18-Definite Integrals and Applications of Integrals

EE1030 : Matrix Theory
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I. E-Subjective Problems

1) Evaluate:
$$\int_0^{\pi/4} \frac{\sin(x)+\cos(x)}{9+16\sin(2x)}dx$$
(1983 – 3 *Marks*)

2) Find the area bounded by the x-axis, part of the curve $y=\left(1+\frac{8}{x^2}\right)$ and the ordinates at x=2 to x=4. If the ordinate at $x=a$ divides the area into two equal parts, (1983 – 3 *Marks*) find a.

3) Evaluate the following
$$\int_0^{1/2} \frac{xsin^{-1}(x)}{\sqrt{1-x^2}}dx$$
(1984 – 2 *Marks*)

4) Find the area of the region bounded by the x-axis and the curves defined by $y=\tan(x), \frac{-\pi}{3}\leq x\leq\frac{\pi}{3}$; (1984 – 4 *Marks*) $y=\cot(x), \frac{\pi}{6}\leq x\leq\frac{3\pi}{2}$

5) Given a function $f(x)$ such that
(i)it is integrable over every interval on a real line and
(ii) $f(t+x)=f(x)$, for every x and a real t, then show that the integral $\int_a^{a+t} f(x)\,dx$ is independent of a. (1984 – 4 *Marks*)

6) Evaluate the following:
$$\int_0^{\pi/2} \frac{x\sin(x)\cos(x)}{\cos^4(x)+\sin^4(x)}dx$$
(1985 – 5/2 *Marks*)

7) Sketch the region bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ and its area. (1985 – 5 *Marks*)

8) Evaluate:
$$\int_0^{\pi} \frac{xdx}{1+\cos(\alpha)\sin(x)},$$
$0<\alpha<\pi$ (1986 – 5/2 *Marks*)

9) Find the area bounded by the curves, $x^2+y^2=25,\ 4y=\left|4-x^2\right|$ and x=0 above the x-axis. (1987 – 6 *Marks*)

10) Find the area of the region bounded by the curve C: y=$\tan(x)$, tangent drawn to C at $x=\pi/4$ and the x-axis. (1988 – 5 *Marks*)

11) Evaluate (1988 – 5 *Marks*)
$\int_0^1 \log[\sqrt{1-x}+\sqrt{1+x}]dx$

12) If f and g are continuous function on [0,a] satisfying $f(x)=f(a-x)$ and $g(x)+g(a-x)=2$, then show that $\int_0^a f(x)\,g(x)\,dx=\int_0^a f(x)\,dx$(1989 – 4 *Marks*)

13) Show that $\int_0^{\pi/2} f(\sin(2x))\sin(x)dx = \sqrt{2}\int_0^{\pi/4} f(\cos(2x))\cos(x)dx (1990 - 4\ Marks)$

14) Prove that for any positive integer k, $\frac{\sin(2kx)}{\sin(x)} = 2[\cos(x) + \cos(3x) + \text{.............} + \cos(2k-1)\,x]$

Hence prove that (1990 – 4 *Marks*) $\int_0^{\pi/2} \sin(2kx)\cot(x)dx = \pi/2$

15) Compute the area of the region bounded by the curves $y = ex\ln x$ and (1990 – 4 *Marks*) $y = \frac{\ln x}{ex}$ where $\ln e = 1$.