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Abstract

In modern structural engineering, mitigation of oscillations induced by environmental forces such as earthquakes and typhoons is critical to preserving the integrity of high-rise buildings. A promising solution involves the use of Pendulum Tuned Mass Dampers (PTMDs), where the dynamic interaction between a pendulum and the oscillating structure provides an efficient means of vibration control. This report explores the optimization of the PTMD system through an extensive experimental, analytical, and numerical study, focusing on the effect of the pendulum rod length on the damping efficiency.

The work includes detailed theoretical modeling, experimental setups, measurements, data analysis, derivation of differential equations of motion, numerical solution via the Runge-Kutta method, and comparative analysis between theory and experiment. The results demonstrate that optimal damping is achieved when the natural damped frequencies of the pendulum and the structure are closely matched, confirming theoretical predictions. This document provides a comprehensive blueprint for understanding, designing, and optimizing PTMD systems.

Acknowledgments

This project would not have been possible without the invaluable guidance and mentorship of Dr. Dhanasri M Joglekar, whose deep insights in vibration and noise control concepts greatly shaped the scope and execution of this study. We are grateful to the Department of Mechanical and Industrial Engineering at IIT Roorkee for providing access to the labs and equipment required for us to proceed with our project.

We also acknowledge the online open-source communities Matlab and Arduino IDE for providing essential resources and libraries that greatly helped with calculations and recording the readings from our prototype.Our gratitude is extended to Lab assistants and friends for assisting in data collection and experimental setups.Also special thanks to Tinkering Labs for letting us use their equipment to 3D print the parts of our setu

Introduction

Overview

Oscillations in tall structures represent a significant engineering challenge. Wind, seismic activity, and dynamic loading conditions can induce resonant oscillations that not only cause structural damage but also discomfort to occupants. To mitigate such effects, a variety of damping systems are employed, one of the most innovative being the Pendulum Tuned Mass Damper (PTMD).

The PTMD concept involves a pendulum attached to the structure, carefully tuned to the building's natural frequency. When the structure oscillates, the pendulum counteracts the motion, reducing amplitude through energy dissipation mechanisms. This project aims to explore and optimize the PTMD configuration, particularly focusing on how varying the pendulum rod length affects the damping performance.

The investigation is performed through:

- A detailed theoretical analysis based on classical mechanics and differential equations.
- Construction of an experimental model using a wooden structure and an electronic gyroscope for measurement.
- Experimental data collection and error analysis.
- Derivation of mathematical models incorporating damping forces and spring-like behavior of the structure.
- Numerical simulations using the fourth-order Runge-Kutta method.
- Comparative analysis between experimental results and numerical predictions.

Ultimately, this research seeks to validate the hypothesis that maximum damping occurs when the pendulum's natural damped frequency matches that of the structure.

Hypothesis

It is hypothesized that:

- There exists an optimal pendulum rod length where the damping effect is maximized.
- The optimal damping occurs when the natural damped frequency of the pendulum matches that of the structure.
- Deviations from this optimal length will result in a decreased damping effect.

- This hypothesis will be tested experimentally and theoretically. The system will also be examined for different lengths of pendulum.

Objectives

- **Objective 1:** Construct a physical model representing a simplified structure with a PTMD.
- **Objective 2:** Derive the equations of motion incorporating damping and spring forces.
- **Objective 3:** Measure oscillation data using MPU6050 accelerometer and Arduino Uno.
- **Objective 4:** Calculate system constants such as damping coefficients and spring constants.
- **Objective 5:** Solve the differential equations numerically using the Runge-Kutta method.
- **Objective 6:** Compare numerical predictions with experimental results.

- **Objective 7:** Use Julia code and existing outputs to make a simulation to find out the output for different initial values without changing the system.

Experimental Setup

In this experiment, a pendulum was attached to a top frame which is attached to two steel rulers. The top frame and bottom are 3d printed and setup rests on a wooden base.
The pendulum is made from a threaded rod and two masses attached to it. An mpu6050 electronic accelerometer is attached to the top frame of the setup.The wires from the sensor are connected to the arduino uno sensor resting at the wooden base.
The finished physical model had a top that can only oscillate *noticeably* along the x-direction(y-axis motion negligible),and a pendulum that can only swing in the plane created by the vertical projection of x-axis on to the ground.

Technical data of experimental Setup

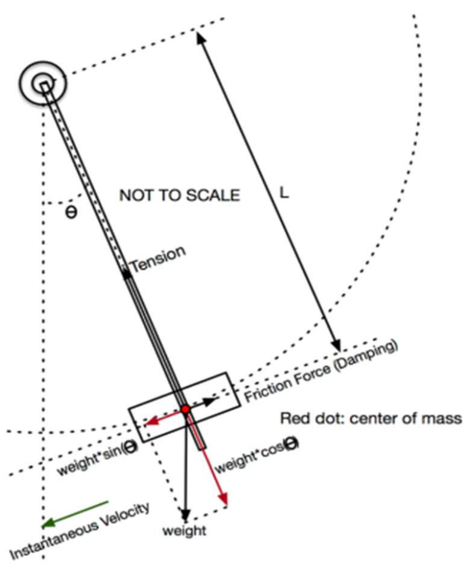
Parameter Measured	Measurement
Width	**(0.250 ± 0.005) m**
Height of Building	**(0.600 ± 0.01) m**
Top Mass	**(0.200 ± 0.001) kg**
Pendulum Mass	**(0.050 ± 0.001) kg**

Mathematical Model

The mechanical friction at a physical-pendulum pivot has to be considered. In linear motion, the dynamic frictional force is proportional to objects' relative velocity; similarly, at a pivot, when the inner bearing rotates clockwise, it experiences an anticlockwise twisting action proportional to its angular velocity. (The twisting action is called a torque. Therefore, the frictional torque can be written as :

$$\tau_{friction} = -b\frac{d\theta}{dt}$$

where b is the pendulum damping torque constant (PDTC), the constant to be calculated in this section. Newton's Second Law (NSL) is a convenient way to mathematically model mechanical systems. To use the law on the pendulum mass for free-body-force analysis, the torque must be defined in terms of a force. By definition, torque has this magnitude if the level arm and the Figure 6 Pendulum (Without Pivot Moving) Free-bodyforce Diagram 9 force are perpendicular : ' = 23 where r is the length of the level arm, F is the force. For the pendulum, r was taken as L, the length from the center of mass (COM) of the pendulum mass to its pivot. The frictional force can be treated as a force acting on the COM of the pendulum mass in the tangential direction along its circular trajectory . In this way, frictional force is:



$$f_{friction} = \frac{\tau_{friction}}{L} = -\frac{b}{L}\frac{d\theta}{dt}$$

Apply NSL and COM of the pendulum:

$$\sum F_{pen} = m_{pen}a_{pen}$$

$$m_{pen}a = -m_{pen}g\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}$$

Instantaneous linear acceleration is equivalent to the second derivative of linear displacement with respect to time:

$$m_{pen}\frac{d^2x}{dt^2} = -m_{pen}g\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}$$

To make the equation in terms of theta and t:

$$m_{pen}\frac{d^2\theta}{dt^2}L = -m_{pen}g\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}$$

$$\frac{d^2\theta}{dt^2} + \frac{b}{m_{pen}L^2}\frac{d\theta}{dt} + \frac{g}{L}\sin(\theta) = 0$$

For ±0.52 radians (about 30 degrees), sin θ ≈ θ (within 5% of error). This EOM of pendulum is a derived completely from practically-measurable variables (PMVs):

$$\frac{d^2\theta}{dt^2} + \frac{b}{m_{pen}L^2}\frac{d\theta}{dt} + \frac{g}{L}\theta = 0$$

Constant b cannot be calculated from this equation alone. Therefore, we decided to compare equation with the general differential equation of motion (GDEOM) of damped harmonic oscillators, which is :

$$\frac{d^2\theta}{dt^2} + 2\omega_{npen}\zeta_{pen}\frac{d\theta}{dt} + \omega_{npen}{}^2\theta = 0 \quad \textbf{(2)}$$

Comparing eqns 1 and 2 since the pendulum Is a damped harmonic oscillator:

$$\frac{d^2\theta}{dt^2} + 2\omega_{npen}\zeta_{pen}\frac{d\theta}{dt} + \omega_{npen}{}^2\theta = \frac{d^2\theta}{dt^2} + \frac{b}{m_{pen}L^2}\frac{d\theta}{dt} + \frac{g}{L}\theta$$

Comparing coefficients:

$$2\omega_{npen}\zeta_{pen} = b\frac{1}{m_{pen}L^2}$$

$$\omega_{npen}{}^2 = \frac{g}{L}$$

Merging two equations, cancelling natural frequency of pendulum which is not directly computable:

$$2\sqrt{\frac{g}{L}}\zeta_{pen} = b\frac{1}{m_{pen}L^2}$$

the pendulum damping coefficient, can be calculated using logarithmic decrement,which is the natural log of the ratio of the amplitudes of two successive peaks:

$$\delta = \frac{1}{n}\ln\left(\frac{\theta(t)}{\theta(t+nT)}\right) \qquad \zeta_{pen} = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}}$$

where n is the number of periods between the two successive peaks, and T is the natural damped period of oscillation of the pendulum.

Combining both equations:

$$2\sqrt{\frac{g}{L}}\frac{1}{\sqrt{1+\left(\frac{2\pi}{\frac{1}{n}\ln\left(\frac{\theta(t)}{\theta(t+nT)}\right)}\right)^2}} = b\frac{1}{m_{pen}L^2}$$

Let y = $2\sqrt{\frac{g}{L}}\frac{1}{\sqrt{1+\left(\frac{2\pi}{\frac{1}{n}\ln\left(\frac{\theta(t)}{\theta(t+nT)}\right)}\right)^2}}$ and x= $\frac{1}{m_{pen}L^2}$.

Computing constant b using experimental results

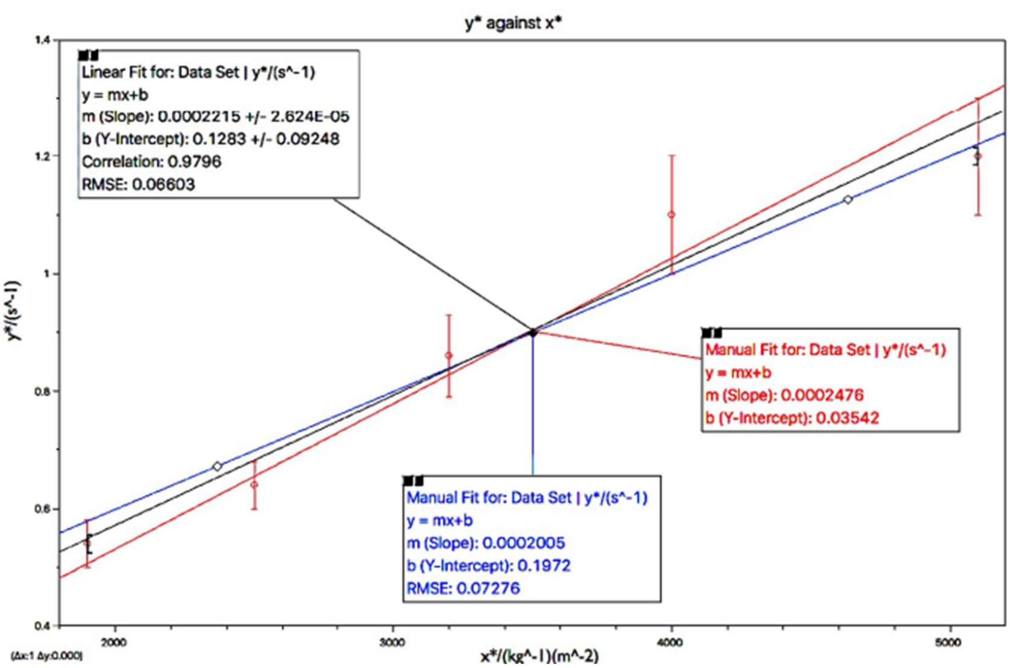
Hypothetically, the linear relationship between y and x should be a straight line through the origin with a positive slope b. Initial parameters includependulum mass = (0.050±0.001)kg and g = 9.818m/s2. The raw data includes pendulum rod lengths and corresponding two successive crests that were 5 periods apart (n = 5). The error of L was obtained from technical instruction paper; the error in angular displacement was obtained from my experience in using the sensor. The gyroscope was programmed in Arduino language.

L±0.00002/m	theta (t) ±0.5/Deg	theta (t+5T) ±0.5/Deg
0.10177	27.1	11.5
0.08917	29.6	11.3
0.07914	28.0	8.3
0.07111	26.5	6.4
0.06289	23.9	5.2

From this raw data y and x can be calculated.

L±0.00002/m	theta (t) ±0.5/Deg	theta(t+5T) ±0.5/Deg	y	x
0.10177	27.1	11.5	0.535	1900
0.08917	29.6	11.3	0.643	2500
0.07914	28.0	8.3	0.86	3200
0.07111	26.5	6.4	1.1	4000
0.06289	23.9	5.2	1.2	5100

Using this table and errors in arduino ide, a plot was made y against x.The black line represented the best linear fit for the data set, and the red and the blue line each represented the maximum and minimum slope line.



The pendulum damping torque constant, b, is the slope of the graph. From the graph, we can see that the best linear fit line has a slope of 0.00022 (2s.f.), and that the maximum and the minimum slope line each has a slope of 0.00025 (2s.f.) and 0.00020 (2s.f.). By convention, we can calculate the absolute uncertainty of b in this way:

$$absolute\ uncertainty = \frac{max - min}{2} = \frac{0.00025 - 0.00020}{2}$$

$$= 0.00003\ kgm^2s^{-1}\ (1s.f.)$$

The percentage uncertainity of the slope can be calculated:

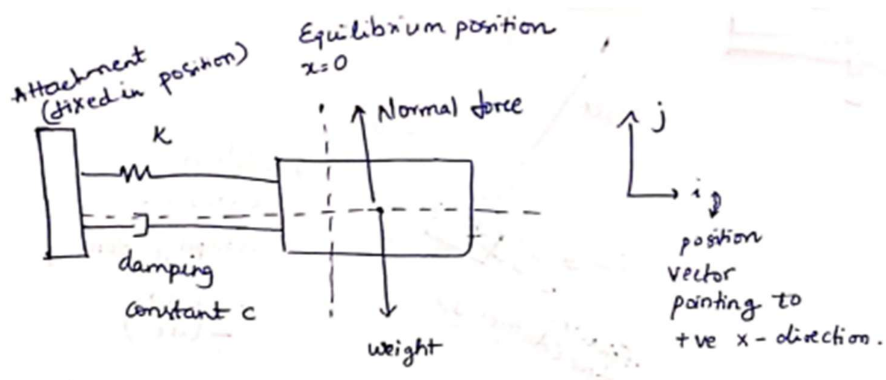
$$percentage\ uncertainty\ in\ slope = \frac{absolute\ uncertainty\ in\ slope}{slope\ of\ best\ fit\ line} = \frac{0.00003}{0.00022}$$

$$\approx 10\%(1s.f.)$$

Finally, the constant b I s found to be 0.00022(2s.f.) and uncertainity is 10%.

Finding constants c and k for the building(without pendulum)

The top of the building was not attached to any springs or dampers, but we can assume the mechanical friction of the building as a damper and the elasticity of the wooden sticks as a spring.



The damping force on the top depends on its velocity (Damped Oscillation）:

$$f_{damp} = -c\frac{dx}{dt}$$

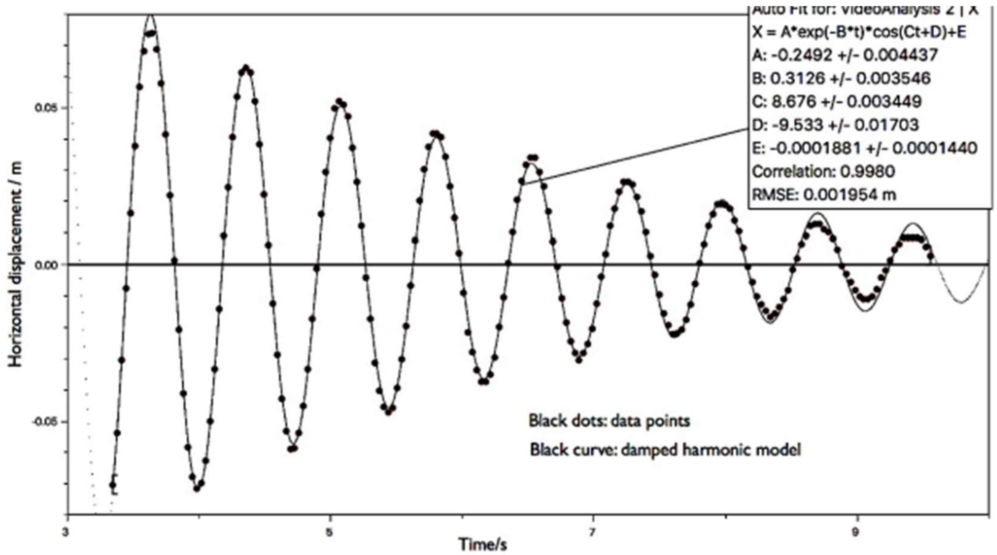
Where c is damping constant.
Using NSL and COM of the top:

$$\sum F_{top} = m_{top} a_{top}$$

$$-kx - c\frac{dx}{dt} = m_{top}\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} + \frac{c}{m_{top}}\frac{dx}{dt} + \frac{k}{m_{top}}x = 0$$

Since c and k cannot be calculated from this equation alone, we assume the top to be a damped harmonic oscillator so that we can compare it to the GDEOM. To verify this assumption we conduct an experiment in which we read the displacement of the top frame of the system.



Comparing with GDEOM:

$$\frac{d^2x}{dt^2} + \frac{c}{m_{top}}\frac{dx}{dt} + \frac{k}{m_{top}}x = \frac{d^2x}{dt^2} + 2\omega_{ntop}\zeta_{top}\frac{dx}{dt} + {\omega_{ntop}}^2 x$$

Matching coefficients:

$$\omega_{ntop}\zeta_{top} = \frac{c}{2m_{top}}$$

$${\omega_{ntop}}^2 = \frac{k}{m_{top}}$$

Solving the GDEOM of damped harmonic oscillator:

$$\frac{d^2x}{dt^2} + 2\omega_{ntop}\zeta_{top}\frac{dx}{dt} + {\omega_{ntop}}^2 x = 0$$

$$x(t) = e^{-\omega_{ntop}\zeta_{top}t} A\cos\left(\omega_{ntop}\sqrt{1 - {\zeta_{top}}^2}\,t + \phi\right) = \quad e^{-\omega_{ntop}\zeta_{top}t} A\cos\left(\omega_{ndtop}t + \phi\right)$$

This equation is the solution of the above equation

Computing c and k using Experimental results:

Using Arduino IDE, the best fit Damped harmonic model of the top's motion is:

$$x(t) = -0.2492e^{-0.3126} \cos(8.676t - 9.533)$$

So,

$$-0.2492e^{-0.3126t} \cos(8.676t - 9.533) = Ae^{-\omega_{ntop}\zeta_{top}t} \cos\left(\omega_{ntop}\sqrt{1-\zeta_{top}{}^2}t + \phi\right)$$

Matching coefficients:

$$0.3126 = \omega_{ntop}\zeta_{top}$$

$$8.676 = \omega_{ntop}\sqrt{1-\zeta_{top}{}^2}$$

$$0.3126 = \frac{c}{2m_{top}}$$

Since we know Mass of the top frame,

$$0.3126 = \frac{c}{2m_{top}}$$

$$c = 0.3126 * 2 * m_{top} = 0.3126 * 2 * 0.198 = 0.124\, kgs^{-1}\ (3s.f.)$$

Algebraically modifying the equation,

$$8.676^2 = \omega_{ntop}^2(1-\zeta_{top}{}^2)$$

$$8.676^2 = \frac{k}{m_{top}}\left(1-\zeta_{top}{}^2\right)$$

For finding k, the top damping coefficient needed to be found from video analysis using logarithmic decrement

$$\delta = \frac{1}{n}\ln\left(\frac{x(t)}{x(t+nT)}\right)$$

$$\zeta_{top} = \frac{1}{\sqrt{1+\left(\frac{2\pi}{\delta}\right)^2}}$$

From experiment	From experiment	Calculated using decrement
x(t)/m	x(t + 5T)/m	ζ_{top}
0.0896 (3s.f)	0.0303 (3s.f.)	0.0345 (3s.f.)

Substituting these values into the equation:

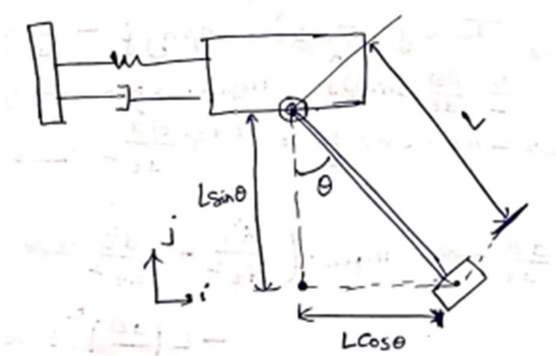
$$8.676^2 = \frac{k}{m_{top}}\left(1 - \zeta_{top}{}^2\right)$$

$$k = \frac{8.676^2 * m_{top}}{\left(1 - \zeta_{top}{}^2\right)} = \frac{8.676^2 * 0.198}{(1 - 0.0345^2)} = 15.0\ Nm^{-1}\ (3s.f.)$$

Value of 2 constants:

c/kgs^{-1}	k/Nm^{-1}
0.124 (3s.f.)	15.0 (3s.f.)

Combined Mathematical Model of the Pendulum Attached to the Building:



$$position_{pen} = x\hat{\imath} + Lsin(\theta)\,\hat{\imath} - Lcos(\theta)\hat{\jmath}$$

$$velocity_{pen} = \frac{dx}{dt}\hat{\imath} + L\frac{d\theta}{dt}\cos(\theta)\,\hat{\imath} + L\frac{d\theta}{dt}\sin(\theta)\,\hat{\jmath}$$

Taking the derivative again:

$$acceleration_{pen} = \frac{d^2x}{dt^2}\hat{\imath} + L\frac{d^2\theta}{dt^2}\cos(\theta)\,\hat{\imath} - L\left(\frac{d\theta}{dt}\right)^2\sin(\theta)\,\hat{\imath} + L\frac{d^2\theta}{dt^2}\sin(\theta)\,\hat{\jmath} + L\left(\frac{d\theta}{dt}\right)^2\cos(\theta)\hat{\jmath}$$

Applying NSL and COM on the top

$$\sum F = m_{top} * acceleration_{top}$$

$$N\hat{\jmath} - m_{top}g\hat{\jmath} + Tsin(\theta)\hat{\imath} - Tcos(\theta)\hat{\jmath} + \frac{b}{L}\frac{d\theta}{dt}\cos(\theta)\,\hat{\imath} + \frac{b}{L}\frac{d\theta}{dt}\sin(\theta)\,\hat{\jmath} - kx\hat{\imath} - c\frac{dx}{dt}\hat{\imath} = m_{top}\frac{d^2x}{dt^2}\hat{\imath}$$

Applying NSL and COM on pendulum mass:

$$\sum F = m_{pen} * acceleration_{pen}$$

$$T\cos(\theta)\hat{j} - T\sin(\theta)\hat{\imath} - m_{pen}g\hat{j} - \frac{b}{L}\frac{d\theta}{dt}\cos(\theta)\,\hat{\imath} - \frac{b}{L}\frac{d\theta}{dt}\sin(\theta)\,\hat{j}$$

$$= m_{pen}\left(\frac{d^2x}{dt^2}\hat{\imath} + L\frac{d^2\theta}{dt^2}\cos(\theta)\,\hat{\imath} - L\left(\frac{d\theta}{dt}\right)^2\sin(\theta)\,\hat{\imath} + L\frac{d^2\theta}{dt^2}\sin(\theta)\,\hat{j} + L\left(\frac{d\theta}{dt}\right)^2\cos(\theta)\,\hat{j}\right)$$

$$T\sin(\theta) + \frac{b}{L}\frac{d\theta}{dt}\cos(\theta) - kx - c\frac{dx}{dt} = m_{top}\frac{d^2x}{dt^2}$$

$$N - m_{top}g - T\cos(\theta) + \frac{b}{L}\frac{d\theta}{dt}\sin(\theta) = 0$$

$$-T\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}\cos(\theta) = m_{pen}\left(\frac{d^2x}{dt^2} + L\frac{d^2\theta}{dt^2}\cos(\theta) - L\left(\frac{d\theta}{dt}\right)^2\sin(\theta)\right)$$

$$T\cos(\theta) - m_{pen}g - \frac{b}{L}\frac{d\theta}{dt}\sin(\theta) = m_{pen}\left(L\frac{d^2\theta}{dt^2}\sin(\theta) + L\left(\frac{d\theta}{dt}\right)^2\cos(\theta)\right)$$

Cancelling out tension by adding two equations since it cannot be computed:

$$\left(T\sin(\theta) + \frac{b}{L}\frac{d\theta}{dt}\cos(\theta) - kx - c\frac{dx}{dt}\right) - T\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}\cos(\theta)$$

$$= \left(m_{top}\frac{d^2x}{dt^2}\right) + m_{pen}\left(\frac{d^2x}{dt^2} + L\frac{d^2\theta}{dt^2}\cos(\theta) - L\left(\frac{d\theta}{dt}\right)^2\sin(\theta)\right)$$

Grouping similar terms:

$$(m_{top} + m_{pen})\frac{d^2x}{dt^2} = m_{pen}L\left(\frac{d\theta}{dt}\right)^2\sin(\theta) - m_{pen}L\frac{d^2\theta}{dt^2}\cos(\theta) - kx - c\frac{dx}{dt}$$

$$T\cos(\theta)\sin(\theta) - m_{pen}g\sin(\theta) - \frac{b}{L}\frac{d\theta}{dt}\sin(\theta)\sin(\theta)$$

$$= m_{pen}\left(L\frac{d^2\theta}{dt^2}\sin(\theta) + L\left(\frac{d\theta}{dt}\right)^2\cos(\theta)\right)\sin(\theta)$$

$$\frac{b}{L}\frac{d\theta}{dt} + m_{pen}\frac{d^2x}{dt^2}\cos(\theta) + m_{pen}L\frac{d^2\theta}{dt^2} + m_{pen}g\sin(\theta) = 0$$

Solving the Differential Equations of Motion:

$$(m_{top} + m_{pen})\frac{d^2x}{dt^2} = m_{pen}L\left(\frac{d\theta}{dt}\right)^2\sin(\theta) - m_{pen}L\frac{d^2\theta}{dt^2}\cos(\theta) - kx - c\frac{dx}{dt}$$

$$\frac{b}{L}\frac{d\theta}{dt} + m_{pen}\frac{d^2x}{dt^2}\cos(\theta) + m_{pen}L\frac{d^2\theta}{dt^2} + m_{pen}g\sin(\theta) = 0$$

The complicatedness of these equations does not allow us to obtain explicit solutions so we use **Range-Kutta method** to obtain numerical solutions.It requires four first order Differential equations, SO we add two more differential equations.

$$\frac{dx}{dt} = v$$

$$\frac{d\theta}{dt} = \omega$$

$$(m_{top} + m_{pen})\frac{dv}{dt} = m_{pen}L(\omega)^2\sin(\theta) - m_{pen}L\frac{d\omega}{dt}\cos(\theta) - kx - cv$$

$$\frac{b}{L}\omega + m_{pen}\frac{dv}{dt}\cos(\theta) + m_{pen}L\frac{d\omega}{dt} + m_{pen}gsin(\theta) = 0$$

The solutions are plotted on graphs using Matlab.

Finding the Optimal Length of Pendulum in theory:

The best fit damped harmonic model on top frame:

$$x(t) = -0.2492e^{-0.3126t}\cos(\,8.676t - 9.533)$$

Comparing it with GDEOM of damped harmonic oscillators:

$$e^{-\omega_{ntop}\zeta_{top}t}A\cos(\omega_{ndtop}t + \phi) = -0.2492e^{-0.3126t}\cos(\,8.676t - 9.533)$$

ωndtop =8.676s

$$\omega_{ndpen} = \omega_{npen}\sqrt{1 - \zeta_{pen}{}^2}$$

$$\omega_{ndpen} = \sqrt{\omega_{npen}{}^2 - \omega_{npen}{}^2\zeta_{pen}{}^2}$$

$$\frac{d^2\theta}{dt^2} + 2\omega_{npen}\zeta_{pen}\frac{d\theta}{dt} + \omega_{npen}{}^2\theta = \frac{d^2\theta}{dt^2} + \frac{b}{m_{pen}L^2}\frac{d\theta}{dt} + \frac{g}{L}\theta$$

Matching coefficients:

$$2\omega_{npen}\zeta_{pen} = \frac{b}{m_{pen}L^2}$$

$$\omega_{npen}{}^2 = \frac{g}{L}$$

$$\omega_{ndpen} = \sqrt{\frac{g}{L} - \left(\frac{b}{2m_{pen}L^2}\right)^2}$$

Substituting all the known values:

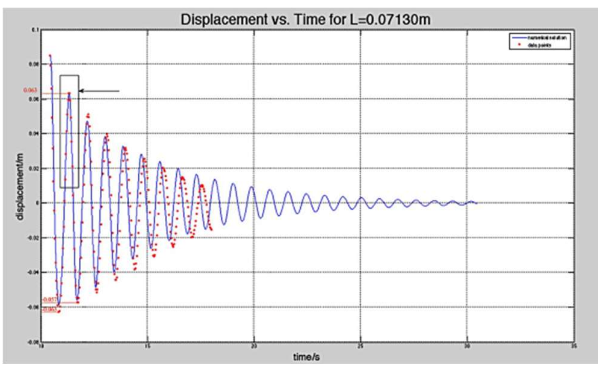
$$8.676^2 = \frac{9.81}{L} - \frac{0.00022^2}{4 * 0.050^2L^4}$$

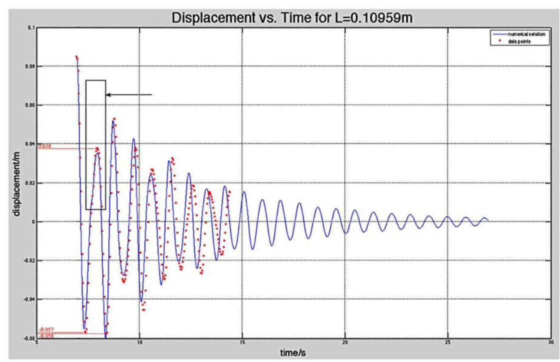
This gives us a **L of 0.13m** .

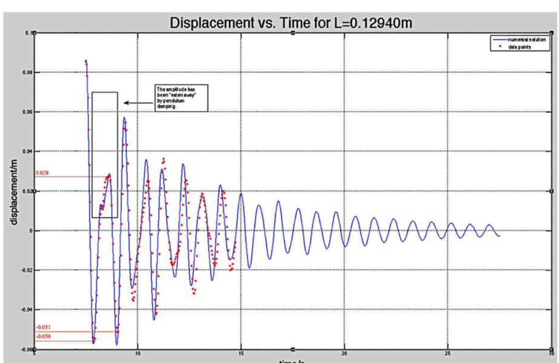
Finding Optimal Length of Pendulum by experiment:

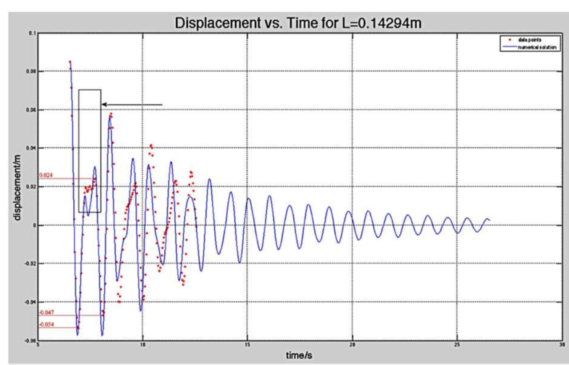
In theory, the optimal length of the pendulum rod for damping my building is about 0.13m (2s.f.), which will produce a natural damped frequency, that is almost identical to the top's natural damped frequency. 5 different lengths of the pendulum were tested. Each time the top frame was displaced by about 0.85m.

Length±0.00002/m				
0.07130	0.10959	0.12940	0.14294	0.15762





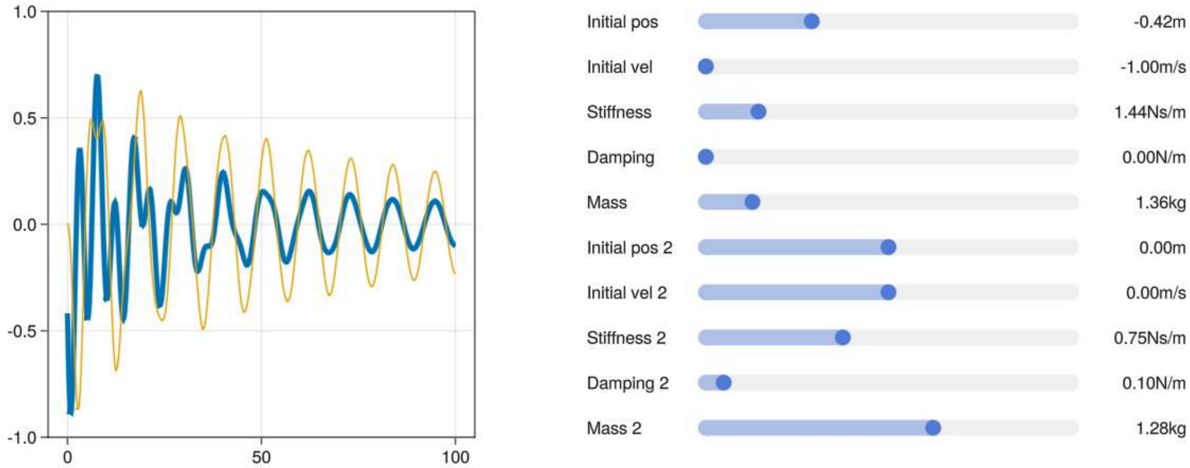




A rod whose length was the closest to theoretical performed the best.

Julia simulation

We also made a Julia code using these available values so it can simulate the setup and give the output for any different initial values given to it.



We wrote code to simulate the coupled tuned mass damper system, and to plot the results.Written in Julia with GLMakie for gpu accelerated plotting.
It contains two scripts:

- simTMD.jl - contains the code to simulate the coupled tuned mass damper system, and returns the results in a gif.

- interactive.jl - contains the code to simulate the coupled tuned mass damper system with an interactive session in GLMakie, allowing the user to change the parameters of the system and see the results.

  This was made to obtain and observe the outputs of the system for different parameters since we can not change the parameters like mass in the system itself.

Conclusion

Our research aimed to discover the relationship between the damping effect of the pendulum on an oscillating mass and the length of the pendulum rod. We first computed the constants, and constructed the mathematical model.

After verifying our mathematical model with experimental data, it is clear that the pendulum would have a good damping effect when its natural damping frequency is close to that of the building. The numerical solution of the mathematical model of the combined structure fitted the experimental data reasonably well, however, the experimental data gradually drifted away from the model.

The flaws of our model could come from measurements.Some measurements were indeed imprecise due to limited access to high-precision instruments, which resulted relatively large uncertainties in constants.We made the model of the bulding with a combination of metal tools,3D printed frames and wood, which could cause some irregularities in the readings.Nevertheless, since the first few oscillations have the largest amplitude and hence more destructive, the ability to model them relatively well should be regarded as a success

For real applications, the dimensions of the building put constraints on the rod length of the pendulum, and some other factors must be tuned to achieve the best damping effect. The buildings also oscillate in more than one direction. How a pendulum can compensate these factors is open to study.

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