An stochastic HIV Epidemic Model Driven with fBM Death Uncertainty

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Abstract

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1. Introduction

2. Model formulation

Here we consider the model reported in [5] as block base to design our stochastic model. The deterministic dynamics obeys the following equation:

$$\begin{aligned}
\frac{dS}{dt} &= \mu K - c\beta(I + bJ)S - \mu S,\\
\frac{dI}{dt} &= c\beta(I + bJ)S - (\mu + k_1)I + \alpha J,\\
\frac{dJ}{dt} &= k_1 I - (\mu + k_2 + \alpha)J,\\
\frac{dA}{dt} &= k_2 J - (\mu + \mu_d)A.
\end{aligned} \tag{1}$$

Our aim is to quantify uncertainty by a fractional Ornstein–Uhlenbeck process. To this end, we fit a mean reverting fractional process to the disease-related AIDS rate μ_d. That is, we estimate the reverting parameters $\gamma_d, \bar{\mu}_d\sigma_d$ and the Hurst parameter H of the fractional stochastic process

$$d\mu_d(t) = \gamma_d\left(\bar{\mu}_d - \mu_d(t)\right)dt + \sigma_d dB_t^H. \tag{2}$$

Here B_t^H denotes a fractional Brownian motion with Hurst parameter $H \in [\frac{1}{2}, 1)$.

3. Hurst parameter estimation

4. Existence of a unique positive invariant solution

5. Numerical results

6. Discussion

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