Let us assume a general quadratic equation in the form;

$$ax^2 + bx + c = 0$$

now for finding the factors of this equation through the " middle term splitting" method or the " factoring by grouping method"

let,

$$a \cdot c = p \cdot q \text{ - } (i)$$

$$b = p + q \text{ - - } (ii)$$

using a common Algebraic equation

$$(p - q)^2 = (p + q)^2 - 4pq$$

using (i),

$$(p - q)^2 = (b)^2 - 4pq$$

$$(p - q)^2 - (b^2 - 4ac) = 0$$

again using another common algebraic identity

$$\left((p-q) + \left(\sqrt{b^2 - 4ac}\right)\right)\left((p-q) - \left(\sqrt{b^2 - 4ac}\right)\right) \text{ ----}(iv)$$

Therefore the solution of the quadratic equation (iii) can be given using the equation (iv)

$$p - q = \pm\sqrt{b^2 - 4ac} \text{ -----}(v)$$

solving for (ii) and (v) by adding them we get;

$$2p = b + \sqrt{b^2 - 4ac}$$

$$p = \frac{b + \sqrt{b^2 - 4ac}}{2}$$

similarly ,solving for (ii) and (v) by subtracting we obtain the value of p as;

$$p = \frac{b - \sqrt{b^2 - 4ac}}{2}$$

we can write both the values of p in one expression as;

$$p = \frac{b \pm \sqrt{b^2 - 4ac}}{2}$$

similarly for solving for q we can subtract (v) from (ii) obtaining;

$$2q = b - \sqrt{b^2 - 4ac}$$

$$q = \frac{b - \sqrt{b^2 - 4ac}}{2}$$

by adding (v) and (ii)
we get;

$$q = \frac{b + \sqrt{b^2 - 4ac}}{2}$$

from all the solutions of 'p' and 'q' derived above we can group them into two pairs or sets of solutions

$$p = \frac{b + \sqrt{b^2 - 4ac}}{2}$$

and

$$q = \frac{b - \sqrt{b^2 - 4ac}}{2}$$

(or)

$$p = \frac{b - \sqrt{b^2 - 4ac}}{2}$$

and

$$q = \frac{b + \sqrt{b^2 - 4ac}}{2}$$

Therefore

The quadratic equation

$$ax^2 + bx + c = 0$$

Can be rewritten as;

$$ax^2 + (p + q)x + c = 0$$

$$ax^2 + px + qx + c = 0 \text{ ------}(vi)$$

This can now be solved as shown below;

Using (i) in (vi),

$$ax^2 + px + qx + \frac{pq}{a} = 0$$

grouping the common terms

$$ax\left(x+\frac{p}{a}\right)+q\left(x+\frac{p}{a}\right)=0$$

$$\left(x+\frac{p}{a}\right)(ax+q)=0$$

$$x=\frac{-p}{a} \qquad \text{(or)} \qquad x=\frac{-q}{a}$$

now substituting the values of p and q as derived we get;

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Hence the formula can even verify the quadratic formula

Conclusion:

Finally it can be conclucded that for splitting any middle term and grouping the following formula can be used:

$$p = \frac{b \pm \sqrt{b^2 - 4ac}}{2}$$

$$q = \frac{b \pm \sqrt{b^2 - 4ac}}{2}$$

inputting it back as:

$$ax^2 + px + qx + c = 0$$

note:
the set of values to be used are given in the derivation