

# An stochastic HIV Epidemic Model Driven with fBM Death Uncertainty

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## Abstract

**Keywords:** fBM, stochastic differential equation.

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## 1. Introduction

## 2. Model formulation

Here we consider the model reported in [5] as block base to design our stochastic model. The deterministic dynamics obeys the following equation:

$$\begin{aligned}\frac{dS}{dt} &= \mu K - c\beta(I + bJ)S - \mu S, \\ \frac{dI}{dt} &= c\beta(I + bJ)S - (\mu + k_1)I + \alpha J, \\ \frac{dJ}{dt} &= k_1 I - (\mu + k_2 + \alpha)J, \\ \frac{dA}{dt} &= k_2 J - (\mu + d)A.\end{aligned}\tag{1}$$

## 3. Hurst parameter estimation

## 4. Existence of a unique positive invariant solution

## 5. Numerical results

## 6. Discussion

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