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Comet Problem:

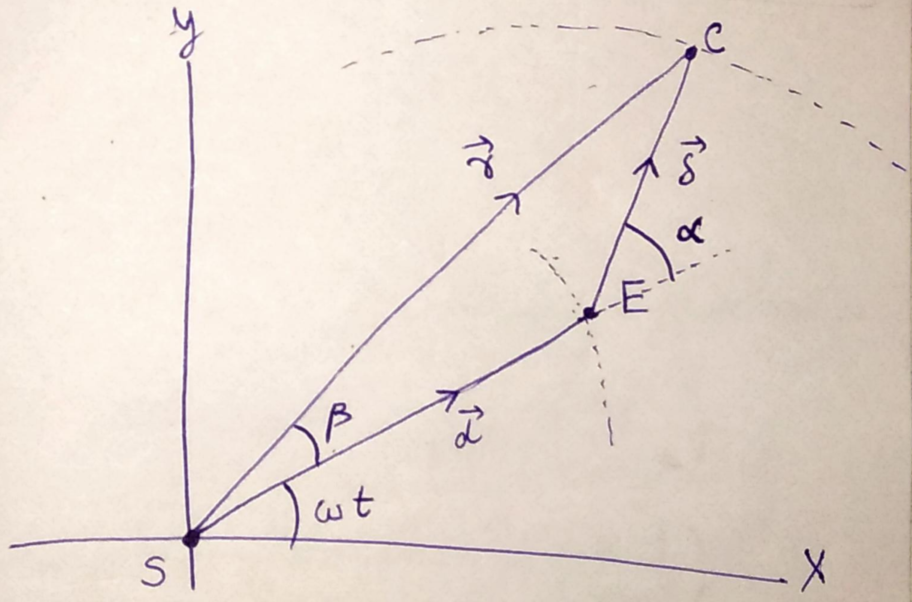
I let the three given quantities:

$\delta(t) \rightarrow$ Distance between observer (on earth) and comet (in AU)

$\alpha(t) \rightarrow$ complimentory angle (NOT the one given in csv but its ~~compliment~~ supplement).

time $\rightarrow$ times given (in years)

first I make a coordinate system with sun at origin:



E $\rightarrow$ Earth ; C $\rightarrow$ Comet ; S $\rightarrow$ Sun,

$|\vec{d}| = 1$, δ and $\vec{r}$ are functions of time

assuming earth revolves around sun in circle with constant angular speed (ω).

I calculated $\beta(t)$ and $\vec{r}(t)$.

as, $\vec{b} + \vec{\delta} = \vec{r}$ and $\vec{b} \cdot \hat{r} = \cos\beta$

$$\Rightarrow \boxed{r(t) = \left[1 + 2\,\delta(t)\cos[\alpha(t)] + (\delta(t))^2\right]^{1/2}} \quad \text{--- (1)}$$

to find $\beta(t)$, we see that,

$$r\cos\beta = 1 + \delta\cos\alpha$$

$$r\sin\beta = \delta\sin\alpha$$

$$\Rightarrow \tan\beta = \frac{\delta\sin\alpha}{1+\delta\cos\alpha}$$

so, polar angle $\theta(t) = \omega t + \beta(t)$

or $$\boxed{\theta(t) = \omega t + \tan^{-1}\left[\frac{\delta\sin\alpha}{1+\delta\cos\alpha}\right]} \quad \text{--- (2)}$$

$\omega = \frac{2\pi(\text{rad})}{1(\text{year})} = 2\pi$

equipped with $r(t)$, $\theta(t)$ and corresponding list of times at which r and θ are calculated, we can do curve fitting.

Programming

1. using pandas to read csv file
2. change angle given by their ~~com~~ suplimentary angle's counter part,

   i.e, $\alpha = \pi$ - given angle (in radians)

3. Calculated polar_theta by using equation (2)
4. Calculated polar_r by using equation (1) at each given time instance.

(5) for curve fitting, defined a function 'r'

that takes in theta (variable), $\underbrace{e, r_0, \theta_0}_{\text{constants}}$.

and returns a value (of r)

in the form, $$\frac{r_0}{1+e\cos(\theta-\theta_0)}.$$

(6) used scipy.optimize's curve fitting function to find the parameters e, r_0 and θ_0.

$e \rightarrow$ eccentricity

$r_0 \rightarrow a(1-e^2)$, $a \rightarrow$ semi major axis

saved the three parameters in popt.

So, e = popt[0]

r_0 = popt[1]

θ_0 = popt[2]

(7) using Kepler's third law (for finding time period):

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2} \approx 7.496\times10^{-6}\left(\frac{AU^3}{days^2}\right)$$

$$\approx 0.9986546\left(\frac{AU^3}{year^2}\right)$$

to find a; used e and r_0 as;

$$\boxed{a = \frac{r_0}{1-e^2}}$$

found T as, $$T = \sqrt{\frac{4\pi^2}{GM}a^3}$$

(5) used

matplotlib.pyplot.figure().add_subplot(projection='polar')
.scatter

to plot scattergraph of comet

results

$$\boxed{\begin{aligned} \text{eccentricity} &= 0.257811 \\ \text{semimajor axis } (a) &= 11.0986692 \text{ AU} \\ \text{Timeperiod} &= 36.99974 \text{ years} \approx 37 \text{ years} \end{aligned}}$$