# Math Notes

**Definition:** Suppose $T \in \mathcal{L}(V, W)$. The dual map of $T$ is the linear map $T' \in \mathcal{L}(W', V')$ defined for each $\varphi \in W'$ by

$$T'(\varphi) = \varphi \circ T$$

**Two remarks:**

1. The image of $T'$ is a linear map from $V$ to $\mathbb{F}$ because $\varphi \circ T$ is a composition of two linear maps.
2. $T'$ itself is also linear:
   - For $\varphi, \psi \in W'$:

     $$T'(\varphi + \psi) = (\varphi + \psi) \circ T = \varphi \circ T + \psi \circ T = T'(\varphi) + T'(\psi)$$

   - For $\lambda \in \mathbb{F}$ and $\varphi \in W'$:

     $$T'(\lambda\varphi) = (\lambda\varphi) \circ T = \lambda(\varphi \circ T) = \lambda T'(\varphi)$$

# Examples

**Example 1:** Let $D : \mathcal{P}(\mathbb{R}) \to \mathcal{P}(\mathbb{R})$ be defined by $D(p) = p'$ (differentiation).
1) Suppose $\varphi$ is the linear functional on $\mathcal{P}(\mathbb{R})$ defined by $\varphi(p) = p(3)$.

# Recommended Reading

- **Tutorium Analysis 2 und Lineare Algebra 2** by Florian Modler, Martin Kreh (Match: 0.68)
- **Dualities and Representations of Lie Superalgebras** by Cheng Wang (Match: 0.68)
- **Lineare Funktionalanalysis Eine Anwendungsorientierte Einfhrung** by Hans Wilhelm Alt (Match: 0.67)