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EE1030 : Matrix Theory
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1 Shift-2(1-15)

1) The point $P(a, b)$ undergoes the following three transformations successively:
   a) reflection about the line $y = x$.
   b) translation through 2 units along the positive direction of x-axis.
   c) rotation through angle $\frac{\pi}{4}$ about the origin in the anti-clockwise direction.

   If the co-ordinates of the final position of the point P are $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$, then the value of $2a + b$ is equal to:
   a) 13
   b) 9
   c) 5
   d) 7

2) A possible value of $'x'$, for which the ninth term in the expansion of $\left\{3^{\log_3 \sqrt{25^{x-1}+7}} + 3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is:
   a) 0
   b) -1
   c) 2
   d) 1

3) For real numbers α and $\beta \neq 0$, if the point of intersection of the straight lines $\frac{x-\alpha}{1} = \frac{y-1}{2} = \frac{z-1}{3}$ and $\frac{x-4}{\beta} = \frac{y-6}{3} = \frac{z-7}{3}$, lies on the plane $x + 2y - z = 8$, then $\alpha - \beta$ is equal to:
   a) 5
   b) 9
   c) 3
   d) 7

4) Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x + y) + f(x - y) = 2f(x) f(y)$, $f\left(\frac{1}{2}\right) = -1$. Then, the value of $\sum_{k=1}^{20} \frac{1}{\sin(k)\sin(k+f(k))}$ is equal to:

a) $\operatorname{cosec}^2(21)\cos(20)\cos(2)$
b) $\sec^2(1)\sec(21)\cos(20)$
c) $\operatorname{cosec}^2(1)\operatorname{cosec}(21)\sin(20)$
d) $\sec^2(21)\sin(20)\sin(2)$

5) Let $\mathbb{C}$ be the set of all complex numbers. Let $S_1 = \{z \in \mathbb{C} : |z-2| \le 1\}$ and $S_2 = \{z \in \mathbb{C} : z(1+i) + \bar{z}(1-i) \ge 4\}$. Then the maximum value of $\left|z - \frac{5}{2}\right|^2$ for $z \in S_1 \cap S_2$ is equal to:
a) $\frac{3+2\sqrt{2}}{4}$
b) $\frac{5+2\sqrt{2}}{2}$
c) $\frac{3+2\sqrt{2}}{2}$
d) $\frac{5+2\sqrt{2}}{4}$

6) A student appeared in an examination consisting of 8 true-false type questions. The student guesses the answers with equal probability. The smallest value of n, so that the probability of guessing at least $'n'$ correct answers is less than $\frac{1}{2}$, is :
a) 5
b) 6
c) 3
d) 4

7) If $\tan\left(\frac{\pi}{9}\right), x, \tan\left(\frac{7\pi}{18}\right)$ are in arithmetic progression and $\tan\left(\frac{\pi}{9}\right), y, \tan\left(\frac{5\pi}{18}\right)$ are also in arithmetic progression, then $|x - 2y|$ is equal to:
a) 4
b) 3
c) 0
d) 1

8) Let the mean and variance of the frequency distribution

$$\begin{array}{lllll} x: & x_1 = 2 & x_2 = 6 & x_3 = 8 & x_4 = 9 \\ f: & 4 & 4 & \alpha & \beta \end{array}$$

be 6 and 6.8 respectively. If x_3 is changed from 8 to 7, then the mean for the new data will be:
a) 4
b) 5
c) $\frac{17}{3}$
d) $\frac{16}{3}$

9) The area of the region bounded by $y - x = 2$ and $x^2 = y$ is equal to:
a) $\frac{16}{3}$
b) $\frac{2}{3}$

c) $\frac{9}{2}$
d) $\frac{4}{3}$

10) Let $y = y(x)$ be the solution of the differential equation $\left(x - x^3\right) dy = \left(y + yx^2 - 3x^4\right) dx$, x ¿ 2. If $y(3) = 3$, then $y(4)$ is equal to:
a) 4
b) 12
c) 8
d) 16

11) The value of $\lim\limits_{x\to 0}\left(\frac{x}{\sqrt[8]{1-\sin(x)}-\sqrt[8]{1+\sin(x)}}\right)$ is equal to:
a) 0
b) 4
c) -4
d) -1

12) Two sides of a parallelogram are along the lines $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals of the parallelogram is $11x + 7y = 9$, then other diagonal passes through the point:
a) $(1, 2)$
b) $(2, 2)$
c) $(2, 1)$
d) $(1, 3)$

13) Let $\alpha = \max\limits_{x\in\mathbf{R}}\left\{8^{2\sin(3x)} \cdot 4^{4\cos(3x)}\right\}$ and $\beta = \min\limits_{x\in\mathbf{R}}\left\{8^{2\sin(3x)} \cdot 4^{4\cos(3x)}\right\}$. If $8x^2 + bx + c = 0$ is a quadratic equation whose roots are $\alpha^{\frac{1}{5}}$ and $\beta^{\frac{1}{5}}$, then the value of $c - b$ is equal to:
a) 42
b) 47
c) 43
d) 50

14) Let $f : [0, \infty) \to [0, 3]$ be a function defined by

$$f(x) = \begin{cases} \max\{\sin(t) : 0 \le t \le x\}, & 0 \le x \le \pi \\ 2 + \cos(x), & x > \pi \end{cases}$$

Then which of the following is true ?
a) f is continuous everywhere but not differentiable exactly at one point in $(0, \infty)$.
b) f is differentiable everywhere in $(0, \infty)$.
c) f is not continuous exactly at two points in $(0, \infty)$.
d) f is continuous everywhere but not differentiable exactly at two points in $(0, \infty)$.

15) Let $\mathbf{N}$ be the set of natural numbers and a relation R on $\mathbf{N}$ be defined by

$$R = \{(x, y) \in \mathbf{N} \times \mathbf{N} : x^3 - 3x^2y - xy^2 + 3y^3 = 0\}.$$

Then the relation R is :

a) symmetric but neither reflexive nor transitive
b) reflexive but neither symmetric nor transitive
c) reflexive and symmetric, but not transitive
d) an equivalence relation