Connectedness

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Definition.: If X is a topological space, a separation of X is a pair of nonempty, disjoint open subsets $U, V \subset X$ such that $U \cup V = X$.

X is disconnected if there exists a separation of X, and connected otherwise.

Examples:

1. X is the union of the two disjoint closed disks $\overline{B_1(2,0)}$ and $\overline{B_1(-2,0)}$ in $\mathbb{R}^2$. Each of the disks is open in X, so the pair of disks is a separation of X.

2. $Y = \mathbb{R} \setminus \{0\}$, the two sets $(-\infty, 0)$ and $(0, \infty)$ separate Y. ($(-\infty, 0)$ is short for $\{x \in Y \mid x < 0\}$.)

3. $Z =$ the set of points with rational coordinates. A separation is given by $U = \{(x, y) \mid x < \pi\}$ and $V = \{(x, y) \mid x > \pi\}$.

Recommended Reading

- **Topology** by James Munkres (Match: 0.69)
- **Lecture Notes** by Topologie (2018) - Andreas Cap (Match: 0.68)
- **Lecture Notes** by Topologie - Andreas Kriegl (Match: 0.68)