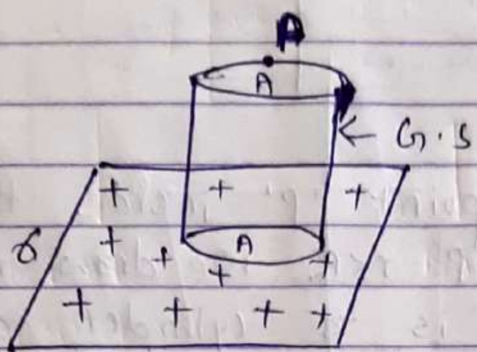
$\therefore E = 0$

(2) At the surface

$r = R$

$\therefore E = \frac{\rho R}{2\varepsilon_0}$

Electric field due to a charged conductor (Charges on the surface) (conductor)



Consider a charged conductor having surface charge density 'σ'. We have to find the electric field at a point 'P', for this we draw a Gaussian surface which is a cylinder then by Gauss law,

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

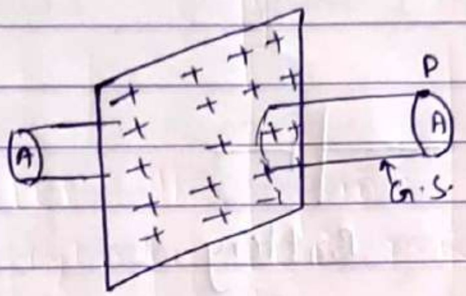
or, $\oint_s \vec{E} \cdot \vec{dA} = \frac{1}{\varepsilon_0} \sigma A$

or, $E \oint_s \vec{dA} = \frac{\sigma A}{\varepsilon_0}$

or, $E \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$

or, $\boxed{E = \frac{\sigma}{\varepsilon_0}}$ — V.V. Imp (Theory/Num - Capacitor, long Q) Num

(6) Electric field due to infinite sheet of charge (charges on the surface (insulator))



Consider an infinite sheet of charge having surface charge density 'σ'. The electric effect is on the both sides of it. Through the point 'P' draw a Gaussian surface which is a cylinder piercing the sheet. Then by Gauss law

$$\phi_E = \frac{1}{\varepsilon_0} q_{encl}$$

$$\oint_s \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \sigma A$$

$$\text{or, } EA + EA = \frac{1}{\varepsilon_0} \sigma A$$

$$\text{or, } 2EA = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{\therefore E = \frac{\sigma}{2\varepsilon_0}}$$

Electricity & Magnetism (Theory & Num.)

Electricity

Electric current (I)

It is the net flow of charge or rate of flow of charge & is given by 'q/t' or $\frac{dq}{dt}$

Unit → $I \xrightarrow{S.I.}$ Amphere (A)

$$\boxed{1A = 1Cs^{-1}}$$

It is a scalar quantity

Current density (J):

It is defined as the current per unit area (cross - sectional area)

i.e $J = \frac{I}{A}$

Unit: Am^{-2}

It is a vector quantity

Note:

Current is a macrosopic quantity and current density is microscopic quantity.

Since $J = \frac{I}{A}$

$\Rightarrow I = JA$

$\Rightarrow I = \oint_s \vec{J} \cdot \vec{dA}$ or, $I = \oint_s \vec{J} \cdot \vec{ds}$

Recall:

Maxwell Eq^n

$\phi_E = \oint_s \vec{E} \cdot \vec{dA}$

$\phi_B = \oint_s \vec{B} \cdot \vec{dA}$

Note: Scalar: $q, \rho, I, \phi_E, \phi_B$

vector:

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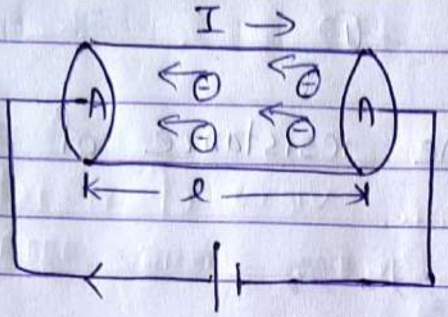
Relation between I & V_d

V_d = drift velocity

$\boxed{I = V_d enA}$



$I = \frac{q}{t} = \frac{Ne}{t}$, N = total no. of electrons

Since n = no. of electros per unit volume or concentration of charge carriers or electrons

i.e. $n = \frac{N}{Vol}$

$\Rightarrow N = n \times vol$

$= n \cdot Al$

$\therefore I = \frac{nAle}{t}$

$= \left(\frac{l}{t}\right) enA$

$\boxed{\therefore I = V_d enA}$

$\therefore \boxed{J = \frac{I}{A} = V_d en}$

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Relation between J & E :

(J = current density, E = electric field)

Since $I \propto V$

or, $V \propto I$

or, $\boxed{V = IR}$ — (1)

Where R is the resistance of the wire or conductor

Also,

$R \propto l$

$R \propto \frac{1}{A}$

$\therefore R \propto l/A$

$$\boxed{R = \rho\, l/A} \quad \text{— (2)}$$

Where ρ = resistivity of the material

from (1) & (2)

$V = I \rho\, l/A$

$$\left[\begin{array}{l} E = -dv/dr \\ E = V/l \end{array}\right]$$

or, $\frac{V}{l} = \frac{I}{A}\rho$

or, $E = J\rho$

or, $\boxed{E = \rho J}$ रोजेको E

Also, $J = \frac{1}{\rho}E$

$\therefore \boxed{J = \sigma E}$

where σ = conductivity

$= 1/\rho$

In vector form

$$\boxed{\vec{J} = \sigma \vec{E}}$$

Q. Using ohm's law derive the relation between J and E.

Note:
Wave : $\mu \Rightarrow$ linear mass density
Optic : $\mu =$ refractive index
Electricity : $\mu \Rightarrow$ mobility
magnetism $\mu \Rightarrow$ permeability

Mobility (μ)

Since $J = V_d e n = \sigma E$

$\Rightarrow V_d \propto E$ $\left(\begin{matrix} e \\ n \\ \sigma \end{matrix} \right. \Rightarrow$ constant$\left. \right)$

$\Rightarrow \boxed{V_d = \mu E}$

where μ is mobility

Defination:

Since $\mu = \frac{V_d}{E}$, mobility is defined as the drift velocity per unit electric field applied.

Unit: $\frac{ms^{-1}}{Vm^{-1}}$
$= m^2 V^{-1} s^{-1}$

$E = F/q : NC^{-1}$
$E = -\frac{dv}{dr} \Rightarrow Vm^{-1}$

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Relation between σ & μ

E ज्ला भाग दि

$J = V_d e n = \sigma E$

$\Rightarrow \frac{V_d}{E} e n = \sigma$

$\Rightarrow \boxed{\mu e n = \sigma}$

V.V.I
Q. Discuss the atomic view of resistivity & prove

Theory/Num $\sigma = \frac{n e^2 \tau}{m}$ [ne square टाउको मुनि m]

$\rho = \frac{m}{n e^2 \tau}$ $\left[\because \sigma = \frac{1}{\rho}\right]$

→ Conduction in metal is due to the uni-directional flow of conduction electrons in the presence of an external electric field (source or battery). During their motion electrons encountered the positive ion ~~tour~~ cores as a result their velocity is reduced to zero. Again they gain acceleration due to the presence of a battery. In this way current flows through the conductor ~~and~~ amidst the consecutive collisions.

Now, the acceleration of a single electron is given by

$$a = \frac{F}{m} \quad [\because F = ma]$$

$$= \frac{eE}{m} \quad \left[\because E = \frac{F}{q} = \frac{F}{e}\right]$$

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Now, the drift velocity is given by,

$$V_d = a\tau \quad [\because v = \cancel{u}^{0} + at = at]$$

where τ = relaxation time
i.e. the time between the two successive collisions

And $\boxed{\tau = \frac{\lambda}{\bar{v}}}$ ← v. Imp

$$\left[\begin{aligned} Vel &= \frac{distance}{time} \\ \bar{v} &= \frac{\lambda}{\tau} \end{aligned} \right]$$

where $\bar{v}$ = free or average speed
λ = mean free path
i.e. distance between the two successive collision

Then,

$$V_d = a\tau = \left(\frac{eE}{m}\right)\tau$$

Also, $J = V_d en = \sigma E$

$$\Rightarrow V_d en = \sigma E$$

$$\Rightarrow \left(\frac{eE}{m}\right) en\tau = \sigma E$$

$$\Rightarrow \frac{ne^2\tau}{m} = \sigma$$

$$\Rightarrow \boxed{\sigma = \frac{ne^2\tau}{m}}$$ → v. v. Imp

or, resistivity $(\rho) = \frac{1}{\sigma}$

$$= \frac{m}{ne^2\tau}$$

where,
m = mass of electron
e = charge of electron
n = No. of electron per unit vol
τ = relaxation time

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Temperature dependence of resistance and resistivity

If R_1 and R_2 are the resistance of a conductor at θ_1 and θ_2 respectively.

Then, $R_2 = R_1 [1 + \alpha \Delta\theta]$

where $\Delta\theta = \theta_2 - \theta_1$

α = temp coeff of resistance

For R_0 at $0°C$

& R_θ at $\theta°C$

$$R_\theta = R_0 [1 + \alpha\theta]$$

Similarly for resistivity

$$\rho_2 = \rho_1 [1 + \alpha \Delta\theta]$$

$$\rho_\theta = \rho_0 [1 + \alpha\theta]$$

This shows that resistance of a conductor or metal or metallic conductor or ohmic conductor increases with the increase in temperature and same is for resistivity.

Here α is '+ve' for metallic conductor

For semiconductor, resistivity decreases. Hence conductivity increase with the increase in temperature and is given by

$$\rho_2 = \rho_1 (1 - \alpha\Delta\theta)$$

$$\rho_\theta = \rho_0 (1 - \alpha\theta)$$

α is '-ve' for semi-conductor.

Joule's law of heating

$\cancel{V = W}$

$$V = W/q$$

$$\Rightarrow W = Vq$$

Also $I = \frac{dq}{dt}$

$$dq = I\,dt$$

For dq charge,

$$dW = V\,dq$$
$$= VI\,dt$$

$$\Rightarrow \frac{dw}{dt} = IV = \text{power}(P)$$

$$\therefore \boxed{P = IV}$$

This amount of workdone is dissipitated in the form of Joule's heat.

Thus heat produced is given by

$$H = IVt$$

$$\therefore \boxed{H = I^2Rt} \quad [\because V = IR]$$

Here, $H \propto I^2$
$H \propto R$
$H \propto t$

$$\Rightarrow \frac{H}{t} = P_R = I^2R$$
$$= IV$$
$$= V^2/R$$

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Magnetism

Force on a moving charge inside a magnetic field or due to magnetic field:

Consider a charge 'q' moving with velocity 'v' inside magnetic field 'B' then the force experienced by charge is such that:

① Directly proportional to B
i.e. $F \propto B$

② Directly proportional to q
i.e $F \propto q$

③ Directly proportional to v
i.e $F \propto v$

④ Directly proportional to sine of the angle between v & B
i.e $F \propto \sin\theta$

Combing,
$F \propto Bqv\sin\theta$
$\Rightarrow \boxed{F = Bqv\sin\theta}$ constant = 1
↳ magnitude

In vector force,
$\vec{F} = q(\underbrace{\vec{v} \times \vec{B}}_{vB\sin\theta})$

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Cases:

(1) F will be maximum
When $\theta = 90^\circ$
$\Rightarrow v \perp B$

(2) F will be minimum
When $\theta = 0^\circ$ or 180°
$\rightarrow v \parallel B$
or antiparallel

Unit:
$B \Rightarrow$ vector
$\rightarrow$ Tesla(T) : SI unit
$\rightarrow$ gauss : cgs unit

$$\boxed{1T = 10^4 \text{ gauss}}$$

Q. Compare and contrast between the electric force & magnetic force

$\rightarrow$ The electric force is given by,
$$F_E = qE$$

& magnetic force is given by
$$F_M = Bqv\sin\theta$$

This shows that both force act on a charge 'q' due to their respective fields

In addition the magnetic force is executed only upon the moving charge and it

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further depends upon the angle between v and B.

Imp Lorentz force [short notes / long question]
Eg: Hall effect

It is the sum of electric force and magnetic force exerted on a charge and hence given by,

$$\vec{F_L} = \vec{F_E} + \vec{F_M}$$
$$= q\vec{E} + q(\vec{v} \times \vec{B})$$
$$= q[\vec{E} + \vec{v} \times \vec{B}]$$

In magnitude, $F_L = q(E + vB\sin\theta)$

For F_L to be maximum,
$\theta = 90°$ i.e. $v \perp B$

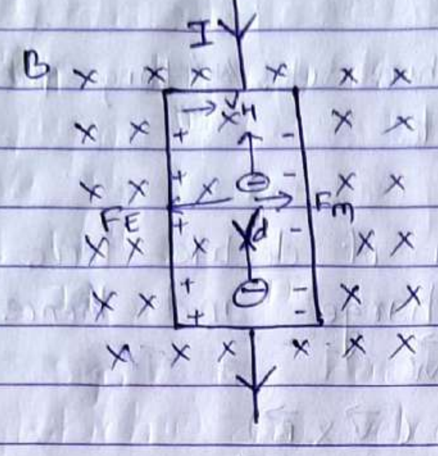
Eg: of Lorentz force is Hall effect
Th long S.N Num

Hall Effect

If a magnetic field is applied perpendicular to the direction of the current, then a p.d is developed in a direction perpendicular to both current and magnetic field. This effect is called the Hall Effect and the p.d developed is called (transverse) hall voltage (V_H) and and electric field developed is called hall field (E_H).

× perpendicular inward
(I A
⊙ outward
do

Theory



Consider a metallic conductor or metal or ohomic (strip) conductor of length 'l', width 'w' and thickness 't'. Current 'I' flows lengthwise from upward to the downward direction. The electrons moved in the opposite direction with drift velocity (v_d).

Now, a magnetic field (B) is applied perpendicular to the direction of current (along the thickness and inward). Now the electrons experience the magnetic force given by, ~~the~~

$$F_m = Bqv\sin\theta$$
$$= Bev_d \quad [\because \theta = 90^\circ]$$

Due to this force the electrons are deflected to the right across the width. As a result '-ve' charges are accumulated to the right side and '+ve' charges are accumulated to the left side across the width. This creates a potential

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across width
difference and is called the Hall voltage (V_H) and the corresponding electric field developed is called Hall field (E_H) which is given by

V.V. Imp Num → $$\boxed{E_H = \frac{V_H}{W}} \quad --- \quad (1)$$

This electric field further exerts the electric force given by

$$F_E = q E_H$$
$$= e E_H$$

Due to this force, an equilibrium is reached then the new incoming electrons pass undeflected.

Hence under equilibrium
$[\vec{F_E} + \vec{F_M} = 0]$ —— 1st way

Or,
$F_E = F_M$ —— 2nd way
$\Rightarrow \cancel{e} E_H = B \cancel{e} V_d$

V. Imp Num
$$\Rightarrow \boxed{V_d = \frac{E_H}{B}} \quad —— \quad (2)$$

Now,
the current density is given by
$J = -V_d e n$ ['-ve' sign is due to the -vely charged particle which are electrons in the metallic conductor]

Using (2)
$$J = -\frac{E_H}{B} e n$$

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$$\frac{E_H}{JB} = -\frac{1}{ne}$$

Here, $R_H = \frac{E_H}{JB}$, is called the Hall co-efficient

$$\therefore R_H = \frac{E_H}{JB} = -\frac{1}{ne} \quad \text{---- (3)}$$

The '-ve' sign is due to the -vely charged particle which are electrons in the metallic conductor.

Uses of Hall Effect

(1) It is used to determine the sign of the charge carriers.

(2) It is used to determine the number of electrons per unit volume. i.e. concentration of the charge carriers 'n'.

To find

(1) n (concentration of charge carriers)

(2) V_H (Hall voltage)

(3) E_H (Hall field)

(4) R (Hall resistance)

(5) μ_H (Hall mobility)

(6) θ_H (Hall angle)

Since $R_H = \frac{E_H}{JB} = \frac{1}{ne}$ - (*), (in magnitude)

Also,

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$$E_H = \frac{V_H}{w}$$

$$J = \frac{I}{A} = \frac{I}{wt} \quad \text{---}(*)(*)$$

Using J in (*)

$$R_H = \frac{\frac{V_H}{\cancel{w}}}{\frac{I}{\cancel{w}t} \cdot B}$$

$$R_H = \frac{V_H t}{IB}$$

$$\frac{1}{ne} = \frac{V_H t}{IB}$$

$$n = \frac{IB}{V_H te}$$

Imp ⇒ $\boxed{n = \frac{IB}{V_H et}}$ नाई IB लाई भेट

(2) $V_H = \frac{IB}{net}$

(3) $E_H = \frac{V_H}{w} = \frac{IB}{netw} = \frac{IB}{ne_A}$

(4) $R = \frac{V_H}{I} \quad [V = IR]$

$$= \frac{B}{net}$$

(5) $\mu_H = \sigma R_H$

(5) $\mu_H = \sigma R_H$ [σ : conductivity]

$$R_H = \frac{1}{ne}$$, Hall co-efficient

$$\sigma = \mu e n$$ [ये जना मानदे]

$$= \mu \frac{1}{R_H}$$

$$\mu_H = \sigma R_H$$

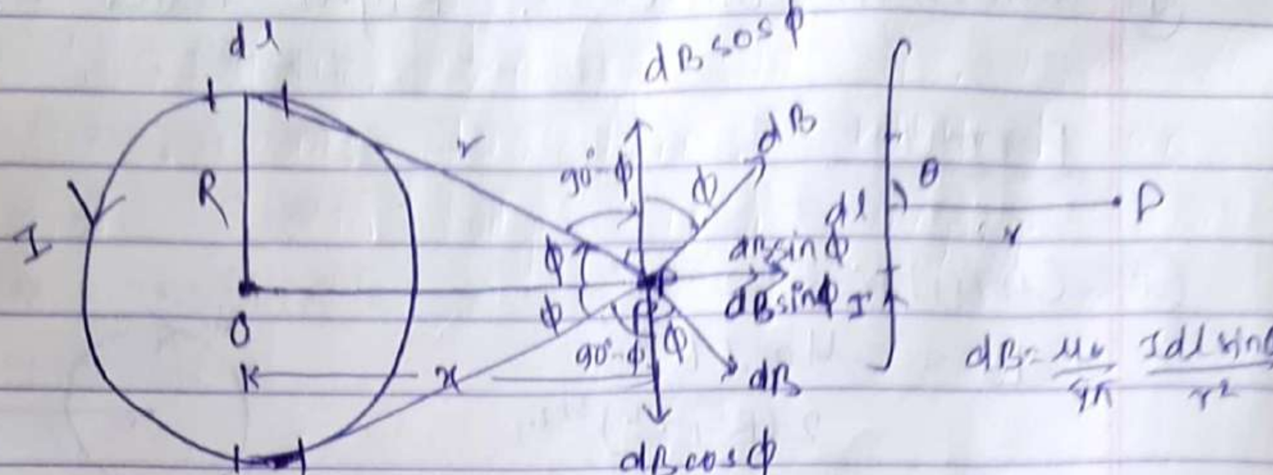
(6) Hall angle (θ_H)

$$= \sigma R_H B$$

Application of Biot savart law

① Magnetic field due to a circular current carrying conductor:

I, R, x → constant → known



Consider a circular conductor (having a single turn) of radius 'R' and carrying current 'I'. We have to find the magnetic field at a point 'P' ~~from~~ on the axis of the conductor such that $OP = x$

For this we consider a small current

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element of length 'dl' then, the magnetic field at a point P due to this element is given by,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

where θ is the angle between dl and r.

$\theta = 90^\circ$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl}{(R^2+x^2)} \quad [\because r^2 = R^2 + x^2]$$

from figure we see that the resultant magnetic field at the point 'P' is contributed by sine components of dB. Hence the resultant magnetic field is given by

$$B = B_{resultant} = B_{Total} = \int_0^{2\pi R} dB \sin\phi$$

$$= \int_0^{2\pi R} \frac{\mu_0}{4\pi} \frac{I dl}{R^2+x^2} \cdot \frac{R}{r}$$

But $r = (R^2+x^2)^{1/2}$

$$\therefore B = \frac{\mu_0}{4\pi} \frac{IR}{(R^2+x^2)^{3/2}} \int_0^{2\pi r} dl$$

$$= \frac{\mu_0}{\cancel{4\pi}\,2} \frac{IR}{(R^2+x^2)^{3/2}} \times \cancel{2\pi} R$$

$$\boxed{B = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}}}$$

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(3) This is the magnetic field due to single turn.

Cases

(1) At the centre $x = 0$

$$\therefore B = \frac{\mu_0 I R^2}{2 R^3}$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2R}}$$

(2) for N turn

$$B = \frac{\mu_0 N I R^2}{2 (R^2 + x^2)^{3/2}}$$

$$\therefore \boxed{B = \frac{\mu_0 N I}{2R}}$$

$\rightarrow$ Goal : $B = \frac{\mu_0 I}{4\pi R}(\sin\phi_1 + \sin\phi_2)$

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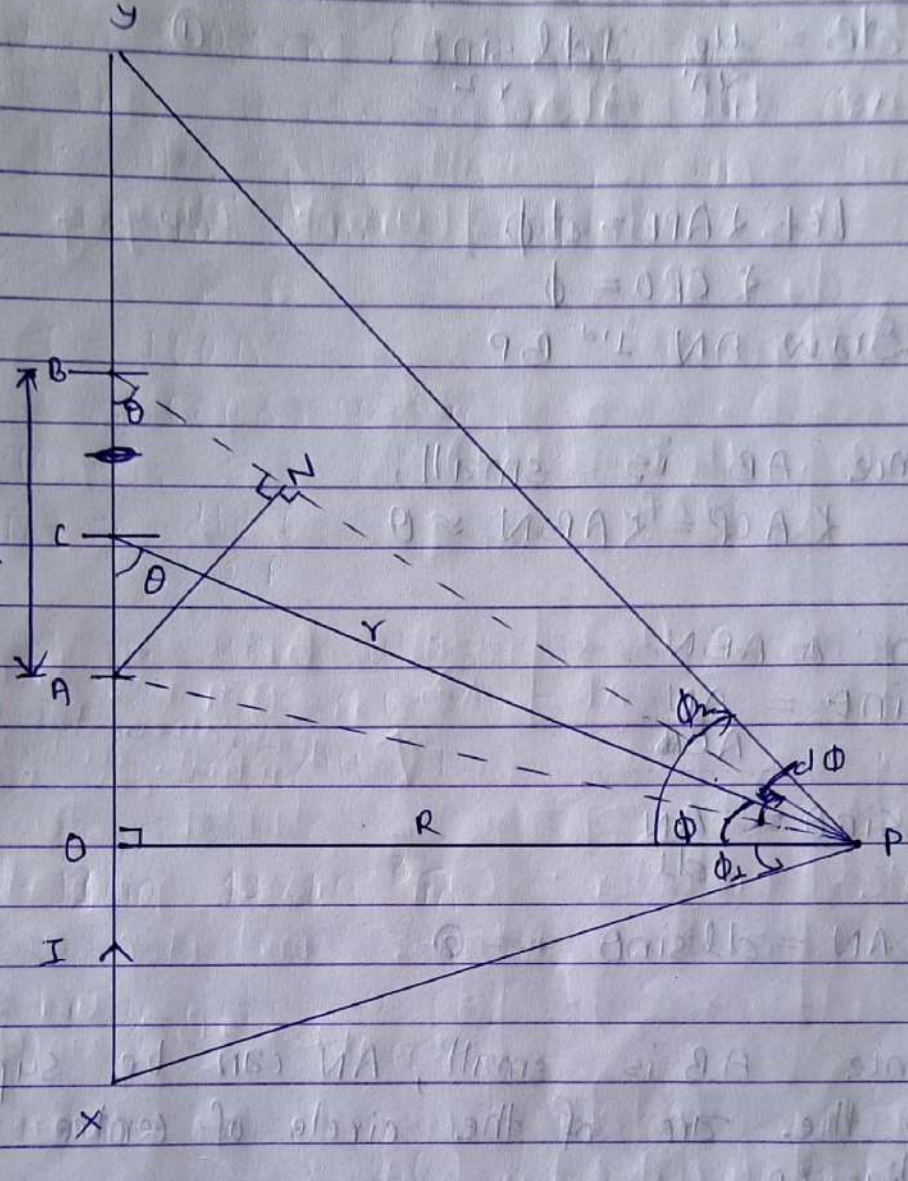
Q. Magnetic ~~Electric~~ field due to a current carrying straight conductor



Consider a finite conductor XY carrying current I. We have to find the magnetic field at the point 'P' such that OP = R.

For this we consider a small element AB of length 'dl' having centre 'C' such that CP = r
$\angle ACP = \theta$.

By Biot Savart's law, the magnetic field due to

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element AB at point P is given by

To convert to ϕ term

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2} \quad \text{--- (1)}$$

Let $\measuredangle APB = d\phi$

$\measuredangle CPD = \phi$

Draw AN $\perp^{rd}$ BP

Since AB is small,

$\measuredangle ACP = \measuredangle ABN \approx \theta$

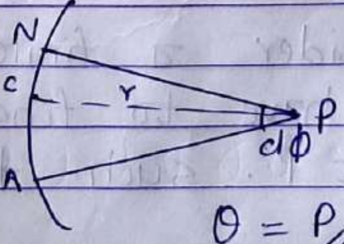
In $\triangle$ ABN,

$$\sin\theta = \frac{AN}{AB}$$

$$\sin\theta = \frac{AN}{dl}$$

$\therefore AN = dl \sin\theta$ --- (2)

Since AB is small, AN can be supposed to be the arc of the circle of centre P and radius CP.



$$\therefore d\phi = \frac{AN}{r}$$

$\therefore AN = r\, d\phi$ --- (3)

$\theta = P/r$

$d\phi = \frac{AN}{r}$

Using (1) (2) and (3)

$$dB = \frac{\mu_0}{4\pi} \frac{I \cancel{r} d\phi}{r^{\cancel{2}}}$$

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$$dB = \frac{\mu_0}{4\pi}\frac{I}{r}d\phi \quad \text{--- (4)}$$

In ΔCPO,

$$\cos\phi = \frac{R}{r}$$

$$\therefore r = \frac{R}{\cos\phi} \quad \text{--- (5)}$$

Using (5) in (4)

$$dB = \frac{\mu_0}{4\pi}\frac{I}{R}\cos\phi\, d\phi$$

Now, Join x with P and x with P such that

$$\angle XPO = \phi_1$$
$$\angle YPO = \phi_2$$

Then the total magnetic field due to the finite conductor is given by,

$$B = \int dB$$

$$= \frac{\mu_0 I}{4\pi R}\int_{-\phi_1}^{\phi_2}\cos\phi\, d\phi$$

$$= \frac{\mu_0 I}{4\pi R}\left|\sin\phi\right|_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_0 I}{4\pi R}\left[\sin\phi_2 - \sin(-\phi_1)\right]$$

$$\boxed{\therefore B = \frac{\mu_0 I}{4\pi R}\left[\sin\phi_1 + \sin\phi_2\right]} \text{ - V.V.Imp}$$

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(3) Magnetic field due to infinite current carrying straight conductor.

Derive till above

For the infinite current carrying st. conductor

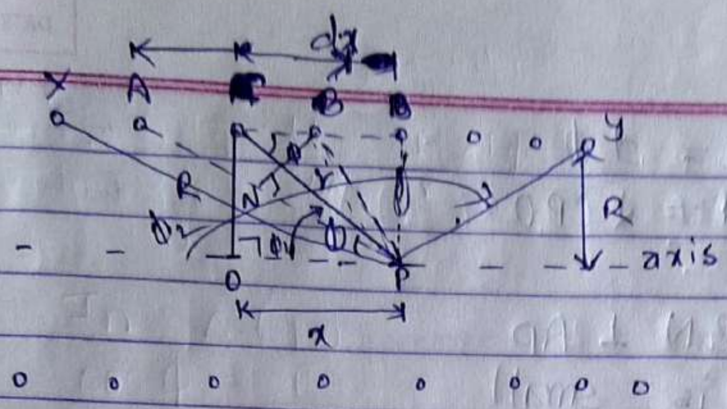
$\phi_1 = 90^\circ$

& $\phi_2 = 90^\circ$

$$\therefore B = \frac{\mu_0 I}{4\pi R}(1+1)$$

$$\boxed{\therefore B = \frac{\mu_0 I}{2\pi R}}$$

(*) Derive till here hence we can



Consider a finite solenoid xy having number of turns per unit length 'n' and carrying current 'I'. We have to find the magnetic field at the point 'P' on the axis of the solenoid.

For this we consider a small element AB of length 'dx' having centre 'C' such that
$CP = r$
& $\angle CPO = \phi$

The number of turns with the element AB is ndx then the magnetic field at P due to element AB is

$$dB = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} \times n dx$$

where $OP = x$ & $CO \perp OP$

Here $r^2 = R^2 + x^2$
$\therefore (R^2+x^2)^{3/2} = r^3$

To convert to ϕ term

$$\therefore dB = \frac{\mu_0 n I}{2} \frac{R^2}{r^3} (dx) \quad \text{--- (1)}$$

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Let $\angle APB = d\phi$

Here, $BCP = \angle CPO =$

Now, draw $BN \perp AP$

since AB is small

$\angle CAP \approx \angle BCP \approx \phi$

In ΔABN,

$$\sin\phi = \frac{BN}{AB} = \frac{BN}{dx}$$

$$\therefore BN = dx \sin\phi \text{ --- (2)}$$

Since AB is small, BN can be supposed to be the arc of the circle of centre P and radius CP.

$$d\phi = \frac{BN}{r}$$

$$\therefore BN = r\,d\phi \text{ --- (3)}$$

from (2) and (3)

$$dx \sin\phi = r\,d\phi$$

$$\therefore dx = \frac{r\,d\phi}{\sin\phi} \text{ --- (4)}$$

Using (4) in (1)

$$dB = \frac{\mu_0 n I}{2} \frac{R^2}{r^{3}} \frac{r\,d\phi}{\sin\phi}$$

$$dB = \frac{\mu_0 n I}{2} \frac{R^2}{r^2} \frac{d\phi}{\sin\phi} \text{ --- (5)}$$

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In Δ COP
$$\sin\phi = \frac{R}{r}$$

So, eq^n (5) becomes

$$dB = \frac{\mu_0 n I}{2} \frac{\sin^2\phi}{\sin\phi} d\phi$$

$$dB = \frac{\mu_0 n I}{2} \sin\phi \, d\phi$$

~~On integrating~~

Then the total magnetic field at P due to the finite solenoid XY is given by

$$B = \int dB$$

$$= \frac{\mu_0 n I}{2} \int_{\phi_1}^{\phi_2} \sin\phi \, d\phi$$

$$= \frac{\mu_0 n I}{2} \left| -\cos\phi \right|_{\phi_1}^{\phi_2}$$

$$= -\frac{\mu_0 n I}{2} (\cos\phi_2 - \cos\phi_1)$$

$$\boxed{\therefore B = \frac{\mu_0 n I}{2} (\cos\phi_1 - \cos\phi_2)}$$

not useful in numerical

<u>Case</u>:
For the infinite solenoid
$\phi_1 = 0$ & $\phi_2 = 180°$

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So,

$$B = \frac{\mu_0 n I}{2} (1 - (-1))$$

$$B = \frac{\mu_0 n I}{\cancel{2}} \cdot \cancel{2}$$

$$\therefore \boxed{B = \mu_0 n I}$$

Imp

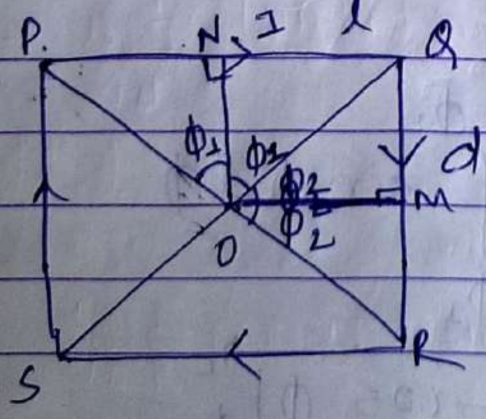
56 Find the strength of magnetic field at the centre of rectangular coil of length l and width d, which carries current I.

Sol^n

current $= I$ known l, d, I

length $= l$

width $= d$



magnetic field due to side PQ at O is given by,

$$B_1 = \frac{\mu_0 I}{4\pi R} (\sin\phi_1 + \sin\phi_2)$$

3.2.21 Self Induction

The induced emf can be produced into the coil itself by changing the current through it

The phenomenon in which the induced emf is produced as a result of change in the current passing through the coil is known as self induction. Consider a coil of inductance L in series with a battery B and Key K. Let I be the current passing through the coil, we have

$$\phi \propto I \Rightarrow \phi = LI \qquad \text{---- (6.138)}$$

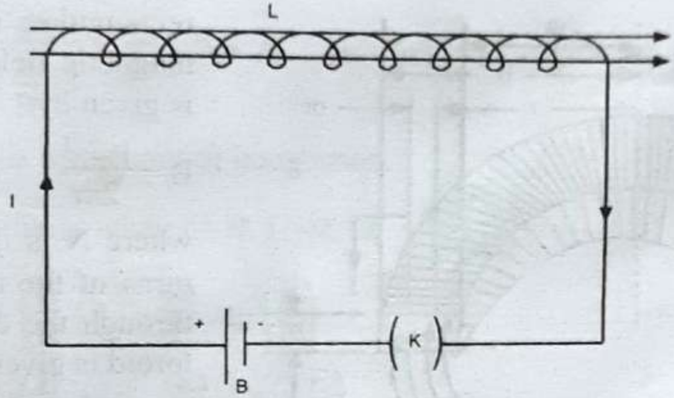


Fig 6.79
Circuit for self induction

where L is a constant known as coefficient of self induction or self inductance of a coil.

Now $L = \frac{\phi}{I}$. If I = 1, then L = ϕ i.e. self inductance of a coil is defined as the flux linked across a coil when the current passing through it is unity. The induced or back emf through the coil is given by

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt} \qquad \text{---- (6.139)}$$

If $\frac{dI}{dt} = 1$, then $L = \varepsilon$ (numerical value)

i.e. self inductance of a coil is numerically equal to the induced or back emf through the coil when the rate of change of current through it is unity. Its unit is Henry (H).

6.2.24 LR Circuit

Here, we study the variation of induced current with time

(a) Rise of Current

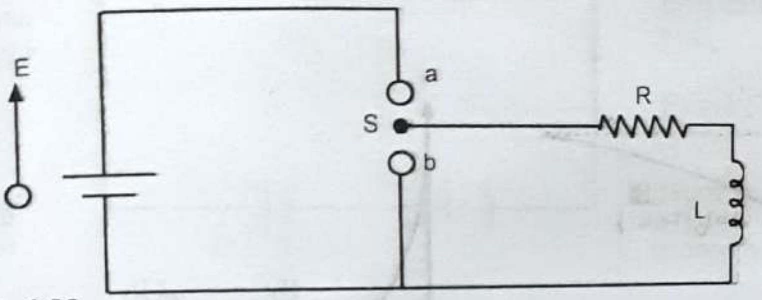


Fig 6.82
Rectangular cross section of a toroid

When switch S in Fig 36.82 is connected to 'a', we have

$$-IR - L\frac{dI}{dt} + E = 0$$

$$L\frac{dI}{dt} + IR = E$$

$$L\frac{dI}{dt} = E - IR$$

$$\frac{dI}{E-IR} = \frac{dt}{L}$$

$$\frac{dI}{R\left(\frac{E}{R} - I\right)} = \frac{dt}{L}$$

$$\frac{dI}{\left(\frac{E}{R} - I\right)} = \frac{R}{L}(dt) \qquad \text{---- (6.146)}$$

$$\int \frac{dI}{\left(\frac{E}{R} - I\right)} = \frac{R}{L}\int dt + A \quad \text{or, } -\ln\left(\frac{E}{R} - I\right) = \frac{Rt}{L} + A$$

A is a constant of integration.

Initially when $t = 0$, $I = 0$, i.e. $-\ln\left(\frac{E}{R}\right) = A$

$$-\ln\left(\frac{E}{R} - I\right) = \frac{Rt}{L} - \ln\left(\frac{E}{R}\right)$$

$$-\ln\left(\frac{E}{R} - I\right) = -\frac{Rt}{L} + \ln\left(\frac{E}{R}\right)$$

$$\ln\left(\frac{E}{R} - I\right) - \ln\left(\frac{E}{R}\right) = -\frac{Rt}{L}$$

$$\ln\frac{\left(\frac{E}{R} - I\right)}{(E/R)} = -\frac{Rt}{L} \qquad \text{or, } \left(\frac{E}{R} - I\right) = \left(\frac{E}{R}\right)e^{-\frac{Rt}{L}}$$

$$I = \frac{E}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

$$I = I_o\left[1 - e^{-\frac{Rt}{L}}\right] \qquad \text{----- (6.147)}$$

This is required expression for rise of a current through LR circuit.

Here, $L/R = \tau_L$ is called the inductive time constant of LR circuit.

When $t = \frac{L}{R}$, then $I = I_o\left(1 - \frac{1}{e}\right) = 0.63\ I_o$

Therefore the inductive time constant is defined as the time in which current increases to 63% times of its maximum value.

The equation (6.136) is plotted as in fig.6.61

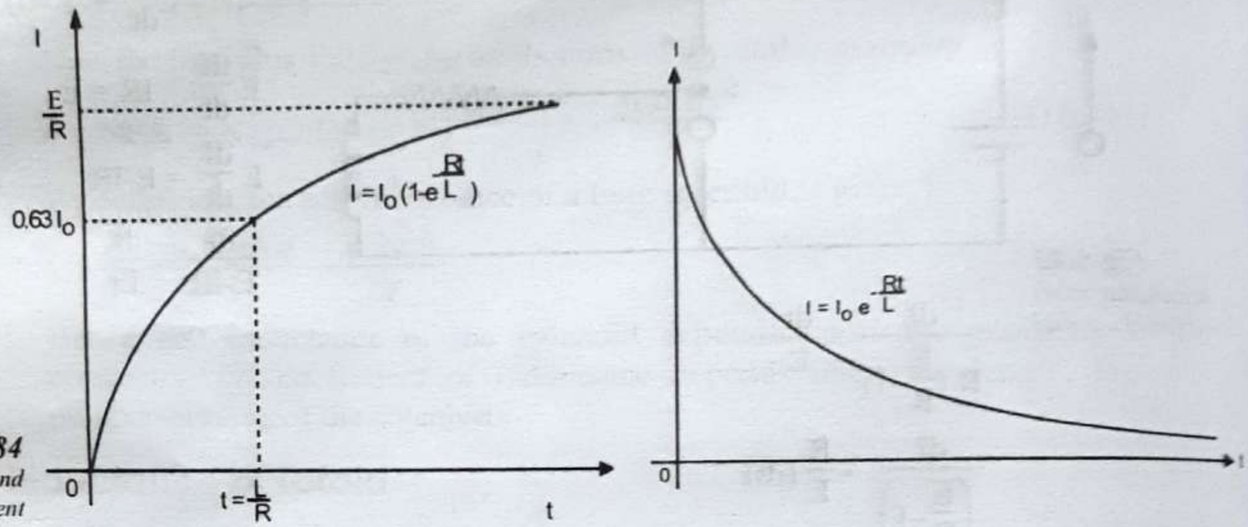


Fig 6.83,84
Graphs for rise and decay of current

(b) Decay of Current:

When switch S in fig. 6.82 is connected to 'b', we have

$$L\frac{dI}{dt} + IR = 0 \qquad \text{or,} \frac{dI}{I} = -\frac{R}{L}\,dt$$

Integrating, we get

$$\int\frac{dI}{I} = -\int\frac{R}{L}dt + B$$

where B being a constant of integration.

$$\ln I = -\frac{Rt}{L} + B$$

When $t = o$, $I = I_o$ and $\ln I_o = B$

Thus, $\ln I = -\frac{Rt}{L} + \ln I_o$

$$\ln I - \ln I_o = -\frac{Rt}{L} \qquad \text{or,} \ln\left(\frac{I}{I_o}\right) = -\frac{Rt}{L}$$

$$I = I_o e^{-\frac{Rt}{L}} \qquad \text{---- (6.148)}$$

This is required expression for decay of current through LR circuits.

When $t = L/R$, we have $I = I_o/e = 0.37\ I_o$

Therefore inductive time constant during decay of current is defined as the time in which the current increases to 0.37 times the maximum value.

Equation (6.148) is plotted as in fig.6.84.

6.2.25 Energy Stored in Magnetic Field

Energy density is directly proportional to the square of flux density

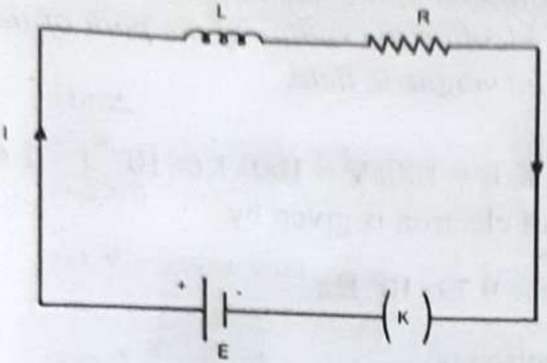


Fig 6.85
L-R circuit

Consider an LR circuit with a battery of emf E and a key K.

By principle of conservation of energy, we have

$$EI = I^2R + LI\frac{dI}{dt} \quad \text{---- (6.149)}$$

Here the term $LI\frac{dI}{dt}$ must be the energy stored per unit time.

i.e.
$$\frac{dU_B}{dt} = LI\frac{dI}{dt} \quad \text{---- (6.150)}$$

Now the total energy stored in the inductance is

$$U_B = \int_0^I \left(\frac{dU_B}{dt}\right) dt = \int_0^I \left(LI\frac{dI}{dt}\right) dt = \int_0^I LIdI = \frac{1}{2}LI^2 \quad \text{---- (6.151)}$$

The energy stored per unit volume of the inductor is called its energy density.

$$u_B = \frac{U_B}{V} = \frac{\frac{1}{2}LI^2}{A\ell} \text{ and } L = \mu_o n^2 A\ell$$

$$u_B = \frac{\frac{1}{2}(\mu_o n^2 A\ell)I^2}{(A\ell)} = \frac{1}{2}\mu_o n^2 I^2 = \frac{(\mu_o nI)^2}{2\mu_o}$$

$$u_B = \frac{B^2}{2\mu_o} \quad \text{---- (6.152)}$$

This is required expression for energy density in the inductor.

14.3 Faraday's Laws of Electromagnetic Induction

The experimental results of Faraday's experiments on electromagnetic induction are called Faraday's laws of electromagnetic induction. Following are the Laws:

1. Whenever magnetic flux linked with a coil or a conductor changes an emf is induced in it, which lasts as long as the change in magnetic flux continues.
2. The magnitude of emf induced in a coil or a conductor is directly proportional to the rate of change of magnetic flux linked with it.

Let ϕ_1 be the magnetic flux linked with a coil or a conductor initially and ϕ_2 be the magnetic flux linked with it after time t. If ε be the emf induced in the coil or conductor, then according to law of electromagnetic induction.

$$\varepsilon \propto \frac{\phi_2 - \phi_1}{t}$$

or, $$\varepsilon = -k\frac{(\phi_2 - \phi_1)}{t} \qquad \text{.......(14.1)}$$

Where k is the proportionality constant and its value depends upon the system of unit in which ε, ϕ and t are measured. In S.I. unit, k = 1. Therefore,

$$\varepsilon = \frac{(\phi_2 - \phi_1)}{t} \qquad \text{.......(14.2)}$$

Negative sign in the formula indicates the opposing nature of induced emf.

If ϕ is the magnetic flux linked with a coil of single turn at any time t, then

The instantaneous rate of change of magnetic flux = $\frac{d\phi}{dt}$

The instantaneous induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt} \qquad \text{.......(14.3)}$$

If there are N number of turns in a coil, which is tightly wound, so that each turn is occupying nearly the same region of space, then flux through each turn will be the same.

Total flux linked with coil, $\phi_T = N\phi$

Total emf induced is the sum of emf's induced in each of the N turns and is given by

$$\varepsilon = \frac{-d\phi_T}{dt}$$

or, $$\varepsilon = \frac{-d(N\phi)}{dt}$$

or, $$\varepsilon = -N\frac{d\phi}{dt} \qquad \text{(14.4)}$$

In S.I. unit, ε is measured in volt. So,

$$1\text{volt} = \frac{1\text{Weber}}{1\text{second}}$$

That is, emf induced in a coil or a conductor is one volt if the magnetic flux linked with it is changing at the rate of one Weber per second.