GRAPH THEORY

GRAPH:-

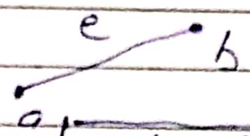
A graph 'G' is a diagram consisting of ~~two thid~~ a collection of words is together with edges joining certain pair of these wordes and it is generally denoted by

$$G = \left(V(G), E(G) \right)$$

where Set $V(G)$ set of Vertices, points or nodes and $E(G)$ set of Edges.

Each edge has two vertices associated with it called end points. and edge is said to connect its end point. i.e

each element e of $E(G)$ is assigned an unordered pair of vertices (a,b) called the end point of edge 'e'.



$\boxed{e=(a,b)}$

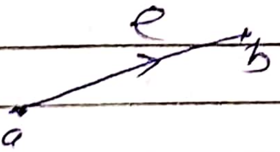
NOTE: $E(G)$ is subset of $V(G) \times V(G)$
$\subset$

i.e a relation on $V(G)$.

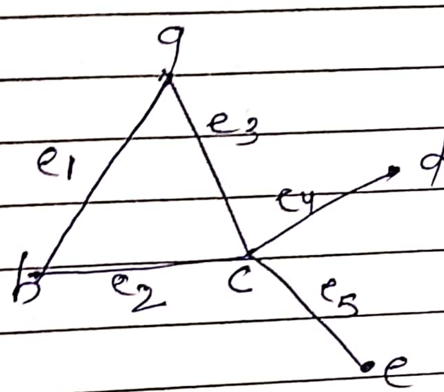
Directed Graph

It is a graph in which each element of 'e' of $E(v)$ is assigned an Ordered pair of vertices (a,b) along with '$\longrightarrow$' starting from a to b.

e of $E(v)$
(a,b)

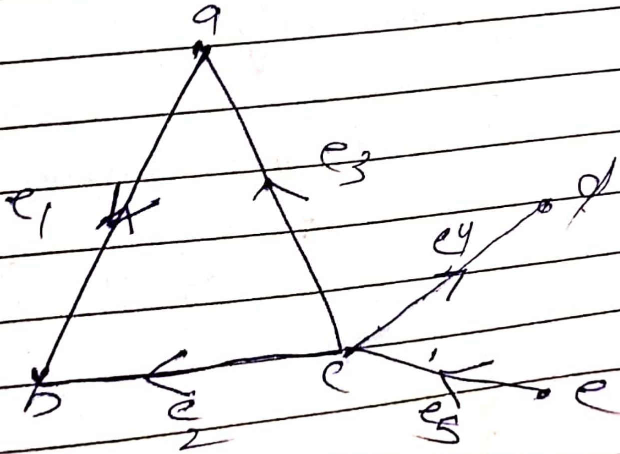


a is called initial vertex.
b is " Terminal " of the edge 'e'.



Undirected graph
$e_1 = (a,b)$ or (b,a)
e_1 is an edge b/w a, b

Directed graph
$e_1 = (a,b)$
e_1 is an edge from a to b.



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NOTE :(1) A graph is represent by mean of a diagram in which a vertices are denoted by points and edges are represent by line segment joining its end vertices and it does not matter the join of two vertices or points in a graph is a straight line or a curves longer or shorter.

Adjacent vertices :

Two vertices a & b of a graph 'G(V,E) are said to be adjacent if there is an edge "e" b/w them.

i.e $e = (a,b)$

The edge 'e' is called incident on each vertex (a,b)

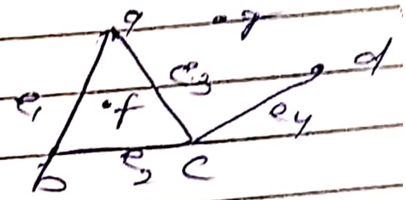
Loop :-

An edge i.e incident from and into itself i.e start and end at the same vertex is called a loop.

$$\boxed{e = (a,a)}$$

Isolate vertex :-

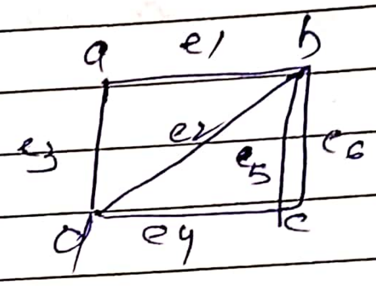
A vertex of graph G which is not connected any other vertex is called an isolate vertex.



The vertex 'f', 'g' are isolate vertex.

Parrallel Edges :-

if two or more edges of graph 'G' have the same end vertices is called parallel edges.



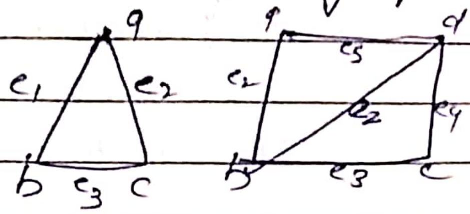
e_5 & e_6 are Parallel Edges

$e_5 = (b, c) = e_6$

Types of Graph

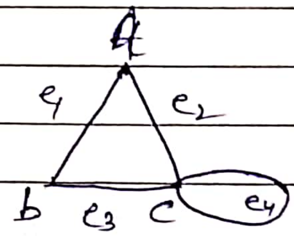
1. Simple Graph :

A graph which has neither loop nor parallel edges is called simple graph.



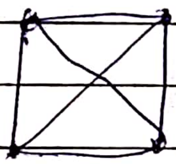
2. Multi Graph / General Graph :

A graph which have either loop or parallel edges or both is called a multi-graph or general graph.

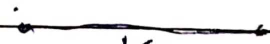


3. Complete Graph :-

A simple graph in which there is an edge between every pair of vertices is called a complete graph, and generally denoted by 'K_n'.
n = No. of vertices


K_4


K_3


K_2

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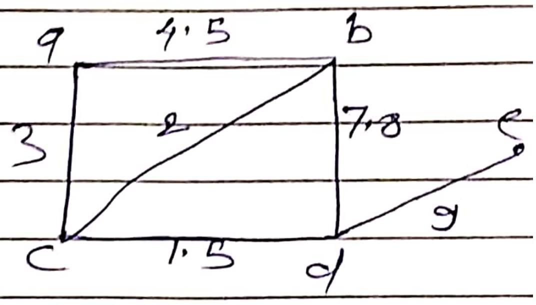
NOTE :- No. of eadges in a complete graph with n - vertices (K_n)

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

Waighted Graph :-

$G(V,E)$ a graph.

if each edge of G is assigned a no. called waight of the edge, G is called a waighted graph.



Finite - infinite graph

a graph 'G' is called finite graph if E is finite set.
V is fintie set.

V is infinite set is called infinite graph.

Order of a graph :-

The no. of vertices in $V(G)$ is called the order of graph G is denoted by.

$|V(G)|$ or mod of $V(G)$

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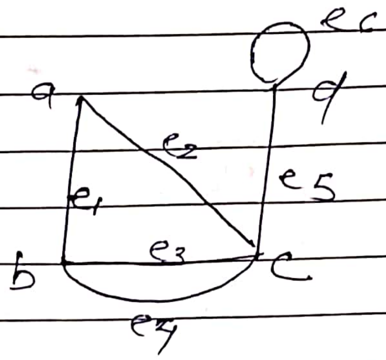
Handshaking Theorem
(1st theorem of graph theory)

The sum of degree of all the Vertices in a graph 'G' is equal to twice the no of edges.



$d(a) = 2$ e_1, e_2
$d(b) = 3$ $e_1 . e_3 . e_4$
$d(c) = 4$ $e_2 . e_3 \; e_4 \; e_5$
$d(d) = 3$ $e_5 \; e_6 \; e_6$

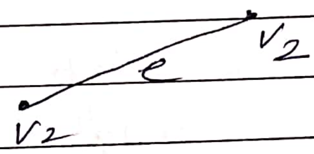
12

$= 2 \times 6$

Prove

let V_1, V_2, V_3 —— V_n vertices of graph 'G'

let e_1 be an edge b/w the Vertices V_1 and V_2



Now

when we count the degree of vertices the edge e_1 counted two times /twice once in the degree of V_1 & again in the degree of V_2.

Also if V_1, V_2 are identical Again e_1 is counted twice

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Hence, in graph 'G' every edge is counted twice
So the sum of degree of all the vertices is twice the no of eadges.

i.e $\boxed{\sum_{i=1}^{n} d(V_i) = 2e}$ = even no.

e = Total no. of Edge in G.

NOTE :- Show that the sum of degree of all the vertices in graph 'G' is always even.

$$\sum_{i=1}^{n} d(V_i) = 2e = \text{Even number}$$

→ Prove that in graph 'G' the odd degree vertex are always even in number.

let V_1, V_2, V_3 ———— $V(n)$ vertices and, e be the no. of eadges in Graph 'G'.

∴ By HS theorem

$$\sum_{i=1}^{n} d(v_i) = 2e \text{ ———— } (1)$$

⇒

$$\sum_{\text{Even}} d(V) + \sum_{\text{Odd}} d(V) = 2e$$

$2+3+4+6+1+5+1+2 = 2e$
$(2+4+6+2)+(3+1+5+1) = 2e$

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$\Rightarrow$ Even no. $+ \sum_{odd} d(v) =$ Even no.

$\Rightarrow \sum_{odd} d(v) =$ Even no $-$ Even no

$\Rightarrow \sum_{odd} d(v) =$ Even no.

i.e the sum of degree of all Vertices having odd degree is even $\Rightarrow$ ~~sum of the~~ odd degree of Vertices are even in no.

Hence the theorem pd

SIMPLE GRAPH

$\rightarrow$ Show that maximum no. of edges in simple Graph with n vertices is $\frac{n(n-1)}{2}$.

let V_1, V_2, V_3 ———— V_n - Vertices and e be the no. of edges in simple graph G.

$$\sum_{i=1}^{n} d(V_i) = 2e \quad \text{—— (1)}$$

$\because$ G is simple Graph

$\therefore$ the maximum degree of any Vertex $= (n-1)$

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The sum of Max^m degree of all the vertices

$= (n-1)+(n-1)+(n-1)+ \text{———— } n \text{ times}$

$\Rightarrow n(n-1)$ —— (2)

Eqn (1) & (2)

$$2e = n(n-1)$$

$$\Rightarrow e = \frac{n(n-1)}{2}$$

$\therefore$ Max^m no. of edges in graph $G = \frac{n(n-1)}{2}$

→

COMPLETE GRAPH

Prove that The no. edges in complete graph with n vertices is $\frac{n(n-1)}{2}$.

Let V_1, V_2, V_3 ———— $V(n)$ vertices and e be the no. of edges in complete graph.

$$\sum_{i=1}^{n} d(v_i) = 2e \text{ —— (1)}$$

$\because$ G is complete graph

$\therefore$ the degree of any vertex is $= (n-1)$

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The sum of degree of all the vertices

$= (n-1) + (n-1) + (n-1) + \text{———— } n \text{ times}$

$\Rightarrow n(n-1)$ —— (2)

from eqn (1) & (2)

$2e = n(n-1)$

$\Rightarrow e = \frac{n(n-1)}{2}$

$\therefore$ no. of eadges in Graph 'G' is $\frac{n(n-1)}{2}$.

Q:- A graph G has 21 eadges three vertices of degree four and all other vertices of degree 3. find the vertices of G.

Soln:- No. of eadges = 21
let no. of vertices = n

By handshaking,

$\sum d(v) = 2e$

$\Rightarrow 4+4+4+(3+3+ \text{——} (n-3) \text{ times}) = 2 \times 21$

$\Rightarrow 3 \times 4 + 3(n-3) = 42$

$3(n-3) = 30$

$n-3 = 10$

$\boxed{n = 13}$

$\Rightarrow 12 + 3n - 9 = 42$

$\Rightarrow 3 + 3n = 45$

$n =$

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→ prove that there does not exist a graph with 5 vertices of degree 1, 3, 4, 2, 3

$$\Sigma d(v) = 2e$$
$$1+3+4+2+3 = 2e$$
$$13 = 2e$$
$$\boxed{e = \frac{13}{2}} \notin N$$

* Find k if k-regular graph with 8 vertices and has 12 eadges and also draw the graph.

$$\Sigma d(v) = 2e$$
$$8k = 2 \times 12$$
$$k = \frac{24}{8}$$
$$\boxed{k = 3}$$

Isomorphic graph

let $G(V,E)$ & $G'(V',E')$ be two graphs then G is isomorphic to G' written as

$\boxed{G \cong G'}$ if there exist a bijection f from V to V' such that

$$(v_i, v_j) \in E \text{ iff } (f(v_i), f(v_j)) \in E'$$

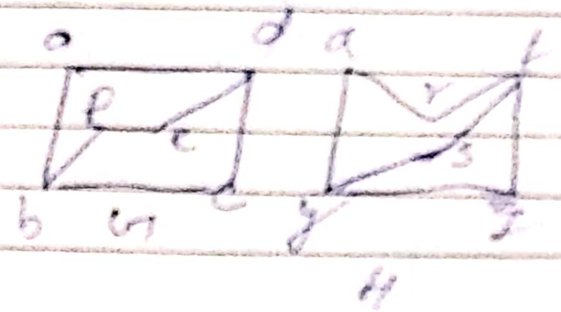
i.e. two graph are isomorphic to each other if there exist a one-one correspondence b/w there vertices and edges such that incidence relationship is preserved.

NOTE: Two graph which are isomorphic to each other have

(i) Same no. of vertices
(ii) Same no of eadges
(iii) Same degree sequence but the converse need not be true.

Q. Whether the graph 'G' and 'H' are isomorphic or not.

Sol.



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Isomorphic graph

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MCQ NOTE: Two graph which are isomorphic to each other have

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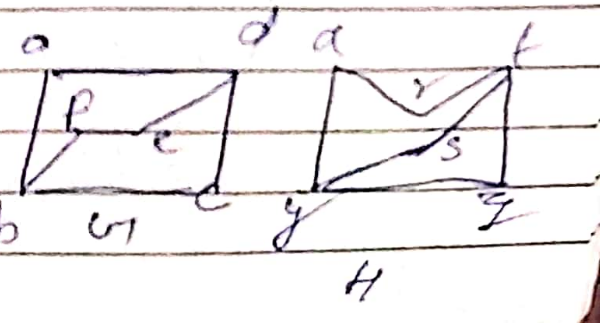


Soln

	G	H
Vertices	6	6
Edges	7	7
Sequence	2,2,2,2,3,3 a,c,f,e,b,d	2,2,2,2,3,3 x,r,s,z,y,t

$\because$ G, H have same vertices, eadges and degree sequence then may or may not be isomorphic to each other.

→ Define a mapping f from $V(G)$ to $V(H)$ as

$f(a) = s$
$f(b) = y$
$f(c) = z$
$f(d) = t$
$f(e) = r$
$f(f) = x$

To prove : f is One One & Onto.

For this we will draw the adjancy matrix for G & H.

	a	b	c	d	e	f
a	0	1	0	1	0	0
b	1	0	1	0	0	1
c	0	1	0	1	0	0
d	1	0	1	0	1	0
e	0	0	0	1	0	1
f	0	1	0	0	1	0

$$A_G = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

∴	s	y	z	t	r	x
$f(a)=s$	0	1	0	1	0	0
$f(b)=y$	1	0	1	0	0	1
$f(c)=z$	0	1	0	1	0	0
$f(d)=t$	1	0	1	0	1	0
$f(e)=r$	0	0	0	1	0	1
$f(f)=x$	0	1	0	0	1	0

$$A_H = \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}_{6\times 6}$$

$\therefore A_G = A_H$

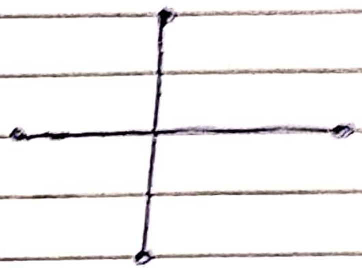
It follows that 'f' preserve edges (incident Relationship)

$\therefore \underline{G \cong H}$

Homeomorphic Graph

let 'G' be any graph Obtain a new graph from 'G' by dividing the edges of G with additional vertices then the new graph is G is called Homeomorphic to each other.

graph b is Homeomorphic to a.



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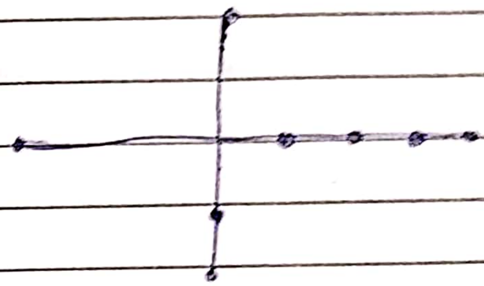

(b)

Subgraph.

let 'G' 'H' be two graphs such that G(V,E), H(V,E)
the ~~vertex~~
$$V(H) \subseteq V(G) \text{ and } E(H) \subseteq E(G)$$

H is called Subgraph of G.

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NOTE :– (I) if $V(H) \subseteq V(G)$ & $E(H) \subset E(G)$

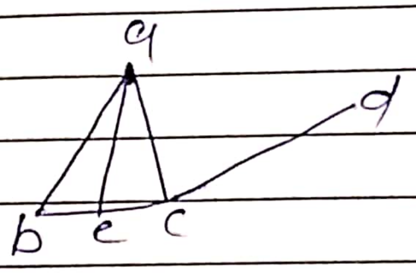
H is proper subgraph of G.

(II) if $V(H) = V(G)$ and $E(H) \subset E(G)$

H is spanning sub graph of G.

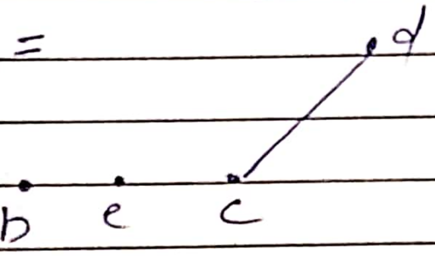
$G - v$

$G-v$ is a subgraph of G obtain by deleting the vertex 'v' from the vertex set of 'G' and also the edges which are incident on vertex v.

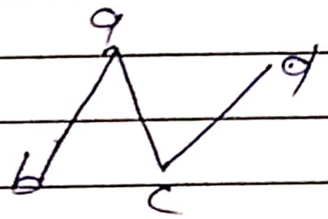


(G)

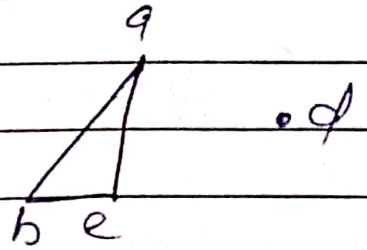
• $G - a$ =



• $G - e$ =



• $G - c$ =



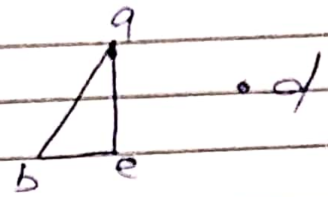
→ <u>Cut Vertex</u> :-

a vertex v is said to be cut vertex if $(G-v)$ is disconnected graph.

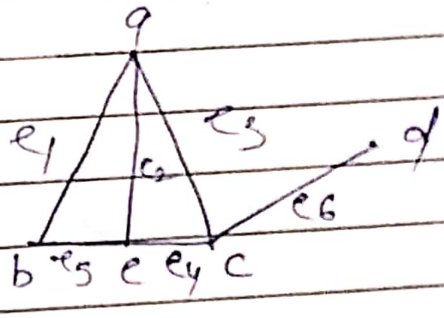
$(G-v$ is divided into parts$)$

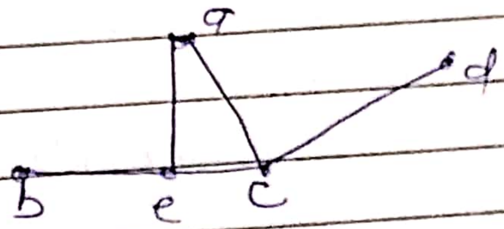
'c' is cut vertex.
$G-c$ is disconnected.

<u>$G-c$</u>

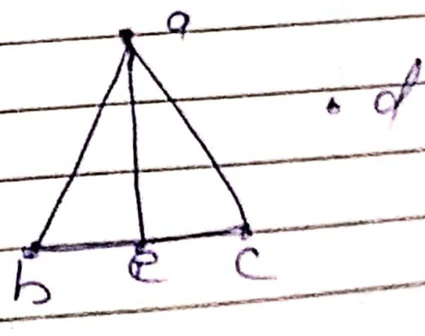


→ $G-e$:- is a sub graph of G obtain by simply deleted eadge 'e' from graph 'G'.



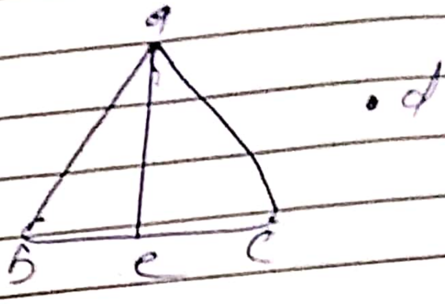


<u>$G-e_6$:-</u>



Cut edge :- an edge 'e' is called cut edge if 'G-e' is called cut edge.

ec is cut edge.



Operation of Graph

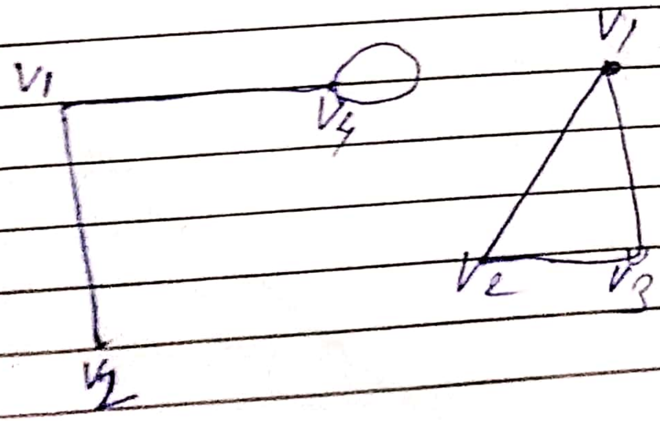
(I) union :- let $G_1(V,E)$ & $G_2(V,E)$ be two graph then

(1) The union of G_1, G_2 denoted by

$$G_1 \cup G_2 = G(V(G_1) \cup V(G_2), E(G_1) \cup E(G_2))$$

(2) Intersection :-

$$G_1 \cap G_2 = G(V(G_1) \cap V(G_2), E(G_1) \cap E(G_2))$$



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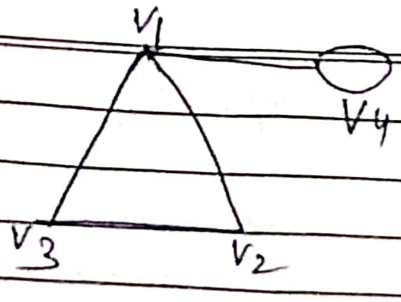
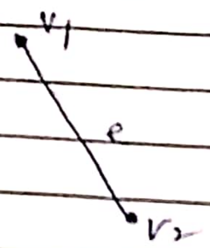


$G_1 \cup G_2$

(2).

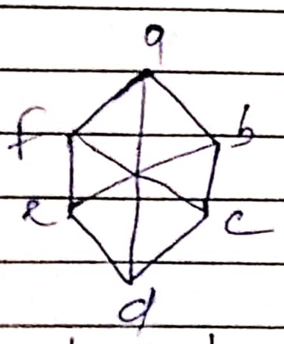


$G_1 \cap G_2$

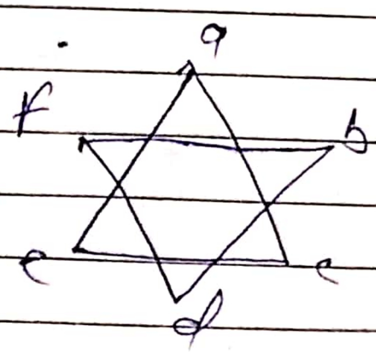
(3) Complement of a graph :-

A complement of a G is denoted by $\overline{G}$ and it is defined as a simple graph with the vertex set same as that of G. and with the eadge set is satisfying. ~~[illegible]~~ the property there is an eadge b/w the vertices in $\overline{G}$

if or when there is no eadge b/w these vertices G.



($\overline{G}$)



(G)

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NOTE :- (I) $G \cup \overline{G}$ = Complete graph

(II) if the degree of a vertex v in a Simple graph G having n - vertices is k. then the degree of same vertex in $\boxed{\overline{G} \text{ is } n-k-1}$

Imp.
Q- Can a graph with 7 vertices be isomorphic to its Complement. justify your answer.

Sol^n let G be the given graph and $\overline{G}$ its complement.

Now
we know if $e \in E(G)$ the $e \notin E(\overline{G})$

So
the total no. of eadges in G and $\overline{G}$ = the max^m no. of possible eadges in Complete graph

$= \frac{n(n-1)}{2}$

$= \frac{7(7-1)}{2}$ $\left\{ \because n = \text{no. of vertices} = 7 \right\}$

$= 7 \times 3$

$= 21$

$\therefore$ total no. of eadges in G & $\overline{G} = 21$

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Which means

$\Rightarrow$ no. of eadges $G \neq$ No of eadge in $\bar{G}$

($\because$ 21 is odd, so we can not divide in equal parts)

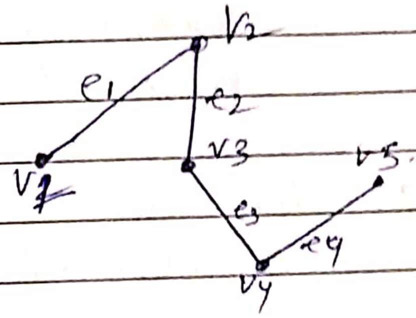
So G & $\bar{G}$ is not isomorphic.

WALK

A walk in a graph G is a finite sequence

$$w = v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4 \text{ ------ } v_n e_n$$

a finite sequence who terms are alternating vertices and eadges.



A v_1 is initial and v_n is called terminal. and all other $v_2, v_3, v_4, \text{---} v_{n-1}$ is called internal vertices.

NOTE: in a walk each eadge is appear once However vertex may repeat.

$\rightarrow$ Open walk & closed walk :-

v_1 & v_n are diff.

v_1 & v_n are identical.

Path

An open walk in which each vertex appear once is called a path.

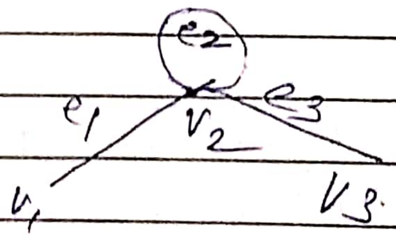
→ length of path :-

The no. of eadges appearing in the Sequence of path is called length. How many eadges is there is called length.

Note :

(I) An eadge which is not a self loop is a path of length one

(II) A loop can be included in a walk but not in path.



Walk

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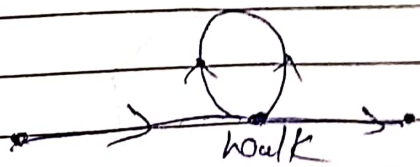
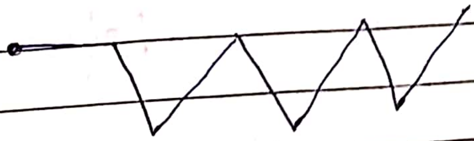
(iii) The terminal vertices of a path are of degree one and all internal vertices of degree two.





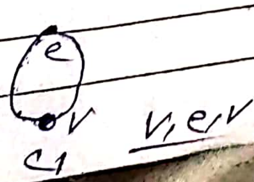
CIRCUIT (closed Path)

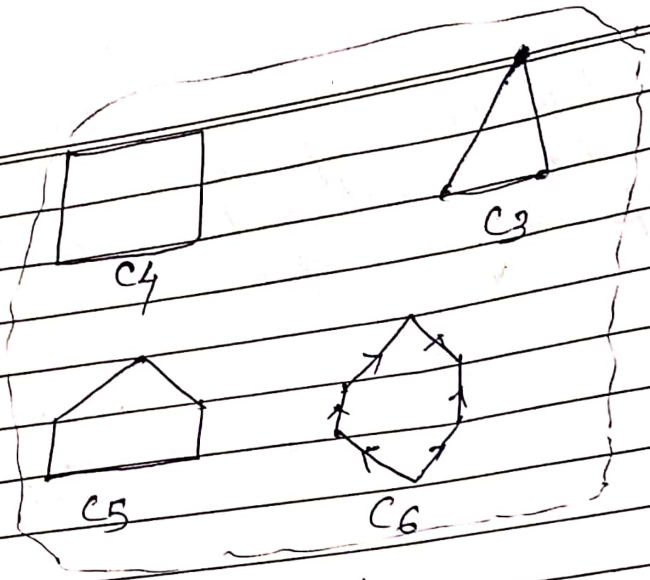
A circuit is a closed path. It is also called cycle, elementary cycle, circular path, polygon.

- K-cycle :- cycle with k-edges (length k) denoted by C_k or C_n.

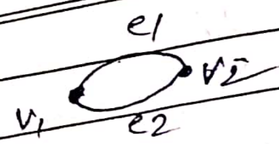
NOTE: (i) A self loop is also a cycle but converse is not true.

(ii) The degree of every vertex in a cycle is 2.





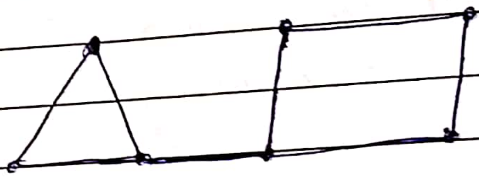
Polygons



$v_1 e_2 v_2 e_1 v_1$

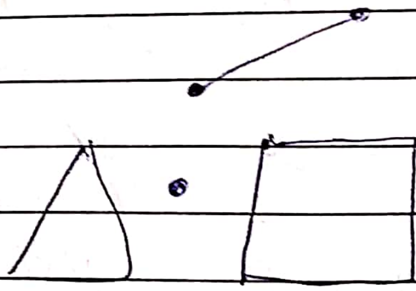
→ Connected Graph :-

A graph G is said to be connected if there is atleast one path b/w every pair of vertices. is reachable.



→ Disconnected Graph

A graph G is said to be disconnect if there is no path b/w vertices is not reachable.



→ Component :-

Each connected subgraph of a disconnected graph is called its component.

→ 3 components.

NOTE :-
No. of component of a connected graph = 1 component that is graph itself.

Matrix Representation of a Graph

Adjency Matrix — Incident Matrix

Adjency Matrix → Directed, undirected

① Adjancy Matrix For undirected Graph

G be an undirected graph with n vertices has no vertices then Graph G represented as $n \times n$ vertices defined as

$$A_G = [a_{ij}]_{n \times n}$$

$$a_{ij} = \begin{cases} k = \text{no of edges b/w } a_i \text{ \& } a_j \\ 0 \text{ if no edge is b/w } a_i \text{ \& } a_j \end{cases}$$

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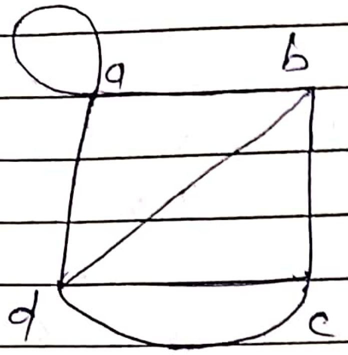
Note :- Order of adjancy Matrix is $n \times n$ (n = no. of vertices)

eg:- No. of vertices = 4
order of A_G = 4×4



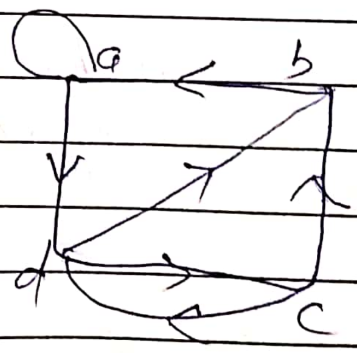
$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 1 \\ c & 0 & 1 & 0 & 2 \\ d & 1 & 1 & 2 & 0 \end{array}$$

$$A_G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 2 \\ 1 & 1 & 2 & 0 \end{bmatrix}_{4 \times 4}$$

(II) **A_G For Directed Graph :-**

$$A_G = [a_{ij}]_{n \times n}$$

$$a_{ij} = \begin{cases} k & : k \text{ is no. of eadges from vertex } a_i \text{ to } a_j \\ 0 & : \text{No eadge} \end{cases}$$



$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 1 \\ b & 1 & 0 & 0 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 0 & 1 & 1 & 0 \end{array}$$

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Incidence Matrix :-

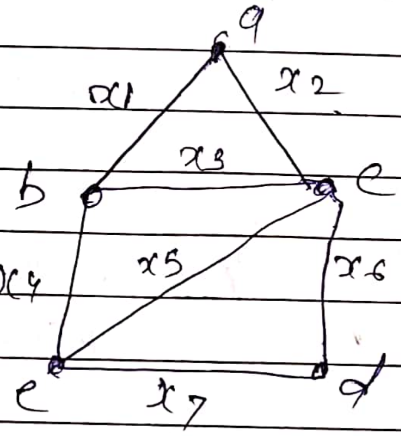
Let G be a graph having 'm' vertices and 'n' eadges then incidence matrix of graph G is a matrix of order $m \times n$.

$$M = [a_{ij}]_{m \times n}$$

and defined as 'a_{ij}' =
- 1 : if jth edge is incidence on ith vertex.
- 0 : No connection (otherwise)

Ex - No. of vertices = 5
" " eadges = 7

Order of incidence Matrix = 5×7



	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Row totel
a	1	1	0	0	0	0	0	2
b	1	0	1	1	0	0	0	3
c	0	1	1	0	1	1	0	4
d	0	0	0	0	0	1	1	2
e	0	0	0	1	1	0	1	3
	2	2	2	2	2	2	2	

∵ Every eadge connected with 2 vertex.

The row of total give the degree of correspon matrix.

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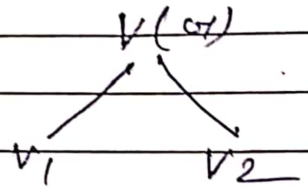
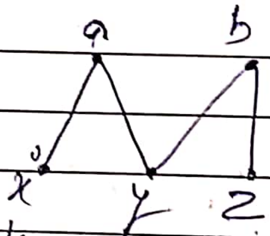
Bipartite Graph

let 'G' be a graph. if the vertex set of 'G' can be partition into two disjoint set V_1 & V_2 such that every edge in G join a vertex V_1 to a vertex in V_2. then 'G' is called a Bipartite graph.



$V_1 \cup V_2 = V$
$V_1 \cap V_2 = \phi$

$\forall e = (v_1, v_2) \in E(G)$
when $v_1 \in V_1$
$v_2 \in V_2$



$V_1 = \{a, b\}$
$V_2 = \{x, y, z\}$

Complete Bipartite Graph :-

A graph is said to be complete if every vertex of V_1 is connected with every vertex of V_2.
It is denoted by $K_{m,n}$.

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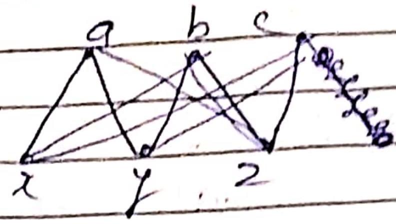
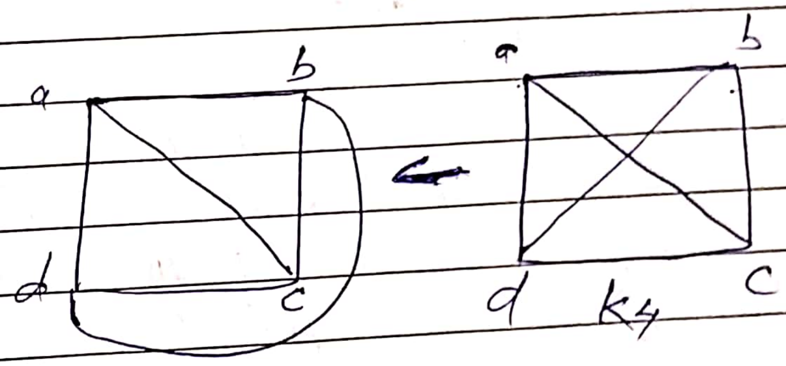
$K_{3,4}$

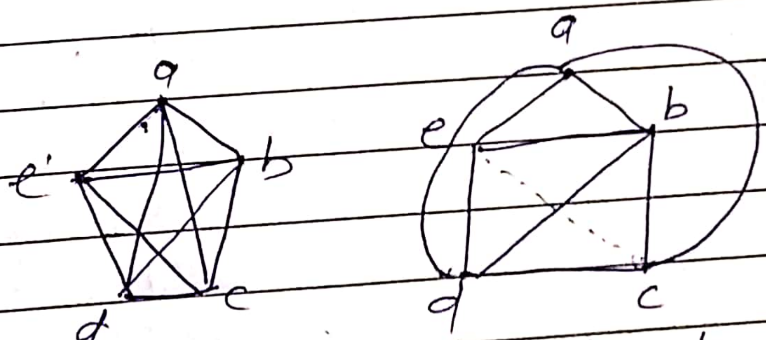


→ **Planar Graph :-**

It is a graph drawn in a plane in such a way that no two eadge cross each other.



K_4 is planer graph.



K_5 is not planer graph

note :- $K_{3,3}$ - Non-planar graph

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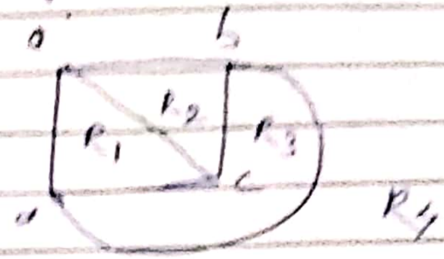
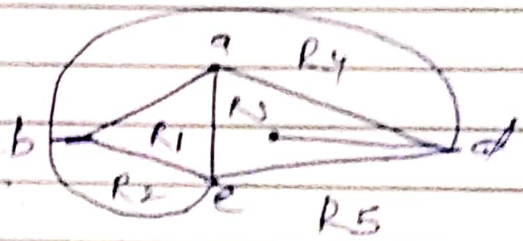
→ **Region**

A planar graph partition the plane into several portion called region.



degree of Face :-

If 'G' be a graph and 'g' be its face then the no. of edges in the boundary of G with cut edge counting twice is defined as the degree of the face.



No. of Region = 5

$d(R_1) = 3$

$d(R_2) = 2$

$d(R_3) = 5$ (de is cut edge counted twice)

$d(R_4) = 3$

$d(R_5) = 3$

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Euler Theorem

let 'G' be a planer graph having 'r' region 'v' no of vertices and 'e' no. of eadges in the planer represention of graph 'G' then

$$\boxed{r = e - v + 2}$$

in previous picture :

$v = 5$

$e = 8$

$r = e - v + 2$

$r = 8 - 5 + 2$

$$\boxed{r = 5}$$

We prove the result by using Mathematical induction on no. of regions :

First we show that the result is true for $r = 1$!-

let 'G' be a planer graph having 4 vertex, $e = 3$ and $r = 1$

$r = e - v + 2$

$= 3 - 4 + 2$

$$\boxed{r = 1}$$

∴ Result is true for $r = 1$

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let's assume the result is true for $r = k$

$\therefore$ $e - v + 2 = k$ ——— ①

let 'G' be a connected planer graph having $(k+1)$ regions
To remove an eadge from the boundary of two region we obtain a new graph 'G'' having 'k' regions.

Now,

$v' = V(G') = V(G) = v$

$e' = E(G') = E(G-1) = e-1$

$r' =$ no of region in $G' = k$

from ① the result is true for $r = k$ (k-regions)

$\therefore$ we have $e' - v' + 2 = r'$

$\Rightarrow (e-1) - v + 2 = k$

$\Rightarrow e - v + 2 = k+1$

$\Rightarrow e - v + 2 = r$

$\therefore$ The result is true for $r = (k+1)$

$\therefore$ By Mathematical induction the theorem is proved.

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if G is a simple connected planer graph with e edge and v vertices and r regions then prove the following inequalities.

(i) $2e \geq 3r$
(ii) $e \leq 3v-6$
(iii) $e \leq 2v-4$ if G has no cycle of length 3.
(iv) There ~~does not~~ exist a Vertex V in G such that
$$\deg(V) \leq 5.$$
then G has a vertex of degree not exceeding 5.

prove :-

(1) $2e \geq 3r$

Since G is a simple connected planer graph so G has more than one edge.

Because G is planer graph, divide the plane into r regions.

The degree of each region is atleast three.
$\therefore \boxed{\deg(R) \geq 3}$

We know the sum of degree of all the region is twice of the no. of edges.

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$\therefore \sum d(R_i) = 2e$

$\Rightarrow 2e = \sum d(R_i)$

$\Rightarrow 2e = \sum_{i=1}^{r} d(R_i)$

$\Rightarrow d(R_1) + d(R_2) + \cdots d(R_r)$

$\geq 3+3+3+3+ \cdots r$ terms

$= 3r$

$\therefore \boxed{2e \geq 3r}$ pd

$\boxed{e \geq \frac{3}{2} r}$ $\boxed{r \leq \frac{2}{3} e}$

(ii) $e \leq 3v - 6$

For a simple connected planer graph 'G' we have

$2e \geq 3r$

$\Rightarrow r \leq \frac{2}{3} e$ — (1)

By Eular theorem we have,

$e - v + 2 = r$

$e - v + 2 = r \leq \frac{2}{3} e$ (By (1))

$\Rightarrow e - v + 2 \leq \frac{2}{3} e$

$\Rightarrow 3e - 2e \leq 3v - 6$

$\Rightarrow \boxed{e \leq 3v - 6}$

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(iii) $e \leq 2v-4$

$\deg(R_i) \geq 4$

$2e \geq 4r$

$e \geq 2r$

$\boxed{r \leq \frac{1}{2} e}$

$e-v+2 = r$

$\Rightarrow e-v+2 = r \leq \frac{1}{2} e$

$e-v+2 \leq \frac{1}{2} e$

$2e-2v+4 \leq e$

$2e-e \leq 2v-4$

$\boxed{e \leq 2v-4}$

Since G is simple connected planer graph having r region and no cycle of degree 3.

$\therefore$ Atleast 4 eadges are required for a region.

$\deg(R_i) \geq 4$

We know the sum of degree of all the region is twice the no of

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edges
$$\sum_{i=1}^{r} d(R_i) = 2e$$

$$\Rightarrow 2e = \sum_{i=1}^{r} d(R_i)$$

$$\Rightarrow d(R_1) + d(R_2) \text{ ——— } d(R_r)$$
$$\Rightarrow \geq 4+4+4 \text{ ——— r times}$$

$$\Rightarrow 4r$$

$$\therefore \boxed{2e \geq 4r}.$$

By Euler Theorem,

$$e - v + 2 = r$$
$$e - v + 2 = r \leq \frac{1}{2} e$$
$$e - v + 2 \leq \frac{1}{2} e$$
$$2e - 2v + 4 \leq e$$
$$2e - e \leq 2v - 4$$
$$\boxed{e \leq 2v - 4}$$

(iv) $deg(v) \leq 5$

let degree of each vertex in graph (v_i) is atleast 6 or more.
ie,
$$d(v_i) \geq 6$$

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By handshaking Theorem,

$$\sum d(v_i) = 2e$$

$\Rightarrow 2e = \sum d(v_i)$

$\Rightarrow 2e \geq 6v$

$\Rightarrow e \geq 3v$

$\Rightarrow \boxed{v \leq \frac{1}{3}e}$

also we have,

$$2e \geq 3r$$

$$\boxed{r \leq \frac{2}{3}e}$$

By Eular Theorem,

$e - v + 2 = r$

$e - v + 2 = \frac{2}{3}e$

$e - \frac{2}{3}e + 2 = v$

$\frac{3e - 2e}{3} + 6 = v$

$\frac{e + 6}{3} = v$

$e + 6 = 3v$

$\Rightarrow$

$e - v + 2 = r$

$\Rightarrow e = r + v - 2$

$\Rightarrow e \leq \frac{2}{3}e + \frac{1}{3}e - 2$

$\Rightarrow e \leq e - 2$

which is not possible.

$\therefore$ Our supposition is wrong.

$\therefore \deg(v_i) \not\geq 6$

$\Rightarrow \deg(v_i) < 6$

$\Rightarrow \deg(v_i) \leq 5$

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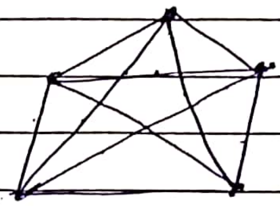
Q: Prove that ~~K~~ K_5 is a non-planer graph.



No. of vertices = 5
edges = 10

let K_5 is a planer graph

$\therefore\ e \leq 3v - 6$

$10 \leq 3 \cdot 5 - 6$

$\Rightarrow 10 \leq 9$

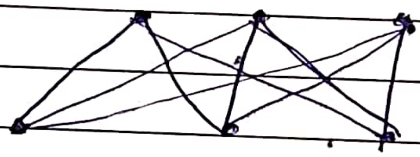
$\therefore$ Not possible

$\therefore$ our supposition is wrong

$\therefore$ K_5 is non planer graph.

Q: $K_{3,3}$ is Non Planer graph

$v = 6$
$e = 9$



let $K_{3,3}$ is planer graph

$\because$ It has no cycle of length three

$\therefore\ e \leq 2v - 4$

$\Rightarrow 9 \leq 2(6) - 4$

$\Rightarrow 9 \leq 8$

$\therefore$ $K_{3,3}$ is not planer graph.

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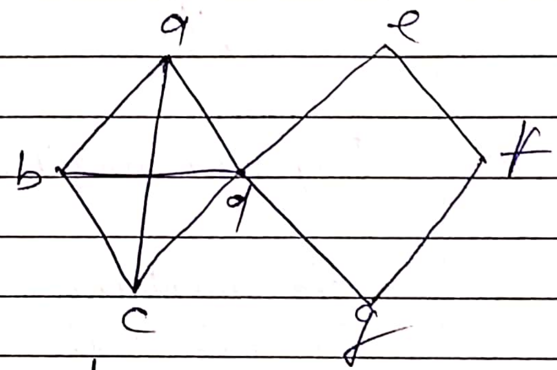
Distance and Diameter

consider a connected graph 'G' distance b/w vertices u & v in graph 'G' is denoted by $d(u,v)$, is the length of the shortest path b/w u & v.

if diameter of graph 'G' is denoted by $dia(G)$ or $d(G)$ or $dim(G)$ is the max^m distance b/w any two vertices of G.

Q:- Find the diameter of the graph?



	a	b	c	d	e	f	g	max^m dist
a	0	1	1	1	2	3	2	3
b	1	0	1	1	2	3	2	3
c	1	1	0	1	2	3	2	3
d	1	1	1	0	1	2	1	2
e	2	2	2	1	0	1	2	2
f	3	3	3	2	1	0	1	3
g	2	2	2	1	2	1	0	2

$\boxed{dia(G) = 3}$

Graph Coloring

It is the process in which we paint the vertices of a graph in such a way that no ~~two~~ adjacent vertex have same color.

Chromatic Number :-

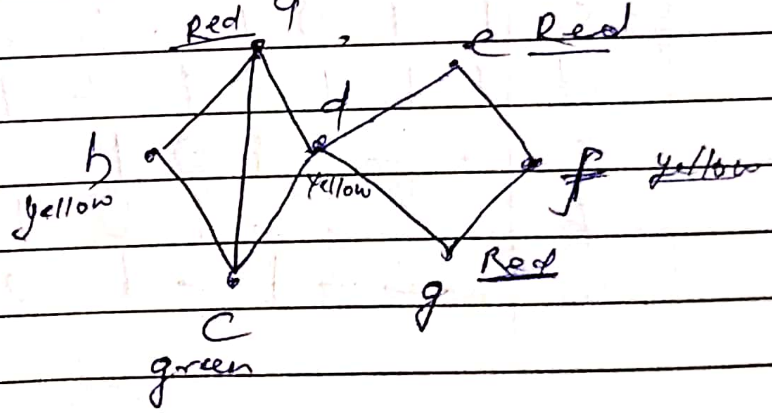
The minimum no. of colors are needed to paint the vertices of a graph such that no two adjacent vertex have same color is called chromatic no.

$\chi(G)$

Edge chromatic Number :-

The minimum no. of colors are needed to paint the eadges of a graph such that no two adjacent eadges have same color is called Edge chromatic No.

$\chi(G) = 3$



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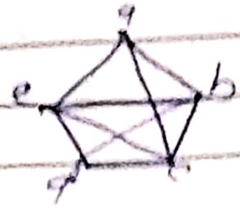
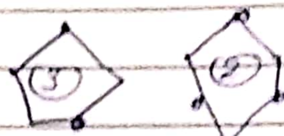
Note:- (i) $\boxed{\chi(K_n) = n}$
↓
Complete graph



(ii) $\chi(C_n)$ $n \geq 3$

C_3 C_4

$$\chi(C_n) = \begin{cases} 2 : & n \text{ is even} \\ 3 : & n \text{ is odd} \end{cases}$$



C_5 C_6

(iii) $\chi(W_n)$

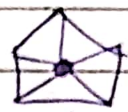
W_n = wheel graph with $(n+1)$ vertices.

$W_n = W_n$ . . — with n vertices

W_3

$$\chi(W_n) = \begin{cases} 4 : & n \text{ is even} \\ 3 : & n \text{ is odd} \end{cases}$$

W_4



W_5

(iv) χ(Bipartite graph)

$$\boxed{\chi(K_{m,n}) = 2}$$

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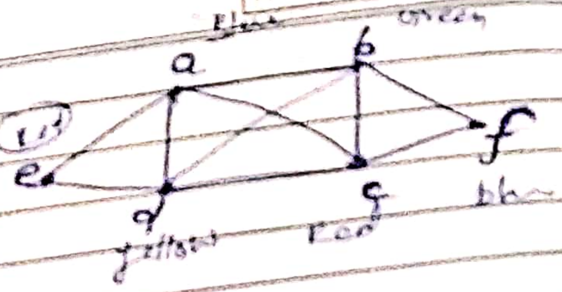
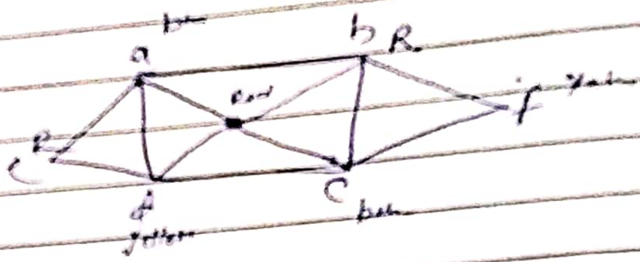
Q. Find chromatic no. of the graph

(i)



$\boxed{\chi(G) = 4}$

Q2.



$\boxed{\chi(G) = 3}$

Q3

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Welch Powell Algorithm

→ To ^the^ chromatic number.

NOTE:- This algorithm does not always yield a minimal coloring G.

Step:-1 Order the vertices of given graph G in decreasing order of their degrees.

Step:2: Assign the first color C_1 to the first vertex having max^m^ degree. and same color to each vertices which is not connected (adjacent) to that vertex / each other. the previous vertices which was assigned C_1.

Step:3: Repeat step:2 with a second color C_2 and subsequence of non-colored vertices and so on.

Q, ~~A ques~~ By using Powell Algorithm find the chromatic no. of the graph

The degree of the vertices

$d(a) = 4$
$d(b) = 4$
$d(c) = 5$
$d(d) = 4$
$d(e) = 6$
$d(f) = 3$
$d(g) = 5$
$d(h) = 3$

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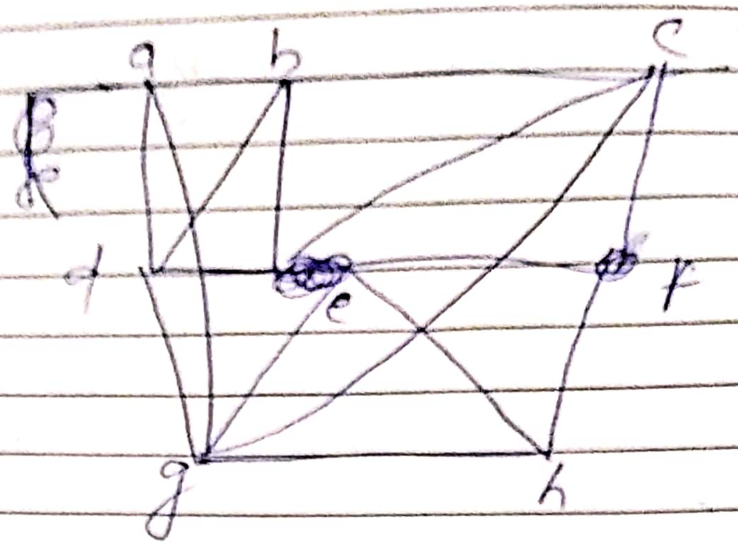
Now arrange the vertex in the decreasing order of their degrees:



degree	6	5	5	4	4	4	3	3
vertex	e	c	g	a	b	d	f	h
colors	Red	Yellow	Blue	Red	Blue	Yellow	Blue	Yellow

- Assign Red color to vertex 'e' with deg(e) = 6 & 'e' is not connected with 'a' so assign same color to 'a'.

- deg(c) = 5 assign Yellow (c2) to vertex c. and Now
  c, d, h are not connected with 'c' so assigned same color.

deg(g) = 5

g, b, f and not adjacent so assign same color to all.

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$x(vr) = 3$

```
const element() =>
{   const element = () => {
        return (
            <div>
            <h1>Hello</h1>
            </div>
        )
    export default element;
    }
```