Optimal piecewise constant vaccination and lockdown

policies for COVID-19,

June 11, 2022

Toy example and classic vaccination OC

To fix ideas:

$$\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
&S(0) = S_0, I(0) = I_0, R(0) = 0 \\
&S(t) + I(t) + R(t) = 1
\end{aligned}$$

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
& S(0) = S_0, I(0) = I_0, R(0) = 0 \\
& S(t) + I(t) + R(t) = 1
\end{aligned}
$$

With vaccination

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(t) \\
& S(0) = S_0, I(0) = I_0, \\
& R(0) = 0, V(0) = 0 \\
& S(t) + I(t) + R(t) + V(t) = 1
\end{aligned}
$$

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
& S(0) = S_0, I(0) = I_0, R(0) = 0 \\
& S(t) + I(t) + R(t) = 1
\end{aligned}
$$

“Classic” Vaccination

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(x,t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(x,t) \\
& S(0) = S_0, I(0) = I_0, \\
& R(0) = 0, V(0) = 0 \\
& S(t) + I(t) + R(t) + V(t) = 1 \\
& x^\top = (S, I, R, V)
\end{aligned}
$$

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
& S(0) = S_0, I(0) = I_0, R(0) = 0 \\
& S(t) + I(t) + R(t) = 1
\end{aligned}
$$

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(x,t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(x,t) \\
& S(0) = S_0, I(0) = I_0, \\
& R(0) = 0, V(0) = 0 \\
& S(t) + I(t) + R(t) + V(t) = 1 \\
& x^\top = (S, I, R, V)
\end{aligned}
$$

“Classic” Vaccination

Gumel,

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
&S(0) = S_0, I(0) = I_0, R(0) = 0 \\
&S(t) + I(t) + R(t) = 1
\end{aligned}
$$

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(x,t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(x,t) \\
&S(0) = S_0, I(0) = I_0, \\
&R(0) = 0, V(0) = 0 \\
&S(t) + I(t) + R(t) + V(t) = 1 \\
&x^\top = (S, I, R, V)
\end{aligned}
$$

“Classic” Vaccination

Gumel,

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
& S(0) = S_0, I(0) = I_0, R(0) = 0 \\
& S(t) + I(t) + R(t) = 1
\end{aligned}
$$

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(x,t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(x,t) \\
& S(0) = S_0, I(0) = I_0, \\
& R(0) = 0, V(0) = 0 \\
& S(t) + I(t) + R(t) + V(t) = 1 \\
& x^\top = (S, I, R, V)
\end{aligned}
$$

“Classic” Vaccination

Gumel,

$$\lambda_V := \underbrace{\xi}_{cte.} \cdot S(t)$$

Toy example and classic vaccination OC

To fix ideas:

$$
\begin{aligned}
S'(t) &= -\beta IS \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
&S(0) = S_0, I(0) = I_0, R(0) = 0 \\
&S(t) + I(t) + R(t) = 1
\end{aligned}
$$

$$
\begin{aligned}
S'(t) &= -\beta IS - \lambda_V(x,t) \\
I'(t) &= \beta IS - \gamma I \\
R'(t) &= \gamma I \\
V'(t) &= \lambda_V(x,t) \\
&S(0) = S_0, I(0) = I_0, \\
&R(0) = 0, V(0) = 0 \\
&S(t) + I(t) + R(t) + V(t) = 1 \\
&x^\top = (S, I, R, V)
\end{aligned}
$$

“Classic” Vaccination

Gumel,

$$\lambda_V := \underbrace{\xi}_{cte.} \cdot S(t)$$

Optimal Controlled:

The Basic Optimization Question

Mexico Section

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

The Basic Optimization Question

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

SIAM Mexico Section

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

$$\begin{aligned} X_{cov} &= X(T) \\ &\approx 1 - \exp(-\lambda_V T). \end{aligned}$$

The Basic Optimization Question

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

$$\begin{aligned} X_{cov} &= X(T) \\ &\approx 1 - \exp(-\lambda_V T). \end{aligned}$$

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T} \ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

$$\begin{aligned} X_{cov} &= X(T) \\ &\approx 1 - \exp(-\lambda_V T). \end{aligned}$$

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T}\ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: **one** year

$\lambda_V \approx 0.00329$

SIAM Mexico Section

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

$$\begin{aligned} X_{cov} &= X(T) \\ &\approx 1 - \exp(-\lambda_V T). \end{aligned}$$

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T} \ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: one year

$\lambda_V \approx 0.003\,29$

If $S(0)N$ corresponds to HMS $(812\,229\,\text{inhabitants})$ $\approx 2668\,\text{jabs}/\text{day}$.

The Basic Optimization Question

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T} \ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: **one** year

$\lambda_V \approx 0.00329$

How to optimize vaccination?

Mexico Section

Hypothesis

Cost The **effort** expended in “**preventing-mitigating** an epidemic” by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

Common Objectives

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T} \ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: one year

$\lambda_V \approx 0.00329$

How to optimize vaccination?

The Basic Optimization Question

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

Common Objectives

Who to vaccine first? (Allocation)

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T}\ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: one year

$\lambda_V \approx 0.00329$

How to optimize vaccination?

The Basic Optimization Question

siam Mexico Section

Hypothesis

Cost The **effort** expended in "**preventing-mitigating** an epidemic" by vaccination is **proportional** to the vaccination rate λ_V.

Jabs Counter If $S(0) \approx 1$, $X(\cdot)$: counts vaccine doses, then

$$X(t) = 1 - \exp(-\lambda_V t),$$

estimates the fraction of vaccinated individuals. Thus, for time horizon T and vaccination coverage

Common Objectives

Who to vaccine first? (Allocation)

How and when? (Administration)

Given X_{cov}, T

$$\lambda_V = -\frac{1}{T}\ln(1 - X_{cov})$$

estimates the constant vaccination rate s.t., afther time T, we reach X_{cov}.

X_{COV} : 70%, T: one year

$\lambda_V \approx 0.00329$

How to optimize vaccination?

Common Objectives

- ⋆ Who to vaccine first? (Allocation)

Vaccine optimiztion for COVID-19

Common Objectives

- Who to vaccine first? (Allocation)
- How and when? (Administration)

Cost

Vaccine optimiztion for COVID-19

Common Objectives

- Who to vaccine first? (Allocation)
- How and when? (Administration)
- Vaccination jointly with NPIs (Administration)

Cost

$$J(u) := \varphi(x(T)) + \int_0^T f(t, x(t), u(t))$$

Vaccine optimiztion for COVID-19

Common Objectives

- ⋆ Who to vaccine first? (Allocation)
- ⋆ How and when? (Administration)
- ⋆ Vaccination jointly with NPIs (Administration)

Optimal Control Problem

$$
\begin{aligned}
\min_{\mathbf{u}\in\mathscr{U}} J(u) &= \varphi(x(T)) + \int_0^T f(t, x(t), u(t)) \\
\dot{x}(t) &= b(t, u(t), x(t)), \qquad \text{a.e. } t \in [0, T], \\
x(0) &= x_0
\end{aligned}
$$

Vaccine optimiztion for COVID-19

Common Objectives

- ⋆ Who to vaccine first? (Allocation)
- ⋆ How and when? (Administration)
- ⋆ Vaccination jointly with NPIs (Administration)

Optimal Control Problem

$$
\begin{aligned}
\min_{\mathbf{u}\in\mathscr{U}} J(u) =& \varphi(x(T)) + \int_0^T f(t, x(t), u(t)) \\
\dot{x}(t) =& b(t, u(t), x(t)), \qquad \text{a.e. } t \in [0, T], \\
x(0) =& x_0
\end{aligned}
$$

Vaccine optimiztion for COVID-19

Common Objectives

- ⋆ Who to vaccine first? (Allocation)
- ⋆ How and when? (Administration)
- ⋆ Vaccination jointly with NPIs (Administration)

Optimal Control Problem

$$
\begin{aligned}
\min_{\mathbf{u}\in\mathscr{U}} J(u) &= \varphi(x(T)) + \int_0^T f(t, x(t), u(t)) \\
\dot{x}(t) &= b(t, u(t), x(t)), \qquad \text{a.e. } t \in [0, T], \\
x(0) &= x_0
\end{aligned}
$$

Vaccine optimiztion for COVID-19

SIAM Mexico Section

Common Objectives

- ⋆ Who to vaccine first? (Allocation)
- ⋆ How and when? **(Administration)**
- ⋆ Vaccination jointly with NPIs (Administration)

Optimal Control Problem

$$\begin{aligned} \min_{\mathbf{u}\in\mathscr{U}} J(u) =& \varphi(x(T)) + \int_0^T f(t, x(t), u(t)) \\ \dot{x}(t) =& b(t, u(t), x(t)), \qquad \text{a.e. } t \in [0, T], \\ x(0) =& x_0 \end{aligned}$$

Vaccine optimiztion for COVID-19

Common Objectives

- ⋆ Who to vaccine first? (Allocation)
- ⋆ How and when? (Administration)
- ⋆ **Vaccination jointly with NPIs (Administration)**

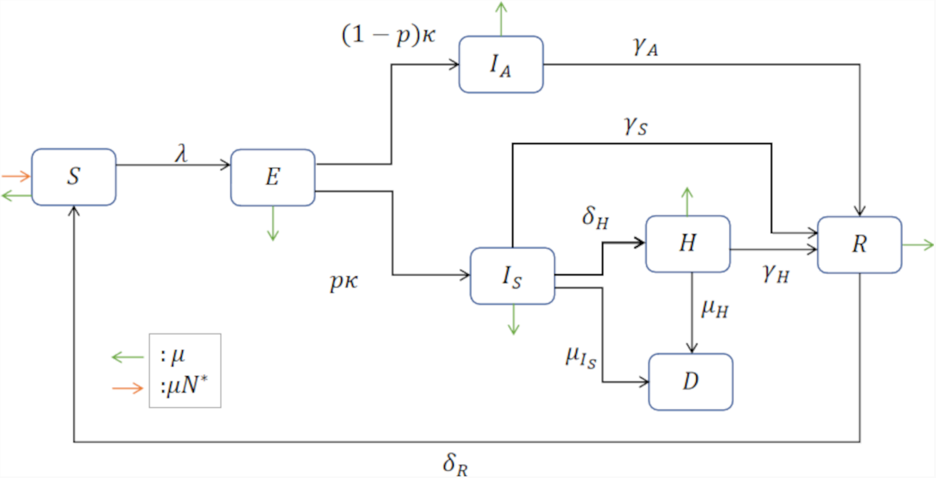
Optimal Control Problem

$$\min_{\mathbf{u}\in\mathscr{U}} J(u) = \varphi(x(T)) + \int_0^T f(t, x(t), u(t))$$
$$\dot{x}(t) = b(t, u(t), x(t)), \qquad \text{a.e. } t \in [0, T],$$
$$x(0) = x_0$$

Aim of this talk

To illustrate the formulation of optimal Lockdown-vaccination policies based in piecewise constant poicies.

The Model Without Vaccination



$$\lambda := \frac{\beta_A I_A + \beta_S I_S}{N^\star},$$

$$N^\star(t) = L + S + E + I_S + I_A + H + R.$$

Model Scheme

$$
\begin{aligned}
L' &= \theta\mu N^\star - (\varepsilon\lambda + \delta_L + \lambda_V + \mu)L \\
S' &= (1-\theta)\mu N^\star + \delta_L L + \delta_V V + \delta_R R \\
&\quad - (\lambda + \lambda_V + \mu)\, S \\
E' &= \lambda(\varepsilon L + (1-\varepsilon)V + S) - (\kappa + \mu)E \\
I_S' &= p\kappa E - (\delta_H + \gamma_S + \mu_{I_S} + \mu)I_S \\
I_A' &= (1-p)\kappa E - (\gamma_A + \mu)I_A \\
H' &= \delta_H I_S - (\gamma_H + \mu_H + \mu)H \\
R' &= \gamma_S I_S + \gamma_A I_A + \gamma_H H - (\delta_R + \mu)R \\
D' &= \mu_{I_S} I_S + \mu_H H \\
V' &= \lambda_V(S + L) - [(1-\varepsilon)\lambda + \delta_V + \mu]\, V \\
\lambda &:= \frac{\beta_A I_A + \beta_S I_S}{N^\star} \\
N^\star(t) &= L + S + E + I_S + I_A + H + R + V.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dX_{vac}}{dt} &= \lambda_V\, [L + S + E + I_A + R] \\
\frac{dY_{I_S}}{dt} &= p\kappa E \\
L(0) &= L_0,\ S(0) = S_0,\ E(0) = E_0, \\
I_S(0) &= I_{S_0}, I_A(0) = I_{A_0}, H(0) = H_0, \\
R(0) &= R_0,\ D(0) = D_0,\ V(0) = 0, \\
X_{vac}(0) &= 0, \quad X_{vac}(T) = x_{coverage},
\end{aligned}
$$

Model Scheme

$$
\begin{aligned}
L' &= \theta\mu N^{\star} - (\varepsilon\lambda + \delta_L + \lambda_V + \mu)L \\
S' &= (1-\theta)\mu N^{\star} + \delta_L L + \delta_V V + \delta_R R \\
&\quad - (\lambda + \lambda_V + \mu)\, S \\
E' &= \lambda(\varepsilon L + (1-\varepsilon)V + S) - (\kappa + \mu)E \\
I_S' &= p\kappa E - (\delta_H + \gamma_S + \mu_{I_S} + \mu)I_S \\
I_A' &= (1-p)\kappa E - (\gamma_A + \mu)I_A \\
H' &= \delta_H I_S - (\gamma_H + \mu_H + \mu)H \\
R' &= \gamma_S I_S + \gamma_A I_A + \gamma_H H - (\delta_R + \mu)R \\
D' &= \mu_{I_S} I_S + \mu_H H \\
V' &= \lambda_V(S + L) - [(1-\varepsilon)\lambda + \delta_V + \mu]\, V \\
\lambda &:= \frac{\beta_A I_A + \beta_S I_S}{N^{\star}} \\
N^{\star}(t) &= L + S + E + I_S + I_A + H + R + V.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dX_{vac}}{dt} &= \lambda_V\,[L + S + E + I_A + R] \\
\frac{dY_{I_S}}{dt} &= p\kappa E \\
L(0) &= L_0,\ S(0) = S_0,\ E(0) = E_0, \\
I_S(0) &= I_{S_0}, I_A(0) = I_{A_0}, H(0) = H_0, \\
R(0) &= R_0,\ D(0) = D_0,\ V(0) = 0, \\
X_{vac}(0) &= 0, \quad X_{vac}(T) = x_{coverage},
\end{aligned}
$$

Model Scheme

$$
\begin{aligned}
L' &= \theta \mu N^{\star} - (\varepsilon\lambda + \delta_L + \lambda_V + \mu) L \\
S' &= (1-\theta)\mu N^{\star} + \delta_L L + \delta_V V + \delta_R R \\
&\quad - (\lambda + \lambda_V + \mu) S \\
E' &= \lambda(\varepsilon L + (1-\varepsilon)V + S) - (\kappa + \mu) E \\
I_S' &= p\kappa E - (\delta_H + \gamma_S + \mu_{I_S} + \mu) I_S \\
I_A' &= (1-p)\kappa E - (\gamma_A + \mu) I_A \\
H' &= \delta_H I_S - (\gamma_H + \mu_H + \mu) H \\
R' &= \gamma_S I_S + \gamma_A I_A + \gamma_H H - (\delta_R + \mu) R \\
D' &= \mu_{I_S} I_S + \mu_H H \\
V' &= \lambda_V (S + L) - [(1-\varepsilon)\lambda + \delta_V + \mu] V \\
\lambda &:= \frac{\beta_A I_A + \beta_S I_S}{N^{\star}} \\
N^{\star}(t) &= L + S + E + I_S + I_A + H + R + V.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dX_{vac}}{dt} &= \lambda_V [L + S + E + I_A + R] \\
\frac{dY_{I_S}}{dt} &= p\kappa E \\
L(0) &= L_0,\ S(0) = S_0,\ E(0) = E_0, \\
I_S(0) &= I_{S_0}, I_A(0) = I_{A_0}, H(0) = H_0, \\
R(0) &= R_0,\ D(0) = D_0,\ V(0) = 0, \\
X_{vac}(0) &= 0, \quad X_{vac}(T) = x_{coverage},
\end{aligned}
$$

Vaccine Hypotheses

- Imperfect preventive
- One dose
- Symptomatic exception
- Action over susceptible

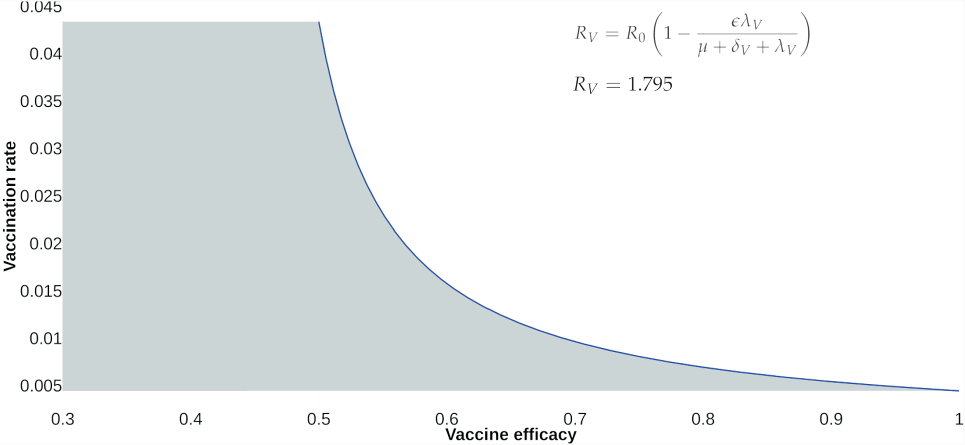
LOCKDOWN: Reduced infection and bidirectional migration to Susceptible class.

Reproductive number

$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1 + \delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

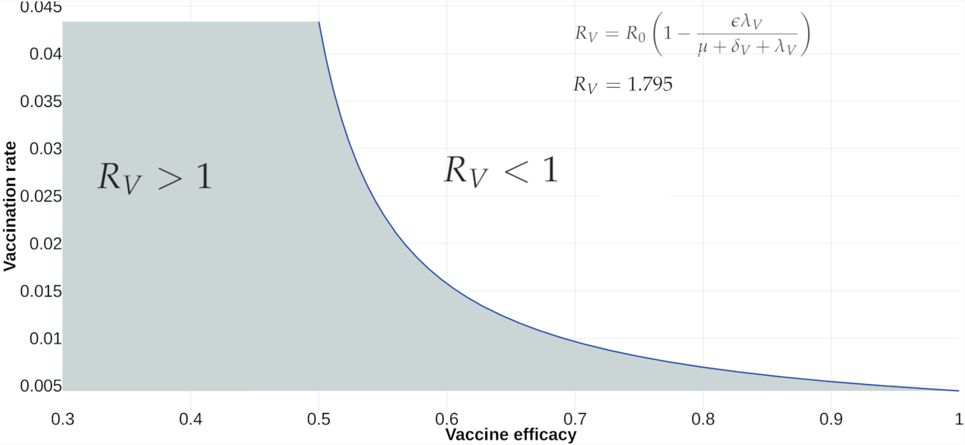
$$R_2 = \mu + \delta_H + \gamma_S + \mu_{I_S}.$$



$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1 + \delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

$$R_2 = \mu+\delta_H+\gamma_S+\mu_{I_S}.$$

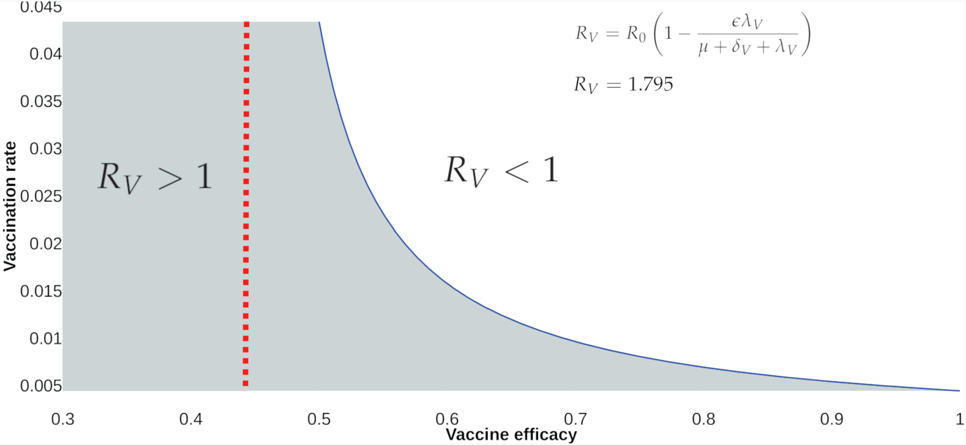


Reproductive number

$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1 + \delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

$$R_2 = \mu+\delta_H+\gamma_S+\mu_{I_S}.$$

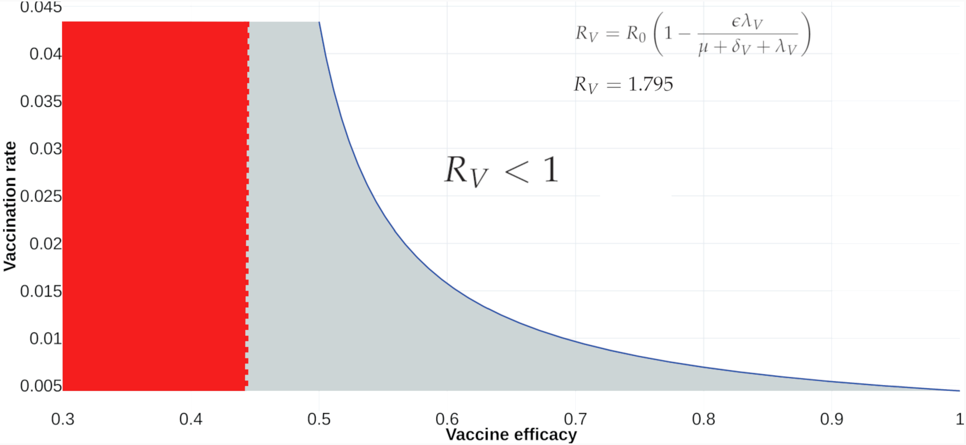


Reproductive number

$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1 + \delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

$$R_2 = \mu+\delta_H+\gamma_S+\mu_{I_S}.$$

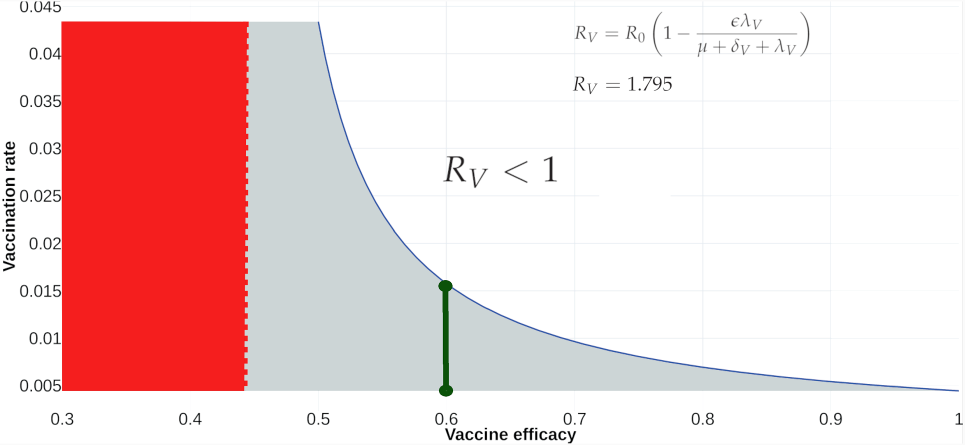


Reproductive number

$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1 + \delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

$$R_2 = \mu+\delta_H+\gamma_S+\mu_{I_S}.$$

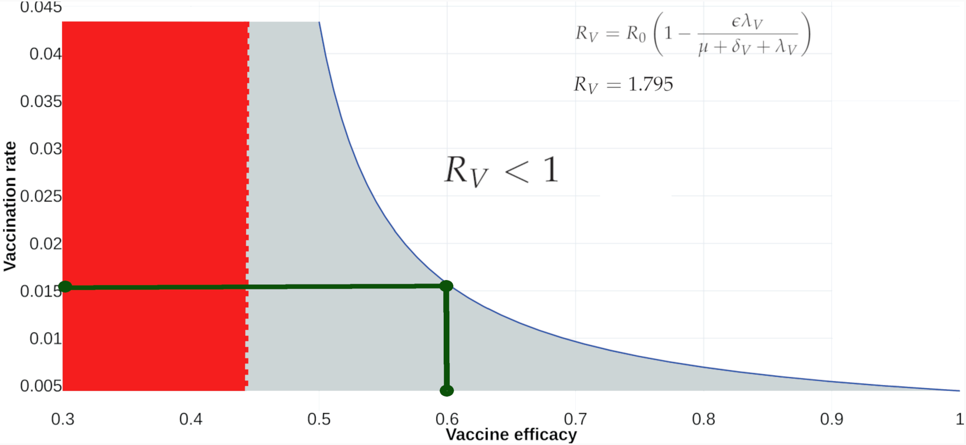


Reproductive number

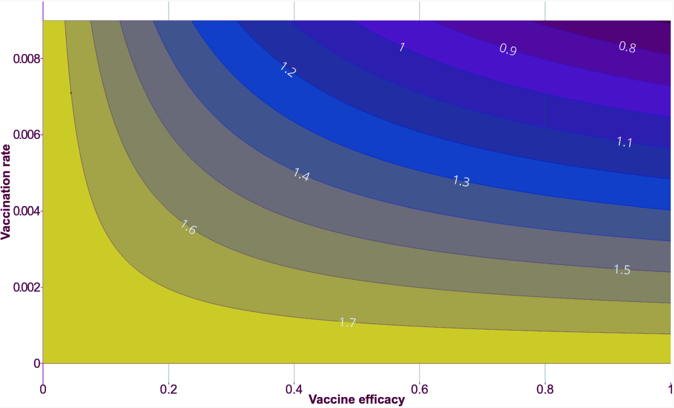
$$R_0^{L,V} := \left[1 - \frac{\varepsilon\lambda_V}{\mu+\lambda_V+\delta_V} - \frac{\theta\mu(1-\varepsilon)}{\mu+\delta_L+\lambda_V}\right](\mu R_1+\delta_L)R_0,$$

$$R_1 = 1-\theta(1-\varepsilon),$$

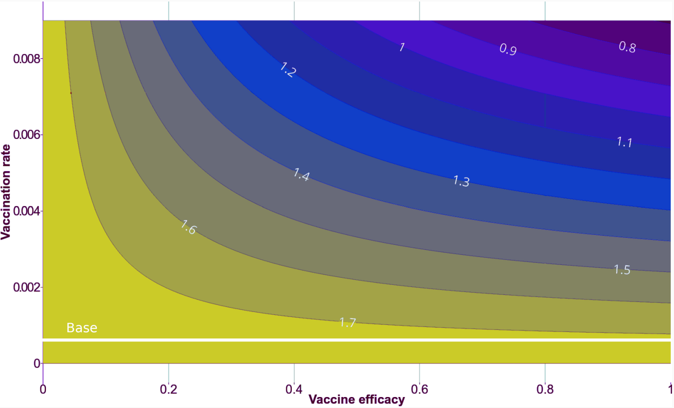
$$R_2 = \mu+\delta_H+\gamma_S+\mu_{I_S}.$$



Reproductive number



Reproductive number

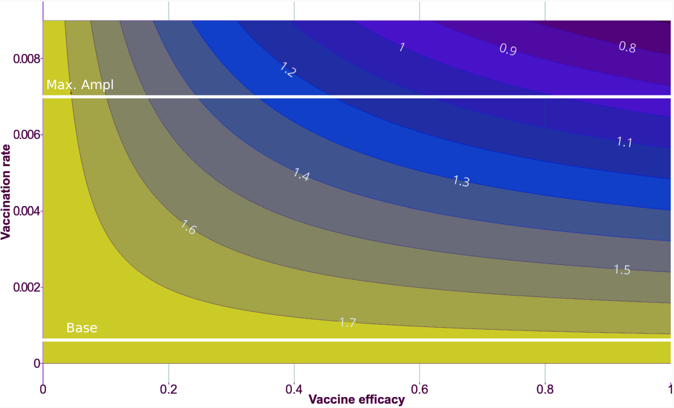


λ_V estimation

Given a fixed coverage X_{cov} and time horizon T

$$1 - \exp(-\lambda_V T) = X_{cov}$$

Reproductive number



X_{COV} : 20 %
T : 1 year

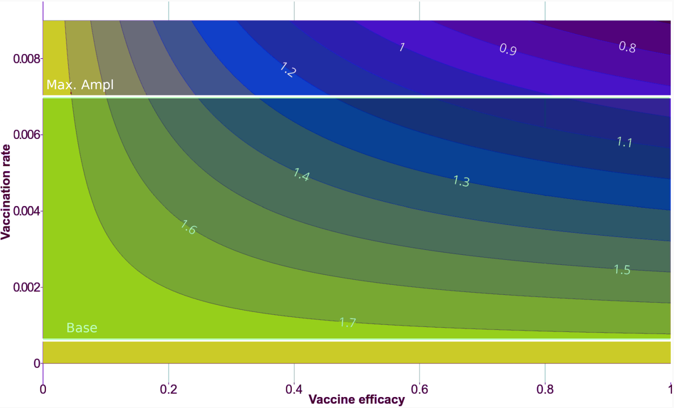
$\lambda_V \approx 0.000611352$

λ_V estimation

Given a fixed coverage X_{cov} and time horizon T

$$1 - \exp(-\lambda_V T) = X_{cov}$$

Reproductive number



X_{COV} : 20 %
T : 1 year

$$\lambda_V \approx 0.000611352$$

To decrease $R_V \leq 1$, we require

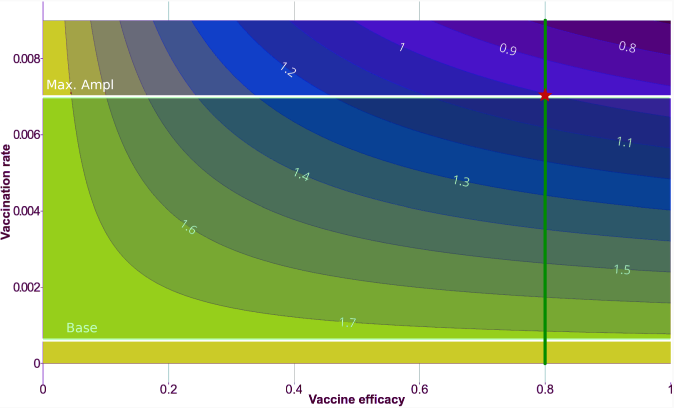
$$\varepsilon \geq 0.8, \quad \lambda_V \geq 0.007$$

λ_V estimation

Given a fixed coverage X_{cov} and time horizon T

$$1 - \exp(-\lambda_V T) = X_{cov}$$

Reproductive number



X_{COV} : 20 %
T : 1 year

$$\lambda_V \approx 0.000611352$$

To decrease $R_V \leq 1$, we require

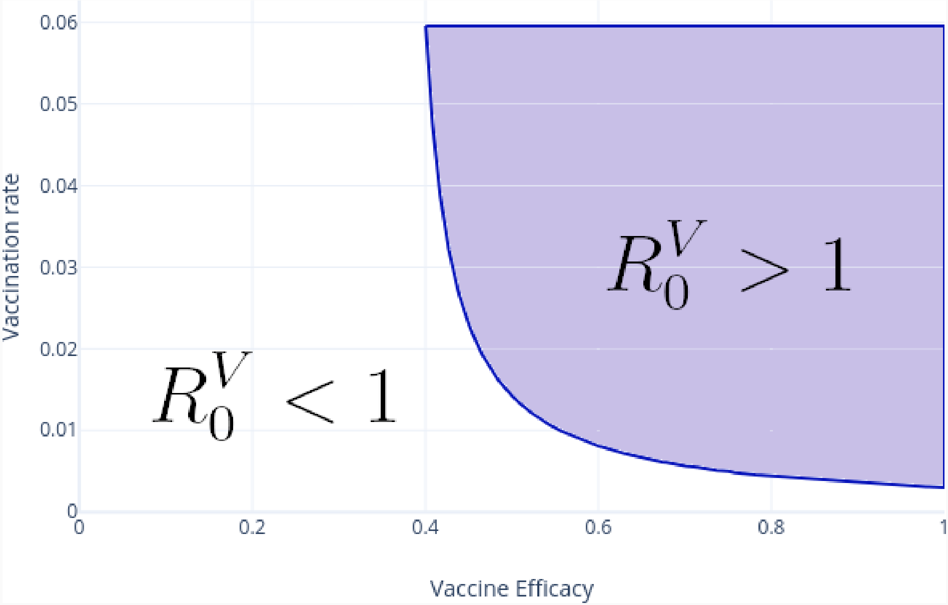
$$\varepsilon \geq 0.8, \quad \lambda_V \geq 0.007$$

λ_V estimation

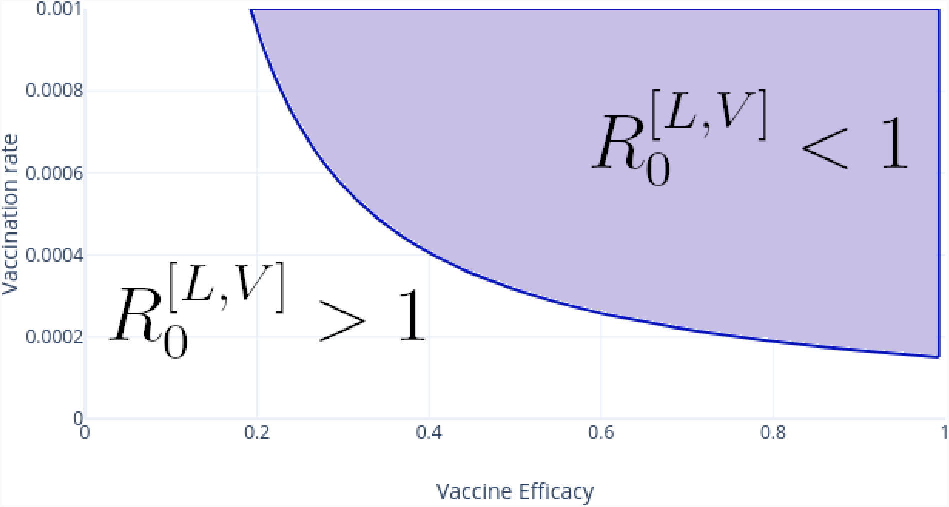
Given a fixed coverage X_{cov} and time horizon T

$$1 - \exp(-\lambda_V T) = X_{cov}$$

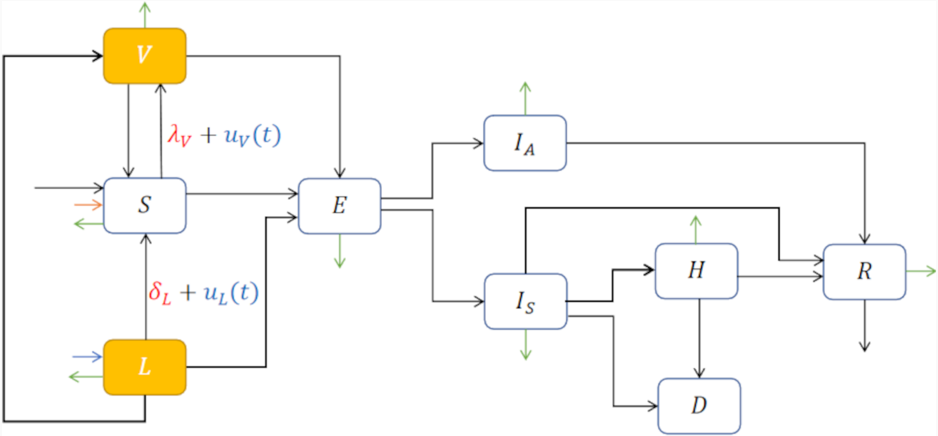
Lockdown-Vaccination Policy



Lockdown-Vaccination Policy



The Optimal Control Problem



$$\min_{u \in \mathcal{U}} J(u_L, u_V)$$
$$X_{vacc}(T) = x_{coverage}$$
$$\Phi(x,t) := \kappa I_S(t) \leq B, \forall t \in [0,T]$$

The disability-adjusted life year (DALY)

$DALY(c,s,a,t) = YLL(c,s,a,t) + YLD(c,s,a,t)$

For given cause c, age a, sex s and year t

YLL : Years of life lost due to premature death.

$$YLL(c,s,a,t) = N(c,s,a,t) \times L(s,a)$$

$N(c,s,a,t)$: is the number of deaths due to the cause c
$L(s,a)$: is a standard loss function specifying years of life lost

YLD : Years of life list due to disability

$$YLD(c,s,a,t) = I(c,s,a,t) \times DW(c,s,a) \times L(c,s,a,t)$$

$I(c,s,a,t)$: number of incident cases for cause c
$DW(c,s,a)$: disability weight for cause c
$L(c,s,a,t)$: average duration of the case until remission or death (years)

$$J(u_V) :=$$

The disability-adjusted life year (DALY)

$DALY(c,s,a,t) = YLL(c,s,a,t) + YLD(c,s,a,t)$

For given cause c, age a, sex s and year t

YLL : Years of life lost due to premature death.

$$YLL(c,s,a,t) = N(c,s,a,t) \times L(s,a)$$

$N(c,s,a,t)$: is the number of deaths due to the cause c
$L(s,a)$: is a standard loss function specifying years of life lost

YLD : Years of life list due to disability

$$YLD(c,s,a,t) = I(c,s,a,t) \times DW(c,s,a) \times L(c,s,a,t)$$

$I(c,s,a,t)$: number of incident cases for cause c
$DW(c,s,a)$: disability weight for cause c
$L(c,s,a,t)$: average duration of the case until remission or death (years)

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) :=$$

The disability-adjusted life year (DALY)

$DALY(c,s,a,t) = YLL(c,s,a,t) + YLD(c,s,a,t)$

For given cause c, age a, sex s and year t

YLL : Years of life lost due to premature death.

$$YLL(c,s,a,t) = N(c,s,a,t) \times L(s,a)$$

$N(c,s,a,t)$: is the number of deaths due to the cause c
$L(s,a)$: is a standard loss function specifying years of life lost

YLD : Years of life list due to disability

$$YLD(c,s,a,t) = I(c,s,a,t) \times DW(c,s,a) \times L(c,s,a,t)$$

$I(c,s,a,t)$: number of incident cases for cause c
$DW(c,s,a)$: disability weight for cause c
$L(c,s,a,t)$: average duration of the case until remission or death (years)

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p \kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr$$

The disability-adjusted life year (DALY)

$$DALY(c,s,a,t) = YLL(c,s,a,t) + YLD(c,s,a,t)$$

For given cause c, age a, sex s and year t

YLL : Years of life lost due to premature death.

$$YLL(c,s,a,t) = N(c,s,a,t) \times L(s,a)$$

$N(c,s,a,t)$: is the number of deaths due to the cause c
$L(s,a)$: is a standard loss function specifying years of life lost

YLD : Years of life list due to disability

$$YLD(c,s,a,t) = I(c,s,a,t) \times DW(c,s,a) \times L(c,s,a,t)$$

$I(c,s,a,t)$: number of incident cases for cause c
$DW(c,s,a)$: disability weight for cause c
$L(c,s,a,t)$: average duration of the case until remission or death (years)

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p \kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr +$$

$$\int_0^T \frac{1}{2}\left[c_L u_L^2(r) + c_V u_V^2(r)\right] dr.$$

Optimal Control Problem

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p \kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr +$$

$$\int_0^T \frac{1}{2}\left[c_L u_L^2(r) + c_V u_V^2(r)\right] dr$$

$$u_V(\cdot) \in [u_{\min}, u^{\max}],$$

$$\kappa I_S(t) \leq B, \quad \forall t \in [0,T],$$

Optimal Control Problem

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p \kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr +$$

$$\int_0^T \frac{1}{2} \left[c_L u_L^2(r) + c_V u_V^2(r)\right] dr$$

$$u_V(\cdot) \in [u_{\min}, u^{\max}],$$

$$\kappa I_S(t) \leq B, \quad \forall t \in [0, T],$$

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p \kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr +$$

$$\int_0^T \frac{1}{2}\left[c_L u_L^2(r) + c_V u_V^2(r)\right] dr$$

$$u_V(\cdot) \in [u_{\min}, u^{\max}],$$

$$\kappa I_S(t) \leq B, \quad \forall t \in [0, T],$$

Optimal Control Problem

$$\min_{u_V \in \mathscr{U}[0,T]} J(u_V) := \int_0^T \underbrace{a_S p\kappa E(r) + a_H \delta_H I_s(r)}_{:=YLL} + \underbrace{a_D \left[\mu_{I_S} I_S(r) + \mu_H H(r)\right]}_{:=YLD} dr +$$

$$\int_0^T \frac{1}{2}\left[c_L u_L^2(r) + c_V u_V^2(r)\right] dr$$

s.t.

$$\begin{aligned}
L' &= \theta\mu N^\star - \varepsilon\lambda L - (u_L(t)+\delta_L)L - \mu L \\
S' &= (1-\theta)\mu N^\star + (u_L(t)+\delta_L)L + \delta_V V \\
&\quad + \delta_R R - [\lambda + (\lambda_V + u_V(t)) + \mu]\, S \\
E' &= \lambda(\varepsilon L + (1-\varepsilon)V + S) - (\kappa+\mu)E \\
I_S' &= p\kappa E - (\gamma_S + \mu_{I_S} + \delta_H + \mu) I_S \\
I_A' &= (1-p)\kappa E - (\gamma_A + \mu) I_A \\
H' &= \delta_H I_S - (\gamma_H + \mu_H + \mu) H \\
R' &= \gamma_S I_S + \gamma_A I_A + \gamma_H H - (\delta_R + \mu) R \\
D' &= \mu_{I_S} I_S + \mu_H H \\
V' &= (\lambda_V + u_V(t)) S \\
&\quad - [(1-\varepsilon)\lambda + \delta_V + \mu]\, V
\end{aligned}$$

$$\frac{dX_{vac}}{dt} = (u_V(t) + \lambda_V)[L + S + E + I_A + R]$$

$$\frac{dY_{I_S}}{dt} = p\kappa E$$

$$\lambda := \frac{\beta_A I_A + \beta_S I_S}{N^\star}$$

$$\begin{aligned}
&L(0) = L_0,\ S(0) = S_0,\ E(0) = E_0,\ I_S(0) = I_{S_0}, \\
&I_A(0) = I_{A_0}, H(0) = H_0,\ R(0) = R_0,\ D(0) = D_0, \\
&V(0) = 0,\ X_{vac}(0) = 0, \quad u_V(.) \in [u_{\min}, u^{\max}], \\
&X_{vac}(T) = x_{coverage}, \quad \kappa I_S(t) \le B, \qquad \forall t \in [0,T], \\
&N^\star(t) = L + S + E + I_S + I_A \\
&\qquad + H + R + V
\end{aligned}$$

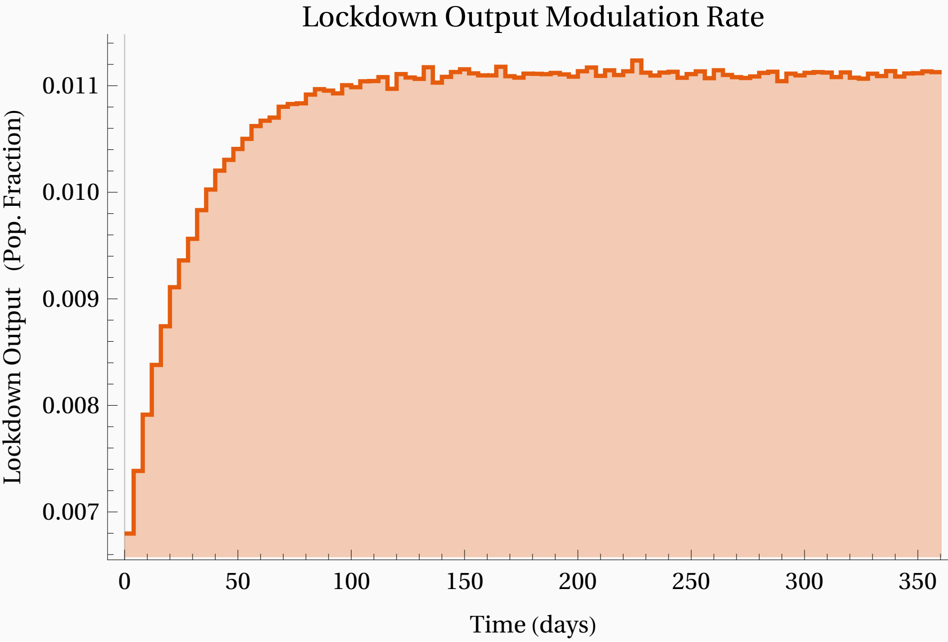
Vaccine efficacy

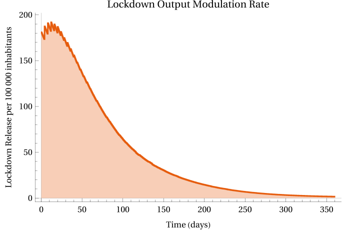
Developer	Vaccine Name	Efficacy %, (95% CI)	Reference
Pfizer-BioNTech	BNT162b2	95 (90.3–97.6)	[?]
Gamaleya Institute	Sputnik V	91.6 (85.6–95.2)	[?]
Oxford University-AztraZeneca	AZD1222	74.6 (41.6-88.9)	[?]
Johnson & Johnson*	Ad26.COV2.S	57 %, 66 % or 72 %	[?]
Sinovac Biotech*	CoronaVac	50.4 %	[?]

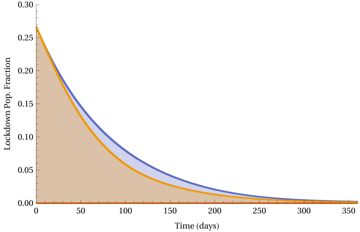
Table: Vaccine efficacy of some of the approved developments for emergency use. (*) No available information about the confidence intervals.

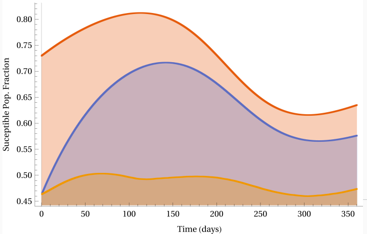
Algorithm 1 Differential Evolution Algorithm

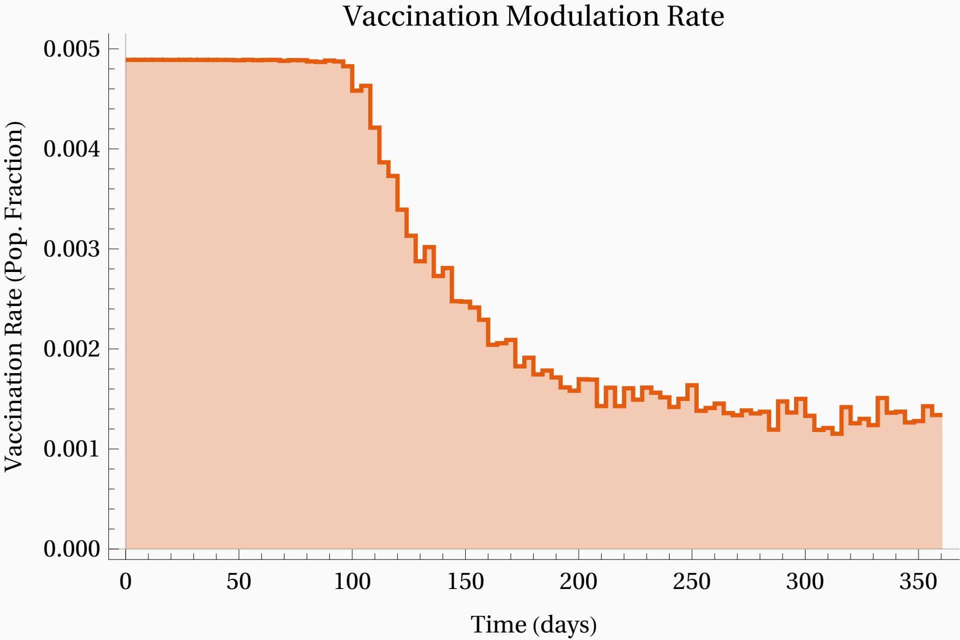
$X \leftarrow \mathbf{X}_0(Np, \mathcal{V})$
while (the stopping criterion has not been met) **do**
  $M \leftarrow \mathbf{M}(X, F, \mathcal{V})$
  $C \leftarrow \mathbf{C}(X, M, C_r)$
  $X \leftarrow \mathbf{S}(X, C, f_{ob})$
end while
$\mathbf{x}_{best} \leftarrow \mathbf{Best}(X, f_{ob})$

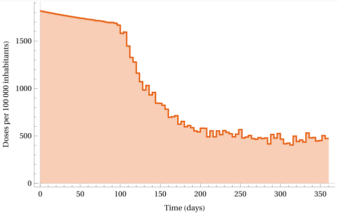

Lockdown Output Modulation Rate
Lockdown Output (Pop. Fraction)
0.007
0.008
0.009
0.010
0.011
0
50
100
150
200
250
300
350
Time (days)


Lockdown Output Modulation Rate
Lockdown Release per 100 000 inhabitants
200
150
100
50
0
0
50
100
150
200
250
300
350
Time (days)

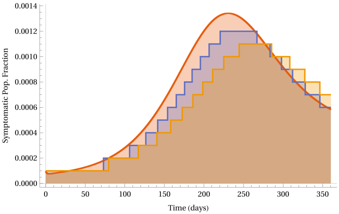

Lockdown Pop. Fraction
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
50
100
150
200
250
300
350
Time (days)


Suceptible Pop. Fraction
0.80
0.75
0.70
0.65
0.60
0.55
0.50
0.45
0
50
100
150
200
250
300
350
Time (days)

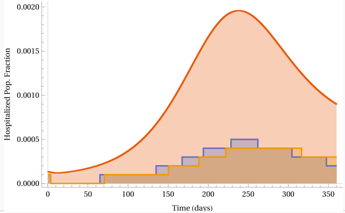

Vaccination Modulation Rate
Vaccination Rate (Pop. Fraction)
0.005
0.004
0.003
0.002
0.001
0.000
0
50
100
150
200
250
300
350
Time (days)



Doses per 100 000 inhabitants)
1500
1000
500
0
0
50
100
150
200
250
300
350
Time (days)



Symptomatic Pop. Fraction
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
0
50
100
150
200
250
300
350
Time (days)



Hospitalized Pop. Fraction
0.0020
0.0015
0.0010
0.0005
0.0000
0
50
100
150
200
250
300
350
Time (days)

Team CONACYT, UNISON, ITSON, UNAM

Working group	
Dr. Saúl Díaz Infante Velasco	CONACYT-UNISON
Dr. Manuel A. Acuña Zegarra	UNISON
Dr. Daniel Olmos Liceaga	UNISON
Dr. David Baca Carrasco	ITSON
Dr. David González-Sánchez	CONACYT-UNISON
Dr. Francisco Peñuñuri	UADY
M.C. Gabriel Salcedo-Varela	UNISON

Adviser	
Dr. Jorge X. Velasco Hernández	UNAM-Juriquilla

Git Hub

References I