

Artificial neural network (ANN) based model predictive control (MPC) and optimization of HVAC systems: A state of the art review and case study of a residential HVAC system



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ABSTRACT

In this paper, a comprehensive review of the artificial neural network (ANN) based model predictive control (MPC) system design is carried out followed by a case study in which ANN models of a residential house located in Ontario, Canada are developed and calibrated with the data measured from site. A new algorithm called best network after multiple iterations (BNMI) is introduced to help in determining the appropriate ANN architecture. The prediction performance of the developed models using BNMI algorithm was significantly better (between 6% and 59% better goodness of fit for various models) when compared to a previous study carried out by the authors which used the default single iteration ANN training algorithm of MATLAB®. The ANN models were further used to design the supervisory MPC for the residential HVAC system. The MPC generated the dynamic temperature set-point profiles of the zone air and buffer tank water which resulted in the operating cost reduction of the equipment without violating the thermal comfort constraints. When compared to the fixed set-point (FSP), MPC was able to save operating cost between 6% and 73% depending on the season.

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1. Introduction

Building sector consumes about 40% and 30% of total energy in U.S. and Canada respectively [1]. Space heating can consume up to 60% of total sector energy in countries with extreme weather conditions such as Canada [2]. Therefore, it is essential to investigate the methods for reducing energy consumption and operating cost of these systems. Heating, ventilation and air-conditioning (HVAC) systems are very complex and nonlinear systems due to the interaction of a large number of subsystems (e.g., chillers, boilers, heat pumps, pipes, ducts, fans, pumps and heat exchangers) and thermal inertia of the buildings. In order to calculate the total energy consumed by HVAC system, each of these subsystems need to be modeled precisely taking into account all the mass and energy transfer across each subsystem [3]. Using conventional forward or physics-based modeling methods, developing precise dynamic models of each subsystem is very difficult and a significant effort

is required for a detailed understanding of system physics [4]. On top of that, these models have a large number of parameters e.g., thermal capacitance and thermal conductance for heat transfer component in each subsystem. These parameters need to be either extracted from the manufacturer supplied data or estimated using the parameter estimation techniques. Using manufacturer supplied data, the estimated parameters are less precise if this data was recorded under different set of operating conditions. Using parameter estimation techniques require measurements of system performance data further complicating the development of forward models. Limitations of these forward modeling methods become very evident when models for a variety of HVAC system configurations need to be developed and tuned impeding the deployment of such methods in real world. Forward modeling methods are useful for simpler systems and research based analysis of complex systems but fail to satisfy industrial users. Traditionally, industry has been very reluctant to adapt complicated methods in modeling and control of HVAC systems and due to this reason majority of HVAC systems are still using very simple on/off or proportional-integral-derivative (PID) controllers instead of more modern controllers such as model predictive control (MPC). Lack of an advanced controller results in many performance and economic penalties such as higher energy consumption, higher operating cost, higher thermal

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discomfort and higher equipment wear and tear [5]. Researchers have shown that by adding a supervisory MPC controller to HVAC systems could result anywhere from 7% to more than 50% reduction in energy consumption and operating cost reduction [6–11]. On some so called advanced HVAC systems, rule-based supervisory controllers [12] are implemented which use the operator knowledge and apply it to reduce the energy and operating cost such as appropriate start/stop time of HVAC, night set-back, precooling and preheating etc. These rule-based controllers are simple to implement and require no modeling and design effort and can provide significant savings when applied correctly. Problem with these systems is that operator has to constantly monitor and adjust the HVAC operation to meet the objectives of reducing energy consumption while maintaining thermal comfort. Rule-based controllers generally are not anticipatory controllers i.e., they cannot look into the future and decide the course of action appropriately and rather work based on the current state of system. Maintaining a rule-based controller is a tedious process and advanced HVAC control task is better suited for an intelligent controller such as supervisory MPC which can automatically take into account the variations in weather parameters over a future horizon and control the appropriate settings of HVAC systems at present [13]. Furthermore, supervisory MPC can take into account the dynamic electricity price and adjust the set-points of local level controllers for active and passive thermal energy storage to offset the peak load to off-peak hours.

A simpler alternative to forward models is inverse or data-driven models [4,14]. Inverse models can be developed comparatively easily since they do not require the understanding of system physics. In order to train the inverse models, a comprehensive set of input-output data of system is needed under all possible working conditions. Therefore, the ease of development of inverse models comes at the cost of reduced generalization capability compared to the forward models. Accuracy of inverse models decreases when training data deviates from testing data. Therefore, it is critical to train the inverse models with a training data that covers all the operating conditions which could be challenging especially for large scale systems such as HVAC systems which operate under a wide range of weather conditions throughout the year. For such systems, models trained under one set of conditions may not be accurate enough under different set of test conditions therefore the adaptive models are sometimes used. Alternatively, many researchers use different models in heating and cooling seasons for HVAC systems. Researchers have developed many types of inverse models such as frequency domain models (first and second order over-damped process with dead time) [15], data mining algorithms (artificial neural network-ANN [16], support vector machine-SVM [17]), fuzzy logic models (fuzzy adaptive network [18], Takagi-Sugeno fuzzy models [19], adaptive network based fuzzy inference system [20]), statistical models (regression [21], auto regression exogenous [22], auto regression moving average exogenous [23], auto-regressive integrated moving average [24]), state-space models (sub-space state space identification [25]), geometric models (thin plate spline approximation [26]), case-based reasoning (topological case base modeling [26]), stochastic models (probability density function approximation [27]) and instantaneous models (just in time models [28]). Out of all these modeling methods, ANN is the most popular method due to its high accuracy to model nonlinear systems compared to other methods. ANN mimics the human brain by using several neurons in multiple layers. The weights of these neurons are generally trained by using supervised learning methods. Appropriately trained ANN can approximate any nonlinear process to a high degree of accuracy. There are many different types of ANN structures but multi-layer perceptron (MLP) feedforward structure is the most popular one. Other types of ANN structures include radial basis function neural network, recurrent

neural network and feedforward neural network with dynamic neutrons etc.

Since models developed with ANN are nonlinear, therefore, MPC approaches based on ANN are nonlinear. Linear MPC problems have guaranteed solution [29]. Minima of a linear optimization problem can be efficiently found by using optimization approaches such as active set method or interior-point method. On the other hand, nonlinear optimization problem may be non-convex and it may have many local minima. An algorithm that guarantees global minima of a nonlinear optimization problem does not exist in the literature yet. Another problem with nonlinear MPC is that the optimization solution might take a long time to converge which could be critical for fast moving processes though in the case of supervisory MPC for HVAC systems, this may not be an issue since supervisory MPC output only updates once every few minutes to every few hours. Global optimization methods such as evolutionary algorithms and simulated annealing can work with nonlinear optimization problems but present convergence and computational complexity issues.

The following are the main objectives and contributions of this paper:

- i Highlighting the current research trends in ANN based MPC and its applications to HVAC control systems;
- ii Development of a new ANN training algorithm to appropriately tune the network weights and aid in the selection of network architecture;
- iii Comparison of developed ANN models with a previous research paper from the authors to highlight the ANN prediction performance improvements as a result of appropriate training algorithm;
- iv Simulation of the ANN based MPC controller on the accurately calibrated residential HVAC system model; and
- v Analysis of potential energy and cost savings using MPC compared to the fixed set-points on a residential HVAC system.

2. Review of ANN based MPC and optimization of HVAC systems

This section presents an in-depth review of ANN based MPC and optimization research for various types of HVAC systems. These approaches differ based on the types of buildings and HVAC systems, control objectives, modeling data generation, ANN architecture and optimization method selection. Following subsections discuss each of these factors in detail with specific examples from literature. At the end of this section, the energy and cost savings potential of ANN-MPC approaches is presented.

2.1. Building and HVAC system

Supervisory and local level ANN-MPC for HVAC systems have been developed for a public university building (HVAC system with an outdoor air cooled inverted compressor unit and one internal unit for each of the four rooms) [6,30], a commercial university building (five rooms sensitive to thermal load with one air handling unit (AHU) for three floor building with variable air volume (VAV) terminal in each zone) [31], a commercial airport building (check-in hall with four thermal zones and constant air volume AHU) [32,7], an experimental research facility office building (commercial HVAC system with AHU and VAV terminals) [8,9,33,34–36], an automotive application (autonomous air conditioning system with a variable speed compressor) [37], an office building (integrated daylighting HVAC system) [11], an office building with warehouse and manufacturing area (four zone building with one HVAC subsystem for each zone) [10] and a school building [38].

Though ANN-MPC have been developed for a variety of buildings by HVAC researchers, it has not been used for a residential building and particularly a house. Many commercial buildings have some sort of supervisory control but most of the houses are lacking any form of such supervisory controller. Therefore, there is an opportunity to develop such controller for residential HVAC systems to reduce the energy consumption and operating cost. The second part of this paper discusses the development of ANN-MPC for residential HVAC system of a house.

2.2. Control objective

MPC is often employed at supervisory level to optimize the energy use at the building scale. The idea of using MPC to save building energy stems from the concept of supervisory control [7]. When MPC is implemented in the local level controller, the control objective is generally to minimize the tracking error and control effort. But when ANN-MPC is implemented in the supervisory layer, the control objective is commonly stated as one or a combination of the following:

- Minimize energy consumption [6,7,33–35,10,11,39,40],
- Maintain thermal comfort [6,33–35,10,11],
- Maintain indoor air quality (IAQ) at acceptable level [9],
- Minimize operating cost [10,41,7,42,30],
- Maintain visual comfort at acceptable level [11],
- Minimize retrofit cost [38], and
- Minimize thermal discomfort hours [38].

When control objective is stated as only one of the above statements, then it is called a single objective optimization and when more than one objectives are to be achieved simultaneously (e.g., simultaneous minimization of energy consumption and maintenance of thermal comfort [6]), then the optimization is called a multi-objective optimization.

Generally, HVAC systems comprise of many subsystems and the energy consumption of all of them is calculated and minimized simultaneously. Thermal comfort is calculated using the predicted mean vote (PMV) index. PMV index predicts the mean response of a group of people according to ASHRAE thermal sensation scale according to which a PMV of +3 is hot and -3 is cold. The neutral value of PMV is 0 where most of the people will feel comfortable. When used as control objective, the PMV is generally targeted in the [-0.5, +0.5] range [10]. PMV is calculated using Fanger's thermal comfort model based on the factors that determine the heat gain and loss (i.e., metabolic rate, clothing insulation, air temperature, mean radiant temperature, air velocity and relative humidity).

IAQ generally refers to the indoor air temperature, humidity and carbon dioxide (CO_2) level. Air temperature and humidity depends on internal (occupants and equipment) and external (weather) disturbances acting on the system. The air CO_2 level depends on the occupancy and occupant's activity. In order to control the temperature, humidity and CO_2 level, the HVAC systems use heating/cooling, humidification/dehumidification and ventilation systems respectively. A comfortable indoor temperature and humidity is different for summer and winter seasons and can be adjusted according to ANSI/ASHRAE Standard 55–2013 Thermal Environmental Conditions for Human Occupancy [43] and ANSI/ASHRAE Standard 62.1–2013 Ventilation for Acceptable Indoor Air Quality [44], respectively. According to Standard 55, the temperature could be approximately in the range [19.5, 27.8] °C. According to Standard 62.1, the relative humidity in occupied spaces should be controlled less than 65%. A higher humidity can increase the likelihood of conditions that can lead to microbial growth. The Standard 62.1 also determines the ventilation rate based on the CO_2 generation rate in the space.

If electricity price changes during the day, it is more beneficial to minimize the operating cost of the HVAC system compared to minimizing the energy consumption. In this case, the supervisory MPC stores the energy in the building mass by pre-cooling or pre-heating during the off-peak hours and tries to minimize the energy use during peak hours. It may increase the energy consumption of the HVAC system but the operating cost will be reduced [5].

Generally, maintenance of the visual comfort and minimization of retrofit cost is not considered while minimizing the HVAC system energy consumption. In this regards [11,38] present unique perspectives on these aspects. In [11], the energy consumption of HVAC system and daylight lighting system is minimized using a multi-objective optimization. In [38], the retrofit cost (cost of upgrading windows, adding more insulation and upgrading the HVAC system) is considered along with the energy consumption and thermal comfort and trade-offs between these three competing objectives are studied. It was shown in [38] that optimizing only one objective individually results in poor performance in other areas therefore all three of them need to be optimized simultaneously.

2.3. Modeling data

In order to develop data-driven models such as ANN, a rich data set is required which comprises of all possible working conditions. Data can be measured from the site under normal operation or under special data gathering test conditions by applying the pseudo-random binary sequence (PRBS) at the controlled inputs. Data can also be generated from the building energy simulation programs such as Transient System Simulation Tool (TRNSYS) and EnergyPlus Energy Simulation Software using the design of experiments method.

For example, PRBS was applied for zone set-points and air-conditioner on/off times in [6]. Similarly, in [31] during a 10-day test period PRBS was applied to generate random supply air temperature and static pressure set-points of AHU. Two controlled set-points were modified every 15 min and data was recorded at 1 min intervals. Data was averaged to 15 min intervals for modeling and was divided into 80% training and 20% validation data sets. In [35], more than 500 parameters comprising of air temperature, humidity, flow rate and energy consumption were recorded at 1 min intervals for 17 days. Each set-point was observed for 1 h. In [37], amplitude modulated PRBS was used to generate a series of amplitude-and-hold time-varying compressor set speeds over the entire operating space. A data set of 2000 points was recorded out of which 75% data was used for training and 25% data was used for testing.

In [11], the database was generated by the EnergyPlus model. In total 28 simulations were carried out to generate the database and 70% data was used for training and 30% was used for testing and validation. In [38], a comprehensive building model was developed in TRNSYS and simulation database was generated from this model to train and validate the ANN models.

The forecast of weather parameters can be estimated using the prediction models or downloaded from government servers where available. Three auto-regressive predictive models were used in [6] to predict the outdoor air temperature, humidity and global solar radiation. In [10], the outdoor air temperature and solar radiation was forecasted for 8 h into the future based on the previous day values corrected by current values. In [30], an intelligent energy autonomous weather station provided the outside air temperature, relative humidity and global radiation measurements as well as their forecasts over the user defined prediction horizon. One-day ahead weather forecast data was downloaded from Australian Bureau of Meteorology in [32,7]. In Canada and U.S., weather data can be downloaded from Meteorological Service of Canada's (MSC) hypertext transfer protocol (HTTP) data server and

National Oceanic and Atmospheric Administration (NOAA) data server, respectively.

Occupancy can be forecasted using schedules and can be measured using occupancy sensors such as passive infrared (PIR) sensors. Occupancy can also be estimated through the CO₂ generation rate in the indoor space. In [30], the occupancy forecast was assumed to be known due to the existence of schedules. Real-time occupancy was measured using PIR sensors and a movement signal was calculated based on PIR sensors to be used in predictive control. In [32,7], occupancy forecast was estimated from flight schedules.

Variable electricity price is generally known in advance. For example, in [30], price had a fixed structure comprising of 4 periods of taxation i.e., peak, normal, saver and super saver periods. In Toronto, electricity price has three-tiered structure i.e., off-peak, mid-peak and peak which was used in the next part of this paper.

All input/output data should be scaled to [-1,1] range prior to the training of ANN to improve the efficiency of training. Data normalization gives equal weight to all variables and suppresses any dominant variables due to magnitude differences in the data [38]. Some researchers have used the normalization of data within [0,1] range [45] but it is less common.

Appropriately selected parameters or variables can improve comprehensibility, scalability and possibly accuracy of the models [33]. If too many parameters are selected, they may contain redundant parameters which may not add any new information to the model accuracy but could increase its dimensionality resulting in increased computational expense during training. If too few parameters are used, some important process dynamics may be left un-modeled. Appropriate number of input variables for a model can be selected based on the physical properties [32], domain knowledge, forward selection method [32], boosting tree algorithm [8], wrapper algorithm [33] and correlation coefficient matrix [9]. For example, in [35], a combination of domain knowledge, boosting tree algorithm and wrapper algorithm was used to select 11 parameters out of 500 total measured parameters to develop four models for HVAC total energy consumption, average indoor temperature, average indoor humidity and average indoor CO₂ concentration.

2.4. ANN architecture

ANN models were used to predict the PMV index [6,30], outside weather parameters (air temperature, air humidity and global solar radiation) [6], room air temperature [6,30,32], radiant temperature, room air humidity [6,30], cooling valve position [7], outdoor air damper position [7], energy consumed (by chillers [8], fans [8], pumps [8], VAV box [8], domestic hot water heater and HVAC system [9,34]), indoor daylight illuminance [11], average CO₂ concentration [35] and chilled water head for pipe networks [39]. The following types of ANN were used for ANN-MPC developments for HVAC systems:

- MLP [34,31,10,11,38,40],
- MLP ensemble [8,9,33,35],
- Radial basis function (RBF) ANN [6],
- Nonlinear auto-regressive model with exogenous input (NARX) [7,32,37],
- RBFs used as NARX [30],
- Feedforward multi-layer self-growing ANN [10], and
- Artificial Neuro-Fuzzy Inference System (ANFIS) [39].

MLP is the most common type of ANN used in control systems [7]. It has an input layer, one or more hidden layers and an output layer. ANN is usually trained with back-propagation algorithm [38], Levenberg-Marquardt (LM) algorithm [6,37,11], Bayesian regularization algorithm [7,32] and Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [8,34,35]. Back-propagation is the most common

algorithm which uses gradient descent to find the minima. Back-propagation suffers from local minima and over fitting problems reducing the generalization of ANN. Bayesian regularization algorithm has slow convergence speed compared to back-propagation but improves the generalization of ANN and prevents over fitting. LM algorithm has faster convergence speed compared to other common optimization techniques such as steepest descent, Gauss-Newton and quasi-Newton. BFGS algorithm has good performance for very large numbers of variables and non-smooth optimizations. It also achieves both fast decrease and fast convergence compared to back propagation algorithm [40] (Zaheeruddin & Ning) (Zaheeruddin & Ning [45]).

When measured, data is used for ANN training, it is called supervised learning otherwise it is called unsupervised learning. Generally, a cost function such as mean squared error (MSE) or root mean squared error (RMSE) is used to terminate the training of ANN. The training of ANN terminates when the MSE or RMSE either stabilizes or drops below a certain desired value or a pre-defined number of epochs have elapsed. At the end of training, resulting ANN weights are optimal and its predictions are very close to measured data. Measured data is divided into training and validation data sets usually with 70% training data and 30% validation data. An appropriately trained ANN performs equally well on both training and testing data sets. Performance of ANN against measured data is analyzed using absolute error (AE), mean absolute error (MAE) [6,8], maximum absolute error (\max_{AE}) [6], normalized relative mean squared error (NRMSE) fitness [32], mean absolute percentage error (MAPE) [8], standard deviation of absolute error (Std_{AE}) [8], standard deviation of absolute percentage error (Std_{APE}) [8], MSE [37], RMSE [7] and linear correlation coefficient [37]. A more detailed discussion and formulation of these various types of performance comparison metrics was provided in [4].

Since the output of a single MLP is dependent on ANN weights which are sensitive to the initial conditions, sometimes a group of MLPs called the MLP ensemble is used. The MLP ensemble output is determined by majority vote. MLP ensemble was shown to outperform other modeling methods such as single MLP, classification and regression tree (CART), chi-square interaction detector (CHAID), boosting tree, random forest, multivariate adaptive regression splines (MARSplines) and SVM in [8] and [16].

Activation functions are used for computing the output of a neuron in each layer. Commonly used types of activation functions include sigmoidal, hyperbolic, RBF and linear. If RBFs (a Gaussian) are used as the activation functions of ANN, then it is called RBF ANN. RBFs usually have only one hidden layer. The output of RBF ANN is a linear combination of RBFs in hidden layer. Compared to MLP; RBF ANN does not suffer from local minima.

MLP is a simple feed forward network which means that there is no feedback in the network and output signals are entirely calculated from the present values of input signals. Since many dynamical systems rely on the previous values of input signal and output signal, a new network structure can be formed by passing the input signals and output signals through time delay networks before connecting to the network inputs. Such ANNs which use previous values of input and output signals are called recurrent networks. NARX is a type of recurrent network whose model is based on the linear auto-regressive exogenous model [7]. NARX has feedback connections enclosing several layers of the network.

ANN architecture (number of neurons in input, output and hidden layers) is generally identified based on trial and error [38,40], suggestion in previous empirical studies [37], chosen randomly [34,35], forward selection method [32], or automatic identification method [6,30,10,11]. Feedforward multi-layer self-growing ANN is simple MLP structure with one hidden layer whose architecture is defined automatically during the training phase by cascade-correlation algorithm. The number of hidden neurons

determines the model complexity of ANN. Increasing the number of hidden layer neurons increases the chances of over-fitting training data, reduces the generalization capability of ANN and also minimizes the training error [38]. Therefore, it is important to choose the number of hidden layer neurons appropriately either manually or using an automatic architecture selection algorithm such as cascade-correlation algorithm and multi-objective genetic algorithm (MOGA). Cascade-correlation algorithm starts ANN architecture with only one hidden layer neuron and minimizes the training error. After ANN is trained, another neuron is added to the hidden layer and ANN is trained again. If ANN has lower training error after adding the neuron compared to the older ANN, the ANN keeps growing until training error stops reducing any further. This method is very useful in finding the appropriate structure of ANN but is computationally intensive. For example, in [10] cascade-correlation algorithm was used to find the ANN structure for approximating air temperature, radiant temperatures and electrical power consumed by HVAC subsystems. Resulting models had 18–24 neurons in the hidden layer. Though automatic tuning can be computationally expensive; it is still more efficient than commonly used tedious trial and error method for ANN structure identification when a large number of ANNs are to be trained. MOGA defines the ANN input/output structure and number of neurons. MOGA was used in [6] to find the ANN architecture for the models of PMV index, outside weather parameters (i.e., air temperature, air humidity and global solar radiation) and indoor air parameters (i.e., temperature and humidity). The models found with MOGA had RBF activation functions and different number of hidden layer neurons for each model (i.e., 5 neurons for PMV, 14 neurons for room air temperature in summer, 11 neurons for room air humidity in summer, 7 neurons for room air temperature in winter and 11 neurons for room air humidity in winter).

ANFIS is a more complicated multilayer hybrid model structure based on both fuzzy logic and ANN. It has no limitation on its network structures and node functions and can be implemented in the framework of adaptive ANNs. Unlike ANN, each layer in ANFIS performs a different function although the node architecture and properties remain unchanged in a single layer but vary across the layers. Genetic algorithm (GA) based ANFIS was reported to perform better than ANN to predict the energy use of two buildings in [46]. Due to more complicated structure, training time for ANFIS is also greater than ANN on large datasets.

2.5. Optimization method

Unlike linear systems, finding optimal solutions for nonlinear systems is a difficult task, since analytical solution usually is not available for nonlinear systems. Therefore, for nonlinear systems numerical techniques, such as dynamic programming and gradient methods have to be employed. Projected and augmented Lagrange multiplier methods do not perform well because of the equality constraints used in the problem formulation and the generalized reduced gradient method provides consistent results, only if it starts with a feasible solution [45]. The following nonlinear optimization methods are used for ANN-MPC research:

- GA [10,11],
- Modified GA or MOGA [39,38],
- Newton-Raphson method [37],
- Interior-point method [34],
- Branch and bound (BaB) method [6,30],
- Particle swarm optimization (PSO) algorithm [8,31],
- Modified or multi-objective PSO (MOPSO) [35],
- Strength Pareto evolutionary algorithm (SPEA) [31],
- SPEA with local search (SPEA-LS) [9],

- Strength MOPSO (S-MOPSO) algorithm [9],
- Firefly algorithm [36], and
- Harmony search [31].

GA is the most common nonlinear optimization method among ANN-MPC researchers. GA is a numerical optimizer able to deal with integer values. It is efficient in the search for optimal solution and solves both constrained and unconstrained optimization problems. GA is empirically proven to provide robust searches in complex spaces [10]. In GA, the objective function is minimized without calculating the derivatives and it is not restricted to the estimation of uncorrelated parameters. GA is less sensitive to initialization due to its probabilistic nature. GA produces more than one solution (individual) or a population of good individuals. The drawback of GA is that it is slow to converge for complex problems and therefore, is not suitable for real-time implementations. GA is also less efficient and slower than traditional optimization methods for simple problems [10]. MOGA is used to solve optimization problems with multiple objectives such as simultaneous minimization of energy consumption and thermal discomfort which are often conflicting objectives.

Newton-Raphson method was used in [37] which is a quadratically converging algorithm. It converges in lower number of iterations compared to other techniques and is faster and more efficient algorithm for real time control applications. Interior-point method solves the continuously differentiable cost function and constrains and guarantees the optimal solution [34]. BaB is an optimization technique which is used for the optimization of discrete systems. BaB has guaranteed optimal solution. It is not negatively affected by the constraints and is not sensitive to poor initialization [6].

PSO is a well-balanced mechanism to enhance and adapt to global and local exploration problems. PSO is excellent in solving single-objective optimization problems with fast convergence [8]. Canonical PSO can only be used to solve the single-objective optimization problems whereas MOPSO can solve the multi-objective optimization problems [35]. In general, due to the conflicting objective functions there is no unique solution to multi-objective optimizations but a set of non-dominated (Pareto optimal) solutions [38]. SPEA has advantage in searching global solution while PSO is applicable in searching for optimal local solutions. S-MOPSO is the combination of SPEA and PSO algorithms and is effective in finding optimal solutions of nonlinear multi-objective models [33].

Firefly algorithm is efficient in finding global optima. Firefly algorithm outperformed PSO and evolutionary strategy in minimizing the energy consumption of HVAC system in [36]. Firefly algorithm was more effective, efficient, robust and used less central processing unit (CPU) time to converge compared to PSO and evolutionary strategy [36].

Harmony search was used in [31] to minimize the HVAC energy consumption and room temperature ramp rate. Harmony search was compared to PSO and SPEA in [31]. Both PSO and Harmony search had lower computational time and higher computational frequency compared to SPEA. Therefore, SPEA is not suitable for online optimization.

2.6. Energy and cost savings potential

ANN-MPC have shown to outperform the more basic controllers such as PID and on/off controllers. Due to the supervisory nature of MPC, the energy and cost savings range anywhere from 7% to 50%. Following are the examples from literature which use ANN-MPC to reduce the energy and cost of operating HVAC systems. A summary of these ANN-MPC approaches and their resulting energy and cost savings is provided in Table 1.

Table 1

A summary of ANN-MPC approaches.

Reference	System	Supervisory Control	ANN model for MPC development	Cost savings investigation	Energy (Cost) Savings
[47,32]	Airport terminal building	✓	x	x	18%
[7]	Airport terminal building	✓	x	✓	-5.4% (13%)
[48]	Residential building	✓	x	x	18%
[49]	Apartment building	x	x	x	-4.5%
[6]	University building	✓	✓	x	50%
[42,30]	University building	✓	✓	✓	13.5% (7.8%)
[35,36,31]	Office building	✓	✓	x	8.13%
[10]	Office building	✓	✓	x	14.7%
[41]	Office building	x	✓	✓	(31%)

Note: A negative energy savings value means that the higher energy was consumed compared to baseline and vice versa.

In [32] and [47], ANN model was used to simulate the behavior of an airport terminal building whereas the resistor-capacitor (RC) network model was used for the controller development. The RC network based MPC supervisory controller was used to evaluate the energy-savings potential. Compared to the baseline fixed set-point (FSP) of 22 °C, MPC resulted in 5%, 18% and 13% energy savings when used alone, with optimized start-stop control strategy and with precooling strategy respectively. In this paper, ANN was not used for the development of supervisory controller and cost savings potential were not investigated under the variable price structure.

The method presented in [47] was further developed in [7]. The nonlinear building and HVAC system was modeled using the RC networks. The zone and HVAC system models contain bilinear terms where the control states were multiplied by the control inputs and disturbances. In order to avoid the use of non-linear

optimization methods, the models were linearized by using the ANN based feedback linearization technique. This resulted in linear models which were used with MPC based on linear programming technique. The cost savings investigations were carried out under a two-tiered price structure of Sydney. MPC resulted in 13% cost savings when compared to baseline control. Since thermal energy was stored in the building mass during the off-peak period, the energy consumption increased about 5.4% compared to baseline control but since this energy was consumed during off-peak period when electricity price was cheap, it resulted in 13% lower costs. This paper did not use ANN model during optimization.

Researchers in [48] developed a random neural network (RNN) based controller to control the temperature of four rooms in a single story residential building. The intelligent controller manipulated the flowrate of hot water through a radiator consisting of

**Fig. 1.** Major subsystems of residential HVAC system of TRCA-ASH.

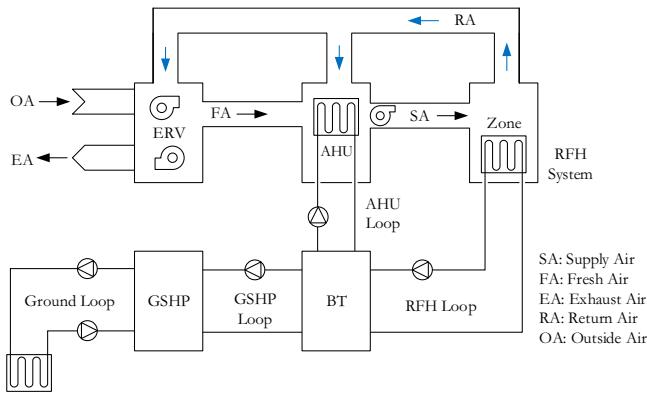


Fig. 2. Block diagram of residential HVAC system of TRCA-ASH.

temperature regulating valve to control the heating in rooms. RNN based controller was compared to ANN based controller and MPC. MPC was developed using the subspace state space system identification (4SID) model. RNN controller was better at maintaining the PMV-based set-points compared to ANN controller. ANN controller performed poorly when met with the set-points not included in the training data whereas RNN was able to maintain those set-points properly. The performance of RNN was comparable to 4SID based MPC controller in terms of energy consumption and process regulation. Both RNN and MPC were able to save 18% more energy compared to ANN controller.

In [49], ANN based predictive controller was developed to control the radiant floor heating (RFH) system in an apartment building in South Korea. The developed ANN based predictive controller was compared to a simple two-position on/off controller. Simple on/off controller was unable to regulate the zone temperature within a desired band due to the time delay in process response for high thermal inertia of apartment building resulting in discomfort of occupants and higher energy consumption. ANN based predictive controller due to its anticipatory action, was able to accurately regulate the process at its set-point and easily handled the time delay in the response. It is important to note that ANN based predictive controller proposed in [49] is not based on MPC algorithm. It rather uses ANN model to predict the on and off times of heat supply so that the zone temperature does not over or undershoot the desired differential temperature range. ANN based predictive controller was able to regulate the room temperature more precisely compared to on/off controller but at the cost of consuming about 4.5% more energy. This shows us that the implementation of local

level predictive controller could result in higher energy consumption if the objective is to improve the process regulation instead of reducing the energy consumption.

A university building was controlled using ANN-MPC controller in [6]. Researchers developed ANN based models for PMV, zone temperature and humidity, outside temperature, humidity and solar radiation to be used for controller development. Four zones were controlled by split air conditioners each having an outdoor air cooled inverter compressor unit and an indoor unit. RBF ANNs were used to develop all the models. The approach is limited in the sense that it does not use cost savings potential and instead focused on the energy savings only. Last but not least, the weather forecast data was predicted on-site instead of downloading from a government weather data server which potentially has more accurate and reliable prediction. Nevertheless, the researchers in [6] showed that the implementation of ANN-MPC resulted in savings above 50%. For optimization, the BaB method was used instead of non-linear optimization techniques since it always guarantees an optimal solution.

The researchers in [42,30] presented ANN-MPC controller to control the indoor air temperature at a university building and tried to improve the controller design presented in [6]. In contrary to most other approaches, the research in [42,30] considered the economic cost function representing the cost of operating HVAC system instead of energy consumption. Using the economic cost function resulted in 13.5% lower energy consumption as well as 7.8% higher cost savings when compared with the approach in [6] which only minimized the energy consumption.

In [35], MLP ensemble was used to model the AHU energy consumption, AHU supply air temperature, AHU static pressure and IAQ of an office building. MLP ensemble based MPC controller was used to predict the optimal settings of AHU static pressure and supply air set-points while minimizing the energy consumption and maximizing the thermal comfort. An evolutionary computation algorithm based on modified PSO was used to solve the multiple objective nonlinear optimization problem. The design of MLP ensemble based MLP resulted in an average estimated energy savings of 8.13% compared with the baseline control. The development of MLP type ANN-MPC for the same system was also reported in [36] where researchers investigated the use of a meta-heuristic search algorithm (i.e., firefly algorithm) during optimization instead of PSO. Firefly algorithm was efficient in finding global optima and outperformed PSO in minimizing the energy consumption. Firefly algorithm also had lower average CPU time of 12.3 s per data point compared to PSO's 13.0 s. Strength of modified-PSO reported in [35] compared to firefly algorithm lies in solving multiple

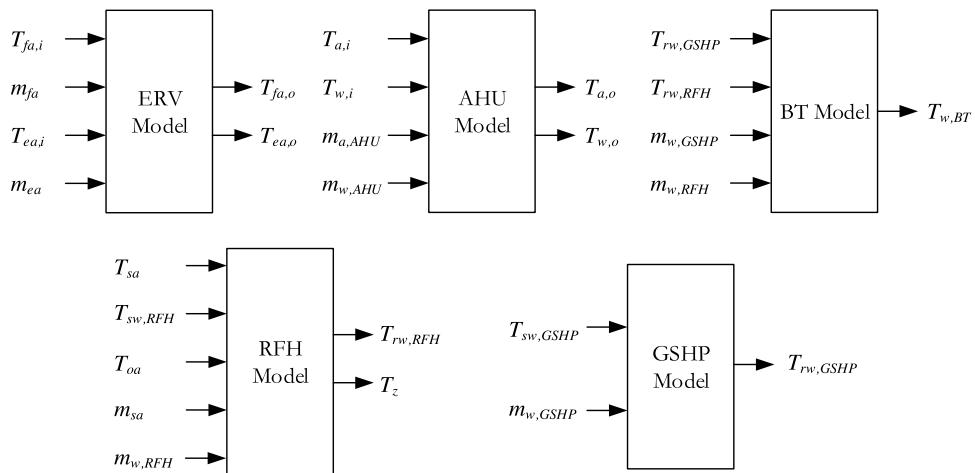


Fig. 3. Inputs and outputs of each subsystem.

Table 2
Nomenclature.

Symbol	Description
E_{TOU}	Electricity time-of-use price
G	Goodness of fit
$m_{a,AHU}$	Flow rate of air passing through AHU
m_{ea}	Flow rate of exhaust air
m_{fa}	Flow rate of fresh air
$m_{ERV,sp}$	Set-point of ERV air
m_{ra}	Flow rate of return air
m_{sa}	Flow rate of supply air
$m_{w,AHU}$	Flow rate of water in AHU loop
$m_{w,GSHP}$	Flow rate of water in GSHP loop
$m_{w,RFH}$	Flow rate of water in RFH loop
n	Number of samples
N	Horizon size of MPC optimization
t	Time
$T_{a,i}$	Temperature of mixed air at the inlet of AHU
$T_{a,o}$	Temperature of air at the outlet of AHU
$T_{ea,i}$	Temperature of exhaust air at the inlet of ERV
$T_{ea,o}$	Temperature of exhaust air at the outlet of ERV
T_{fa}	Temperature of fresh air entering the zone
$T_{fa,i}$	Temperature of fresh air at the inlet of ERV
$T_{fa,o}$	Temperature of fresh air at the outlet of ERV
T_{oa}	Temperature of the outside air
T_{ra}	Temperature of the return air
$T_{rw,GSHP}$	Temperature of return water from GSHP to BT
$T_{rw,RFH}$	Temperature of return water from RFH to BT
T_s	Execution time of supervisory MPC optimization
T_{sa}	Temperature of supply air
$T_{sw,GSHP}$	Temperature of supply water from BT to GSHP
$T_{sw,RFH}$	Temperature of supply water from BT to RFH
$T_{w,BT}$	Temperature of water inside BT
$T_{w,BT,sp}$	Set-point temperature of water inside BT
$T_{w,i}$	Temperature of water at the inlet of AHU cooling coil
$T_{w,o}$	Temperature of water at the outlet of AHU cooling coil
T_z	Zone temperature
$T_{z,sp}$	Set-point of zone temperature
u_{AHU}	Control signal of AHU
u_{RFH}	Control signal of RFH
u_{GSHP}	Control signal of GSHP
u_{RFH}	Control signal of RFH
W_{AHU}	Power consumption of AHU
W_{ERV}	Power consumption of ERV
W_{GSHP}	Power consumption of GSHP
W_{RFH}	Power consumption of RFH
y	Measured data
\hat{y}	Estimated data
Abbreviations	
4SID	Subspace State Space System Identification
AE	Absolute Error
AHU	Air Handling Unit
ANFIS	Artificial Neuro-Fuzzy Inference System
ANN	Artificial Neural Network
ANSI	American National Standards Institute
ASH	Archetype Sustainable House
ASHA	Archetype Sustainable House A
ASHB	Archetype Sustainable House B
ASHRAE	American Society of Heating, Refrigerating and Air-Conditioning Engineers
BaB	Branch and Bound
BFGS	Broyden-Fletcher-Goldfarb-Shanno
BNMI	Best Network after Multiple Iterations
BT	Buffer Tank
CART	Classification and Regression Tree
CHAID	Chi-square Interaction Detector
CO_2	Carbon dioxide
CPU	Central Processing Unit
DAQ	Data Acquisition
ERV	Energy Recovery Ventilator
FSP	Fixed-Set-Point
GA	Genetic Algorithm
GSHP	Ground Source Heat Pump
HTTP	Hypertext Transfer Protocol
HVAC	Heating, Ventilation and Air-Conditioning

Table 2 (Continued)

Symbol	Description
IAQ	Indoor Air Quality
LM	Levenberg-Marquardt
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MARSplines	Multivariate Adaptive Regression Splines
MLP	Multi-Layer Perceptron
MOGA	Multi-Objective Genetic Algorithm
MOPSO	Multi-Objective Particle Swarm Optimization
MPC	Model Predictive Control
MSC	Meteorological Service of Canada
MSE	Mean Squared Error
NARNET	Nonlinear Auto-Regressive Neural Network
NARX	Nonlinear Auto-Regressive Exogenous
NOAA	National Oceanic and Atmospheric Administration
NRMSE	Normalized Relative Mean Squared Error
PID	Proportional-Integral-Derivative
PIR	Passive Infrared
PSO	Particle Swarm Optimization
RC	Resistor-Capacitor
RFH	Radiant Floor Heating
RNN	Random Neural Network
SVM	Support Vector Machine
PMV	Predicted Mean Vote
PRBS	Pseudo-Random Binary Sequence
RBF	Radial Basis Function
RMSE	Root Mean Squared Error
S-MOPSO	Strength Multi-Objective Particle Swarm Optimization
SPEA	Strength Pareto Evolutionary Algorithm
SPEA-LS	Strength Pareto Evolutionary Algorithm with Local Search
TRCA	Toronto and Region Conservation Authority
TRNSYS	Transient System Simulation Tool
VAV	Variable Air Volume

objective optimization problem. In yet another paper [31], same system was investigated and PSO was compared with harmony search optimization method for the development of MLP type ANN-MPC. Harmony search converged in fewer iterations and had lower computational time per iteration as well when compared to PSO. All of these researches [35,36,31] did not carry out the cost savings investigations and only focused on the energy savings.

Adaptive online ANN-MPC was designed for automotive air conditioning system with a variable speed compressor in [37]. Compared with offline ANN-MPC and a PID controller; online ANN-MPC had superior reference tracking and disturbance rejection. Newton-Raphson method was used to minimize the cost function. There was no supervisory controller and cost savings investigations in this paper.

In [10], ANN-MPC was developed for the HVAC system of an office building in France. The building had four zones in it comprising of two office zones, one manufacturing area and one warehouse zone. Researchers developed lower order ANN models for the development of controller and used GA to minimize the cost function. Supervisory MPC measured the air and radiant temperature of four zones and generated the set-points for local zone temperature controllers. This paper did not investigate the cost savings. The whole study was based on the simulation model generated in EnergyPlus and no real-world data was used for training or validation of models. Compared to a baseline strategy used in real non-residential buildings in France, ANN-MPC resulted in 5.2% and 14.7% reduced energy consumption during heating and cooling seasons respectively.

A model of an office building in US was developed using nonlinear auto-regressive neural network (NARNET) in [41]. This model was used for the development of MPC to minimize the energy cost considering a dynamic demand response signal. The building also had an on-site energy storage and generation. The cost function was

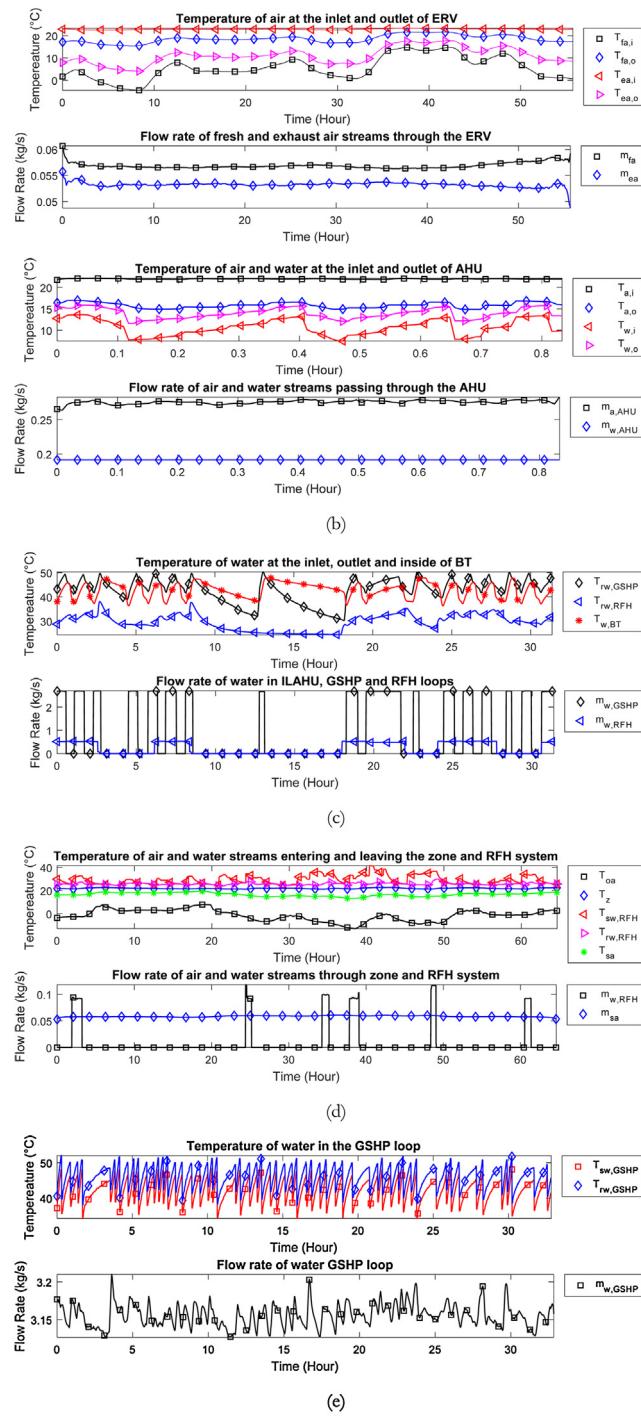


Fig. 4. Modeling and validation data used for each subsystem ANN model (a) ERV, (b) AHU, (c) BT, (d) RFH and (e) GSHP.

minimized by mixed-integer non-linear programming solved with IBM-ILOG CPLEX solver. Compared to a baseline night setup strategy (i.e., two different FSPs during day and night), ANN-MPC with energy storage and energy generation resulted in 31% cost savings.

3. Case study: residential HVAC system

Following section describes the simulation of ANN-MPC on a residential HVAC system of Toronto and Region Conservation Authority (TRCA) Archetype Sustainable House (ASH) located in Vaughan, Ontario, Canada. There are two identical semi-detached

houses called House A (ASHA) and House B (ASHB). ASHA has the HVAC system found in many Canadian households comprising of heat recovery ventilator, AHU and air source heat pump. Whereas, ASHB has the futuristic HVAC system focused on reducing the energy consumption and operating cost reduction of the system. The ASHB system is explained in more detail in the following subsection. This paper only focuses on the HVAC system of ASHB. For further details of ASHA and ASHB systems, reader is referred to [50–59].

3.1. System description

Residential HVAC system of TRCA-ASH is shown in [Figs. 1 and 2](#). Residential HVAC system comprises of many subsystems called energy recovery ventilator (ERV), AHU, ground source heat pump (GSHP), buffer tank (BT), RFH and zone. Fans are used to move the fresh air, return air, supply air, exhaust air and outside air while pumps are used to circulate the water in RFH, AHU, GSHP and ground loops.

ERV preheats the fresh air during winter and precools the fresh air during summer by exchanging energy between outside air and exhaust air streams. During summer outside air is at a higher temperature compared to return air from the zone whereas in winter it is reversed. Fresh air enters AHU where it is mixed with return air from the zone. Mixed air passes through the cooling coil and is supplied to the zone during summer. During winter season, the zone temperature is maintained by RFH system. Both AHU and RFH systems receive cold and hot water from BT respectively. During summer BT stores cold water whereas, during winter season it stores hot water. Temperature of BT water is maintained by GSHP during both seasons. GSHP has a ground loop where heat is rejected to ground during summer and gained during winter and transferred to the BT water.

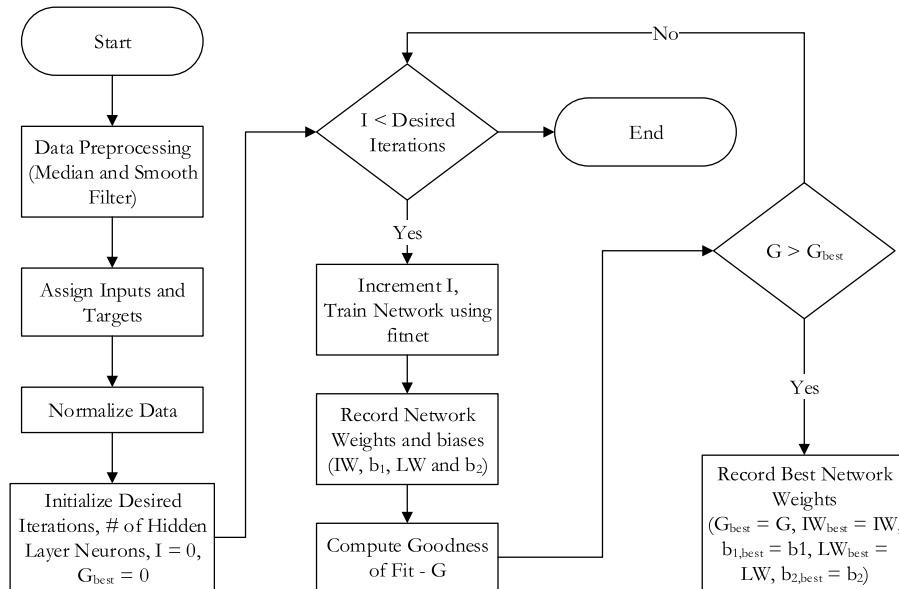
Data acquisition (DAQ) system measures the temperature and flow rate of each air and water stream at the inlet and outlet of each subsystem as well as the energy consumed by fans, pumps and GSHP compressor at an interval of 5 s and stores it into a database.

3.2. System modeling

ERV, AHU and RFH are multiple input and multiple output systems whereas BT and GSHP are multiple input and single output systems. Data was extracted from database for modeling and validation. Inputs and outputs of each of the subsystem are shown in [Fig. 3](#). The symbols are defined in [Table 2](#).

3.2.1. Pre-processing of data

Since DAQ system records data at 5 s' interval, the data captures all the process dynamics at a very high resolution. The dynamics of HVAC system are comparatively slow and the temperature and flow rate changes do not occur at such a fast rate. Therefore, the data was down sampled to reduce the number of data points and improve the convergence speed of ANN training algorithm. The data for each subsystem was down sampled at a different resolution depending on the system dynamics. For example, the temperature inside the ERV changes very slowly since it is dependent on the outside air temperature and zone air temperature both of which are changing at a slow rate, therefore, a much larger sampling time of 500 s was used for down sampling the ERV data. In contrast, the temperature and flow rate inside the AHU changes at a much faster rate, therefore, a shorter sampling time of 30 s was chosen. The sampling time for each system was chosen such that the down sampled data captures all the process dynamics while reducing the unnecessary and redundant data. Sampling time for each of the BT, RFH and GSHP datasets was 50 s. In order to remove the spiked noise and random noise; median and moving average smoothing

**Fig. 5.** Flowchart of BNMI ANN Training Algorithm.

filters were applied to the data respectively. The data for ANN model development and validation is shown in Fig. 4.

3.2.2. ANN training algorithm and architecture selection

MLP with one input layer, one hidden layer and one output layer was chosen to model each subsystem. Since two major factors i.e., starting weights and number of neurons in the hidden layer have most influence on ANN performance, they were determined very carefully. In order to find best weights for ANN, a new algorithm called Best Network after Multiple Iterations (BNMI) was developed to train the ANN multiple times and record the best weights.

Fig. 5 shows the flow chart of BNMI training algorithm. At the start of algorithm, pre-processing of data (i.e., median and smooth filtering) is carried out. After data preprocessing; inputs and targets are identified and data is normalized to $[-1,1]$ range. Then desired number of iterations and number of hidden layer neurons are specified. BNMI algorithm trains the ANN using 'fitnet' function in MATLAB®, and records weight and bias matrices of trained ANN. After that the goodness of fit (G) is computed as follows to evaluate the performance of trained ANN against measured data:

$$G = \left(1 - \frac{\sqrt{\sum_{i=1}^n (\hat{y}_i - y_i)^2}}{\sqrt{\sum_{i=1}^n (y - \frac{\Sigma y}{n})^2}} \right) \times 100. \quad (1)$$

where y , \hat{y} and n are measured data, estimated data and total number of data points respectively.

BNMI algorithm compares the value of G for each trained ANN against best recorded value of G i.e., G_{best} during previous training iterations. When value of G of current iteration is better than G_{best} , the algorithm updates G_{best} with current value and records the best ANN weights as well. The algorithm then keeps repeating until the specified number of iterations are completed. In this paper, the desired number of iterations was set at 1000. Therefore, BNMI algorithm trains 1000 ANNs with random starting weights and records the weight and bias matrices of best trained ANN out of them which has highest value of G .

Selection of the number of neurons in hidden layer was also made systematically by training the ANN of each subsystem with number of neurons between 1 and 50. The value of G increases as the number of neurons increase in hidden layer as seen in Fig. 6. Beyond a certain point, increasing the number of neurons in hidden layer, the value of G does not increase any further. Based on this, the optimum number of neurons can be selected when the value of G stops to increase appreciably. Therefore 20 hidden layer neurons were selected for each of the ERV, AHU and BT models, 40 neurons were selected for RFH zone model and 5 neurons were selected for GSHP model.

3.2.3. Performance of ANN models

The performance of ANN models is summarized in Table 3. The value of G for each model is very high and \max_{AE} and MAE is very low. Compared to ANN models developed in a previous paper

Table 3
Performance of ANN Models on the Validation Dataset.

Model		ANN Architecture	G (%)	\max_{AE} (°C)	MAE (°C)	Improvement over previous paper (%)
ERV	T_{eo}	4-20-2	96.8%	1.025	0.077	12.7%
	T_{fao}		96.3%	0.413	0.048	11.3%
AHU	$T_{w,o}$	4-20-2	86.1%	0.901	0.099	23.9%
	$T_{a,o}$		85.6%	0.315	0.067	24.7%
BT	$T_{w,BT}$	4-20-1	97.2%	0.699	0.062	10.0%
RFH	T_z	5-40-2	79.3%	0.728	0.053	59.7%
	$T_{rw,RFH}$		87.5%	2.365	0.069	57.0%
GSHP	$T_{rw,GSHP}$	2-5-1	89.4%	4.004	0.185	6.85%

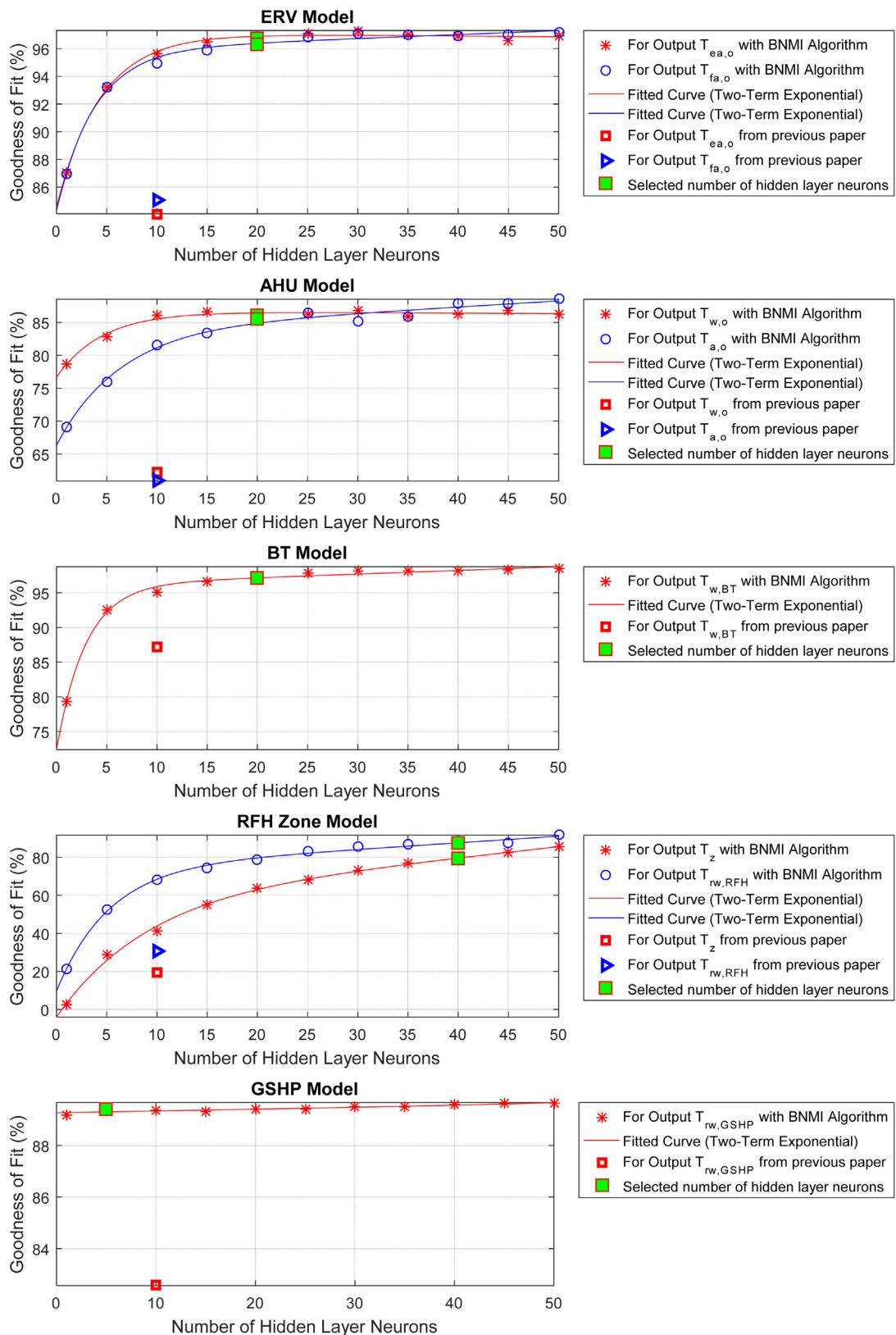


Fig. 6. Number of hidden layer neurons vs. the value of G for each subsystem model with BNMI Algorithm.

by the authors [14] without using BNMI algorithm, the improvements reported in this paper are significant due to data filtering, new BNMI training algorithm and appropriate selection of hidden layer neurons. The previous paper [14] mainly focused on the

comparison of different black-box modeling methods available by default in MATLAB® System Identification Toolbox™ and Neural Network Toolbox™. The current paper shows how the authors have significantly improved the ANN modeling results compared to the

default ANN training method by introducing the BNMI algorithm and appropriate selection of hidden layer neurons.

Visual comparisons of the performance of models along with AE plots are presented in Fig. 7. It can be seen that the developed models have very high accuracy and follow the measured data closely.

3.3. Controller design

Residential HVAC controller has a hierarchical structure with MPC used in supervisory control layer and PID control used for local level control. Supervisory MPC generates the set-points of local level controllers and local level PID controllers regulate these set-points. The detailed architecture of residential HVAC controller is shown in Fig. 8.

3.3.1. Supervisory MPC

The purpose of supervisory MPC is to reduce the operating cost of residential HVAC system. Since different set-points may cause great differences in the energy usage, MPC generates the set-points of local level controllers in order to store energy in building mass during off-peak hours and reduce equipment use during mid-peak and peak price hours.

Fig. 9 shows inputs and outputs of MPC controller. MPC uses a system model to predict the future states of system and generates a control vector that minimizes a certain cost function over the prediction horizon in the presence of disturbances and constraints. The first element of the computed control vector at any sampling instant is applied to the system input, and the remainder is discarded. The entire process is repeated in the next time instant. Cost function can take the form of tracking error, control effort, energy cost, demand cost, power consumption, or a combination of these factors. Constraints can be placed on the rate and range limits of actuators and manipulated variables (e.g., upper and lower limits of zone temperature, supply air flow rate limits, and range and speed limits for damper positioning). External and internal disturbances acting on the system due to weather, occupant activities, and equipment use are also modeled, and their predicted effects on the system are used during control vector computation. This effort results in a controller that is robust to both time-varying disturbances and system parameters and regulates the process tightly within given bounds.

At each time instant, the system output is also measured and is used as initial condition in the optimization process. This helps in eliminating any un-modeled disturbances or modeling errors. The system and disturbance model can be written in a variety of forms such as differential equations, state-space representation, transfer function representation or discrete difference equations. In this paper, nonlinear ANN system model is used.

MPC algorithm works as follows:

- At time t , solve an optimal control problem over a finite future horizon of N steps:

$$\begin{aligned} \min_{u_t, \dots, u_{t+N-1}} \quad & J = \sum_{k=0}^{N-1} \|y_{t+k} - r(t)\|^2 + \|u_{t+k} - u_r(t)\| \\ \text{s.t.} \quad & x_{t+k+1} = f(x_{t+k}, u_{t+k}), \\ & y_{t+k} = g(x_{t+k}, u_{t+k}), \\ & u_{\min} \leq u_{t+k} \leq u_{\max}, \\ & y_{\min} \leq y_{t+k} \leq y_{\max}, \\ & x_t = x(t), k = 0, \dots, N-1, \end{aligned} \quad (2)$$

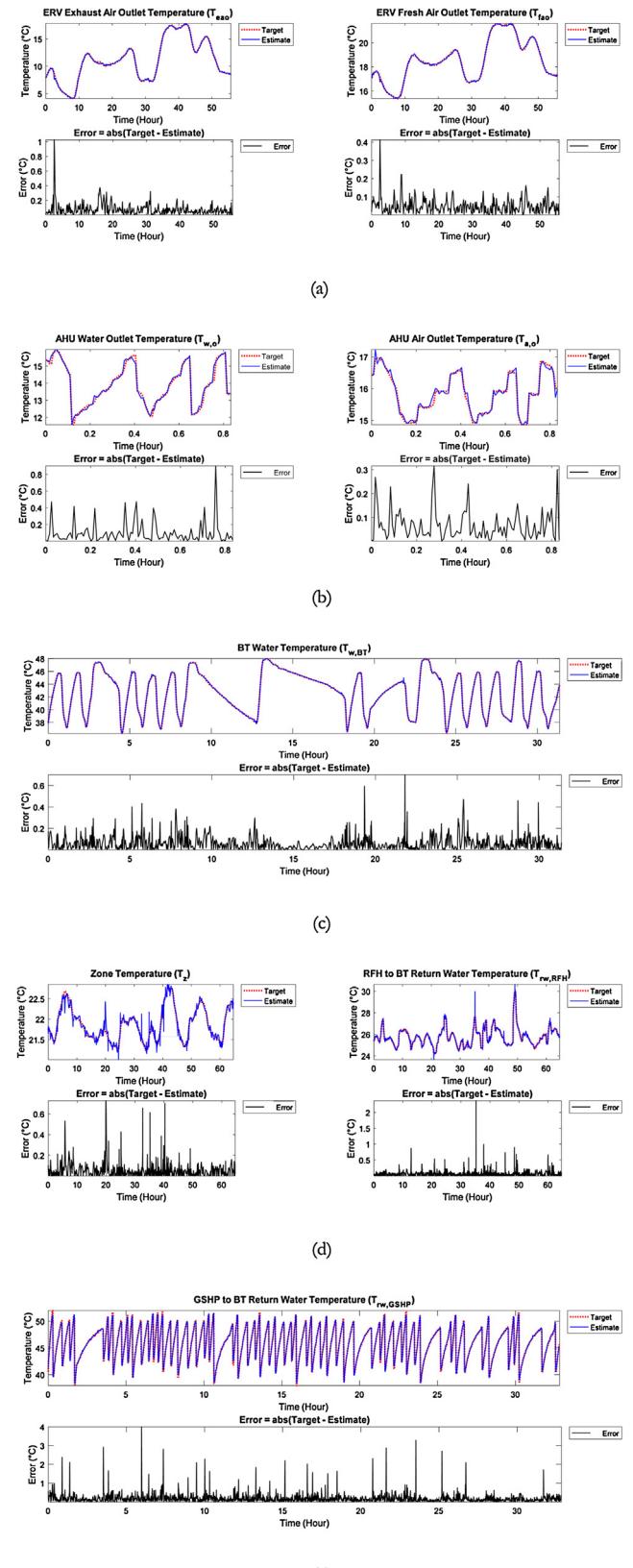


Fig. 7. Measured vs. estimated outputs of (a) ERV, (b) AHU, (c) BT, (d) RFH and (e) GSHP.

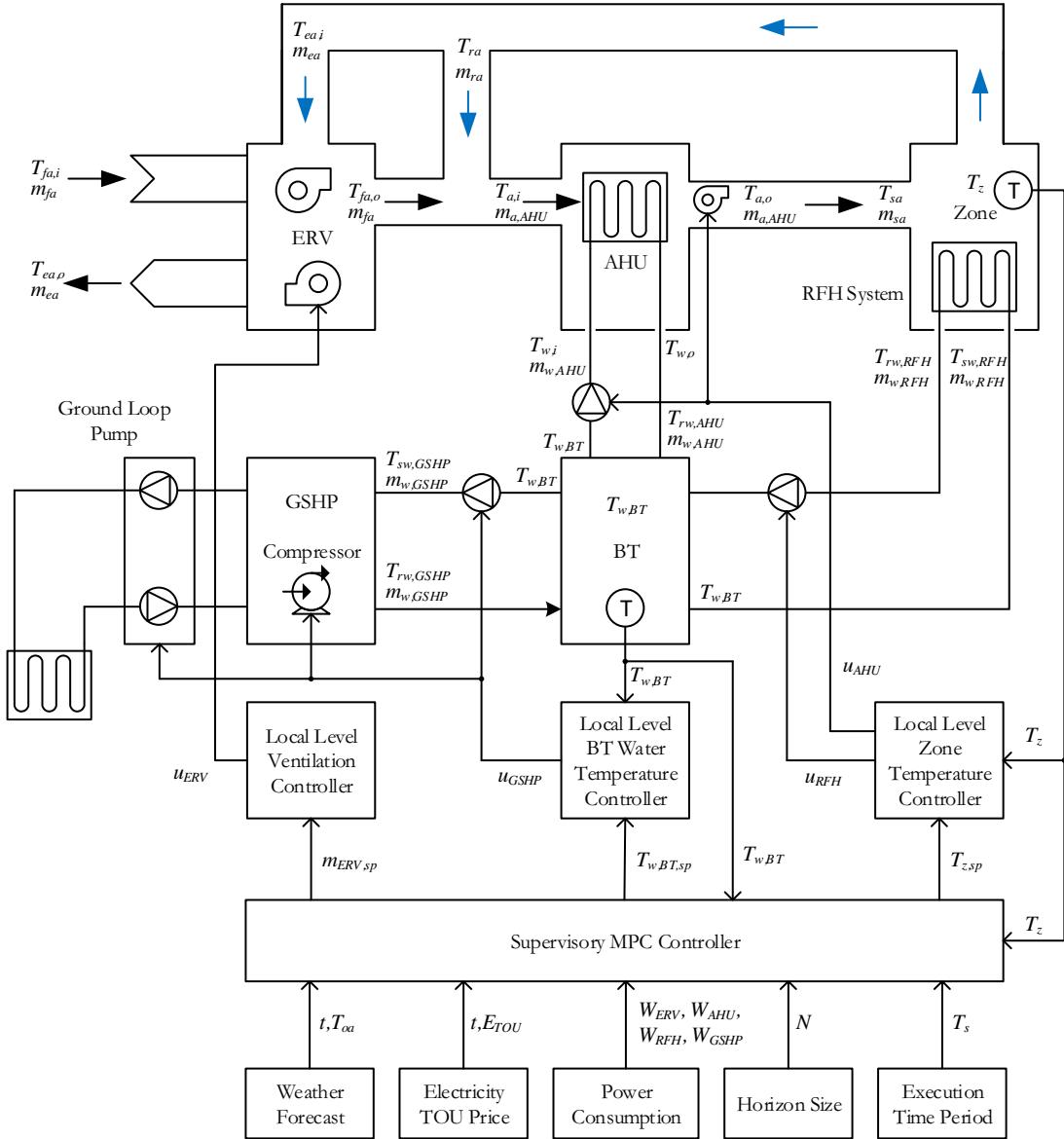


Fig. 8. Residential HVAC controller architecture.

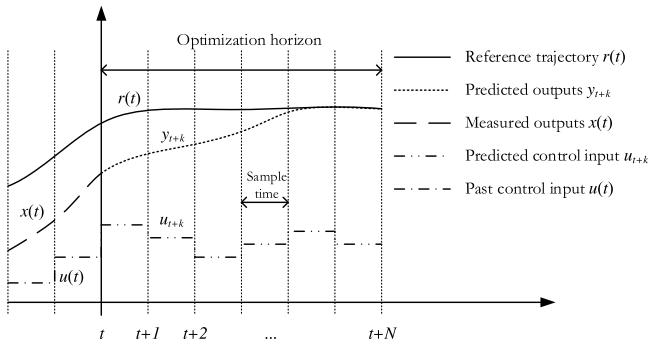


Fig. 9. Inputs and outputs of MPC based controller.

where J is the cost function, N is the prediction horizon, u , y , x and r are the input, output, state and reference trajectory of the system, and $x(t)$ is the measured state of the system at time t .

- Only apply the first optimal move.

- At time $t + 1$, get new measurements and repeat the optimization process.

Cost function in the above formulation takes the form of summation of tracking error and control effort. The optimization tries to reduce the tracking error and control effort over the optimization horizon and produces the optimal control vector in the presence of constraints on system input and output. The limits on control input (i.e., u_{\min} and u_{\max}) and system output (i.e., y_{\min} and y_{\max}) are defined by the actuator constraints and required specifications of control objective, e.g., in case of zone temperature control, the zone temperature can be specified to vary to a certain degree around the reference set-point.

Above form of cost function is not suitable for supervisory control and is only useful for local level control. In a case where supervisory MPC optimization objective is to reduce the operating cost of HVAC system, the cost function is re-written to represent the operating cost of the system as shown in the following subsection.

3.3.1.1. Cost function. Cost function represents the total cost of operating the HVAC system. It comprises of four terms which are

Table 4

Electricity time-of-use price in Ontario during summer and winter 2016.

Day	Time	Hours	Summer (May 1 to Oct 31)		Winter (Nov 1 to Apr 30)	
			Period	Price (\$/kWh)	Period	Price (\$/kWh)
Mon to Fri	07:00 P.M.–07:00 A.M.	12	Off-Peak	0.087	Off-Peak	0.087
	07:00 A.M.–11:00 A.M.	4	Mid-Peak	0.132	On-Peak	0.180
	11:00 A.M.–5:00 P.M.	6	On-Peak	0.180	Mid-Peak	0.132
	5:00 P.M.–07:00 P.M.	2	Mid-Peak	0.132	On-Peak	0.180
Weekends and Holidays	12:00 A.M.–12:00 A.M.	24	Off-peak	0.087	Off-Peak	0.087

the cost of operating AHU, RFH, GSHP and ERV. During winter season, the control signal corresponding to AHU is zero and; therefore, the corresponding term becomes zero; whereas, during heating season, the control signal for RFH is zero therefore, the corresponding term becomes zero. GSHP and ERV work during all seasons. Cost function is written as follows:

$$J = \int_1^N (u_{ERV} W_{ERV} + u_{AHU} W_{AHU} + u_{RFH} W_{RFH} + u_{GSHP} W_{GSHP}) E_{TOU} dt \quad (3)$$

Power consumption of each subsystem was measured from the TRCA-ASHB. The power of AHU, and GSHP is divided into pump, fan and compressor power components as follows:

$$\begin{aligned} W_{ERV} &= W_{ERV, \text{supplyfan}} + W_{ERV, \text{returnfan}}, \\ W_{AHU} &= W_{AHU, \text{pump}} + W_{AHU, \text{fan}}, \\ W_{RFH} &= W_{RFH, \text{pump}}. \end{aligned} \quad (4)$$

Ontario electricity time-of-use price (E_{TOU}) was used. The price structure is given in [Table 4](#). The price during summer and winter seasons is also plotted in [Fig. 10](#) for the first week of July and January respectively. Notice that electricity is cheap during mornings, evenings, and nights on weekdays and whole day on weekends. Therefore, in order to maximize the cost savings, supervisory controller should shift the load to early mornings during weekdays by storing energy in the building mass and should not store energy during weekends due to the lack of any economic incentive during that time. Summer and winter prices are applicable for each of the 26 week periods during May 1–October 31 and November 1–April 30 respectively.

Table 5

Comparison of energy consumption and operating cost of HVAC system with MPC and FSP.

Month	Energy Consumption (kWh)			Operating Cost (\$)		
	MPC	FSP	% Difference	MPC	FSP	% Difference
May	0.00	0.00	0.00	0.00	0.00	0.00
June	385.93	367.39	5.05	40.51	43.37	-6.61
July	520.35	471.81	10.29	54.98	54.48	0.92
August	357.77	333.81	7.18	37.44	40.10	-6.62
September	95.72	138.90	-31.08	10.13	16.93	-40.16
October	6.79	23.08	-70.59	0.74	2.81	-73.79

3.3.1.2. *Constraints.* Constraints were placed on the zone and BT temperature set-points, and control signals as follows:

$$\begin{aligned} T_{z,sp} &\in [22,25] \text{ for cooling} \\ &\in [20,22] \text{ for heating} \\ T_{BT,sp} &\in [5,6] \text{ for cooling} \\ &\in [45,50] \text{ for heating} \end{aligned} \quad (5)$$

Control signals were constrained to vary between 0 and 1 as follows:

$$\begin{aligned} u_{AHU} &\in [0,1], \\ u_{RFH} &\in [0,1], \\ u_{GSHP} &\in [0,1]. \end{aligned} \quad (6)$$

ERV runs all the time at low speed. Its control signal was fixed at 1, i.e.,

$$u_{ERV} = 1. \quad (7)$$

3.4. Results

In order to simulate the performance of supervisory MPC, measured data of outside air was downloaded from Environment

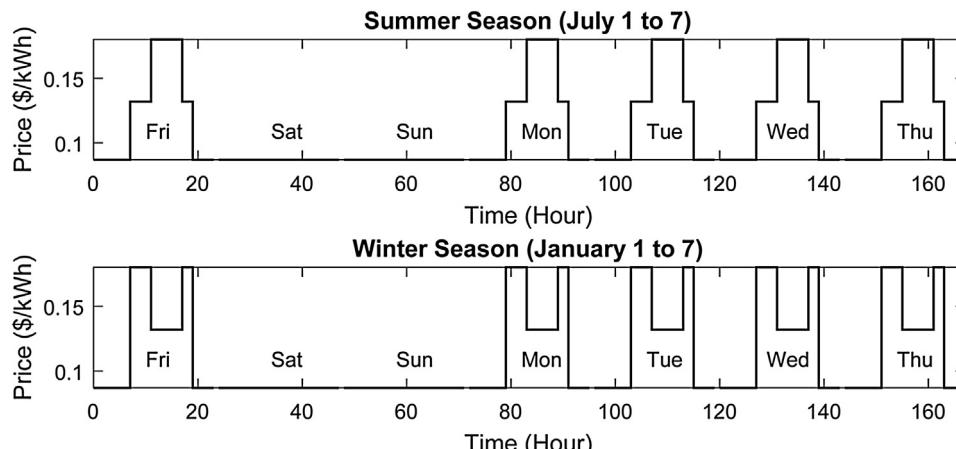


Fig. 10. Electricity time-of-use price in Ontario during a week in both summer and winter seasons.

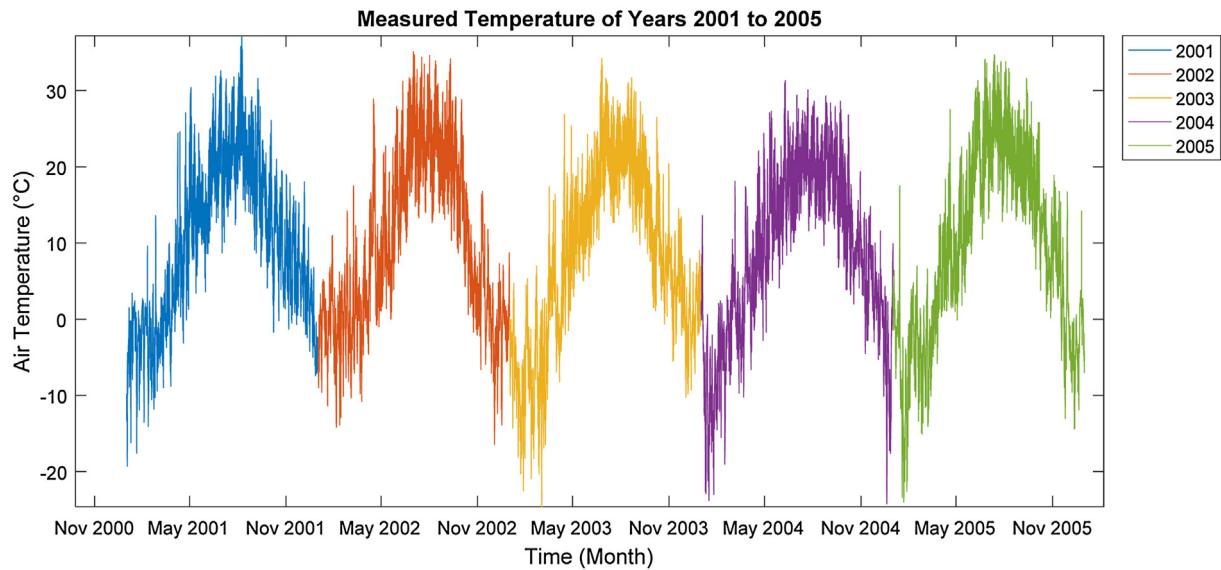


Fig. 11. Measured outside air temperature in Toronto during the years 2001–2005.

Canada website. In order to see the variations in year over year air temperature, data for the years between 2001 and 2005 was plotted in Fig. 11. It can be seen that the outside air temperature is quite repeatable year after year so it can be assumed that the performance of supervisory MPC will also be repeatable year after year. Therefore, only the data of year 2005 was selected for the simulation of controller and estimation of cost savings. Outside air temperature and corresponding electricity price during this period is shown in Fig. 12.

MPC was compared to FSPs where zone and BT water temperature set-points were fixed at 25 °C and 6 °C respectively.

Summer season starts in May and ends in October. Energy consumption and operating cost of residential HVAC system during summer season is shown in Table 5 and Fig. 13. Weather is moderate in May in Toronto and cooling is not needed during this month so both MPC and FSP based controllers do not turn on the cooling and energy consumption and cost is zero during this month. Most of the cooling is supplied between June and early October. MPC consumes more energy compared to FSP during the hottest months of June, July and August but since this energy is consumed during off-peak hours therefore, MPC overall results in lower operating

cost except in July when MPC consumes about 10.29% more energy and results in 0.92% higher cost compared to FSP. This is because the weather is very extreme in July and the house is not able to hold enough cooling to shift the load to off-peak hours. Nevertheless, MPC tries to shift the load but fails which results in a higher cost and higher energy consumption although the cost penalty is not very high. Therefore, when the weather is extreme, MPC is not practical with passive-only thermal storage. Adding active thermal energy storage to the house could potentially result in significant cost savings during such periods but could also result in higher equipment installation costs to store the thermal energy. Another study needs to be carried out to find out the benefits and drawbacks of adding active thermal storage to the residential HVAC system of ASHB. During the moderate months of September and October, MPC consumes significantly lower energy and results in very high cost savings compared to FSP. Since weather is moderate during these months, MPC is able to fully shift the load to off-peak hours resulting in maximum savings.

Performance of supervisory MPC during the month of June is shown in Fig. 14. Based on the weather forecast, zone temperature set-point is gradually dropped by MPC during off-peak hours on

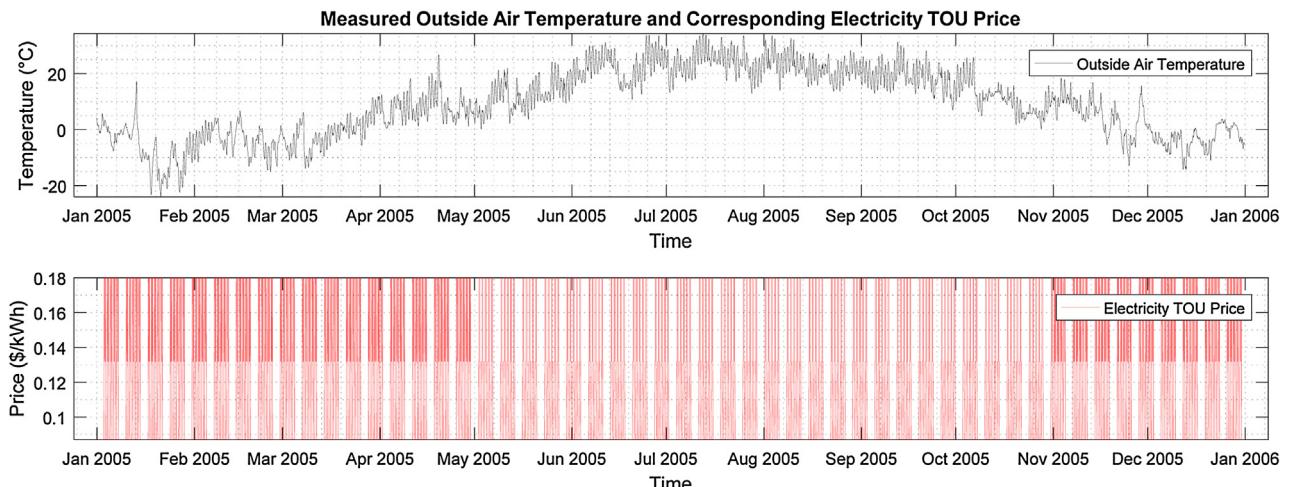


Fig. 12. Measured outside air temperature and corresponding electricity time-of-use price during the whole year.

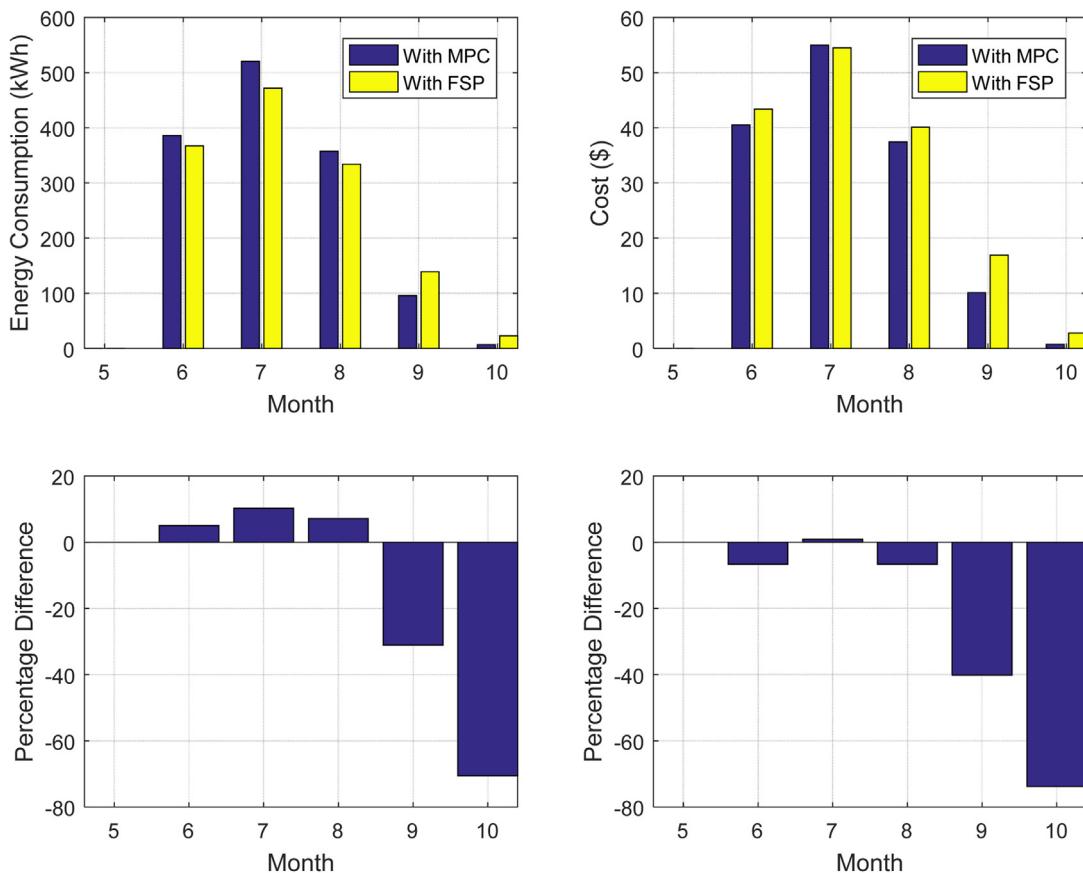


Fig. 13. Comparison of residential HVAC system energy consumption and operating cost during summer with MPC and FSP.

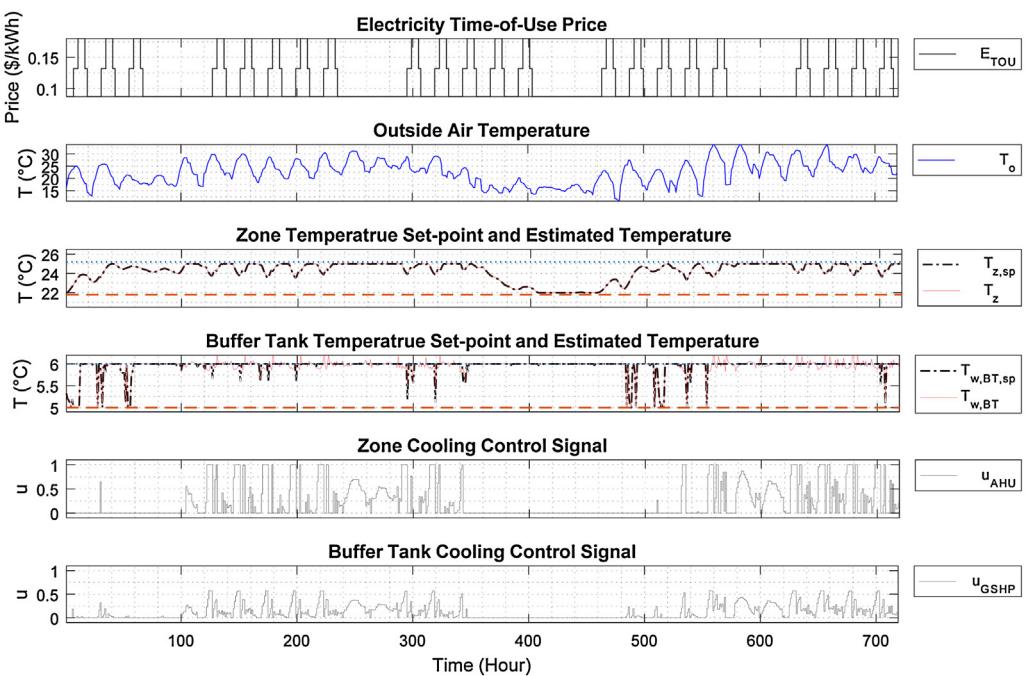


Fig. 14. Zone and BT water temperatures and control signals during the month of June with supervisory MPC.

weekdays and energy is stored in the zone. During weekend, MPC did not store any energy. Local controllers were able to regulate the desired set-point tightly and zone and BT water temperatures remain within the specified constraints as well.

Commercial users would benefit even more than the residential users due to their comparatively higher electricity usage. If a large chunk of commercial energy usage can be shifted to off-peak hours, it could result in significant operating cost savings.

Since the developed MPC controller in this paper does not require any new mechanical HVAC equipment (e.g., ERV, AHU, RFH or BT) therefore, it is a low-cost investment. The MPC proposed in this paper is a supervisory controller which can be added to the existing system by changing the thermostats in a typical household with a digital control system comprising of a computer to run the optimization algorithm and low level control layer to directly send the on/off signals to the equipment and receive feedback from them. The implementation of the MPC as a supervisory controller will not increase the maintenance cost of the system significantly as all the main mechanical systems will still be in-tact and serviced at their regular intervals. The electronics (digital controller and sensors) introduced into the system as a result of the implementation of MPC is much more robust than the mechanical systems and will require little to no maintenance in the long run.

4. Conclusions

In this paper, a comprehensive review of existing ANN based MPC approaches was carried out. Existing ANN-MPC approaches have been applied by HVAC researchers to various buildings including the university, airport, residential complex and office building but they have not been used for the most common residential houses found in all parts of Canada and North America. The researchers have generally considered the minimization of energy consumption while ignoring the operating cost of HVAC systems. In the presence of variable electricity prices such as in Canada, minimization of energy consumption is not an appropriate objective and rather focus should be on the reduction of operating cost of HVAC system. Therefore, ANN based models of the residential HVAC systems were developed and supervisory MPC was applied. A new algorithm was implemented to train the ANNs and select the best ANN among multiple trained ANNs. The algorithm was used to generate the plots for improvement in goodness of fit with the increase in number of hidden layer neurons and appropriate number of neurons were selected for each model. MPC was applied to the full summer season and the research showed that MPC consumes more energy when compared to fixed set-points but results in lower operating cost due to its ability to store the energy in building mass during off-peak hours. Since a house has a small thermal mass, passive thermal storage only works in the moderate weather conditions. For MPC to transfer the load to off-peak hours and maximize the cost savings during all seasons, active thermal storage needs to be used.

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