GATE - 2014- MA

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

Satyanarayana Gajjarapu
AI24BTECH11009

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1) Suppose that X is a population random variable with probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter. In order to test the null hypothesis $H_0 : \theta = 2$, against the alternative hypothesis $H_1 : \theta = 3$, the following test is used: Reject the null hypothesis if $X_1 \geq \frac{1}{2}$ and accept otherwise, where X_1 is a random sample of size 1 drawn from the above population. Then the power of the test is ____

2) Suppose that $X_1, X_2, \cdots, X_n$ is a random sample of size n drawn from a population with probability density function

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter such that $\theta > 0$. The maximum likelihood estimator of θ is

a) $\frac{\sum_{i=1}^{n} X_i}{n}$
b) $\frac{\sum_{i=1}^{n} X_i}{n-1}$
c) $\frac{\sum_{i=1}^{n} X_i}{2n}$
d) $\frac{2\sum_{i=1}^{n} X_i}{n}$

3) Let $\overrightarrow{F}$ be a vector field defined on $\mathbb{R}^2 \backslash \{(0,0)\}$ by

$$\overrightarrow{F}(x,y) = \frac{y}{x^2+y^2}\hat{i} - \frac{x}{x^2+y^2}\hat{j}.$$

Let $\gamma, \alpha : [0,1] \to \mathbb{R}^2$ be defined by

$$\gamma(t) = (8\cos(2\pi t), 17\sin(2\pi t)) \text{ and } \alpha(t) = (26\cos(2\pi t), -10\sin(2\pi t)).$$

If $3\int\limits_{\alpha} \overrightarrow{F} \cdot d\overrightarrow{r} - 4\int\limits_{\gamma} \overrightarrow{F} \cdot d\overrightarrow{r} = 2m\pi$, then m is ____

4) If $g : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$g(x, y, z) = ((3y + 4z, 2x - 3z, x + 3y)$$

and let $S = \{(x, y, z) \in \mathbb{R}^3 : 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\}$. If

$$\iiint\limits_{g(S)} (2x + y - 2z)\, dx\, dy\, dz = \alpha \iiint\limits_{S} z\, dx\, dy\, dz,$$

then α is ____

5) Let $T_1, T_2 : \mathbb{R}^5 \to \mathbb{R}^3$ be linear transformations such that $rank(T_1) = 3$ and $nullity(T_2) = 3$. Let $T_3 : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$. Then $rank(T_3)$ is ____

6) Let $\mathbb{F}_3$ be the field of 3 elements and let $\mathbb{F}_3 \times \mathbb{F}_3$ be the vector space over $\mathbb{F}_3$. The number of distinct linearly dependent sets of the form $\{u, v\}$, where $u, v \in \mathbb{F}_3 \times \mathbb{F}_3 \backslash \{0, 0\}$ and $u \neq v$ is ____

7) Let $\mathbb{F}_{125}$ be the field of 125 elements. The number of non-zero elements $\alpha \in \mathbb{F}_{125}$ such that $\alpha^5 = \alpha$ is ____

8) The value of $\iint\limits_{R} xy\, dx\, dy$, where R is the region in the first quadrant bounded by the curves $y = x^2$, $y + x = 2$ and $x = 0$ is ____

9) Consider the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \ 0 < x < \pi, \ t > 0,$$

with the boundary conditions $u(0, t) = 0$, $u(\pi, t) = 0$ for $t > 0$, and the initial condition $u(x, 0) = \sin(x)$. Then $u\left(\frac{\pi}{2}, 1\right)$ is ____

10) Consider the partial order in $\mathbb{R}^2$ given by the relation $(x_1, y_1) < (x_2, y_2)$ EITHER if $x_1 < x_2$ OR if $x_1 = x_2$ and $y_1 < y_2$. Then in the order topology on $\mathbb{R}^2$ defined by the above order

a) $[0, 1] \times \{1\}$ is compact but $[0, 1] \times [0, 1]$ is NOT compact
b) $[0, 1] \times [0, 1]$ is compact but $[0, 1] \times \{1\}$ is NOT compact
c) both $[0, 1] \times [0, 1]$ and $[0, 1] \times \{1\}$ are compact
d) both $[0, 1] \times [0, 1]$ and $[0, 1] \times \{1\}$ are NOT compact

11) Consider the following linear programming problem:
Minimize: $x_1 + x_2 + 2x_3$

Subject to

$$\begin{aligned} x_1 + 2x_2 &\geq 4, \\ x_2 + 7x_3 &\leq 5, \\ x_1 - 3x_2 + 5x_3 &= 6, \\ x_1, x_2 &\geq 0, \;\; x_3 \text{ is unrestricted} \end{aligned}$$

The dual to this problem is:
Maximize: $4y_1 + 5y_2 + 6y_3$
Subject to

$$\begin{aligned} y_1 + y_3 &\leq 1, \\ 2y_1 + y_2 - 3y_3 &\leq 1, \\ 7y_2 + 5y_3 &= 2 \end{aligned}$$

and further subject to:

a) $y_1 \geq 0$, $y_2 \leq 0$ and y_3 is unrestricted
b) $y_1 \geq 0$, $y_2 \geq 0$ and y_3 is unrestricted
c) $y_1 \geq 0$, $y_3 \leq 0$ and y_2 is unrestricted
d) $y_3 \geq 0$, $y_2 \leq 0$ and y_1 is unrestricted

12) Let $X = C^1[0,1]$. For each $f \in X$, define

$$\begin{aligned} p_1(f) &:= \sup\{|f(t)| : t \in [0,1]\} \\ p_2(f) &:= \sup\{\left|f'(t)\right| : t \in [0,1]\} \\ p_3(f) &:= p_1(f) + p_2(f). \end{aligned}$$

Which of the following statements is **TRUE** ?

a) (X, p_1) is a Banach space
b) (X, p_2) is a Banach space
c) (X, p_3) is NOT a Banach space
d) (X, p_3) does NOT have denumerable basis

13) If the power series

$$\sum_{n=0}^{\infty} a_n (z + 3 - i)^n$$

converges at $5i$ and diverges at $-3i$, then the power series

a) converges at $-2 + 5i$ and diverges at $2 - 3i$
b) converges at $2 - 3i$ and diverges at $-2 + 5i$
c) converges at both $2 - 3i$ and $-2 + 5i$
d) diverges at both $2 - 3i$ and $-2 + 5i$