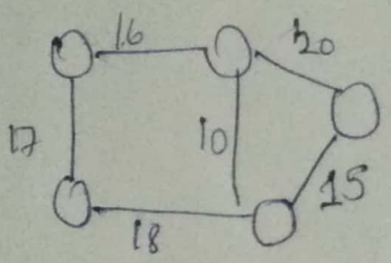
Satyam Rawat
2017006
G2
16 (Class Roll No)

DAA
Tutorial - 6

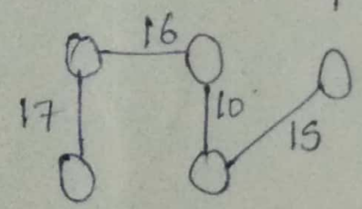
Ans 1:> Minimum Spanning Tree :

A spanning tree of an undirected graph is a subgraph that is a tree & joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning tree.

For eg :



Minimum cost Spanning Tree :



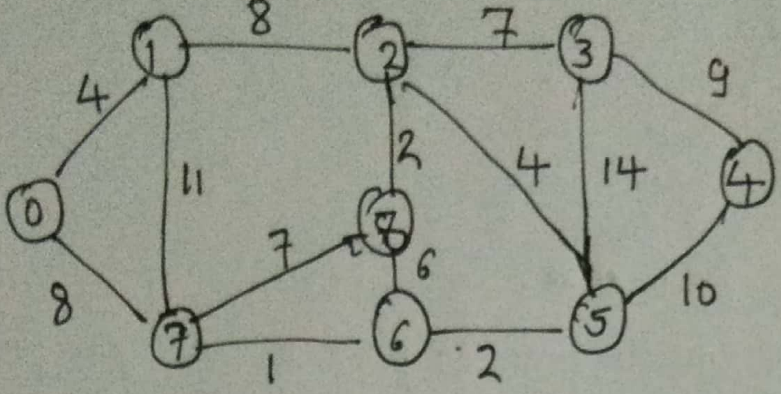
Applications of MST

- It has direct applications in design of network including computer networks, telecommunication network, etc.

Ans 2:>

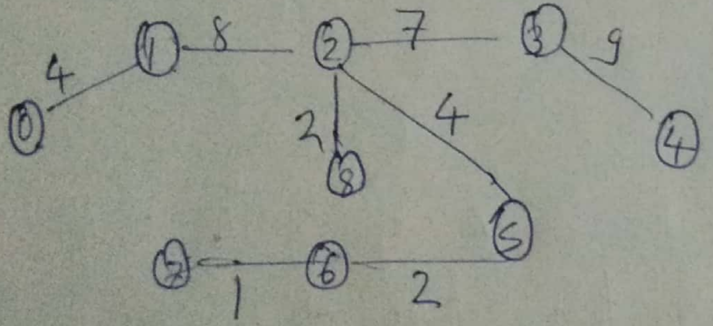
	Prim's Algo	Kruskal's Algo	Dijkstra's Algo	Bellmanford's Algo
T.C.	$O(V^2)$	$O(E\log V)$	$O(V+E\log V)$	$O(VE)$
S.C.	$O(V+E)$	$O(\lvert E\rvert+\lvert V\rvert)$	$O(V^2)$	$O(V^2)$

Q3>



Prim's Algo :

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	∞	∞	∞	∞	∞	~~∞~~	∞
	$\boxed{4}$							
		$\boxed{8}$					~~8~~	
			$\boxed{7}$		$\boxed{4}$	~~6~~	~~7~~	$\boxed{2}$
				~~10~~		$\boxed{2}$		
				$\boxed{9}$			$\boxed{1}$	



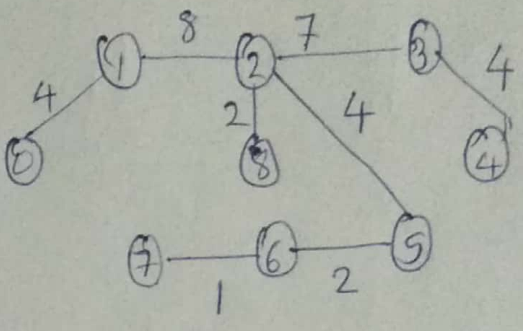
Min Weight = 37

Parent :	0	1	2	3	4	5	6	7	8
	-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	~~-1~~	~~-1~~	~~-1~~
		0	1	2		2		~~0~~	2
							~~8~~	~~8~~	
					~~5~~		5	6	
					3				

Parent : -1 0 1 2 3 2 5 6 2

Kruskal's Algo →

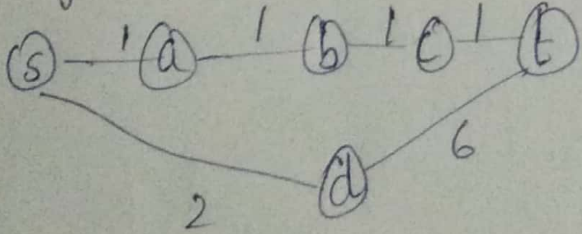
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	X
7	8	7	X
2	3	7	✓
1	2	8	✓
0	7	8	X
3	4	9	✓
5	4	10	X
1	7	11	X
3	5	14	X



Weight = 37

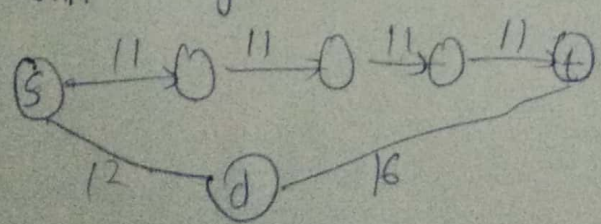
Ans 4.) If 10 units are added to each edge the overall weight of path may change.

eg



Shortest path is s → a → b → c → t weight = 1+1+1+1 = 4

Now if 10 unit weight is added,

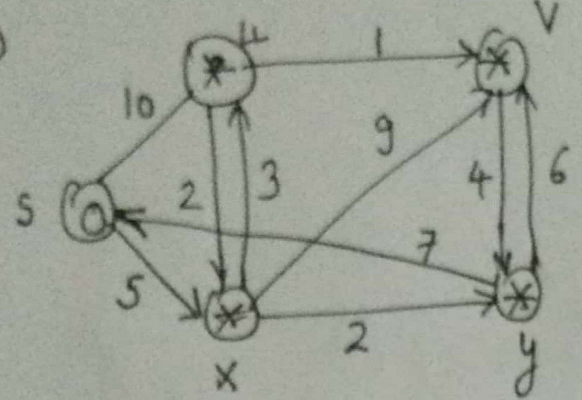


⇒ Shortest path = s→d→t

Wt = 28

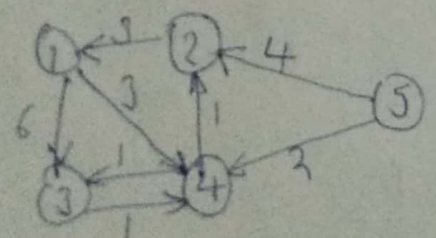
Multiplying the weight of each edge by 10 will have no impact on shortest path.

Ans 5:)



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	7

Ans 6:> All pair shortest path algo - Floyd Warshall



$$A^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & \infty & \infty & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$$A^1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ \infty & 4 & \infty & 2 & 0 \end{bmatrix} \end{array}$$

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3 + 6 = 9 < \infty$

Scanned with CamScanner

Similarly $A^0[2,4] = \infty$

$$A^0[2,1] + A^0[1,4] = 3+3 = 6 < \infty$$

$$A^0[2,5] = \infty$$

$$A^0[2,1] + A^0[1,5] = 3 + \infty$$

$$A^2 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^1[1,3] = 6$$

$$A^1[1,2] + A^1[2,3] = \infty + 9$$

$$6 < \infty + 9$$

$$A^3 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & \infty & 6 & 3 & \infty \\ 3 & 0 & 9 & 6 & \infty \\ \infty & \infty & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 4 & 13 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A_5 = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$