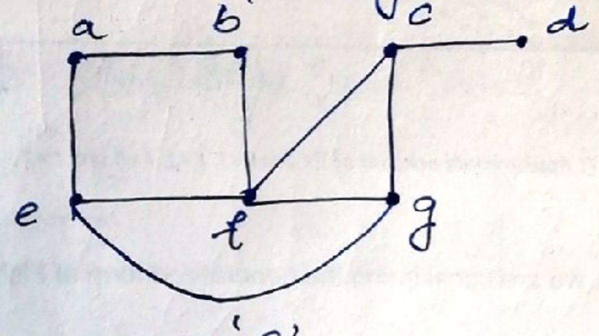
Lecture-4 (Tree Graphs)

Spanning Trees :→

Let G be a simple graph. A spanning tree of G is a subgraph of G that is a tree containing every vertex of G.

A simple graph with a spanning tree must be connected because there is a path in the spanning tree between any two vertices. The converse is also true. i.e every connected simple graph has a spanning tree

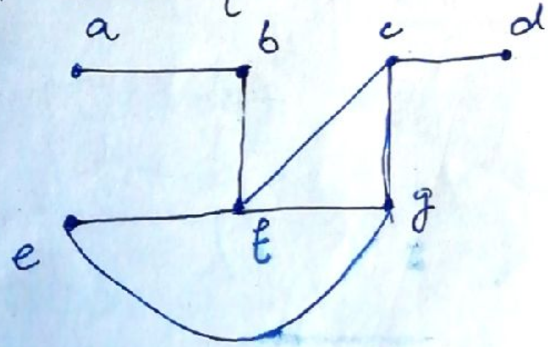
Example:→ Find a spanning tree of the simple graph 'G'



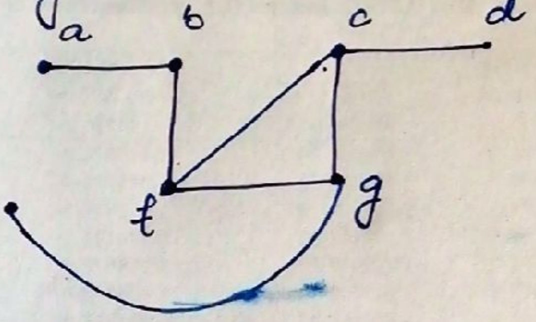
'G'

As G is connected but it is not tree because it contains simple circuits

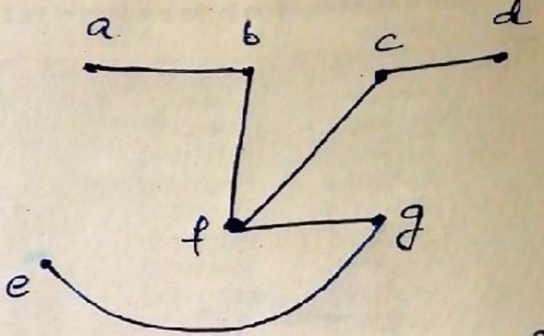
Remove the edge {a,e} This eliminates one circuit and the resultant subgraph is still connected and still contains every vertex of G. as shown.



Next remove the edge $\{e, f\}$ to eliminate the second circuit. Finally remove edge $\{c, g\}$ to produce a simple graph with no simple circuit. This subgraph is a spanning tree



Removal of edge $\{e, f\}$



Removal of $\{c, g\}$

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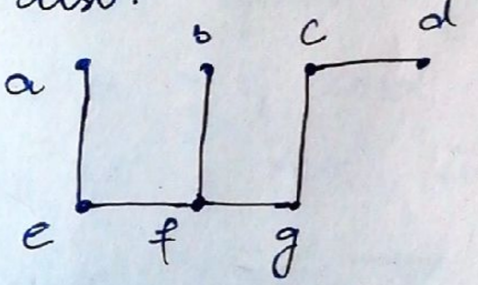
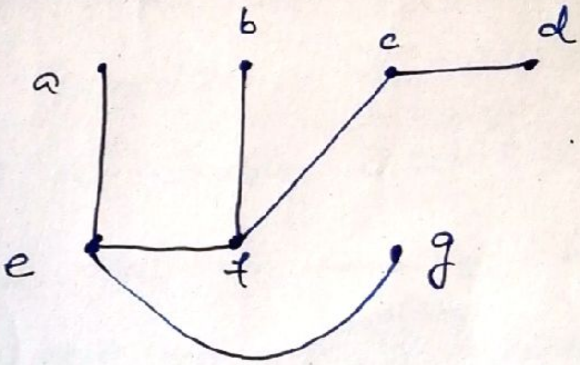
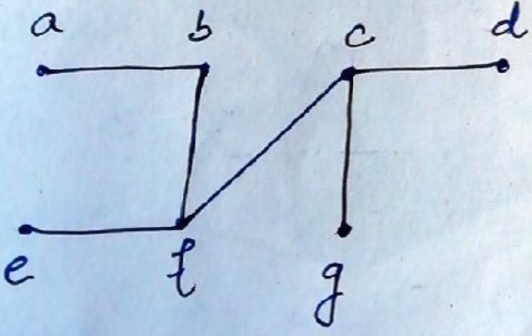
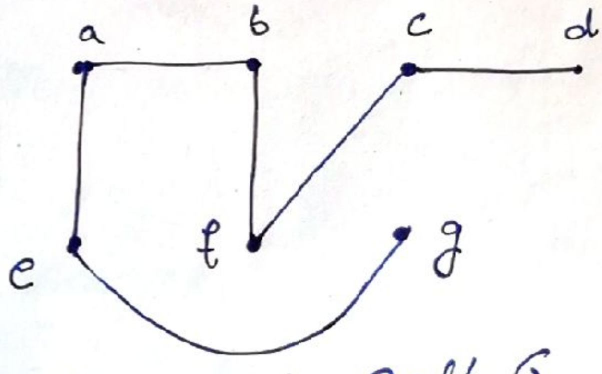
Spanning Tree.

We can remove other edges and have other spanning trees also.









Spanning trees of the graph G.

⇒ We can have more than one spanning tree of a graph G.

Theorem 1 :→ A simple graph is connected if and only if it has a spanning tree.

Proof :→ Suppose that a simple graph G has a spanning tree T. T contains every vertex of G. implies there is a path in T between any two of its vertices. because T is a subgraph of G, there is a path in G between any two of its vertices. Hence G is connected

Now suppose that G is connected. If G is not a tree, it must contains a simple circuit Remove an edge from one of these simple circuits. The resultant subgraph has one fewer edge but still contains all the ~~edges~~ vertices of G and is connected. If this subgraph is not a tree, it has a simple circuit. so as before, remove an edge that is in the simple circuit. Repeat this process until no simple circuit remains This is possible because there are only a finite no. of edges in the graph. The process must terminates when no simple circuit remains. This resultant subgraph is a spanning tree because it contains every vertex of G.