Algorithm 2 Forward Backward Sweep

Input: $t_0, t_f, x_0, h, \text{tol}, \lambda_f$

Output: x^*, u^*, λ

procedure FORWARD_BACKWARD_SWEEP($g, \lambda_{\text{function}}, u, x_0, \lambda_f, h, n_{max}$)

while $\epsilon > \text{tol}$ **do**

$u_{\text{old}} \leftarrow u$

$x_{\text{old}} \leftarrow x$

$x \leftarrow$ RUNGE_KUTTA_FORWARD(g, u, x_0, h)

$\lambda_{\text{old}} \leftarrow \lambda$

$\lambda \leftarrow$ RUNGE_KUTTA_BACKWARD$(\lambda_{\text{function}}, x, \lambda_f, h)$

$u_1 \leftarrow$ OPTIMALITY_CONDITION(u, x, λ)

$u \leftarrow \alpha u_1 + (1-\alpha) u_{old}, \qquad \alpha \in [0,1]$ ▷ convex combination

$\epsilon_u \leftarrow \dfrac{||u - u_{\text{old}}||}{||u||}$

$\epsilon_x \leftarrow \dfrac{||x - x_{\text{old}}||}{||x||}$ ▷ relative error

$\epsilon_\lambda \leftarrow \dfrac{||\lambda - \lambda_{\text{old}}||}{||\lambda||}$

$\epsilon \leftarrow \max\{\epsilon_u, \epsilon_x, \epsilon_\lambda\}$

end while

return x^*, u^*, λ ▷ Optimal pair

end procedure