GATE - 2017- MA

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

Satyanarayana Gajjarapu
AI24BTECH11009

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1) Consider the vector space $V = \{a_0 + a_1x + a_2x^2 : a_i \in \mathbb{R} \text{for} i = 0, 1, 2\}$ of polynomials of degree at most 2. Let $f : V \to \mathbb{R}$ be a linear function such that $f(1 + x) = 0$, $f\left(1 - x^2\right) = 0$ and $f\left(x^2 - x\right) = 2$. Then $f\left(1 + x + x^2\right)$ equals ____.

2) Let A be a 7×7 matrix such that $2A^2 - A^4 = I$, where I is the identity matrix. If A has two distinct eigenvalues and each eigenvalue has geometric multiplicity 3, then the total number of nonzero entries in the Jordan canonical form of A equals ____.

3) Let $f(z) = \left(x^2 + y^2\right) + i2xy$ and $g(z) = 2xy + i\left(y^2 - x^2\right)$ for $z = x + iy \in \mathbb{C}$. Then, in the complex plane $\mathbb{C}$,
   a) f is analytic and g is NOT analytic
   b) f is NOT analytic and g is analytic
   c) neither f nor g is analytic
   d) both f and g are analytic

4) If $\sum\limits_{n=-\infty}^{\infty} a_n (z - 2)^n$ is the Laurent series of function $f(z) = \frac{z^4+z^3+z^2}{(z-2)^3}$ for $z \in \mathbb{C}\backslash\{2\}$, then a_{-2} equals ____.

5) Let $f_n : [0, 1] \to \mathbb{R}$ be given by

$$f_n(x) = \frac{2x^2}{x^2 + (1 - 2nx)^2}, n = 1, 2, \cdots$$

   Then the sequence (f_n)
   a) converges uniformly on $[0, 1]$
   b) does NOT converge uniformly on $[0, 1]$ but has a subsequence that converges uniformly on $[0, 1]$
   c) does NOT converge pointwise on $[0, 1]$
   d) converges pointwise on $[0, 1]$ but does NOT have a subsequence that converges uniformly on $[0, 1]$

6) Let $C : x^2 + y^2 = 9$ be the circle in $\mathbb{R}^2$ oriented positively. Then

$$\frac{1}{\pi} \oint_C \left(3y - e^{\cos x^2}\right) dx + \left(7x + \sqrt{y^4 + 11}\right) dy$$

equals _____.

7) Consider the following statements:
(P): There exists an unbounded subset of $\mathbb{R}$ whose Lebesgue measure is equal to 5.
(Q): If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $g : \mathbb{R} \to \mathbb{R}$ is such that $f = g$ almost everywhere on $\mathbb{R}$, then g must be continuous almost everywhere on $\mathbb{R}$.

Which of the above statements hold TRUE ?
a) Both P and Q
b) Only P
c) Only Q
d) Neither P nor Q

8) If x^3y^2 is an integrating factor of

$$\left(6y^2 + axy\right) dx + \left(6xy + bx^2\right) dy = 0,$$

where $a, b \in \mathbb{R}$, then
a) $3a - 5b = 0$
b) $2a - b = 0$
c) $3a + 5b = 0$
d) $2a + b = 0$

9) If $x(t)$ and $y(t)$ are the solutions of the system $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = -x$ with the initial conditions $x(0) = 1$ and $y(0) = 1$, then $x\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ equals _____.

10) If $y = 3e^{2x} + e^{-2x} - \alpha x$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + \beta y = 4\alpha x, \;\; y(0) = 4 \text{ and } \frac{dy}{dx}(0) = 1, \text{ where } \alpha, \beta \in \mathbb{R},$$

then
a) $\alpha = 3$ and $\beta = 4$
b) $\alpha = 1$ and $\beta = 2$
c) $\alpha = 3$ and $\beta = -4$
d) $\alpha = 1$ and $\beta = -2$

11) Let G be a non-abelian group of order 125. Then the total number of elements in

$$Z(G) = \{x \in G : gx = xg \text{ for all } g \in G\}$$

equals _____.

12) Let F_1 and F_2 be subfields of a finite F consisting of 2^9 and 2^6 elements, respectively. Then the total number of elements in $F_1 \cap F_2$ equals _____.

13) Consider the normed linear space $\mathbb{R}^2$ equipped with the norm given by $||(x,y)|| = |x| + |y|$ and the subspace $X = \{(x,y) \in \mathbb{R}^2 : x = y\}$. Let f be the linear functional on X given by $f(x,y) = 3x$. If $g(x,y) = \alpha x + \beta y$, α, $\beta \in \mathbb{R}$, is a Hahn-Banach extension of f on $\mathbb{R}^2$, then $\alpha - \beta$ equals _____.