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EE1030 : Matrix Theory
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1 Shift-2(1-15)

1) If $f(x) = \frac{x^3}{x-1} + \frac{x^3}{x+1}$ and $g(x) = \sqrt{x}$, then $fog : [0,2] - \{1\} \to \mathbb{R}$ is:
   a) one-one but not onto
   b) one-one and onto
   c) onto but not one-one
   d) neither one-one nor onto

2) The sum of squares of the values of $\alpha \in \mathbb{R}$ such that the argument of $\frac{\alpha+i}{\alpha-i}$ is $60°$, $i = \sqrt{-1}$, is:
   a) 3
   b) $\frac{10}{3}$
   c) $\frac{11}{3}$
   d) 4

3) For the system of equations

$$\begin{aligned} x + \lambda y - z &= 1 \\ x + 2y + \lambda z &= 2 \\ x + 2y + z &= 2, \end{aligned}$$

   which one of the following is **NOT** correct?
   a) it has unique solution if λ is not a root of the equation $t^2 - 3t + 2 = 0$
   b) it has unique solution if λ is not a root of the equation $t^2 - t - 2 = 0$
   c) it has infinitely many solutions if $\lambda = 1$
   d) it has no solution if $\lambda = 2$

4) If $a_n = \left(2n^2 - n + 2\right)(n!)$ then $\sum_{n=1}^{20} a_n$ is equal to:
   a) $37(20!) - 1$
   b) $37(20!) + 1$
   c) $39(21!) + 1$
   d) $39(21!) - 1$

5)

$$\lim_{x\to 0^+} \frac{1}{\sqrt{x}}\left(\frac{1}{\sin(x)} - \frac{1}{x}\right)$$

a) is equal to 0
b) is equal to $\frac{1}{5}$
c) is equal to 1
d) does not exist

6) Let $x = x(t)$ be the solution curve of the differential equation $\frac{dx}{dt} = -kx$, and $x(0) = 100$, $x\left(\frac{1}{2}\right) = 80$. If $x(t_\alpha) = 5$, then t_α is equal to:

a) $\frac{\log_e 5+\log_e 4}{2(\log_e 5-\log_e 4)}$
b) $\frac{\log_e 5+\log_e 4}{\log_e 5-\log_e 4}$
c) $\frac{\log_e 5-\log_e 4}{2(\log_e 5+\log_e 4)}$
d) $\frac{\log_e 5-\log_e 4}{\log_e 5+\log_e 4}$

7) The slope of the tangent to the curve

$$= y(x) = \int_{\sin^{-1}(x)}^{\cos^{-1}(x)} \sqrt{1 + 4\sin^2 t}dt, 0 \le x \le 1$$

at the point $\left(\frac{1}{\sqrt{2}}, 0\right)$ on the curve is:

a) $-2\sqrt{6}$
b) $2\sqrt{6}$
c) $-4\sqrt{3}$
d) $4\sqrt{3}$

8) Let $\alpha, \beta, \gamma \left(0 < \alpha, \beta, \gamma < \frac{\pi}{2}\right)$ be the angles between non-zero vectors **a** and **b**, **b** and **c**, **c** and **a** respectively. If θ is the angle that the vector **a** makes with the plane containing **b** and **c**, then

a) $\cos^2\theta = \operatorname{cosec}^2\beta\left(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\right)$
b) $\cos^2\theta = \sec^2\beta\left(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\right)$
c) $\sin^2\theta = \operatorname{cosec}^2\beta\left(\cos^2\alpha + \cos^2\gamma - 2\cos\alpha\cos\beta\cos\gamma\right)$
d) $\sin^2\theta = \sec^2\beta\left(\cos^2\alpha + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma\right)$

9) The domain of the function

$$f(x) = \sin^{-1}\left(\log_2((x-1)(x-2))\right)$$

is:

a) $[0, 3]$
b) $\left[0, \frac{3-\sqrt{3}}{2}\right] \cup \left[\frac{3+\sqrt{3}}{2}, 3\right]$

c) $\left(\frac{3-\sqrt{3}}{2}, 1\right) \cup \left(2, \frac{3+\sqrt{3}}{2}\right)$
d) $\left[0, \frac{3-\sqrt{6}}{2}\right] \cup \left[\frac{3+\sqrt{6}}{2}, 3\right]$

10) Let α and β be the roots of the equation $2x^2 - 5x - 1 = 0$. For $n \in \mathbb{N}$, let $P_n = \alpha^n + \beta^n$. Then $\frac{2P_{11}(2P_{10}-5P_9)}{P_8(5P_{10}+P_9)}$ is equal to:
a) $-\frac{1}{2}$
b) $\frac{1}{2}$
c) -1
d) 1

11) If the image of the point $(1, 1, 2)$ in the plane $2x - y + z + 3 = 0$ is the point P, then the distance of P from origin is
a) $2\sqrt{3}$
b) $3\sqrt{2}$
c) 4
d) 6

12) For three non-coplanar vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, if $(\mathbf{b}+\mathbf{c}) \cdot \{(\mathbf{c}+\mathbf{a}) \times (\mathbf{a}+\mathbf{b})\} = \alpha\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$ and $(\mathbf{a}+\mathbf{b}) \cdot \{(\mathbf{b}+\mathbf{c}) \times (\mathbf{a}+\mathbf{b}+\mathbf{c})\} = \beta\,[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$, then $\alpha + \beta$ is equal to:
a) -3
b) -1
c) 1
d) 3

13) If $I(x) = \int \frac{dx}{1-2\sin^2 x \cos^2 x}$, then $\tan\left(\sqrt{2}\left(I\left(\frac{\pi}{8}\right) - I(0)\right)\right)$ is equal to:
a) $\frac{1}{\sqrt{2}}$
b) 1
c) $\sqrt{2}$
d) 2

14) Let X have a binomial distribution $B(6, p)$. If the sum of the mean and the variance of X is $\frac{21}{8}$, then $\frac{P(2 \le X < 4)}{P(4 < X < 6)}$ is equal to:
a) 65
b) 195
c) $\frac{195}{2}$
d) $\frac{225}{2}$

15) If the statement

$$((p * (\sim q)) \wedge (p \vee q)) \iff p$$

is a tautology, then $*$ is:
a) $\wedge$

b) $\vee$
c) $\Longrightarrow$
d) $\Longleftrightarrow$