Banach space and Hilbert Space

- A sequence (x_n) in X is called a Cauchy sequence if, for each $\varepsilon > 0$, there exists a $N \in N$ such that $d(x_n, x_m) < \varepsilon$ for all $m, n \geq N$.
- A metric space X is called complete if every Cauchy sequence in X converges.
- A complete normed vector space is called Banach space.
- A complete inner product space is called a Hilbert space.

Notes:

- Every convergent sequence is a Cauchy sequence.
- Every Cauchy sequence is bounded.
- If a Cauchy sequence has a convergent subsequence, then it is itself convergent.
- R^m and C^m are Banach spaces.
- Let X be a nonempty set and $E = (E, ||\cdot||)$ a Banach space. Then $B(X, E)$ is also a Banach space.

Recommended Reading

- **Calculus On Normed Vector Spaces** by Rodney Coleman (Match: 0.71)
- **Analysis in Banach Spaces : Volume I** by Tuomas Hytönen, Jan van Neerven, Mark Veraar, Lutz Weis (Match: 0.70)
- **Analysis in Banach Spaces : Volume II** by Tuomas Hytönen, Jan van Neerven, Mark Veraar, Lutz Weis (Match: 0.70)