# 28-07-2022 Shift-1

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

Satyanarayana Gajjarapu
AI24BTECH11009

## 1 Shift-1(16-30)

1) The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1$, $z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves?
   a) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$
   b) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1$, $z = 6 - 2x - 3y$
   c) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1$, $z = 6 - 2x - 3y$
   d) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$

2) If the minimum value of

$$f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}, x > 0$$

   is 14, then the value of $\alpha$ is equal to
   a) 32
   b) 64
   c) 128
   d) 256

3) Let $\alpha, \beta, \gamma$ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x, x \in \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be such that $g(f(x)) = x$ for all $x \in \mathbb{R}$. If $a_1, a_2, a_3, \cdots, a_n$ be in arithmetic progression with mean zero, then the value of

$$f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$$

   is equal to
   a) 0
   b) 3
   c) 9
   d) 27

4) Consider the sequence $a_1, a_2, a_3, \cdots$ such that $a_1 = 1$, $a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}+a_n}$ for $n = 1, 2, 3, \cdots$. If

$$\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right)\left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right)\left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right)\cdots\left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right),$$

then $\alpha$ is equal to

a) -30
b) -31
c) -60
d) -61

5) The minimum value of the twice differentiable function

$$f(x) = \int_0^x e^{x-t} f'(t) - \left(x^2 - x + 1\right)e^x, x \in \mathbb{R}$$

is

a) $-\frac{2}{\sqrt{e}}$
b) $-2\sqrt{e}$
c) $-\sqrt{e}$
d) $\frac{2}{\sqrt{e}}$

6) Let $S$ be the set of all passwords which are six to eight chaacters long, where each character is either an alphabet from $\{A, B, C, D, E\}$ or a number from $\{1, 2, 3, 4, 5\}$ with the repetition of characters allowed. If the number of passwords in $S$ whose at least one character is a number from $\{1, 2, 3, 4, 5\}$ is $\alpha \times 5^6$, then $\alpha$ is equal to _____

7) Let $P(-2, -1, 1)$ and $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus $PRQS$. If the direction ratios of the diagonal $RS$ are $\alpha$, -1, $\beta$, where both $\alpha$ and $\beta$ are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to _____

8) Let $f : [0, 1] \to \mathbf{R}$ be a twice differentiable function in $(0, 1)$ such that $f(0) = 3$ and $f(1) = 5$. If the line $y = 2x + 3$ intersects the graph of $f$ at only two distinct points in $(0, 1)$ then the least number of points $x \in (0, 1)$ at which $f''(x) = 0$, is _____

9) If

$$\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1 + x^2 + \sqrt{(1 + x^2)^3}}} dx = \alpha\sqrt{2} + \beta\sqrt{3}$$

where $\alpha$, $\beta$ are integers, then $\alpha + \beta$ is equal to

10) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & \alpha \end{bmatrix}$ and $B = \begin{bmatrix} \beta & 1 \\ 1 & 0 \end{bmatrix}$, $\alpha, \beta \in R$. Let $\alpha_1$ be the value of $\alpha$ which satisfies

$$(A + B)^2 = A^2 + \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

and $\alpha_2$ be the value of $\alpha$ which satisfies

$$(A + B)^2 = B^2.$$

Then $|\alpha_1 - \alpha_2|$ is equal to _____

11) For $p, q \in R$, consider the real valued function $f(x) = (x - p)^2 - q, x \in R$ and $q > 0$. Let $a_1$, $a_2$, $a_3$ and $a_4$ be in an arithmetic progression with mean $p$ and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$, then the absolute difference between the roots of $f(x) = 0$ is

12) For the hyperbola $H : x^2 - y^2 = 1$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0$, let the
a) eccentricity of $E$ be reciprocal of the eccentricity of $H$, and
b) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of $E$ and $H$.

Then $4\left(a^2 + b^2\right)$ is equal to _____

13) Let $x_1, x_2, x_3, \cdots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each $x_i$ by $(x_i - i)^2$. If $\overline{x}$ is the mean of new data, then the greatest integer less than or equal to $\overline{x}$ is _____

14)

$$\lim_{x \to 0} \left( \frac{(x + 2\cos(x))^3 + 2(x + 2\cos(x))^2 + 3\sin(x + 2\cos(x))}{(x + 2)^3 + 2(x + 2)^2 + 3\sin(x + 2)} \right)^{\frac{100}{x}}$$

is equal to _____

15) The sum of all real value of $x$ for which

$$\frac{3x^2 - 9x + 17}{x^2 + 3x + 10} = \frac{5x^2 - 7x + 19}{3x^2 + 5x + 12}$$

is equal to _____