Vector Space Axiom Proofs and Function Spaces

February 11, 2026

) Show that in the definition of a vector space, the additive inverse condition can be replaced with the condition that: $\mathbf{0}v = \mathbf{0}$ for all $v \in V$.

With $\mathbf{0}v = \mathbf{0}$ for all $v \in V$ follows:

$$(1-1)v = 0v + (-1)v = 0 \quad \text{and therefore for}$$
$$w := (-1)v \quad \text{with} \quad v + w = \mathbf{0} \quad \text{for all } v \in V \text{ exists.}$$

) Let ∞ and $-\infty$ denote two distinct objects, neither of which is in $\mathbb{R}$. Define an addition and multiplication on $\mathbb{R} \cup \{\infty, -\infty\}$ as you could guess from the notation:

$$\mathbf{0} = \begin{cases} -\infty : & t < 0 \\ 0 : & t = 0 \\ \infty : & t > 0 \end{cases} \qquad t \cdot (-\infty) = \begin{cases} \infty : & t < 0 \\ 0 : & t = 0 \\ -\infty : & t > 0 \end{cases}$$

$$t + \infty = \infty + t = \infty$$
$$t + (-\infty) = (-\infty) + t = (-\infty) + (-\infty) = -\infty$$
$$\infty + (-\infty) = (-\infty) + \infty = 0$$
$$\infty + (\infty - \infty) = \infty \quad (\infty + \infty) - \infty = 0$$

so this addition is not associative.

) Suppose S is a nonempty set. Let V^S denote the set of functions from S to V. Define a natural addition and scalar multiplication on V^S and show that V^S is a vector space with these definitions:

$$f, g \in V^S : f : S \to V, \quad g : S \to V$$
$$(f+g)(x) := f(x) + g(x) \qquad (\lambda f)(x) := \lambda f(x)$$

Recommended Reading

- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.68)
- **Prüfungstraining Lineare Algebra : Band I** by Thomas C. T. Michaels (Match: 0.67)
- **Lineare Algebra 1** by Menny-Akka (Match: 0.67)