The Space of Bounded Functions

X is a set, $(E, \|\cdot\|)$ is a normed vector space. A function $u \in E^X$ is called bounded if the image of u in E is bounded. For $u \in E^X$ define:

$$\|u\|_\infty := \|u\|_{\infty,X} := \sup_{x \in X} \|u(x)\| \in R^+ \cup \{\infty\}$$

For $u \in E^X$ are equivalent:

* u is bounded. * $u(X)$ is bounded in E. * $\exists r > 0 : \|u(x)\| \leq r, \quad x \in X$. * $\|u\|_\infty < \infty$.

$$B(X, E) := \{u \in E^X : u \text{ is bounded}\}$$

is called the space of bounded functions from X to E.

Notes:

1. $B(X, E)$ is a subspace of E^X and $\|\cdot\|_\infty$ is a norm, called the supremum norm, on $B(X, E)$.

2. If $X = N$, then $B(X, E)$ is the normed vector space of bounded sequences in E. In the case $E = R$ or $E = C$:

$$l_\infty := l_\infty(C) := B(N, C)$$

In the normed vector space of bounded sequences with the supremum norm:

$$\|(x_n)\|_\infty = \sup_{n \in N} |x_n|$$

3. $c_0 \subseteq c \subseteq l_\infty$.

Recommended Reading

- **Lineare Funktionalanalysis Eine Anwendungsorientierte Einfhrung** by Hans Wilhelm Alt (Match: 0.70)
- **Linear Functional Analysis** by Alt Nuernberg (Match: 0.70)
- **Functional Analysis, Spectral Theory, and Applications** by Manfred Einsiedler, Thomas Ward (Match: 0.70)