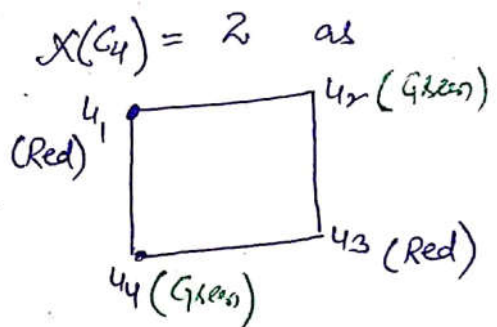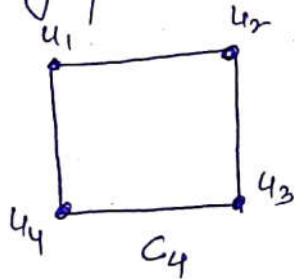# Graph Coloring

A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

A graph can be colored by assigning a different color to each of its vertices. but we need to use minimum colors to colour the graph.

Chromatic no. of graph 'G'

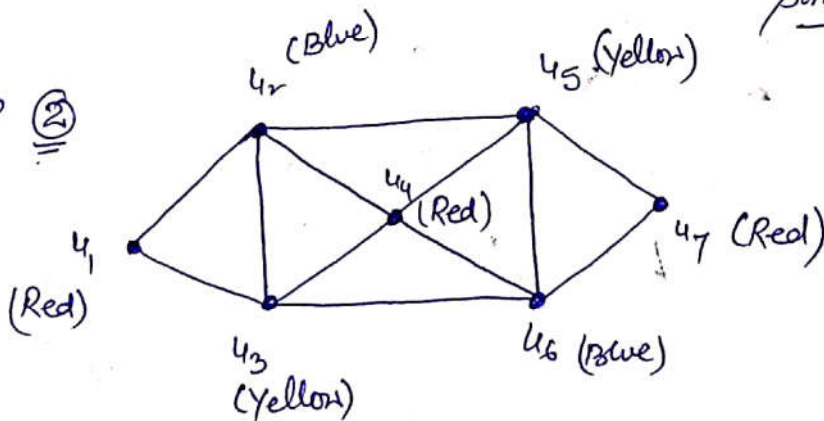The chromatic no. of a graph is denoted by $X(G)$. It is the least no. of colors needed for a coloring of this graph.

for example ①

u_1 u_2

u_4 u_3

C_4

$X(C_4) = 2$ as

u_1 (Red) u_2 (Green)

u_4 (Green) u_3 (Red)

No two adjacent vertices are of the same color.

Example ②

u_2 (Blue) u_5 (Yellow)

u_4 (Red)

u_1 (Red)

u_7 (Red)

u_3 (Yellow) u_6 (Blue)

So $X(G) = 3$.

Welch-Powell Algorithm for Coloring of a graph G

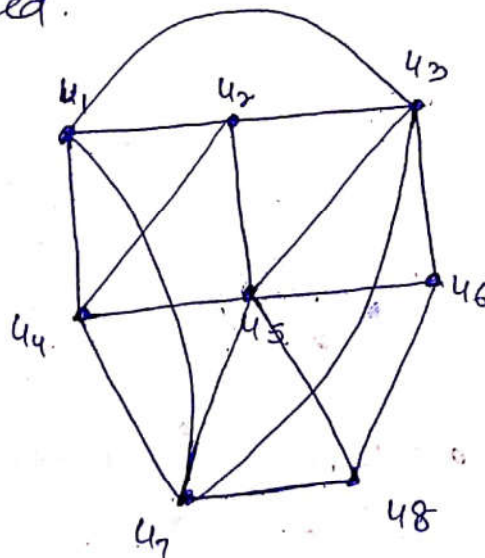(Important This algorithm does not always yield a minimal
coloring of G)

Step 1 Order the vertices of the given graph G according to decreasing degrees.

Step 2 Assign the first color c_1 to the first vertex and then assign c_1 to each vertex which is not adjacent to a previous vertex which was assigned c_1

Step 3 Repeat Step 2 with a second color c_2 and the subsequence of non colored vertices.

Step 4 Repeat Step 3 with a third color c_3, then a fourth color c_4 and so on until all vertices are colored.

Example

$$deg(u_1) = 4$$
$$dy(u_2) = 4$$
$$dy(u_3) = 5$$
$$dy(u_4) = 4$$
$$dy(u_5) = 6$$
$$dy(u_6) = 3$$
$$dy(u_7) = 5$$
$$dy(u_8) = 3$$

Arrange the vertices
in non increasing degrees

u_5	u_3	u_7	u_4	u_2	u_1	u_6	u_8
Red	Green	Blue	Green	Blue	Red	Blue	Green

We can given first color to u_5 and u_1

So. $X(G) = 3$.