Chapter 6

Electrostatics

Electric Charge

When two bodies are rubbed, there is transference of electrons in the outermost orbit from atoms of one body to another. The body which gains the electron will be negatively charged and which losses electron will be positively charged.

According to modern electron theory, the state of an atom after loss or gain of electron is called the charged state. And the new form of atom is called charge.

It's S.I. unit is Coulomb.

c.g.s. unit is e.s.u or stat coulomb.

Examples:

1. Positive charge - proton = 1.6×10^{-19} coulomb
2. Negative charge - electron = -1.6×10^{-19} coulomb

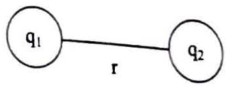
Electric Force: Coulomb's Law

The force of attraction or repulsion between two charges is directly proportional to the product of the magnitude of charges and inversely proportional to the square of the distance between them.

If distance between two charges q_1 and q_2 is r then Coulomb's law states that.

$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$



Combining,

$$F \propto \frac{q_1 q_2}{r_2} = K \frac{q_1 q_2}{r^2}$$

Where, K is propertionality constant known as Coulomb's constant. It depends upon the nature of the medium and system of unit used.

In S.I system and air medium (or vaccum).

$$K = \frac{1}{4\pi\epsilon_o}$$

Therefore, $F = \frac{1}{4\pi\epsilon_o} \frac{q_1 q_2}{r^2}$

Where, $\epsilon_o = 8.85 \times 10^{-12}$ Farad m^{-1}, known as permitivity of free space.

In C.G.S. system (for air or vacuum), K = 1

Therefore, $F = \frac{q_1 q_2}{r^2}$

But, if there is a medium of permitivity, ϵ, $F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$

When more than two charges are present. The resultant force on any one of them is given by vector sum of the forces exerted by various individual charges.

i.e. $F = \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \ldots\ldots$ for force act on first charge.

Permitivity

It is found that when the two charges are placed in medium other than air or vacuum, the force between the charges is reduced. "This is due to the higer permitivity of medium. Thus permitivity is the characteristics of medium because of which the force between the charges in the medium is affected". Higher the value of permitivity lower will be the force and vice- versa.

When the charges are located in the medium of absolute permitivity ϵ, the Coulomb force will be.

$$F = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$$

The relative permitivity of medium is defined as the ratio of absolute permittivity of medium to the absolute permittivity of free space,

$$\epsilon_r = \frac{\epsilon}{\epsilon_o}$$

It is also known as dielectric constant.

Note: Relative permitivity of vaccum = 1
Relative permitivity of air = 1.005
Relative permitivity of water = 80

Electric Field Intensity

The electric field intensity at a point in an electric field is defined as the force experienced by unit test charge at that point.

i.e. $E = \frac{F}{q_0}$

Unit, in S.I - N C^{-1}

C.G.S. - dyne / e.s.u

Note: 1 Newton = 10^5 dyne.

Electric Field Intensity due to a Point Charge

When a test charge is placed in the Electric field of charge q, then force experienced by q_o due to q (or vice versa) is,

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

Where, r is distance of q_0 from q.

Electric field intensity due to charge q is given by $E = \frac{F}{q_0}$.

$$= \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2} \times \frac{1}{q_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

If there is group of charges, the electric field intensity at any point is given by.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots\dots\dots + \vec{E}_n$$

Electric field Intensity due to Continuous Charge Distribution

In case of group of charges, the distances between them are much smaller. In such case, the system of charges can be considered as continuous charge distribution.

In such situation, the continuous charge distribution can be divided into small elements of small charge (dq) in terms of linear charge density (λ) or surface charge density (σ) or volume charge density (ρ), on the basis of dimension of the charged body.

Then the electric field at any point due to one charge element 'dq' is given by

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

Total electric field due to all element (or continuous charge distribution) is

$$E = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r^2}$$

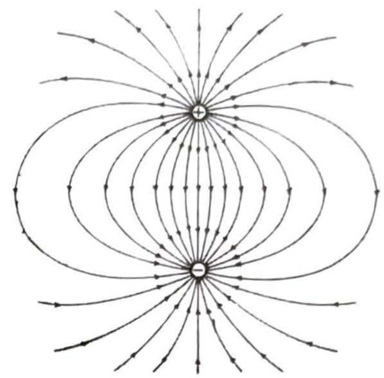
Lines of Force

An electric lines of force is the path along which a unit positive charge would move, if it is free to do so.

The concept of electric lines of force help us to visualize the electric field.

Properties

1. Electric lines of force starts from a positive charge and end at an equal quantity of negative charge.
2. The tangent drawn to any point on electric lines of force gives the direction of electric field at that point.
3. The number of electric lines of force per unit area, measured in a plane that is perpendicular to the lines is proportional to the magnitude of E.
4. Two lines of force never cross each other.
5. It gives the strength of electric field. If the lines are closer, the strength of E will be more and vice versa.



Electric Flux

The electric flux at a point in electric field is defined as the number of electric lines of force passing per unit area perpendicular to the direction of lines of force.

Generally speaking, the number of lines of electric field is electric flux.

This means the electric field at a point is given by electric flux passing per unit area perpendicular to direction of lines of force at that point.

i.e. $\vec{E} = \frac{d\phi}{dA}$

or, $d\phi = \vec{E}\cdot d\vec{A}$

or, $\phi = \int \vec{E}\cdot d\vec{A}$

[Note: For uniform Electric field $\phi = EA$ & for non uniform electric field $\phi = \int \vec{E}\cdot d\vec{A}$]

Gauss Law

It states that the total electric flux through a closed surface enclosing a charge is equal to $\frac{1}{\epsilon_0}$ times the magnitude of the charge enclosed.

i.e. $\phi = \frac{q}{\epsilon_0}$

or, $\oint \vec{E}\cdot\vec{dA} = \frac{q}{\epsilon_0}$

Proof

Consider a closed surface A, enclosing a charge q. Let area element $d\vec{A}$ be at a distance $\vec{r}$ from the charge. If $\vec{E}$ is electric field at the area element, then the electric flux passing through area element is given by .

$$\phi = \oint \vec{E}.d\vec{A}$$

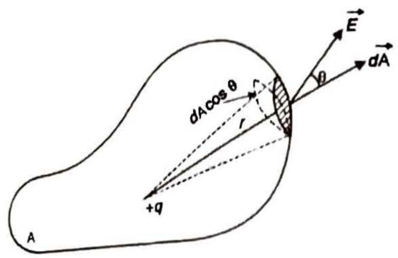
let θ is the angle between $\vec{E}$ and $d\vec{A}$ then

$$\phi = \oint E\, dA \cos\theta$$

$$= \oint \frac{q}{4\pi\epsilon_o r^2}.\, dA \cos\theta$$

$$= \frac{q}{4\pi\epsilon_o} \oint \frac{dA\cos\theta}{r^2}$$

$$= \frac{q}{4\pi\epsilon_o} \int d\omega$$



Where dω is solid angle subtended by area element dA.

or, $\phi = \frac{q}{4\pi\epsilon_o}.\omega = \frac{q}{4\pi\epsilon_o}.4\pi$ [Since, ω = 4π]

$$\phi = \frac{q}{\epsilon_o}$$

Note:

1. When the calculation of Electric field become difficult, in such cases an imaginary closed surface enclosing a charge is taken called Gaussian surface.
2. The Gaussian surface is constructed so that the lines of force are perpendicular to it and it contains the point where the $\vec{E}$ is to calculate.

Application of Gauss's law

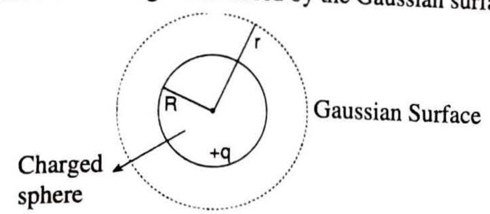
1. **Field due to Non conducting spherical symmetric charge distribution.**

(Electric field due to charged sphere)

Consider a spherical charge distribution of total charge q and radius R. For spherical charge distribution the charge density 'ρ' remains same for all the points lying at equidistance from the centre but 'ρ' can vary with the variation of radial distance.

To find an expression for $\vec{E}$ for points 1) out side 2) inside 3) and on the surface of charge distribution we use Gauss law.

i) ***Electric field out side the sphere:*** for this we construct a Gaussian surface of radius 'r' such that r > R. Since all the charge is enclosed by the Gaussian surface.



So, flux, $\int \vec{E}.\vec{dA} = \frac{q}{\epsilon_o}$

or, $E.4\pi r^2 = \frac{q}{\epsilon_o}$

or, $E = \frac{q}{4\pi\epsilon_o r^2}$

This is the electric field intensity due to spherical charge distribution of total charge 'q' and of radius R at any point on the Gaussian surface.

ii) ***Electric field inside the sphere:*** In this case, Gaussian surface of radius 'r' lies inside the symmetrically charged sphere such that r < R. In this case the part of charge lying outside the Gaussian surface does not contribute to set up electric field E.

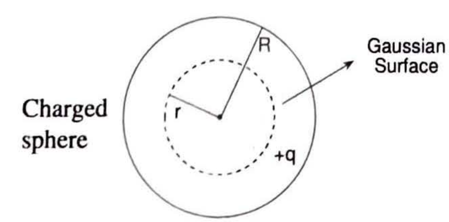
Let q' represents the part of q enclosed by Gaussion surface, then applying gauss law,

$$\int \vec{E}.\vec{dA} = \frac{q'}{\epsilon_o}$$

$$E.4\pi r^2 = \frac{q'}{\epsilon_o} \Rightarrow E = \frac{q'}{4\pi\epsilon_o r^2}$$

Since, charge density,

$$\rho = \frac{q}{\frac{4\pi R^3}{3}} = \frac{q'}{\frac{4\pi r^3}{3}}$$



$$\Rightarrow q' = \frac{qr^3}{R^3}$$

Therefore,

$$E = \frac{1}{4\pi\epsilon_o r^2} \times \frac{qr^3}{R^3}$$

$$E = \frac{qr}{4\pi\epsilon_o R^3}$$

iii. **Electric field on the surface of the sphere.**

At surface of the sphere, r = R.

Therefore, $E = \frac{q}{4\pi\epsilon_o R^2}$

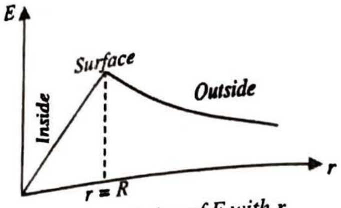


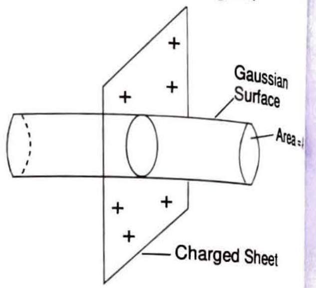
Figure: Variation of E with r

2) **Field due to planner symmetric distribution of charge**
(Electric field due to infinite sheet of charge)
(Electric field due to charged plane lamina)

Figure shows a portion of thin, infinite, non - conducting sheet of charge with uniform charg density σ.

To find the electric field $\vec{E}$ at a distan 'r' from the sheet, a closed cyllinder with cross ection area 'A' perpendicular to the sheet can be taken as Gaussian surface as shown in figure.

Since the charge is positive, $\vec{E}$ is directed away from the sheet, and thus the electric field lines pass through the cross section of the cylinder. Because the field lines do not cross the curved surface so there is no flux through this section.



Applying gauss law

$$\oint E.dA = \frac{q}{\epsilon_0}$$

$$\Rightarrow \quad EA + EA = \frac{q}{\epsilon_0}$$

[Since, for two cross section $\int \vec{E}.\vec{dA} = E.A + E.A$]

$$\Rightarrow \quad 2EA = \frac{q}{\epsilon_0}$$

$$\Rightarrow \quad 2EA = \frac{\sigma A}{\epsilon_0} \quad [\sigma = \frac{q}{A} \Rightarrow q = \sigma A]$$

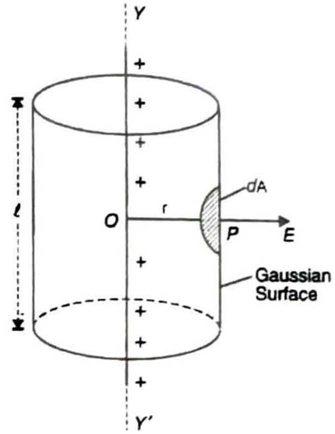
$$\Rightarrow \quad \vec{E} = \frac{\sigma}{2\epsilon_0}$$

Since, we consider the σ as uniform charge density, this result hold for any point at a fini distance from the sheet.

3) **Electric field due to linear symmetric charge distribution (Electric field due t cyllindrical charge distribution)**
(Electric field due to an infinite line of charge)
(Electric field due to a charged rod)

Figure shows a section of an infinitely long cyllindrical rod made up of non conducting material with linear charge density λ.

To find the electric field intensity $(\vec{E})$ at a point 'p' at a distance 'r' from the line charge, we consider a circular cylinder of radius 'r' and height 'l' as a Gaussian surface.



Since the charge distribution is symmetrical, thus $(\vec{E})$ have same magnitude on every point of Gaussian surface.

Since $2\pi r$ is the circumference of cylinder and 'l' is its height so area (A) = $2\pi rl$

Applying gauss law

$$\int \vec{E}.d\vec{A} = \frac{q}{\epsilon_0} \Rightarrow E.\,2\pi rl = \frac{q}{\epsilon_0}$$

$$\Rightarrow \quad E.\,2\pi rl = \frac{\lambda l}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

This is the electric field due to infinite line of charge at a point that is at radial distance 'r' from the line of charge.

Electric Potential Energy

The amount of work done required to bring a unit positive test charge (q_o) from infinity to any point in the electric field of another charge (q) is called electric potential energy.

i.e. Electric potential energy

$$U = -W = -\int_{\infty}^{r} F.\,dr = \frac{-qq_o}{4\pi\epsilon_0}\int_{\infty}^{r} r^{-2}dr = \frac{qq_o}{4\pi\epsilon_0 r}$$

Here U is positive if q_o and q both have Same sign and -ve when they have apposite sign. For a system of charges q_1, q_2, q_3 .... Separated by distance r_{12}, r_{23}, r_{31}, ... The electric potential energy is given by

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_3q_1}{r_{31}} + \ldots\right]$$

$$= \frac{1}{4\pi\epsilon_0}\sum \frac{q_iq_j}{r_{ij}}, \; i \neq j$$

Electric Potential

The electric potential energy per unit test charge at a point inside an electric field is called electric potential

$$V = \frac{U}{q_o} = \frac{1}{q_o}\frac{qq_o}{4\pi\epsilon_o r} = \frac{q}{4\pi\epsilon_o r}$$

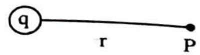
unit $= \frac{J}{C}$ = Volt [1 Volt = 1/300 stat Volt]

Electric potential due to a point charge

(Potential due to electric monopole)

let a point charge q. 'P' is the point where potential due to point charge q is to be determi[ned] which is at a distance 'r' from the charge q.

When a test charge q_o is moved from infinity and placed at 'P', the coulomb force between the[m] is $F = \frac{qq_o}{4\pi\epsilon_o r^2}$



Now electric potential energy is given by,

$$U = -w = -\int_{\infty}^{r} F.dr = \frac{-qq_o}{4\pi\epsilon_o}\int_{\infty}^{r} r^{-2}\,dr = \frac{qq_o}{4\pi\epsilon_o r}$$

Therefore, electric potential due to a point charge at a point 'P' at distance 'r' from it is given by

$$V = \frac{U}{q_o} = \frac{1}{q_o}\left(\frac{qq}{4\pi\epsilon_o r}\right) = \frac{q}{4\pi\epsilon_o r}$$

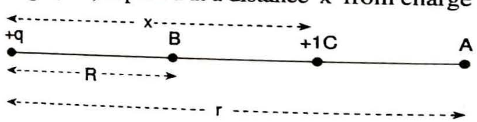
Note: 1 volt $= \frac{1J}{1 \text{ coulomb}}$

[1 volt $= \frac{10^7 \text{ erg}}{3 \times 10^9 \text{ stat columbs}} = \frac{1}{300}$ state volt]

Potential Difference

The potential difference between two points in an electric field may be defined as the amount o[f] work done in moving a unit positive test charge from one point to another (without acceleration) against electric force.

Let a unit positive test charge (+1c) is placed at a distance 'x' from charge (+q).



The coulomb force between these charges is given by, $F = \frac{q.1}{4\pi\epsilon_o x^2} = \frac{q}{4\pi\epsilon_o x^2}$

Let A and B be any two points on the electric field of 'q' at a distance 'r' and 'R' from 'q' respectively.

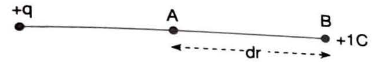
The amount of work done on moving unit test charge from A to B against the coulomb force is

$$W = -\int_{r}^{R} F.dx = -\int_{r}^{R}\frac{q}{4\pi\epsilon_o x^2}dx = -\frac{q}{4\pi\epsilon_o}\int_{r}^{R} x^{-2}\,dx$$

$$= -\frac{q}{4\pi\epsilon_o}\left[-\frac{1}{x}\right]_r^R = \frac{q}{4\pi\epsilon_o}\left[\frac{1}{R}-\frac{1}{r}\right]$$

Potential Gradiant: (Relation between E & V)

Consider a positive point charge (+q). A and B are two points very close to each other separately by small distance dr.



By definition the potential difference between points A and B is equal to work done in moving a unit positive charge from B to A against electric force between +q and unit positive test charge.

$$dV = -F.\,dr$$

or, $dV = -E\,dr$ [since F = E, being one of charge is unit charge]

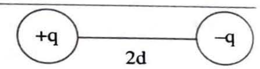
$$E = -\frac{dV}{dr}$$

The ratio $\frac{dV}{dr}$ is called potential gradiant.

Thus electric field intensity at a point in an electric field is equal to rate of change of potential with distance. The -ve sign shows that the direction of $\vec{E}$ is always in the direction of decrease of potential.

Electric Dipole

A set of two equal and opposite charges separated by a distance with no net charge is called an electric dipole.



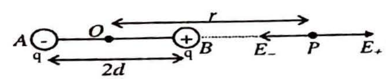
The separation between the two charges is known as length of dipole and is denoted by 2d.

The product of magnitude of one of the charge and separation of charges in dipole is called dipole moment i.e. P = 2qd.

Electric Field due to Dipole

1. **Electric field along the axial line of a dipole:**

   Let E_+ and E_- be the electric fields at a point P along the axial line of the dipole AB with separation 2d.



Using the principle of super position, the resultant electric field at P at a distance 'r' from centre of the dipole is.

$$\vec{E} = \vec{E}_+ + \vec{E}_-$$

$$\text{or, } E = \frac{q}{4\pi\epsilon_o(r-d)^2} + \frac{-q}{4\pi\epsilon_o(r+d)^2}$$

$$\text{or, } E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right]$$

$$E = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-d)^2} - \frac{1}{(r+d)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+d)^2 - (r-d)^2}{(r-d)^2(r+d)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+d-r+d)(r+d+r-d)}{(r^2-d^2)(r^2-d^2)}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{2d.2r}{(r^2-d^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\frac{4rd}{(r^2-d^2)^2}$$

$$E = \frac{q\,rd}{\pi\epsilon_0(r^2-d^2)^2} = \frac{q.\,2d.\,r}{2\pi\epsilon_0(r^2-d^2)^2} = \frac{P.r}{2\pi\epsilon_0(r^2-d^2)^2}$$

$$\vec{E} = \frac{P.\,r}{2\pi\epsilon_0(r^2-d^2)^2}$$

Where, $P = q.\,2d$ is dipole moment

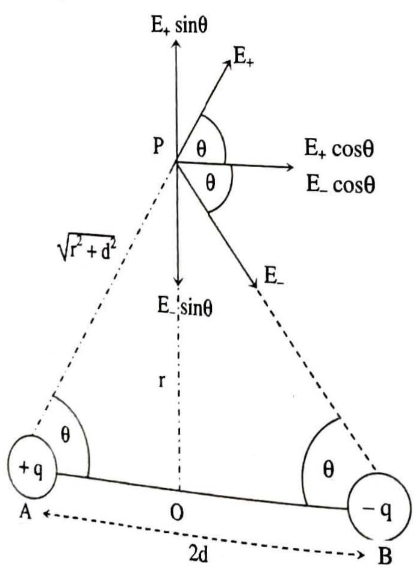
For short dipole $r >> d$, then we get

$$E = \frac{P}{2\pi\epsilon_0 r^3}$$

Hence, electric field varies with $1/r^3$.

2. **Electric field along the equatorial line of a dipole**

Let $\vec{E}_+$ and $\vec{E}_-$ be the electric fields due to +q and -q at a point 'P' on the equaitorial line of dipole AB with separation 2d.



The distance of point 'P' from the centre of dipole is r.

The distance of point P from each charge is

$$AP = BP = \sqrt{r^2 + d^2}$$

The vertical components of $\vec{E}_+$ and $\vec{E}_-$ cancel each other and the resultant electric field at point 'P' is due to the horizontal components of E_+ and E_-.

$$E = |E_+| \cos\theta + |E_-| \cos\theta$$

$$\text{Since } |E_+| = |E_-| = \frac{q}{4\pi\epsilon_0(r^2+d^2)}$$

Therefore, $E = 2\,|E_+| \cos\theta$

$$= 2\frac{q}{4\pi\epsilon_0(r^2+d^2)}\cdot\cos\theta$$

$$= \frac{2q}{4\pi\epsilon_0(r^2+d^2)}\cdot\frac{d}{\sqrt{r^2+d^2}}$$

$$= \frac{q.\,2d}{4\pi\epsilon_0(r^2+d^2)^{3/2}}$$

$$= \frac{P}{4\pi\epsilon_0(r^2+d^2)^{3/2}}$$

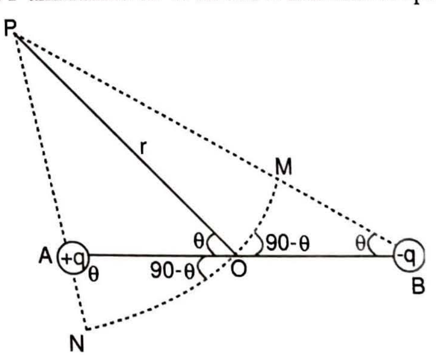
For short dipole, $r >> d$.

$$E = \frac{P}{4\pi\epsilon_0 r^3}$$

Electric Potential due to Dipole

Let A B be an electric dipole of length 2d. P be any point where OP = r and let θ be an angle between r and dipole axis.

An arc is drawn with centre P and radius OP to meet PB at M and PA produced at N, such that



OP = PN = PM = r.

By definition the electric potential at 'P' due to dipole is,

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{PA} - \frac{q}{PB}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{PN - AN} - \frac{q}{PM + BM}\right]$$

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r - AN} - \frac{q}{r + BM}\right]$$

If the dipole is very short i.e. $2d << r$, MN is nearly straight line perpendicular to both PB & PN and OP also

From figure, $\sin(90 - \theta) = \frac{AN}{AO} \Rightarrow AN = AO\cos\theta = d\cos\theta$

Similarly, $BM = d\cos\theta$

Therefore, $V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r - d\cos\theta} - \frac{q}{r + d\cos\theta}\right]$

or, $$V = \frac{q}{4\pi\epsilon_0}\left[\frac{r + d\cos\theta - r + d\cos\theta}{r^2 - d^2\cos^2\theta}\right] = \frac{q}{4\pi\epsilon_0}\cdot\frac{2d\cos\theta}{r^2 - d^2\cos^2\theta}$$

$$V = \frac{P\cos\theta}{4\pi\epsilon_0(r^2 - d^2\cos^2\theta)}$$

Since, for short dipole $r^2 >> d^2$,

$$V = \frac{P\cos\theta}{4\pi\epsilon_0 r^2}$$

$$= \frac{\vec{P}\cdot\hat{r}}{4\pi\epsilon_0 r^2} = \frac{\vec{P}\cdot\vec{r}}{4\pi\epsilon_0 r^3}$$

This is potential due to dipole at any point 'P' at an angle θ to the dipole axis.

Special Cases

i. When the point P lies on the axial line of the dipole, $\theta = 0$

$$V = \frac{1}{4\pi\epsilon_0}\frac{P}{r^2}$$

ii. When the point P lies on the equatorial line of dipole $\theta = 90^\circ$.

$V = 0$

Potential Along the Axial Line of Dipole

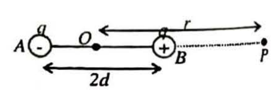
(Alternative Approach)

Consider an electric dipole with separation 2d. P is any point at a distance 'r' from centre of dipole at which electric potential is to be determined.

Using the principle of superposition of electric potential, total potential due to dipole at P is

$V = V_A + V_B$

$$= -\frac{q}{4\pi\epsilon_0(r + d)} + \frac{q}{4\pi\epsilon_0(r - d)}$$



$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r - d} - \frac{1}{r + d}\right]$$

$$= \frac{2qd}{4\pi\epsilon_0(r^2 - d^2)}$$

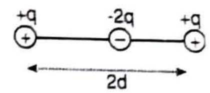
$$= \frac{p}{4\pi\epsilon_0(r^2 - d^2)}$$

In case of very short dipole, we have $r >> d$

$$V = \frac{P}{4\pi\epsilon_0 r^2}$$

Electric Quadrupole

The arrangement of four equal and opposite charges or arrangement of two dipole is called a quadrupole.

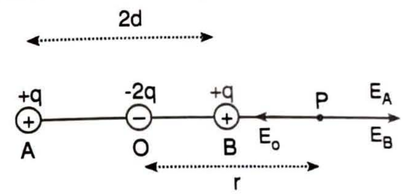


The quadrapole moment is defined by the relation,

$Q = 2qd^2$, where 'q' is magnitude of each charge in quadrupole and '2d' is quadrupole separation.

Electric Field due to Quadrupole

Consider a linear quadrupole of separation 2d with magnitude of each charges q as shown in figure. 'P' is the point on the axial line of quadrupole at distance 'r' from it's centre.



According to superposition principle the net field at point 'P' is the resultant of electric fields due to all charges at A, O and B i.e. $E = E_A + E_O + E_B$.

$$= \frac{q}{4\pi\epsilon_0(r + d)^2} - \frac{2q}{4\pi\epsilon_0 r^2} + \frac{q}{4\pi\epsilon_0(r - d)^2}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r+d)^2} + \frac{1}{(r - d)^2} - \frac{2}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2(r - d)^2 + r^2(r+d)^2 - 2(r + d)^2(r - d)^2}{r^2(r + d)^2(r - d)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2(r^2 - 2rd + d^2 + r^2 + 2rd + d^2) - 2(r^2 - d^2)^2}{r^2(r^2 - d^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{r^2(2r^2 + 2d^2) - 2(r^2 - d^2)^2}{r^2(r^2 - d^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{2r^4 + 2r^2d^2 - 2r^4 + 4r^2d^2 - 2d^4}{r^2(r^2 - d^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{6r^2d^2 - 2d^4}{r^2(r^2 - d^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}.2d^2\left[\frac{3r^2 - d^2}{r^2(r^2 - d^2)^2}\right]$$

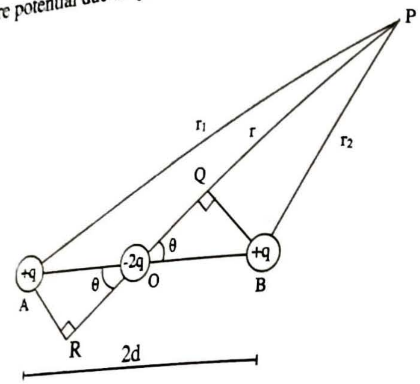
$$E = \frac{Q}{4\pi\epsilon_0}\left[\frac{3r^2 - d^2}{r^2(r^2 - d^2)^2}\right]$$

Since, quadra pole moment $Q = q.\,2d^2$ and for a short Quadra pole, $r >> d$.

Therefore, $E = \frac{Q}{4\pi\epsilon_0}\cdot\frac{3r^2}{r^2.r^4} = \frac{3Q}{4\pi\epsilon_0 r^4}$

Electric potential due to linear Quadru pole:

Let AB be a linear quadrupole of separation 2d. P is the point at a distance r from centre 'O' of quadrupole where potential due to quadru pole is to be determined.



PO is extended to R. BQ and RA are drawn perpendicular on RP, so that, $<BOQ = <AOR = \theta$.

Now the electric potential at P due to quadrupole is:

$$V = \frac{q}{4\pi\epsilon_0 AP} + \frac{q}{4\pi\epsilon_0 BP} - \frac{2q}{4\pi\epsilon_0 OP}$$

or, $$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{AP} + \frac{1}{BP} - \frac{2}{r}\right]$$

Now, in Δ APR, $AP^2 = PR^2 + AR^2$

or, $$AP^2 = (PO + OR)^2 + AR^2$$
$$= (r + d\cos\theta)^2 + (d\sin\theta)^2$$
$$= r^2 + 2rd\cos\theta + d^2\cos^2\theta + d^2\sin^2\theta$$
$$= r^2 + d^2 + 2rd\cos\theta$$
$$AP = (r^2 + d^2 + 2rd\cos\theta)^{1/2}$$

Similarly, in Δ BPQ,

$$BP^2 = PQ^2 + BQ^2$$
$$= (OP - OQ)^2 + BQ^2$$
$$= (r - d\cos\theta)^2 + (d\sin\theta)^2$$
$$= r^2 - 2rd\cos\theta + d^2\cos^2\theta + d^2\sin^2\theta$$
$$= r^2 + d^2 - 2rd\cos\theta$$
$$BP = (r^2 + d^2 - 2rd\cos\theta)^{1/2}$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r^2 + d^2 + 2rd\cos\theta)^{1/2}} + \frac{1}{(r^2 + d^2 - 2rd\cos\theta)^{1/2}} - \frac{2}{r}\right]$$

or, $$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r\left(1 + \frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right)^{1/2}} + \frac{1}{r\left(1 + \frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right)^{1/2}} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\left[\left(1 + \frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right)^{-1/2} + \left(1 + \frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right)^{-1/2} - 2\right]$$

Using binomial expression.

$$V = \frac{q}{4\pi\epsilon_0 r}\left[\left\{1 - \frac{1}{2}\left(\frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right)^2 + ... + 1 - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right) + \frac{\frac{-1}{2}\left(\frac{-1}{2} - 1\right)}{2!}\left(\frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right)^2 + ...\right\} - 2\right]$$

Neglecting the higher terms, containing d^3 and higher power of d.

$$V = \frac{q}{4\pi\epsilon_0 r}\left[2 - \frac{1}{2}\left(\frac{d^2}{r^2} + \frac{2d\cos\theta}{r}\right) - \frac{1}{2}\left(\frac{d^2}{r^2} - \frac{2d\cos\theta}{r}\right) + \frac{3}{8}\left(\frac{2d\cos\theta}{r}\right)^2 + \frac{3}{8}\left(\frac{2d\cos\theta}{r}\right)^2 - 2\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\left[\frac{-d^2}{2r^2} - \frac{d^2}{2r^2} - \frac{d\cos\theta}{r} + \frac{d\cos\theta}{r} + \frac{6}{8}\left(\frac{2d\cos\theta}{r}\right)^2\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\left[\frac{-d^2}{r^2} + \frac{6}{8}\cdot\frac{4d^2\cos^2\theta}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\left[\frac{3d^2\cos^2\theta}{r^2} - \frac{d^2}{r^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0 r}\cdot\frac{d^2}{r^2}(3\cos^2\theta - 1)$$

$$V = \frac{q.d^2}{4\pi\epsilon_0 r^3}(3\cos^2\theta - 1)$$

This is the electric potential at any point 'P' at an angle θ at a distance r from the centre of the quadrupole.

1. **For axial line, θ = 0**

$$V = \frac{q.d^2.2}{4\pi\epsilon_0 r^3} = \frac{Q}{4\pi\epsilon_0 r^3} \quad [\text{Since, } q.2d^2 = Q]$$

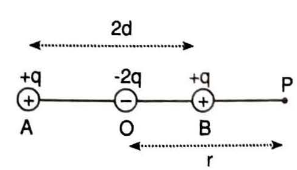
2. **For equatorial line, θ = 90°.**

$$V = \frac{q.d^2}{4\pi\epsilon_0 r^3}.(1) = \frac{Q}{8\pi\epsilon_0 r^3}$$

Potential along the axial line of Quadrupole

(Alternation approach)

Consider an electric quadrupole with separation 2d. P is any point at a distance 'r' from centre of quadrupole at which electric potential is to be determined. From figure, total electric potential at P is



$$V = V_A + V_B + V_o$$

$$= \frac{q}{4\pi\epsilon_o (r+d)} + \frac{q}{4\pi\epsilon_o (r-d)} - \frac{2q}{4\pi\epsilon_o r}$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{1}{r+d} + \frac{1}{r-d} - \frac{2}{r}\right]$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{r(r-d) + r(r+d) - 2(r+d)(r-d)}{r(r^2-d^2)}\right]$$

$$= \frac{q}{4\pi\epsilon_o}\left[\frac{r^2 - rd + r^2 + rd - 2r^2 + 2d^2}{r(r^2-d^2)}\right]$$

$$= \frac{2q\,d^2}{4\pi\epsilon_o r(r^2-d^2)} = \frac{Q}{4\pi\epsilon_o r(r^2-d^2)}$$

In case of very short quadrupole we have, $r >> d$

$$V = \frac{Q}{4\pi\epsilon_o r^3}$$

Solved Examples

1. **What is the magnitude of point charge choosen so that the electric field 50 cm away has magnitude of 2 N/C.**

Solution:

Here, $E = 2N/C$, $r = 50\text{ cm} = 0.5\text{ m}$, $q = ?$

We have, $E = \dfrac{q}{4\pi\epsilon_o r^2} = \dfrac{9 \times 10^9 \times q}{(0.5)^2}$

$q = 5.5 \times 10^{-11}\text{ C}$

2. **Two metal spheres are 3 cm in radius and carry charges of + 1 × 10^{-8} C and - 3 × 10^{-8} C respectively, assumed to be at centre of the sphere. If their centres are 2 meter apart calculate i) the potential of the point half way between their centers, and ii) the potential of each sphere.**

Solution:

Here, $q_1 = 1 \times 10^{-8}$ C, $q_2 = -3 \times 10^{-8}$ C, radius of each sphere, $R = 3\text{ cm} = 0.03\text{ m}$, distance between two charges $r = 2\text{ m}$.

i. Here, $r_1 = r_2 = 1\text{ m}$

$$V = \frac{q_1}{4\pi\epsilon_o r_1} + \frac{q_2}{4\pi\epsilon_o r_2} = \frac{1}{4\pi\epsilon_o}(q_1 + q_2)$$

$$= 9 \times 10^9 (1 \times 10^{-8} - 3 \times 10^{-8}) = -180\text{ Volts.}$$

ii. The potential at the surface of first sphere is due to its own charge q_1 plus potential due to second sphere.

$$V_1 = \frac{q_1}{4\pi\epsilon_o R} + \frac{q_2}{4\pi\epsilon_o r} = \frac{1}{4\pi\epsilon_o}\left(\frac{q_1}{r} + \frac{q_2}{R}\right)$$

$$= 9 \times 10^9 \left(\frac{1 \times 10^{-8}}{0.03} - \frac{3 \times 10^8}{2}\right) = 2865\text{ volts}$$

The potential at the surface of second sphere is due to its own charge q_2 plus potential due to first sphere.

$$V_2 = \frac{q_2}{4\pi\epsilon_o R} + \frac{q_1}{4\pi\epsilon_o r} = \frac{1}{4\pi\epsilon_o}\left(\frac{q_2}{R} + \frac{q_1}{r}\right)$$

$$= 9 \times 10^9 \left(\frac{-3 \times 10^8}{0.03} + \frac{1 \times 10^{-8}}{2}\right) = -8995\text{ volts}$$

3. **Three charges +q, +2q and - 4q are placed at the three vertices of an equilateral triangle of side 10 cm. What is the mutual potential energy of the system of the charges?**

OR

Three charges + 1 × 10^{-7} C, - 4 × 10^{-7} C and + 2 ×10^{-7} C are placed at the three vertices of an equilateral triangle of side 0.1 m. Find the minimum amount of work required to dismantle this structure.

Solution:

i. Here, $q_1 = +q$, $q_2 = +2q$ and $q_3 = -4q$, $a = 0.1$ m

The total energy of configuration is given by,

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\epsilon_o}\left[\frac{q_1q_2}{a} + \frac{q_2q_3}{a} + \frac{q_3q_1}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_o}\left[\frac{(+q)(+2q)}{a} + \frac{(+2q)(-4q)}{a} + \frac{(-4q)(+q)}{a}\right]$$

$$U = -\frac{10q^2}{4\pi\epsilon_o a}$$

ii) Here $q = 1 \times 10^{-7}$ C

$$U = -\frac{10 \times 9 \times 10^9 \times (1 \times 10^{-7})^2}{0.1} = -9 \times 10^{-3}\text{ J}$$

4. **Twenty seven identical drops of mercury are charged simultaneously to the same potential of 10 Volt. What will be the potential if all the drops are made to combine to form one large drop? Assume the drops to be spherical.**

Solution:

Let, r be the radius of each small drop, R be the radius of large drop and q be the charge on each drop.

then, $\dfrac{4\pi}{3}R^3 = 27 \times \dfrac{4\pi}{3}r^3$

$\Rightarrow R = 3r$

The electrical potential of each small drop, $V_s = \dfrac{q}{4\pi\epsilon_o r}$ and that of large drop,

$$V_L = \frac{27q}{4\pi\epsilon_0 R} = \frac{27q}{4\pi\epsilon_0 (3r)} = \frac{9q}{4\pi\epsilon_0 r}$$

$V_L = 9\,V_S = 9 \times 10 = 90$ Volts.

5. **What is the potential gradient in volts/ meter at a distance of 10^{-12} m from the centre of the gold nucleus? What is the gradient at the nuclear surface of radius $R = 5 \times 10^{-15}$ m? [Atomic number of gold = 79 and $e = 1.6 \times 10^{-19}$ C]**

Solution:

i. Since the potential gradient is also known as electric field.

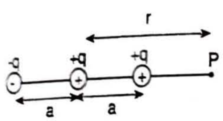
$$E = \frac{dV}{dr} = \frac{q}{4\pi\epsilon_0 r^2} = \frac{Ze}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0}\frac{79 \times 1.6 \times 10^{-9}}{(10^{-12})^2} = 1.14 \times 10^{17} \text{ V/m}$$

ii. The potential gradient at the surface of gold nucleus of radius $R = 5 \times 10^{-15}$ m is given by,

$$E = \frac{dV}{dr} = \frac{q}{4\pi\epsilon_0 R^2} = \frac{Ze}{4\pi\epsilon_0 R^2} = \frac{79 \times 1.6 \times 10^{-19}}{(5 \times 10^{-15})^2} = 4.55 \times 10^{21} \text{ V/m}$$

6. **For the charge configuration of the figure, show that V (r) at a point 'P' on the line (assuming r >> a) is given by**

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r} + \frac{2qa}{r^2}\right).$$



Solution:

From figure, the total potential at P is given by

$$V = \frac{(-q)}{4\pi\epsilon_0 (r+a)} + \frac{q}{4\pi\epsilon_0 r} + \frac{q}{4\pi\epsilon_0 (r-a)}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{1}{r-a} - \frac{1}{r+a}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{r+a-r+a}{r^2-a^2}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{2a}{r^2-a^2}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} + \frac{2a}{r^2}\right] (\because r >> a)$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{r} + \frac{2qa}{r^2}\right]$$

7. **A spherical drop of water carrying a charge of 30 PC has potential of 500 V at its surface (with V = 0 at infinity) a) what is the radius of the drop? b) If two such drops of the same charge and radius combine to form a single spherical drop, what is the potential at the surface of the new drop?**

Solution:

Here, $q = 30$ PC $= 30 \times 10^{-12}$ C, $V = 500$ Volts.

a. $V = \frac{q}{4\pi\epsilon_0 r} \Rightarrow r = \frac{q}{4\pi\epsilon_0 V} = \frac{9 \times 10^9 \times 30 \times 10^{-12}}{500} = 5.4 \times 10^{-4}$ m

b. After the drops are combined, the total volume is double of the volume of an original drop.

$$\Rightarrow \frac{4\pi}{3}R^3 = 2 \times \frac{4\pi}{3}r^3 \Rightarrow R^3 = 2r^3 \Rightarrow R = 2^{1/3}\,r = 6.8 \times 10^{-4} \text{m}$$

Therefore, potential of New drop, $V = \frac{2q}{4\pi\epsilon_0 R} = \frac{2 \times 9 \times 10^9 \times 30 \times 10^{-12}}{6.8 \times 10^{-4}} = 794.12$ Volts

8. **A small conducting sphere of mass m having charge +q is suspended by a thread of length *l*. The sphere is placed in uniform electric field of strength E directed vertically upward. Show that this sphere oscillate with period. $T = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$, if the electrostatic force acting on the sphere is less than gravitational force.**

Solution:

The force acting on the sphere are:

i. Tension T along the string.

ii. Weight mg acting vertically downwards.

iii. Electrical force qE vertically upwards.

The resultant force acting vertically down wards is (mg - qE) i.e resultant force. F = mg - qE

T = mg - qE

mg' = mg - qE

$$g' = g - \frac{qE}{m}$$

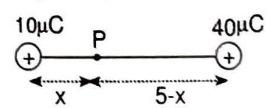
Effective acceleration, $g' = g - \frac{qE}{m}$

Time period, $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g - \frac{qE}{m}}}$

9. **Two small spheres of charge 10 μC and 40 μC are placed 5 cm apart. Find the location of a point between them where the field strength is zero.**

Solution:

Here, $q_1 = 10\ \mu C = 10 \times 10^{-6}$ C, $q_2 = 40\ \mu C = 40 \times 10^{-6}$ C, r = 5 cm = 0.05 m



Let, the electric field be zero at the distance x from charge 10 μC.

⇒ The net electric field is given by, $E = E_1 - E_2 = 0$

⇒ $E_1 = E_2$

$$\frac{1}{4\pi\epsilon_0}\frac{10\times10^{-6}}{x^2} = \frac{1}{4\pi\epsilon_0}\frac{40\times10^{-6}}{(5-x)^2}$$

$4x^2 = (5-x)^2$

$2x = 5 - x$

$3x = 5$

$x = 5/3 = 1.67$ cm

10. Assume that earth has surface charge density of electron per meter square. Calcula the earth's electric field and potential on the earth surface. Given that radius of th earth is 6400 km.

Solution:

Here, $\sigma = 1.6 \times 10^{-19}$ C/m², R = 6400 km = 6.4×10^6 m

If q is charge on earth, the electric field on it's surface is

$$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{1}{\epsilon_0}\frac{q}{4\pi R^2} = \frac{1}{\epsilon_0}\frac{q}{A} = \frac{\sigma}{\epsilon_0} = \frac{1.6\times10^{-19}}{8.85\times10^{-12}} = 180.79 \text{ N/C}$$

And, the potential on the surface of earth is given by,

$$V = \frac{q}{4\pi\epsilon_0 R} = \frac{R}{\epsilon_0}\cdot\frac{q}{4\pi R^2}$$

$$= \frac{\sigma R}{\epsilon_0} = \frac{1.6\times10^{-19}\times6.4\times10^{6}}{8.85\times10^{-12}} = 1.157\times10^{9} \text{ Volts}$$

11. The electric potential V (in volts) varies with x (in metre) according to the relation V = $5 + 4x^2$. Calculate the force experienced by a negative charge of 2×10^{-6} C located at x =0.5 m.

Solution:

Here, $V = 5 + 4x^2$, $q = -2\times10^{-6}$ C and x = 0.5 m

$$\Rightarrow E = -\frac{dV}{dx} = -8x$$

$F = qE = -2\times10^{-6}\times(-8x)$

At x = 0.5 m, force, $F = 8\times2\times0.5\times10^{-6} = 8\times10^{-6}$ N

12. Two positive point charges of 12 and 8 micro coulombs respectively are placed 10 cm apart in air. Calculate the amount of work done to bring them 4 cm closer.

Solution:

Here, $q_1 = 12\times10^{-6}$ C, $q_2 = 8\times10^{-6}$ C.

i. Electrostatic potential energy when the charges are 10 cm = 0.1 apart is

$$W_1 = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\epsilon_0\times0.1} = \frac{9.6\times10^{-10}}{4\pi\epsilon_0}$$

ii. Potential energy when the charges are brought 4 cm closer i.e. when they are (10 - 4) cm = 6 cm = 0.06 m apart is,

$$W_2 = \frac{12\times10^{-6}\times8\times10^{-6}}{4\pi\epsilon_0\times0.06} = \frac{16\times10^{-10}}{4\pi\epsilon_0}$$

$\therefore$ Work done = $W_2 - W_1 = (16 - 9.6)\times10^{-10}\times9\times10^{9} = 5.8$ J

13. Two concentric spheres of radii r_1 and r_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is the same for both spheres. Show that, the electric potential at the common centre is $V = \frac{\sigma}{\epsilon_0}(r_1 + r_2)$.

Solution:

The electric potential at the common centre is

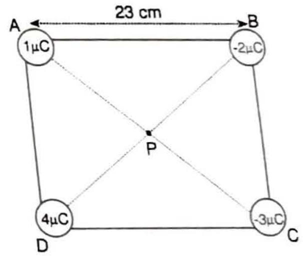
$$V = \frac{q_1}{4\pi\epsilon_0 r_1} + \frac{q_2}{4\pi\epsilon_0 r_2}$$

$$= \frac{q_1}{4\pi r_1^2}\cdot\frac{r_1}{\epsilon_0} + \frac{q_2}{4\pi r_2^2}\cdot\frac{r_2}{\epsilon_0} = \frac{\sigma r_1}{\epsilon_0} + \frac{\sigma r_2}{\epsilon_0}$$

$$= \frac{\sigma}{\epsilon_0}(r_1 + r_2)$$

14. What will be the potential at the centre of the square of side 23 cm, if the charges 1 μC, -2μC, -3μC and 4 μC are placed at the corners.

Solution:



From figure

$AC = \sqrt{23^2 + 23^2} = 32.5269$

$AP = \frac{AC}{2}$

AP = 16.26 cm = 0.163 m

AP = BP = CP = DP = 0.163 m

The total potential at P is

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{1\times10^{-6}}{AP} + \frac{(-2\times10^{-6})}{BP} + \frac{(-3\times10^{-6})}{CP} + \frac{4\times10^{-6}}{DP}\right]$$

$$= \frac{1}{4\pi\epsilon_0}[1 - 2 - 3 + 4]\times\frac{10^{-6}}{0.163}$$

= 0 Volt

15. A metallic sphere A of radius 'a' carries a charge Q. It is brought in contact with an uncharged sphere B of radius b. Calculate the amount of charge left on the sphere A interms of a, b and Q.

Solution:

Charge will flow from A to B until their potentials become equal. If charge q flows from A to B, then

$$V_1 = V_2 \Rightarrow \frac{Q-q}{4\pi\epsilon_0 a} = \frac{q}{4\pi\epsilon_0 b}$$

$$\Rightarrow Q - q = \frac{a}{b}\cdot q \Rightarrow Q = \left(1 + \frac{a}{b}\right)q = \left(\frac{a+b}{b}\right).q$$

$\Rightarrow q = \frac{bQ}{(a+b)}$

Charge left on A $= Q - q = Q - \frac{bQ}{(a+b)} = \left(\frac{a+b-b}{a+b}\right). Q = \frac{aQ}{(a+b)}$

16. **Charge of uniform volume density 3.2 $\mu C/m^3$ fill a non conducting solid sphere of radius 5 cm. What is the magnitude of electric field at (a) 3.5 cm (b) 8 cm from the centre of the sphere.**

Solution:

Here,

$\rho = 3.2 \times 10^{-6}\ C/m^3,\ R = 5\ cm = 0.05\ m$

a. Here, $r = 3.5\ cm = 0.035\ m$

$E = \frac{q'}{4\pi\epsilon_o r^2}$

$q' = \frac{4\pi}{3} r^3 . \rho$

Therefore, $E = \frac{4\pi}{3} r^3 . \rho . \frac{1}{4\pi\epsilon_o r^2} = \frac{\rho r}{3\epsilon_o}$

$\Rightarrow E = \frac{3.2 \times 10^{-6} \times 0.035}{3 \times 8.85 \times 10^{-12}} = 4.22 \times 10^3\ N/C$

b. Here, $r = 8\ cm = 0.08\ m$

$E = \frac{q}{4\pi\epsilon_o r^2},\ q = \frac{4\pi}{3} R^3 . \rho$

$\Rightarrow E = \frac{4\pi}{3} R^3 \rho . \frac{1}{4\pi\epsilon_o r^2} = \frac{\rho R^3}{3\epsilon_o r^2}$

$= \frac{3.2 \times 10^{-6} \times (0.05)^3}{3 \times 8.85 \times 10^{-12} \times (0.08)^2} = 2.35 \times 10^3\ N/C$

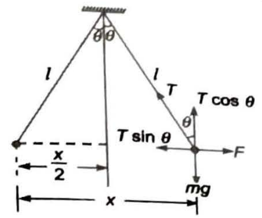
17. **Two identical tiny spheres, each of mass m are hung from insulating string of equal length *l*. when an equal charge q is given to each sphere, they repel each other. Show that the separation x between them is given by $x = \left(\frac{q^2 l}{2\pi\epsilon_o mg}\right)^{1/3}$**

Solution:

From figure, at equilibrium condition,

$T \sin\theta = F_E$ .....(1)

And $T \cos\theta = mg$ .....(2)



Where, T is tension on string, $F_E = \frac{q^2}{4\pi\epsilon_o x^2}$ is the electric force between two charges.

Dividing equation (1) by equation (2), $\tan\theta = \frac{F_E}{mg} = \frac{q^2}{4\pi\epsilon_o x^2 mg}$

For small θ, $\tan\theta \approx \theta \Rightarrow \theta = \frac{q^2}{4\pi\epsilon_o x^2 mg}$ ....(3)

Again from figure, $\sin\theta = \frac{x}{2l}$

For small θ, $\sin\theta \approx \theta \Rightarrow \theta = \frac{x}{2l}$ ...(4)

From equation (3) and equation (4) $\frac{x}{2l} = \frac{q^2}{4\pi\epsilon_o x^2 mg} \Rightarrow x^3 = \frac{q^2 l}{2\pi\epsilon_o mg}$

$x = \left[\frac{q^2 l}{2\pi\epsilon_o mg}\right]^{1/3}$

18. **A charge of 5×10^{-5} C is distributed between two spheres. It is found that they repel each other with a force of 1N when their centres are 2m apart. Find the charge on each sphere.**

Solution:

Here, $q_1 + q_2 = 5 \times 10^{-5}$ c.........(1)

$F = \frac{q_1 q_2}{4\pi\epsilon_o r^2} \Rightarrow 1 = \frac{q_1 q_2}{4\pi\epsilon_o 2^2} \Rightarrow q_1 q_2 \times 9 \times 10^9 = 4$

$\Rightarrow q_1 q_2 = 4.44 \times 10^{-10}$ .....(2)

We have, $(q_1 - q_2) = [(q_1 + q_2)^2 - 4 q_1 q_2]^{1/2}$

$= [(5 \times 10^{-5})^2 - 4 \times 4.44 \times 10^{-10}]^{1/2}$

$q_1 - q_2 = 2.69 \times 10^{-5}$ .....(3)

Adding equation (1) and (3)

$2q_1 = 7.69 \times 10^{-5}$

$q_1 = 3.84 \times 10^{-5}$ C

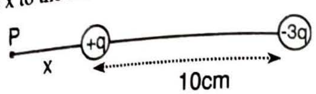
From equation (3), $q_2 = q_1 - 2.69 \times 10^{-5}$

$= 1.15 \times 10^{-5}$ C.

19. **Two point charges + q and - 3q are separated by a distance d = 10 cm. Locate the points on the line passing through the two charges where a) electric potential is zero and b) electric field is zero.**

Solution:

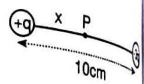
At a point P at a distance x to the left of charge + q, the electric potential is,



$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{x} - \frac{3q}{d+x}\right)$$

If V = 0, $\frac{q}{x} = \frac{3q}{d+x} \Rightarrow 3x = d + x \Rightarrow x = \frac{d}{2} = \frac{10}{2} = 5$ cm

At a point 'P' between two charges, at a distance x to the right of charge +q,



$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{x} - \frac{3q}{d-x}\right)$$

If V = 0, $\frac{q}{x} = \frac{3q}{d-x} \Rightarrow 3x = d - x \Rightarrow x = \frac{d}{4} = \frac{10}{4} = 2.5$cm

At a point P at a distance x to the left of +q, the electric field is

$$E = \frac{q}{4\pi\epsilon_0 x^2} - \frac{3q}{4\pi\epsilon_0 (x+d)^2}$$

If E = 0,

$$\frac{q}{4\pi\epsilon_0 x^2} = \frac{3q}{4\pi\epsilon_0 (x+d)^2} \Rightarrow \frac{1}{x^2} = \frac{3}{(x+d)^2}$$

$$x^2 + 2xd + d^2 = 3x^2$$

$$2x^2 - 2xd - d^2 = 0$$

$$x = \frac{2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{20 \pm 34.64}{4}$$

The positive root is, x = 13.66 cm.

20. **Two charges 4q and - q are placed at a distance r apart. A charge Q is placed exactly mid way between them, what will be the value of Q so that charge - q experiences no net force?**

Solution:

Here, net force on charge -q is

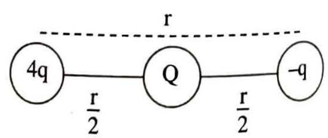
$$F = \frac{1}{4\pi\epsilon_0}\frac{(-q)\times 4q}{r^2} + \frac{1}{4\pi\epsilon_0}\frac{(-q).Q}{(r/2)^2}$$

If, F = 0,

$$\frac{-4q^2}{r^2} - \frac{4Qq}{r^2} = 0$$

$$-q - Q = 0$$

$$Q = -q$$



21. **Find the electric field intensity at the centre of a square of side 5 cm consisting of 2 μC charges on each vertex.**

Solution.

Let q = 2μC charge in each vertex of square of side a = 5 cm as shown in figure.

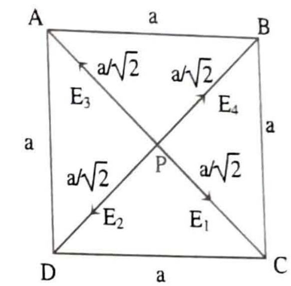
Then,

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{\left(\frac{a}{\sqrt{2}}\right)^2} = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2} \text{ (along PC)}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2} \text{ (along PD)}$$

$$E_3 = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2} \text{ (along PA)}$$

$$E_4 = \frac{1}{4\pi\epsilon_0}\frac{2q}{a^2} \text{ (along PB)}$$



The resultant of E_1 and E_3 are given by,

$E' = E_3 - E_1 = 0$ [E_1 and E_3 are in opposite direction]

Also, the resultant of E_2 and E_4 are given by

$E'' = E_4 - E_2 = 0$ [E_2 and E_4 are in opposite direction]

Hence, the resultant electric field intensity at the centre of square having equal charge on the vertex is zero.

22. **Find the potential at the centre of the square having charges 2×10^{-6} C, 3×10^{-6} C, 4×10^{-12} C and -4×10^{-12} C at four corners.**

Solution.

The distance r of each of charge from centre P of the square is $r = \frac{a}{\sqrt{2}}$

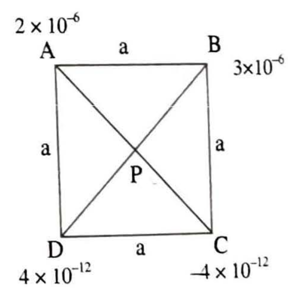
Now, $V = V_1 + V_2 + V_3 + V_4$

$$= \frac{1}{4\pi\epsilon_0 r}[q_1 + q_2 + q_3 + q_4]$$

$$= \frac{1}{4\pi\epsilon_0 r}[2\times10^{-6} + 3\times10^{-6} + 4\times10^{-12} - 4\times10^{-12}]$$

$$= \frac{1}{4\pi\epsilon_0 \frac{a}{\sqrt{2}}}[5\times10^{-6}]$$

$$= \frac{6.36}{a}\times10^{-3}\text{ C}$$



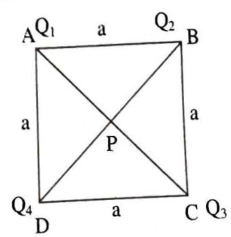
23. **What is the potential at the center at the center of the square of the figure. Assume $Q_1 = +1\times10^{-8}$, $Q_2 = -2\times10^{-8}$ C, $Q_3 = +3\times10^{-8}$ C, $Q_4 = +2\times10^{-8}$ C and a = 1 m.**

Solution.

From figure, $AC = \sqrt{1^2 + 1^2} = \sqrt{2}$ m

$$\Rightarrow AP = \frac{AC}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$AP = BP = CP = DP = \frac{1}{\sqrt{2}}\text{ m}$$



The total potential at P is,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q_1}{AP} + \frac{Q_2}{BP} + \frac{Q_3}{CP} + \frac{Q_4}{DP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{1\times10^{-8}}{1/\sqrt{2}} + \frac{-2\times10^{-8}}{1/\sqrt{2}} + \frac{3\times10^{-8}}{1/\sqrt{2}} + \frac{2\times10^{-8}}{1/\sqrt{2}}\right]$$

$$= 9\times10^{9}\times10^{-8}\times\sqrt{2}\,[1-2+3+2]$$

$$= 509.11 \text{ Volt}$$

Exercises

1. Define electric field intensity, electronic polarization, and electric displacement. Find the relation between them. Explain the physical significance of the three vectors.
2. Find the relation between electric field intensity and potential.
3. Define electric dipole and dipole moment. Obtain an expression for electric field due to dipole.
4. State Gauss law. Apply Gauss law to calculate the intensity due to charged sphere at the points outside, inside and at the surface of the sphere.
5. What is electric dipole? Find out potential and field due to electric dipole at a distance from its centre.
6. Define quadrupole. Find field and potential due to electric quadrupole.
7. Prove that electric field due to short dipole at axial point is twice that at equatorial line.
8. For a given short electric dipole, show that the electric potential at any point at a distance is $V = \frac{P\cos\theta}{r^2}$, where θ is the angle made by r to the dipole axis and P is its dipole moment. Using above relation find an expression for resultant electric field intensity at that point.
9. Find the potential at any point at an angle θ at a distance r from the centre of the short dipole. What result do you obtain if the point is along axial line.
10. Derive an expression for the electric field at any point on the axis of the short linear quadrupole.
11. What is electric quadrupole? Finding an expression for electric potential at any point on axial line at a distance r from centre of short quadrupole, show that electric field at that point is inversely proportional to r^3.
12. Define electric dipole. Find an expression for electric potential at any point in space due to dipole of length 2a. Could you extend this relation to calculate electric field intensity? If so how?
13. What is dipole? Derive an expression for electric field due to dipole at the points (i) axis of the dipole and ii) perpendicular bisector of dipole.
14. Explain Gauss law in free space. How the law is modified for the presence of dielectric material?

15. Two point charges +4q and +q are 30 cm apart. At what point on the line joining them is the electric field zero?
16. Electrostatic force between two charges placed in vaccum is F. If the charges are placed at the same separation, in a medium of dielectric constant (relative permittivity) K, find the force between the charges in medium.
17. Two tiny spheres carrying charges of 1 μC and 3 μC are placed 8 cm apart in air. What is the potential at a point 3 cm from the mid point in a plane normal to the line passing through the mid point.
18. Four point charges +q, +q, - q and -q are placed respectively at the corners of a square of side a. Find the electric potential at the centre of the square.
19. A cube of side 'b' has a charge q at each of its vertices. what is the potential at the centre of the cube.
20. Find the electric potential at the surface of an atomic nucleus (z = 50) of radius 9.0×10^{-3} cm.
21. A point charge $q_1 = +1.0\times10^{-8}$ C placed at a distance of 10 cm from another point charge $q_2 = +2.0\times10^{-8}$ C. At what point on the line joining the two charges is the electric field zero?
22. Two identical oppositely charged spheres, with their centres 0.5 m apart, attract each other with a force of 0.108 N. The spheres are connected by a conducting wire. When the wire is removed, they repel each other with a force of 0.036 N. Find the initial charges on the sphere.
23. Find the work done on assembling four charges +q, -q, +q and - q on the corners of a square of side a.
24. The electric potential V due to a charge in the surrounding space at any point x - meters from the charge is given by the relation, $V = 8x + 3x^2$ volts. Find the electric field intensity at a point 1.5 m from the charge. consider the medium is air.
25. Two equal and opposite charge of magnitude 2×10^{-7} C are 15 cm apart i) What are the magnitude and direction of $\vec{E}$ at a point mid way between the charges? b) what force (magnitude and direction) would act on an electron placed there?
26. Find the potential at the centre of the square having charges 2×10^{-6}C, 3×10^{-6}C, 4×10^{-12}C and -4×10^{-12}C at four corners.
27. What is the initial rate of increase of current and final saturation current in RL circuit with L = 15 mH, R = 24 Ohm and emf = 10 volt?
28. If the earth had a net charge equivalent to 1 electron/m^2 of surface area.
    (i) What will be the earth's potential?
    (ii) What would the electric field due to earth be just just outside its surface?