1-Vector Arithmetic

EE1030:Matrix Theory

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Question:1.7.6
Find a relation between x and y if the points **A**(x, y), **B**(-4,6), and **C**(-2,3) are collinear. (10, 2019)

Solution:

Vertex	Coordinates
A	$\begin{pmatrix} x \\ y \end{pmatrix}$
B	$\begin{pmatrix} -4 \\ 6 \end{pmatrix}$
C	$\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

Table 1.7.6.1 0: Vertex and its coordinates

Points **A**, **B**, **C** are said to be collinear if

$$rank\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = 1 \tag{0.1}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.2}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} -4-x \\ 6-y \end{pmatrix} \tag{0.3}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.4}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} -2-x \\ 3-y \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = \begin{pmatrix} -4-x & -2-x \\ 6-y & 3-y \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} -4-x & -2-x \\ -6-y & 3-y \end{pmatrix} \xrightarrow{R_2 \to R_1 + \frac{2}{3}R_2} \begin{pmatrix} -4-x & -2-x \\ -x-\frac{2}{3}y & -x-\frac{2}{3}y \end{pmatrix} \tag{0.7}$$

$$\tag{0.8}$$

Given **A**, **B**, **C** are collinear, so rank$\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} \end{pmatrix} = 1$ from equation 0.1

Therefore,

$$-x - \frac{2}{3}y = 0 \tag{0.9}$$

$$x = -\frac{2}{3}y \tag{0.10}$$

$$3x = -2y \tag{0.11}$$

$$3x + 2y = 0 \tag{0.12}$$

The relation between x and y is $3x + 2y = 0$.

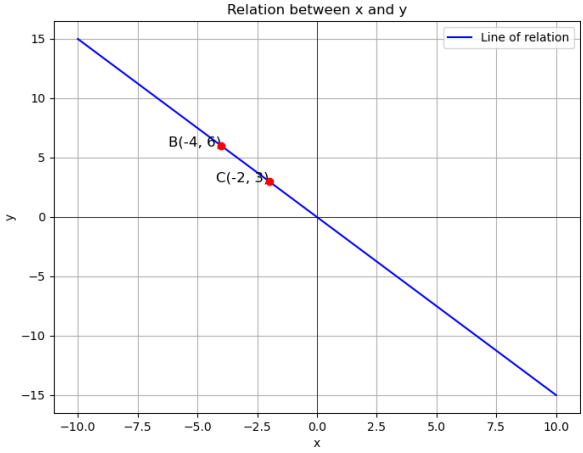

Fig. 0.1: Relation between x and y: 3x + 2y = 0