18-Definite Integrals and Applications of Integrals

EE1030 : Matrix Theory
Indian Institute of Technology Hyderabad

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1 E-Subjective Problems

1) Evaluate:

$$\int_0^{\pi/4} \frac{\sin(x) + \cos(x)}{9 + 16\sin(2x)} dx$$

(1983 – 3 *Marks*)

2) Find the area bounded by the x-axis, part of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates at x=2 to x=4. If the ordinate at $x = a$ divides the area into two equal parts, find a. (1983 – 3 *Marks*)

3) Evaluate the following

$$\int_0^{1/2} \frac{x sin^{-1}(x)}{\sqrt{1 - x^2}} dx$$

(1984 – 2 *Marks*)

4) Find the area of the region bounded by the x-axis and the curves defined by

$$y = \tan(x), \frac{-\pi}{3} \leq x \leq \frac{\pi}{3};$$

$$y = \cot(x), \frac{\pi}{6} \leq x \leq \frac{3\pi}{2}$$

(1984 – 4 *Marks*)

5) Given a function $f(x)$ such that
   a) it is integrable over every interval on a real line and
   b) $f(t + x) = f(x)$, for every x and a real t, then show that the integral $\int_a^{a+t} f(x)\, dx$ is independent of a. (1984 – 4 *Marks*)

6) Evaluate the following:

$$\int_0^{\pi/2} \frac{x\sin(x)\cos(x)}{\cos^4(x) + \sin^4(x)} dx$$

(1985 – 5/2 *Marks*)

7) Sketch the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|$ and its area. (1985 – 5 *Marks*)

8) Evaluate:

$$\int_0^{\pi} \frac{x\,dx}{1+\cos(\alpha)\sin(x)}, 0<\alpha<\pi$$

(1986 – 5/2 *Marks*)

9) Find the area bounded by the curves, $x^2+y^2=25,\ 4y=\left|4-x^2\right|$ and x=0 above the x-axis. (1987 – 6 *Marks*)

10) Find the area of the region bounded by the curve C: y=tan(x), tangent drawn to C at $x=\pi/4$ and the x-axis. (1988 – 5 *Marks*)

11) Evaluate (1988 – 5 *Marks*)

$$\int_0^1 \log\left[\sqrt{1-x}+\sqrt{1+x}\right]dx$$

12) If f and g are continuous function on [0,a] satisfying $f(x)=f(a-x)$ and $g(x)+g(a-x)=2$, then show that

$$\int_0^a f(x)\,g(x)\,dx = \int_0^a f(x)\,dx$$

(1989 – 4 *Marks*)

13) Show that

$$\int_0^{\pi/2} f(\sin(2x))\sin(x)dx = \sqrt{2}\int_0^{\pi/4} f(\cos(2x))\cos(x)dx$$

(1990 – 4 *Marks*)

14) Prove that for any positive integer k,

$$\frac{\sin(2kx)}{\sin(x)} = 2\left[\cos(x)+\cos(3x)+\cdots+\cos(2k-1)x\right]$$

Hence prove that $\int_0^{\pi/2} \sin(2kx)\cot(x)dx = \pi/2$ (1990 – 4 *Marks*)

15) Compute the area of the region bounded by the curves $y = ex\ln x$ and $y = \frac{\ln x}{ex}$ where $\ln e = 1$. (1990 – 4 *Marks*)