1-Vector Arithmetic

EE1030:Matrix Theory

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Question:1.8.24
Find a relation between x and y such that the point (x, y) is equidistant from the points (7,1) and (3,5).
Solution:

Coordinates Given	**Labeled as**
$\begin{pmatrix} x \\ y \end{pmatrix}$	**A**
$\begin{pmatrix} 7 \\ 1 \end{pmatrix}$	**B**
$\begin{pmatrix} 3 \\ 5 \end{pmatrix}$	**C**

Table 1.8.24.1 0: Labeling given coordinates as **A**, **B**, **C**

A is said to be equidistant from **B** and **C** if

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{A}\| \tag{0.1}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 7 \\ 1 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.2}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 7-x \\ 1-y \end{pmatrix} \tag{0.3}$$

$$(\mathbf{B}-\mathbf{A})^\top = \begin{pmatrix} 7-x & 1-y \end{pmatrix} \tag{0.4}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.5}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 3-x \\ 5-y \end{pmatrix} \tag{0.6}$$

$$(\mathbf{C}-\mathbf{A})^\top = \begin{pmatrix} 3-x & 5-y \end{pmatrix} \tag{0.7}$$

Given **A** is equidistant from **B** and **C**, so $\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{A}\|$ from equation 0.1 , Therefore

$$\sqrt{(\mathbf{B}-\mathbf{A})^\top(\mathbf{B}-\mathbf{A})} = \sqrt{(\mathbf{C}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A})} \tag{0.8}$$

Squaring the equation 0.8 on both sides,

$$(\mathbf{B}-\mathbf{A})^\top (\mathbf{B}-\mathbf{A}) = (\mathbf{C}-\mathbf{A})^\top (\mathbf{C}-\mathbf{A}) \tag{0.9}$$

$$\begin{pmatrix} 7-x & 1-y \end{pmatrix} \begin{pmatrix} 7-x \\ 1-y \end{pmatrix} = \begin{pmatrix} 3-x & 5-y \end{pmatrix} \begin{pmatrix} 3-x \\ 5-y \end{pmatrix} \tag{0.10}$$

$$(7-x)^2 + (1-y)^2 = (3-x)^2 + (5-y)^2 \tag{0.11}$$

$$49 + x^2 - 14x + 1 + y^2 - 2y = 9 + x^2 - 6x + 25 + y^2 - 10y \tag{0.12}$$

$$8x - 8y - 16 = 0 \tag{0.13}$$

$$x - y - 2 = 0 \tag{0.14}$$

The relation between x and y is $x - y - 2 = 0$.

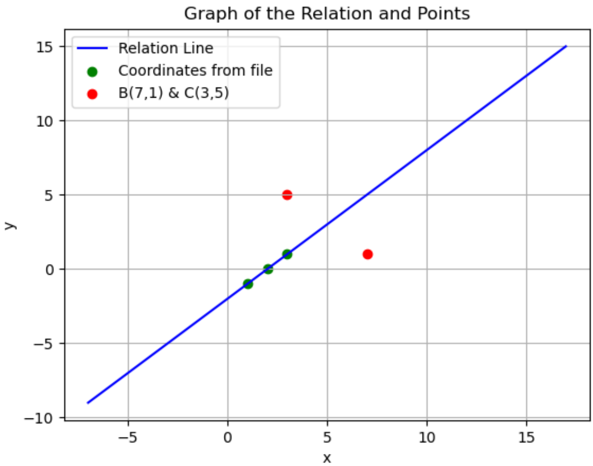

Fig. 0.1: Relation between x and y: $x - y - 2 = 0$