Modelling SDEs in biology

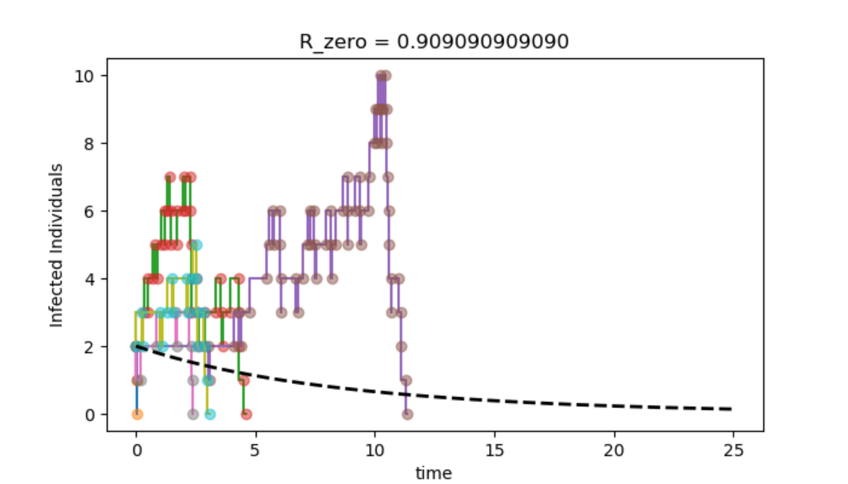
Formulation, Numerical Simulation and Analysis.

Third day: Analysis of epidemic models

Saúl Díaz Infante Velasco

CONACYT-UNIVERSIDAD de SONORA, Cimat, Guanajuato Gto

SIS-CTCM



R_zero = 0.909090909090
Infected Individuals
time
10
8
6
4
2
0
0
5
10
15
20
25

Analysis of an SI-SDE model

$$
\begin{aligned}
\dot{S}(t) &= \Lambda - \mu S(t) - \beta S(t) I(t) - \delta S(t) \\
\dot{I}(t) &= \beta S(t) I(t) - (\mu + \gamma + \varepsilon) I(t) \\
\dot{R}(t) &= \gamma I(t) - \mu R(t) + \delta S(t)
\end{aligned}
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Deterministic threshold

$$\mathscr{R}_0 = \frac{\beta\Lambda}{(\mu+\gamma+\varepsilon)(\mu+\delta)}$$
$$\mathscr{R}_0 < 1 \Rightarrow FDE: \text{ (g.a.s)}$$
$$\mathscr{R}_0 > 1 \Rightarrow EE: \text{ (g.a.s)}$$

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$$\beta dt \rightsquigarrow \beta dt + \sigma dB_t$$

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Stochastic threshold

$$\mathscr{R}_0^S = \ ?$$
$$\mathscr{R}_0^S < 1 \Rightarrow \text{Extinction}$$
$$\mathscr{R}_0^S > 1 \Rightarrow \text{Persistence}$$

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Check:

Yue Zhang, Yang Li, Qingling Zhang, and Aihua Li.
Behavior of a stochastic SIR epidemic model with saturated incidence and vaccination rules.
Physica A: Statistical Mechanics and its Applications, 501:178–187, 2018.

When apply Stochastic Models?

When are interested in

- Small population
- Demographic variability
- Environmental variability

According to

Allen, L. J. (2017). **A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis.** *Infectious Disease Modelling*, 2(2):128–142.

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Example

Transmission, births, recovery, deaths

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Example

Transmission, births, recovery, deaths

Territorial conditions, aquatic: diseases vector, zoonotic Foodborne

Alternatives

Models

- (D/C)−TMCs
- Parameter perturbation
  - Mean reverting processes
  - β_t^H $H \in (0.5, 1)$
- Random Diff. Eq.

MC + ME $\rightarrow$ *SDE*

L. J. Allen.
A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis.
Infectious Disease Modelling, 2(2):128–142, may 2017.

Tools

- Gillespie
- Kloeden-Methods
- Hermite-PC

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$\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$

Gray, A., Greenhalgh, D., Hu, L., Mao, X., and Pan, J. (2011).
A Stochastic Differential Equation SIS Epidemic Model.
SIAM Journal on Applied Mathematics, 71(3):876–902.

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$\varphi dt \rightsquigarrow \varphi dt + F(x)dB_t$

Schurz, H. and Tosun, K. (2015). **Stochastic Asymptotic Stability of SIR Model with Variable Diffusion Rates.** *Journal of Dynamics and Differential Equations*, 27(1):69–82.

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Fractiona BM.: long range memory.

Ma, Y., Zhang, Q., and Ye, M. (2017). **Mean-square dissipativity of numerical methods for a class of resource-competition models with fractional brownian motion.** *Systems Science & Control Engineering*, 5(1):268–277.

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Objetivo

To illustrate ideas $\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$

- Modelling
- Analysis and simulation
- Perspectives

Esquema de Charla

1. Introduction
2. Perturbación con MB
3. Solution properties
4. Trheshold: $R_0^S := R_0^D - f(\sigma, \cdot)$
5. Perspectives and fina comments

Modelo de jugete

$$
\begin{aligned}
\frac{dS(t)}{dt} &= \mu N - \beta S(t)I(t) + \gamma I(t) - \mu S(t), \\
\frac{I(t)}{dt} &= \beta S(t)I(t) - (\mu+\gamma)I(t), \\
R_0^D &= \frac{\beta N}{\mu+\gamma}, \\
R_0^D &< 1 \ \Rightarrow \lim_{t\to\infty} I(t) = 0 \\
R_0^D &> 1 \ \Rightarrow \lim_{t\to\infty} I(t) = N\left(1-\frac{1}{R_0^D}\right)
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- $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t \geq 0}, \mathbb{P})$,
- B_t M.B.
- $\beta S(t) I(t) dt$
  nuevas infecciones en $[t, t + dt)$
- βdt, contactos potencialmente infecciosos

$dI(t) = [\beta S(t) I(t) - (\mu + \gamma) I(t)]\, dt$

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$$\begin{aligned}
\beta dt &\rightsquigarrow \underbrace{\beta dt + \sigma dB_t}_{:=\tilde{\beta} dt} \\
\mathbb{E}[\tilde{\beta} dt] &= \beta dt \\
\mathbb{V}ar[\tilde{\beta} dt] &= \sigma^2 dt \xrightarrow{dt \to 0} 0
\end{aligned}$$

Modelo de jugete

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R_0^D &= \frac{\beta N}{\mu+\gamma},
\end{aligned}
$$

$$N = S(t) + I(t) = \text{cte.}$$

$$dI(t) = I(t)\left([\beta(N - I(t)) - \mu - \gamma]\right)dt - \sigma(N - I(t))dB_t$$

Existence and uniqueness of a positive solution

Theorem

- $\forall I(0) \in (0, N)$

$!\exists$ **global positive invariant solution** I_t

$$\mathbb{P}\left[I(t) \in (0, N) \forall t \geq 0\right] = 1.$$

$$\begin{aligned} dI(t) = I(t)\{&[\beta(N - I(t)) - \mu - \gamma]dt \\ &+ \sigma(N - I(t))dB_t\} \end{aligned}$$

Extinction by noise

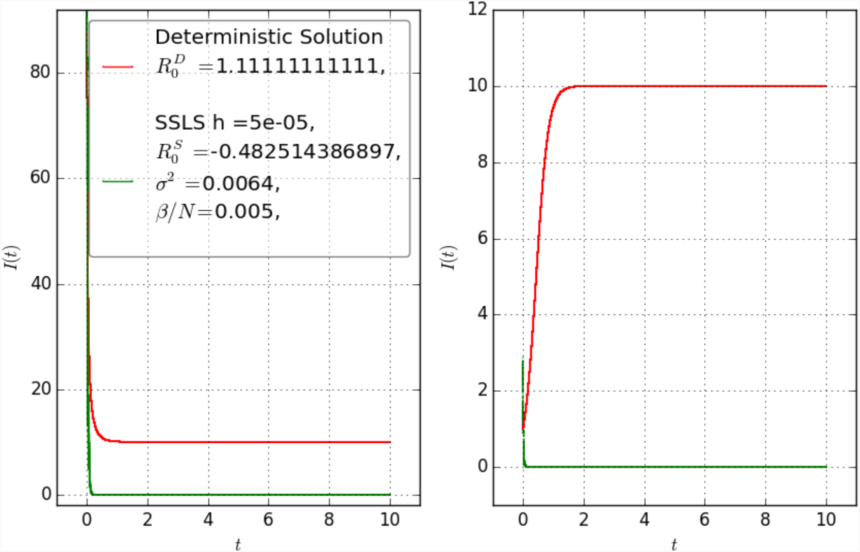
Theorem

- $\sigma^2 > \max\left\{\frac{\beta}{N}, \frac{\beta^2}{2(\mu+\gamma)}\right\}$

for all $I(0) \in (0, N)$ the solution to SDE($\star$) satisfies

$$\limsup_{t\to\infty} \frac{1}{t} \log I(t) \leq \underbrace{-\mu - \gamma + \frac{\beta^2}{2\sigma^2}}_{<0}$$

Simulación: Extinción por ruido ambiental



Theorem (Extinction)

- $R_0^S := R_0^D - \dfrac{\sigma^2}{2(\mu+\gamma)} < 1$
- $\sigma^2 \leq \dfrac{\beta}{N}$

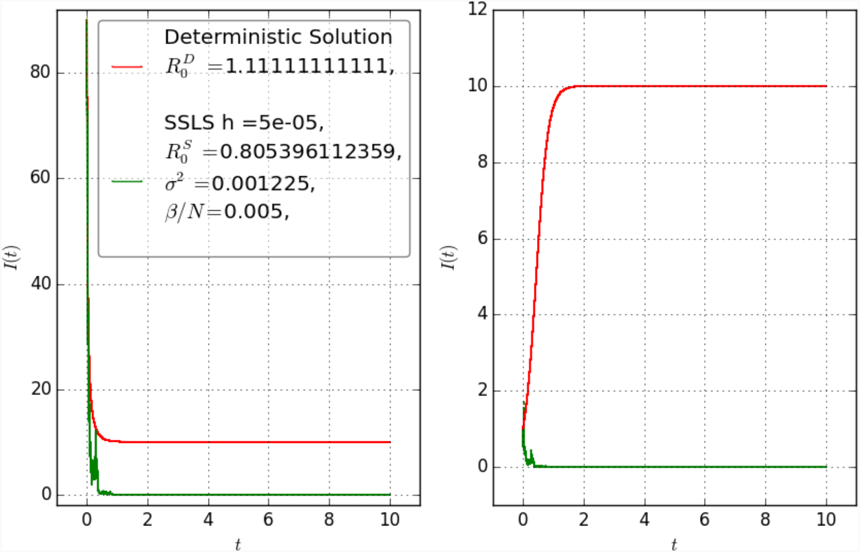
for all $I(0) \in (0, N)$

$$\limsup_{t\to\infty} \frac{1}{t} \log I(t) \leq \kappa, \quad c.s.$$

$$\kappa := \beta N - \mu - \gamma - \frac{\sigma^2 N^2}{2\sigma^2} < 0$$

$$\begin{aligned} dI(t) = I(t)\{&[\beta(N - I(t)) - \mu - \gamma]dt \\ &+ \sigma(N - I(t))dB_t\} \end{aligned} \tag{$\star$}$$

Extinction by a threshold condition



Persistence

Theorem (persistence)

- $R_0^S := R_0^D - \dfrac{\sigma^2}{2(\mu+\gamma)} > 1$

para toda $I(0) \in (0, N)$

$$\limsup_{t\to\infty} \frac{1}{t} \log I(t) \geq \varepsilon,$$

$$\liminf_{t\to\infty} \frac{1}{t} \log I(t) \leq \varepsilon, \quad c.s.$$

$$\varepsilon = \tfrac{1}{\sigma^2}\left(\sqrt{\beta^2 - 2\sigma^2(\mu+\gamma)} - (\beta - \sigma^2 N)\right)$$

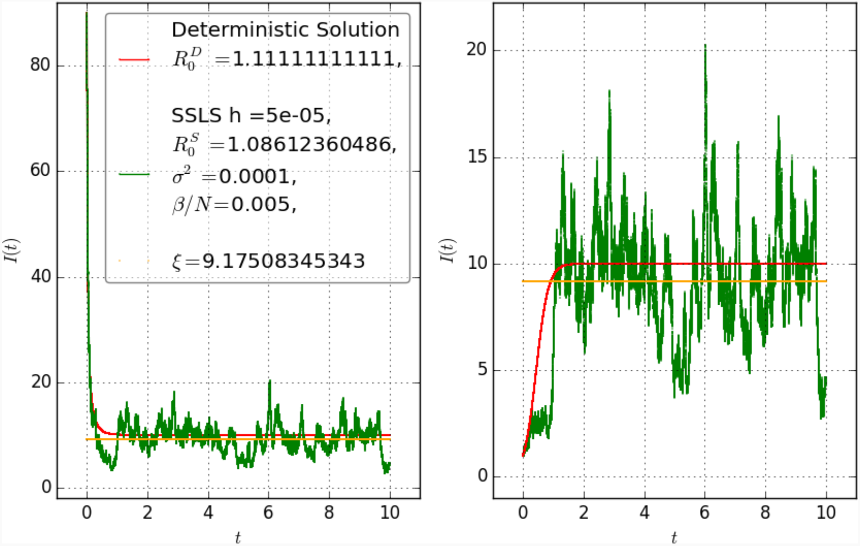
$$\begin{aligned} dI(t) = I(t)\{&[\beta(N - I(t)) - \mu - \gamma]dt \\ &+ \sigma(N - I(t))dB_t\} \end{aligned} \tag{$\star$}$$

$R_0^D : \beta N/(\mu+\gamma)$

$$R_0^D < 1 \Rightarrow \lim_{t\to\infty} I(t) = 0$$

$$R_0^D > 1 \Rightarrow \lim_{t\to\infty} I(t) = N\left(1 - \frac{1}{R_0^D}\right)$$

Simulation: Persistence)



Stationary distribution

Stationary distribution

- $P_{I_0,t}(A) = \mathbb{P}[I(t) \in A]$, $A \in \mathscr{B}((0,N))$
- $\lim_{t\to\infty} P_{I_0,t}(\cdot) = \mathbb{P}_\infty(\cdot)$ in distribution.

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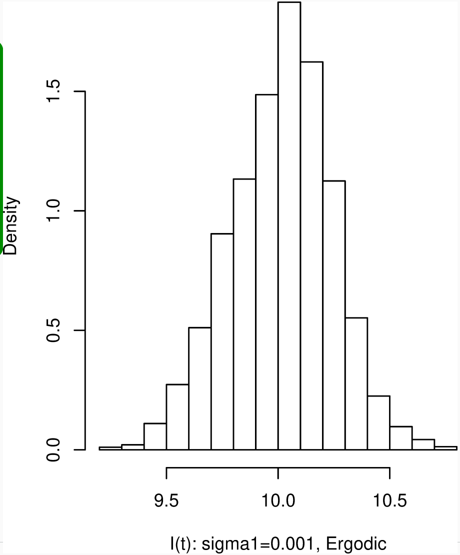

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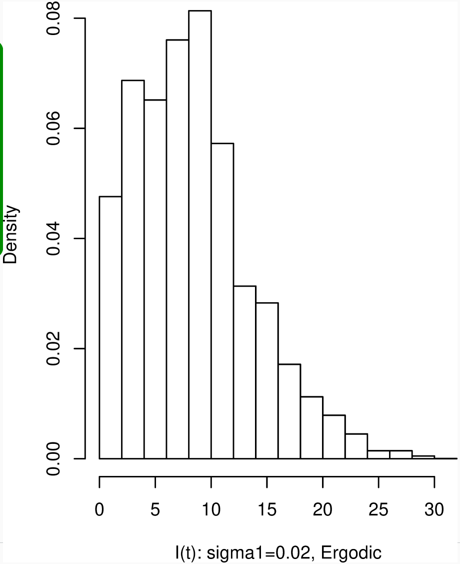

Manuel Adrian Acuña-Zegarra and Saúl Díaz-Infante.
Stochastic asymptotic analysis of a multi-host model with vector transmission.
Physica A: Statistical Mechanics and its Applications, 510:243 – 260, 2018.

S. Jerez, S. Díaz-Infante, and B. Chen.
Fluctuating periodic solutions and moment boundedness of a stochastic model for the bone remodeling process.
Mathematical Biosciences, 299:153 – 164, 2018.

S. Díaz-Infante and S. Jerez.
The linear steklov method for sdes with non-globally lipschitz coefficients: Strong convergence and simulation.
Journal of Computational and Applied Mathematics, 309:408 – 423, 2017.

Modelos

- Mean reverting processes
- $\beta_t^H \; H \in (0.5, 1)$
- Random Diff. Eq.
- Random Dynamical Systems

$$d\varphi_t = (\varphi_e - \varphi_t)dt + \sigma_\varphi dB_t$$

Allen, E. (2016).
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Long time memory: fBM

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Parameters as r.v

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Constructing adaptive generalized polynomial chaos method to measure the uncertainty in continuous models: A computational approach.
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The Bibles:

- Rafail Khasminskii.
  ***Stochastic Stability of Differential Equations*.**
  Stochastic Modelling and Applied Probability 66. Springer-Verlag Berlin Heidelberg, 2 edition, 2012.
- Alexander D. Wentzell Mark I. Freidlin.
  ***Random Perturbations of Dynamical Systems (Grundlehren der mathematischen Wissenschaften)*.**
  Springer, 2nd edition, 1998.

The Bibles:

Peter E. Kloeden.
***Random Ordinary Differential Equations and Their Numerical Solution*.**
Probability Theory and Stochastic Modelling 85. Springer Singapore, 1 edition, 2017.

Eckhard Platen Peter E. Kloeden.
***Numerical solution of stochastic differential equations*.**
Stochastic Modelling and Applied Probability. Springer, corrected edition, 1995.

About the policies

- $u = u(t)$, possible improvement
- $u = (x)$, or $u = u(t, x)$
- uncertinity

Tool: Dynamical Programing-HJB, MDP .

About the policies

$$\min_{u \in \mathscr{U}} \mathbb{E}\left[\int_0^T f(t,x,u)\right]$$

$$s.t. \qquad dx_t = a(t,x,u)dt + b(t,x,u)dw_t$$

Tools: Dynamic Programing - HJB, MDP, Viscosity Solution,

About the policies

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Tools: Dynamic Programing - HJB, MDP, Viscosity Solution,

Peter Grandits, Raimund M. Kovacevic, and Vladimir M. Veliov.
Optimal control and the value of information for a stochastic epidemiological SIS-model.
J. Math. Anal. Appl., 476(2):665–695, 2019.

About the policies

$$\min_{u \in \mathscr{U}} \mathbb{E}\left[\int_0^T f(t,x,u)\right]$$

$$s.t. \qquad dx_t = a(t,x,u)dt + b(t,x,u)dw_t$$

Tools: Dynamic Programing - HJB, MDP, Viscosity Solution,

Tony Huschto and Sebastian Sager.
Solving stochastic optimal control problems by a Wiener chaos approach.
Vietnam J. Math., 42(1):83–113, 2014.

The END

Code: `*.tex, *.py`

https://github.com/SaulDiazInfante/Modelling-Simulation-with-SDE

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Gracias!!!