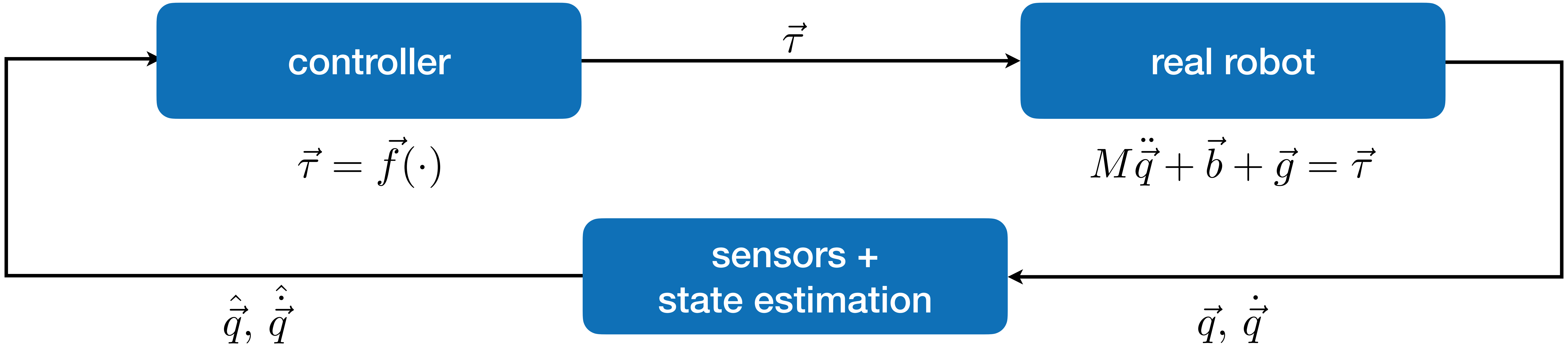
Robot Dynamics Exercise Session 05

last week: EoM $M(\vec{q})\,\ddot{\vec{q}} + \vec{b}(\vec{q},\,\dot{\vec{q}}) + \vec{g}(\vec{q}) = S^\top \vec{\tau} + J^\top \vec{F} \quad \in \mathbb{R}^q$

=> can be used for physical simulations and <u>model-based control</u>

now:

- joint impedance control (+ gravity compensation)
- inverse dynamics control
- operational space control



$= \vec{q},\ \dot{\vec{q}}$ (assume perfect state estimation)

$\vec{\tau} = \vec{f}(\vec{q},\, \dot{\vec{q}},$ references , model)

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1) Joint Impedance Control

- plain PD-control:

$$\vec{\tau} = \underbrace{K_p}_{\substack{>0 \\ \text{diagonal}}} (\vec{q}_{ref} - \vec{q}) + \underbrace{K_d}_{\substack{>0 \\ \text{diagonal}}} (\dot{\vec{q}}_{ref} - \dot{\vec{q}})$$

- linear control law, non-model-based
- purely decentralized approach (independently decoupled joint control)
- can only give zero steady-state error if $K_p \to +\infty$ (or gravity is 0)

At steady state: $\dot{\bar{\vec{q}}} = \ddot{\bar{\vec{q}}} = \vec{0} \quad \to \quad \vec{g}(\bar{\vec{q}}) = K_p(\vec{q}_{ref} - \bar{\vec{q}})$

$\bar{\vec{q}} \to \vec{q}_{ref}$ if $K_p \to +\infty$ (or gravity is 0)

- integrator: tricky, as it might cause instability

- PD-control + gravity compensation:

$$\vec{\tau} = K_p(\vec{q}_{ref} - \vec{q}) + K_d(\dot{\vec{q}}_{ref} - \dot{\vec{q}}) + \hat{\vec{g}}(\vec{q})$$

- non-linear, model-based control law
- can achieve zero steady-state error with low PD-gains
- centralized approach, but only accounts for static couplings

model might not match reality, $\vec{g}(\vec{q})$, perfectly.

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2) Inverse Dynamics Control

idea: linearize robot dynamics using feedback (feedback linearization)

- replace $\vec{\tau}$ in EoM

$$\vec{\tau} = \hat{M}(\vec{q})\underbrace{\ddot{\vec{q}}_{des}}_{\substack{\text{auxiliary variable}\\ \text{(desired acceleration)}}} + \hat{\vec{b}}(\vec{q}, \dot{\vec{q}}) + \hat{\vec{g}}(\vec{q})$$

assuming perfect model: $\{\hat{M}, \hat{\vec{b}}, \hat{\vec{g}}\} = \{M, \vec{b}, \vec{g}\}$

=> $\boxed{\ddot{\vec{q}} = \ddot{\vec{q}}_{des}}$ (decoupled linear system)

- how to choose $\ddot{\vec{q}}_{des}$?

- <u>Option 1:</u> given $\underbrace{\vec{q}_{ref}, \dot{\vec{q}}_{ref}, \ddot{\vec{q}}_{ref}}_{\text{pre-specified (or e.g. inverse dynamics)}}$ (joint space)

$$\ddot{\vec{q}}_{des} = K_p(\vec{q}_{ref} - \vec{q}) + K_d(\dot{\vec{q}}_{ref} - \dot{\vec{q}}) + \ddot{\vec{q}}_{ref}$$

$$\rightarrow \quad \vec{0} = K_p\,\vec{e} + K_d\,\dot{\vec{e}} + \ddot{\vec{e}} \quad (\vec{e} = \vec{q}_{ref} - \vec{q})$$

asymptotically stable 2nd order system

$\vec{e} \rightarrow \vec{0}$ as $t \rightarrow \infty$, $\vec{q} \rightarrow \vec{q}_{ref}$

- <u>Option 2:</u> given $\underbrace{\vec{\chi}_{\mathcal{I}\mathcal{E}_{ref}}}_{\text{pose}}, \underbrace{\vec{w}_{\mathcal{I}\mathcal{E}_{ref}}}_{\text{twist}}, \underbrace{\dot{\vec{w}}_{\mathcal{I}\mathcal{E}_{ref}}}_{\text{accelerations}}$ (task space)

to get the same error dynamics as before, with

$$\vec{e} = \Delta\vec{\chi}_{\mathcal{I}\mathcal{E}} = \begin{bmatrix} \vec{r}_{IE_{ref}} - \vec{r}_{IE} \\ \underbrace{\Phi_{\mathcal{I}\mathcal{E}_{ref}} \boxminus \Phi_{\mathcal{I}\mathcal{E}_{ref}}}_{{}_{\mathcal{I}}\vec{\phi}_{\mathcal{E}\mathcal{E}_{ref}}} \end{bmatrix}$$

we need $\dot{\vec{w}}_{des} = K_p(\Delta\vec{\chi}_{\mathcal{I}\mathcal{E}}) + K_d(\vec{w}_{ref} - \vec{w}) + \dot{\vec{w}}_{ref}$

=> go back to joint accelerations: $\dot{\vec{w}} = J\ddot{\vec{q}} + \dot{J}\dot{\vec{q}}$

$$\ddot{\vec{q}}_{des} = J^{+}\Big(\underbrace{K_p(\Delta\vec{\chi}_{\mathcal{I}\mathcal{E}}) + K_d(\vec{w}_{ref} - \vec{w}) + \dot{\vec{w}}_{ref}}_{\dot{\vec{w}}_{des}} - \dot{J}\dot{\vec{q}}\Big)$$

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3) Operational Space Control

applied by robot on environment

- end-effector dynamics: $$\Lambda \dot{\vec{w}}_e + \vec{\mu} + \vec{p} + \vec{F}_C = \vec{F}_e$$

effective force on end-effector due to $\vec{\tau}$

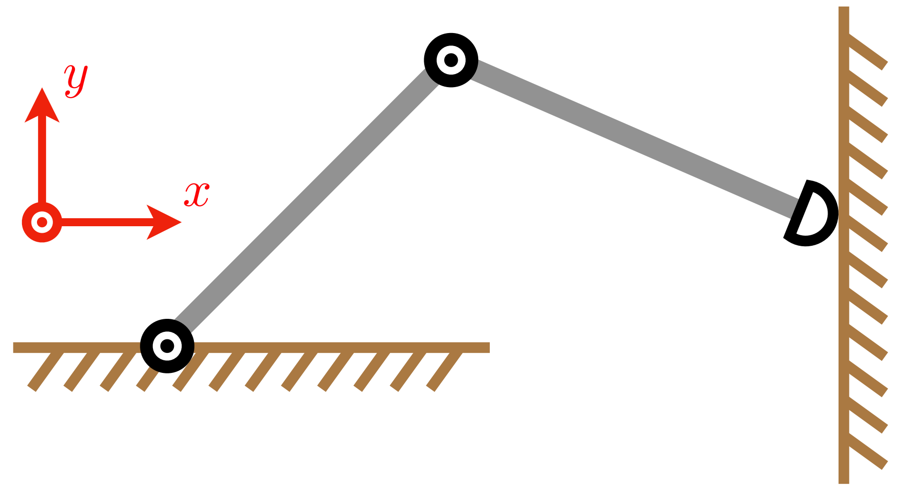
- control law given desired $\dot{\vec{w}}_e^*$: $$\vec{\tau} = J^\top \vec{F}_e = J^\top (\Lambda \dot{\vec{w}}_e^* + \vec{\mu} + \vec{p})$$

  difference between previous $\vec{\tau}$ (option 2): mass-weighted pseudo-inverse (see lecture)
  (only equal if $J^+ = J^{-1}$, J is square)

- hybrid force and motion control:

  introduce selection matrices ${}_\mathcal{I}S_M$, ${}_\mathcal{I}S_F$ to decouple motion control directions from force control directions:

  e.g.



(frictionless wall)

$${}_\mathcal{I}S_M = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad {}_\mathcal{I}S_F = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \mathbb{I}_{2\times 2} - {}_\mathcal{I}S_M$$

control law: $$\vec{\tau} = J^\top (\Lambda \, {}_\mathcal{I}S_M \, \dot{\vec{w}}_e^* + {}_\mathcal{I}S_F \, \vec{F}_C^* + \vec{\mu} + \vec{p})$$