CAPE Laboratory

Spring Semester 2024 - 2025

Assignment – 5

Objective: Numerical solution of a Partial Differential Equation.

Problem

Consider the following unsteady-state heat conduction problem in a one-dimensional slab of 1 m thickness.

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}, \qquad T(x, t = 0) = 350\ K,\ \ T(x = 0, t) = 300\ K,\ \ T(x = 1\ \text{m}, t) = 400\ K$$

Determine the unsteady temperature distribution $T(x,t)$ in the slab at different times (t = 1, 5, 10, 50, 100 s) for three different values of thermal diffusivity (α = 1, 10, 100 m^2/s).

Use: Explicit discretization, Implicit discretization, and Crank-Nicholson discretization.

In the following description of discretization, *n* stands for time and *i* stands for space.

Explicit discretization:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^n + T_{i-1}^n - 2T_i^n}{(\Delta x)^2}$$

Implicit discretization:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha \frac{T_{i+1}^{n+1} + T_{i-1}^{n+1} - 2T_i^{n+1}}{(\Delta x)^2}$$

Crank-Nicholson discretization:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{\alpha}{2}\left[\frac{T_{i+1}^{n+1} + T_{i-1}^{n+1} - 2T_i^{n+1}}{(\Delta x)^2} + \frac{T_{i+1}^n + T_{i-1}^n - 2T_i^n}{(\Delta x)^2}\right]$$

NOTE: Solve the above problem using MATLAB function `pdepe` also and compare your results with the output of `pdepe`.