D2014 5.11.20 G1

Thursday, November 5, 2020 1:54 PM

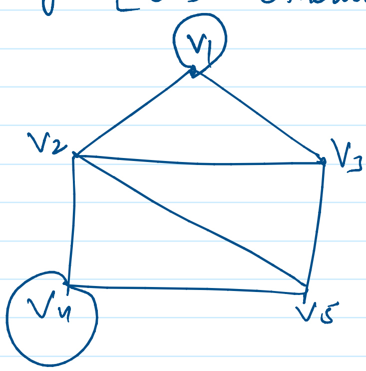
# Matrix Representation of Graph

① Adjacency Matrix ② Incidence Matrix

① **Adjacency Matrix** :- G be a graph (undirected) with n-vertices. then Adjacency Matrix of G is a matrix of order $n \times n$ and defined as $A = [a_{ij}]_{n \times n}$ where

$$a_{ij} = \begin{cases} 1 \, ; & \text{if there is an edge b/w the vertices } v_i \text{ \& } v_j \\ 0 \, ; & \text{otherwise} \end{cases}$$

Ex

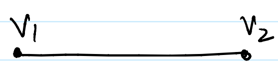




No. of vertices = 5

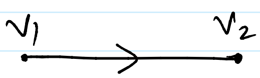
| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 1 | 1 | 1 | 0 | 0 |
| $V_2$ | 1 | 0 | 1 | 1 | 1 |
| $V_3$ | 1 | 1 | 0 | 0 | 1 |
| $V_4$ | 0 | 1 | 0 | 1 | 1 |
| $V_5$ | 0 | 1 | 1 | 1 | 0 |

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}_{5 \times 5}$$

② If G is Directed




| | $V_1$ | $V_2$ |
|---|---|---|
| $V_1$ | | 1 |
| $V_2$ | 1 | |



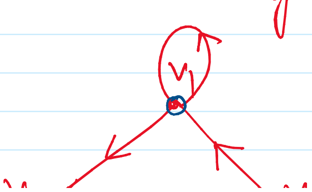

from $V_1$ to $V_2$

| | $V_1$ | $V_2$ |
|---|---|---|
| $V_1$ | | 1 |
| $V_2$ | 0 | |

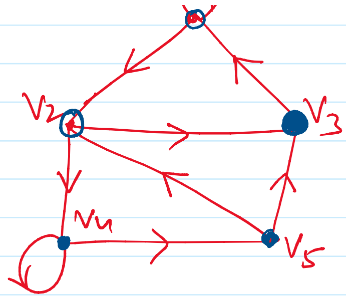

from $V_2 \to V_1$

| | $V_1$ | $V_2$ |
|---|---|---|
| $V_1$ | | 0 |
| $V_2$ | 1 | |

$$a_{ij} = \begin{cases} 1 \, ; & \text{if there is an Edge from } v_i \text{ to } v_j \\ 0 \, ; & \text{otherwise} \end{cases}$$



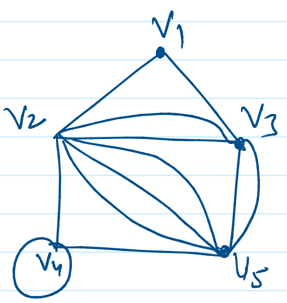

| | $V_1$ | $V_2$ | $V_3$ | $V_4$ | $V_5$ |
|---|---|---|---|---|---|
| $V_1$ | 1 | 1 | 0 | 0 | 0 |




| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| $v_1$ | 1 | 1 | 0 | 0 | 0 |
| $v_2$ | 0 | 0 | 1 | 1 | 0 |
| $v_3$ | 1 | 0 | 0 | 0 | 0 |
| $v_4$ | 0 | 0 | 0 | 1 | 1 |
| $v_5$ | 0 | 1 | 1 | 0 | 0 |

③ if G is Multigraph (has parallel Edges)

$$a_{ij} = \begin{cases} m \ ; & m \text{ is the number of edges b/w the Vertices } v_i \,\&\, v_j \\ 0 \ ; & \text{otherwise} \end{cases}$$




| | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| $v_1$ | 0 | 1 | 1 | 0 | 0 |
| $v_2$ | 1 | 0 | 2 | 1 | 3 |
| $v_3$ | 1 | 2 | 0 | 0 | 2 |
| $v_4$ | 0 | 1 | 0 | 1 | 1 |
| $v_5$ | 0 | 3 | 2 | 1 | 0 |

$\left[ \quad \right]_{5\times 5}$

② Incidence Matrix: Let G be a graph having m-Vertices and n-edges then the Incidence Matrix is a $m \times n$ matrix defined as

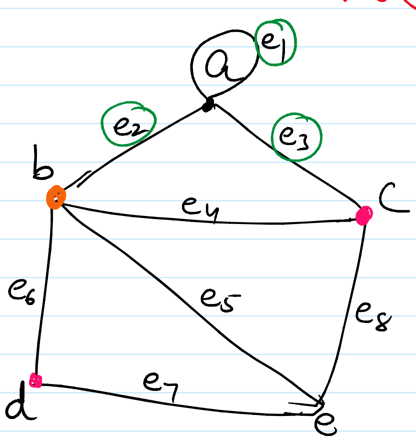
$$[a_{ij}]_{m\times n}$$

$$a_{ij} = \begin{cases} 1 \ ; & \text{if } j^{th} \text{ edge is Incident on } i^{th} \text{ Vertex ie } v_i \\ 0 \ ; & \text{otherwise} \end{cases}$$

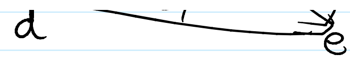
$$I = [a_{ij}]_{m\times n}$$

$m$ = No. of Rows = No. of Vertices

$n$ = No. of Column = No. of edges




| Vertices \ Edges | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $e_7$ | $e_8$ | Total |
|---|---|---|---|---|---|---|---|---|---|
| a | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 3 |
| b | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 4 |
| c | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 3 |
| d | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 2 |
| e | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 3 |




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| e | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 3 |
| Total | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | |

Note 1 if G is Simple Row Sum (Total) = degree of the Vertex

Monday — 9/11/20

CA — 3

Unit — ~~4~~, 5, 6 upto Today Class