9 - Intersection of Conics

EE1030:Matrix Theory

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Question:9.3.21

If the area of the region bounded by the line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration. (12, 2022)

Solution:

Variables	Description
$\mathbf{V}, \mathbf{u}, f$	Parameters of the conic
$\mathbf{h}, \mathbf{m}$	Parameters of line
κ_i	$\frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top\left(\mathbf{V}\mathbf{h}+\mathbf{u}\right) \pm \sqrt{\left[\mathbf{m}^\top\left(\mathbf{V}\mathbf{h}+\mathbf{u}\right)\right]^2 - g(\mathbf{h})\left(\mathbf{m}^\top\mathbf{V}\mathbf{m}\right)}\right)$
$g(\mathbf{h})$	$\mathbf{h}^\top\mathbf{V}\mathbf{h} + 2\mathbf{u}^\top\mathbf{h} + f$

Table 9.3.21.1 0: Variables and their description

The parameters of the given line equation are

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.1}$$

The parameters of given curve when expressed as a conic

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}, f = 0 \tag{0.2}$$

Using these parameters,

$$g(\mathbf{h}) = 0 \tag{0.3}$$

$$\kappa_i = \frac{1}{1}\left(-\begin{pmatrix} 1 & m \end{pmatrix}\begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \pm \begin{pmatrix} 1 & m \end{pmatrix}\begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix}\right) \tag{0.4}$$

$$\kappa_1 = 0, \kappa_2 = m \tag{0.5}$$

The points of intersection of line with conic are given by

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \tag{0.6}$$

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 0 \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.7}$$

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.8}$$

$$\mathbf{x}_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + m \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.9}$$

$$\mathbf{x}_2 = \begin{pmatrix} m \\ m^2 \end{pmatrix} \tag{0.10}$$

The area bounded by the curve and the line is $\frac{32}{3}$

$$\int_0^m (mx - x^2)dx = \frac{32}{3} \tag{0.11}$$

$$\left[\frac{mx^2}{2} - \frac{x^3}{3}\right]_0^m = \frac{32}{3} \tag{0.12}$$

$$\frac{m^3}{2} - \frac{m^3}{3} = \frac{32}{3} \tag{0.13}$$

$$m^3 = 64 \tag{0.14}$$

$$m = 4 \tag{0.15}$$

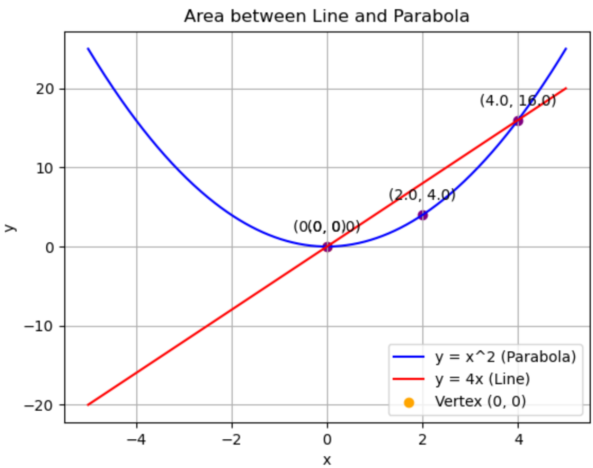

Fig. 0.1: Line and Parabola