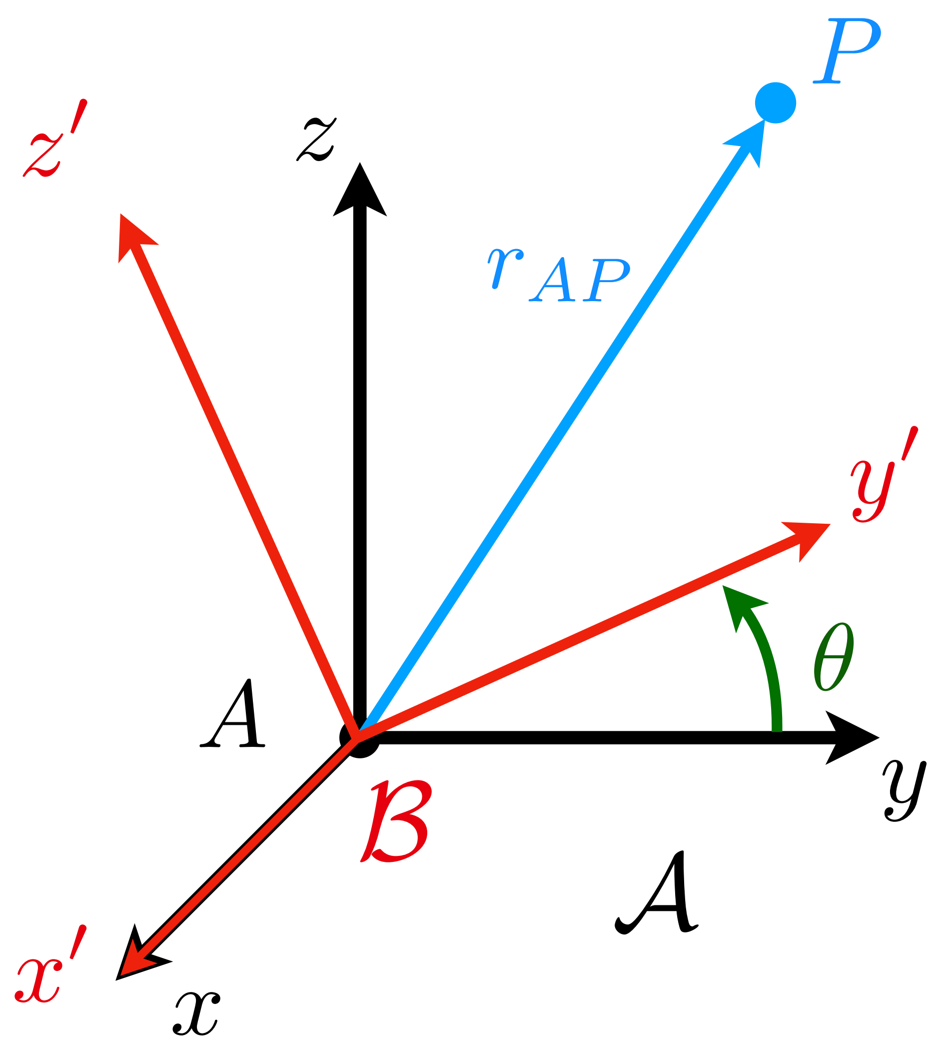
Robot Dynamics Exercise Session 01

position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

$$_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ _{\mathcal{A}}}}\vec{r}_{AP}$$

$C_{\mathcal{AB}}$ is a change of representation (or basis)

$$C_{\mathcal{AB}} = \begin{bmatrix} _{\mathcal{A}}\vec{e}_x^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_y^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_z^{\mathcal{B}} \end{bmatrix}$$

$$C_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

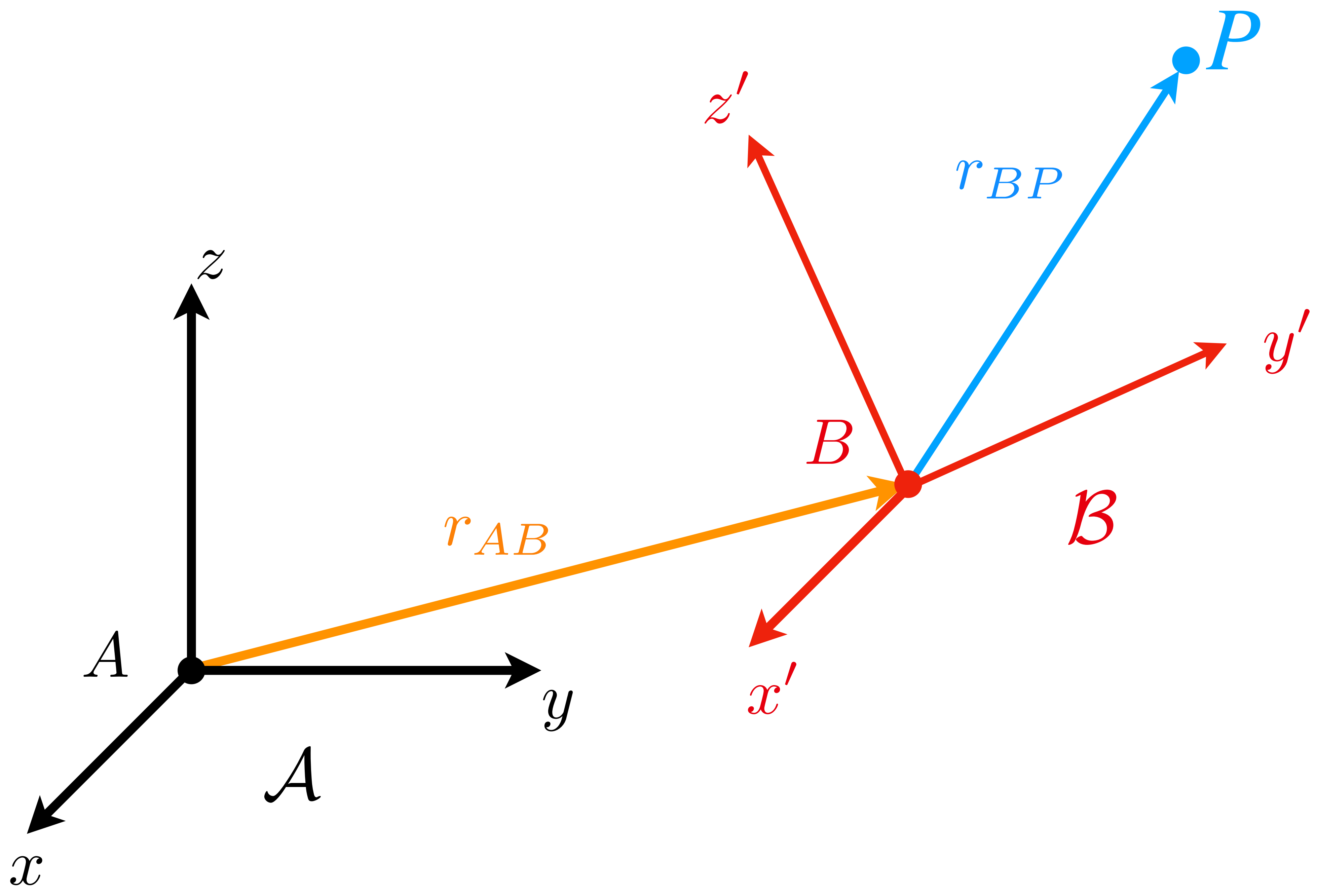
inverse: $C_{\mathcal{BA}} = C_{\mathcal{AB}}^{-1} = C_{\mathcal{AB}}^{\top} \quad \leftrightarrow \quad C_{\mathcal{AB}} \underbrace{C_{\mathcal{AB}}^{\top}}_{C_{\mathcal{BA}}} = \mathbb{I}_{3\times 3}$

$C_{\mathcal{AB}}$ is an orthogonal matrix: $\det(C_{BA}) = 1$ (length preserving)

composition: $C_{\mathcal{AC}} = C_{\mathcal{AB}} C_{\mathcal{BC}}$

Robot Dynamics Exercise Session 01

homogeneous transformations



given: $_{\mathcal{A}}\vec{r}_{AB},\ C_{\mathcal{AB}},\ _{\mathcal{B}}\vec{r}_{BP}$

find: $_{\mathcal{A}}\vec{r}_{AP}$

$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
$$_{\mathcal{A}}\vec{r}_{AP} = {_{\mathcal{A}}\vec{r}_{AB}} + {_{\mathcal{A}}\vec{r}_{BP}}$$
$$_{\mathcal{A}}\vec{r}_{AP} = {_{\mathcal{A}}\vec{r}_{AB}} + C_{\mathcal{AB}}\, {_{\mathcal{B}}\vec{r}_{BP}}$$

homogeneous coordinates:

$$\begin{bmatrix} _{\mathcal{A}}\vec{r}_{AP} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} C_{\mathcal{AB}} & _{\mathcal{A}}\vec{r}_{AB} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{T_{\mathcal{AB}}\ (4\times 4)} = \begin{bmatrix} _{\mathcal{B}}\vec{r}_{BP} \\ 1 \end{bmatrix}$$

change of representation and origin

inverse: $$T_{\mathcal{AB}}^{-1} = T_{\mathcal{BA}} = \begin{bmatrix} C_{\mathcal{BA}} & _{\mathcal{B}}\vec{r}_{BA} \\ 0_{1\times 3} & 1 \end{bmatrix}$$
$$= \begin{bmatrix} C_{\mathcal{AB}}^{\top} & C_{\mathcal{BA}}\, {_{\mathcal{A}}\vec{r}_{BA}} \\ 0_{1\times 3} & 1 \end{bmatrix}$$
$$= \begin{bmatrix} C_{\mathcal{AB}}^{\top} & -C_{\mathcal{AB}}^{\top}\, {_{\mathcal{A}}\vec{r}_{AB}} \\ 0_{1\times 3} & 1 \end{bmatrix}$$

$$\boxed{\vec{r}_{AB} = -\vec{r}_{BA}}$$

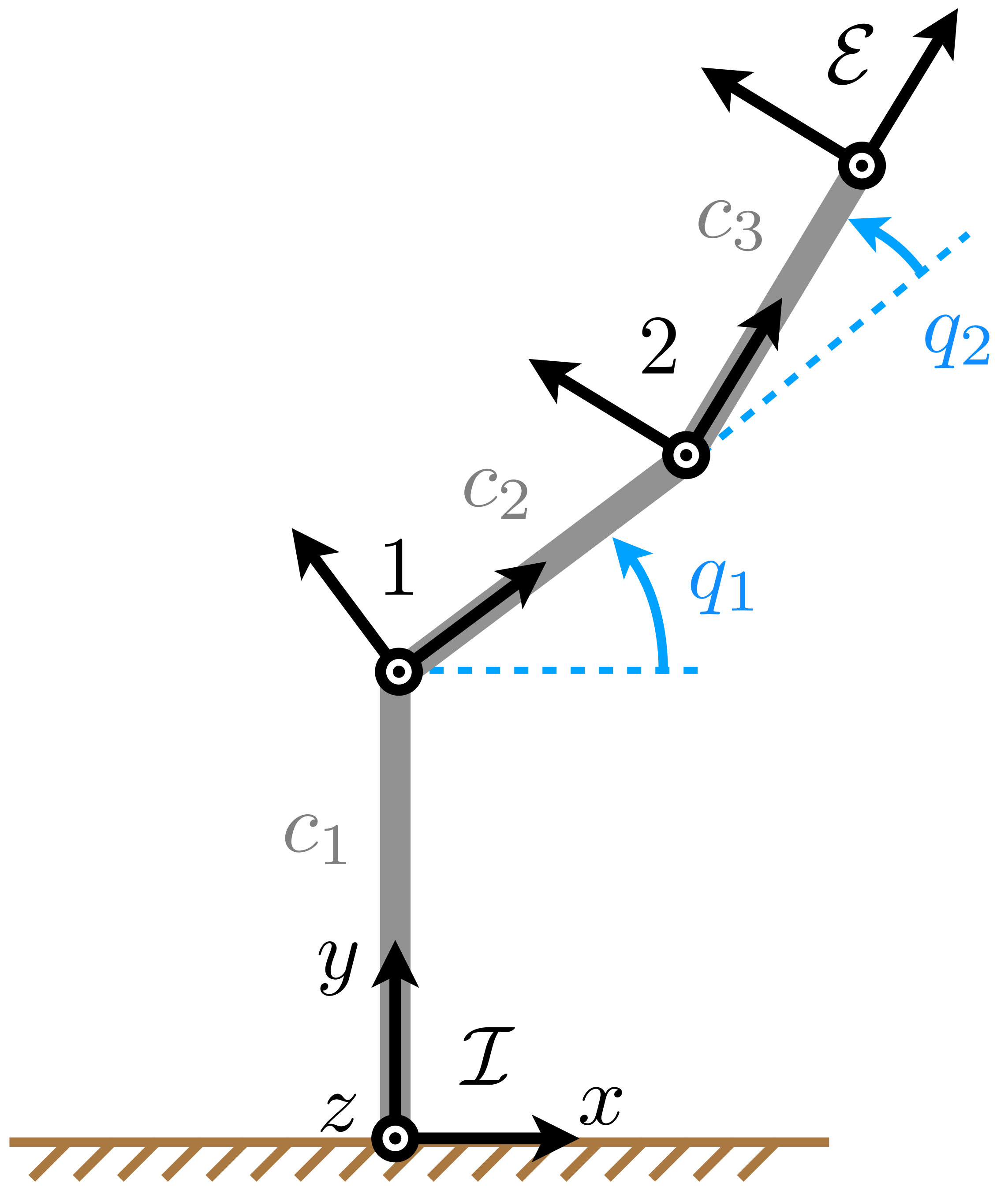
composition:

$$T_{\mathcal{AC}} = T_{\mathcal{AB}}\, T_{\mathcal{BC}}$$

Robot Dynamics Exercise Session 01

example: kinematics of a planar manipulator

$\boxed{\text{find: } \quad C_{\mathcal{I}\mathcal{E}},\ {}_{\mathcal{I}}\vec{r}_{IE}}$



Step 1: find $T_{\mathcal{I}1},\ T_{12},\ T_{2\mathcal{E}}$

$$T_{\mathcal{I}1} = \begin{bmatrix} \overbrace{C_z(q_1)}^{C_{\mathcal{I}1}} & \overbrace{\begin{pmatrix} 0 \\ c_1 \\ 0 \end{pmatrix}}^{{}_{\mathcal{I}}\vec{r}_{I1}} \\ 0_{1\times 3} & 1 \end{bmatrix} \qquad T_{12} = \begin{bmatrix} C_z(q_2) & \begin{pmatrix} c_2 \\ 0 \\ 0 \end{pmatrix} \\ 0_{1\times 3} & 1 \end{bmatrix} \qquad T_{2\mathcal{E}} = \begin{bmatrix} \mathbb{I}_{3\times 3} & \begin{pmatrix} c_3 \\ 0 \\ 0 \end{pmatrix} \\ 0_{1\times 3} & 1 \end{bmatrix}$$

Step 2: composition $T_{\mathcal{I}\mathcal{E}} = T_{\mathcal{I}1}\, T_{12}\, T_{2\mathcal{E}}$

Step 3: get $C_{\mathcal{I}\mathcal{E}},\ {}_{\mathcal{I}}\vec{r}_{IE}$ from $T_{\mathcal{I}\mathcal{E}} = \begin{bmatrix} \boxed{C_{\mathcal{I}\mathcal{E}}} & \boxed{{}_{\mathcal{I}}\vec{r}_{IE}} \\ 0_{1\times 3} & 1 \end{bmatrix}$