Vector Space

Definition: Let V be a set and $(F, +, \cdot)$ a field. Define the operations:

$$+ : V \times V \to V$$
$$\cdot : F \times V \to V$$

The set V is a vector space over F if:

- $(V, +)$ is an abelian group.
- $\lambda \cdot (\mu \cdot w) = (\lambda \cdot \mu) \cdot w$ for all $\lambda, \mu \in F$ and $w \in V$.
- $\lambda \cdot (u + w) = \lambda u + \lambda w$ for all $\lambda \in F$ and $u, w \in V$.
- $(\lambda + \mu) \cdot w = \lambda w + \mu \cdot w$ for all $\lambda \in F$ and $u, w \in V$.
- $1 \cdot w = w$ for all $w \in V$, and 1 is the multiplicative identity in F.

Metric Space

Definition: A set X, whose elements we shall call points, is said to be a metric space if for any two points p and q of X are elements of the domain of a distance function $d(p, q) : X \times X \to R^+$ such that:

- $d(p, q) > 0$ if $p \neq q$; $d(p, p) = 0$.
- $d(p, q) = d(q, p)$.
- $d(p, q) + d(q, r) \leq d(p, r)$ for any $r \in X$.

Recommended Reading

- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.70)
- **Lineare Algebra 1** by Menny-Akka (Match: 0.70)
- **Tutorium Analysis 2 und Lineare Algebra 2** by Florian Modler, Martin Kreh (Match: 0.69)