17-03-2021 Shift-1

EE1030 : Matrix Theory
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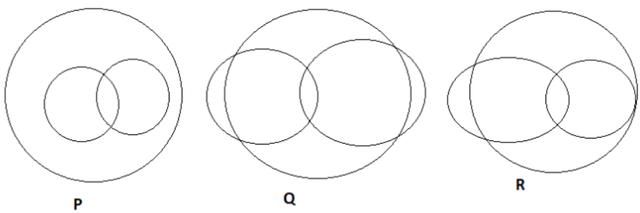
1 Shift-1(16-30)

1) Two dice are rolled. If both dices have six faces numbered 1, 2, 3, 5, 7 and 11, then the probability that the sum of the numbers on the top faces is less than or equal to 8 is:
   a) $\frac{17}{36}$
   b) $\frac{4}{9}$
   c) $\frac{5}{12}$
   d) $\frac{1}{2}$

2) The inverse of $y = 5^{\log x}$ is:
   a) $x = 5^{\log y}$
   b) $x = y^{\log 5}$
   c) $x = y^{\frac{1}{\log 5}}$
   d) $x = 5^{\frac{1}{\log y}}$

3) In a school, there are three types of games to be played. Some of the students play two types of games, but none play all three games. Which Venn diagrams can justify the above statements.



a) P and R
b) P and Q
c) None of these
d) Q and R

4) The area of the triangle with vertices $A(z)$, $B(iz)$ and $C(z+iz)$ is:
   a) $\frac{1}{2}|z+iz|^2$
   b) 1
   c) $\frac{1}{2}$
   d) $\frac{1}{2}|z|^2$

5) The value of $\lim_{x\to 0+}\frac{[\cos^{-1}(x-[x]^2)\cdot\sin^{-1}(x-[x]^2)]}{[x-x^3]}$, where $[x]$ denote the greatest integer $\leq x$ is:
   a) 0
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{2}$
   d) π

6) Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let $'p'$ denote the probability of none of the events that occur that satisfies the equations $(\alpha-2\beta)\,p=\alpha\beta$ and $(\beta-3\gamma)\,p=2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$. Then, $\frac{[\text{probability of occurrence of} E_1]}{[\text{probability of occurrence of} E_3]}$ is equal to ______

7) If the equation of the plane passing through the line of intersection of the planes $2x-7y+4z-3=0$, $3x-5y+4z+11=0$ and the point $(-2,1,3)$ is $ax+by+cz-7=0$, then the value of $2a+b+c-7$ is ______

8) If $A=\begin{bmatrix}2 & 3\\ 0 & -1\end{bmatrix}$, then the value of $\det\left(A^4\right)+\det\left(A^{10}-adj\,(2A)^{10}\right)$ is equal to ______

9) The minimum distance between any two points P_1 and P_2 while considering point P_1 on one circle and point P_2 one the other circle for the given circles' equations $x^2+y^2-10x-10y+41=0$ and $x^2+y^2-24x-10y+160=0$ ______

10) If $(2021)^{3762}$ is divided by 17, then the remainder is ______

11) If $[\cdot]$ represents the greatest integer function, then the value of $\left|\int_0^{\frac{\sqrt{\pi}}{2}}\left[\left[x^2\right]-\cos x\right]dx\right|$ is ______

12) If $f(x)=\sin\left[\cos^{-1}\frac{(1-2^{2x})}{(1+2^{2x})}\right]$ and its first derivative with respect to x is $-\frac{b}{a}\log_e 2$ when $x=1$, where a and b are integers, then the minimum value of $\left|a^2-b^2\right|$ is ______

13) If the function $f(x)=\frac{[\cos(\sin x)-\cos x]}{x^4}$ is continuous at each point in its domain and $f(0)=\frac{1}{k}$, then k is ______

14) The maximum value of z in the following equation $z=6xy+y^2$, where $3x+4y\leq 100$ and $4x+3y\leq 75$ for $x\geq 0$ and $y\geq 0$ is ______

15) If $\mathbf{a} = \alpha i + \beta j + 3k$, $\mathbf{b} = -\beta i - \alpha j - k$ and $\mathbf{c} = i - 2j - k$, such that $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{b} \cdot \mathbf{c} = -3$, then $\frac{1}{3}\left((\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}\right)$ is equal to ______