**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

The work begins after 10 min, so the average time increase from 45min to 55min.

for normal distribution:-

z = (X-μ)/б

= (60-55)/8

= 0.625

P(E) = 0.7323from z-table

The probability of events not happening is 1- p(E)

=1-0.7323

=**0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

N= 400

Mean = 38

SD = 6

Z score = (Value - Mean)/SD

Z score for 44 is (44 - 38)/6 = 1 => 84.13 %

=> People above 44 age = 100 - 84.13 = 15.87% ≈ 63    out of 400

Z score for 38 is (38 - 38)/6 = 0 => 50%

Hence People between 38 & 44 age = 84.13 - 50 = 34.13 % ≈ 137 out of 400

Hence More employees at the processing centre are older than 44 than between 38 and 44. is **FALSE**

Z score for 30 is(30 - 38)/6 =  -1.33  =  9.15  %   ≈ 36 out of 400

Hence A training program for employees under the age of 30 at the centre would be expected to attract about 36 employees - **TRUE**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

**As we know, if X-N(μ1, σ12), and Y~ *N*(μ, σ22) are two independent random variable then X+Y~ N(μ1+ μ2, σ12+ σ22)**

**And X-Y~ N (μ1- μ2, σ12+ σ22)**

**Similarly, if z= aX + bY, whereas X and Y are as explained above. Z is linear to X and Y**

**Therefore, in the equation**

**2X1~N (2 μ,4 σ2) and**

**X1+X2~N (μ + μ, σ2+ σ2) which is N (2 μ,2 σ2)**

**2X1-(x1+x2) = N (4 μ+6 σ12)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

**Z value is given as stats.norm.ppf(pvalue)**

**Z value at 0.5th percentile is given as**

**Z(0.5) = stats.norm.ppf(0.005)= -2.576**

**Z value at 99.5 percentile is given as**

**Z(99.5) = stats.norm.ppf(0.995) = 2.576**

**Z = (x - 100)/20 = > x = 20z+100**

**a = -(20\*2.576) + 100= 48.5**

**b = (20\*2.576)+100= 151.5**

**Two values symmetric about mean for the given standard normal distribution are [48.5,151.5]**

**So, Answer is D. 48.5, 151.5.**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

1. **Mean = 5+7=12 after converting to rupees it will be 12\*45=540**

**Standard deviation = sqrt(9+16) = 5 and after rupee conversion it will be 5\*45=225**

**Z score = X-mean/Standard Deviation (SD)**

**From z table, z-score of probability of 0.025 is -1.96**

**So -1.96=x-540/225**

**X= -1.96\*225+540=99**

**From z table, z-score of probability of 0.975 is 1.96**

**So 1.96=x-540/225**

**X= 1.96\*225+540=981**

**From the above 2 analysis it is confirmed that X lies between 99 and 981.**

1. **From z table, z-score of probability of 0.05 is -1.645**

**Z score = X-mean/Standard Deviation (SD)**

**X=-1.645\*225+540**

**=169.875**

1. **Probability of of division 1 making loss**

**Z= 0.5/3-1.66**

**P(z)=0.04846**

**Probability of division 2 making loss**

**Z=0.7/4 =-1.75**

**P(z)=0.04006**

**Hence, probability of division 1 making loss is more than the probability of division making loss.**