Theorem: In a finite-dimensional vector space, the length of every linearly independent list of vectors is less than or equal to the length of every spanning list of vectors. (A, page 35)

Theorem: Every subspace of a finite-dimensional vector space is finite-dimensional. (A, page 36)

Bases and Dimension

Definition: A basis is a list of vectors in V that is linearly independent and spans V.

Theorem: A list $v_1, \ldots, v_n$ of vectors is a basis of V if and only if every $v \in V$ can be written uniquely in the form

$$V = a_1 v_1 + \cdots + a_n v_n, \quad a_1, \ldots, a_n \in F$$

(A, page 39)

Theorem: Every spanning list in a vector space can be reduced to a basis of the vector space. (A, page 40)

Theorem: Every finite-dimensional vector space has a basis. (A, page 41)

Theorem: Every linearly independent list of vectors in a finite-dimensional vector space can be extended to a basis of the vector space. (A, page 41)

Theorem: Suppose V is finite dimensional and U is a subspace of V. Then there is a subspace W of V such that $V = U \oplus W$.

Recommended Reading

- **Lineare Algebra 1** by Menny-Akka (Match: 0.72)
- **Groups, Matrices, and Vector Spaces** by James B. Carrell (Match: 0.71)
- **Prüfungstraining Lineare Algebra : Band I** by Thomas C. T. Michaels (Match: 0.71)