16.) $\dim V = \dim \operatorname{range}(T) \leq \dim W$

$\dim V \leq \dim W$: There exists a basis B_V of V and B_W a basis of W, with $|B_V| \leq |B_W|$, define $T(b_{i_V}) = b_{i_W}$ for all $b_{i_V} \in B_V$, where $b_{i_W} \in B_W$.
T is injective:

$$T(v) = 0 \implies v = 0$$

Let $v = a_1 b_{1V} + \cdots + a_m b_{mV}$.

$$T(a_1 b_{1V} + \cdots + a_m b_{mV}) = a_1 b_{1W} + \cdots + a_m b_{mW} = 0$$

Since $b_{1W}, \ldots, b_{mW}$ are linearly independent, so $a_1 = \cdots = a_m = 0$ and $v = 0$, and $\dim \operatorname{null}(T) = 0$ and T injective.

17.) Suppose V and W are both finite-dimensional. Prove that there exists a surjective linear map from V onto W if and only if $\dim V \geq \dim W$.
$\Rightarrow$: $\dim V = \dim \operatorname{null} T + \dim \operatorname{range} T = \dim \operatorname{null} T + \dim W$, and since $\dim \operatorname{null} T \geq 0$, follow $\dim V \geq \dim W$.
$\Leftarrow$: Same idea as in 16.)

Recommended Reading

- **Linear algebra problem book** by Paul R. Halmos (Match: 0.68)
- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.68)
- **Lineare Algebra 1** by Menny-Akka (Match: 0.67)