# Span, Linear Independence, Linear Dependence

**Definition:** A linear combination of a list $v_1, \ldots, v_m$ of vectors in $V$ of the form $a_1v_1 + \cdots + a_mv_m$ with $a_1, \ldots, a_m \in \mathbb{F}$ is called a linear combination.

**Definition:** The set of all linear combinations of a list of vectors $v_1, \ldots, v_m$ in $V$ is called the span of $v_1, \ldots, v_m$, denoted by $\text{span}(v_1, \ldots, v_m)$. In other words, $\text{span}(v_1, \ldots, v_m) = \{a_1v_1 + \cdots + a_mv_m : a_1, \ldots, a_m \in \mathbb{F}\}$. The span of the empty list () is defined to be $\{0\}$.

**Definition:** If $\text{span}(v_1, \ldots, v_m) = V$, we say the list $v_1, \ldots, v_m$ spans $V$.

**Definition:** A vector space is called finite dimensional if some vectors in it span the space.

**Definition:** A vector space is called infinite dimensional if it is not finite dimensional.

**Definition:** A list $v_1, \ldots, v_m$ of vectors in $V$ is called linearly independent if the only choice of $a_1, \ldots, a_m \in \mathbb{F}$ that makes $a_1v_1 + \cdots + a_mv_m = 0$ is $a_1 = \cdots = a_m = 0$. The empty list () is also declared to be linearly independent.

**Definition:** A list of vectors in $V$ is called linearly dependent if it is not linearly independent.

In other words: a list $v_1, \ldots, v_m$ of vectors in $V$ is linearly dependent if there exist $a_1, \ldots, a_m \in \mathbb{F}$, not all 0, such that $a_1v_1 + \cdots + a_mv_m = 0$.

# Recommended Reading

- **Lineare Algebra 1** by Menny-Akka (Match: 0.70)
- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.70)
- **Linear Algebra** by Meckes Meckes (Match: 0.69)