A Deterministic Cellular Automaton Model of Action Potential Propagation

submitted by

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1 Introduction

1.1 Cellular Automata in Biology

Cellular automata (CA) are discrete dynamical model in automata theory defined on regular lattices with local interaction rules. In biology most of the system are continuous, CA has shown to be a valuable approach when we try to deal with the the propagation of action potential of neuron in the biological tissue.

The main advantage of CA is it can replicate the spatial and temporal dynamics simultaneously. In this project I trying to show how complex wave like neural activity can emerge from local, deterministic rules.

1.2 The Action Potential Propagation Model

The model's cells are arranged on a two-dimensional lattice representing the states of the neurons in the brain. When one stimulus is applied, it propagates like a wave and unlike Conway's Game of Life[2] it does not permit stable configurations or still-life structures. The main goal to design this model are to capture the qualitative features of neuronal firing, action potential propagation, excitation and recovery dynamics in an excitable media[1].

2 Mathematical Framework

2.1 Lattice

The system is defined on a two-dimensional square lattice:

$$L = \{(i, j) \mid i, j \in \{1, 2, \ldots, N\}\}$$

In our simulation we used a 100×100 grid where each grid represent as one neuron. To propagate the simulation we applied a periodic boundary condition where the boundary cell will act like a neighbor of the exactly opposite cell like the grid converts into a torus. So the boundary condition represent as:

$$(i \pm 1) \mod N$$

2.2 State Space

Each lattice site takes values from:

$$S = \{0, 1, 2\}$$

State	Interpretation	Neural analogy
0	OFF (Resting)	Resting membrane potential
1	ON (Excited)	Action potential firing
2	REFRACTORY (Recovering)	Channel inactivation / recovery

The global configuration at time t is:

$$G_t : L \to S$$

2.3 Neighborhood

Here we utilize the Moore neighborhood (8-neighborhood), which is the particular cell's 8 neighbor cells in contact(4 with edge and 4 with corner) except that cell represents as:

$$N_{i,j} = \{(i + dx, j + dy) \mid dx, dy \in \{-1, 0, 1\}, (dx, dy) \neq (0, 0)\}$$

3 Rules of Propagation

3.1 Initial Conditions

First every cells are in a static state. Then we introduced stimulus on the random 10% of the population. According the rule the stimulus is propagated like an wave in each step. Now the rules of propagation are:

3.2 Rule 1: Excitation

A resting cell becomes excited if exactly two neighbors of that particular cell are excited. This defines as a threshold condition as more than two excited neighbor does not display the same.The threshold condition (exactly two excited neighbors) can be viewed as a simplified coincidence detection mechanism. This represents as:

$$\text{If } G_t(i,j) = 0 \text{ and } \sum_{(k,l) \in N_{i,j}} \mathbf{1}_{G_t(k,l)=1} = 2 \Rightarrow G_{t+1}(i,j) = 1$$

3.3 Rule 2: Decay Rule

The excitation period only lasts for one step. This prevents the continuous firing and we represent this state as 2 in our simulation:

$$\text{If } G_t(i,j) = 1 \Rightarrow G_{t+1}(i,j) = 2$$

3.4 Rule 3: Recovery Rule

This is the most crucial rule as it recover the cells from excited state to the initial state and again make it ready for firing another action potential. This represents as:

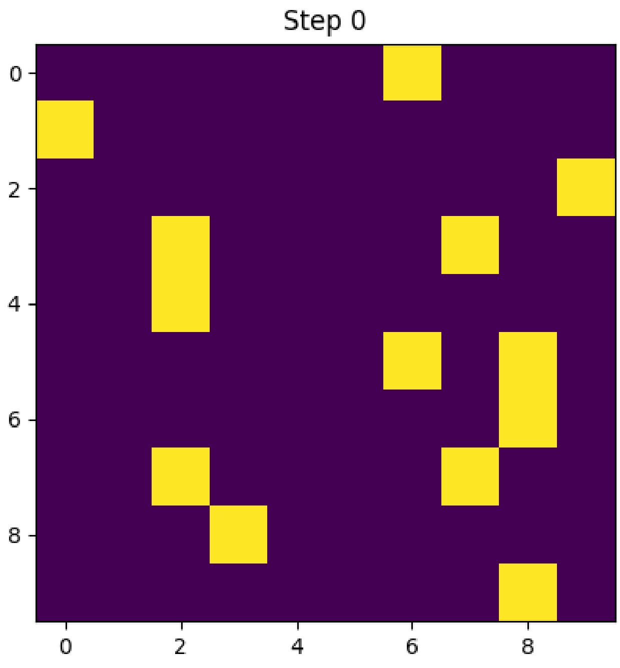
$$\text{If } G_t(i,j) = 2 \Rightarrow G_{t+1}(i,j) = 0$$

4 Simulation Results

For the better understanding the dynamics of the model the grid size reduced to 10×10.

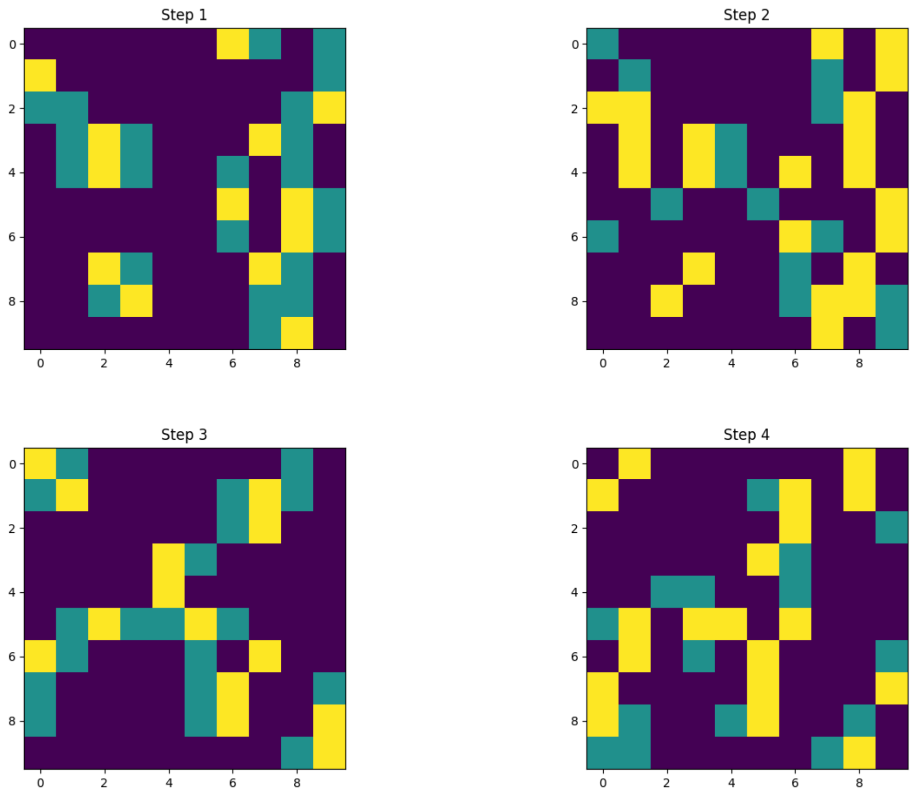
4.1 The Initial state:

At t=0 or the step 0 the 10 % of the grid is excited randomly.



4.2 Propagation of the Action Potential:

To show the propagation of the action potential here 4 steps are shown. The yellow color cells are represent as excited, the green color cells are represented as the cells having two excited contact neighbor cells. The green cells become yellow and the yellow cell become recover to in the initial cell.



5 Key Properties of the Model

- There is no randomness in the rule itself. all the future states are completely determine by the initial configurations.
- Unlike Game of Life, this model does not support small static oscillators. This model generates waves and spatiotemporal chaos.
- Due to finite excitation duration, presence of recovery period and threshold activation, this model belongs to the class of discrete excitable systems and shows some analogy with Hodgkin–Huxley excitability Reaction–diffusion systems and Greenberg–Hastings model.

6 Conclusion

This model is a minimal yet dynamically rich cellular automaton which captures the qualitative structure of excitable systems. The three step rule is taken for the simplicity of the model but it produces complex spatiotemporal behavior and serves as an effective bridge between discrete dynamical systems and biological modeling. From this model we understand the wave like neural activity in a discrete way.

7 Limitation

This model makes several assumptions which simplifies the outcome but limits its biological realism like

i. Excitation and Refractory each lasts exactly 1 step.
ii. Fully deterministic rule which is not observed in the real neuron.
iii. The inhibitory and plasticity features of neuron is not included.
iv. Also there is no energy or metabolic constraints presents.

References

[1] Greenberg, J. M., & Hastings, S. P. *Spatial patterns for discrete models of diffusion in excitable media*, SIAM Journal on Applied Mathematics, 1978.

[2] Bays, C., *Candidates for the Game of Life in Three Dimensions*, 1989.