NoMMA
Nodo Multidisciplinario de Matemáticas Aplicadas
Instituto d

Extinción, Persistencia y Comportamiento Umbral en Modelos Compartimentales

Estocásticmente Perturbados

UNAM, Juriquilla, Queretaro
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Introducción

Para fijar ideas

$$\dot{S}(t) = \Lambda - \mu S(t) - \beta S(t)I(t) - \delta S(t)$$
$$\dot{I}(t) = \beta S(t)I(t) - (\mu + \gamma + \varepsilon)I(t)$$
$$\dot{R}(t) = \gamma I(t) - \mu R(t) + \delta S(t)$$

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Umbral determinista

$$\mathscr{R}_0 = \frac{\beta \Lambda}{(\mu + \gamma + \varepsilon)(\mu + \delta)}$$
$$\mathscr{R}_0 < 1 \;\Rightarrow\; FDE: \;\text{(g.a.s)}$$
$$\mathscr{R}_0 > 1 \;\Rightarrow\; EE: \;\text{(g.a.s)}$$

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$\mathscr{R}_0 > 1 \Rightarrow$ *EE* : (g.a.s)

Umbral estocástico

$\mathscr{R}_0^S =$?

$\mathscr{R}_0^S < 1 \Rightarrow$ extinción

$\mathscr{R}_0^S > 1 \Rightarrow$ persistencia

Para fijar ideas

mm 40 60 80 100 120

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Ver:

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Y. Zhang, Y. Li, Q. Zhang, and A. Li.
Behavior of a stochastic SIR epidemic model with saturated incidence and vaccination rules.
Physica A: Statistical Mechanics and its Applications, 501:178–187, 2018.

¿Cuando considerar Modelos Estocásticos?

mm 40 60 80 100 120

Sean importantes

- Poblaciones pequeñas
- Variabilidad demográfica
- Variabilidad ambiental

Según:

Allen, L. J. (2017). A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis. *Infectious Disease Modelling*, 2(2):128–142.

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Ejemplo

Transmisión, recuperación, nacimientos, muertes.

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Transmisión, recuperación, nacimientos, muertes.

Condiciones territoriales, acuáticas: enfermedades vectoriales, zoonóticas transmitidas por alimentos.

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Alternativas

Modelos

- (D/C)–TMCs
- Perturbación de parámetros
  - Procesos reversibles en media
  - $\beta_t^H \; H \in (0.5, 1)$
- Random Diff. Eq.

Herramientas

- Gillespie
- Kloeden-Methods
- Hermite-PC

MC + ME → *SDE*

L. J. Allen.
A primer on stochastic epidemic models: Formulation, numerical simulation, and analysis.
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$\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$

Gray, A., Greenhalgh, D., Hu, L., Mao, X., and Pan, J. (2011). A Stochastic Differential Equation SIS Epidemic Model. *SIAM Journal on Applied Mathematics*, 71(3):876–902.

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$\varphi dt \rightsquigarrow \varphi dt + F(x) dB_t$

Schurz, H. and Tosun, K. (2015). Stochastic Asymptotic Stability of SIR Model with Variable Diffusion Rates. *Journal of Dynamics and Differential Equations*, 27(1):69–82.

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$d\varphi_t = (\varphi_e - \varphi_t)dt + \sigma_\varphi dBt$

Allen, E. (2016).
Environmental variability and mean-reverting processes.
Discrete and Continuous Dynamical Systems - Series B, 21(7):2073–2089.

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Ma, Y., Zhang, Q., and Ye, M. (2017). Mean-square dissipativity of numerical methods for a class of resource-competition models with fractional brownian motion. *Systems Science & Control Engineering*, 5(1):268–277.

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parametros son v.a.

Chen-Charpentier, B.-M., Cortés, J.-C., Licea, J.-A., Romero, J.-V., Roselló, M.-D., Santonja, F.-J., and Villanueva, R.-J. (2015).
Constructing adaptive generalized polynomial chaos method to measure the uncertainty in continuous models: A computational approach.
Mathematics and Computers in Simulation, 109:113 – 129.

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Objetivo

Ilustrar las ideas de $\varphi dt \rightsquigarrow \varphi dt + \sigma dB_t$

- Modelación
- Análisis y Simulación
- Ideas al aire

Esquema de Charla

1. Introducción
2. Perturbación con MB
3. Propiedades del proceso soulución
4. Parámetro Umbral: $R_0^S := R_0^D - f(\text{noise})$

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Perturbación con MB

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Modelo de jugete

$$\frac{dS(t)}{dt} = \mu N - \beta S(t)I(t) + \gamma I(t) - \mu S(t),$$

$$\frac{I(t)}{dt} = \beta S(t)I(t) - (\mu + \gamma)I(t),$$

Modelo de jugete

$$dI(t) = [\beta S(t)I(t) - (\mu + \gamma)I(t)]\, dt$$

- $(\Omega, \mathscr{F}, \{\mathscr{F}\}_{t\geq 0}, \mathbb{P})$,
- B_t M.B.
- $\beta S(t)I(t)dt$ nuevas infecciones en $[t, t+dt)$
- βdt, contactos potencialmente infecciosos

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$$dI(t) = [\beta S(t)I(t) - (\mu + \gamma)I(t)]\, dt$$

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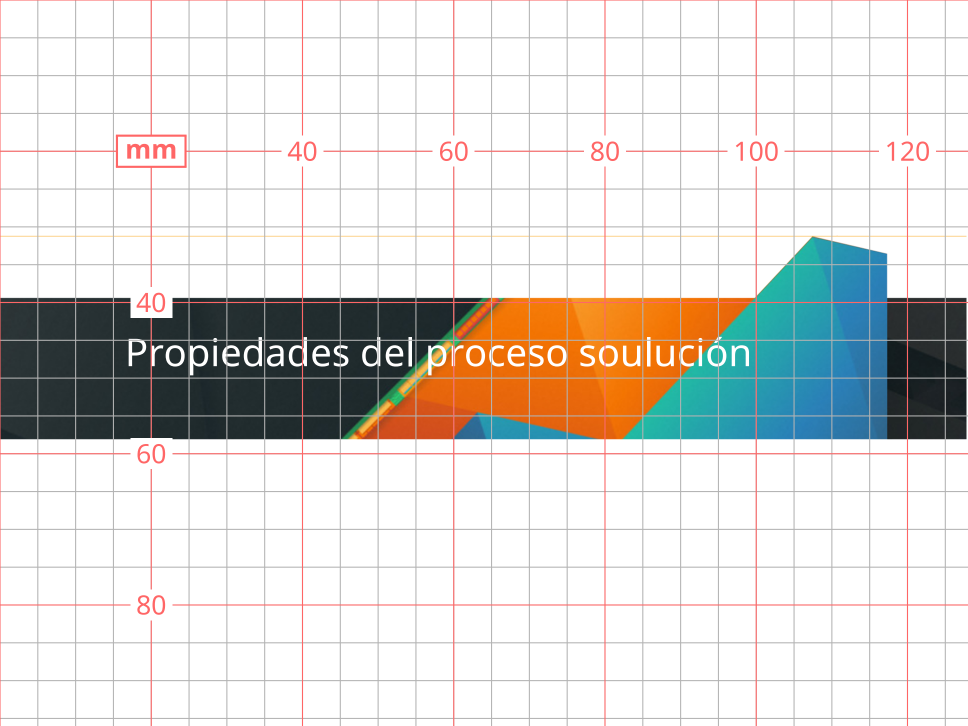
$$\frac{dI(t)}{dt} = I(t)([\beta(N - I(t)) - \mu - \gamma])dt, - \sigma(N - I(t))dB_t$$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Propiedades del proceso soulución
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