7 - Circle

EE1030:Matrix Theory

Gajjarapu Satyanarayana AI24BTECH11009

Question:7.2.23

If the lines 2x - 3y = 5 and 3x - 4y = 7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Solution:

Variables	Description
c	centre
r	radius
u	-с
f	$ \mathbf{u} ^2 - r^2$
X	$\begin{pmatrix} x \\ y \end{pmatrix}$

Table 7.2.23.1 0: Variables and their description

The augmented matrix formed by the given equations of diameter is

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xrightarrow{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix}$$
 (0.1)

$$\xrightarrow{R_1 \to R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{0.2}$$

$$\xrightarrow{R_1 \to \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{0.3}$$

Therefore from equation 0.3 centre of the circle is

$$\mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.4}$$

$$\mathbf{u} = \begin{pmatrix} -1\\1 \end{pmatrix} \tag{0.5}$$

$$\mathbf{u}^{\mathsf{T}} = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{0.6}$$

$$\|\mathbf{u}\|^2 = \mathbf{u}^\mathsf{T}\mathbf{u} \tag{0.7}$$

$$\|\mathbf{u}\|^2 = \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.8}$$

$$\|\mathbf{u}\|^2 = 2\tag{0.9}$$

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Given area is 154 square units and taking $\pi = \frac{22}{7}$ as approximation

$$\pi r^2 = 154 \tag{0.10}$$

$$r^2 = 49 (0.11)$$

$$r = 7 \tag{0.12}$$

$$f = 2 - 49 \tag{0.13}$$

$$f = -47 \tag{0.14}$$

The equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\mathsf{T}\mathbf{x} + f = 0 \tag{0.15}$$

$$\mathbf{x}^{\mathsf{T}}\mathbf{x} + 2(-1 \quad 1)\mathbf{x} + (-47) = 0 \tag{0.16}$$
$$x^2 + y^2 - 2x + 2y - 47 = 0 \tag{0.17}$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 (0.17)$$

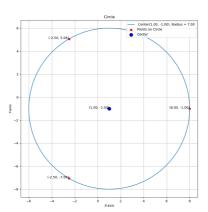


Fig. 0.1: Circle