EE1030:Matrix Theory

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Question: 1.7.6

Find a relation between x and y if the points A(x, y), B(-4,6), and C(-2,3) (10, 2019) are collinear.

Solution:

Vertex	Coordinates
A	$\begin{pmatrix} x \\ y \end{pmatrix}$
В	$\begin{pmatrix} -4 \\ 6 \end{pmatrix}$
C	$\begin{pmatrix} -2\\3 \end{pmatrix}$

Table 1.7.6.1 0: Vertex and its coordinates

Points A, B, C are said to be collinear if

$$rank\left(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}\right) = 1 \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4\\6 \end{pmatrix} - \begin{pmatrix} x\\y \end{pmatrix} \tag{0.2}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -4 - x \\ 6 - y \end{pmatrix} \tag{0.3}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -2\\3 \end{pmatrix} - \begin{pmatrix} x\\y \end{pmatrix} \tag{0.4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -2 - x \\ 3 - y \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} -4 - x & -2 - x \\ 6 - y & 3 - y \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} -4 - x & -2 - x \\ -6 - y & 3 - y \end{pmatrix} \xrightarrow{R_2 \to R_1 + \frac{2}{3}R_2} \begin{pmatrix} -4 - x & -2 - x \\ -x - \frac{2}{3}y & -x - \frac{2}{3}y \end{pmatrix}$$
(0.7)

Given A, B, C are collinear, so rank (B - A C - A) = 1 from equation 0.1

Therefore,

$$-x - \frac{2}{3}y = 0 ag{0.8}$$

$$x = -\frac{2}{3}y$$
 (0.9)

$$3x = -2y$$
 (0.10)

$$3x = -2y \tag{0.10}$$

$$3x + 2y = 0 ag{0.11}$$

The relation between x and y is 3x + 2y = 0.

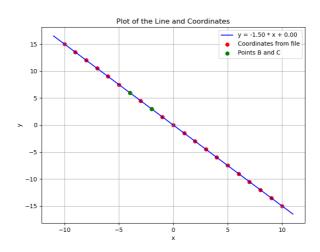


Fig. 0.1: Relation between x and y: 3x + 2y = 0