Presentation - Matgeo

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AI24BTECH11009
EE1030 - Matrix Theory

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Problem Statement

If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Description of Variables used

Variables	Description
$\mathbf{c}$	centre
r	radius
$\mathbf{u}$	$-\mathbf{c}$
f	$\|\mathbf{u}\|^2 - r^2$
$\mathbf{x}$	$\begin{pmatrix} x \\ y \end{pmatrix}$

Row Reduction: Finding $\mathbf{c}$

The augmented matrix formed by the given equations of diameter is

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xrightarrow{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.1}$$

$$\xrightarrow{R_1 \to R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.2}$$

$$\xrightarrow{R_1 \to \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \tag{3.3}$$

Therefore from equation 3.3

$$\mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{3.4}$$

Finding $\mathbf{u}$, r and f

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{3.5}$$

$$\mathbf{u}^\mathsf{T} = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{3.6}$$

$$\|\mathbf{u}\|^2 = \mathbf{u}^\mathsf{T}\mathbf{u} \tag{3.7}$$

$$\|\mathbf{u}\|^2 = 2 \tag{3.8}$$

Given area is 154 square units

$$\pi r^2 = 154 \tag{3.9}$$

$$r = 7 \tag{3.10}$$

$$f = 2 - 49 \tag{3.11}$$

$$f = -47 \tag{3.12}$$

Equation of Circle

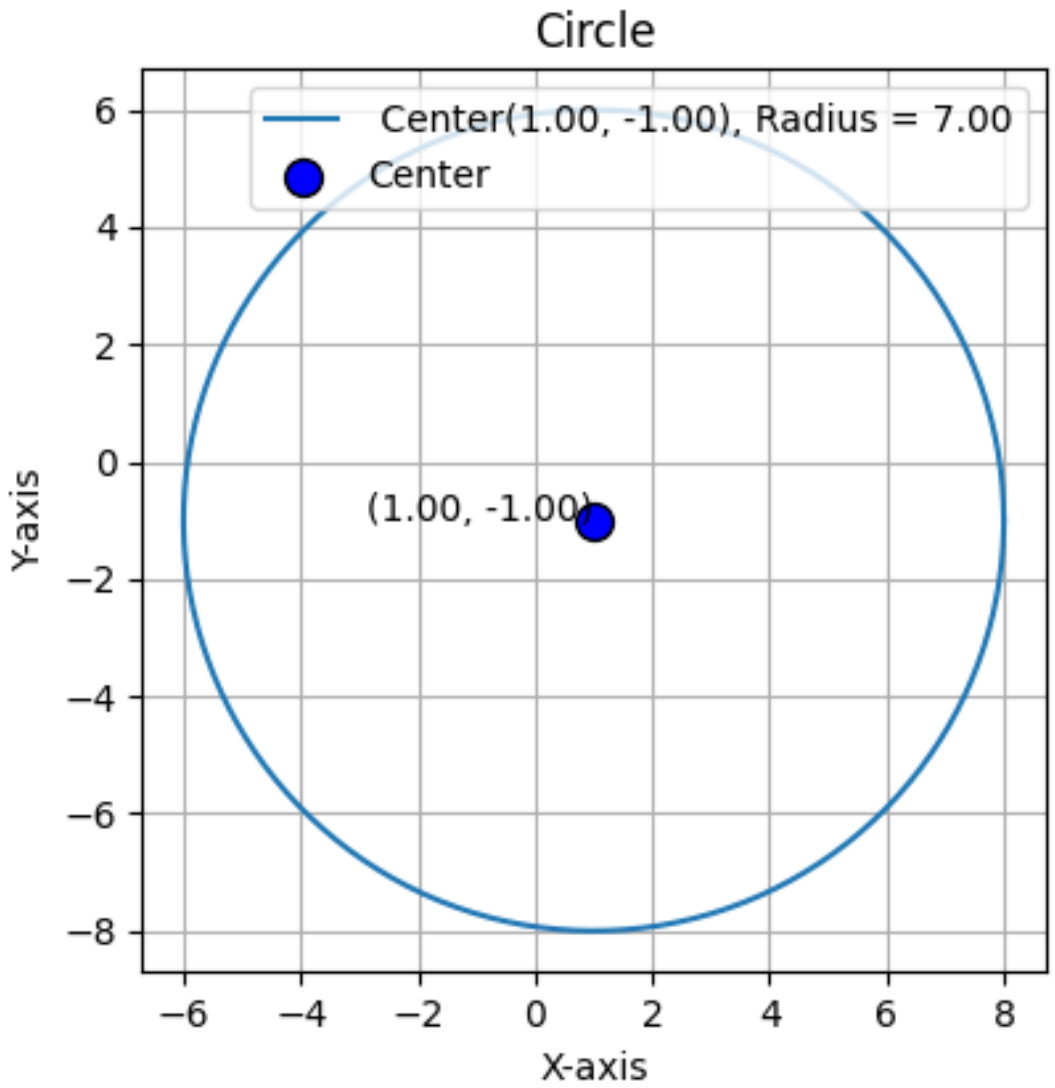
The equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\mathsf{T}\mathbf{x} + f = 0 \tag{3.13}$$

$$\mathbf{x}^\mathsf{T}\mathbf{x} + 2\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} + (-47) = 0 \tag{3.14}$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 \tag{3.15}$$

Plot


Circle
Center(1.00, -1.00), Radius = 7.00
Center
(1.00, -1.00)
Y-axis
X-axis
6
4
2
0
−2
−4
−6
−8
−6
−4
−2
0
2
4
6
8

Codes

The equation of circle (3.15) can be found through

https://github.com/Satyanarayana-123456/EE1030/blob/8e602868638ce68d254508a0a3312206087937d4/presentation-matgeo/codes/main.c

The plot of the circle can be obtained by

https://github.com/Satyanarayana-123456/EE1030/blob/8e602868638ce68d254508a0a3312206087937d4/presentation-matgeo/codes/circle.py