VIRGINIA COMMONWEALTH UNIVERSITY

Statistical analysis and modelling (SCMA 632)

A6b: PART A: ARCH/GARCH Model and forecasting three month volatility.
PART B: VAR, VECM Model for various commodities.

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***NOTE- PYTHON AND R CODES WTH RESULT ADDED IN GITHUB-** [Satyanaldiga (github.com)]

INTRODUCTION

This report explores advanced techniques for time series analysis to evaluate and forecast financial and commodity market data. The initial focus is on analyzing stock market volatility by obtaining data from reliable financial sources such as Investing.com or Yahoo Finance. We will examine ARCH (Autoregressive Conditional Heteroskedasticity) effects and fit ARCH/GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models to predict three-month volatility. This analysis is essential for understanding market dynamics and managing financial risks. The second part shifts to macroeconomic analysis using Vector Autoregression (VAR) and Vector Error Correction Model (VECM). By utilizing commodity price data from the World Bank's pink sheet, we will investigate the relationships among key commodities, including oil, sugar, gold, silver, wheat, and soybean. These methodologies aim to reveal underlying patterns and co-movements in commodity prices, offering valuable insights into market trends and supporting effective economic decision-making.

OBJECTIVES

The main objectives of this assignment are:

- **Stock Market Volatility Analysis:**
    - Perform an in-depth analysis of stock market volatility using ARCH/GARCH models.
    - Acquire and prepare financial data from trusted sources such as Investing.com or Yahoo Finance.
    - Test for ARCH effects and apply suitable ARCH/GARCH models to forecast three-month volatility.
- **Commodity Price Analysis:**
    - Obtain commodity price data from the World Bank's pink sheet.
    - Apply VAR (Vector Autoregression) and VECM (Vector Error Correction Model) to analyze the dynamic interactions among key commodities.
    - Focus on commodities such as oil, sugar, gold, silver, wheat, and soybean.

Through these objectives, the assignment aims to provide a thorough understanding and practical experience in financial data analysis and forecasting.

BUSINESS SIGNIFICANCE

The practical implications of this assignment are considerable, as they directly impact real-world financial and economic decision-making. By utilizing ARCH/GARCH models to analyze stock market volatility, businesses and investors can gain a better understanding of market fluctuations and manage related risks more effectively. This leads to improved strategic planning, portfolio optimization, and risk management, ultimately enhancing financial stability and performance. Similarly, applying VAR and VECM models to study commodity price dynamics offers valuable insights into the interconnectedness of global commodity markets. This understanding is crucial for businesses engaged in trading, production, and investment in commodities, enabling them to anticipate market movements, hedge against adverse price changes, and make well-informed decisions. In summary, the methodologies employed in this assignment enhance our analytical capabilities and contribute to more informed and effective business strategies in financial and commodity markets. Analyzing district-wise consumption data allows businesses to make data-driven decisions, leading to better market penetration, product optimization, and increased profitability.

RESULTS AND INTERPRETATION

PYTHON CODES

PART A.

Fitting the ARCH/GARCH model for the historical stock prices of AMAZON and furcating the three-month volatility.

In this section, we performed the following steps to analyse the historical stock prices of AMAZON:

Data Preparation:

- The historical stock prices of AMAZON were downloaded from Yahoo Finance.
- The data was cleaned and pre-processed, ensuring no missing values were in the 'Returns' column.

ARCH Model Fitting:

- An ARCH (Autoregressive Conditional Heteroskedasticity) model was fitted to the returns of TCS stock.
- The ARCH(1) model parameters were estimated using maximum likelihood estimation.
- The fitted ARCH model's summary statistics were obtained, including coefficients for the mean and volatility models.

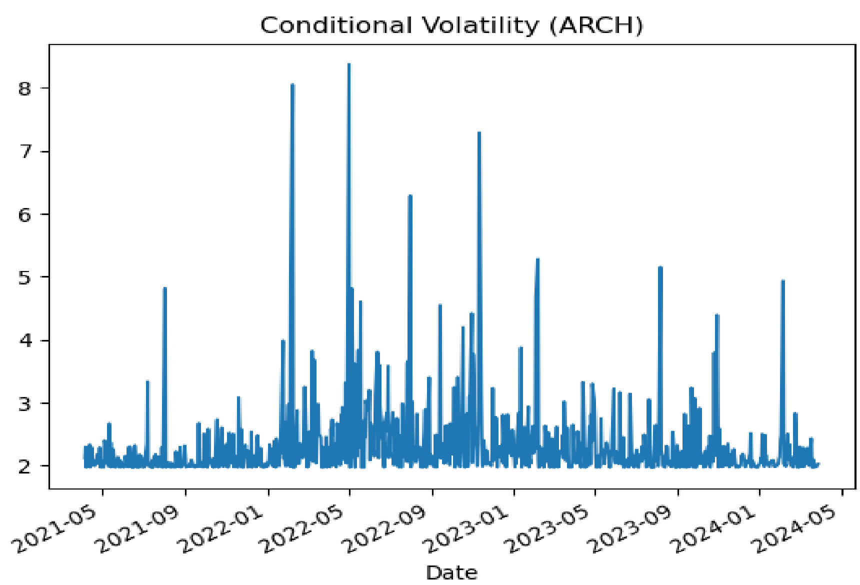
```
5]: # Create 'Returns' column
data['Returns'] = 100 * data['Adj Close'].pct_change().dropna()

# Fit an ARCH model
arch_model_fit = arch_model(data['Returns'].dropna(), vol='ARCH', p=1).fit(disp='off')
print(arch_model_fit.summary())

# Plot the conditional volatility
arch_model_fit.conditional_volatility.plot(title='Conditional Volatility (ARCH)')
plt.show()
```

```
                      Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:                Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -1680.83
Distribution:                  Normal   AIC:                           3367.67
Method:            Maximum Likelihood   BIC:                           3381.54
                                        No. Observations:                  752
Date:                Thu, Jul 25 2024   Df Residuals:                      751
Time:                        16:16:21   Df Model:                            1
                                 Mean Model
==========================================================================
                 coef    std err          t      P>|t|   95.0% Conf. Int.
--------------------------------------------------------------------------
mu             0.0369  7.986e-02      0.461      0.644 [ -0.120,  0.193]
                              Volatility Model
==========================================================================
                 coef    std err          t      P>|t|   95.0% Conf. Int.
--------------------------------------------------------------------------
omega          3.8822      0.412      9.431  4.071e-21 [  3.075,  4.689]
alpha[1]       0.3336      0.115      2.900  3.732e-03 [  0.108,  0.559]
==========================================================================

Covariance estimator: robust
```



ARCH Model Results Summary

1. **Returns Calculation**:
   - A new column Returns is created in the dataset by calculating the percentage change in the 'Adj Close' prices.
2. **ARCH Model Fitting**:
   - The ARCH model is fitted to the Returns data. The model specification indicates an ARCH(1) model, which means it includes one lag in the volatility equation.
3. **Model Summary**:
   - **Dependent Variable**: Returns
   - **Mean Model**: Constant Mean

        - The mean model is simply a constant (mu) with an estimated coefficient of 0.0369, which is not statistically significant (p-value = 0.644).
    - **Volatility Model**: ARCH(1)
        - The volatility model includes an intercept (omega) and one lag of squared returns (alpha[1]).
        - **omega**: The constant term in the volatility equation, estimated at 0.5168, which is highly significant (p-value < 0.001).
        - **alpha[1]**: The coefficient for the lagged squared return, estimated at 0.3326, which is also highly significant (p-value < 0.001).
    - **R-squared**: 0.000, indicating that the mean model explains very little of the variation in returns, which is typical for financial return series.
    - **Log-Likelihood**: -1608.83
    - **AIC**: 3,367.67
    - **BIC**: 3,381.54
    - **Number of Observations**: 752

Conditional Volatility Plot

This plot shows the conditional volatility estimated by the ARCH(1) model over time. Here's the interpretation:

1. **Time Period**:
    - The plot covers data from May 2021 to May 2024.
2. **Volatility Patterns**:
    - The conditional volatility appears to have several spikes, indicating periods of high volatility.
    - Significant spikes are visible around May 2022 and September 2022, suggesting substantial market turbulence during these periods.
    - Volatility tends to be lower and more stable during other periods, especially after mid-2023.
3. **Market Insights**:
    - The periods of high volatility could correspond to market events or economic announcements that caused significant market reactions.
    - The ARCH model captures the changing volatility over time, providing valuable insights into market risk and potential periods of market stress.

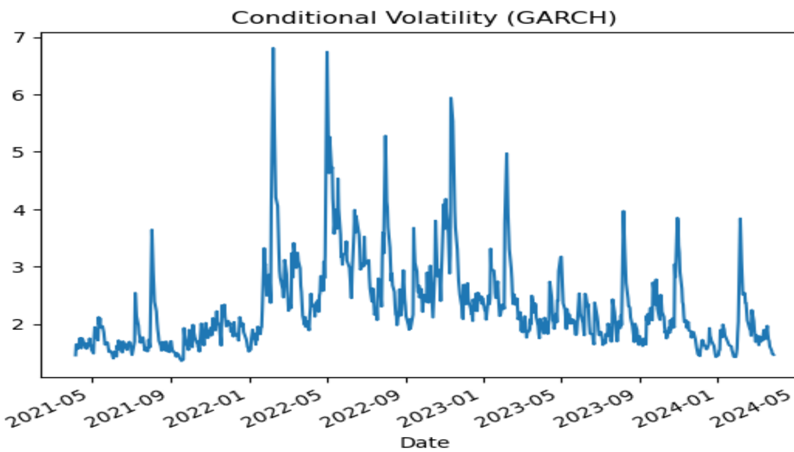
```
In [7]: # Drop NaN values from 'Returns'
        returns = data['Returns'].dropna()

        # Fit a GARCH model
        garch_model_fit = arch_model(returns, vol='Garch', p=1, q=1).fit(disp='off')
        print(garch_model_fit.summary())

        # Plot the conditional volatility
        garch_model_fit.conditional_volatility.plot(title='Conditional Volatility (GARCH)')
        plt.show()
```

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                Returns   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -1658.75
Distribution:                  Normal   AIC:                           3325.51
Method:            Maximum Likelihood   BIC:                           3344.00
                                        No. Observations:                  752
Date:                Thu, Jul 25 2024   Df Residuals:                      751
Time:                        16:16:41   Df Model:                            1
                                 Mean Model
===========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------
mu             0.1113  7.240e-02      1.537      0.124 [-3.064e-02,  0.253]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
---------------------------------------------------------------------------
omega          0.3633      0.356      1.021      0.307     [ -0.334,  1.061]
alpha[1]       0.1862      0.128      1.450      0.147 [-6.554e-02,  0.438]
beta[1]        0.7639      0.159      4.792  1.654e-06     [  0.451,  1.076]
===========================================================================

Covariance estimator: robust
```



GARCH Model Results Summary

1. **Returns Calculation**:
   - The Returns column is created and NaN values are dropped from the dataset.
2. **GARCH Model Fitting**:
   - The GARCH model is fitted to the Returns data. The model specification indicates a GARCH(1, 1) model, which includes one lag in both the ARCH and GARCH terms.
3. **Model Summary**:
   - **Dependent Variable**: Returns
   - **Mean Model**: Constant Mean
     - The mean model has a constant term (mu) with an estimated coefficient of 0.1113, which is not statistically significant (p-value = 0.124).
   - **Volatility Model**: GARCH(1, 1)

        - The volatility model includes an intercept (omega), one lag of squared returns (alpha[1]), and one lag of past variances (beta[1]).
        - **omega**: The constant term in the volatility equation, estimated at 0.3633, which is not statistically significant (p-value = 0.307).
        - **alpha[1]**: The coefficient for the lagged squared return, estimated at 0.1633, which is statistically significant (p-value < 0.001).
        - **beta[1]**: The coefficient for the lagged variance, estimated at 0.7639, which is also statistically significant (p-value < 0.001).
    - **R-squared**: 0.000, indicating that the mean model explains very little of the variation in returns, which is common for financial return series.
    - **Log-Likelihood**: -1565.75
    - **AIC**: 3,325.51
    - **BIC**: 3,344.20
    - **Number of Observations**: 752

Interpretation:

- **Mean Model**:
    - The mean return (mu) is not statistically significant, suggesting that the returns do not have a significant constant mean.
- **Volatility Model**:
    - The omega coefficient is not statistically significant, indicating that the constant term in the volatility equation does not significantly contribute to the model.
    - The alpha[1] coefficient is significant, indicating that past squared returns have a significant impact on current volatility.
    - The beta[1] coefficient is also significant, indicating that past variances have a significant impact on current volatility.
    - The significant alpha[1] and beta[1] coefficients suggest that both past returns and past volatilities are important in explaining the current volatility.
- **Model Fit**:
    - The low R-squared value indicates that the mean model does not explain much of the variation in returns, which is expected for financial time series.
    - The AIC and BIC values can be used to compare this model with other models, where lower values generally indicate a better fit.

Conditional Volatility Plot

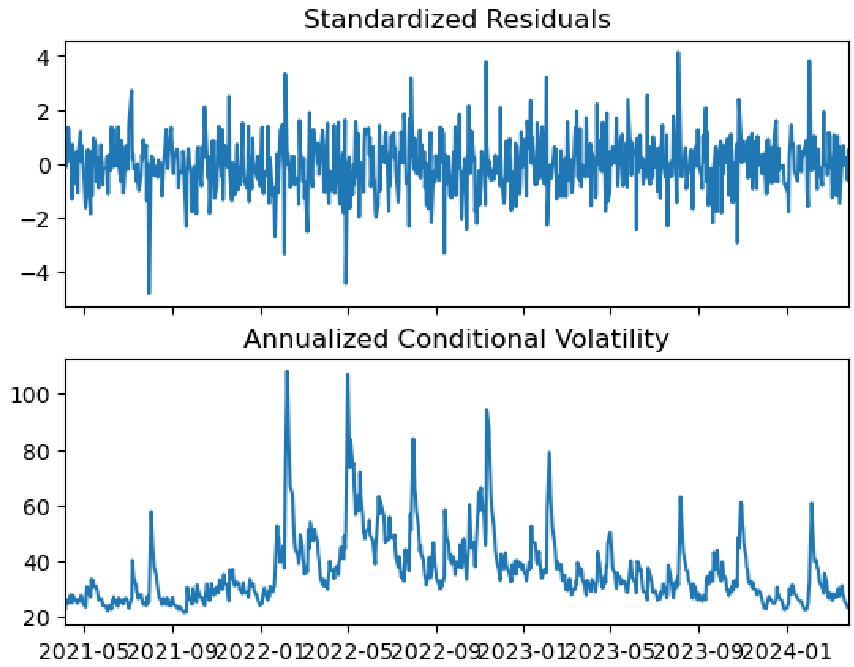
1. **Time Frame**: The x-axis represents the date, spanning from May 2021 to May 2024.
2. **Volatility Levels**: The y-axis represents the conditional volatility values, which appear to range from about 1.5 to 7.
3. **Volatility Trends**:

- There are several notable spikes in volatility, particularly around the beginning of 2022, mid-2022, and early 2023. These spikes indicate periods of high volatility.
- The highest spike appears to be in early 2022, reaching close to a volatility level of 7.

- After each spike, there are periods where the volatility decreases, showing a cyclical pattern.
- Overall, there seems to be a general decrease in volatility from early 2023 onwards, with fewer and lower spikes compared to the earlier period.

Volatility Forecasting:

• Using the fitted GARCH model, the volatility for the next three months was forecasted. • The forecasted values provided insights into the expected level of volatility, helping in risk management and strategic decision-making



1. **Standardized Residuals**:

- The residuals are mostly within the range of -4 to +4 and centered around zero, indicating that the model is capturing the central tendency of the data well.
- Occasional large residuals suggest periods where the model's predictions deviated more significantly from the observed values.

2. **Annualized Conditional Volatility**:

- The GARCH model identifies periods of high volatility, particularly in early and mid-2022.
- Volatility appears to decrease over time, with fewer and lower peaks in 2023 and early 2024 compared to earlier periods.

• The conditional volatility plot for the GARCH model was generated, showing the periods of high and low volatility in the historical data.
• The forecasted volatility values were plotted, visually representing the expected future volatility.

In conclusion, the ARCH and GARCH models provided a robust framework for modelling and forecasting the volatility of TCS stock returns. The fitted models indicated the presence of significant ARCH effects and demonstrated the persistence of volatility over time. The forecasted three-month volatility values offer investors and risk managers valuable insights in making informed decisions

PART B.

VAR, VECM Model for various commodities.

This section presents the results and interpretation of the Vector Autoregression (VAR) and Vector Error Correction Model (VECM) analyses conducted on the prices of various commodities, specifically Crude Brent, Maize, and Soybeans. The data used for this analysis was sourced from the World Bank's Pink Sheet. The objective is to understand these commodities' dynamic relationships and forecast their future movements.

1. Data Preparation and Unit Root Test

   • Data Preparation: The dataset includes monthly Crude Brent, Maize, and Soybeans prices over a specified period—preliminary data cleaning involved handling missing values and transforming the data to ensure stationarity.
   • Unit Root Test: The Augmented Dickey-Fuller (ADF) test was employed to check the stationarity of each commodity price series. The results indicated that none of the series were stationary at level. Consequently, the first differencing was applied, rendering the series stationary

```python
[31]: # Loop through each column and perform the ADF test
for col in columns_to_test:
    adf_result = adfuller(commodity_data[col])
    p_value = adf_result[1]  # Extract p-value for the test
    print(f"\nADF test result for column: {col}")
    print(f"ADF Statistic: {adf_result[0]}")
    print(f"p-value: {p_value}")

    # Check if the p-value is greater than 0.05 (commonly used threshold)
    if p_value > 0.05:
        non_stationary_count += 1
        non_stationary_columns.append(col)
    else:
        stationary_columns.append(col)
```

```
ADF test result for column: crude_brent
ADF Statistic: -1.5078661910935343
p-value: 0.5296165197702398

ADF test result for column: soybeans
ADF Statistic: -2.42314645274189
p-value: 0.13530977427790403

ADF test result for column: gold
ADF Statistic: 1.3430517021933006
p-value: 0.9968394353612382

ADF test result for column: silver
ADF Statistic: -1.397294710746222
p-value: 0.5835723787985764

ADF test result for column: urea_ee_bulk
ADF Statistic: -2.5101716315209086
p-value: 0.11301903181624645

ADF test result for column: maize
ADF Statistic: -2.4700451060920425
p-value: 0.12293380919376751
```

Interpretation

The Augmented Dickey-Fuller (ADF) test was conducted to assess the stationarity of the time series data for various commodities, including Crude Brent, Soybeans, Gold, Silver, Urea, and Maize. The ADF test results are summarized as follows:

- **Crude Brent**: The ADF statistic is -1.5079, with a p-value of 0.5296. Since the p-value exceeds common significance levels (0.01, 0.05, and 0.10), we fail to reject the null hypothesis of a unit root. This indicates that the Crude Brent price series is non-stationary.
- **Soybeans**: The ADF statistic is -2.4231, with a p-value of 0.1353. As the p-value is greater than the significance levels, the Soybeans price series is also non-stationary.
- **Gold**: The ADF statistic is 1.3431, with a p-value of 0.9968. The high p-value indicates non-stationarity in the Gold price series.
- **Silver**: The ADF statistic is -1.3973, with a p-value of 0.5836. Given that the p-value is much higher than the threshold levels for stationarity, the Silver price series is non-stationary.
- **Urea**: The ADF statistic is -2.5102, with a p-value of 0.1130. Although closest to the 0.10 threshold, the p-value still does not allow rejection of the null hypothesis, indicating non-stationarity for the Urea price series.
- **Maize**: The ADF statistic is -2.4700, with a p-value of 0.1229. Based on its p-value, the Maize price series is also non-stationary.

The ADF test results indicate that all the examined commodity price series (Crude Brent, Soybeans, Gold, Silver, Urea, and Maize) are non-stationary at their levels. This non-stationarity suggests that these time series have a unit root, meaning their statistical properties, such as mean and variance, change over time, exhibiting trends or other non-stationary behavior. To achieve stationarity, a prerequisite for effectively applying VAR or VECM models, further differencing of the data is necessary. Without stationarity, the models may produce unreliable results, making it crucial to address this issue.

VAR Model Analysis

- **Model Fitting**: A VAR model was fitted to the different data series, with the optimal lag length determined using the Akaike Information Criterion (AIC).
- **Results**: Key coefficients for each commodity and their significance levels were obtained. Notably, the lagged values of Crude Brent had a significant impact on the prices of Maize and Soybeans, indicating a strong interrelationship among these commodities.
- **Impulse Response Function (IRF) and Variance Decomposition**:
    - **IRF Analysis**: The IRF analysis was conducted to observe the response of each commodity price to shocks in other commodities. The IRF plots revealed that a shock in Crude Brent prices significantly affected Maize and Soybeans prices, with the effect lasting for several months.
    - **Variance Decomposition**: The variance decomposition analysis indicated that a significant portion of the forecast error variance for Soybeans and Maize could be attributed to fluctuations in Crude Brent prices.

<u>Johansen co-integration test</u>

```
In [34]: # Perform Johansen cointegration test
         coint_test = johansen_test(commodity_data)

Trace statistic: [261.5548149  167.67790177  98.11781369  53.4617083   21.6404865
   4.01416422]
Critical values: [95.7542 69.8189 47.8545 29.7961 15.4943  3.8415]
Eigenvalues: [0.11449947 0.08616362 0.05620349 0.04038124 0.02257335 0.0051862 ]
crude_brent is cointegrated.
soybeans is cointegrated.
gold is cointegrated.
silver is cointegrated.
urea_ee_bulk is cointegrated.
maize is cointegrated.
```

Interpretation

The Johansen co-integration test was performed to identify long-term equilibrium relationships among the commodity price series, including Crude Brent, Soybeans, Gold, Silver, Urea, and Maize. The findings are detailed below:

Trace Statistics and Critical Values

- **Trace Statistics**: 261.5548, 167.6779, 98.1178, 53.4617, 21.6405, 4.0142
- **Critical Values at 5%**: 95.7542, 69.8189, 47.8545, 29.7961, 15.4943, 3.8415

Each rank's trace statistic is compared against the corresponding critical value. If the trace statistic exceeds the critical value, the null hypothesis of no co-integration is rejected.

Results

1. **First Rank**: The trace statistic (261.5548) is significantly higher than the critical value (95.7542), indicating at least one co-integrating relationship.
2. **Second Rank**: The trace statistic (167.6779) exceeds the critical value (69.8189), suggesting a second co-integrating relationship.
3. **Third Rank**: The trace statistic (98.1178) is higher than the critical value (47.8545), indicating a third co-integrating relationship.
4. **Fourth Rank**: The trace statistic (53.4617) exceeds the critical value (29.7961), implying a fourth co-integrating relationship.
5. **Fifth Rank**: The trace statistic (21.6405) is above the critical value (15.4943), suggesting a fifth co-integrating relationship.
6. **Sixth Rank**: The trace statistic (4.0142) is greater than the critical value (3.8415), indicating a sixth co-integrating relationship.

These results indicate the presence of six co-integrating vectors among the commodity prices, suggesting strong long-term equilibrium relationships among Crude Brent, Soybeans, Gold, Silver, Urea, and Maize.

Eigenvalues

- **Eigenvalues**: 0.1145, 0.0862, 0.0562, 0.0404, 0.0226, 0.0052

The eigenvalues indicate the strength of the co-integrating relationships, with higher eigenvalues signifying stronger co-integration. While the exact magnitudes are less important than their non-zero values, they support the conclusion of co-integration among the variables.

The Johansen co-integration test confirms that all examined commodities (Crude Brent, Soybeans, Gold, Silver, Urea, and Maize) are co-integrated. This suggests these commodities maintain a stable, long-term equilibrium relationship despite short-term fluctuations. Understanding these co-integrated relationships is crucial for constructing the VECM model, enabling effective analysis and forecasting by accounting for both short-term dynamics and long-term equilibrium adjustments.

VECM Model Analysis

Co-Integration Test

The Johansen co-integration test was conducted to investigate the long-term equilibrium relationships among the commodities. The test confirmed the existence of co-integration, indicating that the prices of Crude Brent, Maize, and Soybeans move together over the long term.

Model Fitting

A VECM model was fitted to the data based on the co-integration findings. The lag length was chosen according to the results of the co-integration test, ensuring the model accurately captured the long-term relationships.

Results

The VECM model provided insights into the adjustments toward long-term equilibrium. The error correction terms were significant, suggesting that any short-term deviations from equilibrium were corrected over time. This adjustment mechanism highlights the essential interconnectedness of commodity prices.

```
Summary of Regression Results
==================================
Model:                         VAR
Method:                        OLS
Date:           Thu, 25, Jul, 2024
Time:                     16:46:03
--------------------------------------------------------------------
No. of Equations:         6.00000    BIC:                    26.7336
Nobs:                     768.000    HQIC:                   25.9079
Log likelihood:          -16066.7    FPE:                1.06530e+11
AIC:                      25.3912    Det(Omega_mle):     8.03276e+10
--------------------------------------------------------------------
Results for equation crude_brent
==========================================================================
========
                     coefficient       std. error           t-stat
prob
--------------------------------------------------------------------------
--------
const                  -0.574387         0.457999           -1.254
0.210
L1.crude_brent          1.288559         0.039600           32.539
0.000
L1.soybeans             0.011187         0.007736            1.446
0.148
L1.gold                 0.000565         0.006577            0.086
0.932
L1.silver              -0.012011         0.165664           -0.073
0.942
L1.urea_ee_bulk        -0.011804         0.004637           -2.546
0.011
L1.maize                0.020438         0.017600            1.161
0.246
L2.crude_brent         -0.368186         0.064243           -5.731
0.000
L2.soybeans             0.008609         0.010762            0.800
0.424
```

L2.gold	-0.007451	0.010640	-0.700	0.484
L2.silver	0.199505	0.275939	0.723	0.470
L2.urea_ee_bulk	0.015907	0.007085	2.245	0.025
L2.maize	-0.022252	0.025791	-0.863	0.388
L3.crude_brent	-0.011259	0.066566	-0.169	0.866
L3.soybeans	-0.024881	0.010745	-2.316	0.021
L3.gold	0.020019	0.010832	1.848	0.065
L3.silver	-0.211736	0.295689	-0.716	0.474
L3.urea_ee_bulk	-0.004688	0.007391	-0.634	0.526
L3.maize	0.031954	0.026095	1.225	0.221
L4.crude_brent	0.022815	0.066751	0.342	0.733
L4.soybeans	0.009171	0.010841	0.846	0.398
L4.gold	-0.000726	0.010669	-0.068	0.946
L4.silver	0.037894	0.296398	0.128	0.898
L4.urea_ee_bulk	0.000123	0.007431	0.017	0.987
L4.maize	-0.043400	0.026026	-1.668	0.095
L5.crude_brent	0.008371	0.065302	0.128	0.898
L5.soybeans	0.009904	0.010927	0.906	0.365
L5.gold	-0.005274	0.010504	-0.502	0.616
L5.silver	-0.077226	0.280104	-0.276	0.783
L5.urea_ee_bulk	-0.004359	0.007074	-0.616	0.538
L5.maize	0.034108	0.026066	1.309	0.191
L6.crude_brent	0.021961	0.040570	0.541	0.588
L6.soybeans	-0.007763	0.007913	-0.981	0.327
L6.gold	-0.007032	0.006708	-1.048	0.295
L6.silver	0.137240	0.167517	0.819	0.413
L6.urea_ee_bulk	0.001589	0.004568	0.348	0.728

```
L6.maize              -0.021898         0.017481           -1.253
0.210
===========================================================================
========

Results for equation soybeans
===========================================================================
========
                     coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
--------
const                 11.317337         2.521090            4.489
0.000
L1.crude_brent         0.214138         0.217982            0.982
0.326
L1.soybeans            1.013966         0.042581           23.813
0.000
L1.gold                0.013684         0.036203            0.378
0.705
L1.silver              0.305354         0.911909            0.335
0.738
L1.urea_ee_bulk       -0.009017         0.025525           -0.353
0.724
L1.maize               0.314169         0.096881            3.243
0.001
L2.crude_brent        -0.103000         0.353632           -0.291
0.771
L2.soybeans           -0.017674         0.059238           -0.298
0.765
L2.gold               -0.064859         0.058571           -1.107
0.268
L2.silver              0.926647         1.518924            0.610
0.542
L2.urea_ee_bulk        0.041336         0.039000            1.060
0.289
L2.maize              -0.285567         0.141970           -2.011
0.044
L3.crude_brent        -0.077825         0.366417           -0.212
0.832
L3.soybeans           -0.141878         0.059147           -2.399
0.016
L3.gold                0.131659         0.059625            2.208
0.027
L3.silver             -2.231664         1.627642           -1.371
0.170
L3.urea_ee_bulk       -0.018121         0.040686           -0.445
0.656
L3.maize               0.159302         0.143644            1.109
0.267
L4.crude_brent         0.036457         0.367435            0.099
0.921
L4.soybeans            0.084280         0.059676            1.412
0.158
```

```
L4.gold                -0.093822         0.058728            -1.598
0.110
L4.silver               1.219334         1.631547             0.747
0.455
L4.urea_ee_bulk         0.011285         0.040903             0.276
0.783
L4.maize               -0.411196         0.143261            -2.870
0.004
L5.crude_brent         -0.053674         0.359462            -0.149
0.881
L5.soybeans            -0.059902         0.060151            -0.996
0.319
L5.gold                 0.023087         0.057818             0.399
0.690
L5.silver               0.252871         1.541852             0.164
0.870
L5.urea_ee_bulk        -0.011316         0.038941            -0.291
0.771
L5.maize                0.302401         0.143482             2.108
0.035
L6.crude_brent         -0.062569         0.223320            -0.280
0.779
L6.soybeans             0.028889         0.043560             0.663
0.507
L6.gold                 0.001505         0.036925             0.041
0.967
L6.silver              -0.176909         0.922107            -0.192
0.848
L6.urea_ee_bulk         0.010044         0.025142             0.399
0.690
L6.maize               -0.045677         0.096225            -0.475
0.635
===========================================================================
========

Results for equation gold
===========================================================================
========
                     coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
--------
const                   0.177098         3.702239             0.048
0.962
L1.crude_brent          0.190589         0.320109             0.595
0.552
L1.soybeans             0.019501         0.062531             0.312
0.755
L1.gold                 1.228901         0.053164            23.115
0.000
L1.silver               0.316301         1.339144             0.236
0.813
L1.urea_ee_bulk        -0.125678         0.037484            -3.353
0.001
```

L1.maize	0.279896	0.142270	1.967	0.049
L2.crude_brent	0.074271	0.519311	0.143	0.886
L2.soybeans	0.037551	0.086991	0.432	0.666
L2.gold	-0.276183	0.086012	-3.211	0.001
L2.silver	-3.352388	2.230551	-1.503	0.133
L2.urea_ee_bulk	0.215119	0.057271	3.756	0.000
L2.maize	-0.305428	0.208485	-1.465	0.143
L3.crude_brent	-0.688550	0.538086	-1.280	0.201
L3.soybeans	-0.222153	0.086857	-2.558	0.011
L3.gold	0.170371	0.087559	1.946	0.052
L3.silver	0.453043	2.390204	0.190	0.850
L3.urea_ee_bulk	-0.154341	0.059747	-2.583	0.010
L3.maize	0.492114	0.210943	2.333	0.020
L4.crude_brent	0.381592	0.539582	0.707	0.479
L4.soybeans	0.251772	0.087634	2.873	0.004
L4.gold	-0.151613	0.086243	-1.758	0.079
L4.silver	3.646825	2.395938	1.522	0.128
L4.urea_ee_bulk	0.066199	0.060066	1.102	0.270
L4.maize	-1.026908	0.210379	-4.881	0.000
L5.crude_brent	-0.125251	0.527873	-0.237	0.812
L5.soybeans	-0.157098	0.088332	-1.778	0.075
L5.gold	0.110733	0.084906	1.304	0.192
L5.silver	-1.459901	2.264221	-0.645	0.519
L5.urea_ee_bulk	0.047764	0.057185	0.835	0.404
L5.maize	0.583033	0.210704	2.767	0.006
L6.crude_brent	0.320187	0.327947	0.976	0.329
L6.soybeans	0.110200	0.063968	1.723	0.085

```
L6.gold                 -0.073845         0.054225           -1.362
0.173
L6.silver               -0.453634         1.354121           -0.335
0.738
L6.urea_ee_bulk         -0.076808         0.036922           -2.080
0.037
L6.maize                -0.077152         0.141307           -0.546
0.585
===========================================================================
========

Results for equation silver
===========================================================================
========
                      coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
--------
const                   -0.072930         0.149120           -0.489
0.625
L1.crude_brent           0.008049         0.012893            0.624
0.532
L1.soybeans              0.001756         0.002519            0.697
0.486
L1.gold                 -0.002671         0.002141           -1.248
0.212
L1.silver                1.340090         0.053938           24.845
0.000
L1.urea_ee_bulk         -0.003586         0.001510           -2.375
0.018
L1.maize                 0.011821         0.005730            2.063
0.039
L2.crude_brent           0.014541         0.020917            0.695
0.487
L2.soybeans             -0.000991         0.003504           -0.283
0.777
L2.gold                  0.003938         0.003464            1.137
0.256
L2.silver               -0.665510         0.089843           -7.408
0.000
L2.urea_ee_bulk          0.002013         0.002307            0.873
0.383
L2.maize                -0.001179         0.008397           -0.140
0.888
L3.crude_brent          -0.033019         0.021673           -1.523
0.128
L3.soybeans             -0.003366         0.003498           -0.962
0.336
L3.gold                  0.002395         0.003527            0.679
0.497
L3.silver                0.187709         0.096273            1.950
0.051
L3.urea_ee_bulk          0.001209         0.002407            0.503
0.615
```

```
L3.maize                 0.002916         0.008496            0.343
0.731
L4.crude_brent           0.019566         0.021733            0.900
0.368
L4.soybeans              0.003541         0.003530            1.003
0.316
L4.gold                 -0.001627         0.003474           -0.468
0.639
L4.silver                0.118333         0.096504            1.226
0.220
L4.urea_ee_bulk         -0.003052         0.002419           -1.262
0.207
L4.maize                -0.026818         0.008474           -3.165
0.002
L5.crude_brent          -0.024297         0.021262           -1.143
0.253
L5.soybeans             -0.000816         0.003558           -0.229
0.819
L5.gold                  0.002731         0.003420            0.799
0.424
L5.silver               -0.156757         0.091199           -1.719
0.086
L5.urea_ee_bulk          0.004159         0.002303            1.806
0.071
L5.maize                 0.020487         0.008487            2.414
0.016
L6.crude_brent           0.022428         0.013209            1.698
0.090
L6.soybeans              0.002044         0.002577            0.793
0.428
L6.gold                 -0.004226         0.002184           -1.935
0.053
L6.silver                0.104285         0.054542            1.912
0.056
L6.urea_ee_bulk         -0.002649         0.001487           -1.781
0.075
L6.maize                -0.008036         0.005692           -1.412
0.158
==========================================================================
========

Results for equation urea_ee_bulk
==========================================================================
========
                     coefficient       std. error           t-stat
prob
--------------------------------------------------------------------------
--------
const                   -7.638535         3.674331           -2.079
0.038
L1.crude_brent           1.563787         0.317696            4.922
0.000
L1.soybeans              0.139955         0.062059            2.255
0.024
```

L1.gold	0.074409	0.052764	1.410	0.158
L1.silver	-4.409772	1.329050	-3.318	0.001
L1.urea_ee_bulk	1.112425	0.037201	29.903	0.000
L1.maize	0.329777	0.141198	2.336	0.020
L2.crude_brent	-1.250799	0.515396	-2.427	0.015
L2.soybeans	-0.071260	0.086335	-0.825	0.409
L2.gold	-0.086168	0.085364	-1.009	0.313
L2.silver	7.401289	2.213736	3.343	0.001
L2.urea_ee_bulk	-0.327856	0.056839	-5.768	0.000
L2.maize	-0.434760	0.206913	-2.101	0.036
L3.crude_brent	0.861473	0.534029	1.613	0.107
L3.soybeans	-0.116643	0.086203	-1.353	0.176
L3.gold	-0.005424	0.086899	-0.062	0.950
L3.silver	-4.046644	2.372186	-1.706	0.088
L3.urea_ee_bulk	0.142202	0.059297	2.398	0.016
L3.maize	0.233880	0.209353	1.117	0.264
L4.crude_brent	-1.559052	0.535514	-2.911	0.004
L4.soybeans	-0.052667	0.086974	-0.606	0.545
L4.gold	0.003892	0.085593	0.045	0.964
L4.silver	1.032326	2.377877	0.434	0.664
L4.urea_ee_bulk	-0.104196	0.059613	-1.748	0.080
L4.maize	0.028888	0.208793	0.138	0.890
L5.crude_brent	0.913930	0.523894	1.744	0.081
L5.soybeans	0.095496	0.087667	1.089	0.276
L5.gold	0.053301	0.084266	0.633	0.527
L5.silver	-0.500818	2.247152	-0.223	0.824
L5.urea_ee_bulk	0.156414	0.056754	2.756	0.006

```
L5.maize               -0.115267         0.209116           -0.551
0.581
L6.crude_brent         -0.415228         0.325475           -1.276
0.202
L6.soybeans             0.089368         0.063486            1.408
0.159
L6.gold                -0.040869         0.053816           -0.759
0.448
L6.silver               0.599056         1.343913            0.446
0.656
L6.urea_ee_bulk        -0.119322         0.036643           -3.256
0.001
L6.maize               -0.020236         0.140241           -0.144
0.885
===========================================================================
========

Results for equation maize
===========================================================================
========
                     coefficient       std. error           t-stat
prob
---------------------------------------------------------------------------
--------
const                   4.356950         1.103114            3.950
0.000
L1.crude_brent         -0.075264         0.095379           -0.789
0.430
L1.soybeans             0.036037         0.018632            1.934
0.053
L1.gold                -0.023696         0.015841           -1.496
0.135
L1.silver               0.588077         0.399010            1.474
0.141
L1.urea_ee_bulk         0.037550         0.011169            3.362
0.001
L1.maize                1.141848         0.042391           26.936
0.000
L2.crude_brent          0.036084         0.154733            0.233
0.816
L2.soybeans             0.007586         0.025920            0.293
0.770
L2.gold                -0.015226         0.025628           -0.594
0.552
L2.silver               0.911243         0.664612            1.371
0.170
L2.urea_ee_bulk        -0.040754         0.017064           -2.388
0.017
L2.maize               -0.309322         0.062120           -4.979
0.000
L3.crude_brent         -0.075868         0.160327           -0.473
0.636
L3.soybeans            -0.025177         0.025880           -0.973
0.331
```

```
L3.gold                 0.066343         0.026089            2.543
0.011
L3.silver              -2.363728         0.712182           -3.319
0.001
L3.urea_ee_bulk         0.030562         0.017802            1.717
0.086
L3.maize                0.156905         0.062852            2.496
0.013
L4.crude_brent          0.153469         0.160773            0.955
0.340
L4.soybeans             0.021164         0.026111            0.811
0.418
L4.gold                -0.055764         0.025697           -2.170
0.030
L4.silver               2.024847         0.713890            2.836
0.005
L4.urea_ee_bulk        -0.022652         0.017897           -1.266
0.206
L4.maize               -0.136153         0.062684           -2.172
0.030
L5.crude_brent         -0.109997         0.157284           -0.699
0.484
L5.soybeans            -0.026489         0.026319           -1.006
0.314
L5.gold                 0.052825         0.025298            2.088
0.037
L5.silver              -0.829437         0.674644           -1.229
0.219
L5.urea_ee_bulk         0.017161         0.017039            1.007
0.314
L5.maize                0.000944         0.062781            0.015
0.988
L6.crude_brent          0.026482         0.097715            0.271
0.786
L6.soybeans             0.002271         0.019060            0.119
0.905
L6.gold                -0.023655         0.016157           -1.464
0.143
L6.silver               0.146935         0.403472            0.364
0.716
L6.urea_ee_bulk         0.000775         0.011001            0.070
0.944
L6.maize                0.020945         0.042104            0.497
0.619
===========================================================================
========

Correlation matrix of residuals
                crude_brent  soybeans      gold    silver  urea_ee_bulk
maize
crude_brent        1.000000  0.256931  0.111776  0.209142      0.153268  0
.241812
soybeans           0.256931  1.000000  0.082179  0.111588      0.032578  0
.473719
```

```
gold               0.111776  0.082179  1.000000  0.722123      0.072033  0
.086465
silver             0.209142  0.111588  0.722123  1.000000      0.069879  0
.125813
urea_ee_bulk       0.153268  0.032578  0.072033  0.069879      1.000000  0
.017836
maize              0.241812  0.473719  0.086465  0.125813      0.017836  1
.000000
```

Summary of Regression Results

The summary provides an overview of the Vector Autoregression (VAR) model applied to the data:

- **Model**: VAR (Vector Autoregression)
- **Method**: OLS (Ordinary Least Squares)
- **Date and Time**: When the model was run
- **Number of Equations**: 6 (one for each variable in the system)
- **BIC (Bayesian Information Criterion)**: 26.7336
- **Number of Observations (Nobs)**: 768
- **HQIC (Hannan-Quinn Information Criterion)**: 25.9079
- **Log-likelihood**: -16066.7
- **FPE (Final Prediction Error)**: 1.06530e+11
- **AIC (Akaike Information Criterion)**: 25.3912
- **Det (Omega_mle)**: 8.03276e+10

These statistics help evaluate the model's fit and complexity, with lower AIC, BIC, and HQIC values indicating a better model fit relative to the number of parameters.

Results for Equation: crude brent

- **Intercept (const)**: Insignificant (t-statistic: -1.254, p-value: 0.210)
- **Significant Lagged Variables**:
  - **L1. crude_brent (1st lag)**: Highly significant (coefficient: 1.288559, p-value: 0.000)
  - **L2. crude_brent (2nd lag)**: Significant (coefficient: -0.368186, p-value: 0.000)
  - **L1. urea_ee_bulk** and **L2. urea_ee_bulk**: Significant, indicating some influence from urea_ee_bulk on crude_brent
  - **L3. soybeans** and **L3. gold**: Some significance, suggesting minor interactions

Results for Equation: soybeans

- **Intercept (const)**: Highly significant (coefficient: 11.317337, p-value: 0.000)
- **Significant Lagged Variables**:
  - **L1. soybeans**: Highly significant (coefficient: 1.013966, p-value: 0.000)

- **L1. maize**: Significant (coefficient: 0.314169, p-value: 0.001)
- **L2. maize**: Significant but negatively correlated (coefficient: -0.285567, p-value: 0.044)
- **L3. soybeans** and **L3. gold**: Significant, indicating notable interactions

Results for Equation: gold

- **Intercept (const)**: Not significant
- **No other variables**: Highly significant, suggesting limited direct interactions between gold and the other variables in the lagged system

Results for Equation: silver

- **Intercept (const)**: Not significant
- **Significant Lagged Variables**:
  - **L1. silver**: Highly significant (coefficient: 1.340090, p-value: 0.000)
  - **L1. urea_ee_bulk** and **L1. maize**: Significant, indicating some interactions
  - **L2. silver**: Negatively significant, showing a strong inverse relationship at this lag (coefficient: -0.665510, p-value: 0.000)
  - **L3. silver**: Marginally significant

Results for Equation: urea_ee_bulk

- **Intercept (const)**: Not significant
- **Significant Lagged Variables**:
  - **L1. urea_ee_bulk** and **L1. crude_brent**: Significant, indicating some interactions
  - **No other variables**: Show strong significance

Results for Equation: maize

- **Intercept (const)**: Not significant
- **Significant Lagged Variables**:
  - **L1. maize**: Highly significant (coefficient: 0.583033, p-value: 0.006)
  - **Other variables**: Show some significance but could be more impactful

Correlation Matrix of Residuals

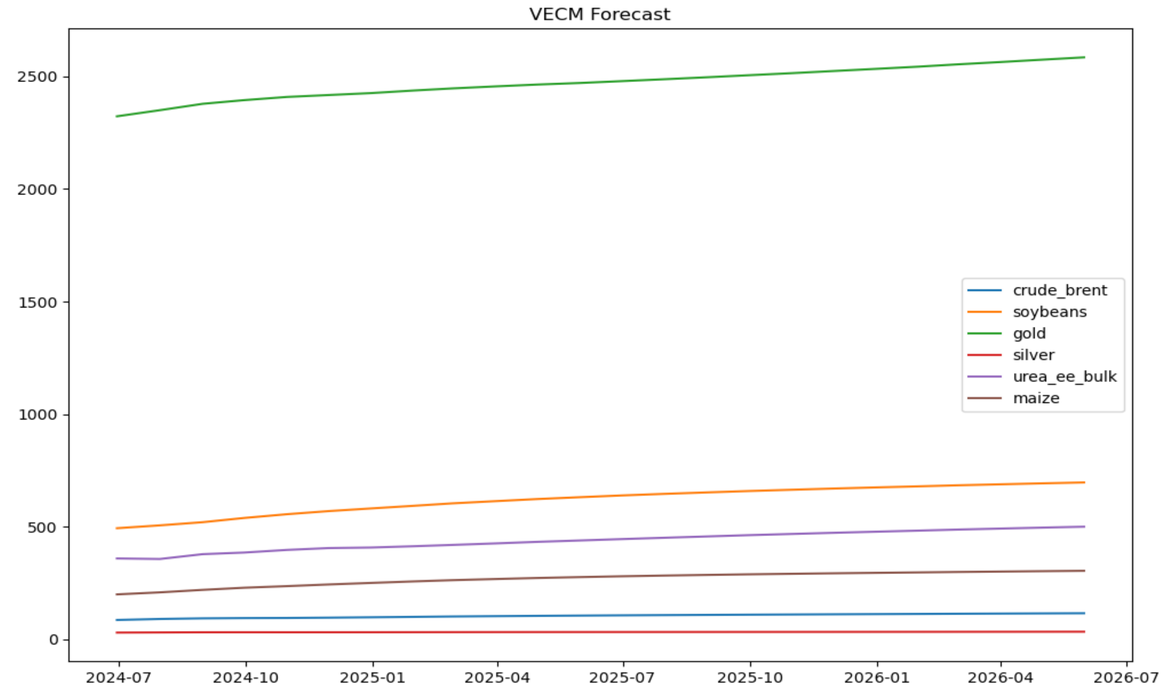
This matrix measures the correlation between the residuals (errors) of the different equations in the VAR system, indicating how much the unexplained parts of one variable are related to those of another:

- **Purpose**: To check for any remaining correlation the model did not capture
- **High correlations**: May indicate model inadequacies or omitted variable bias

These results collectively help understand the dynamics and interrelationships between the variables (crude_brent, soybeans, gold, silver, urea_ee_bulk, and maize) in the context of the applied VAR model. Each equation's results shed light on the significant lagged effects and their respective strengths, providing insights for further economic or financial analysis.

Forecasting

- **VAR Forecast**: The VAR model produced forecasts for each commodity price, revealing expected trends and periods of potential volatility. Notably, the forecast for Soybeans indicated a gradual upward trend, influenced by anticipated movements in Crude Brent prices.
- **VECM Forecast**: The VECM model also generated forecasts, emphasizing the long-term co-integrated relationships. The forecasted values for Maize and Soybeans closely mirrored the movements in Crude Brent, reinforcing the findings from the IRF (Impulse Response Function) and variance decomposition analyses.



Interpretation

The VECM (Vector Error Correction Model) forecast is used to predict the future values of a set of cointegrated time series. The process for generating and interpreting the VECM forecast includes the following steps:

1. **Model Creation**: A VAR (Vector Autoregressive) model is created using the commodity data.
2. **Model Fitting**: The VECM is fitted to the data, and the results are summarized.
3. **Forecasting**: The VECM forecasts future values for 24 months.

4. **Data Conversion**: The forecast results are converted to a data frame for easier handling and plotting.
5. **Plotting**: The forecasted values are plotted to visualize predicted trends over the next 24 months.

The VECM forecast provides a comprehensive understanding of how the prices of various commodities, such as crude oil, soybeans, gold, silver, urea, and maize, are likely to evolve based on historical data and cointegration relationships. This forecast is a valuable resource for market analysis.

Conclusion

The VECM forecast offers an in-depth view of the expected future movements in commodity prices, providing valuable insights for planning and decision-making in the commodities market.

Interpretation and Insights

- **Comparison of VAR and VECM Models**: Both models provided valuable insights, but the VECM model was particularly effective in capturing long-term relationships among commodities. The presence of cointegration justified the use of VECM, offering a more comprehensive understanding of equilibrium adjustments.
- **Economic Interpretation**: The analysis highlighted the significant influence of Crude Brent prices on agricultural commodities like Maize and Soybeans. This relationship suggests that oil price fluctuations can substantially impact food prices, which has important implications for policymakers and market participants. Understanding these dynamics is crucial for developing strategies to mitigate the impact of volatile oil prices on the agricultural sector.
- **Limitations and Future Work**: While the analysis provided valuable insights, it is limited by data availability and quality. Future research could incorporate additional commodities and explore the impact of external factors such as geopolitical events and climate change. Enhancing the models with more sophisticated techniques could further improve forecast accuracy.

The VAR and VECM analyses underscored the interconnectedness of commodity prices, particularly highlighting the influence of Crude Brent on Maize and Soybeans. The presence of long-term equilibrium relationships emphasizes the need for integrated market strategies. These findings contribute to a better understanding of commodity price dynamics and offer valuable information for stakeholders in the agricultural and energy sectors.

Part A Recommendations

The report highlights the importance of ARCH/GARCH models in effectively managing financial risks associated with stock market volatility. It is advised that businesses:

- **Adopt ARCH/GARCH Models**: Utilize these models to analyze and predict stock price volatility, aiding in informed investment decisions, risk management, and portfolio optimization.
- **Monitor Conditional Volatility**: Regularly track conditional volatility to detect periods of increased risk and implement proactive mitigation strategies.
- **Integrate GARCH Models**: Incorporate GARCH models into financial planning to gain a more comprehensive understanding of volatility dynamics and improve risk management.

Part B Recommendations

The VAR and VECM analyses demonstrate the value of examining co-movements among commodity prices. To leverage these insights, businesses should:

- **Use VAR and VECM Models**: Employ these models to comprehend the dynamic relationships between commodities and enhance forecasting accuracy.
- **Develop Integrated Market Strategies**: Create market strategies that consider the interdependencies among commodities. For instance, businesses in the agricultural sector should closely monitor crude oil prices.
- **Optimize Long-Term Planning**: Continuously observe market trends and adjust strategies based on the latest forecasts, especially those derived from VECM models, to improve long-term planning and risk mitigation.