Robot Dynamics Quiz 1

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Duration: 1h 15min
Permitted Aids: The exam is open book, which means you can use the script, slides, exercises, etc; The use of internet (besides for licenses) is forbidden; no communication among students during the test is allowed.

1 Instructions

1. Download the ZIP file `RobotDynamics_Quiz1_2022.zip` from Moodle. Extract all contents of this file into a new folder and set MATLAB's[1] current path to this folder.

2. Run `init_workspace` in the Matlab command line.

3. All problem files that you need to complete are located in the `problems` folder.

4. Run `evaluate_problems` to check if your functions run. This script does not test for correctness. You will get 0 points if a function does not run (e.g., for syntax errors).

5. When the time is up, zip the entire folder and name it
   `ETHStudentID_StudentName.zip`
   Submit this zip-file through Moodle under **Midterm Exam 1 Submission**. You should receive a confirmation email.

6. If the previous step did not succeed, you can email your file to
   `robotdynamics@leggedrobotics.com`
   from your ETH email address with the subject line
   `[RobotDynamics] ETHStudentID - StudentName`

7. **Important:**

   (a) Implementations outside the provided templates will not be graded and receive 0 points.

   (b) Helper functions included in the `solutions/pcode` directory are specifically for Question 4. Using these functions in the solutions for other questions is prohibited and will receive 0 points.

[1] Online version of MATLAB at `https://matlab.mathworks.com/`

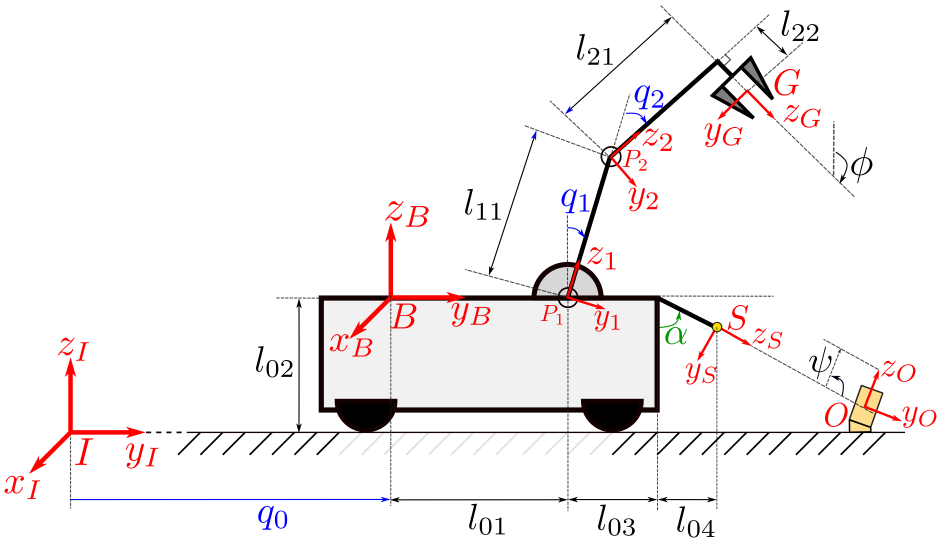

Figure 1: Schematic of a robot mobile manipulator with a two degrees of freedom robotic arm attached to a mobile base. The base can only move along y_I direction, while all joints on the arm can rotate around the positive x_I axis. The x axis of the frames $\{P_1\}, \{P_2\}$ is parallel to the x_I axis.

2 Questions

In this quiz, you will model the forward and differential kinematics for the robotic manipulator shown in Fig. 1. It is a 2 Degrees-of-Freedom (DoF) arm connected to a mobile base. The robot has a gripper attached to the last link of the arm, and a camera sensor attached to the base.

The base can only move linearly along the y_I axis. The reference frame attached to the base is denoted as $\{B\}$. The arm is composed of two links. The reference frames attached to each link are denoted as $\{P_1\}, \{P_2\}$. The frame attached to the gripper is denoted as $\{G\}$. *Note:* The transformation between $\{P_2\}$ and $\{G\}$ is fixed, i.e. there is a fixed 90 deg rotation between the gripper and the second link of the arm. As shown in Fig. 1, a camera sensor is rigidly mounted at point S on the base, rotated by a constant angle α. The corresponding reference frame is denoted as $\{S\}$.

Additionally, a target object is located at point O, with the corresponding frame denoted as $\{O\}$. The task is to grasp the object using the robot manipulator.

The generalized coordinates are defined as

$$\boldsymbol{q} = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \end{bmatrix} \in \mathbb{R}^3 . \tag{1}$$

Clarification 1: The angles q_1, q_2, ϕ, ψ are measured using right-hand thumb rule. For the state of the scene shown in Fig. 1, q_1, q_2 and ϕ would have negative values, while ψ would have a positive value.
Clarification 2: The angle α is measured with respect to negative z-axis in frame B. An angle of zero would mean that the sensor is facing towards the ground.

In the following questions, all required parameters are passed to your functions in a structure called `params`. You can access it as follows:

```
l01 = params.l01;
l02 = params.l02;
l03 = params.l03;
l04 = params.l04;
l11 = params.l11;
l21 = params.l21;
l22 = params.l22;
alpha = params.alpha;
```

Question 1. 6 P.

Find the homogeneous transformation between the inertial frame $\{I\}$ and the gripper frame $\{G\}$, i.e., the matrix $\mathbf{T}_{IG}$ as a function of the generalized coordinates $\boldsymbol{q}$.

You should implement your solution in the function `jointToGripperPose.m`

Question 2. 4 P.

Compute the position Jacobian ${}_I\mathbf{J}_P \in \mathbb{R}^{3\times3}$, that fulfills:

$$ {}_I\boldsymbol{v}_{IG} = {}_I\mathbf{J}_P(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{2} $$

where ${}_I\boldsymbol{v}_{IG} \in \mathbb{R}^3$ is the linear velocity of point G (the gripper) with respect to a fixed point expressed in frame $\{I\}$.
You should implement your solution in the function `jointToPositionJacobian.m`

Question 3. 2 P.

Compute the rotation Jacobian ${}_I\mathbf{J}_R \in \mathbb{R}^{3\times3}$, that fulfills:

$$ {}_I\boldsymbol{\omega}_{IG} = {}_I\mathbf{J}_R(\boldsymbol{q})\dot{\boldsymbol{q}}, \tag{3} $$

where ${}_I\boldsymbol{\omega}_{IG} \in \mathbb{R}^3$ is the angular velocity of frame $\{G\}$ with respect to the inertial frame $\{I\}$, expressed in frame $\{I\}$.
You should implement your solution in the function `jointToRotationJacobian.m`

Question 4. 3 P.

Given a desired gripper pose ${}_I\boldsymbol{p}^*$, use inverse kinematics formulation to compute the generalized coordinates required to have the gripper frame $\{G\}$ coinciding and aligned with the desired pose.

We indicate with ${}_I\boldsymbol{p}^* \in \mathbb{R}^3$ the following vector:

$$ {}_I\boldsymbol{p}^* = \begin{bmatrix} {}_Iy_G^* \\ {}_Iz_G^* \\ \phi^* \end{bmatrix}, \tag{4} $$

where the angle ϕ is indicated in Fig. 1.

For this question, we provide:

- a function to calculate the current gripper position in the plane ${}_Iyz$:

$$ {}_I\boldsymbol{p}_{yz} = \begin{bmatrix} {}_Iy_G \\ {}_Iz_G \end{bmatrix} \in \mathbb{R}^2. \tag{5} $$

  You can call it with `jointTo2DGripperPosition_solution(q, params);`

- the analytical Jacobian $\mathbf{J}_A \in \mathbb{R}^{3\times 3}$, that fulfills:

$$
{}_I\boldsymbol{w} = \mathbf{J}_A\dot{\boldsymbol{q}}. \tag{6}
$$

  You can call it with `jointToGripperAnalyticalJacobian_solution(q, params)`;

- a function to compute the damped pseudo-inverse of a matrix `A`. You can call it with `pseudoInverseMat_solution(A, lambda)`.

You should implement your solution in the function `inverseKinematics.m`.

Question 5. 3 P.

The sensor S on the mobile manipulator now detects the pose of a target object O. The sensor provides the position of this point O in sensor frame and the angle, ψ, between the z-axis of the sensor frame and the object frame.

We indicate this sensor measurement with ${}_S\boldsymbol{p}_{SO} \in \mathbb{R}^3$ the following vector:

$$
{}_S\boldsymbol{p}_{SO} = \begin{bmatrix} {}_Sy_O \\ {}_Sz_O \\ \psi \end{bmatrix}, \tag{7}
$$

where the angle ψ is indicated in Fig. 1.

Given as input the pose of the object in the sensor frame, compute the desired gripper pose (Eq. (4)) to execute grasping of the object.

Hint: For proper grasping, the z-axis of the gripper frame needs to be aligned opposite to the z-axis of the object frame. What will be the relative rotation between $\{O\}$ and $\{G_{des}\}$ in that case?

You should implement your solution in the function `gripperToObject.m`.