

1-Vector Arithmetic

EE1030:Matrix Theory

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Question:1.7.6

Find a relation between x and y if the points $\mathbf{A}(x, y)$, $\mathbf{B}(-4, 6)$, and $\mathbf{C}(-2, 3)$ are collinear. (10, 2019)

Solution:

Vertex	Coordinates
A	$\begin{pmatrix} x \\ y \end{pmatrix}$
B	$\begin{pmatrix} -4 \\ 6 \end{pmatrix}$
C	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Table 1.7.6.1 0: Vertex and its coordinates

Points \mathbf{A} , \mathbf{B} , \mathbf{C} are said to be collinear if

$$\text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1 \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \quad (0.2)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 - x \\ 6 - y \end{pmatrix} \quad (0.3)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \quad (0.4)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -2 - x \\ 3 - y \end{pmatrix} \quad (0.5)$$

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = \begin{pmatrix} -4 - x & -2 - x \\ 6 - y & 3 - y \end{pmatrix} \quad (0.6)$$

$$\begin{pmatrix} -4 - x & -2 - x \\ -6 - y & 3 - y \end{pmatrix} \xrightarrow{R_2 \rightarrow R_1 + \frac{2}{3}R_2} \begin{pmatrix} -4 - x & -2 - x \\ -x - \frac{2}{3}y & -x - \frac{2}{3}y \end{pmatrix} \quad (0.7)$$

Given \mathbf{A} , \mathbf{B} , \mathbf{C} are collinear, so $\text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1$ from equation 0.1

Therefore,

$$-x - \frac{2}{3}y = 0 \quad (0.8)$$

$$x = -\frac{2}{3}y \quad (0.9)$$

$$3x = -2y \quad (0.10)$$

$$3x + 2y = 0 \quad (0.11)$$

The relation between x and y is $3x + 2y = 0$.

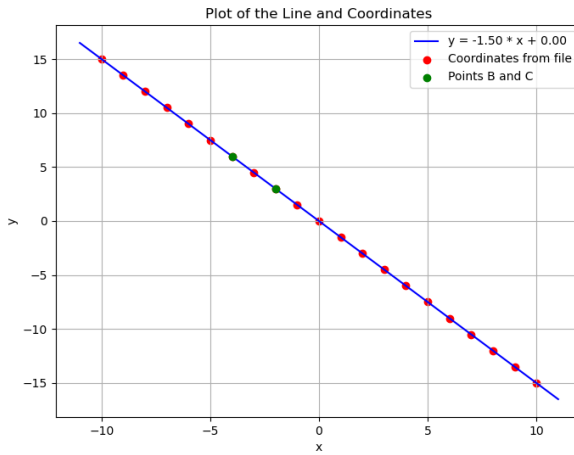


Fig. 0.1: Relation between x and y : $3x + 2y = 0$