Problems

1) Prove that $-(-v) = v$ for every $v \in V$. With 1.32: $(-1)v = -v$

$$-(-v) = (-1)(-v) = (-1)((-1)v) = (-1)(-1)v = 1v = v$$

2) Suppose $\alpha \in F$, $v \in V$, and $\alpha v = 0$. Prove that $\alpha = 0$ or $v = 0$. If $\alpha \neq 0$, then there exists α^{-1} with $\alpha\alpha^{-1} = 1$,

$$\alpha^{-1}(\alpha v) = \alpha^{-1} \cdot 0(\alpha^{-1}\alpha)v = 01v = 0v = 0$$

If $v \neq 0$, then α has to be zero, otherwise it will contradict the previous part.

3) Suppose $v, w \in V$. Explain why there exists a unique $x \in V$ such that $v + 3x = w$. 1) For $(w - v) \cdot \frac{1}{3} = x$ the statement is true. 2) If x' is an other element of V with $x' \neq (w - v) \cdot \frac{1}{3}$ and

$$v + 3x' = w : \quad v + 3x' = v + 3x3x' = 3xx' = x$$

So it's because of $a = b \iff a + x = b + x$, and $a = b \iff x \cdot a = x \cdot b$ for $x \in F$ and $x \neq 0$.

4) The empty set is no vector space because of:

$$\exists x \in V : v + x = x \text{ for all } v \in V.$$

Recommended Reading

- **Linear algebra problem book** by Paul R. Halmos (Match: 0.69)
- **Prüfungstraining Lineare Algebra : Band I** by Thomas C. T. Michaels (Match: 0.68)
- **Lineare Algebra 1** by Menny-Akka (Match: 0.68)