NoMMA
Nodo Multidisciplinario de Matemáticas Aplicadas
Instituto de

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# Extinción, Persistencia y Comportamiento Umbral en Modelos Compartimentales Estocásticmente Perturbados

UNAM, Juriquilla, Queretaro
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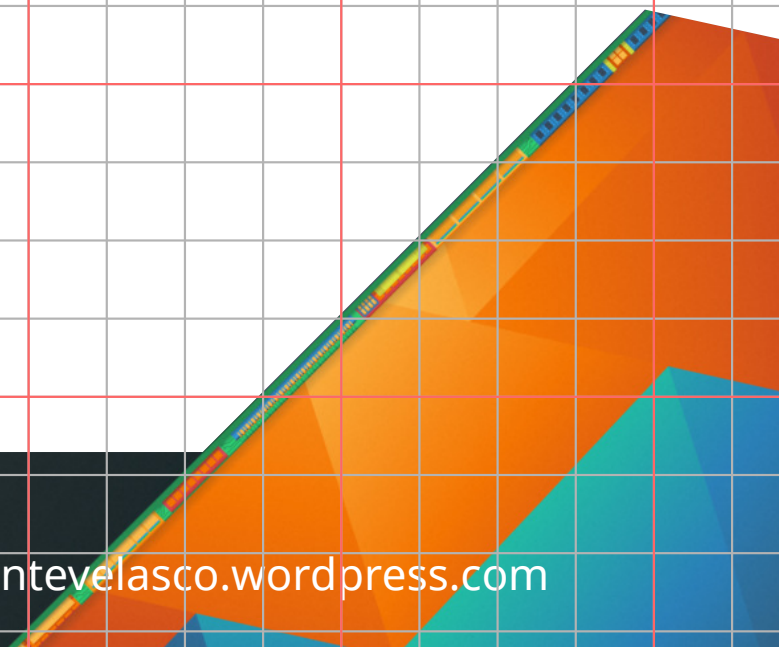

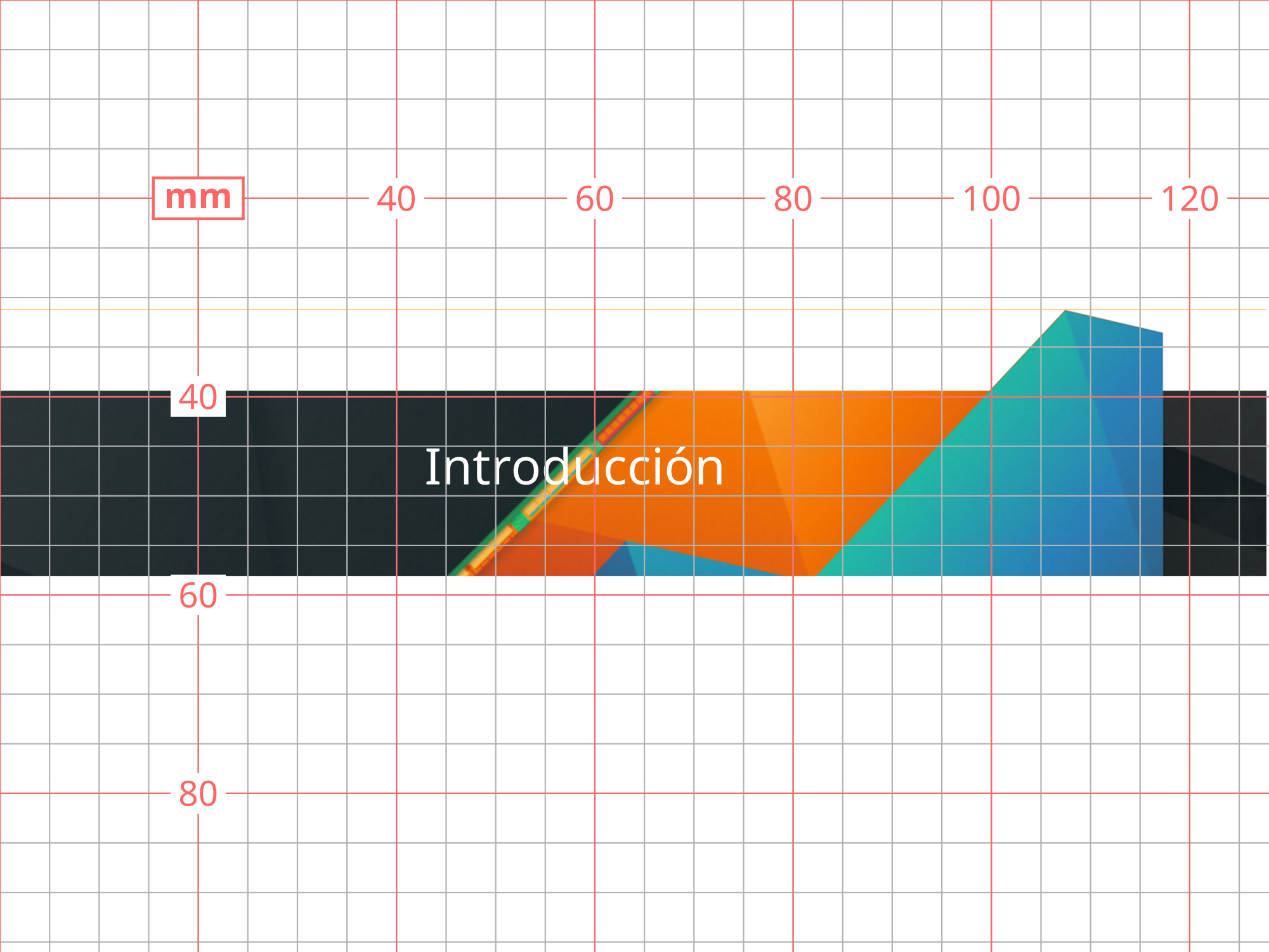
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Introducción

# Para fijar ideas

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$$\dot{S}(t) = \Lambda - \mu S(t) - \beta S(t) I(t) - \delta S(t)$$

$$\dot{I}(t) = \beta S(t) I(t) - (\mu + \gamma + \varepsilon) I(t)$$

$$\dot{R}(t) = \gamma I(t) - \mu R(t) + \delta S(t)$$

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## Umbral determinista

$$\mathscr{R}_0 = \frac{\beta \Lambda}{(\mu + \gamma + \varepsilon)(\mu + \delta)}$$

$$\mathscr{R}_0 < 1 \Rightarrow FDE: \text{ (g.a.s)}$$

$$\mathscr{R}_0 > 1 \Rightarrow EE: \text{ (g.a.s)}$$

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$$\dot{S}(t) = \Lambda - \mu S(t) - \beta S(t)I(t) - \delta S(t) - \sigma S(t)I(t)dB_t$$
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## Umbral estocástico

$\mathscr{R}_0^S = ?$

$\mathscr{R}_0^S < 1 \Rightarrow$ extinción

$\mathscr{R}_0^S > 1 \Rightarrow$ persistencia

# Para fijar ideas

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Ver:

Zhang, Y., Li, Y., Zhang, Q., and Li, A. (2018). Behavior of a stochastic SIR epidemic model with saturated incidence and vaccination rules. *Physica A: Statistical Mechanics and its Applications*, 501:178–187.

# ¿Cuando considerar Modelos Estocásticos?

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## Sean importantes

- Poblaciones pequeñas
- Variabilidad demográfica
- Variabilidad ambiental

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## Según:

Allen, L. J. (2017). A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis. *Infectious Disease Modelling*, 2(2):128–142.

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## Ejemplo

Transmisión, recuperación, nacimientos, muertes.

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## Ejemplo

Transmisión, recuperación, nacimientos, muertes.

Condiciones territoriales, acuáticas: enfermedades vectoriales, zoonóticas transmitidas por alimentos.

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# Alternativas

## Modelos

- (D/C)–TMCs
- Perturbación de parámetros
  - Procesos reversibles en media
  - $\beta_t^H$ $H \in (0.5, 1)$
- Random Diff. Eq.

## Herramientas

- Gillespie
- Kloeden-Methods
- Hermite-PC

## MC + ME $\rightarrow$ *SDE*

L. J. Allen.
A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis.
*Infectious Disease Modelling*, 2(2):128–142, may 2017.

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## $\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$

Gray, A., Greenhalgh, D., Hu, L., Mao, X., and Pan, J. (2011). A Stochastic Differential Equation SIS Epidemic Model. *SIAM Journal on Applied Mathematics*, 71(3):876–902.

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## $\varphi dt \rightsquigarrow \varphi dt + F(x) dB_t$

Schurz, H. and Tosun, K. (2015).
Stochastic Asymptotic Stability of SIR Model with Variable Diffusion Rates.
*Journal of Dynamics and Differential Equations*, 27(1):69–82.

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$$d\varphi_t = (\varphi_e - \varphi_t)dt + \sigma_\varphi dBt$$

Allen, E. (2016). Environmental variability and mean-reverting processes. *Discrete and Continuous Dynamical Systems - Series B*, 21(7):2073–2089.

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Ma, Y., Zhang, Q., and Ye, M. (2017).
Mean-square dissipativity of numerical methods for a class of resource-competition models with fractional brownian motion.
*Systems Science & Control Engineering*, 5(1):268–277.

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## parametros son v.a.

Chen-Charpentier, B.-M., Cortés, J.-C., Licea, J.-A., Romero, J.-V., Roselló, M.-D., Santonja, F.-J., and Villanueva, R.-J. (2015). Constructing adaptive generalized polynomial chaos method to measure the uncertainty in continuous models: A computational approach. *Mathematics and Computers in Simulation*, 109:113 – 129.

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# Objetivo



**Ilustrar las ideas de $\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$**

- Modelación
- Análisis y Simulación
- Perspectivas

# Esquema de Charla

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1. Introducción

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2. Perturbación con MB

3. Propiedades del proceso soulución

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4. Umbral: $\mathscr{R}_0^S := \mathscr{R}_0^D - f(\text{noise})$

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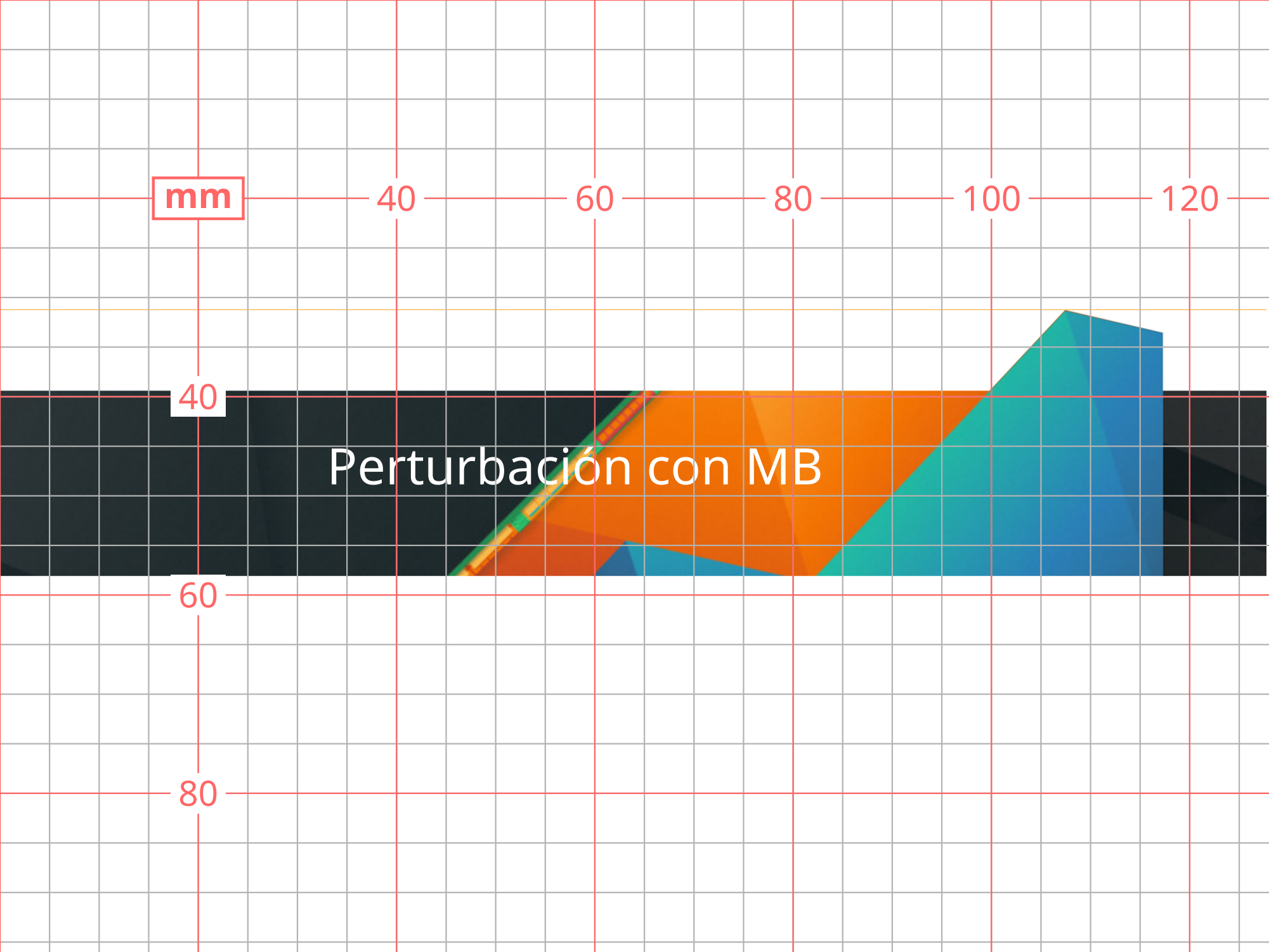
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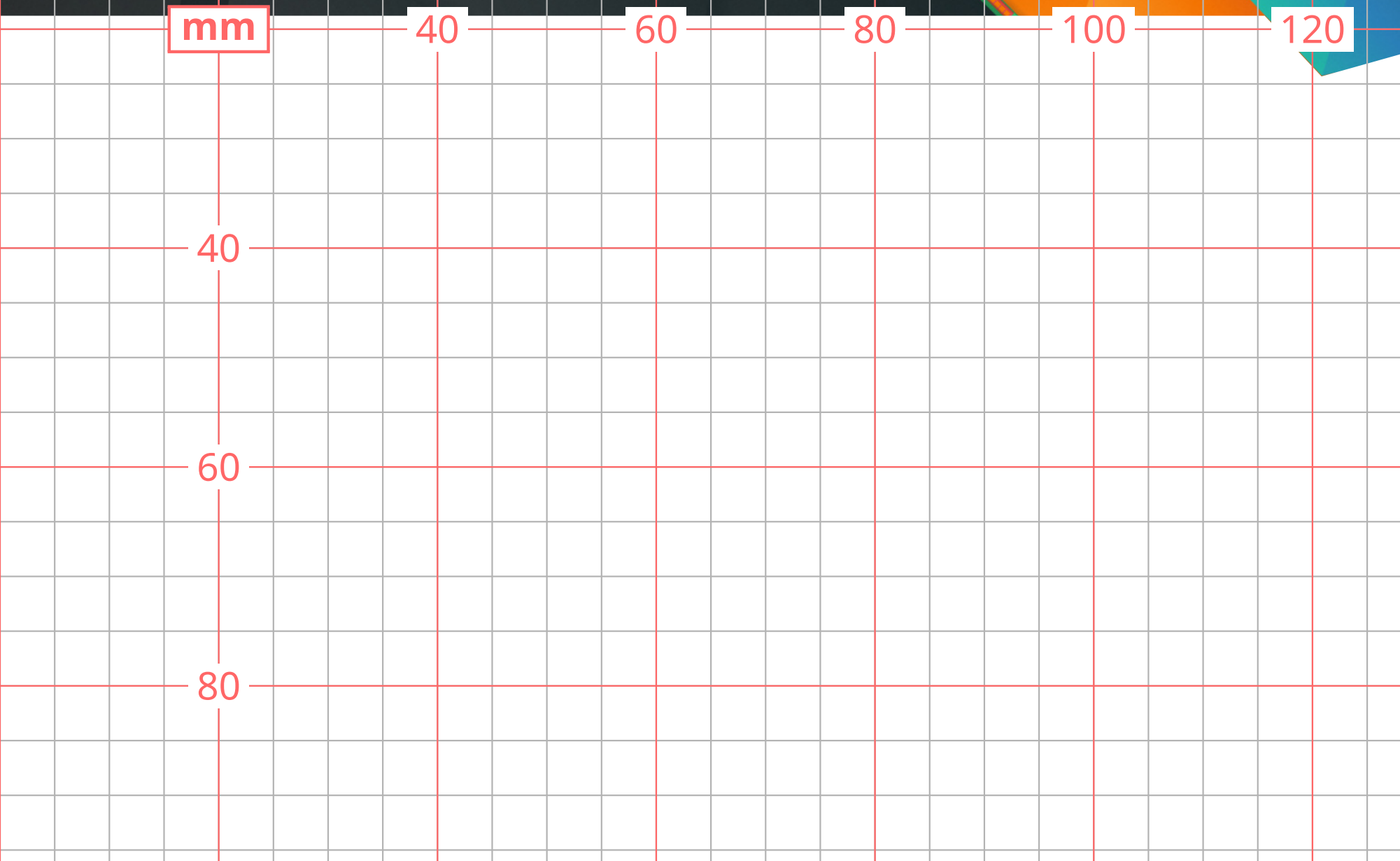
Perturbación con MB

# Consideremeos la siguiente ede

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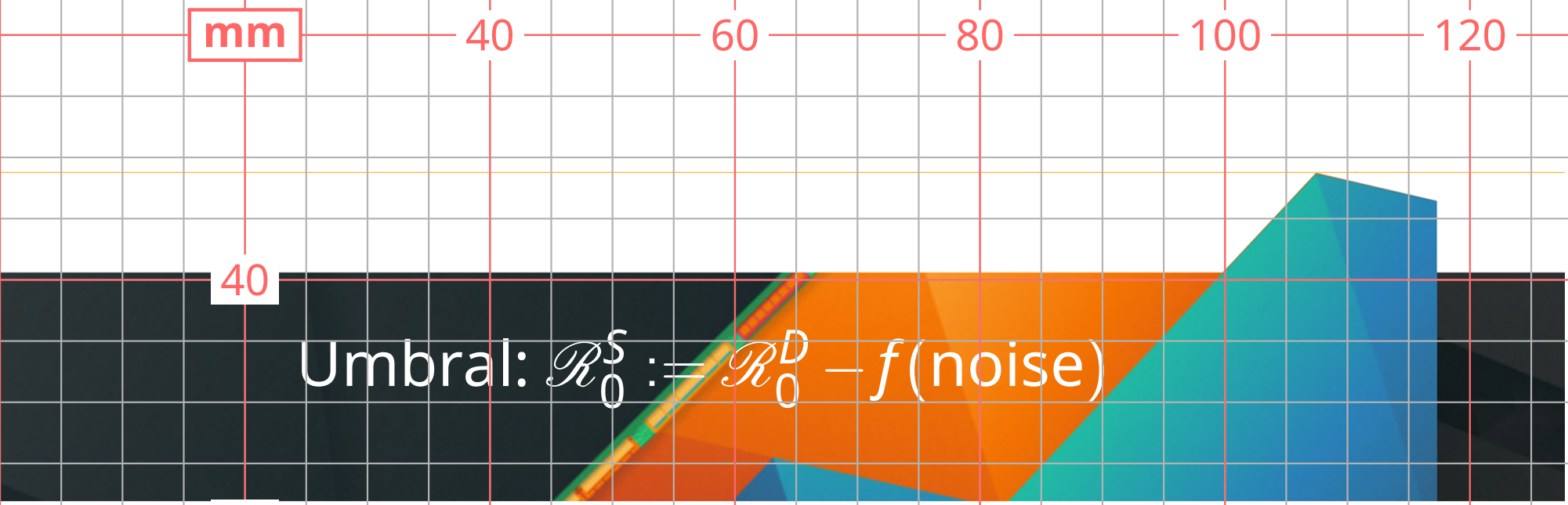
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