GATE - 2007 - XE

EE1030 : Matrix Theory

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1) The volume of the prism whose base is the triangle in the xy - plane bounded by the x - axis and the lines $y = x$ and $x = 2$ and whose top lies in the plane $z = 5 - x - y$ is
   a) 2
   b) 4
   c) 6
   d) 10

2) The general solution of

$$x\left(z^2 - y^2\right)\frac{\partial z}{\partial x} + y\left(x^2 - z^2\right)\frac{\partial z}{\partial y} = z\left(y^2 - x^2\right)$$

   is
   a) $F\left(x^2 + y^2 + z^2, xyz\right) = 0$
   b) $F\left(x^2 + y^2 - z^2, xyz\right) = 0$
   c) $F\left(x^2 - y^2 + z^2, xyz\right) = 0$
   d) $F\left(-x^2 + y^2 + z^2, xyz\right) = 0$

3) Choose a point uniformly distributed at random on the disc $x^2 + y^2 \leq 1$. Let the random variable X denote the distance of this point from the center of the disc. Then the variance of X is
   a) $\frac{1}{16}$
   b) $\frac{1}{17}$
   c) $\frac{1}{18}$
   d) $\frac{1}{19}$

4) If Runge-Kutta method of order 4 is used to solve the differential equation $\frac{dy}{dx} = f(x)$, $y(0) = 0$ in the interval $[0, h]$ with step size h, then
   a) $y(h) = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right]$
   b) $y(h) = \frac{h}{6}\left[f(0) + f(h)\right]$
   c) $y(h) = \frac{h}{2}\left[f(0) + f(h)\right]$

d) $y(h) = \frac{h}{6}\left[f(0) + 2f\left(\frac{h}{2}\right) + f(h)\right]$

5) If a polynomial of degree three interpolates a function $f(x)$ at the points $(0,3)$, $(1,13)$, $(3,99)$ and $(4,187)$, then $f(2)$ is

a) 20
b) 36
c) 43
d) 58

Common Data for Questions 23, 24:
Let $f : \Re \to \Re$ be defined by $f(x) = x^2$ for $-\pi \le x \le \pi$ and $f(x + 2\pi) = f(x)$.

6) The Fourier series of f in $[-\pi, \pi]$ is

a) $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}$
b) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n \cos(nx)}{n^2}$
c) $\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^2 \cos(nx)}{n^2}$
d) $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{\cos(nx)}{n^2}$

7) The sum of the absolute values of the Fourier coefficients of f is

a) $\frac{\pi^2}{6}$
b) $\frac{\pi^2}{3}$
c) $\frac{2\pi^2}{3}$
d) π^2

Statement for Linked Answer Questions 25 & 26:
Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ be a solution of the differential equation $\frac{d^2y}{dx^2} + xy = 0$.

8) The value of a_{11} is

a) 0
b) 1
c) 2
d) 3

9) The solution of the differential equation given above satisfying $y(0) = 1$ and $y'(0) = 0$ is

a) $y(x) = 1 + \frac{1}{2.3}x^2 - \frac{1}{2.3.5.6}x^4 + \frac{1}{2.3.5.6.8.9}x^6 - \cdots$
b) $y(x) = 1 - \frac{1}{2.3}x^2 + \frac{1}{2.3.5.6}x^4 - \frac{1}{2.3.5.6.8.9}x^6 + \cdots$
c) $y(x) = 1 + \frac{1}{2.3}x^3 - \frac{1}{2.3.5.6}x^6 + \frac{1}{2.3.5.6.8.9}x^9 - \cdots$
d) $y(x) = 1 - \frac{1}{2.3}x^3 + \frac{1}{2.3.5.6}x^6 - \frac{1}{2.3.5.6.8.9}x^9 + \cdots$

Statement for Linked Answer Questions 27 & 28:
The potential $u(x, y)$ satisfies the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the square $0 \le x \le \pi$, $0 \le y \le \pi$. Three of the edges $x = 0$, $x = \pi$ and $y = 0$ of the square are kept at zero potential and the edge $y = \pi$ is kept at nonzero potential.

10) The potential $u(x, y)$ is given by
   a) $u(x, y) = \sum_{n=1}^{\infty} A_n \cosh(nx) \sin(ny)$
   b) $u(x, y) = \sum_{n=1}^{\infty} A_n \sin(nx) \cosh(ny)$
   c) $u(x, y) = \sum_{n=1}^{\infty} A_n \sinh(nx) \sin(ny)$
   d) $u(x, y) = \sum_{n=1}^{\infty} A_n \sin(nx) \sinh(ny)$

11) If the edge $y = \pi$ is kept at the potential $\sin(x)$, then the potential $u(x, y)$ is given by
   a) $u(x, y) = \sum_{n=1}^{\infty} \frac{\sin(nx)\sinh(ny)}{\sinh(n\pi)}$
   b) $u(x, y) = \frac{\sin(x)\sinh(y)}{\sinh(\pi)}$
   c) $u(x, y) = \frac{\sin(x)\cosh(y)}{\cosh(\pi)}$
   d) $u(x, y) = \sum_{n=1}^{\infty} \frac{\cosh(nx)\sin(ny)}{\cosh(n\pi)}$

12) If the 7-base representation of a number is 123, then its octal representation is
   a) 102
   b) 103
   c) 111
   d) 112

13) Consider the following four FORTRAN statements

$$S_1 : X = 5^{**}3$$
$$S_2 : X = (-5)^{**}\ 3.0$$
$$S_3 : X = 5^{**}\ (-3)$$
$$S_4 : X = 5^{**}3.0$$

   Which one of the following sets contains the set of valid statements from above?
   a) $\{S_1, S_3\}$
   b) $\{S_1, S_4\}$
   c) $\{S_2, S_3\}$
   d) $\{S_2, S_4\}$

14) Which one of the following sets contains the set of the basic data types in C?
   a) {char, int, float, logical}
   b) {char, boolean, int, float}

c) {char, int, long, short, float, double}
d) {char, int, float, void}

15) If a root of $f(x) = x^2 - 2x + 1 = 0$ is obtained by using the iterative scheme

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

with initial value $x_0 = 0.5$, then the convergence rate is

a) 1
b) 1.62
c) 1.84
d) 2

16) Let S_1 be the sum of the eigen values of a 2×2 matrix P and S_2 be the sum of the eigen values of another 2×2 matrix Q. If $S_1 = S_2$, then P and Q are

a) $\begin{pmatrix} 4 & 1 \\ 3 & 5 \end{pmatrix}$ and $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$
b) $\begin{pmatrix} 3 & 4 \\ 5 & 1 \end{pmatrix}$ and $\begin{pmatrix} 2 & 4 \\ 3 & 1 \end{pmatrix}$
c) $\begin{pmatrix} 4 & 1 \\ 3 & 5 \end{pmatrix}$ and $\begin{pmatrix} 3 & 4 \\ 1 & 5 \end{pmatrix}$
d) $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}$ and $\begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$

17) If y_i denotes the value of $y(x)$ at $x = x_i$ in $x_0 < x_1 < \cdots < x_i < \cdots < x_n$ and $x_i - x_{i-1} = h$ for $1 \le i \le n$, then $\frac{d^2y}{dx^2}$ at $x = x_i$, $1 \le i \le n-1$ is approximated using finite difference scheme by

a) $\frac{1}{2h}(y_{i+1} - 2y_i + y_{i-1})$
b) $\frac{1}{2h}(y_{i+1} - y_i + y_{i-1})$
c) $\frac{1}{h^2}(y_{i+1} - 2y_i + y_{i-1})$
d) $\frac{1}{h^2}(y_{i+1} - y_i + y_{i-1})$