06 Implement SGD

June 6, 2019

```
In [1]: import warnings
        warnings.filterwarnings("ignore")
        from sklearn.datasets import load_boston
        from random import seed
        from random import randrange
        from csv import reader
        from math import sqrt
        from sklearn import preprocessing
        import numpy as np
        import matplotlib.pyplot as plt
        from sklearn.linear_model import SGDRegressor
        from sklearn import preprocessing
        from sklearn.metrics import mean_squared_error
        import time
```

```
In [27]: X = load_boston().data
         Y = load_boston().target
```

```
In [30]: print(X.shape)
         # 506 samples and 13 features
```

```
(506, 13)
```

```
In [31]: print("Mean : {}".format(X.mean()))
         print("Max  : {}".format(X.max()))
         print("Min  : {}".format(X.min()))
```

```
Mean : 70.07396704469443
Max  : 711.0
Min  : 0.0
```

```
In [33]: print(Y.shape)
         # Target is the house price in each sample
```

```
(506,)
```

```
In [34]: # scaling the data for faster gradient descent
         scaler = preprocessing.StandardScaler().fit(X)
         X = scaler.transform(X)

In [35]: print("Mean : {}".format(X.mean()))
         print("Max  : {}".format(X.max()))
         print("Min  : {}".format(X.min()))

Mean : -1.1147462804871136e-15
Max  : 9.933930601860268
Min  : -3.9071933049810337


In [171]: clf = SGDRegressor()
          clf.fit(X, Y)
          print(mean_squared_error(Y, clf.predict(X)))

23.028034816777073
```

Implementing our own gradient descent function

```
In [37]: samples = X.shape[0]
         iterations = 300

In [38]: # Adding the 'bias' column with the features columns
         X = np.c_[np.ones(samples), X]

In [39]: # shuffle data X and Y
         # https://stackoverflow.com/questions/4601373/better-way-to-shuffle-two-numpy-arrays-
         randomize = np.arange(len(X))
         np.random.shuffle(randomize)
         X = X[randomize]
         Y = Y[randomize]

In [40]: Y = Y.reshape(Y.shape[0], 1)

In [41]: X.shape

Out[41]: (506, 14)

In [13]: def gradient_descent(X, Y, learning_rate=0.01, iterations=400, batch_Size=0):
             # X is matrix used for predictions along with the added bias column
             # Y target variable
             # learning rate determines the step size in the direction of gradient
             # iterations are the num of times for the algo to run
             # batch_Size=1 means SGD, 0 means GD and specify batch for mini-batch

             # sanity check
             assert(Y.shape == (X.shape[0], 1))
```

```
            assert(batch_Size >= 0)
            samples = X.shape[0]

            start = time.time()

            # theta initialised with random
            theta = np.random.randn(X.shape[1], 1)
            # storing the cost values per iteration to plot later
            cost_hist = np.zeros(iterations)

            for i_num in range(iterations):
                if batch_Size == 0:
                    prediction = np.dot(X,theta)
                    theta -= (1/samples) * learning_rate * np.dot(X.T, prediction-Y)
                    prediction = np.dot(X,theta)
                    cost_hist[i_num] = np.sum(np.square(prediction-Y)) / (2*samples)

                elif batch_Size == 1:
                    cost = 0.0
                    for s in range(samples):
                        # iterate over each of the samples as they are already shuffled
                        X_i = X[s,:].reshape(1,X.shape[1])
                        Y_i = Y[s].reshape(1,1)
                        prediction = np.dot(X_i,theta)
                        theta -= learning_rate * np.dot(X_i.T, prediction-Y_i)
                        prediction = np.dot(X_i,theta)
                        cost += np.sum(np.square(prediction-Y_i)) / 2
                    cost_hist[i_num] = cost/samples

                else:
                    cost = 0.0
                    n_batches = samples//batch_Size
                    batches = np.random.permutation(samples - batch_Size +1)[:n_batches]
                    for bat in batches:
                        # iterate over each of the batches
                        X_i = X[bat: bat + batch_Size]
                        Y_i = Y[bat: bat + batch_Size]
                        prediction = np.dot(X_i,theta)
                        theta -= (1/batch_Size)*learning_rate*np.dot(X_i.T,prediction-Y_i)
                        prediction = np.dot(X_i,theta)
                        cost += np.sum(np.square(prediction-Y_i)) / (2*batch_Size)
                    cost_hist[i_num] = cost/batch_Size

            end = time.time()
            print("Run time : {} ms".format(round((end-start)*1000, 2)))
            return theta, cost_hist
```

```
In [42]: # Batch gradient descent
```

```
         theta_gd, cost_hist_gd = gradient_descent(X, Y,
                                 iterations=iterations, batch_Size=0)
```

```
Run time : 22.72 ms
```

```
In [43]: # stochastic gradient descent
         theta_sgd, cost_hist_sgd = gradient_descent(X, Y,
                                 iterations=iterations, batch_Size=1)
```

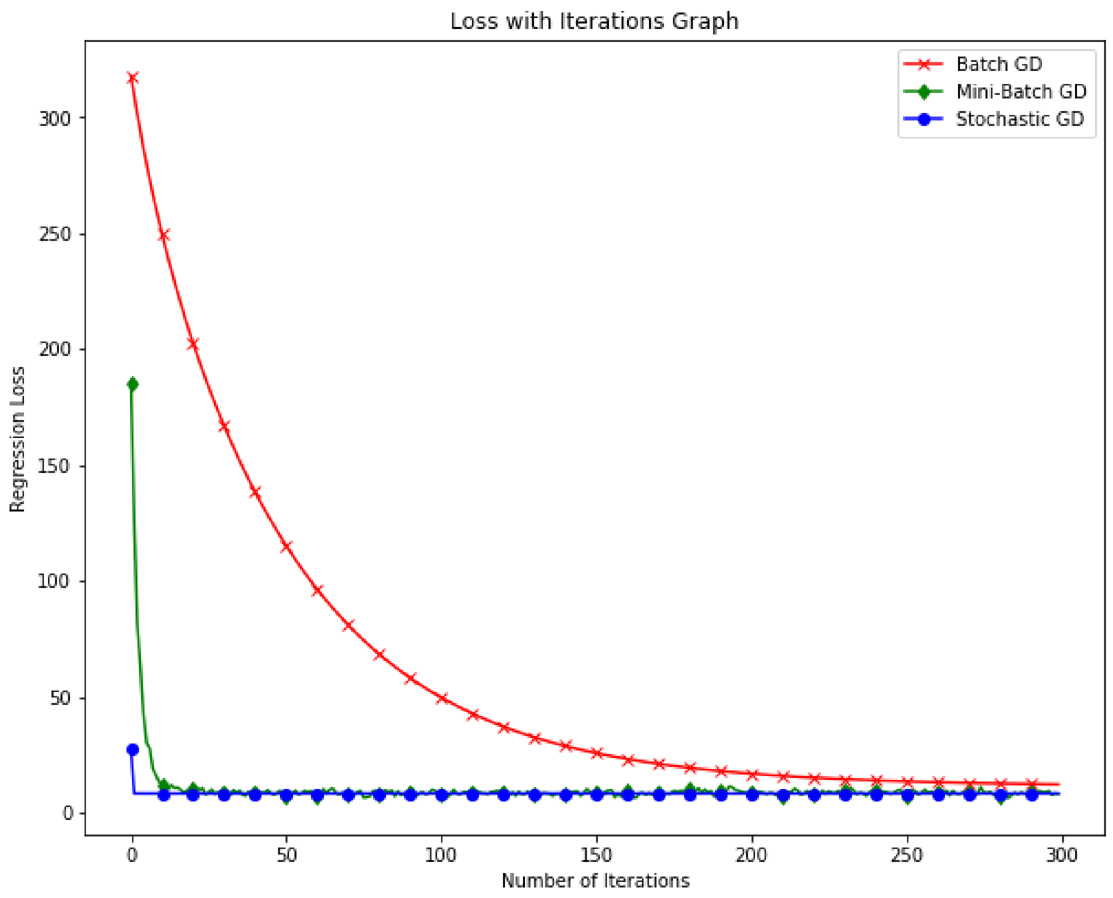
```
Run time : 2072.1 ms
```

```
In [44]: # mini-batch gradient descent
         theta_mbgd, cost_hist_mbgd = gradient_descent(X, Y,
                                 iterations=iterations, batch_Size=25)
```

```
Run time : 118.32 ms
```

Comparing the cost history graphs

```
In [45]: # Plotting the cost values with iterations graph
         x_axis = range(iterations)
         plt.figure(figsize=(10.0, 8.0))
         markers_on = [i for i in range(iterations) if i%10 ==0]
         plt.plot(x_axis, cost_hist_gd, '-rx', markevery=markers_on,
                               label='Batch GD', )
         plt.plot(x_axis, cost_hist_mbgd, '-gd', markevery=markers_on,
                               label='Mini-Batch GD')
         plt.plot(x_axis, cost_hist_sgd, '-bo', markevery=markers_on,
                               label='Stochastic GD')
         plt.xlabel("Number of Iterations")
         plt.ylabel("Regression Loss")
         plt.title("Loss with Iterations Graph")
         plt.legend()
         plt.show()
```



```
In [46]: clf = SGDRegressor()
         clf.fit(X, Y)
         pred_sklearnSGD = clf.predict(X)
         print(mean_squared_error(Y, pred_sklearnSGD))
```

```
22.82971463980024
```

Finding most optimal theta

```
In [47]: # Since the number of features are not too large and our computational
         # power is sufficient, lets directly find the optimal value of theta by
         # using the vectorised normal equation

         theta_best = np.linalg.inv(X.T.dot(X)).dot(X.T).dot(Y)
         print(theta_best)
```

```
[[ 2.25328063e+01]
 [-9.28146064e-01]]
 [ 1.08156863e+00]
 [ 1.40899997e-01]
 [ 6.81739725e-01]
 [-2.05671827e+00]
 [ 2.67423017e+00]
 [ 1.94660717e-02]
 [-3.10404426e+00]
 [ 2.66221764e+00]
 [-2.07678168e+00]
 [-2.06060666e+00]
 [ 8.49268418e-01]
 [-3.74362713e+00]]
```

```
In [48]: # Lets compare the Cost value from our gradient descent functions and the
         # one we get using the most optimal theta value, 'theta_best'

         pred_best = X.dot(theta_best)
         print("Error from Optimal Theta : {}".format(mean_squared_error(Y,
                                                     pred_best)))
         pred_gd = X.dot(theta_gd)
         print("Error from Calculated Theta for Batch GD: {}".format(
                                                 mean_squared_error(Y, pred_gd)))
         pred_mbgd = X.dot(theta_mbgd)
         print("Error from Calculated Theta for Mini-Batch GD: {}".format(
                                                 mean_squared_error(Y, pred_mbgd)))
         pred_sgd = X.dot(theta_sgd)
         print("Error from Calculated Theta for Stochastic GD: {}".format(
                                                 mean_squared_error(Y,pred_sgd)))

         print("Error from Calculated Theta for skLearn SGD: {}".format(
                                                 mean_squared_error(Y,pred_sklearnSGD)))
```

```
Error from Optimal Theta : 21.894831181729206
Error from Calculated Theta for Batch GD: 24.698483543800545
Error from Calculated Theta for Mini-Batch GD: 21.96812758471529
Error from Calculated Theta for Stochastic GD: 22.845951445106508
Error from Calculated Theta for skLearn SGD: 22.82971463980024
```

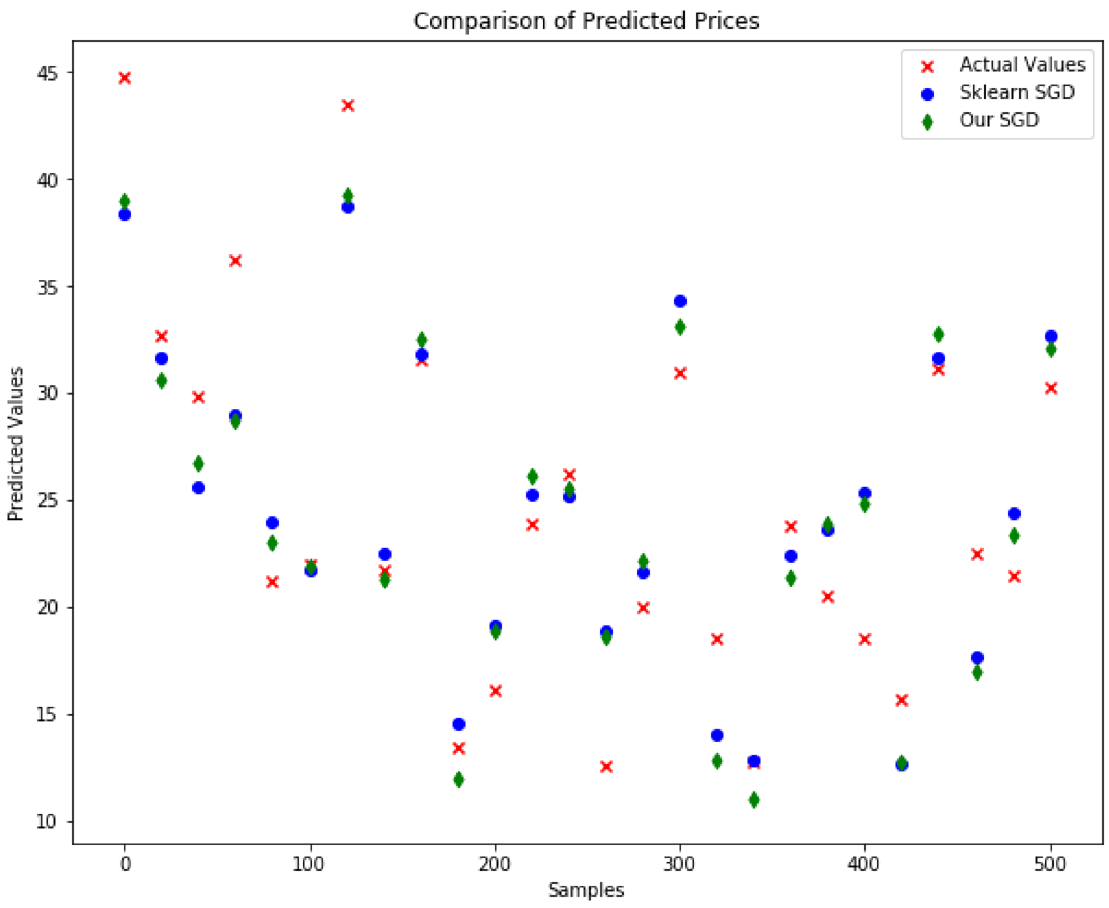
```
In [49]: print("Comparing the weights from our SGD and skLearn Regressor: ")
         sklSGD = clf.coef_
         for i in range(len(sklSGD)):
             print("  {0:2d} :  {1: 10.8f}   -   {2: 10.8f}". format(
                                 i+1,
                                 theta_sgd[i][0],
                                 sklSGD[i]))
```

```
Comparing the weights from our SGD and skLearn Regressor:
   1 :   22.46789267   -    11.31434460
```

```
 2 :  -0.66849967   -   -0.66531255
 3 :   1.14203826   -    0.48237328
 4 :   0.29153106   -   -0.39389998
 5 :   0.64938312   -    0.81531448
 6 :  -1.95053319   -   -1.06691459
 7 :   2.82412889   -    3.25478057
 8 :  -0.28268455   -   -0.14535041
 9 :  -2.94866511   -   -2.10835523
10 :   2.86720407   -    0.86533220
11 :  -2.06102117   -   -0.39270045
12 :  -1.61062724   -   -1.82439325
13 :   1.28868717   -    0.93486820
14 :  -4.43665434   -   -3.27933427
```

Comparing the predictions of our SGD, sklearn SGD and the actual Values

```
In [58]: # Take 25 samples from the 506 samples
         x_axis = [i for i in range(samples) if i%20==0]
         plt.figure(figsize=(10.0, 8.0))
         plt.scatter(x_axis, Y[x_axis], marker='x',
                     color='r', label='Actual Values')
         plt.scatter(x_axis, pred_sklearnSGD[x_axis], marker='o',
                     color='b', label='Sklearn SGD')
         plt.scatter(x_axis, pred_sgd[x_axis], marker='d',
                     color='g', label='Our SGD')
         plt.xlabel("Samples")
         plt.ylabel("Predicted Values")
         plt.title("Comparison of Predicted Prices")
         plt.legend()
         plt.show()
```



```
In [59]: # Hence, we see we were able to beat the SGD regressor score but we couldn't
         # surpass the score we got from the optimal value of theta
```

Conclusions

```
- Batch Gradient descent is the fastest to execute, followed by mini-batch
gradient descent. The stochastic gradient descent is immensely slow by around
95 times compared to batch.

- On the other hand, we see stochastic gradient descent is fastest to converge
around the optimal theta and batch gradient descent is the slowest. This might
be an observation specific to this dataset. In a linear regression dataset a
few randomly sampled points can give a good approximation of the optimal plane(which
we are trying to fit) and that helps the stochastic and mini-batch to converge faster.
Whereas, the batch gradient while trying to fit the theta plane over all the points
in each iteration, becomes slow. In more complex datasets the stochastic and mini-
batch are expected to show a much more random behaviour.

- The mini-batch gradient descent came closest to the optimum values of theta,
```

which we calculated directly by solving the vectorised normal equation. The mean squared error in this case was even better than the sklearn SGDRegressor error value. The stochastic gradient descent also came close, but the batch gradient descent has little amount of error.

- Overall, we can say considering the time and optimal value of theta, that the mini-batch gradient descent with batch size of 25 showed the best performance.

```
In [ ]:
```