

$$\begin{aligned}
\tilde{\mu}_j &= \frac{\mu_j}{r_1}, \quad \tilde{e}_i = \frac{e_i}{r_1}, \quad \delta_i = \frac{a_i}{r_1}, \quad r = \frac{r_2}{r_1}, \quad \sigma_H = \left( \frac{\pi_V}{r_1} \right), \quad \alpha_1 = \left( \frac{\tilde{\pi}_h K_1}{r_1 H_0} \right), \\
\tilde{\sigma}_2 &= \left( \frac{\tilde{\pi}_V}{r_1} \right) \left( \frac{A_2}{(1-\eta_1)\omega_1 A_1 + A_2} \right), \quad \alpha_2 = \left( \frac{\pi_h K_2}{r_1 (1-\eta_1)\omega_1 A_1 + r_1 A_2} \right), \\
\tilde{\sigma}_1 &= \left( \frac{\tilde{\pi}_V}{r_1} \right) \left( \frac{(1-\eta_1)\omega_1 A_1}{(1-\eta_1)\omega_1 A_1 + A_2} \right), \quad \sigma_{11} = \left( \frac{\tilde{\pi}_V \eta_1 \omega_1}{r_1 (1-\omega_1) + \eta_1 \omega_1 r_1} \right), \\
\sigma_{12} &= \left( \frac{\tilde{\pi}_V \omega_2 (1-\omega_1)}{r_1 (1-\omega_1) + \eta_1 \omega_1 r_1} \right), \quad \tilde{\alpha}_{11} = \left( \frac{\pi_h \eta_1 \omega_1}{(1-\omega_1) A_1 + \eta_1 \omega_1 A_1} \right) \left( \frac{K_1}{r_1} \right), \\
\tilde{\alpha}_{12} &= \left( \frac{\pi_h (1-\omega_1) \omega_2}{(1-\omega_1) A_1 + \eta_1 \omega_1 A_1} \right) \left( \frac{K_1}{r_1} \right), \quad \tilde{\alpha}_2 = \left( \frac{\pi_h (1-\eta_1) \omega_1}{(1-\eta_1) \omega_1 A_1 + A_2} \right) \left( \frac{K_2}{r_1} \right)
\end{aligned}$$

The ODEs is,

$$\begin{aligned}
\frac{dx_1}{dt} &= z_1 \alpha_1 x_3 (1-x_1) - \tilde{\mu}_H x_1 \\
\frac{dx_2}{dt} &= ((\tilde{z}_1 \tilde{\alpha}_{11} + \tilde{z}_2 \tilde{\alpha}_{12}) x_3 + \tilde{z}_1 \tilde{\alpha}_2 x_6) (1-x_2) - \tilde{\mu}_1 x_2 \\
\frac{dx_3}{dt} &= (z_1 \sigma_H x_1 + (\tilde{z}_1 \sigma_{11} + \tilde{z}_2 \sigma_{12}) x_2) (x_4 - x_3) - (\tilde{e}_1 + x_4) x_3 \\
\frac{dx_4}{dt} &= x_4 (1-x_4) \\
\frac{dx_5}{dt} &= \tilde{z}_2 \alpha_2 x_6 (1-x_5) - \tilde{\mu}_2 x_5 \\
\frac{dx_6}{dt} &= (\tilde{z}_1 \tilde{\sigma}_1 x_2 + \tilde{z}_2 \tilde{\sigma}_2 x_5) (x_7 - x_6) - (\tilde{e}_2 + r x_7) x_6 \\
\frac{dx_7}{dt} &= r x_7 (1-x_7)
\end{aligned} \tag{1}$$

We define the random variables,

$$\begin{aligned}
z_{s1} dt &= z_1 dt + \beta_1 dW_1(t) \\
\tilde{z}_{s1} dt &= \tilde{z}_1 dt + \beta_2 dW_2(t) \\
\tilde{z}_{s2} dt &= \tilde{z}_2 dt + \beta_3 dW_3(t)
\end{aligned}$$

Now, we replace  $z_1 dt$ ,  $\tilde{z}_1 dt$  and  $\tilde{z}_2$  in system 2 by  $z_{s1} dt$ ,  $\tilde{z}_{s1} dt$  and  $\tilde{z}_{s2} dt$ , respec-

tively. Thus, we obtain the follow SDEs,

$$\begin{aligned}
dx_1 &= [z_1 \alpha_1 x_3 (1 - x_1) - \tilde{\mu}_H x_1] dt + \beta_1 \alpha_1 x_3 (1 - x_1) dW_1(t) \\
dx_2 &= [(\tilde{z}_1 \tilde{\alpha}_{11} + \tilde{z}_2 \tilde{\alpha}_{12}) x_3 + \tilde{z}_1 \tilde{\alpha}_2 x_6 (1 - x_2) - \tilde{\mu}_1 x_2] dt + \\
&\quad [(\beta_2 \tilde{\alpha}_{11} dW_2(t) + \beta_3 \tilde{\alpha}_{12} dW_3(t)) x_3 + \beta_2 \tilde{\alpha}_2 x_6 dW_2(t)] (1 - x_2) \\
dx_3 &= [z_1 \sigma_H x_1 + (\tilde{z}_1 \sigma_{11} + \tilde{z}_2 \sigma_{12}) x_2 (x_4 - x_3) - (\tilde{e}_1 + x_4) x_3] dt + \\
&\quad [\beta_1 \sigma_H x_1 dW_1(t) + (\beta_2 \sigma_{11} dW_2(t) + \beta_3 \sigma_{12} dW_3(t)) x_2] (x_4 - x_3) \\
dx_4 &= x_4 (1 - x_4) dt \\
dx_5 &= [\tilde{z}_2 \alpha_2 x_6 (1 - x_5) - \tilde{\mu}_2 x_5] dt + \beta_3 \alpha_2 x_6 (1 - x_5) dW_3(t) \\
dx_6 &= [\tilde{z}_1 \tilde{\sigma}_1 x_2 + \tilde{z}_2 \tilde{\sigma}_2 x_5 (x_7 - x_6) - (\tilde{e}_2 + r x_7) x_6] dt + \\
&\quad (\beta_2 \tilde{\sigma}_1 x_2 dW_2(t) + \beta_3 \tilde{\sigma}_2 x_5 dW_3(t)) (x_7 - x_6) \\
dx_7 &= r x_7 (1 - x_7) dt
\end{aligned} \tag{2}$$