Subspaces, Sum of Subspaces and Direct Sums

Definition: A subset M of V is called a subspace of V if M is also a vector space with the same additive identity, addition, and scalar multiplication as on V.

Definition: Suppose $V_1, \dots, V_m$ are subspaces of V. The sum of $V_1, \dots, V_m$, denoted by $V_1 + \cdots + V_m$ is the set of all possible sums of elements of $V_1, \dots, V_m$. More precisely,

$$V_1 + \cdots + V_m = \{v_1 + \cdots + v_m : v_1 \in V_1, \dots, v_m \in V_m\}$$

Definition: Suppose $V_1, \dots, V_m$ are subspaces of V.

- The sum $V_1 + \cdots + V_m$ is called a direct sum if each element of $V_1 + \cdots + V_m$ can be written in only one way as a sum $v_1 + \cdots + v_m$, where each $v_k \in V_k$.
- If $V_1 + \cdots + V_m$ is a direct sum, then $V_1 \oplus \cdots \oplus V_m$ denotes $V_1 + \cdots + V_m$, with the $\oplus$ notation serving as an indication that this is a direct sum.

Recommended Reading

- **Lineare Algebra 1** by Menny-Akka (Match: 0.70)
- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.69)
- **Prüfungstraining Lineare Algebra : Band I** by Thomas C. T. Michaels (Match: 0.69)