Satyam Rawat
2017006
G2

Tutorial 4

Solve using Master's Theorem ;

1) $T(n) = 3T\left(\frac{n}{2}\right)^2 + n^2$

$T(n) = aT\left(\frac{n}{b}\right) + f(n) \Rightarrow a=3 \quad b=2 \quad f(n)=n^2$

$n^{\log_b a} = n^{\log_2 3} < n^2$

$\Rightarrow T(n) = \Theta(n^2)$

2) $T(n) = 4T(n/2) + n^2$

$a=4 \quad b=2 \quad f(n)=n^2$

$\Rightarrow n^{\log_2 4} = n^2 = n^2$

$\Rightarrow T(n) = \Theta(n^2 \cdot \log n)$

3) $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a=1 \quad b=2 \quad f(n)=2^n$

$\Rightarrow n^{\log_2 1} = 1 < 2^n$

$\Rightarrow T(n) = \Theta(2^n)$

4) $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$

$a=2^n \quad b=2 \quad f(n)=n^n$

$\Rightarrow n^{\log_2 2^n} = n < n^n$

$\Rightarrow T(n) = \Theta(n^n)$

∵ a & b are constants we cannot apply Masters here.

5) $T(n) = 16\,T\left(\frac{n}{4}\right) + n$

$a=4^2 \quad b=4 \quad f(n)=n \Rightarrow n^{\log_4 16} = n^2 > n$

$\Rightarrow T(n) = \Theta(n^2)$

Q6.) $T(n) = 2T(n/2) + n \log n$ We will apply extended Master's

$n^{\log_2 2} = n$ $\quad n^k (\log n)^p \quad k=p=1$

$\Rightarrow T(n) = \Theta(n^{\log_b a} \cdot \log^{p+1} n) = \Theta(n \cdot (\log n)^2)$

Q7.) $T(n) = 2T(n/2) + \frac{n}{\log n}$

By Extended Master's Theorem,

$T(n) = 2T(n/2) + n (\log n)^{-1}$

$a=b=2$ $\quad f(n) = n^k \log^p n$ where $p=-1, k=1$

$\Rightarrow T(n) = \Theta(n^{\log_b a} \log \log n)$

$= \Theta(n \log \log n)$

Q8.) $T(n) = 2T(n/4) + n^{0.51}$

$n^{\log_4 2} = n^{1/2}$ & $f(n) = n^{0.51} = n^{0.5+\varepsilon}$ $\quad \varepsilon = 0.01 > 0$

$\Rightarrow T(n) = \Theta(n^{0.51})$

Q9.) $T(n) = 0.5\, T\left(\frac{n}{2}\right) + \frac{1}{n}$

Does not apply

Q10.) $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$n^{\log_4 16} = n^2 < n!$

$\Rightarrow T(n) = \Theta(n!)$

Q11.) $T(n) = 4T(n/2) + \log n \rightarrow n^0 (\log n)^1$

$n^{\log_2 4} = n^2$

$k=0 \quad p=1 \Rightarrow a > b^k$

$\Rightarrow T(n) = \Theta(n^2 \cdot$ [redacted] $)$

Q12.> $T(n) = \sqrt{n}\, T\left(\frac{n}{2}\right) + \log n$

Does not apply

Q13.> $T(n) = 3T\left(\frac{n}{2}\right) + n$

$n^{\log_2 3} > n$

$\Rightarrow T(n) = \Theta(n^{\log_2 3})$

Q14.> $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$n^{\log_3 3} = n > \sqrt{n}$

$\Rightarrow T(n) = \Theta(n)$

Q15.> $T(n) = 4T(n/2) + cn$

$n^{\log_2 4} = n^2 > cn$

$\Rightarrow T(n) = \Theta(n^2)$

Q16.> $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$n^{\log_4 3}$ $\quad k=1 \quad p=1$

$\Rightarrow 3 < 4^1$ & $p \geq 0 \Rightarrow T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^{\log_4 3} \log n)$

Q17.> $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$n^{\log_3 3}$

$= n^1 = \Theta\left(\frac{n}{2}\right) \Rightarrow T(n) = \Theta(n \log n)$

Q18.> $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$n^{\log_3 6}$ $\quad k=2 \quad p=1$

$\Rightarrow 6 < 3^2$ and $p \geq 0 \Rightarrow T(n) = \Theta(n^k \log^p n)$

$= \Theta(n^2 \log n)$

Q19.) $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$T(n) = 4T\left(\frac{n}{2}\right) + n^1 (\log n)^{-1}$

$n^{\log_2 4} = n^2$ $\quad k=1 \quad p=-1$

$4 > 2^1 \Rightarrow T(n) = \Theta(n^2)$

Q20.) $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

~~$n^{\log_8 64} = n^2$~~ Does not apply

Q21.) $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$n^{\log_3 7} < n^2$

$\Rightarrow T(n) = \Theta(n^2)$

Q22.) $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

Does not apply