

18-Definite Integrals and Applications of Integrals

EE1030 : Matrix Theory

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I. E-SUBJECTIVE PROBLEMS

1) Evaluate:

$$\int_0^{\pi/4} \frac{\sin(x) + \cos(x)}{9 + 16 \sin(2x)} dx$$

(1983 – 3 Marks)

2) Find the area bounded by the x-axis, part of the curve $y = \left(1 + \frac{8}{x^2}\right)$ and the ordinates at $x=2$ to $x=4$. If the ordinate at $x = a$ divides the area into two equal parts, (1983 – 3 Marks) find a .

3) Evaluate the following

$$\int_0^{1/2} \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} dx$$

(1984 – 2 Marks)

4) Find the area of the region bounded by the x-axis and the curves defined by $y = \tan(x)$, $\frac{\pi}{3} \leq x \leq \frac{\pi}{2}$; (1984 – 4 Marks)
 $y = \cot(x)$, $\frac{\pi}{6} \leq x \leq \frac{3\pi}{4}$

5) Given a function $f(x)$ such that

(i) it is integrable over every interval on a real line and

(ii) $f(t+x) = f(x)$, for every x and a real t , then show that the integral $\int_a^{a+t} f(x) dx$ is independent of a . (1984 – 4 Marks)

6) Evaluate the following:

$$\int_0^{\pi/2} \frac{x \sin(x) \cos(x)}{\cos^4(x) + \sin^4(x)} dx$$

(1985 – 5/2 Marks)

7) Sketch the region bounded by the curves $y = \sqrt{5-x^2}$ and $y = |x-1|$ and its area. (1985 – 5 Marks)

8) Evaluate:

$$\int_0^{\pi} \frac{xdx}{1 + \cos(\alpha) \sin(x)},$$

$$0 < \alpha < \pi$$

(1986 – 5/2 Marks)

9) Find the area bounded by the curves, $x^2 + y^2 = 25$, $4y = |4 - x^2|$ and $x=0$ above the x-axis. (1987 – 6 Marks)

10) Find the area of the region bounded by the curve C: $y = \tan(x)$, tangent drawn to C at $x = \pi/4$ and the x-axis. (1988 – 5 Marks)

11) Evaluate (1988 – 5 Marks)
 $\int_0^1 \log[\sqrt{1-x} + \sqrt{1+x}] dx$

12) If f and g are continuous function on $[0, a]$ satisfying $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$, then show that $\int_0^a f(x) g(x) dx = \int_0^a f(x) dx$ (1989 – 4 Marks)

13) Show that $\int_0^{\pi/2} f(\sin(2x)) \sin(x) dx = \sqrt{2} \int_0^{\pi/4} f(\cos(2x)) \cos(x) dx$ (1990 – 4 Marks)

14) Prove that for any positive integer k ,

$$\frac{\sin(2kx)}{\sin(x)} = 2 [\cos(x) + \cos(3x) + \cdots + \cos(2k-1)x]$$

Hence prove that (1990 – 4 Marks)

$$\int_0^{\pi/2} \sin(2kx) \cot(x) dx = \pi/2$$

15) Compute the area of the region bounded by the curves $y = ex \ln x$ and $y = \frac{\ln x}{ex}$ where $\ln e = 1$. (1990 – 4 Marks)