Bounded Sets

Let $Y \subseteq X$, where X is a metric space with metric function d.

Y is called bounded in X if

$$\exists M > 0 : d(x, y) \leq M \text{ for all } x, y \in Y.$$

The diameter of Y, denoted $\operatorname{diam}(Y)$, is defined as:

$$\operatorname{diam}(Y) = \sup_{x,y \in Y} d(x, y),$$

and is finite.

Notes:

1. $\forall a \in X \wedge r > 0 : B(a, r)$ and $\overline{B}(a, r)$ are bounded.
2. $Y \subseteq X$ bounded $\Leftrightarrow \exists x_0 \in X \wedge r > 0 : Y \subseteq B_X(x_0, r)$.
3. $Y \subseteq R$ bounded $\Leftrightarrow \exists M > 0 : |y| \leq M : \forall y \in Y$.

Normed Vector Spaces

Let E be a vector space over R or C. A function $\|\cdot\| : E \to R^+$ is called a norm, if:

- N1 $\|x\| = 0 \Leftrightarrow x = 0$.
- N2 $\|\lambda x\| = |\lambda| \|x\|$, $x \in E, \lambda \in R/C$.
- N3 $\|x + y\| \leq \|x\| + \|y\|$, $x, y \in E$.

Notes:

1. $d : E \times E \to R^+$, $(x, y) \mapsto \|x - y\|$ is a metric on E.
2. The reversed triangle inequality: $\|x - y\| \geq |\|x\| - \|y\||$ holds.

Recommended Reading

- **Calculus On Normed Vector Spaces** by Rodney Coleman (Match: 0.71)
- **Lecture Notes** by Topologie - Andreas Kriegl (Match: 0.70)
- **Basic Analysis III** by James K. Peterson (Match: 0.70)