

7 - Circle

EE1030:Matrix Theory

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Question:7.2.23

If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Solution:

Variables	Description
\mathbf{c}	centre
r	radius
\mathbf{u}	$-\mathbf{c}$
f	$\ \mathbf{u}\ ^2 - r^2$
\mathbf{x}	$\begin{pmatrix} x \\ y \end{pmatrix}$

Table 7.2.23.1 0: Variables and their description

The augmented matrix formed by the given equations of diameter is

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xrightarrow{R_2 \rightarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \quad (0.1)$$

$$\xrightarrow{R_1 \rightarrow R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \quad (0.2)$$

$$\xrightarrow{R_1 \rightarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \quad (0.3)$$

Therefore from equation 0.3 centre of the circle is

$$\mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (0.4)$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (0.5)$$

$$\mathbf{u}^T = \begin{pmatrix} -1 & 1 \end{pmatrix} \quad (0.6)$$

$$\|\mathbf{u}\|^2 = \mathbf{u}^T \mathbf{u} \quad (0.7)$$

$$\|\mathbf{u}\|^2 = \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (0.8)$$

$$\|\mathbf{u}\|^2 = 2 \quad (0.9)$$

Given area is 154 square units and taking $\pi = \frac{22}{7}$ as approximation

$$\pi r^2 = 154 \quad (0.10)$$

$$r^2 = 49 \quad (0.11)$$

$$r = 7 \quad (0.12)$$

$$f = 2 - 49 \quad (0.13)$$

$$f = -47 \quad (0.14)$$

The equation of circle is given by

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0 \quad (0.15)$$

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -1 & 1 \end{pmatrix}\mathbf{x} + (-47) = 0 \quad (0.16)$$

$$x^2 + y^2 - 2x + 2y - 47 = 0 \quad (0.17)$$

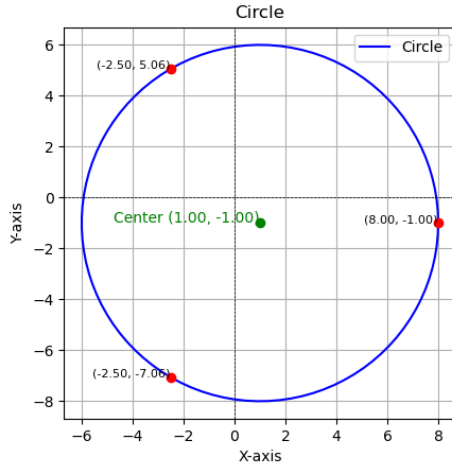


Fig. 0.1: Circle