GATE - 2023 - PH

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

Satyanarayana Gajjarapu
AI24BTECH11009

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1) A spin $\frac{1}{2}$ particle is in a spin up state along the x-axis (with unit vector $\hat{x}$) and is denoted as $\left|\frac{1}{2}, \frac{1}{2}\right\rangle_x$. What is the probability of finding the particle to be in a spin up state along the direction $\hat{x}'$, which lies in the xy-plane and makes an angle θ with respect to the positive x-axis, if such a measurement is made ?
   a) $\frac{1}{2}\cos^2\frac{\theta}{4}$
   b) $\cos^2\frac{\theta}{4}$
   c) $\frac{1}{2}\cos^2\frac{\theta}{2}$
   d) $\cos^2\frac{\theta}{2}$

2) Different spectral lines of the Balmer series (transitions $n \to 2$, with n being the principal quantum number) fall one at a time on a Young's double slit apparatus. The separation between the slits is d and the screen is placed at a constant distance from the slits. What factor should d be multiplied by to maintain a constant fringe width for various lines, as n takes different allowed values ?
   a) $\frac{n^2-4}{4n^2}$
   b) $\frac{n^2+4}{4n^2}$
   c) $\frac{4n^2}{n^2-4}$
   d) $\frac{4n^2}{n^2+4}$

3) Under parity and time reversal transformations, which of the following statements is(are) TRUE about the electric dipole moment $\mathbf{p}$ and the magnetic dipole moment $\boldsymbol{\mu}$?
   a) $\mathbf{p}$ is odd under parity and $\boldsymbol{\mu}$ is odd under time reversal
   b) $\mathbf{p}$ is odd under parity and $\boldsymbol{\mu}$ is even under time reversal
   c) $\mathbf{p}$ is even under parity and $\boldsymbol{\mu}$ is odd under time reversal
   d) $\mathbf{p}$ is even under parity and $\boldsymbol{\mu}$ is even under time reversal

4) Consider the complex function

$$f(z) = \frac{z^2\sin(z)}{(z-\pi)^4}$$

At $z = \pi$, which of the following options is(are) CORRECT ?

a) The order of the pole is 4
b) The order of the pole is 3
c) The residue at the pole is $\frac{\pi}{6}$
d) The residue at the pole is $\frac{2\pi}{3}$

5) Consider the vector field $\overrightarrow{V}$ consisting of the velocities of points on a thin horizontal disc of radius $R = 2$ m, moving anticlockwise with uniform angular speed $\omega = 2$ rad/sec about an axis passing through its center. If $V = \left|\overrightarrow{V}\right|$, then which of the following options is(are) CORRECT ? (In the options, $\hat{r}$ and $\hat{\theta}$ are unit vectors corresponding to the plane polar coordinates r and θ).

You may use the fact that in cylindrical coordinates (s, ϕ, z) (s is the distance from the z-axis), the gradient, divergence, curl and Laplacian operators are:

$$\begin{aligned}
\overrightarrow{\nabla} f &= \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}; \\
\overrightarrow{\nabla} \cdot \overrightarrow{A} &= \frac{1}{s}\frac{\partial}{\partial s}(sA_s) + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}; \\
\overrightarrow{\nabla} \times \overrightarrow{A} &= \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} \\
&\quad + \frac{1}{s}\left(\frac{\partial}{\partial s}\left(sA_\phi\right) - \frac{\partial A_s}{\partial \phi}\right)\hat{z}; \\
\overrightarrow{\nabla}^2 f &= \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}.
\end{aligned}$$

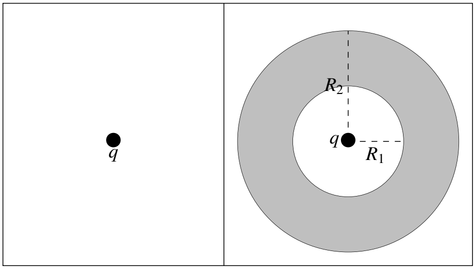
a) $\overrightarrow{\nabla} V = 2\hat{r}$
b) $\overrightarrow{\nabla} \cdot \overrightarrow{V} = 2$
c) $\overrightarrow{\nabla} \times \overrightarrow{V} = 4\hat{z}$, where $\hat{z}$ is a unit vector perpendicular to the (r, θ) plane
d) $\overrightarrow{\nabla}^2 V = \frac{4}{3}$ at $r = 1.5$ m

6) A slow moving π^- particle is captured by a deuteron (d) and this reaction produces two neutrons (n) in the final state, i.e., $\pi^- + d \to n + n$. Neutron and deuteron have even intrinsic parities, whereas π^- has odd intrinsic parity. L and S are the orbital and spin angular momenta, respectively of the system of two neutrons. Which of the following statements regarding the final two-neutron state is(are) CORRECT ?

a) It has odd parity
b) $L + S$ is odd
c) $L = 1, S = 1$
d) $L = 2, S = 0$

7) Two independent electrostatic configurations are shown in the figure. Configuration (I) consists of an isolated point charge $q = 1$ C, and configuration (II) consists

of another identical charge surrounded by a thick conducting shell of inner radius $R_1 = 1$ m and outer radius $R_2 = 2$ m, with the charge being at the center of the shell. $W_I = \frac{\epsilon_0}{2}\int E_I^2\, dV$ and $W_{II} = \frac{\epsilon_0}{2}\int E_{II}^2\, dV$, where E_I and E_{II} are the magnitudes of the electric fields for configurations (I) and (II) respectively, ϵ_0 is the permittivity of vacuum, and the volume integrations are carried out over all space. If $\frac{8\pi}{\epsilon_0}|W_I - W_{II}| = \frac{1}{n}$, what is the value of the integer n ?



(I) (II)

8) In pion nucleon scattering, the pion and nucleon can combine to form a short lived bound state called the Δ particle ($\pi + N \to \Delta$). The masses of the pion, nucleon and the Δ particle are 140 MeV/c^2, 938 MeV/c^2 and 1230 MeV/c^2, respectively. In the lab frame, where the nucleon is at rest, what is the minimum energy (in MeV/c^2, rounded off to one decimal place) of the pion to produce the Δ particle ?

9) Consider an electromagnetic wave propagating in the z-direction in vacuum, with the magnetic field given by $\overrightarrow{B} = \overrightarrow{B_0} e^{i(kz-\omega t)}$. If $B_0 = 10^{-8}$ T, the average power passing through a circle of radius 1.0 m placed in the xy plane is P (in Watts). Using $\epsilon_0 = 10^{-11}\ \frac{C^2}{Nm^2}$, what is the value of $\frac{10^3 P}{\pi}$ (rounded off to one decimal place) ?

10) An α-particle is emitted from the decay of Americium (Am) at rest, i.e., ${}^{241}_{94}\text{Am} \to {}^{237}_{92}\text{U} + \alpha$. The rest masses of ${}^{241}_{94}$Am, ${}^{237}_{92}$U and α are 224.544 GeV/c^2, 220.811 GeV/c^2 and 3.728 GeV/c^2 respectively. What is the kinetic energy (in MeV/c^2, rounded off to two decimal places) of the α-particle ?

11) Consider 6 identical, non-interacting, spin $\frac{1}{2}$ atoms arranged on a crystal lattice at absolute temperature T. The z-component of the magnetic moment of each of these atoms can be $\pm\mu_B$. If P and Q are the probabilities of the net magnetic moment of the solid being $2\mu_B$ and $6\mu_B$ respectively, what is the value of $\frac{P}{Q}$ (in integer) ?

12) Two identical, non-interacting ${}^4He_2$ atoms are distributed among 4 different non-degenerate energy levels. The probability that they occupy different energy levels is p. Similarly, two ${}^3He_2$ atoms are distributed among 4 different non-degenerate energy levels, and the probability that they occupy different levels is q. What is the

value of $\frac{p}{q}$ (rounded off to one decimal place) ?

13) Two identical bodies kept at temperatures 800 K and 200 K act as the hot and the cold reservoirs of an ideal heat engine, respectively. Assume that their heat capacity (C) in Joules/K is independent of temperature and that they do not undergo any phase change. Then, the maximum work that can be obtained from the heat engine is $n \times C$ Joules. What is the value of n (in integer) ?