PRESCRIPTIVE ANALYTICS

ASSIGNMENT 1 : NOTATIONS AND SPREADSHEET MODELS

Math Notation Exercise

1. Provide a matrix representation of the following system of equations.
   x1 − x2 = 4,
   2x1 + x2 = 6,
   x1 + 3x2 = 8.

   Ans: Let the following system of equations be represented as
   AX = B
   Where, A = $\begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ X = $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ 6 \\ 8 \end{bmatrix}$

2. Write out all of the terms in each of the following sums:
   (a) $\sum_{i=1}^{5} z_i$
   (b) $\sum_{i=1}^{5} z_{ij}$
   (c) $\sum_{j=0}^{4} \sum_{k=6}^{8} x_{ij}$
   (d) $\sum_{j \in S} y_j$ where S = {2, 3, 6, 8}

   Ans: (a) $\sum_{i=1}^{5} z_i = z_1 + z_2 + z_3 + z_4 + z_5$

   (b) $\sum_{i=1}^{5} z_{ij} = z_{1j} + z_{2j} + z_{3j} + z_{4j} + z_{5j}$

   (c) $\sum_{j=0}^{4} \sum_{k=6}^{8} x_{ij} = x_{06} + x_{07} + x_{08} + x_{16} + x_{17} + x_{18} + x_{26} + x_{27} + x_{28} + x_{36} + x_{37} + x_{38} + x_{46} + x_{47} + x_{48}$

   (d) $\sum_{j \in S} y_j$ where S $= \{2, 3, 6, 8\} = y_2 + y_3 + y_6 + y_8$

3. Give the third term in each sum:
   (a) $\sum_{i=4}^{8} z_i$
   ⇨ z_6
   (b) $\sum_{q=100}^{200} z_{iq}$
   ⇨ z_{i102}

4. Does $\sum_{i \in P} \sum_{j \in Q} x_{ij} = \sum_{j \in P} \sum_{i \in Q} x_{ji}$ for any definition of P and Q?

   Ans: Yes. The above equation, $\sum_{i \in P} \sum_{j \in Q} x_{ij} = \sum_{j \in P} \sum_{i \in Q} x_{ji}$ holds true for any definition of P and Q since the pair (i, j) on the LHS is same as the pair (j, i) on the RHS which is just indexed differently. i.e, $i \in P, j \in Q = j \in P, i \in Q$.

5. Let A = {−1, 0, 1, 2, 3, 4}, B = {−1, 0, 2, 3, 5}, and C = {(i, j) : i ∈ A, j ∈ B, i + j = 3}.

(a) List all of the elements in C

⇨ C = { (0,3), (1,2), (3,0), (4,-1) }

(b) Write out the following inequalities. $x_i + x_j \le 1, (i,j) \in C$

⇨ $\boldsymbol{x_0 + x_3 \le 1}$

⇨ $\boldsymbol{x_1 + x_2 \le 1}$

⇨ $\boldsymbol{x_3 + x_0 \le 1}$

⇨ $\boldsymbol{x_4 + x_{-1} \le 1}$

(c) Write out the following equations.

i. $\sum_{(i,j)\in C} x_{ij} = 1$

$$x_{03} + x_{12} + x_{30} + x_{4(-1)} = 1$$

ii. $\sum_{(i,j)\in C: i<j} x_{ij} = 1$

$$x_{03} + x_{12} = 1$$

iii. $\sum_{i\epsilon A} \sum_{j\in B} x_{ij} = 1$

$$x_{-1-1} + x_{-10} + x_{-12} + x_{-13} + x_{-15} + x_{0-1} + x_{00} + x_{02} + x_{03} + x_{05} + x_{1-1} + x_{10} + x_{12} + x_{13} + x_{15} + x_{2-1} + x_{20} + x_{22} + x_{23} + x_{25} + x_{3-1} + x_{30} + x_{32} + x_{33} + x_{35} + x_{4-1} + x_{40} + x_{42} + x_{43} + x_{45}$$

(d) Evaluate the following expression.

$\sum_{(i,j)\in C} i^2$

⇨ $0^2 + 1^2 + 3^2 + 4^2 = 1 + 9 + 16$

⇨ 26

6. Put the following linear program in written-out notation:

Min $\sum_{j=1}^{3} c_j x_j,\ such\ that\ \sum_{j=1}^{3} a_{ij} x_j \le b_i, i \in \{1,2,3,4\}, x_j \ge 0, j \in \{1,2,3\}$

⇨ Minimize

$$c_1x_1 + c_2x_2 + c_3x_3$$

Such that

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \le b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \le b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \le b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 \le b_4$$

7. Put the following linear program in written out notation:
   Min $\sum_{(i,j)\in A} c_{ij}x_{ij}, such\ that\ \sum_{j\in V:(i,j)\in A} x_{ij} - \sum_{j\in V:(j,i)\in A} x_{ji} = b_i, i \in V, l_{ij} \le x_{ij} \le u_{ij}, (i,j) \in A$
   Where $V = \{1,2,3,4,5\}\ and\ A = \{(1,3),(1,4),(2,3),(2,5),(3,4),(3,5)\}$.

   ⇨ Minimize
   $c_{13}x_{13} + c_{14}x_{14} + c_{23}x_{23} + c_{25}x_{25} + c_{34}x_{34} + c_{35}x_{35}$

   Such that

Spreadsheet Exercise

1. Advertising budget allocation

Problem

The Sea Wharf Restaurant would like to determine the best way to allocate a monthly advertising budget of $1,000 between newspaper advertising and radio advertising. Management decided that at least 25% of the budget must be spent on each type of media and that the amount of money spent on local newspaper advertising must be at least twice the amount spent on radio advertising. A marketing consultant developed an index that measures audience exposure per dollar of advertising on a scale from 0 to 100, with higher values implying greater audience exposure. If the value of the index for local newspaper advertising is 50 and the value of the index for spot radio advertising is 80, how should the restaurant allocate its advertising budget to maximize the value of total audience exposure?

⇨ A={newspaper,radio}
$x_1 = newspaper\ advertising$
$x_2 = radio\ advertising$

Objective

To allocate the advertising budget between newspaper advertising and radio advertising
Such that the value of total audience exposure is maximized
Subject to the constraints:
Total budget = $1000
Minimum budget for newspaper and radio = 25% of total budget
Money spent on newspaper should be atleast twice the money spent on radio

i.e,
maximize $50x_1 + 80x_2$, subject to the following constraints

$x_1 + x_2 \leq 1000$
$x_1 \geq 250$
$x_2 \geq 250$
$x_1 \geq 2x_2$
$x_1, x_2 \geq 0$

Results
The optimal solution for Sea Wharf Restaurant to maximize the value of total exposure is by allocating a budget of $666.66 to newspaper and $333.33 to radio which will return an exposure of 60,000.

(excel sheet attached)

2. Bit-Byte

Problem

Develop a spreadsheet model to derive an optimal solution. BitByte, Inc. is a computer laptop manufacturer which makes 2 product models - Basic and Pro. Each model requires a specific set of inputs and generates a different profit. There is a limited capacity for each input element. The table below shows the inputs required per laptop and the total available capacity for each input.

Inputs required to make 1 unit

Inputs	**Basic**	**Pro**	**Total Capacity**
CPU Cores	1	2	100
Peripherals	3	5	220
Display	1	1	50
Labor	3	5	210
Other	2	3	130

Each Basic model laptop sold produces a profit of $125 and each Pro model laptop sold produces a profit of $200.

⇨ P={basic,pro}
$x_1 = basic$
$x_2 = pro$

Objective

to decide the number of basic and pro products to be sold

such that the profit of the company is maximized,

i.e,

maximize the objective function $125x_1 + 200x_2$

subject to the following constraints:

$x_1 + 2x_2 \leq 100$

$3x_1 + 5x_2 \leq 220$

$x_1 + x_2 \leq 50$

$3x_1 + 5x_2 \leq 210$

$2x_1 + 3x_2 \leq 130$

$x_1, x_2 \geq 0$

Results

The optimal solution for BitByte Inc to maximize their profit is to sell 20 units of basic product and 30 units of pro products which would in turn give the profit of $8500.