

Tutorial - 2 (Radip Datta)

201751

Q) 1) Describe the working principle of a very simple CPU that can execute the following instructions.

Instruction	Instruction Code	Operation
ADD	00AAAAAAA	$AC \leftarrow AC + M[AR]$
AND	01AA AAAA	$AC \leftarrow AR \& M[AAAAAA]$
JMP	10AAAAAA	GO TO AAAAAAA
INC	11xxxxxx	$AC \leftarrow AC + 1$

Draw the state diagram and explain how each instruction is executed.

⇒ Answer.

In addition to AC, this CPU needs several additional registers to perform the internal operations necessary to fetch, decode, and execute instructions. The registers in this CPU are fairly standard and are found in many CPUs; their sizes vary depending on the CPU in which they are used. This CPU contains the following registers:

- ↳ A 6-bit address register, AR, which supplies an address to memory via $A[5..0]$.
- ↳ A 6-bit program Counter, PC, which contains the address of the next instruction to be executed.
- ↳ An 8-bit data register, DR, which receives instructions and data from memory via $D[7..0]$.

↳ A 2-bit instruction register, IR, which stores the opcode portion of the instruction code fetched from memory.

→ Fetching instruction from memory:

↳ The address of instruction to be fetched is stored in the PC. Since, address register (AR) supplies an address to memory through A[5...0] pins, firstly copy the content of PC to AR.

Fetch 1: AR \leftarrow PC

↳ The CPU must read the instruction from the memory and store in DR.

Fetch 2: DR \leftarrow M, PC \leftarrow PC + 1

↳ Finally, two higher order bits of DR are copied into IR: these two bit indicate which instruction is to be executed, and the CPU copies the lower order 6-bit of DR into AR.

Fetch 3: IR \leftarrow DR[7, 6], AR \leftarrow DR[5...0]

→ Decoding Instruction:

↳ The CPU must determine which instruction it has fetched so as to invoke the current routine.

↳ The value of IR i.e., 00, 01, 10 and 11 determine which routine is to be invoked.

→ Executing Instructions:

↳ ADD instruction:

⇒ first it must fetch one operand from the memory.

⇒ Then, it must add this operand to the current content of accumulator and store the result back into the accumulator.

$$\text{ADD1: DR} \leftarrow M$$

$$\text{ADD2: AC} \leftarrow \text{AC} + \text{DR}$$

↳ AND instruction:

$$\text{AND1: DR} \leftarrow M$$

$$\text{AND2: AC} \leftarrow \text{AC} \wedge \text{DR}$$

↳ JMP instruction:

⇒ The address to which the CPU must jump is copied into the PC.

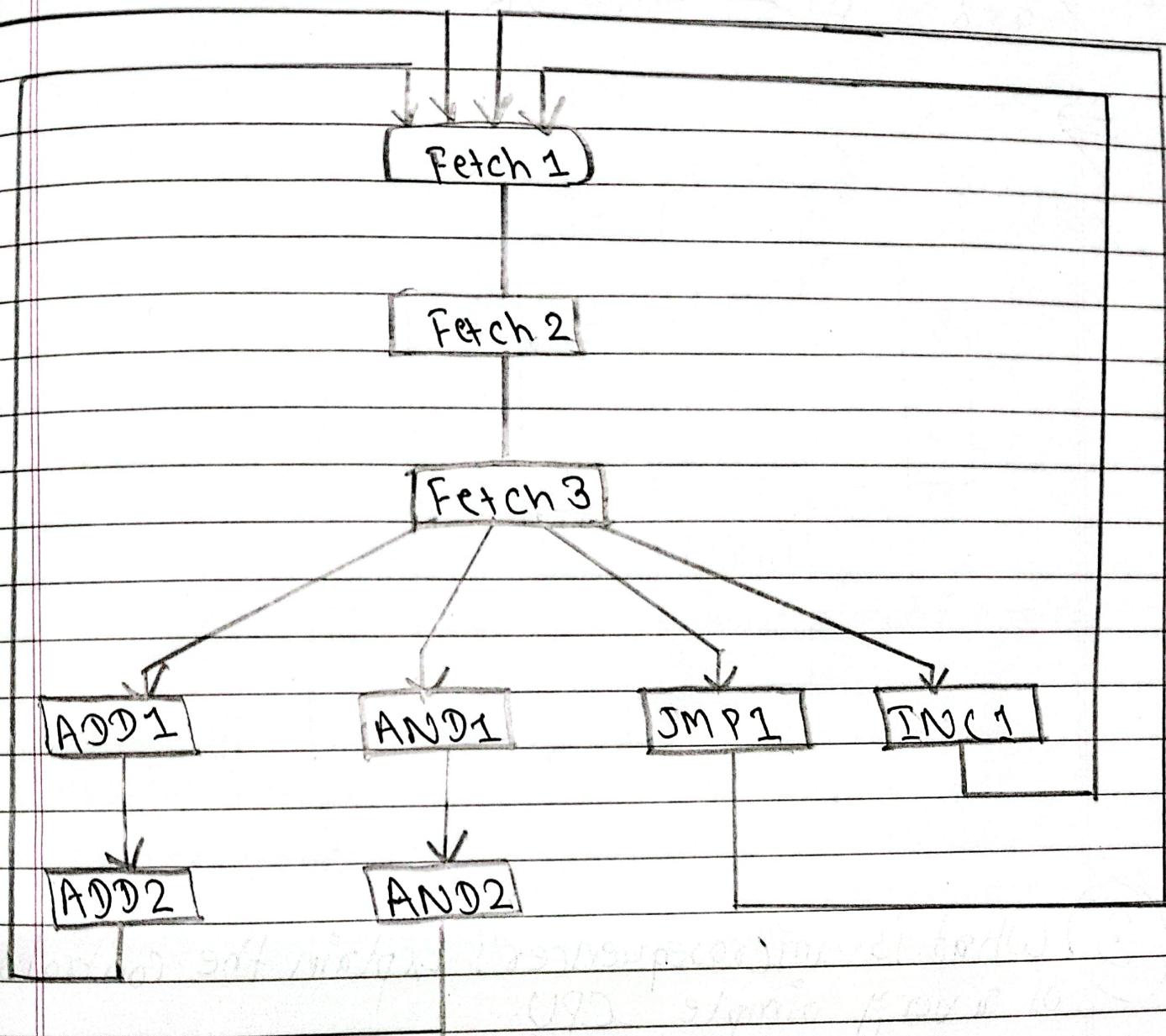
⇒ Since, the result is already stored in DR[5 ... 0], we simply copy this value to PC.

$$\text{JMP1: PC} \leftarrow \text{DR}[5 \dots 0]$$

⇒ INC instruction:

$$\text{INC1: AC} \leftarrow \text{AC} + 1$$

State diagram:



→ Establishing required data path:

Fetch 1: $AR \leftarrow PC$

Fetch 2: $DR \leftarrow M, PC \leftarrow PC + 1$

Fetch 3: $IR \leftarrow DR[7, 6], AR \leftarrow DR[5 \dots 0]$

ADD1: $DR \leftarrow M$

ADD2: $AC \leftarrow AC + DR$

AND1: $DR \leftarrow M$

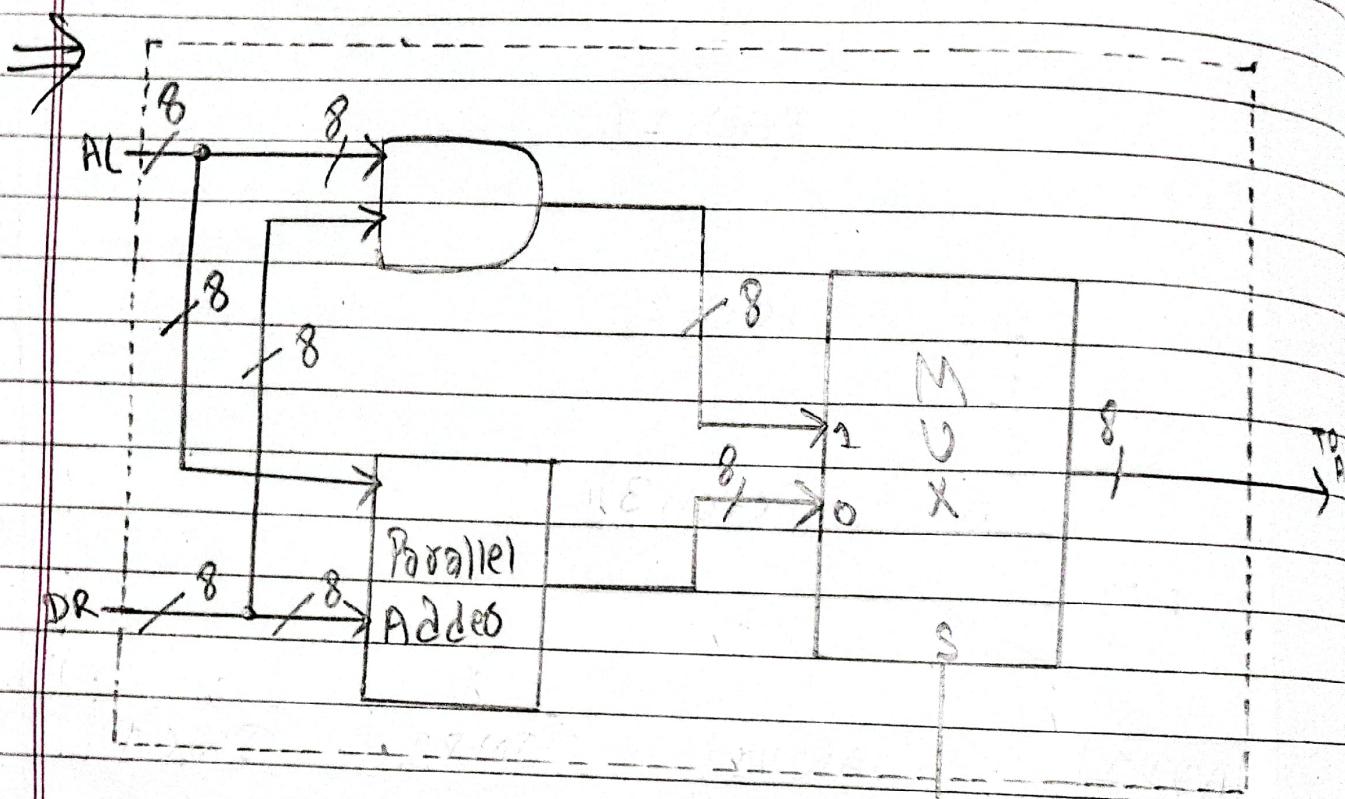
AND2: $AC \leftarrow AC \wedge DR$

JMP1: $PC \leftarrow DR[5 \dots 0]$

INC1: $AC \leftarrow AC + 1$

(2)

Design the ALU that performs $AC \leftarrow AC + DR$
and $AC \leftarrow AC \cdot DR$



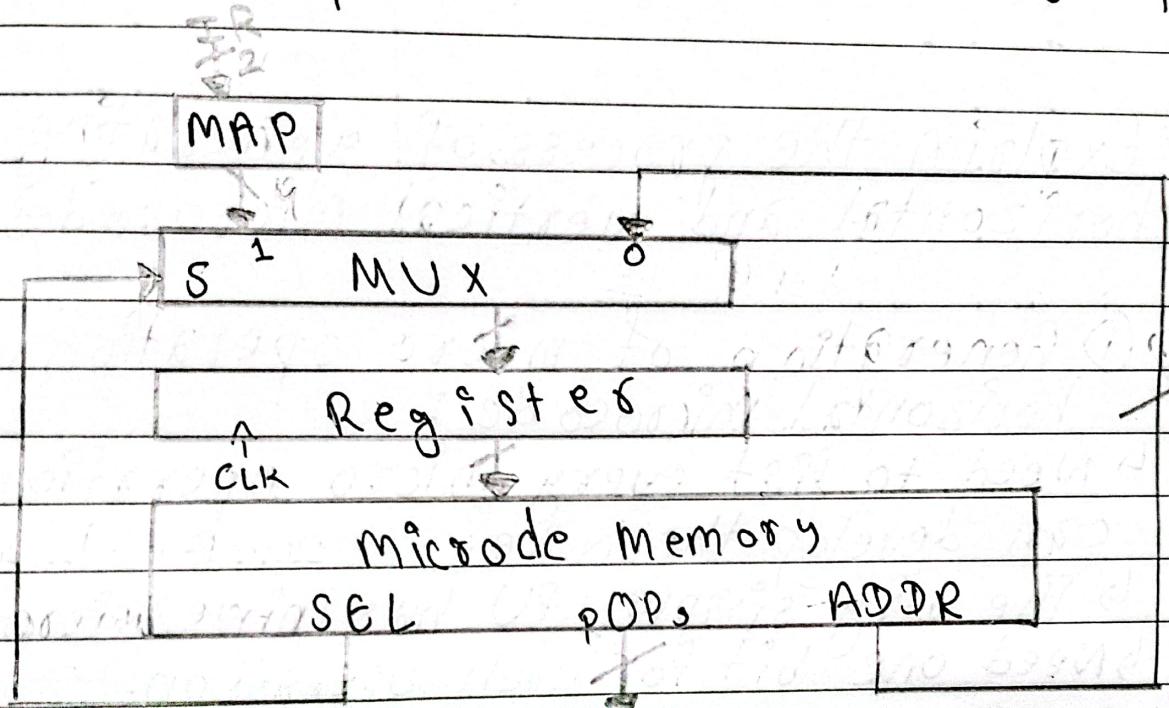
Control Signal
(from control unit)

(3)

What is microsequencer? Explain the control signals of a very simple CPU.

⇒ A microsequencer is a component or circuitry within a computer's central processing unit (CPU) that controls the execution of microinstructions. It retrieves, decodes, and sequences the microinstructions, which are low-level instructions that specify the individual operations performed by the CPU. The microsequencer generates the necessary control signals to execute the microinstructions in the correct order, ensuring proper operation of the CPU.

- Consider the state diagram in the CPU design.
- ④ In this only two possible next addresses are used; the opcode mapping and an absolute jump.
 - ↳ fetch 3 goes to one of the four execute routines; implemented via the mapping input.
 - ↳ Remaining states must go to next specific address; implemented via absolute jump.



- ↳ MUX is used to select one of the two possible address
- ↳ Select bit generated by microsequencer is used to choose correct next address.
- ↳ A total of 9 states in the state diagram, and hence 4-bits needed to represent them.

→ Control Signal values for the very simple CPU:

ARLOAD	ARPC V AIDR
PCLOAD	PCDR
PCINC	PCIN
DRLOAD	DRM
	PLUSVAND

ACINC
IRLOAD
ALUSEL
MEMBUS
PCBUS
DRBUS
READ

ACIN
AIDR
AND
DRM
ARPC
AIDR UP CDRV PLUS V AND
DRM

(4)

Explain the process of generating horizontal and vertical microcodes.

⇒ ① Generation of micro operation using horizontal microcode:

- ↳ Need to list every micro operation so we can develop the code.
- ↳ The very simple CPU has nine micro operations.
- ↳ Need one bit for each operation.

Mnemonics

ARPC
ARDR
PCIN
PRDR
DRM
IRDR
PLUS
AND
ACIN

Micro Operation

$AR \leftarrow PC$
$AR \leftarrow DR[5 \dots 0]$
$PC \leftarrow PC + 1$
$PC \leftarrow DR[5 \dots 0]$
$DR \leftarrow M$
$IR \leftarrow DR[7 \dots 6]$
$AC \leftarrow AC + DR$
$AC \leftarrow AC \wedge DR$
$AC \leftarrow AC + 1$

→ Optimized horizontal microcode for the very simple microsequencers

State	Address	S E L	A B C	A I R	P F N	P C R	P R M	P L U	A N G	A A J	AADR
FETCH 1	0000 (0)	0	1	0	0	0	0	0	0	0	0001
FETCH 2	0001 (1)	0	0	0	1	0	1	0	0	0	0010
FETCH 3	0010 (2)	1	0	1	0	0	0	0	0	0	xxxx
ADD 1	1000 (8)	0	0	0	0	0	0	1	0	0	1001
ADD 2	1001 (9)	0	0	0	0	0	0	1	0	0	0000
ADD 1	1010 (10)	0	0	0	0	0	0	1	0	0	1011
AND 2	1011 (11)	0	0	0	0	0	0	0	0	1	0000
JMP 1	1100 (12)	0	0	0	0	1	0	0	0	0	0000
INC 1	1110 (14)	0	0	0	0	0	0	0	0	1	0000

II Generation of microoperation using Vertical microCode:

→ Micro-operation field assignments & values

M1		M2	
Value	Micro-operation	Value	Micro-operation
000	NOP	0	NOP
001	DRM	1	PCIN
010	ARPC		
011	ADIR		
100	PCDR		
101	PLUS		
110	AND		
111	ACIN		

Vertical microcode for the very simple microsequencer

State	Address	SEL	M1	M2	ADD
FETCH1	0000 (0)	0	010	0	001
FETCH2	0000 (1)	0	001	1	0010
FETCH3	0010 (2)	1	011	0	XXXX
ADD1	0000 (8)	0	001	0	1001
ADD2	1001 (9)	0	101	0	0000
AND1	1010 (10)	0	001	0	1011
AND2	1011 (11)	0	110	0	0000
SMP1	1100 (12)	0	100	0	0000
TNC1	1110 (14)	0	111	0	0000

⑥ Compare Hardwired Control Unit & Microprogrammed

⇒ Hardwired Control Unit

The control unit whose control signals are generated by the hardware through a sequence of instruction is called a hardwired control unit.

(i) The control logic of a hardwired control unit is implemented with gate, flip flops, decoders etc.

(ii) Wiring changes are made in the hardwired control unit if there are any changes required in the design.

(iii) It is faster and known to be complex structure.

⇒ Microprogrammed Control Unit

The control unit whose control signals are generated by the data stored in control memory and constitutes a program on the small scale is called a microprogrammed control unit.

(i) The control logic of a micro-programmed control is the instructions that are stored in control memory to initiate the required sequence of microoperations.

(ii) Changes in a microprogrammed control unit are done by updating the microprogram in control memory.

(iii) Slow compared but are simple.

⑤ Design a CPU that ~~meets~~ :

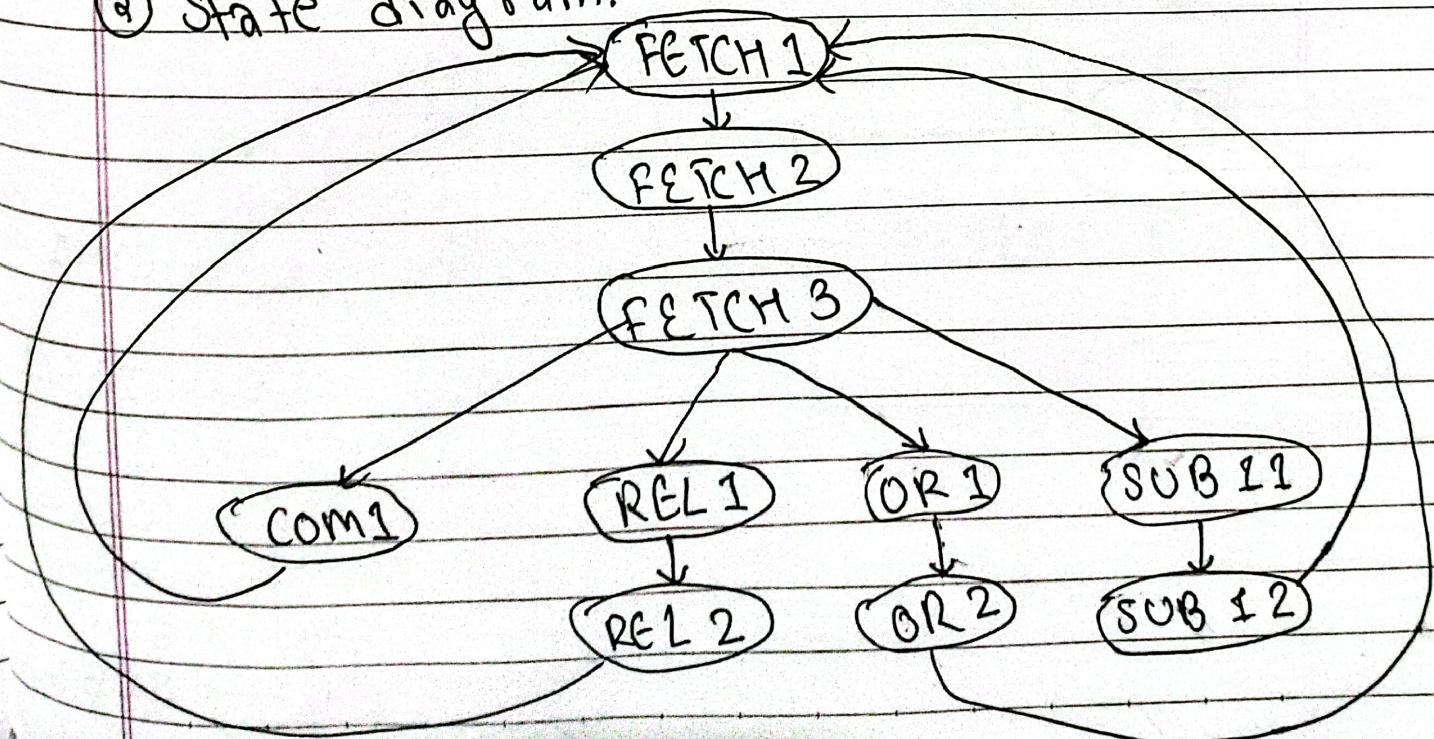
⑥ The instruction set of a very simple CPU is given as:

Instruction	Instruction Code	Operation
COMP	00XXXXXX	$AC \leftarrow AC'$
JREL	01AAAAAA	$PC \leftarrow PC + AAAAAA$
OR	10AAAAAA	$AC \leftarrow AC \text{ UM}[AAAAAA]$
SUB	11AAAAAA	$AC \leftarrow AC - m[AAAAAA] - 1$

- ① Design & implement state diagram.
- ② Design & implement register section & ALU.
- ③ Design & implement Hardwired control Unit
- ④ Design & implement microsequencer & control unit.

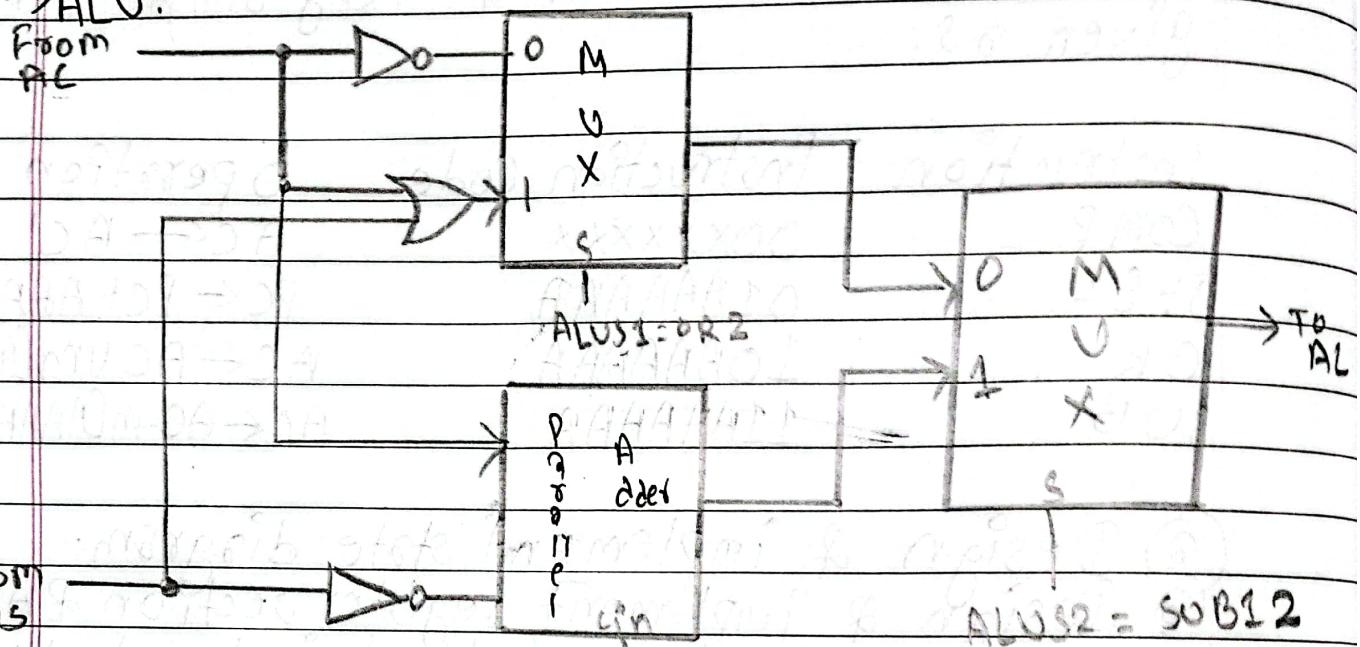
→ Solution:

① State diagram:

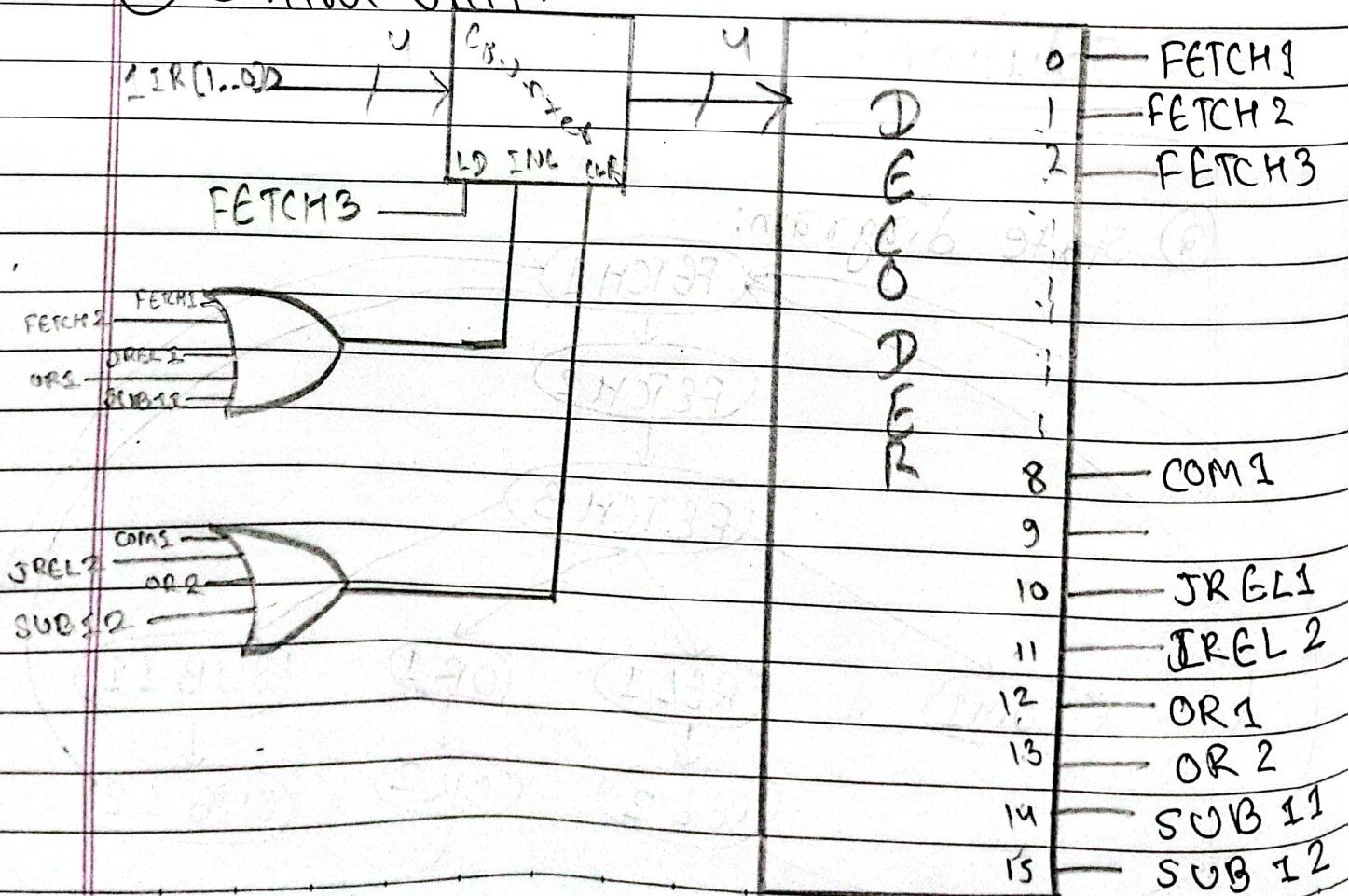


(b) Register section & ALU:

→ ALU:



(c) Control Unit:



(12) Explain the Booth's algorithm for division with an example.

⇒ Here,

let's take an example,

For $112 \div 7$ with $n=4$, $UV = 0111\ 0000$ and $X = 0111$. Since, $U \geq X$, the algorithm exists immediately. Had it proceeded, it should have produced a result of $16(10000)$ and a remainder of 0. The value 10000 cannot be stored in a 4-bit register, this is the reason for the overflow;

Now,

G1: FINISH $\leftarrow 1$, OVERFLOW $\leftarrow 1$

2: $Y \leftarrow 0$, $C \leftarrow 0$, OVERFLOW $\leftarrow 0$, $i \leftarrow n$

3: $\text{shl}(UV)$, $\text{shl}(Y)$, $i \leftarrow i - 1$

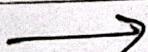
(G1) 4: $Y_0 \leftarrow 1$, $UV \leftarrow UV \times' + 1$

z' 4: GOTO 3

z 4: FINISH $\leftarrow 1$

→ Trace of the RTL code for the operation $147 \div 13$.

Here, $U = 1001$, $V = 0011$, $X = 1101$ & $n = 4$



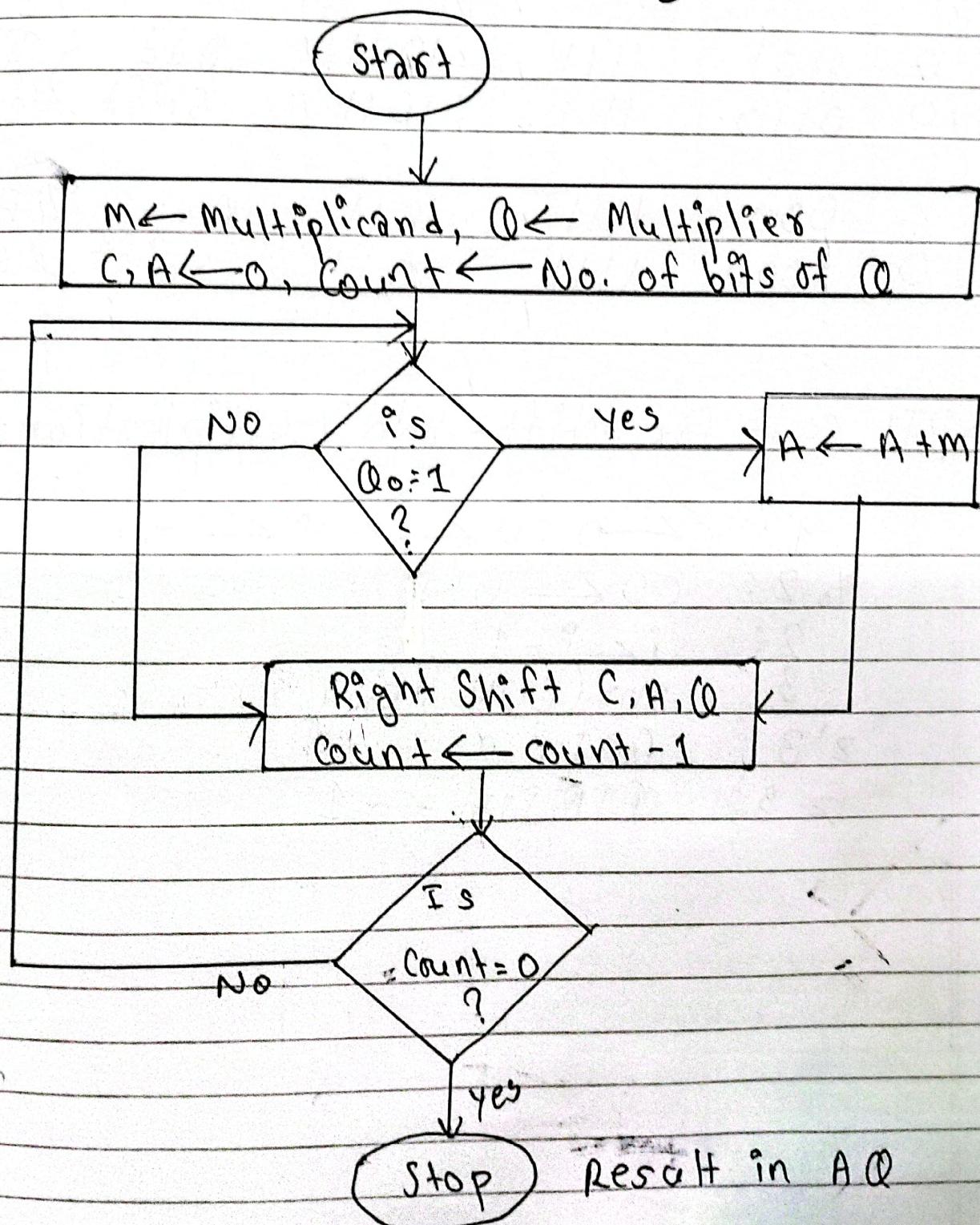
Conditions	Micro-operations	i	C	U	V	Y	Z	Finish
START		x	x	1001	0011	xxxx		
1	NONE							
2	$y \leftarrow 0; C \leftarrow 0,$ OVERFLOW $\leftarrow 0,$ $i \leftarrow i + 1$	4	0			0000	0	
3	$Shl(CUV), Shl(Y)$ $i \leftarrow i - 1$	3	1	0010	0110	0000	0	
(C+G)4,24	$y_0 \leftarrow 1,$ $U \leftarrow Ut x^1 + 1,$ GOTO 3			0101		0001		
3	$Shl(CUV), Shl(Y)$ $i \leftarrow i - 1$	2	0	1010	1100	0010	0	
2'4	GOTO 3							
3	$Shl(CUV), Shl(Y)$ $i \leftarrow i - 1$	1	1	0101	1000	0100	0	
(C+G)4,24	$y_0 \leftarrow 1,$ $U \leftarrow Ut x^1 + 1,$ GOTO 3			1000		0101	1	
3	$Shl(CUV), Shl(Y)$ $i \leftarrow i - 1$	0	1	10001	0000	1010	1	
(C+G)4,24	$y_0 \leftarrow 1$ $U \leftarrow Ut x^1 + 1,$ FINISH $\leftarrow 1$			0100		1011		1

⑦ Explain the shift add multiplication algorithm with an example. Also write down the RTL.

⇒ Shift add multiplication:

⇒ multiplication of unsigned numbers.

⇒ multiplication can be performed by repeated multiplicand 'A' and shifting.



C A CP M
 0 0000 1101 1011 Initial values

0 1011 1101 1011 Add } First
 0 0101 1110 1011 shift } cycle

0 0010 1111 1011 shift } second cycle

0 1101 1111 1011 Add } Third
 0 0110 1111 1011 shift } cycle

1 0001 1111 1011 Add } Fourth
 0 1000 1111 1011 shift } cycle

→ RTL code for shift-add multiplication:

1: $C \leftarrow 0, V \leftarrow 0, i \leftarrow n$

2: $CU \leftarrow V + X$

2: $i \leftarrow i - 1$

3: $shs(CUV), c_{18}(Y)$

2'3: GOTO 2

2 3: FINISH $\leftarrow 1$

Example: $UV \leftarrow X+Y; X=1101 \& Y=1011$

Conditions	Micro-operations	1	C	U	V	Y	Z	FINISH
START		X	X	XXXX	XXXX	1011	0	0
1	$C \leftarrow 0, U \leftarrow 0, i \leftarrow 4$	4	0	0000			0	
$Y_0 2,2$	$CU \leftarrow U + X$ $i \leftarrow i - 1$	3	0	1101			0	
$3, Z'3$	$Shs(CUV), cir(Y)$ GOTO 2		0	0110	1XXX	1101		
$Y_0 2,2$	$CU \leftarrow U + X,$ $i \leftarrow i - 1$	2	1	0011			0	
$3, Z'3$	$Shs(CUV), cir(Y)$ GOTO 2		0	1001	11XX	1110		
2	$i \leftarrow i - 1$	1					0	
$3, Z'3$	$Shs(CUV), cir(Y)$ GOTO 2		0	0100	111X	0111		
$Y_0 2,2$	$CU \leftarrow U + X,$ $i \leftarrow i - 1$	0	1	0001			1	
$3, Z'3$	$Shs(CUV), cir(Y)$ FINISH $\leftarrow 1$		0	1000	1111	1011		1

8) Trace the shift add algorithm for 10×6 .

Here,
for shift add algorithm for 10×6 ;
we have,

$$X = 1010, Y = 0110$$

Now,

Function	i	C	U	V	Y	Comments
$U=0$			0000	0000	0110	y.
if $Y_0 = 1$	1	0	1010			$Y_0 = 0$
$Shs(CUV)$		0	0101	0000	0011	
$Cir(Y)$						
if $Y_0 = 1$	2	0	1111			$Y_0 = 1, Add$
$Shs(CUV)$						
Shs(CUV)		0	0111	1000		
$Cir(Y)$					1001	
if $Y_0 = 1$	3	1	1001			$Y_0 = 1, Add$
$Shs(CUV)$		0	1100	1100		
$Cir(Y)$					1100	
if $Y_0 = 1$	4					$Y_0 = 0$
$Shs(CUV)$		0	0110	0110		
$Cir(Y)$					0110	original

RTL:

1: $U \leftarrow 0, i \leftarrow x$ 2: $CU \leftarrow U + x$ 2: $i \leftarrow i - 1$ 3: $Shs(CUV), Cir(Y)$

2'3: GOTO 2

23: FINISH $\leftarrow 1$

Under,

2=1. when $i=0 \& 1, 2, \& 3$ are consecutive states

⑨ Trace the RTL code for shift add multiplication of 5×5 .

Conditions	Micro-operations	i	C	U	V	Y	Z	Finish
START		x	x	xxxx	xxxx	0101		0
1	$U \leftarrow 0, i \leftarrow 4$	4		0000				0
2	$i \leftarrow i - 1$	3						0
3, 2' 3	Shs(UVV), cir(Y), goto 2	0	0000	0xxx	1010			
Y ₀ 2, 2	$(U \leftarrow U + X, i \leftarrow i - 1)$	2	0	0101				0
3, 2' 3	Shs(UVV), cir(Y), goto 2	0	0010	10xx	0101			
Y ₀ 2, 2	$(U \leftarrow U + X, i \leftarrow i - 1)$	1	0	0111				0
3, 2' 3	Shs(UVV), cir(Y), goto 2	0	0011	110x	1010			
Y ₀ 2, 2	$(U \leftarrow U + X, i \leftarrow i - 1)$	0	0	1000				1
3, 2' 3	Shs(UVV), cir(Y), goto 2	0	0100	0110	0101			1

Result: $5 * 5 = 25$, or, $0101 * 0101 = 0100\ 0110$

⑩ Explain Booth's algorithm for multiplication of unsigned two's complement numbers with example.

⇒ Booth Algorithm:

- ↳ It reduces the number of addition.
- ↳ Negative multiplier, positive multiplier are treated same.
- ↳ For negative numbers multiplication, the operation is performed by representing the negative numbers in its 2's complement form.

ADVANTAGES:

- ⇒ When there are lot of ones, all will be cancelled and replaced with zeros.

⇒ More Zeros means, it reduces the addition
(no operation is done with zeros)

⇒ But shift remains the same, can't be reduced.

* Example: -5×4

Perform ~~Binary~~ multiplication using Booth algorithm.

Here, $X = -5$

$$\begin{array}{r} X = \cancel{0} \cancel{-} \cancel{1} \cancel{0} \cancel{0} \\ Y = \cancel{1} \cancel{0} = \end{array}$$

$$X = -5 = 01011$$

$$Y = 4 = 00100$$

$$\therefore X = 1011, Y = 0100$$

$$\begin{array}{r} x_1 = 0100 \\ 01011 \\ 10100 \\ + \\ \hline 10101 \end{array}$$

Now,

Conditions	Micro-operations	i	U	V	$\times Y$	$Y-1$	Finish
START			x	xxxx	$xxx \cancel{x}$	0100	X
1	$U \leftarrow 0, Y-1 \leftarrow 0, i \leftarrow n$		5	000000			0 0
$Y \neq 1, 2, 2$	$U \leftarrow U + X^i + 1$		4	10101			0
	$i \leftarrow i-1$				11010		
3, 2, 3	ashr(UV), cir(Y), $Y \leftarrow Y$		1	1XX	00010	1	
	goto 2						
$Y \neq 1, 2, 2$	$U \leftarrow U + X$		3	00101			0
	$i \leftarrow i-1$						
3, 2, 3	ashr(UV), cir(Y), $Y \leftarrow Y$		1	0010	11XX0000	0	
	goto 2						
$Y \neq 1, 2, 2$	$U \leftarrow U + X^i + 1$		2	00111			0
	$i \leftarrow i-1$						
3, 2, 3	ashr(UV), cir(Y), $Y \leftarrow Y$		1	0011	111XX10000	1	
	goto 2						
2	$i \leftarrow i-1$		1				
:	:	:	1				



3, z'3	ashr(UV), cir(Y), Y ₋₁ ← Y ₀	11001	1111X	01000	1
2	goto 2 i ← i - 1	0			1
3, z'3	ashr(UV), cir(Y), Y ₋₁ ← Y ₀	11100	11111	00100	

Here UV = 11100 1111

There is end around carry so, we find 2's complement and place 've' sign before the equivalent decimal.

→ The RTL of Booth's algorithm for multiplication:

1: U ← 0, Y₋₁ ← 0, I ← n

Y₀, Y₋₁, 2: U ← U + X + 1

Y₀, Y₋₁, 2: U ← U + X

2: i ← i - 1

3: ashx(UV), cir(Y), Y₋₁ ← Y₀

z'3 & goto 2

z3: FINISH ← 1

(1)

⇒ Done in 10 → example.