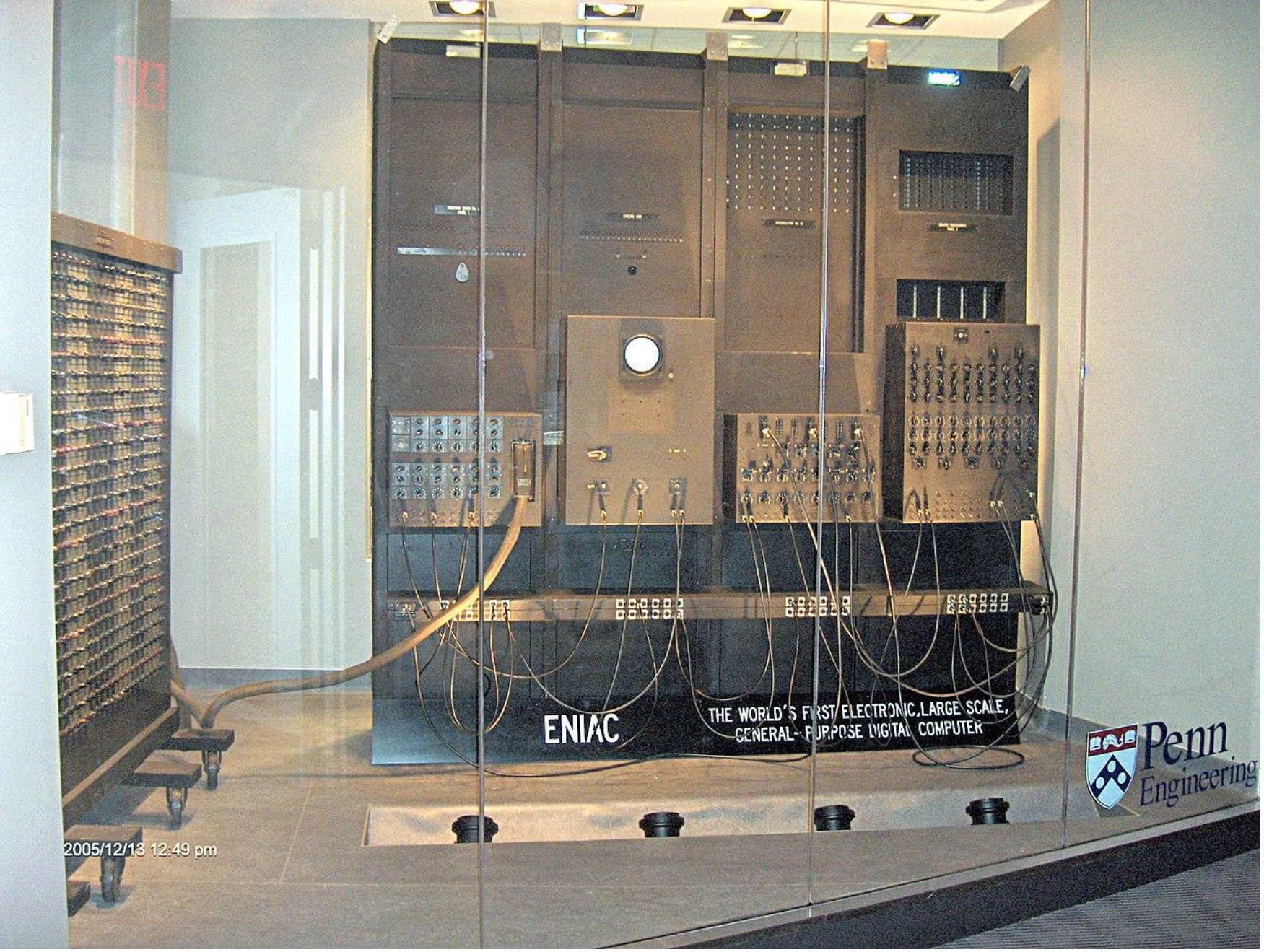
Monte Carlo Simulations

Chapter 18

Monte Carlo Simulations

- Execute a series of randomized trials and average results to approximate probability
  - We have already been doing this!
  - Leverages the speed and power of modern computers
- Most early theory revolves around games of chance
- Hence the nod to Monte Carlo by Ulam and Metropolis
  - Besides Las Vegas was just a little city in the desert then



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Pascal's Problem

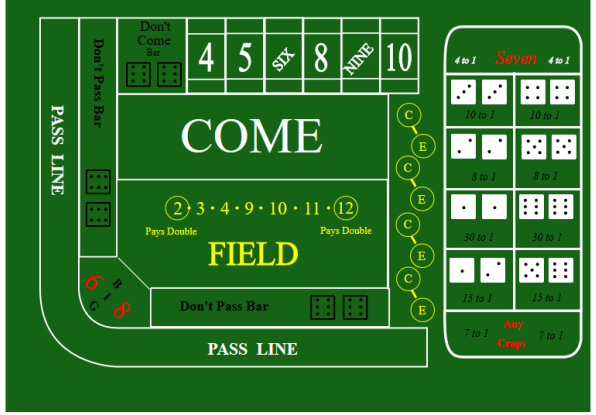
- Should I bet that I can roll at 12 (box cars) within 24 rolls?
  - Probability of rolling a 6 is $\frac{1}{6} = 0.16667$
  - So the probability of rolling 2 6's is $\frac{1}{6^2} = \frac{1}{36} = 0.027778$
  - With 24 rolls I get $24 \times \frac{1}{36} = 0.66667$
  - So it's a good bet, right?
  - Or is it ${}^{1}/_{36}{}^{24} = 4.453876 \times 10^{-38}$
  - So it's not a good bet!
  - Right?

Pascal's Problem - deux

- So what do we do?
  - The probability of rolling a 12 ($\frac{1}{36}$) is the same as 1 – the probability of *not* rolling a 12 ($1 - \frac{1}{36}$ or $1 - {}^{35}/_{36}$)
  - The probability of *not* rolling a 12 in 24 rolls (negative bet) is $({}^{35}/_{36})^{24} = 0.508596$
  - Then the probability of rolling a 12 in 24 rolls is $1 - {}^{35}/_{36}{}^{24} = 0.4914038$
  - So keep your money or bet with the odds

- Let's simulate!

Craps



Pass Bet

- First Roll
  - 7 or 11 = Win
  - 2, 3 or 12= Lose “Craps”
  - Otherwise – establish point
- Until Win or Loss
  - Roll
    - Point = Win
    - 7 = Lose “Crapped out”

Don’t Pass Bet

- First Roll
  - 7 or 11 = “craps”
  - 2 or 3 = win
  - 12 = “push” - tie
  - Otherwise – establish point
- Until Win or Loss
  - Roll
    - 7 wins
    - Point lose

Probability

Pass Bet

- First roll
  - Win = prob(7) + prob(11)
  - Lose = prob(2) + prob(3) + prob(12)
- Subsequent Rolls
  - Lose = prob(7)
  - Win = prob(point?)

Don’t Pass Bet

- First roll
  - Win = prob(2) + prob(3)
  - Lose = prob(7) + prob(11)
  - Tie = prob(12)
- Subsequent Rolls
  - Lose = prob(point?)
  - Win = prob(7)

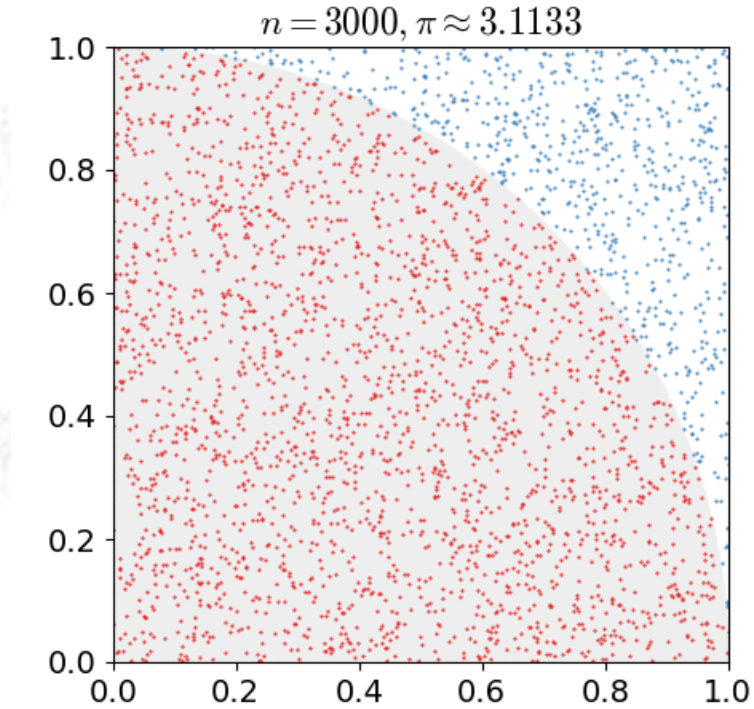
This looks like a job for SIMULATION!

How do we determine whether we have made a good investment?

- **Return on Investment (ROI)**
  - Net Income / Investment
  - $ROI = \frac{current\ value\ of\ investment - cost\ investment}{cost\ of\ investment}$
  - $ROI = \frac{net\ income}{investment}$
  - For Craps $ROI = \frac{wins - losses}{number\ of\ bets}$

Calculating the area of a circle (i.e. finding π

- Inscribe a circle inside a square
- Drop *n* needles into the square
- Count the points inside the circle
- Ratio of in $\frac{in}{dropped} \approx \frac{area\ of\ circle}{area\ of\ square}$
- $\pi = {}^{area}/_{r^2}$



- Finally, with a unit circle we can isolate to one quarter and multiply by 4.

Remember Statistically Valid is not the same thing as Correct

- Depends on our model
- Depends on our implementation
- Depends on our data

Analytical vs. Simulation Models

- Prior the mid-20th century analytical models were built based on mathematical theory
- With the advent of computers, increased complexity of models and randomness of data simulation models became popular
- Permits easier “what if” scenarios and inspection of interim results
- Simulations are **descriptive** not **prescriptive**
  - Tells how a system works under given conditions
  - Does not tell you how to best configure a system

Three axes of simulation

- Deterministic versus stochastic
  - Deterministic – everything is completely defined by the model
  - Stochastic – inputs are random
- Static versus dynamic
  - Time is not an essential component
    - Calculating pi
  - Time or duration is a key element
    - Crab race or chicken drop
- Discrete versus continuous
  - Discrete – pertinent variables are integers
  - Continuous – real numbers (e.g. flow)