

18-Definite Integrals and Applications of Integrals

EE1030 : Matrix Theory

Indian Institute of Technology Hyderabad

Satyanarayana Gajjarapu

AI24BTECH11009

I. E-SUBJECTIVE PROBLEMS

- 1) Evaluate: $\int_0^{\pi/4} \frac{\sin(x) + \cos(x)}{9 + 16 \sin(2x)} dx$ (1983 – 3 Marks)
- 2) Find the area bounded by the x-axis, part of the curve $y = (1 + \frac{8}{x^2})$ and the ordinates at $x=2$ to $x=4$. If the ordinate at $x = a$ divides the area into two equal parts, (1983 – 3 Marks) find a .
- 3) Evaluate the following $\int_0^{1/2} \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx$ (1984 – 2 Marks)
- 4) Find the area of the region bounded by the x-axis and the curves defined by $y = \tan(x)$, $\frac{-\pi}{3} \leq x \leq \frac{\pi}{3}$; $y = \cot(x)$, $\frac{\pi}{6} \leq x \leq \frac{3\pi}{2}$ (1984 – 4 Marks)
- 5) Given a function $f(x)$ such that
 - (i) it is integrable over every interval on a real line and
 - (ii) $f(t+x) = f(x)$, for every x and a real t , then show that the integral $\int_a^{a+t} f(x) dx$ is independent of a . (1984 – 4 Marks)
- 6) Evaluate the following: $\int_0^{\pi/2} \frac{x \sin(x) \cos(x)}{\cos^4(x) + \sin^4(x)} dx$ (1985 – 5/2 Marks)
- 7) Sketch the region bounded by the curves $y = \sqrt{5-x^2}$ and $y = |x-1|$ and its area. (1985 – 5 Marks)
- 8) Evaluate: $\int_0^{\pi} \frac{x dx}{1 + \cos(\alpha) \sin(x)}$, $0 < \alpha < \pi$ (1986 – 5/2 Marks)
- 9) Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x=0$ above the x-axis. (1987 – 6 Marks)
- 10) Find the area of the region bounded by the curve $C: y = \tan(x)$, tangent drawn to C at $x = \pi/4$ and the x-axis. (1988 – 5 Marks)
- 11) Evaluate $\int_0^1 \log[\sqrt{1-x} + \sqrt{1+x}] dx$ (1988 – 5 Marks)
- 12) If f and g are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then show that $\int_0^a f(x)g(x)dx = \int_0^a f(x)dx$ (1989 – 4 Marks)
- 13) Show that $\int_0^{\pi/2} f(\sin(2x)) \sin(x) dx = \sqrt{2} \int_0^{\pi/4} f(\cos(2x)) \cos(x) dx$ (1990 – 4 Marks)
- 14) Prove that for any positive integer k , $\frac{\sin(2kx)}{\sin(x)} = 2[\cos(x) + \cos(3x) + \dots + \cos(2k-1)x]$

Hence prove that $\int_0^{\pi/2} \sin(2kx) \cot(x) dx = \pi/2$ (1990 – 4 Marks)

- 15) Compute the area of the region bounded by the curves $y = ex \ln x$ and $y = \frac{\ln x}{ex}$ where $\ln e = 1$. (1990 – 4 Marks)