Satyam Sharma
Section E
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(Tutorial – 4)

① $T(n) = 3T(n/2) + n^2$

Solⁿ:- $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$a \geq 1, b > 1$

On Comparing

$a = 3, b = 2, \quad f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584,$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$

② $T(n) = 4T(n/2) + n^2$

Solⁿ:- $a \geq 1, b \geq 1.$

$a = 4, b = 2, \quad f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

③ $T(n) = T(n/2) + 2^n$

Solⁿ:- $a = 1, b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1 \qquad f(n) > n^c \quad T(n) = \theta(2^n).$

(4) $T(n) = 2^n T(n/2) + n^n$

→ Sol^n:- (4)

$a = 2^n$,
$b = 2$, $f(n) = n^n$
Here a is not a constant,
So we cannot apply master's theorem.

(5) $T(n) = 16T\left(n/4\right) + n$

→ Sol^n

$a = 16$, $b = 4$
$f(n) = n$
$c = \log_4 16 = \log_4 (4)^2 = 2$

$n^c = n^2$
$f(n) < n^c$
$\therefore T(n) = \theta(n^2)$

(6) $T(n) = 2T\left(n/2\right) + n\log n$

Sol^n → $a = 2, b = 2$
$f(n) = n\log n$
$c = \log_2 2 = 1$
$\therefore n^c = n^1 = n$
Since, $n\log n > n$
$\therefore f(n) > n^c$
$\therefore T(n) = \theta(n\log n)$

(7) $T(n) = 2T(n/2) + n/\log n$

→ Sol^n $a = 2$, $b = 2$, $f(n) = n/\log n$
$c = \log_2 2 = 1$

$\therefore n^c = n^1 = n$

Since, $\dfrac{n}{\log n} < n$

$\therefore n^{0.5} < n^{0.51}$
$f(n) > n^c$
$\therefore T(n) = \theta(n^{0.51})$

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(8) $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

→(8) $a=2, \quad b=4, \quad F(n) = n^{0.51}$

$C = \log_b a = \log_4 2 = 0.5$

$\therefore n^c = n^{0.5}$

Since, $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$\therefore T(n) = \theta(n^{0.51})$

(9) $T(n) = 0.5\,T\left(\frac{n}{2}\right) + 1/n$

→(9) $a = 0.5, \; b = 2$

Since acc. to master Theorem

$a \geq 1$, but here a is 0.5

So we cannot apply master theorem.

$a \geq 1$, but here a is 0.5

So we cannot apply master theorem.

(10) $T(n) = 16T(n/4) + n!$

Solⁿ→(10) $a = 16, \quad b = 4, \quad f(n) = n!$

$\therefore C = \log_b a = \log_4 16 = 2.$

Now, $n^c = n^2$

As $n! > n^2$ $\therefore T(n) = \theta(n!)$

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(11) $4T(n/2) + \log n$

Sol^n ⇒ (11) $a = 4,\ b = 2,\ f(n) = \log n$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n^c)$

$= \theta(n^2)$.

(12) $T(n) = \text{sqrt}(n)\ T(n/2) + \log n$

Sol^n ⇒ (12) $a = \sqrt{n},\ b = 2$

Here a is not a constant, so we cannot apply master's theorem

(13) $T(n) = 3T(n/2) + n$

⇒ (13) $a = 3,\ b = 2,\ f(n) = n$

$c = \log_b a = \log_2 3 \approx 1.5849$

$\therefore n^c = n^{1.5849}$

$\therefore n < n^{1.5849}$

$\Rightarrow f(n) < n^c$

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$\therefore \quad T(n) = \theta(n^{1.5849})$

(14) $T(n) = 3T\lfloor n/3 \rfloor + \text{sqrt}(n)$

→ Solⁿ (14) $a = 3, \; b = 3$

$C = \log_b a = \log_3 3 = 1$

$\therefore \quad n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$\therefore \; f(n) < n^c$

$\therefore$ Time complexity $= \theta(n)$

(15) $T(n) = 4T(n/2) + Cn$

→ Solⁿ (15) $a = 4, \; b = 2$

$c = \log_b a = \log_2 4 = 2$

$\therefore \quad n^c = n^2$

$\therefore \quad cn < n^2$ (for any constant)

$\therefore \quad f(n) < n^c$

$\therefore \quad T(n) = \theta(n^2)$.

(16.) $T(n) = 3T(n/4) + n\log n$

→ (16) $a = 3, \; b = 4, \; f(n) = n\log n$.

$C = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$\therefore \quad n^{0.792} < n\log n$

$\therefore \quad T(n) = \theta(n\log n)$.