

Permutation and Combination

Factorial Notation

Let n be a positive integer. Then, factorial n , denoted $n!$ is defined as:

$$n! = n(n - 1)(n - 2) \dots 3.2.1.$$

Examples:

We define $0! = 1$.

$$4! = (4 \times 3 \times 2 \times 1) = 24.$$

$$5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$$

Permutations

The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Examples:

All permutations (or arrangements) made with the letters a , b , c by taking two at a time are (ab , ba , ac , ca , bc , cb).

All permutations made with the letters a , b , c taking all at a time are:

(abc , acb , bac , bca , cab , cba)

Combinations

Each of the different groups or selections which can be formed by taking some or all objects is called a Combination.

Examples:

- ✓ Suppose we want to select two out of three boys A, B, C. Then possible selections are AB, BC, CA. Here AB and BA represent the same selection.
- ❖ Note that AB and BA are two different permutations but they represent the same combination.

Question Q25.1

In how many ways a 4 digit no. can be formed out of digits 1,2,3,4,5,6,7

Case I. Odd Number (4 digit)

- a) If repetition is not allowed
- b) If Repetition is allowed

Case II. Even Number (4 digit)

- a) If repetition is not allowed
- b) If Repetition is allowed

Answer:

Case-I

a) $6*5*4*4$

b) $7*7*7*4$

Case-II

a) $6*5*4*3$

b) $7*7*7*3$

Question Q25.2

In how many ways a 4 digit no. can be formed out of digits 0,1,2,3,4,5,6,7

Case I. Odd Number (4 digit)

- a) If repetition is not allowed
- b) If Repetition is allowed

Case II. Even Number (4 digit)

- a) If repetition is not allowed
- b) If Repetition is allowed

Answer:

Case-I

a) $6*6*5*4$

b) $7*8*8*4$

Case-II

a) $6*6*5*3+7*6*5*1$

b) $7*8*8*4$

Question Q25.3

In how many ways no. of words can be formed with the letters of the word COMMITTEE.

Case I. All Vowels are together

Case II. Vowels are never together

Answer:

Case-I

$$(6! * 4!) / (2! * 2! * 2!)$$

Case-I

$$9! / (2! * 2! * 2!) - (6! * 4!) / (2! * 2! * 2!)$$

Question Q25.4

In how many ways no. of words can be formed with the letters of the word EXAMINATION.

Case I. All Vowels are together

Case II. Vowels are never together

Answer:

Case-I

$$(6! * 6!) / (2! * 2! * 2!)$$

Case-I

$$11! / (2! * 2! * 2!) - (6! * 6!) / (2! * 2! * 2!)$$

Question Q25.5

How many straight lines and triangle can be formed out of 15 different points in a plane.

Answer

Straight line- $^{15}C_2$

Triangle- $^{15}C_3$

Question P25.7

How many ways a committee of 6 members can be formed out of 7 Men and 6 women so that

Case I. Exactly 5 woman must be selected in each committee

Case II. At least 5 Women must be selected in each committee

Answer:

$$\text{Case-I} = {}^6C_5 * {}^7C_1$$

$$\text{Case-I} = {}^6C_5 * {}^7C_1 + {}^6C_6 * {}^7C_0$$

Question P25.8

The number of ways in which 8 persons can be seated at a round table if 2 particular persons must always sit together

Answer:

Circular Arrangement = $(n-1)!$

$6! * 2!$

Probability

Experiment

An operation which can produce some well-defined outcomes is called an experiment.

Random Experiment:

An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

- Rolling an unbiased dice.
- Tossing a fair coin.
- Drawing a card from a pack of well-shuffled cards.
- Picking up a ball of certain colour from a bag containing balls of different colours.

CARDS

- 52 cards
- There are 4 suits- Diamonds, Hearts, spade, club
- 13 cards for each suit ($13 \times 4 = 52$)
- Diamond and Heart – Red (total 26 Red card)
- Spade and Club- Black (total 26 black card)
- There are 4 honors of each suit (total 16 honours card)
- 4 honour cards are
 - King(4)- 2R, 2B; Queen(4)- 2R,2B;
 - Ace(4)- 2R,2B; Jack(4)- 2R, 2B

Sample Space

When we perform an experiment, then the set S of all possible outcomes is called the *sample space*.

Examples:

In tossing a coin, $S = \{H, T\}$

If two coins are tossed,

$S = \{HH, HT, TH, TT\}$.

In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.

Event:

Any subset of a sample space is called an *event*.

Probability of Occurrence of an Event

Let S be the sample and let E be the event,

Then $P(E) = n(E)/n(S)$

Question Q26.1

Two unbiased coins are tossed. What is probability of getting at most one head?

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) 1

Answer: Option C

Explanation:

$$S = \{ HH, HT, TH, TT \}$$

$$E = \{ TT, HT, TH \}$$

$$P(E) = \frac{3}{4}$$

Question Q26.2

Determine the probability that a digit chosen at random from digits 1, 2, 3, 100 will be even.

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) 1

Answer: Option B

Question Q26.3

A card is drawn from a pack of 52 cards. What is the probability that the card is a king?

- (a) $1/4$
- (b) $1/2$
- (c) $1/13$
- (d) 1

Answer: Option C

Question Q26.4

Two cards are drawn in succession from a pack of 52 cards, without replacement. What is the probability, that the first is a king and the second is a queen of a different suit.

Answer:

$${}^4C_1 * {}^3C_1 / {}^{52}C_1 * {}^{51}C_1$$

Question Q26.5

Two dices are thrown, what is the probability that both the dices are having the same number.

- (a) $1/4$
- (b) $1/6$
- (c) $5/4$
- (d) 1

Answer: Option B

Question Q26.6

If on a paper all the natural numbers from 1 to 100 are written, if a finger is kept on a number, what is the probability that the number is a prime?

(a) $\frac{1}{4}$

(b) $\frac{1}{2}$

(c) $\frac{3}{4}$

(d) 1

Answer: Option A

Question Q26.7

If an alphabet is selected from the whole alphabetic order, what is the probability that it is a vowel?

- (a) $1/26$
- (b) $1/2$
- (c) $5/26$
- (d) $26/5$

Answer: Option C

Question Q26.8

A bag Contains 6 White and 4 RED balls. Two balls are drawn at random, find the probability that they are from same colour.

Answer:

$$({}^6C_2 + {}^4C_2) / {}^{10}C_2$$

Question Q26.8

A bag contains 4 white, 5 Red and 6 blue balls.
Three balls are drawn at random from the bag, the
probability that atleast one is red.

Answer:

$$({}^5C_1 * {}^{10}C_2 + {}^5C_2 * {}^{10}C_1 + {}^5C_3 * {}^{10}C_0) / {}^{15}C_3$$

THANK YOU & ALL THE BEST