Presentation - Matgeo

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EE1030 - Matrix Theory

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Problem Statement

If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Description of Variables used

Variables	Description
$\mathbf{c}$	centre
r	radius
$\mathbf{u}$	$-\mathbf{c}$
f	$\|\mathbf{u}\|^2 - r^2$
$\mathbf{x}$	$\begin{pmatrix} x \\ y \end{pmatrix}$

Row Reduction: Finding $\mathbf{c}$

The augmented matrix formed by the given equations of diameter is

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xrightarrow{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.1}$$

$$\xrightarrow{R_1 \to R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.2}$$

$$\xrightarrow{R_1 \to \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.3}$$

Therefore from equation 3.3

$$\mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{3.4}$$

Finding $\mathbf{u}$, r and f

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{3.5}$$

$$\mathbf{u}^\intercal = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{3.6}$$

$$\|\mathbf{u}\|^2 = \mathbf{u}^\intercal \mathbf{u} \tag{3.7}$$

$$\|\mathbf{u}\|^2 = 2 \tag{3.8}$$

Given area is 154 square units

$$\pi r^2 = 154 \tag{3.9}$$

$$r = 7 \tag{3.10}$$

$$f = 2 - 49 \tag{3.11}$$

$$f = -47 \tag{3.12}$$

Equation of Circle

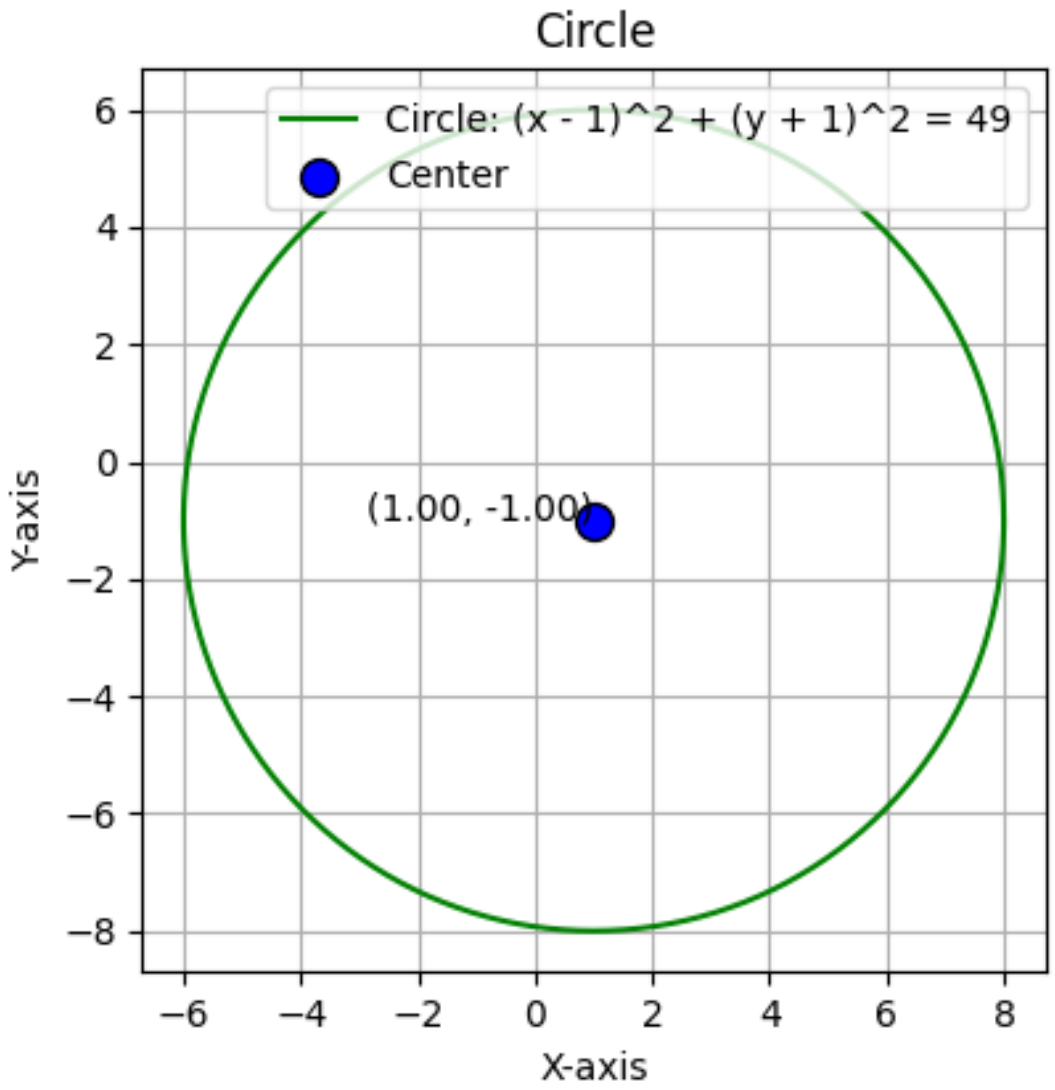
The equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\mathsf{T}\mathbf{x} + f = 0 \tag{3.13}$$

$$\mathbf{x}^\mathsf{T}\mathbf{x} + 2\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} + (-47) = 0 \tag{3.14}$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 \tag{3.15}$$

Plot


Circle
Circle: (x - 1)^2 + (y + 1)^2 = 49
Center
(1.00, -1.00)
Y-axis
X-axis
6
4
2
0
−2
−4
−6
−8
−6
−4
−2
0
2
4
6
8

Code - C

The code to find the equation of circle is

```
#include <stdio.h>
#include <math.h>

#define NUM_POINTS 500

void findCenter(float a1, float b1, float c1, float a2, float b2, float c2,
    float* centerX, float* centerY) {
    float determinant = a1 * b2 - a2 * b1;
    if (determinant != 0) {
        *centerX = (b1 * c2 - b2 * c1) / determinant;
        *centerY = (a2 * c1 - a1 * c2) / determinant;
    } else {
        printf("The lines are parallel, no intersection found.\n");
    }
    }
```

Code - C

```
    float calculateRadius(float area) {
    float pi = 22.0 / 7.0;
    return sqrt(area / pi);
}
    int main() {
    float a1 = 2, b1 = -3, c1 = -5;
    float a2 = 3, b2 = -4, c2 = -7;
    float area = 154.0;
    float centerX, centerY, radius;
    findCenter(a1, b1, c1, a2, b2, c2, &centerX, &centerY);
    radius = calculateRadius(area);

    printf("Center of the circle: (%.2f, %.2f)\n", centerX, centerY);
    printf("Radius of the circle: %.2f\n", radius);
     printf("Equation of the circle: (x - (%.2f))^2 + (y - (%.2f))^2
          = %.2f\n", centerX, centerY, radius * radius);
```

Code - C

```
    FILE *file = fopen("coordinates.txt", "w");
    if (file == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
    for (int i = 0; i < NUM_POINTS; i++) {
        float theta = (2 * M_PI * i) / NUM_POINTS; // Angle in
            radians
        float x = centerX + radius * cos(theta);
        float y = centerY + radius * sin(theta);
        fprintf(file, "%.4f, %.4f\n", x, y);
    }

    fclose(file);
    return 0;
}
```

Code - Python

The code to obtain the required plot is

```
from ctypes import *
import numpy as np
import matplotlib.pyplot as plt
circle_lib = CDLL('./circle.so')
circle_lib.findCenter.argtypes = [c_float, c_float, c_float, c_float, c_float,
    c_float, POINTER(c_float), POINTER(c_float)]
circle_lib.calculateRadius.argtypes = [c_float]
circle_lib.calculateRadius.restype = c_float
a1, b1, c1 = 2.0, -3.0, -5.0
a2, b2, c2 = 3.0, -4.0, -7.0
area = 154.0
centerX = c_float(0)
centerY = c_float(0)
circle_lib.findCenter(a1, b1, c1, a2, b2, c2, byref(centerX), byref(centerY))
centerX = centerX.value
```

Code - Python

```
centerY = centerY.value
radius = circle_lib.calculateRadius(area)
points = np.loadtxt("coordinates.txt", delimiter=",", unpack=False)
plt.figure()
plt.plot(points[:, 0], points[:, 1], color='green', label=f"Circle")
plt.scatter(centerX, centerY, color='blue', s=100, label="Center",
    edgecolor='black')
plt.text(centerX, centerY, f"({centerX:.2f}, {centerY:.2f})", fontsize=10,
    ha='right', color='black')
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.title("Circle")
plt.legend(loc='upper right')
plt.gca().set_aspect('equal', adjustable='box')
plt.grid(True)
plt.show()
```