Greedy Algorithm

Greedy Algorithm

- Greedy algorithm is generally used to solve optimization problems.
- Optimization problems
  - A problem that may have many feasible solutions.
  - Each solution has a value.
  - Optimization problem can be either maximization or minimization problems, where in maximization problem, we wish to find a solution to maximize the value and in the minimization problem, we wish to find a solution to minimize the value.
  - There are a number of approaches to solve optimization problems. Some of them are given below-
    - Branch and Bound
    - Greedy Method
    - Dynamic Programming

Greedy Algorithm

```
Algorithm Greedy (a, n)
{
   Solution : = 0;
  for i = 0 to n do
  {
      x: = select(a);
     if feasible(solution, x)
    {
       Solution: = union(solution , x)
    }
       return solution;
  }
}
```

Greedy Algorithm

- Greedy Method finds out of many options, but you have to choose the best option.
- Greedy Algorithm solves problems by making the best choice that seems best at the particular moment.
- A greedy algorithm works in phases. At each phase:
  - You take the best you can get right now, without regard for future consequences
  - You hope that by choosing a *local* optimum at each step, you will end up at a *global* optimum
- Greedy algorithms are simple and straightforward, and they take decisions on the basis of information at hand without worrying about the effect these decisions may have in the future.
- They are easy to invent, easy to implement and most of the time quite efficient.

Greedy Algorithm

- Only a few optimization problems can be solved by the greedy method. Some of them are given below-
  - Machine scheduling
  - Fractional Knapsack Problem
  - Minimum Spanning Tree
  - Huffman Code
  - Job Sequencing
  - Activity Selection Problem
- Steps for achieving a Greedy Algorithm are:

  Feasible: Here we check whether it satisfies all possible constraints or not, to obtain at least one solution to our problems.

  Local Optimal Choice: In this, the choice should be the optimum which is selected from the currently available

  Unalterable: Once the decision is made, at any subsequence step that option is not altered.

Knapsack Problem using Greedy method

Two main kinds of Knapsack Problems

- **0-1 Knapsack:**
  - N items (can be the same or different)
  - Have **only one** of each
  - Must **leave or take** (i.e., 0-1) each item (ingots of gold)
  - DP works, greedy does not since items cannot be broken

- **Fractional Knapsack:**
  - N items (can be the same or different)
  - Can take **fractional part** of each item (bags of gold dust)
  - Greedy works and DP algorithms work

Fractional Knapsack Problem

Given a set of items, each with a weight and a value/profit, determine a subset of items to include in a collection so that the total weight is less than or equal to a given limit and the total value/profit is as large as possible.

Steps to solve the Fractional Problem:

- Calculate the ratio(value/weight) for each item.
- Sort all the items in decreasing order of the ratio.
- Initialize res =0, curr_cap = given_cap.
- Do the following for every item in the sorted order:
  - If the weight of the current item is less than or equal to the remaining capacity, then add the value of that item into the result
  - Else add the current item as much as we can (fraction) and break out of the loop.
- Return res.

Fractional Knapsack Problem

Example: Consider 5 items along their respective weights and values: -

$I = (I_1, I_2, I_3, I_4, I_5)$

w = (5, 10, 20, 30, 40)

v = (30, 20, 100, 90,160)

The capacity of knapsack W = 60

The greedy algorithm:

Step 1: Sort v_i/w_i into non-increasing order.

Step 2: Put the objects into the knapsack according to the sorted sequence as possible as we can.

Fractional Knapsack Problem

Example: Consider 5 items along their respective weights and values: -

The capacity of knapsack W = 60

- First, we choose the item I_i whose weight is 5.
- Then choose item I_3 whose weight is 20.
- Now, the total weight of knapsack is 20 + 5 = 25
- Now the next item is I_5, and its weight is 40, but
- we want only 35, so we chose the fractional part of it,

I	I1	I2	I3	I4	I5
W	5	10	20	30	40
V	30	20	100	90	160

V/W	6	2	5	3	4

$$\text{i.e., } 5 \times \frac{5}{5} + 20 \times \frac{20}{20} + 40 \times \frac{35}{40}$$

$$\text{Weight} = 5 + 20 + 35 = 60$$

Maximum Value:-

$$30 \times \frac{5}{5} + 100 \times \frac{20}{20} + 160 \times \frac{35}{40}$$

$$= 30 + 100 + 140 = 270$$

I	I1	I3	I5	I4	I2
W	5	20	40	30	10
V	30	100	160	90	20

V/W	6	5	4	3	2

- So, we arrange the value of V/W in decreasing order as given in table and this will be the solution of given problem.
- The algorithm uses sorting to sort the items which takes **O(n×logn)** time complexity and then loops through each item which takes **O(n).**Hence summing up to a time complexity of **O(n×logn+n)=O(n×logn).** If the items are already sorted, then it takes **O(n)** time complexity.

Find Minimum number of Coins

- Suppose we want a change for **M Rs.** and we have an infinite supply of each of the denominations in Indian currency, i.e., we have an infinite supply of **{1, 2, 5, 10, 20, 50, 100, 500, 1000}** valued **coins/notes**, what is the **minimum number of coins** and/or notes needed to make the change?

- **Input:** M = 70
- **Output:** 2
  We need a 50 Rs note and a 20 Rs note.
- **Input:** V = 121
- **Output:** 3
  We need a 100 Rs note, a 20 Rs note and a 1 Rs coin.

Solution: Greedy Approach.

Find Minimum number of Coins

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- **Input:** M = 70
- **Output:** 2
  We need a 50 Rs note and a 20 Rs note.
- **Input:** V = 121
- **Output:** 3
  We need a 100 Rs note, a 20 Rs note and a 1 Rs coin.

Algorithm:

1. Sort the array of coins in decreasing order.
2. Initialize result as empty.
3. Find the largest denomination that is smaller than current amount.
4. Add found denomination to result. Subtract value of found denomination from amount.
5. If amount becomes 0, then print result.
6. Else repeat steps 3 and 4 for new value of V

Find Minimum number of Coins

Input: M = 11, Denominations = {9, 6, 5, 1}
The greedy approach will return denomination 9, 1, and 1
Output-3

But we can solve by taking 5 and 6. So, output will be 2.

Note: This approach may not work for all denominations. For example, it doesn't work for denominations {9, 6, 5, 1} and M = 11. The above approach would print 9, 1 and 1. But we can use 2 denominations 5 and 6. For general input, dynamic programming approach can be used:

Minimum Spanning Tree

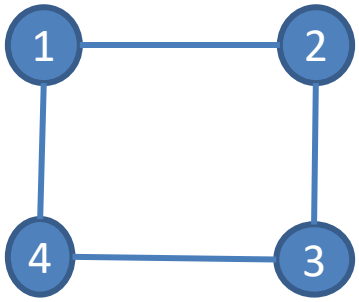
Given an undirected and connected graph *G (V, E),* a spanning tree of the graph *G* is a tree that spans *G* and is a subgraph of *G*.

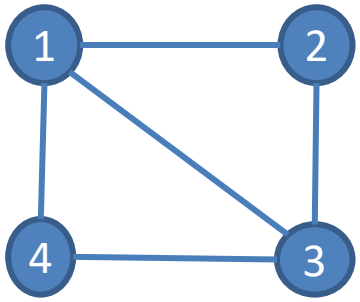
- The cost of the spanning tree is the sum of the weights of all the edges in the tree. There can be many spanning trees. Minimum spanning tree (MST) is the spanning tree where the cost is minimum among all the spanning trees. There also can be many minimum spanning trees.

- Minimum spanning tree has direct application in the design of networks. It is used in approximating the travelling salesman problem. Other practical applications are:
  - Cluster Analysis
  - Handwriting recognition
  - Image segmentation

Minimum Spanning Tree

How many spanning tree will be possible for above graph?
Total number of vertices (V)=**4**
Total number of edges (e)= **4**, Spanning tree can be created by taking **v-1** edges out of **e** edges.
So, there will be $^{4}C_{3}$ spanning trees.



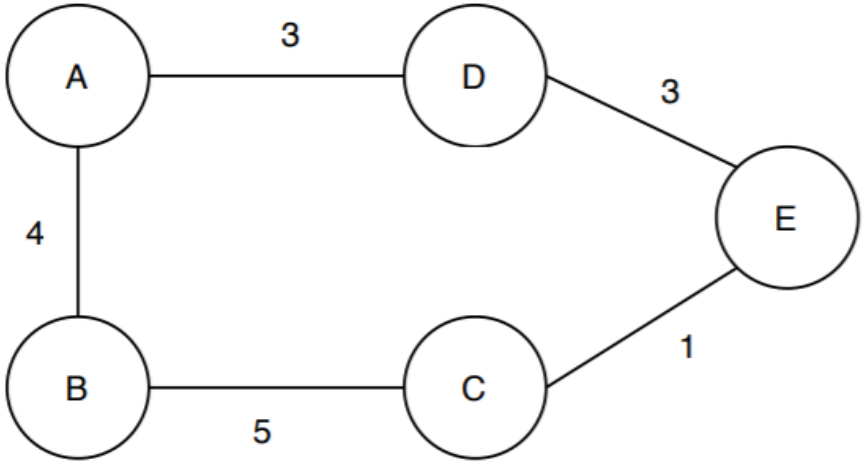


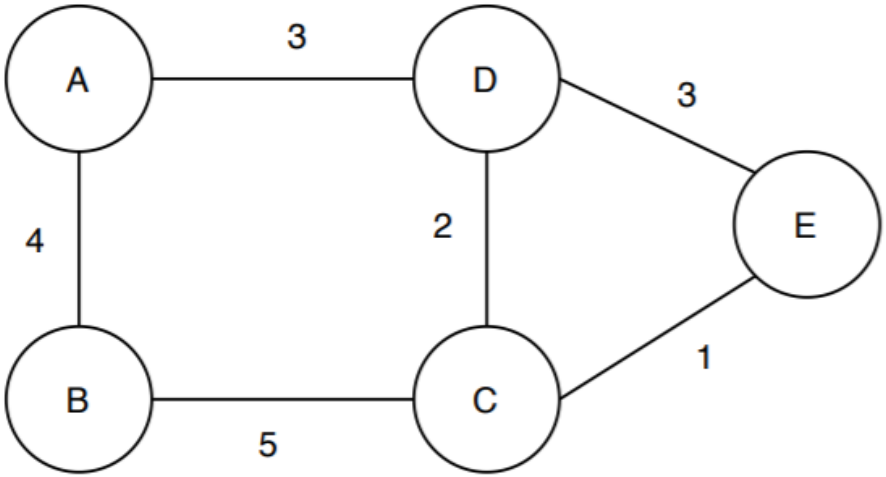
Suppose the above graph contain one more edge b/w **1 and 3**. Then, above formula will be modified as follows.

$$^{5}C_{3} - a$$

Where, **a** is the number of cycles formed. For the above graph, **a=2**.

Minimum Spanning Tree





How many spanning tree will be possible for above graph?

There are two famous algorithms for finding the Minimum Spanning Tree:

- Kruskal's Algorithm
- Prims Algorithm

Minimum Spanning Tree

Kruskal's Algorithm

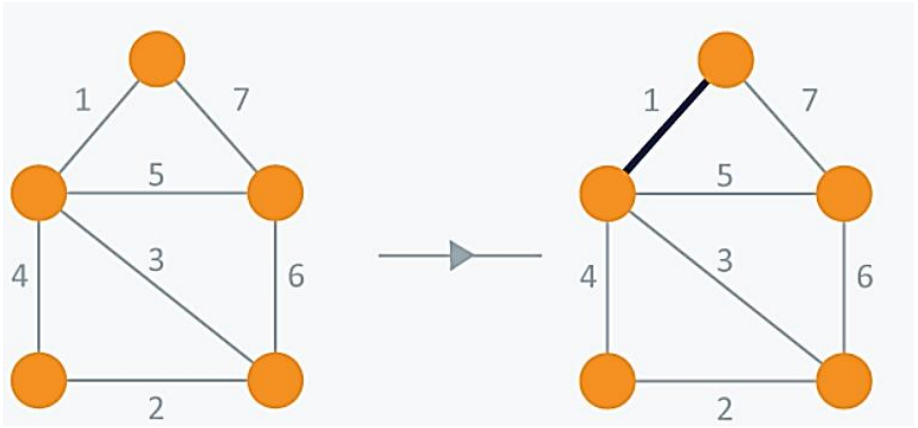
- Kruskal's Algorithm builds the spanning tree by adding edges one by one into a growing spanning tree. Kruskal's algorithm follows greedy approach as in each iteration it finds an edge which has least weight and add it to the growing spanning tree.

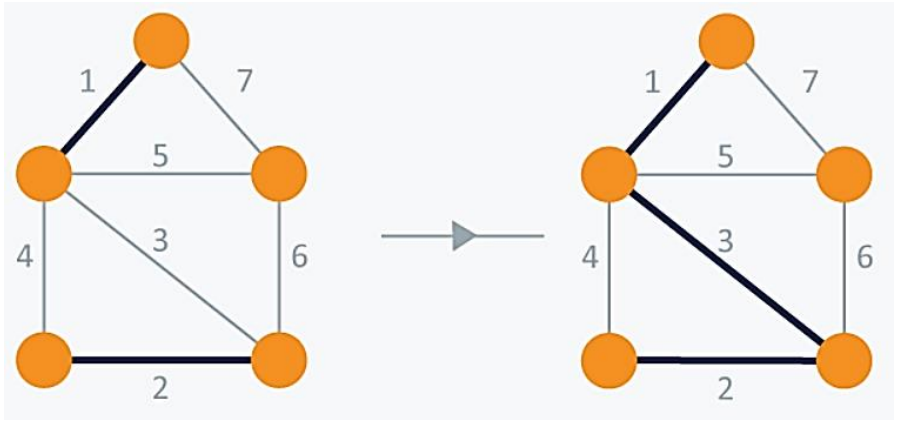
Algorithm Steps:

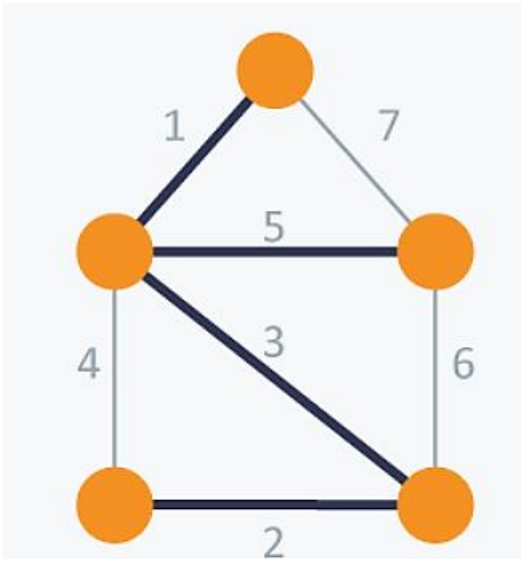
- Sort the graph edges in increasing order with respect to their weights.
- Start adding edges to the MST from the edge with the smallest weight.
- Only add edges which doesn't form a cycle, until (n - 1) edges are used.
- Stop

Minimum Spanning Tree

Kruskal's Algorithm







Minimum spanning tree with total cost= 11
(= 1 + 2 + 3 + 5)

Minimum Spanning Tree

Kruskal's Algorithm

Time Complexity:

As we have to select min cost edge from the graph, so we can use min heap as it gives min values. So, if all the edges are kept in the min heap, then after deleting every time we get the smallest value. The time taken to delete an element from min heap is **O(log n).** So, time to create spanning tree for **n** edge will be-

O(nlog n)

Minimum Spanning Tree

Prims's Algorithm

- Prim's Algorithm also use Greedy approach to find the minimum spanning tree. In Prim's Algorithm we grow the spanning tree from a starting position. Unlike an edge in Kruskal's, we add vertex to the growing spanning tree in Prim's.

General Steps:

1. First, we have to initialize an MST with the randomly chosen vertex.

2. Now, we have to find all the edges which are connected to the vertex(ces). From the edges found, select the minimum edge and add it to the tree.

3. Repeat step 2 until the minimum spanning tree is formed.

Minimum Spanning Tree

Prims's Algorithm

Algorithm Steps:

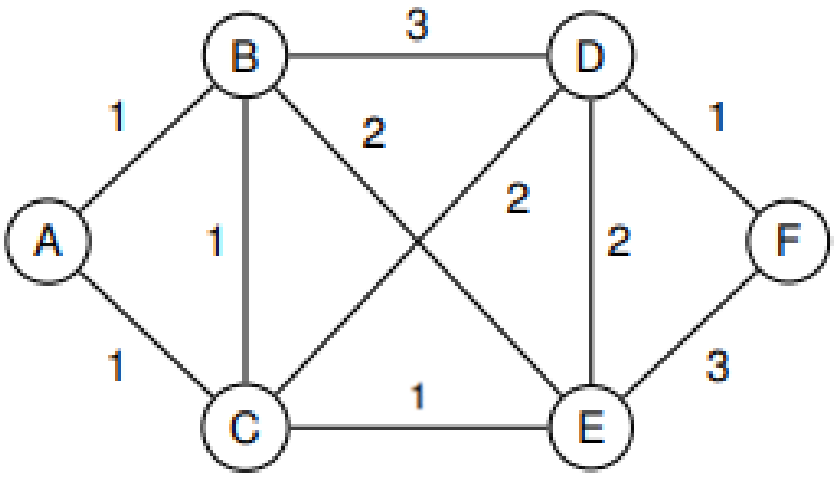
- Create a set mstSet that keeps track of vertices already included in MST.
- Assign a key value to all vertices in the input graph. Initialize all key values as INFINITE. Assign key value as 0 for the first vertex so that it is picked first.
- While mstSet doesn't include all vertices
- Pick a vertex u which is not there in mstSet and has minimum key value.
- Include u to mstSet.
- Update the key value of all adjacent vertices of u. To update the key values, iterate through all adjacent vertices. For every adjacent vertex v, if the weight of edge u-v is less than the previous key value of v, update it else keep it as it is

Minimum Spanning Tree

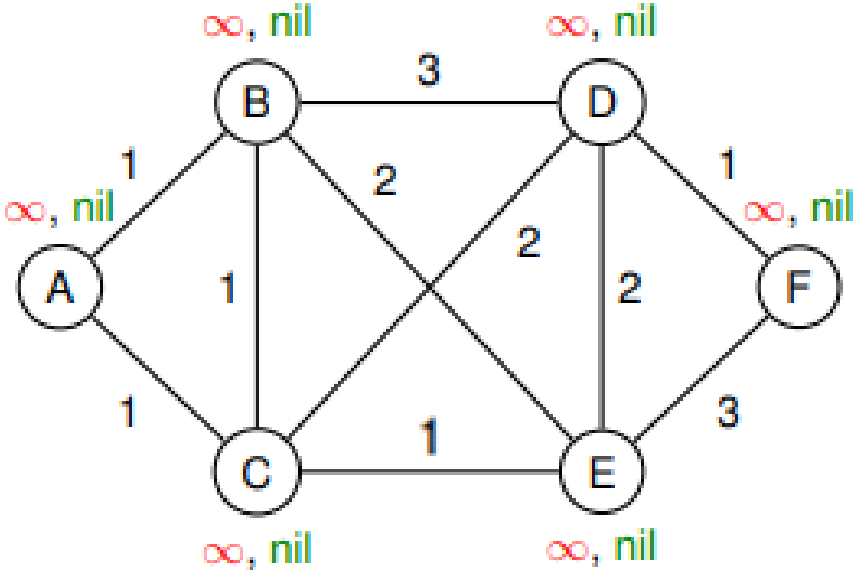
```
MST-PRIM(G, w, r)
1.      for each u ∈ G.V
2.          u.key ← ∞
3.          u.π ← NIL
4.      r.key ← 0
5.      Q ← G.V          ------ (Build_Heap)
6.      while Q ≠ Ø
7.          u ← EXTRACT-MIN(Q)
8.          for each v ∈ G.Adjacent[u]
9.              if v ∈ Q and w(u, v) < v.key
10.                 v.π ← u
11.                 v.key ← w(u, v)
```

Minimum Spanning Tree

Use Prim's algorithm to find an MST in the following graph.



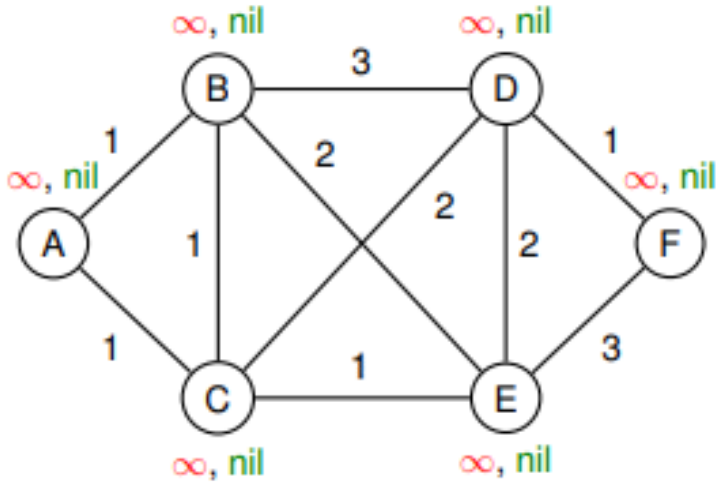
The state at the start of the algorithm:



In the above diagram, the red text is the key values of the vertices (i.e., v.key) and the green text is the predecessor vertex.

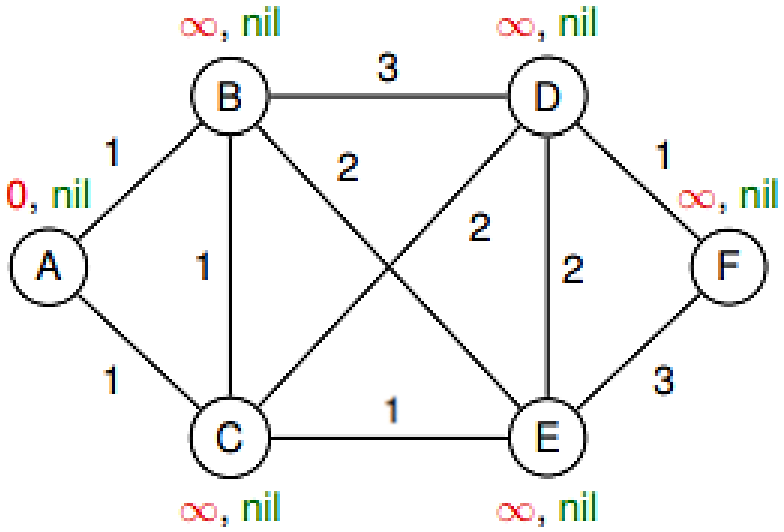
Minimum Spanning Tree

The state at the start of the algorithm:



In the above diagram, the **red text** is the **key values** of the vertices (i.e., v.key) and the **green text** is the **predecessor vertex**.

First the algorithm picks an **arbitrary starting vertex** and updates its key value to **0**.

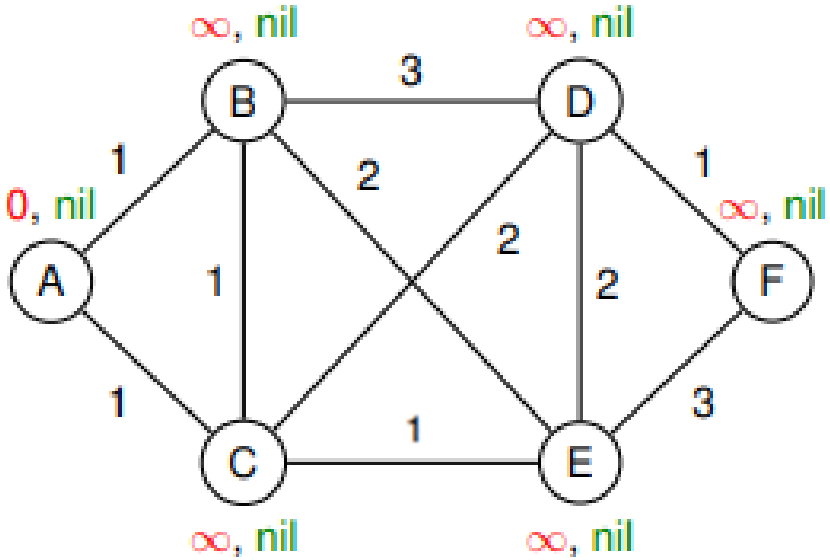


Here we **arbitrarily choose A** as our starting vertex.

Vertex	A	B	C	D	E	F
Key	0	∞	∞	∞	∞	∞
Predecessor	NIL	NIL	NIL	NIL	NIL	NIL

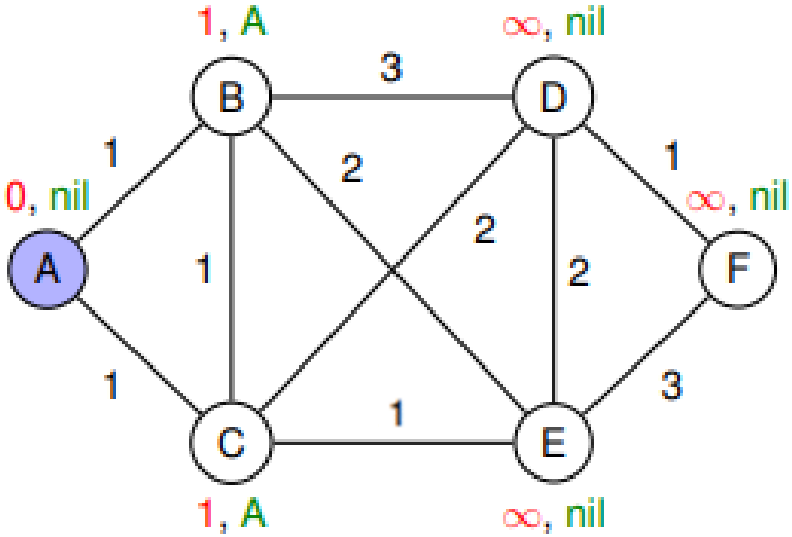
Minimum Spanning Tree

First the algorithm picks an **arbitrary starting vertex** and updates its key value to 0.



Here we **arbitrarily choose A** as our starting vertex.

Then **A** is extracted from the queue, and the keys of its neighbours are updated:

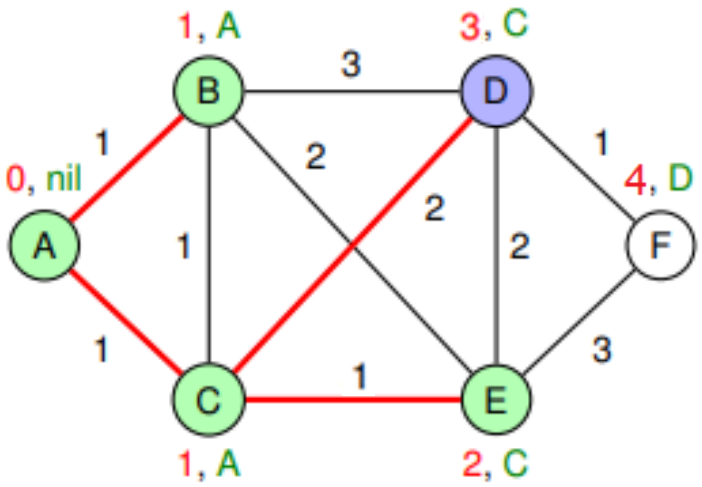


Vertex colours: **Blue: current vertex**, **green: vertices** added to tree.

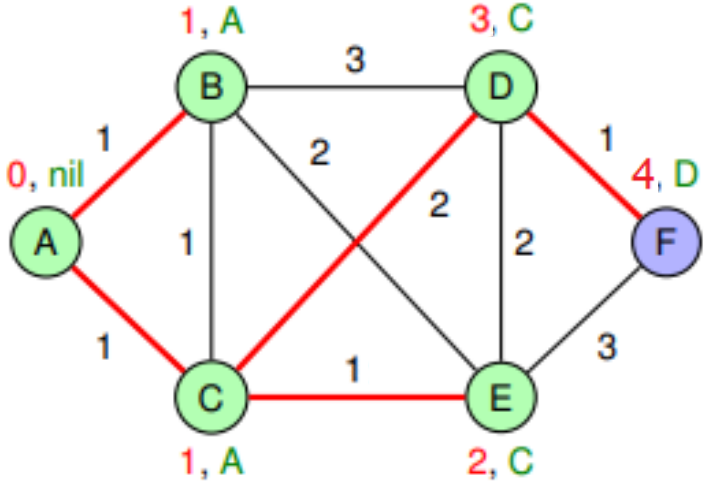
Vertex	A	B	C	D	E	F
Key	0	1	1	∞	∞	∞
Predecessor	NIL	A	A	NIL	NIL	NIL

Minimum Spanning Tree

Then D is extracted from the queue



Finally, F is extracted from the queue and the algorithm is complete



Minimum Spanning Tree

```
MST-PRIM(G, w, r)
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2.          u.key ← ∞
3.          u.π ← NIL
4.      r.key ← 0
5.      Q ← G.V
6.      while Q ≠ Ø
7.          u ← EXTRACT-MIN(Q)
8.          for each v ∈ G.Adjacent[u]
9.              if v ∈ Q and w(u, v) < v.key
10.                 v.π ← u
11.                 v.key ← w(u, v)
```

V times

i. The time complexity required for one call to **EXTRACT-MIN(Q)** is **O(log V)** using a min priority queue. The while loop at **line 6** is executing total **V times**. **EXTRACT-MIN(Q)** is called **V times**. So, the complexity of **EXTRACT-MIN(Q)** is **O(V logV).**

ii. The for loop at **line 8** is executing total **V times** as a vertex may be connected to remaining **|V-1|** vertices in complete graph. The time required to execute line **11 is O(log v)** by using the **DECREASE_KEY** operation on the **min heap**. **Line 11** also executes total **V-1** times.

So, the total time required to execute line **11 is O(|V|*|V-1|*logV) = O(V^2 logV).**

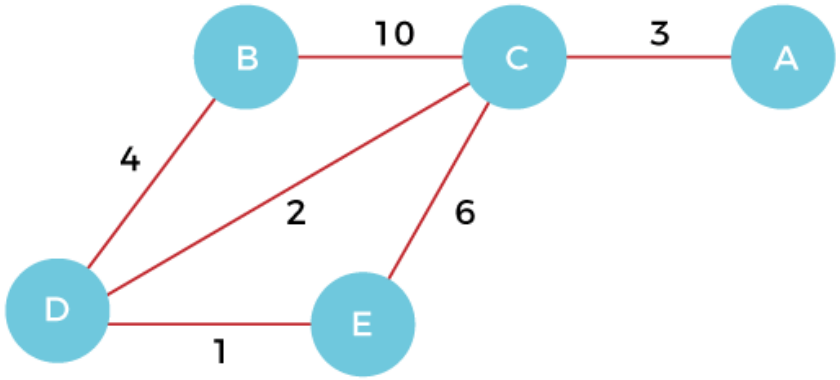
Total time complexity of MST-PRIM is the sum of all time complexities i.e., **O((VlogV) + (V^2 logV) + (V)+ V) .**

The DECREASE_KEY operation will execute for number of edges which are connected to the selected vertex. So, we can replace **V^2** by **E**. Where, **E= V*(V-1)/2** in worst case.

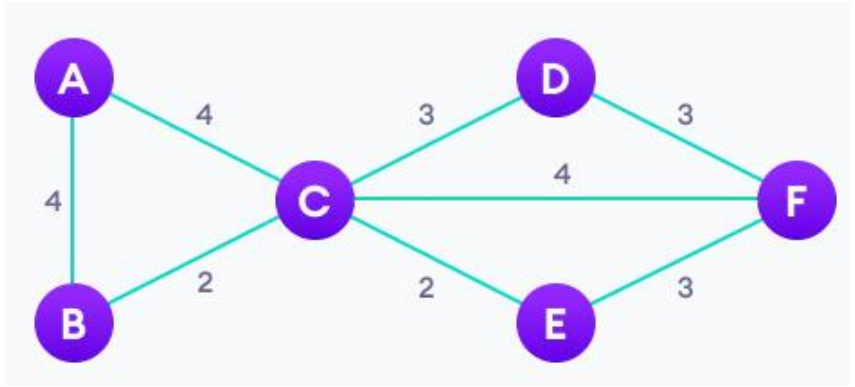
O((VlogV) + (E logV) + (V)+ V) = O((V+E) logV) => O((V+E) logV), since |E| >= |V|.

Minimum Spanning Tree

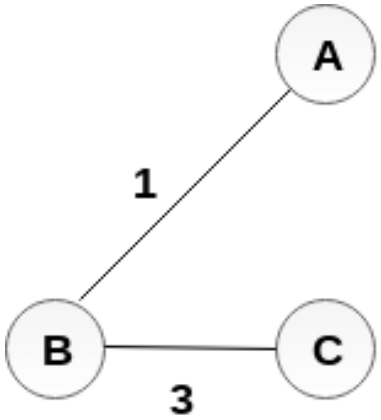
Construct the minimum spanning tree for the following graphs.

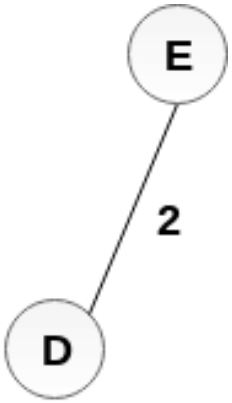


i



ii



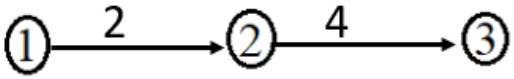


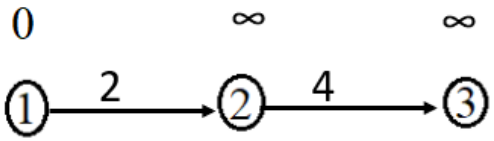
Dijkstra Algorithm

- Dijkstra algorithm is a single-source shortest path algorithm. Here, single-source means that only one source is given, and we have to find the shortest path from the source to all the vertices.
- It is minimization problem and can be solved by greedy method. It only works on weighted graphs with positive weights.
- Dijkstra's algorithm allows us to find the shortest path between any two vertices of a graph.
- It differs from the minimum spanning tree because the shortest distance between two vertices might not include all the vertices of the graph.

Dijkstra Algorithm

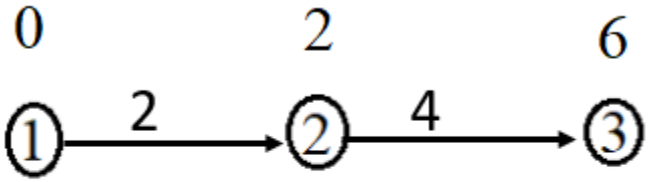
- In Dijkstra, first select the **starting vertex** and set its **cost =0**. The cost of **remaining vertices** will be **set to ∞**.
- Use the **relaxation formula** to update the cost of each vertex. The relaxation formula is-
  - If(d[u]+c[u, v]< d[v] then
    - d[v]=d[u]+ c[u, v]
  - Else
    - No change





d[2]= d[1]+c(1,2)
= 0+2
=2 which is smaller than d[2] i.e. is ∞

So, d[2]=2



Similarly, d[3]= 6

Dijkstra Algorithm

Algorithm

1. Mark the **source node** with a current distance of 0 and the **rest with infinity.**
2. Set the **non-visited node** with the **smallest current distance** as the current node, lets say **C.**
3. For **each neighbour N** of the **current node C**: add the current distance of C with the weight of the edge connecting C-N. If it is smaller than the current distance of N, set it as the new current distance of N.
4. Mark the current node C as visited.
5. Go to step 2 if there are any nodes are unvisited.

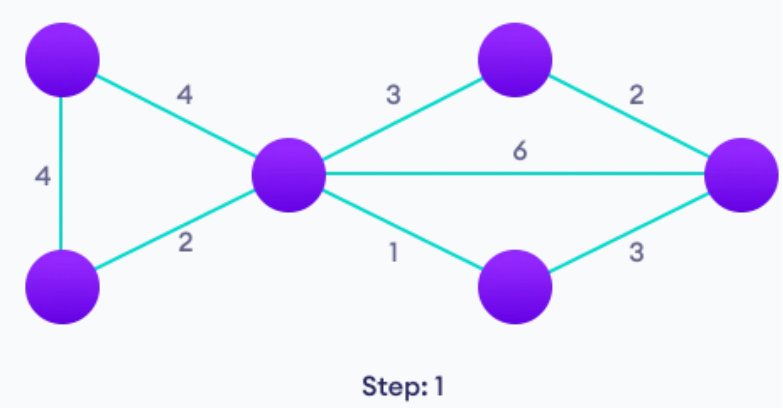
Dijkstra Algorithm

Pseudo-code

```
Dijkstra (G, W, s)                  //uses priority queue Q
  Initialize (G, s)
    S ← φ
    Q ← V [G]                       //Insert into Q (Min Heap)
  while Q != φ
        do u ← EXTRACT-MIN(Q)  //deletes u from Q
        S = S ∪ {u}
        for each vertex v in Adj[u]
             do RELAX (u, v, w) ← this is an implicit DECREASE KEY operation
```

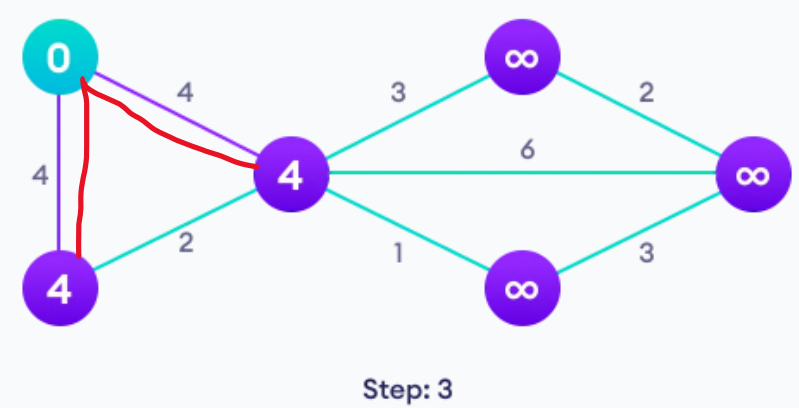
Dijkstra Algorithm

Dijkstra Algorithm



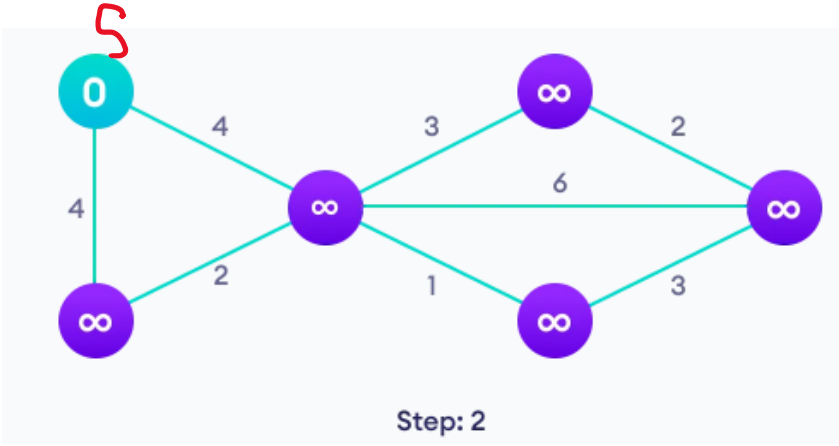
Step: 1

Start with a weighted graph



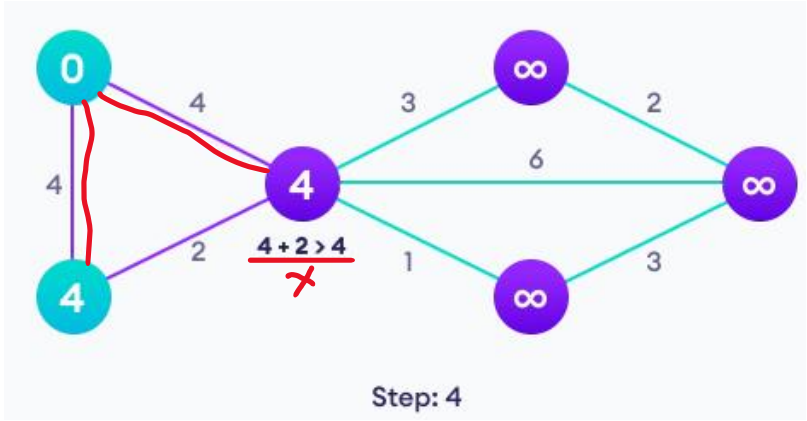
Step: 3

Go to each vertex and update its path length



Step: 2

Choose a starting vertex and assign infinity path values to all other vertices

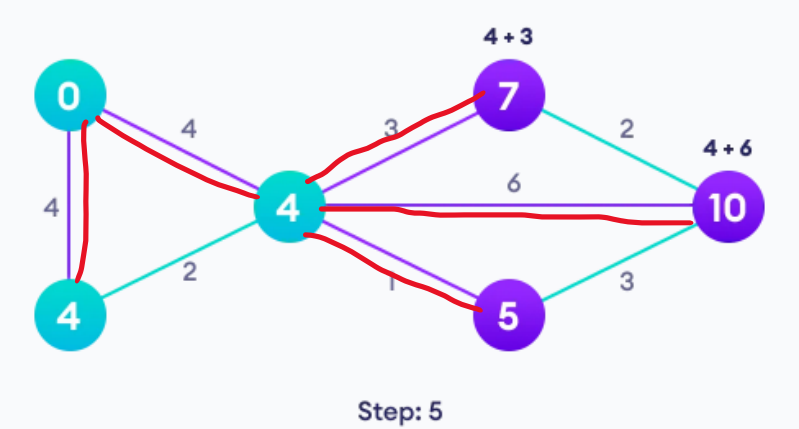


Step: 4

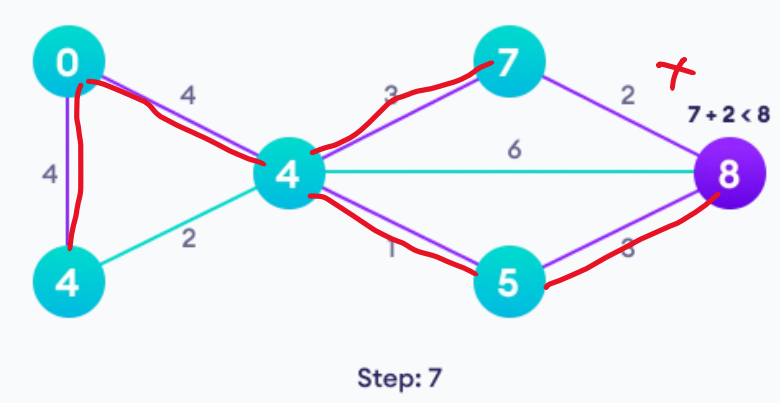
If the path length of the adjacent vertex is lesser than new path length, don't update it

Dijkstra Algorithm

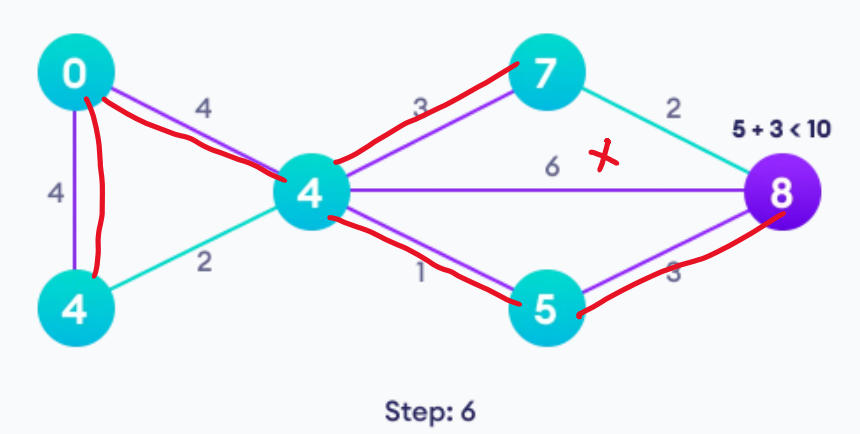
Dijkstra Algorithm



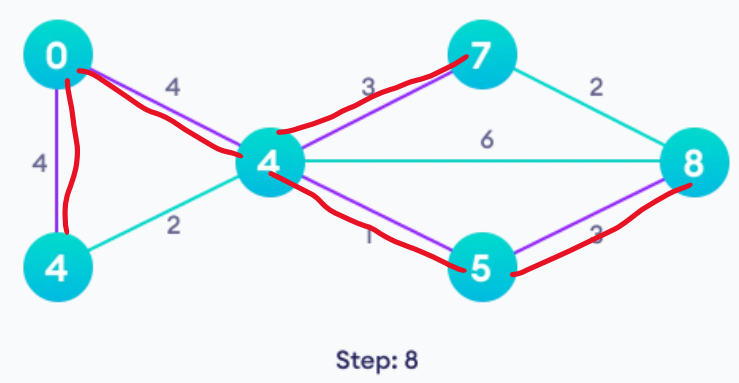
Avoid updating path lengths of already visited vertices



Notice how the rightmost vertex has its path length updated twice



After each iteration, we pick the unvisited vertex with the least path length. So, we choose 5 before 7



Repeat until all the vertices have been visited

Dijkstra Algorithm

Pseudo-code

```
Dijkstra (G, W, s)              //uses priority queue Q
  Initialize (G, s)
  S ← φ                                                    } V times
  Q ← V [G]                     //Insert into Q            ----- V log V times
  while Q != φ
        do u ← EXTRACT-MIN(Q)   //deletes u from Q        ------ V log V times
        S = S ∪ {u}
        for each vertex v in Adj[u]
             do RELAX (u, v, w) ← DECREASE KEY operation   ----- V² log V times
```

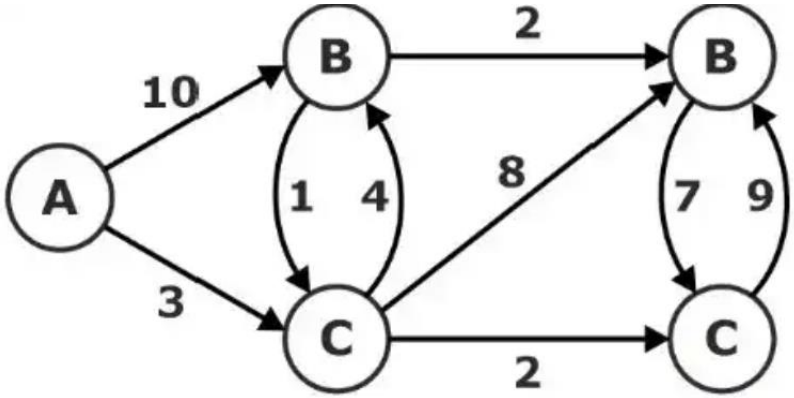
Dijkstra Algorithm

Dijkstra Algorithm Time Complexity-

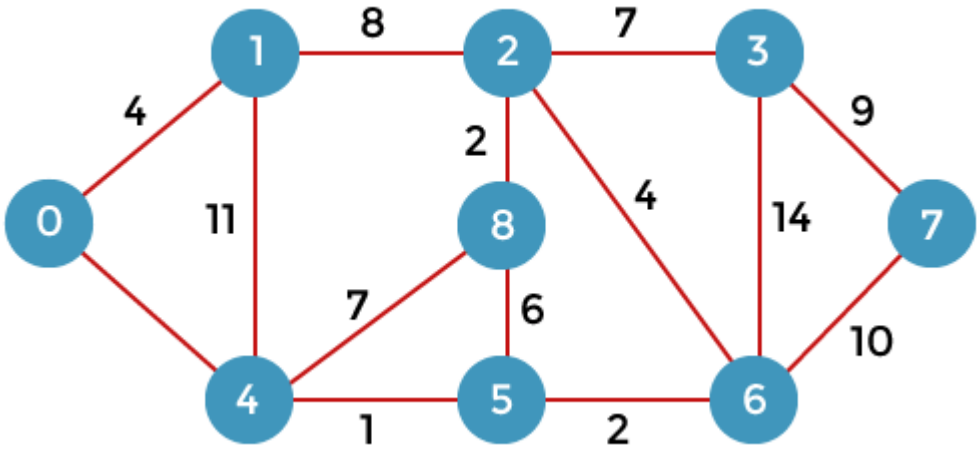
The time complexity of Dijkstra's algorithm can be reduced to **O((V+E)logV)** using **adjacency list representation** of the graph and **a min-heap to store the unvisited vertices**, where **E** is the number of edges in the graph and **V** is the number of vertices in the graph.

Time complexity = **O((V+E)logV)**

Dijkstra Algorithm



i



ii

Job Sequencing with Deadlines

Problem Statement

- In job sequencing problem, the **objective** is to find **a sequence of jobs**, which is **completed within their deadlines** and **gives maximum profit**.

  The simple and brute-force solution for this problem is to generate all the sequences of the given set of jobs and find the most optimal sequence that maximizes the profit.

  Time complexity of this solution would be O(2^n)

To optimize this algorithm, we can make use of a greedy approach that produces an optimal result, which works by selecting the best and the most profitable option available at the moment.

Job Sequencing with Deadlines: Algorithm

Greedy approach

- Sort the jobs based on decreasing order of profit and find the largest deadline
- Create a Gantt chart of maximum deadline size
- Iterate through the jobs and perform the following:
  - Choose a ***Slot i*** if:
    - ***Slot i*** isn't previously selected.
    - ***Slot i*** should be closest to its deadline
    - ***i*** must be as maximum as possible
  - If no such slot exists, ignore the job and continue.

Job Sequencing with Deadlines: Example

In the table below, jobs with their profits and deadlines are given. What would be the optimal sequencing of jobs which will give maximum profit?

JOBS	DEADLINES	PROFITS
Job 1	5	200
Job 2	3	180
Job 3	3	190
Job 4	2	300
Job 5	4	120
Job 6	2	100

Job Sequencing with Deadlines: Example

In the table below, jobs with their profits and deadlines are given. What would be the optimal sequencing of jobs which will give maximum profit?

JOBS	DEADLINES	PROFITS
Job 1	5	200
Job 2	3	180
Job 3	3	190
Job 4	2	300
Job 5	4	120
Job 6	2	100

Step 1:
Sort the jobs in decreasing order of their profit.

JOBS	DEADLINES	PROFITS
Job 4	2	300
Job 1	5	200
Job 3	3	190
Job 2	3	180
Job 5	4	120
Job 6	2	100

Here we can see that value of the maximum deadline is **5.**

Job Sequencing with Deadlines: Example

JOBS	DEADLINES	PROFITS
Job 4	2	300
Job 1	5	200
Job 3	3	190
Job 2	3	180
Job 5	4	120
Job 6	2	100

Step 2:

As maximum deadline is 5, so create a Gantt chart of size 5

Gantt Chart

Step 3:

Now, pick the jobs one by one as presented in step, 1, and place them on the Gantt chart as far as possible from 0 and closest to its deadline.

We will pick Job 4. Its deadline is 2. So, placing the job in the empty slot available just before the deadline.

Job Sequencing with Deadlines: Example

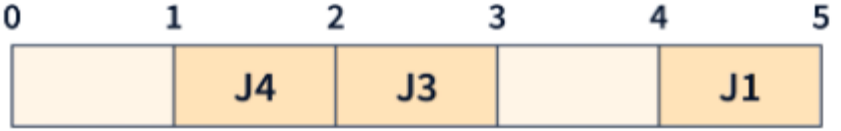
JOBS	DEADLINES	PROFITS
Job 4	2	300
Job 1	5	200
Job 3	3	190
Job 2	3	180
Job 5	4	120
Job 6	2	100

Step 4:

We will now pick Job 1. Its deadline is 5. So, placing the job in the empty slot available just before the deadline.

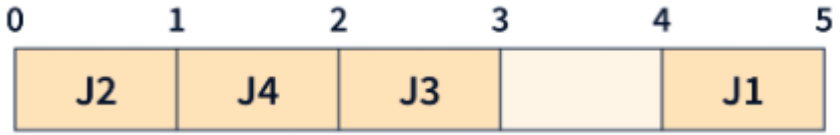
Step 5:

We will now pick Job 3. Its deadline is 3. So, placing the job in the empty slot available just before the deadline.



Step 6:

We will now pick Job 2. Its deadline is 3. Here second and third slots are already filled. So, place job 2 on the next available free slot i.e., first slot.

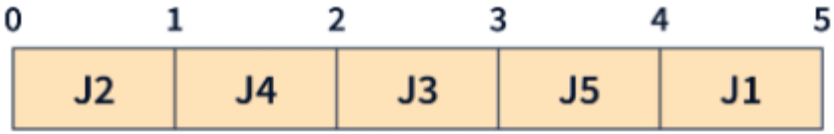


Job Sequencing with Deadlines: Example

JOBS	DEADLINES	PROFITS
Job 4	2	**300**
Job 1	5	**200**
Job 3	3	**190**
Job 2	3	**180**
Job 5	4	**120**
Job 6	2	100

Step 7:

We will now pick Job 5. Its deadline is 4. Place the job in the first empty slot before the deadline, i.e., fourth slot.



We will now pick Job 6. Its deadline is 2. Now we need to place the job in the first empty slot before the deadline. Since, no such slot is available, hence Job 6 can not be completed.

So, the most optimal sequence of jobs to maximize profit is **Job 2, Job 4, Job 3, Job 5, and Job 1.**

And the maximum profit earned can be calculated as:
Profit of Job 2 + Profit of Job 4 + Profit of Job 3 + profit of Job 5 + profit of Job 1

180+300+190+120+200 = 990

Job Sequencing with Deadlines: Analysis

- Sort the jobs based on decreasing order of profit and find the largest deadline--- **nlogn**
- Create a Gantt chart of maximum deadline size ---------- **K**
- Iterate through the jobs and perform the following: ---------- **n**
  - Choose a **Slot i** if:
    - **Slot i** isn't previously selected.
    - ***Slot i*** should be closest to its deadline
    - ***i*** must be as maximum as possible

    ----------**nxm**
  - If no such slot exists, ignore the job and continue.

Since, size of the Gantt chart array will be **m** and max comparison to place an item may be **m** and for **n jobs** it will **nxm**

Time complexity= **nlogn + n+ nxm +k**

= **nxm = O(n^2) if n==m**

Optimal Merge Pattern

Problem Statement

- Given n number of sorted files of different lengths, the task is to find the minimum computations done to reach the Optimal Merge Pattern.

Optimal merge pattern is a pattern that relates to the merging of two or more sorted files in a single sorted file. This type of merging can be done by the two-way merging method.

If we have two sorted files containing ***n*** and ***m*** records respectively then they could be merged together, to obtain one sorted file in time ***O (n+m).***

Optimal Merge Pattern

Let us consider the given files, f1, f2, f3, f4 and f5 with 20, 30, 10, 5 and 30 number of elements, respectively.

If merge operations are performed according to the provided sequence, then

M1 = merge f1 and f2 => 20 + 30 = 50
M2 = merge M1 and f3 => 50 + 10 = 60
M3 = merge M2 and f4 => 60 + 5 = 65
M4 = merge M3 and f5 => 65 + 30 = 95

Hence, the total number of operations is

50 + 60 + 65 + 95 = 270

Now, the question arises is there any better solution?

Optimal Merge Pattern

Let us consider the given files, f1, f2, f3, f4 and f5 with 20, 30, 10, 5 and 30 number of elements, respectively.

Sorting the numbers according to their size in an ascending order, we get the following sequence −
f4, f3, f1, f2, f5
Hence, merge operations can be performed on this sequence
M1 = merge f4 and f3 => 5 + 10 = 15
M2 = merge M1 and f1 => 15 + 20 = 35
M3 = merge M2 and f2 => 35 + 30 = 65
M4 = merge M3 and f5 => 65 + 30 = 95

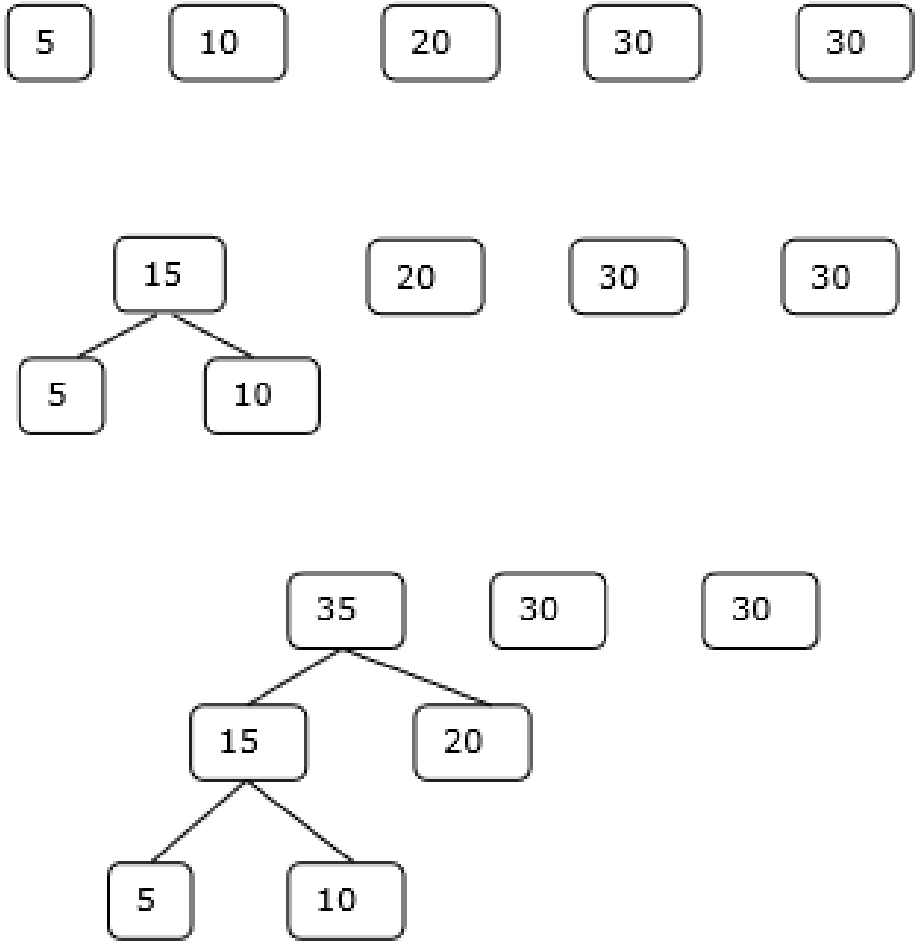
Therefore, the total number of operations is 15 + 35 + 65 + 95 = 210, which is less than first one.

So, we need to find a merge pattern that can merge elements in the least number of comparison.

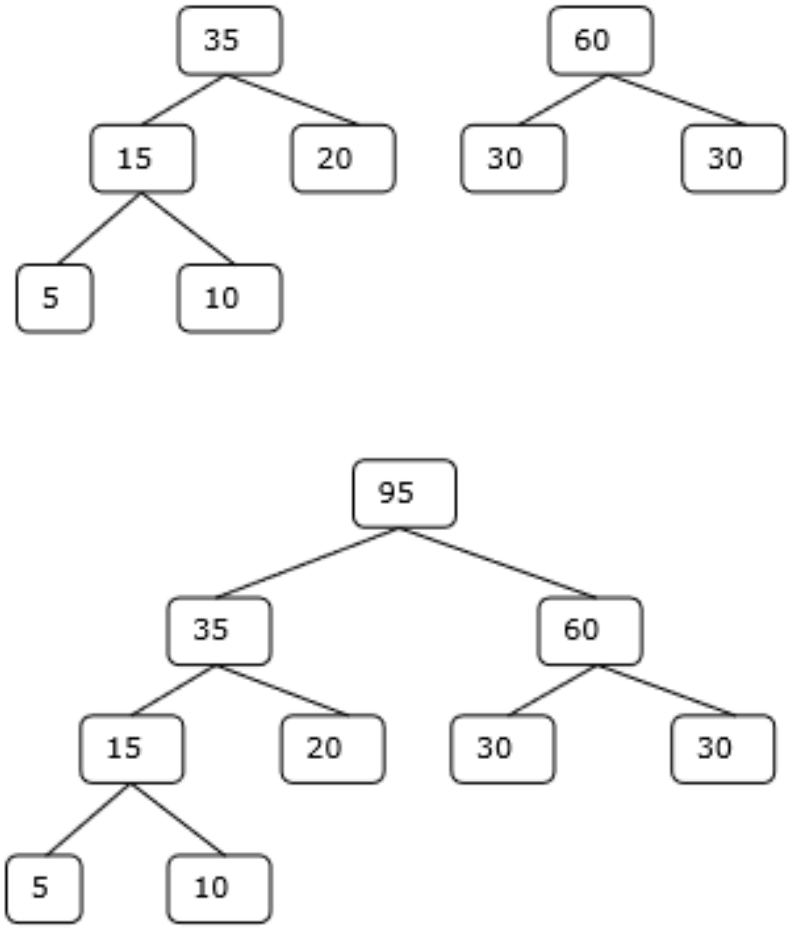
Optimal Merge Pattern: Greedy approach

```
Algorithm OPTIMAL_MERGE_PATTERNS(S) // S is set of sequences
Create min heap H from S
while H.length > 1 do
    min1 ← minDel(H) // returns minimum element from H & delete it
    min2 ← minDel(H)
     NewNode.Data ← min1 + min2
     NewNoode.LeftChild ← min1
     NewNode.RightChild ← min2
     Insert(NewNode, H)// Insert node NewNode in Heap
end
```

Optimal Merge Pattern: Greedy approach



Optimal Merge Pattern: Greedy approach



Hence, the solution takes 15 + 35 + 60 + 95 = 205 number of comparisons.

Optimal Merge Pattern: Greedy approach

```
Algorithm OPTIMAL_MERGE_PATTERNS(S)   // S is set of sequences
Create min heap H from S
while H.length > 1 do
    min1 ← minDel(H)
    min2 ← minDel(H)
    NewNode.Data ← min1 + min2
    NewNoode.LeftChild ← min1
    NewNode.RightChild ← min2
 Insert(NewNode, H)
```

Complexity Analysis

The main loop in this algorithm is executed in **n-1** times. In every iteration, **two delete minimum**, and **one insert operation** is performed. Construction of heap takes **O(logn)** time. The total running time of this algorithm is **O(nlogn)**.

T(n) = (n – 1) * max(O(findmin), O(insert))

Optimal Merge Pattern: Greedy approach

Complexity Analysis-

T(n) = (n – 1) * max(O(findmin), O(insert))

Case 1 : If list is sorted and stored in an array

O(findmin) = O(1) and O(insert) = O(n)
So, T(n) = (n – 1) * n = $O(n^2)$

Case 2 : List is represented as min-heap

O(findmin) = O(1) and O(insert) = O(logn)
So, T(n) = (n – 1) * logn = O(nlogn)

Huffman Coding

Problem Statement

- Huffman Coding is a technique of compressing data to reduce its size without losing any of the details. It was first developed by David Huffman.
- Huffman Coding is generally useful to compress the data in which there are frequently occurring characters.

Suppose the string below is to be sent over a network.

As characters in computers are represented using ASCII code, which is denoted by 8 bits in binary. There are a total of 15 characters in the above string. Thus, a total of 8 * 15 = 120 bits are required to send this string.

To reduce the size (compress) without losing any information, Huffman coding is used. Huffman coding assign variable-length codes to input characters.

Huffman Coding

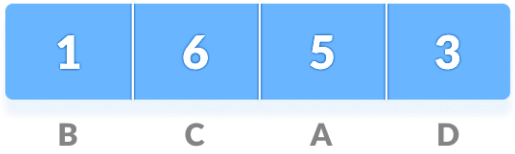
Steps to build Huffman Tree-

1. Calculate the frequency of each character in the string.
2. Sort the characters in increasing order of the frequency. These are stored in a priority queue Q using heap.
3. Make each unique character as a leaf node.
4. Create an empty node z. Assign the minimum frequency to the left child of z and assign the second minimum frequency to the right child of z. Set the value of the z as the sum of the above two minimum frequencies.
5. Remove these two minimum frequencies from Q and add the sum into the list of frequencies (* denote the internal nodes in the figure above).
6. Insert node z into the tree.
7. Repeat steps 3 to 5 for all the characters.
8. For each non-leaf node, assign 0 to the left edge and 1 to the right edge.

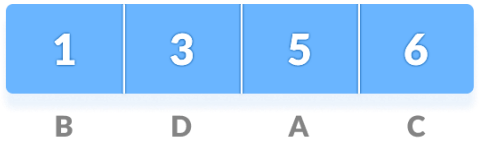
Huffman Coding

Steps to build Huffman Tree-

- Calculate the frequency of each character in the string.



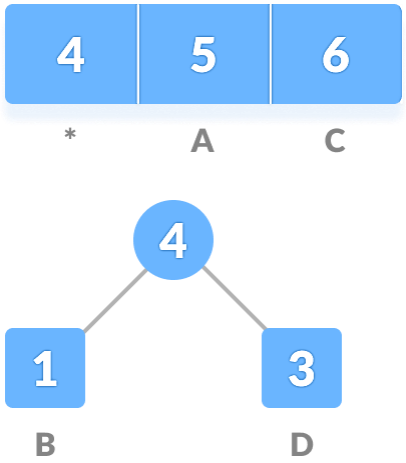
- Sort the characters in increasing order of the frequency. These are stored in a priority queue Q.



Huffman Coding

Steps to build Huffman Tree-

- Create an empty **node z**. Assign the minimum frequency to the **left child of z** and assign the **second minimum frequency to the right child of z**. Set the **value of the z** as the sum of the above two minimum frequencies.

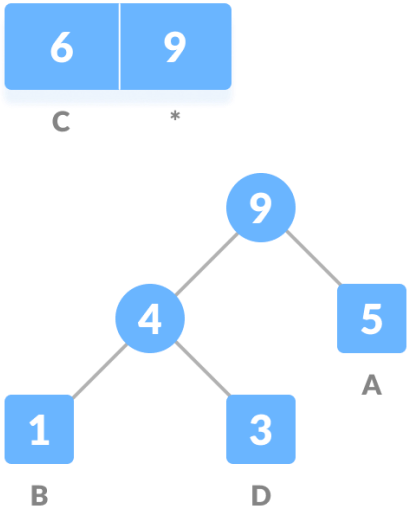


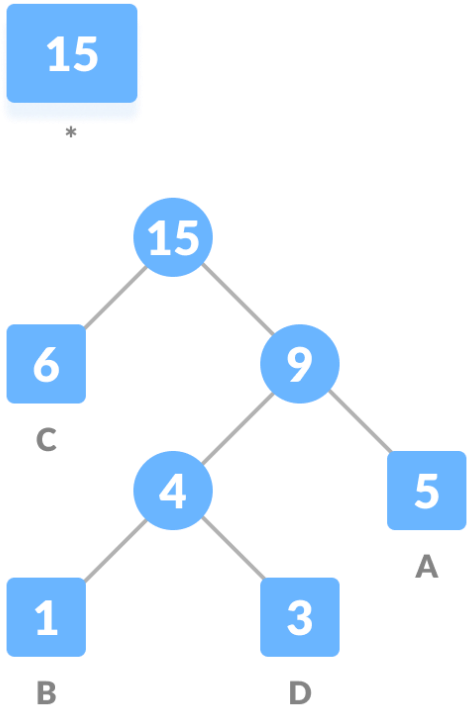
- Remove these two minimum frequencies from **Q** and add the sum into the list of frequencies (* denote the internal nodes in the figure above).

Huffman Coding

Steps to build Huffman Tree-

- Select two minimum elements from Q and add them in tree using the step 3.



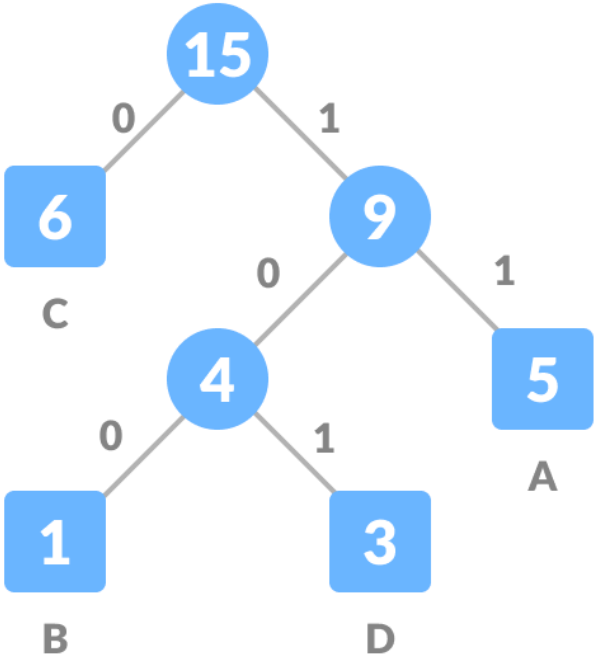


For each non-leaf node, assign 0 to the left edge and 1 to the right edge

Huffman Coding

Steps to build Huffman Tree-

- Assign 0 to the left edge and 1 to the right edge



Huffman Coding

- For sending the string over a network, we have to send the tree as well as the compressed-code. The total size is given by the table below.

Character	Frequency	Code	Size
A	5	11	5*2 = 10
B	1	100	1*3 = 3
C	6	0	6*1 = 6
D	3	101	3*3 = 9
4 * 8 = 32 bits	15 bits		28 bits

Without encoding, the total size of the string was 120 bits. After encoding the size is reduced to 32 + 15 + 28 = 75

Huffman Coding Complexity

create a priority queue Q using heap consisting of each unique character and its frequency means a min heap is constructed for the frequencies.

for all the unique characters:

create a newNode

extract minimum value from Q and assign it to leftChild of newNode

extract next minimum value from Q and assign it to rightChild of newNode

calculate the sum of these two minimum values and assign it to the value of newNode

insert this newNode into the tree and add the sum into the list of frequencies

return rootNode

Time complexity

Time Complexity is **O(nlogn)** because we are extracting **minimum nodes 2*(n-1) times** and each time whenever **a node** is being extracted **from the heap** then a function called **heapify()** is being called to **rearrange** the element **according to their priority**. This function heapify() takes **O(logn)** time.

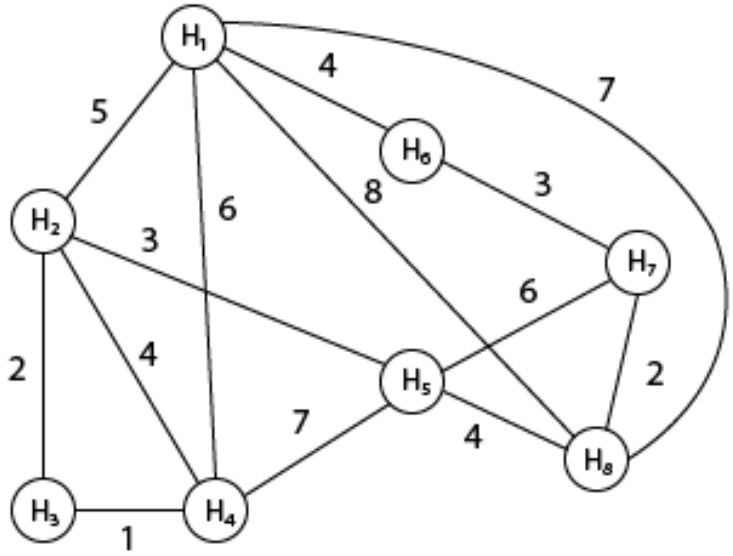
So, overall time= **2*(n-1) *log n = O(n log n)**

Travelling Sales Person Problem

The travelling salesman problems abide by a salesman and a set of cities. The salesman has to visit every one of the cities starting from a certain one (e.g., the hometown) and to return to the same city. The challenge of the problem is that the travelling salesman needs to minimize the total length of the trip.

- **Example:** A newspaper agent daily drops the newspaper to the area assigned in such a manner that he has to cover all the houses in the respective area with minimum travel cost. Compute the minimum travel cost.

- The area assigned to the agent where he has to drop the newspaper is shown in fig:



Travelling Sales Person Problem

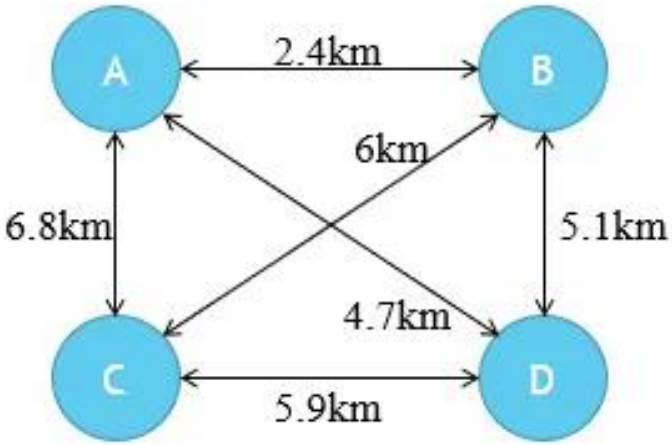
The TSP describes a scenario where a salesman is required to travel between n cities. He wishes to travel to all locations exactly once and he must finish at his starting point. The order in which the cities are visited is not important, but he wishes to minimize the distance traveled.

The greedy algorithm goes as follows:

1. Sort all of the edges in the network.
2. Select the shortest edge and add it to our tour if it does not violate any of the following conditions: there are no cycles in our tour with less than n edges or increase the degree of any node (city) to more than 2.
3. If we have n edges in our tour stop, if not repeat step 2.

Travelling Sales Person Problem

Find the optimal path for the following graph.

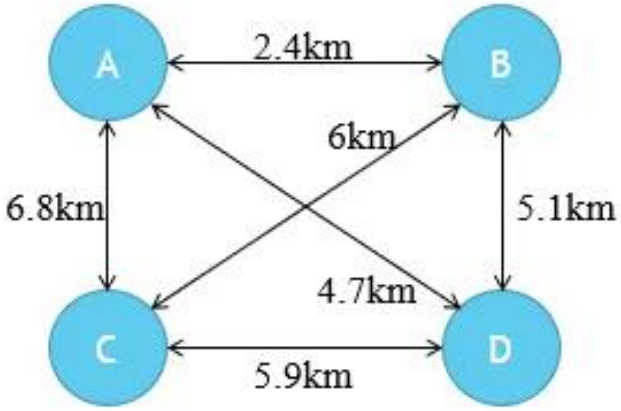


The weights of the edges of the graph is as follows-

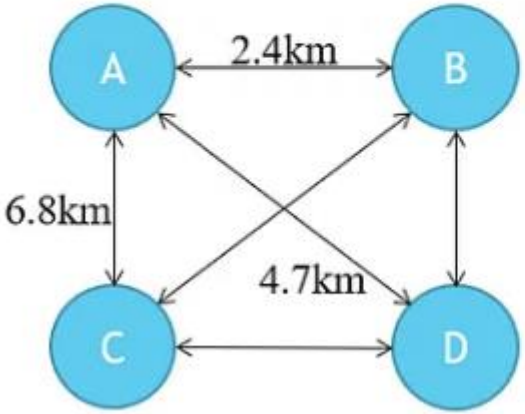
A <-> B = 2.4,
A <-> D =4.7,
A <-> C =6.8,
B <-> D = 5.1,
C <-> B =6,
C <-> D =5.9,

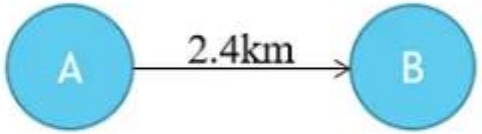
Travelling Sales Person Problem

Find the optimal path for the following graph.



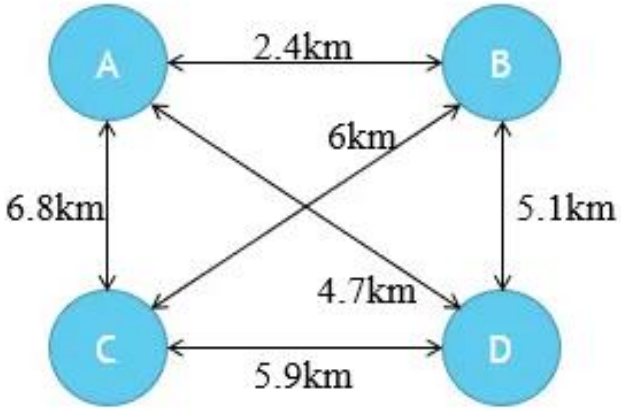
Step 1



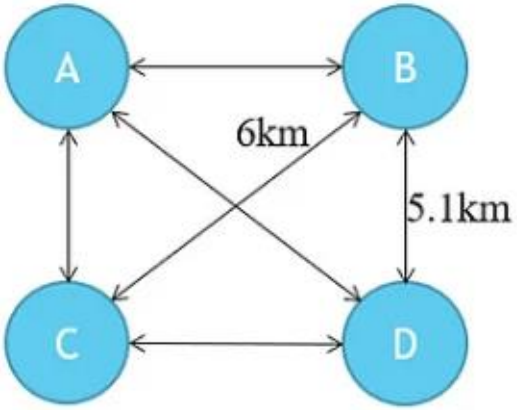


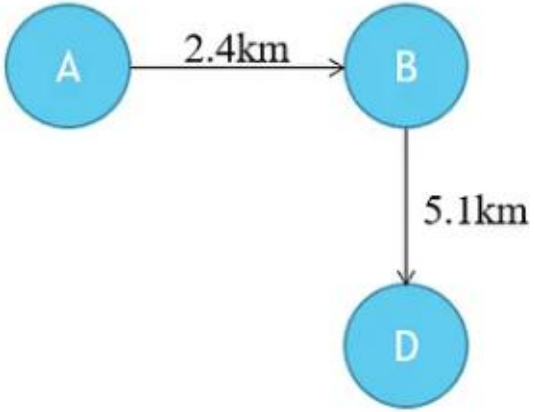
Travelling Sales Person Problem

Find the optimal path for the following graph.



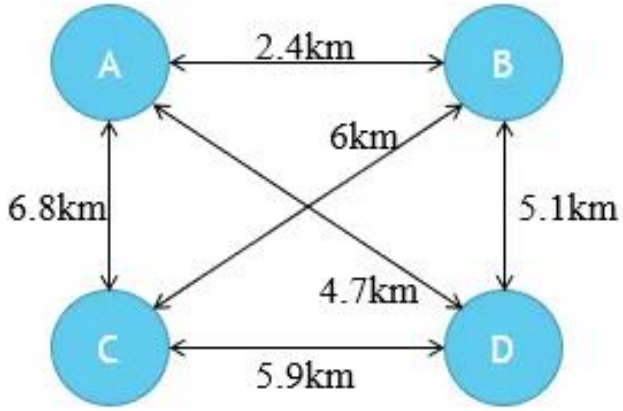
Step 2



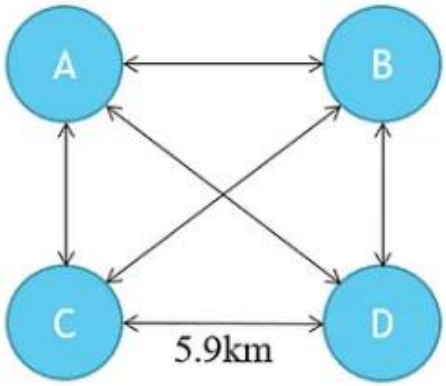


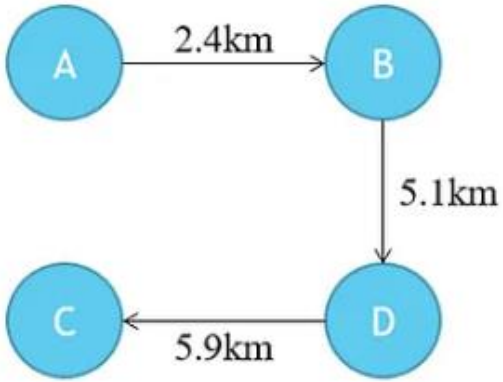
Travelling Sales Person Problem

Find the optimal path for the following graph.



Step 3





Travelling Sales Person Problem

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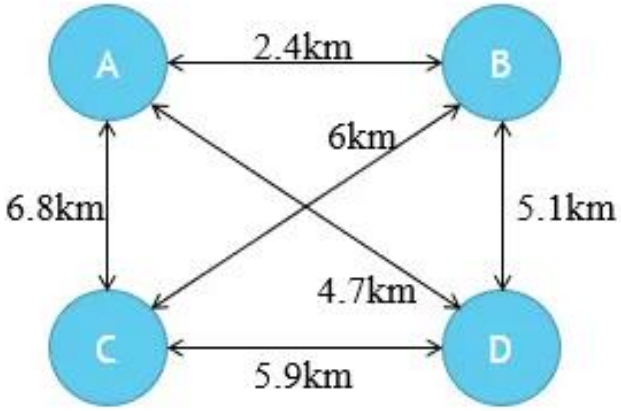
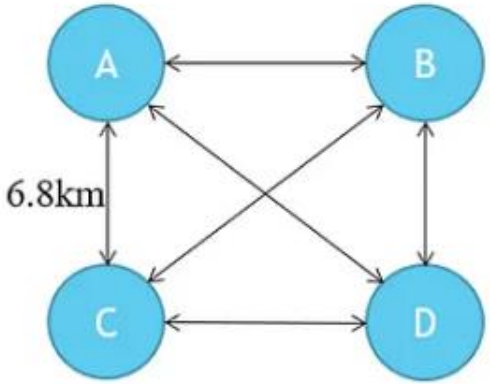
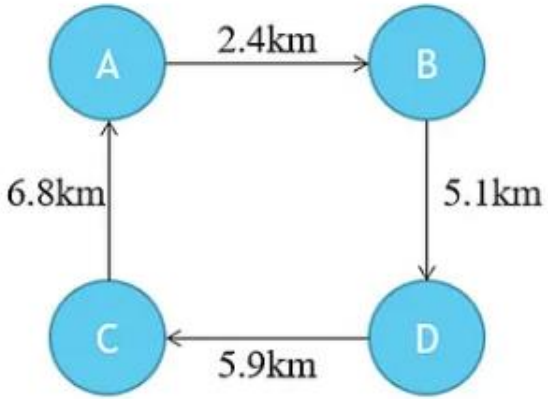


Step 4

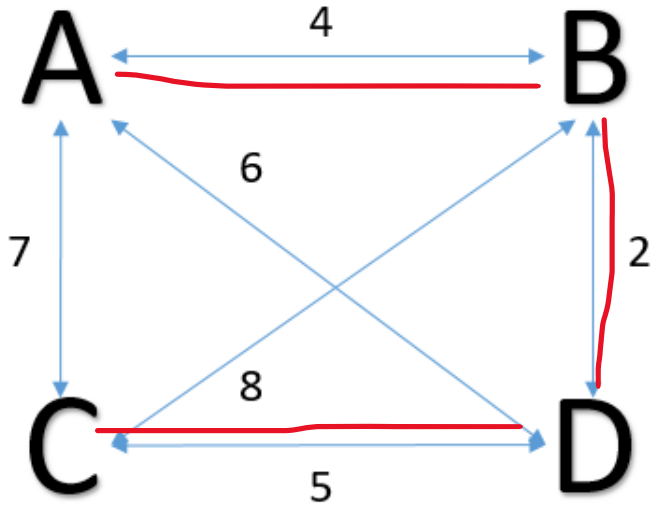
The Final answer is A -> B -> D -> C -> A = 2.4 + 5.1 + 5.9 + 6.8 = **20.2**





Travelling Sales Person Problem

Find the optimal path for the following graph.

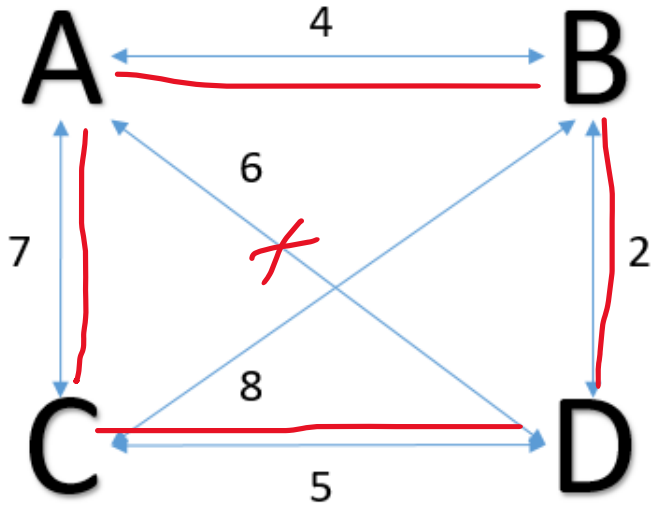


Path weight in ascending order
B <-> D = 2,
A <-> B = 4,
C <-> D =5,
A <-> D =6,
A <-> C =7,
C <-> B =8

Use algorithm and find the path
We can add the routes **B <-> D, B <-> A** and **C <-> D** to our tour without problem

Travelling Sales Person Problem

Find the optimal path for the following graph.



Path weight in ascending order
B <-> D = 2,
A <-> B = 4,
C <-> D =5,
A <-> D =6,
A <-> C =7,
C <-> B =8

Use algorithm and find the path
We can add the routes **B <-> D, A <-> B** and **C <-> D** to our tour without problem

We cannot add **A <-> D** to our tour as it would create a cycle between the nodes **A, B and D** and increase the order of node **D to 3**.
We therefore skip this edge and ultimately add edge **A <-> C** to the tour.

Travelling Sales Person Problem

Complexity: This algorithm searches for the local optima and optimizes the local best solution to find the global optima.

- It begins by sorting all the edges and then selects the edge with the minimum cost.
- It continuously selects the best next choices given a condition that no loops are formed.

The computational complexity of the greedy algorithm is **$O(N^2 \log2(N))$** and there is no guarantee that a global optimum solution is found.