1. Zachary's Karate Club

Calculate relevant centralities for the nodes in the Karate Club network. Partition the network into clusters*. Visualize the network so that the important nodes are highlighted#. Do you find two partitions centered on the administrator (node 34) and the instructor (1)? Node 9 was the only one that Zachary's analysis failed to properly place. Does your analysis say that node 9 should stay with the administrator or the instructor? The original article notes that number 9 was close to receiving his/her blackbelt and suggests that this might have influenced his/her decision.

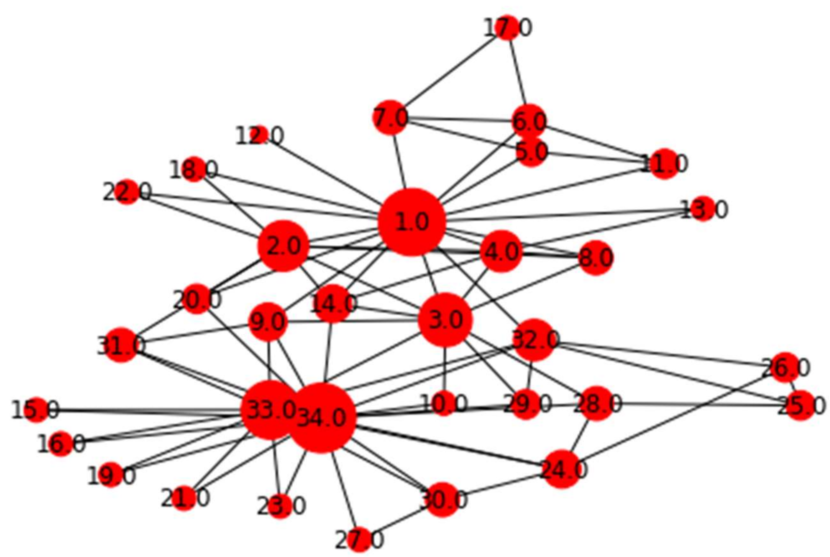
* See *kernighan_lin_bisection* or *girvan_newman*

For examples, see https://networkx.github.io/documentation/latest/auto_examples/index.html

You might try using *draw_kamada_kawai* or *spring_layout*

Figure 1 shows a graph that represent the Karate Club network. The size of the nodes corresponds to the degree of the node. Hence, the bigger the node the more important and influential the node is. As shown, nodes 34 (degree=17) and node 1 (degree=16) are shown to be more important than the rest. Node 33 (degree=12) comes in third place.

Figure 1. Graphical Representation of the Karate Club network.



The following presents the relevant centralities for the nodes in the Karate Club network. Nodes are sorted by each centrality metric in descending order. The administrator (node 34) of the Karate Club is the highest in degree, eigenvector and page rank centrality metrics, second in betweenness and third in closeness centrality metrics. The instructor (node 1) ranks highest in betweenness and closeness centrality metrics, and comes second in degree, eigenvector and page rank centrality metrics. The administrator and the instructor are influential and important in the club in different ways, depending on the centrality metric being measured.

```
Nodes sorted by degree centrality:
('34.0', 0.5151515151515151)
('1.0', 0.48484848484848486)
('33.0', 0.36363636363636365)
('3.0', 0.30303030303030304)
('2.0', 0.2727272727272727)
('4.0', 0.18181818181818182)
('32.0', 0.18181818181818182)
('9.0', 0.15151515151515152)
('14.0', 0.15151515151515152)
('24.0', 0.15151515151515152)
('6.0', 0.12121212121212122)
('7.0', 0.12121212121212122)
('8.0', 0.12121212121212122)
('28.0', 0.12121212121212122)
('30.0', 0.12121212121212122)
('31.0', 0.12121212121212122)
('5.0', 0.09090909090909091)
('11.0', 0.09090909090909091)
('20.0', 0.09090909090909091)
('25.0', 0.09090909090909091)
('26.0', 0.09090909090909091)
('29.0', 0.09090909090909091)
('10.0', 0.06060606060606061)
('13.0', 0.06060606060606061)
('15.0', 0.06060606060606061)
('16.0', 0.06060606060606061)
('17.0', 0.06060606060606061)
('18.0', 0.06060606060606061)
('19.0', 0.06060606060606061)
('21.0', 0.06060606060606061)
('22.0', 0.06060606060606061)
('23.0', 0.06060606060606061)
('27.0', 0.06060606060606061)
('12.0', 0.030303030303030304)

Nodes sorted by betweenness centrality:
('1.0', 0.43763528138528146)
('34.0', 0.30407497594997596)
('33.0', 0.145247113997114)
('3.0', 0.14365680615680618)
('32.0', 0.13827561327561325)
('9.0', 0.05592682780182781)
('2.0', 0.053936688311688304)
('14.0', 0.04586339586339586)
('20.0', 0.03247504810004811)
('6.0', 0.02998737373737374)
('7.0', 0.029987373737373736)
('28.0', 0.02233345358345358)
('24.0', 0.017613636363636363)
('31.0', 0.014411976911976909)
('4.0', 0.011909271284271283)
('26.0', 0.003840488215488215)
('30.0', 0.0029220779220779218)
('25.0', 0.0022095959595959595)
('29.0', 0.0017947330447330447)
('10.0', 0.0008477633477633478)
('5.0', 0.0006313131313131313)
('11.0', 0.0006313131313131313)
('8.0', 0.0)
('12.0', 0.0)
('13.0', 0.0)
('15.0', 0.0)
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('16.0', 0.0)
('17.0', 0.0)
('18.0', 0.0)
('19.0', 0.0)
('21.0', 0.0)
('22.0', 0.0)
('23.0', 0.0)
('27.0', 0.0)

Nodes sorted by closeness centrality:
('1.0', 0.5689655172413793)
('3.0', 0.559322033898305)
('34.0', 0.55)
('32.0', 0.5409836065573771)
('9.0', 0.515625)
('14.0', 0.515625)
('33.0', 0.515625)
('20.0', 0.5)
('2.0', 0.4852941176470588)
('4.0', 0.4647887323943662)
('28.0', 0.4583333333333333)
('31.0', 0.4583333333333333)
('29.0', 0.4520547945205479)
('8.0', 0.44)
('10.0', 0.4342105263157895)
('24.0', 0.39285714285714285)
('6.0', 0.38372093023255816)
('7.0', 0.38372093023255816)
('30.0', 0.38372093023255816)
('5.0', 0.3793103448275862)
('11.0', 0.3793103448275862)
('18.0', 0.375)
('22.0', 0.375)
('25.0', 0.375)
('26.0', 0.375)
('13.0', 0.3707865168539326)
('15.0', 0.3707865168539326)
('16.0', 0.3707865168539326)
('19.0', 0.3707865168539326)
('21.0', 0.3707865168539326)
('23.0', 0.3707865168539326)
('12.0', 0.36666666666666664)
('27.0', 0.3626373626373626)
('17.0', 0.28448275862068967)

Nodes sorted by eigenvector centrality:
('34.0', 0.373371213013235)
('1.0', 0.3554834941851943)
('3.0', 0.31718938996844476)
('33.0', 0.3086510477336959)
('2.0', 0.2659538704545025)
('9.0', 0.2274050914716605)
('14.0', 0.22646969838808148)
('4.0', 0.2111740783205706)
('32.0', 0.19103626979791702)
('31.0', 0.17476027834493085)
('8.0', 0.17095511498035434)
('24.0', 0.15012328691726787)
('20.0', 0.14791134007618667)
('30.0', 0.13496528673866567)
('28.0', 0.13347932684333308)
('29.0', 0.13107925627221215)
('10.0', 0.10267519030637758)
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('15.0', 0.10140627846270832)
('16.0', 0.10140627846270832)
('19.0', 0.10140627846270832)
('21.0', 0.10140627846270832)
('23.0', 0.10140627846270832)
('18.0', 0.09239675666845953)
('22.0', 0.09239675666845953)
('13.0', 0.08425192086558088)
('6.0', 0.07948057788594247)
('7.0', 0.07948057788594247)
('5.0', 0.07596645881657382)
('11.0', 0.07596645881657381)
('27.0', 0.07558192219009324)
('26.0', 0.05920820250279008)
('25.0', 0.05705373563802805)
('12.0', 0.05285416945233648)
('17.0', 0.023634794260596875)

Nodes sorted by page rank:
('34.0', 0.1009179167487121)
('1.0', 0.09700181758983709)
('33.0', 0.0716921300658829)
('3.0', 0.057078423047636745)
('2.0', 0.05287839103742701)
('32.0', 0.03715663592267942)
('4.0', 0.03586064322306479)
('24.0', 0.03152091531163228)
('9.0', 0.029765339186167028)
('14.0', 0.029536314977202986)
('6.0', 0.02911334166344221)
('7.0', 0.02911334166344221)
('30.0', 0.02628726283711208)
('28.0', 0.025638803528350497)
('31.0', 0.02458933653429248)
('8.0', 0.024490758039509182)
('5.0', 0.021979406974834498)
('11.0', 0.021979406974834498)
('25.0', 0.021075455001162945)
('26.0', 0.021005628174745786)
('20.0', 0.019604416711937293)
('29.0', 0.01957296050943854)
('17.0', 0.016785378110253487)
('27.0', 0.015043395360629753)
('13.0', 0.014645186487916191)
('18.0', 0.014558859774243493)
('22.0', 0.014558859774243493)
('15.0', 0.014535161524273825)
('16.0', 0.014535161524273825)
('19.0', 0.014535161524273825)
('21.0', 0.014535161524273825)
('23.0', 0.014535161524273825)
('10.0', 0.014308950284462801)
('12.0', 0.009564916863537148)
```

The Karate Club network was partitioned into two clusters two method: the Kernighan_lin_bisection[1] and the *girvan_newman* algorithms.[2] Both algorithms resulted in two clusters with the administrator (node 34) and the instructor (node 1) as the center of each cluster. In both cases, node 9 stays in the partitioned cluster with administrator (node 34).

Figures 2 and 3 below presents the clusters resulted from Kernighan_lin_bisection algorithm. These clusters have the same number of nodes because the algorithm used is to partition the set of nodes into tow disjoint subsets. Figures 4 and 5 below presents the partitions resulted from Girvan Newman algorithm. With this algorithm, the resulted cluster with the administrator (node 34) is slighter bigger than the one with the instructor (node 1).

The original article shows that node 9 has a stronger relationship with node 34 (the administrator/John A.) than with node 1 (the instructor/Mr. Hi). See Figure 3 "Quantified Matrix of Relative Strengths of the Relationships in the Karate Club: The Matrix C" on page 462. My analysis using either Kernighan_lin_bisection or the *girvan_newman* algorithms on the provided, unweighted network data shows that node 9 stays with node 34. This agrees with run result from the *NETFLOW* presented in the original article (page 465). The NETFLOW results show that node 9 sides with the Sink (the administrator/node 34/John A.). However according to the original report, node 9 joined the instructor's (Mr. Hi's/node 1's) club after the Karate Club split. So, all three algorithms failed to correctly predict node 9's decision. The original article notes that node 9 was close to receiving his/her blackbelt and suggests that this might have influenced his/her decision. It makes sense that node 9 chose to stay with the same instructor till he got his blackbelt and joint Mr. Hi's private karate studio after karate club split. Had this factor been included as one of the contexts that add weight to the connection between node 9 and instructor (node 1), the result from *NETFLOW* might have been different.

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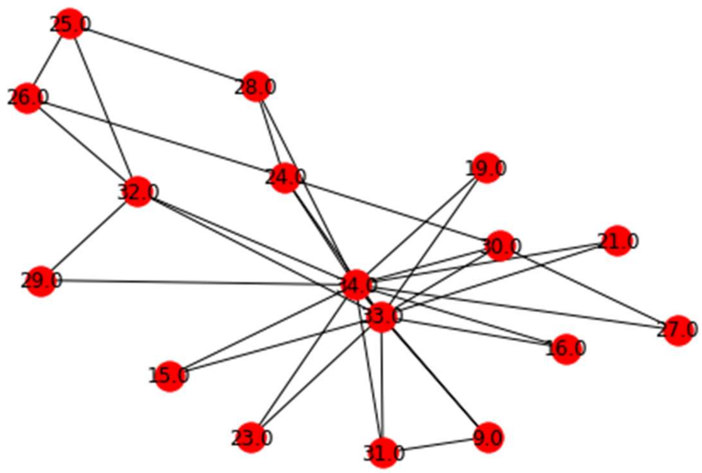
1 The input to the algorithm is an undirected graph $G = (V,E)$ with vertex set V, edge set E, and (optionally) numerical weights on the edges in E. The goal of the algorithm is to partition V into two disjoint subsets A and B of equal (or nearly equal) size, in a way that minimizes the sum T of the weights of the subset of edges that cross from A to B. If the graph is unweighted, then instead the goal is to minimize the number of crossing edges; this is equivalent to assigning weight one to each edge. (Source: Wikipedia.org; link: https://en.wikipedia.org/wiki/Kernighan%E2%80%93Lin_algorithm)

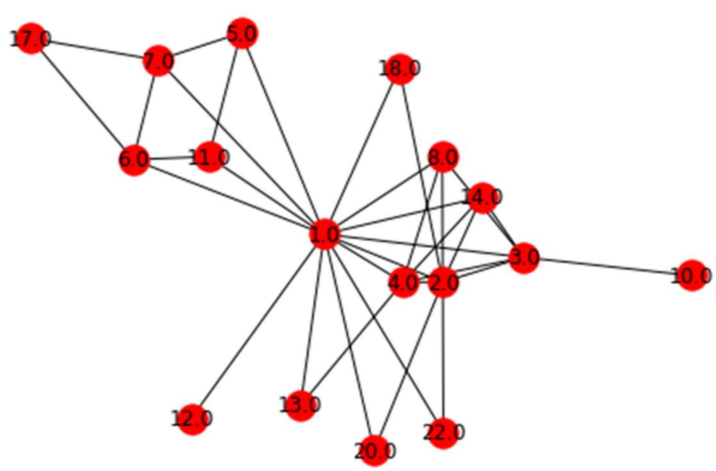
2 The Girvan–Newman algorithm detects communities by progressively removing edges from the original network. The connected components of the remaining network are the communities. Instead of trying to construct a measure that tells us which edges are the most central to communities, the Girvan–Newman algorithm focuses on edges that are most likely "between" communities. (Source: Wikipedia.org; link: https://en.wikipedia.org/wiki/Girvan%E2%80%93Newman_algorithm)

Figure 2. Bisected Cluster Containing Node 34, the Administrator and Node 9.



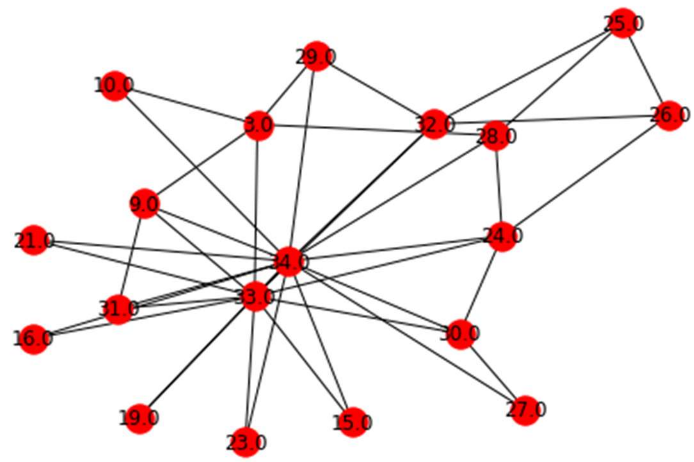

```
set1 = {'16.0', '19.0', '31.0', '23.0', '34.0', '24.0', '25.0', '33.0', '26.0',
'21.0', '30.0', '27.0', '9.0', '28.0', '15.0', '29.0', '32.0'}
Name:
Type: SubGraph
Number of nodes: 17
Number of edges: 34
Average degree:   4.0000
```

Figure 3. Bisected Cluster Containing Node 1, the Instructor.



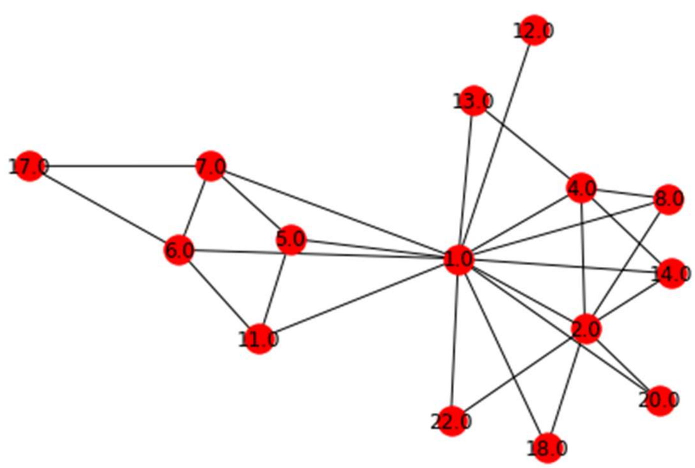

```
Set2 = {'22.0', '1.0', '7.0', '2.0', '4.0', '10.0', '11.0', '20.0', '8.0',
'13.0', '14.0', '5.0', '18.0', '12.0', '6.0', '17.0', '3.0'}
Name:
Type: SubGraph
Number of nodes: 17
Number of edges: 34
Average degree:   4.0000
```

Figure 4. Girvan Newman Algorithm:
Cluster Containing Node 34, the Administrator and Node 9




```
Set1 = {'27.0', '16.0', '30.0', '9.0', '28.0', '19.0', '15.0', '29.0', '31.0',
'23.0', '34.0', '33.0', '32.0', '24.0', '25.0', '26.0', '10.0', '21.0', '3.0'}
Name:
Type: SubGraph
Number of nodes: 19
Number of edges: 40
Average degree:   4.2105
```

Figure 5. Girvan Newman Algorithm: Cluster Containing Node 1, the Instructor.




```
Set2 = {'22.0', '20.0', '11.0', '8.0', '1.0', '13.0', '7.0', '14.0', '5.0',
'2.0', '12.0', '18.0', '6.0', '4.0', '17.0'}
Name:
Type: SubGraph
Number of nodes: 15
Number of edges: 28
Average degree:   3.7333
```