Topological Space

A topological space is a set X, and a family $\mathcal{T}$ of subsets of X (the topology) that obey three axioms:

1. $\emptyset, X \in \mathcal{T}$.
2. If $A_1, \ldots, A_n \in \mathcal{T}$, then $A_1 \cap \cdots \cap A_n \in \mathcal{T}$.
3. If $\{A_\alpha\}_{\alpha \in I} \subset \mathcal{T}$, then $\bigcup_{\alpha \in I} A_\alpha \in \mathcal{T}$.

Base and Subbase

- A base of a topology $\mathcal{T}$ is a subset $\mathcal{B}$ of $\mathcal{T}$ so that any element of $\mathcal{T}$ is a union of sets of $\mathcal{B}$.
- A subbase of a topology is a subset $\mathcal{S}$, so that the family of finite intersections of sets in $\mathcal{S}$ is a base.

This definition is from an analytical point of view.
The topology already defined and $\mathcal{B}$ is a subset of $\mathcal{T}$.

- Suppose X is any set. A basis in X is a collection $\mathcal{B}$ of subsets of X satisfying the following conditions:
  1. Every element of X is in some elements of $\mathcal{B}$, in other words, $X = \bigcup_{B \in \mathcal{B}} B$.
  2. If $B_1, B_2 \in \mathcal{B}$ and $x \in B_1 \cap B_2$, there exists an element $B_3 \in \mathcal{B}$ such that $x \in B_3 \subset B_1 \cap B_2$.

This constructs a topology, as proven in the next proposition.

Recommended Reading

- **Lecture Notes** by Topologie - Andreas Kriegl (Match: 0.72)
- **Lecture Notes** by Topologie (2018) - Andreas Cap (Match: 0.71)
- **Topology** by James Munkres (Match: 0.71)