Algebra

SAT: Math - ~~foundation~~

* Linear equations and inequalities:

Linear equation: $7x+3=80$ => Find x

Linear inequality: $7x+3>80$ => Find the range for x

* Equivilancy *

For linear equations, you need to make sure both sides are equivilant. To do this you need to ~~add~~ work on both sides

* But, be careful when doing this with what is or might be 0 (zero)

$3(x+2)-\frac{6}{3}=12$

$3x+6-\frac{6}{3}=12$

$+2 \quad 3x-2=12 \quad +2$

$\frac{3x}{3}=\frac{14}{3}$

$x=\frac{14}{3}$

↓

equivilant

$-2 \quad 5x+2=6x+2 \quad -2$

$\frac{5x}{x}=\frac{6x}{x}$

↓

$5=6$

↓

NOT equivilant

$x=0$ -> be careful of what might (variable) be zero

Also, adding or subtracting on one side only is stupid, as well as multiplying by zero on both sides.

* Inequality "trick":

if you divide or multiply by a negative value, you switch the signs. For example:

$-9x+6 \geq 12$

$\frac{-9x}{-9} \geq \frac{6}{-9}$

$x \leq -\frac{1}{3}$

There's nothing else of major importance here. Just be careful it is on multiplication and division of negatives only, not addition / subtraction.

* Solutions to linear equations:

Normal equations

- One solution => $x=a$
- No solution => $a=b$
- Infinite solutions => $x=x$

'a' and 'b' are constants

$2x-1=ax-2$	$2y-4=a(x+2)$	$5x+3=ax+b$
$a=2$	$a=2$	
↓	↓	if $a=5$ and $b=3$ = infinite
$2x-1=2x-2$	$2x-4=2x-4$	
(obvious already)	(same = infinite)	if $a \neq 5$ and b: any = one
↓	!	
$-1=-2$ x	$-4=-4$ | $2x=2x$	if $a=5$ and $b \neq 3$ = No
No solution	$2=2$ | $x=x$	
	infinite solutions	

* Linear word problems and equations:

Linear graphs are those that are simply a straight line

* Main formula *

$$y = mx + c$$

m = how steep
↑
For graphs
↓
c = y-intercept

x = distance / height / value / etc.
↑
For word problems
↓
y = final answer

m = recurring number (cost, each, every, etc)
↑
word problems
↓
c = starting value

x = x-axis
↑
graphs
↓
y = y-axis

* Remember:

1) you can always find m / gradient / slope via => $\frac{y_2 - y_1}{x_2 - x_1}$

Look out for this when you don't have m in graphs or word problems

2) You can draw or write a linear equation / graph if you have either
- Two points on the graph
- One point and the rate / slope / m

To find c you first find m and then substitute values of x and y if you have m just substitute immediately or look at the graph

parallel and perpendicular:

Parallel lines = same gradient | Perpendicular lines = negative inverse ($m \rightarrow -\frac{1}{m}$)

You will be given two lines and you would have to deduce whether they are parallel, perpendicular, or unrelated

* Types of questions *

1) Write linear function based on word problem => might need to find m or c

2) Identify the slope or y-intercept and what they represent in a word problem

3) Parallel, perpendicular, or unrelated? => given line a, find line b if it is parallel / perpendicular and passes through (x,y)

↓

e.g. m of $a = \frac{1}{3}$, b is perpendicular and passes through (1, -5), find function b(-2)

- m of $b = \frac{1}{3} \rightarrow -3$
- $y = -3x + c \rightarrow -5 = -3(1) + c$
- $c = -2$
- $b = -3x - 2$
- $b(-1) = -3(-2) - 2 = \boxed{4}$

* Functions *

$g(x) = 3x - 2$

$g(5) = 3(5) - 2 = \boxed{13}$

remember functions

Systems of linear equations:

These are (systems) equations that have:
- Two sets of variables (x and y)
- Two sets of equations are given, both with the same x and y

* Methods of solving *

1) simultaneous / elimination
2) substitution

* Both are easy and known, will bother explaining them

You can use either at any time, it's just personal preference, substitution is generally faster unless the values are already setup or easily multiplied in which case simultaneous would be faster
→ only need to switch signs of multiply by an easy integer

** Intersections and number of solutions

Meaning: The x and y answer to a simultaneous / substitution question is the point where the two linear lines intersect.

However, some linear lines could have one solution, no solution or infinite number of solutions

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i.e. second method involves going through the simultaneous process and checking if it is equal, different, or gives a value to a variable, and

Main method of knowing number of solutions: (x=5, 0=0, 0=20) method, won't go into detail

=> Change to $y = mx + c$ format (no coefficient)

=> If:

* Both lines have the same slope → parallel → no solution / intersect
* Slope is different → one solution
* Slope and y-intercept are both the same → same line → infinite

$5x - 9y = 16$	$-6x + 4y = 2$	$5x - 2y = 6$
$5x - 9y = 36$	$3x - 2y = -1$	$5x + 3y = 1$
↓	↓	↓
$5x - 16 = 9y$	$-6x - 2 = -4y$	$5x - 6 = 2y$
$5x - 36 = 9y$	$3x + 1 = 2y$	$5x - 1 = 3y$
↓	↓	↓
$\frac{5}{9}x - \frac{16}{9} = y$	$\frac{3}{2}x + \frac{1}{2} = y$	$\frac{5}{2}x - 3 = y$
$\frac{5}{9}x - 4 = y$	$\frac{3}{2}x + \frac{1}{2} = y$	$\frac{5}{3}x - \frac{1}{3} = y$
↓	↓	↓
Same slope = no solution	Same slope and y-intercept = infinite solutions	Different slopes = one solution

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Systems of linear inequalities:

Two different sets of inequalities that work together to form a complete analysis

* Translating and understanding *

One of the main aspects in solving linear inequality systems is to set the foundation by translating a worded problem into the relevant inequalities (i.e. find both/all inequalities, not just one)

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The importance of "laying the foundation":

Cargo delivers 50 (a) and 80 (b) pound packages. Should deliver 20 packages minimum. Max weight is 1500 pounds

∴ => $a + b \geq 20$ and $50a + 80b \leq 1500$

How many 80lb packages can it hold? => (15) (16) (17) (18)

why?

- $80(17) + 50a \leq 1500 \Rightarrow 50a \leq 140 \Rightarrow a \leq 2$ | $17 + 2 \neq 20$ doesn't match
- Same as above for 15 and 18
- $80(16) + 50a \leq 1500 \Rightarrow 50a \leq 220 \Rightarrow a \leq 4$ | $16 + 4 \geq 20$ | $80(16) + 50(4) \leq 1500$ — both satisfied

As such, before solving, it is important to set the parameters, in this case, it was $(a + b \geq 20)$ and $(80b + 50a \leq 1500)$

* Vocabulary of inequalities *

Just be careful from bigger than and bigger than or equal to vocabulary and vice versa for smaller than

- More than = $> c$
- no smaller than / max / bigger or equal to = $\geq c$
- Smaller than = $< c$
- no more than / min / smaller or equal to = $\leq c$

* Graphing inequalities *

1) Draw the line as a normal linear equation (in $y = mx + c$ format)

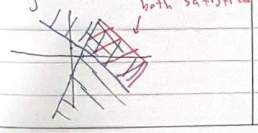
2) If it is $<$ or $>$ and NOT $\leq$ or $\geq$ then the line is dashed

3) If it is less than, then shade below the line, and above if it is more than

⇓

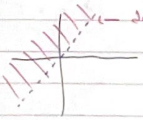
You could be given a system in which case the solution / shaded area is the area that satisfies both inequalities. It is usually the part that is shaded by both the individual inequalities:

- $y \leq 3x - 2$
- $y \geq -\frac{1}{3}x + \frac{4}{3}$



$>$, not $\geq$

$y > x$



Answers on the line are wrong