1-Vector Arithmetic

EE1030:Matrix Theory

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Question:1.11.2

Unit vector along PQ, where coordinates of \mathbf{P} and \mathbf{Q} respectively are (2, 1, -1) and (4, 4, -7), is (12, 2023)

Solution:

Vertex	Coordinates
P	$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$
Q	$\begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix}$

Table 1.11.2.1 0: Vertex and its coordinates

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{Q} = \begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix} \tag{0.2}$$

The vector along PQ is Q - P

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 4 - 2 \\ 4 - 1 \\ -7 - (-1) \end{pmatrix} \tag{0.4}$$

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 2\\3\\-6 \end{pmatrix} \tag{0.5}$$

$$(\mathbf{Q} - \mathbf{P})^{\mathsf{T}} = \begin{pmatrix} 2 & 3 & -6 \end{pmatrix} \tag{0.6}$$

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The magnitude of the vector along PQ is

$$\|\mathbf{Q} - \mathbf{P}\| = \sqrt{(\mathbf{Q} - \mathbf{P})^{\mathsf{T}} (\mathbf{Q} - \mathbf{P})}$$
(0.7)

$$\|\mathbf{Q} - \mathbf{P}\| = \sqrt{\begin{pmatrix} 2 & 3 & -6 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}} \tag{0.8}$$

$$\|\mathbf{Q} - \mathbf{P}\| = \sqrt{(2)^2 + (3)^2 + (-6)^2}$$
 (0.9)

$$\|\mathbf{Q} - \mathbf{P}\| = \sqrt{4 + 9 + 36} \tag{0.10}$$

$$\|\mathbf{Q} - \mathbf{P}\| = \sqrt{49} \tag{0.11}$$

$$\|\mathbf{Q} - \mathbf{P}\| = 7\tag{0.12}$$

The unit vector along PQ is

$$\frac{\mathbf{Q} - \mathbf{P}}{\|\mathbf{Q} - \mathbf{P}\|} = \frac{\begin{pmatrix} 2\\3\\-6 \end{pmatrix}}{7} \tag{0.13}$$

$$\frac{\mathbf{Q} - \mathbf{P}}{\|\mathbf{Q} - \mathbf{P}\|} = \frac{1}{7} \begin{pmatrix} 2\\3\\-6 \end{pmatrix} \tag{0.14}$$

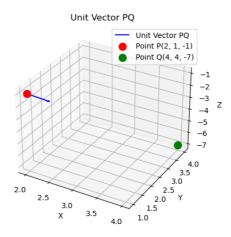


Fig. 0.1: Unit Vector PQ