A OPTIMAL CONTROLLED LEPTOSPIROSIS MODEL

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ABSTRACT. We survey some theoretical results about a family of optimal control problems that arise in epidemiology. We also implement the so-called forward-backward-sweep method in Python to find approximate optimal control policies via the Pontryagin maximum principle. In addition, four specific models are described and simulated.

1. Introduction

2. The uncontrolled leptospirosis model

$$
\begin{aligned}
\lambda_H &:= \frac{\beta_H B}{k+B}, \qquad \lambda_A := \frac{\beta_A B}{k+B} + \beta_{AA} I_A \\
\frac{dS_H}{dt} &= b(S_A + (1-q)I_A + R_A) - \lambda_H S_H - (\mu_H + m)S_H \\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \delta_H + \gamma_H)I_H \\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H \\
\frac{dS_A}{dt} &= \Lambda + mS_H - \lambda_A S_A - \gamma_A d S_A \\
\frac{dI_A}{dt} &= \lambda_A S_A - (\gamma_H + \gamma_A)I_A \\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \gamma_A R_A \\
\frac{dB}{dr} &= \theta_H I_H + \theta_A I_A - kB
\end{aligned}
\tag{2.1}
$$

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We also consider a simplified version of the above model.

$$
\begin{aligned}
\lambda_H &:= \beta_H I_H + \bar{\beta}_{AH} I_A \\
\lambda_A &:= \beta_A I_H + \bar{\beta}_{AA} I_A + \beta_{AA} I_A \\
\frac{dS_H}{dt} &= b(S_A + (1-q)I_A + R_A) - \lambda_H S_H - (\mu_H + m)S_H \\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \mu_{ij} + \gamma_H)I_H \\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H \\
\frac{dS_A}{dt} &= \Lambda + mS_H - \lambda_A S_A - \gamma_A dS_A \\
\frac{dI_A}{dt} &= \lambda_A S_A - (\gamma_H + \gamma_A)I_A \\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \gamma_A R_A
\end{aligned}
\tag{2.2}
$$

3. Control policies for Leptospirosis in Live Stock

Notation n. We denote by $u_1(t)$ the proportion of vaccinated live-stock, and by $u_2(t)$ the bacterial fumigation strategies.... Our controlled model reads:

$$
\begin{aligned}
\lambda_H &:= u_2(t)(\bar{\beta}_{HH} I_H + \bar{\beta}_{AH} I_A) \\
\lambda_A &:= u_2(t)(\bar{\beta}_{HA} + \bar{\beta}_{AA}) I_A + \bar{\beta}_{AA} I_A \\
\frac{dS_H}{dt} &= b(S_A + (1-q)I_A + R_A) - \lambda_H S_H - (\mu_H + m)S_H - u_1(t)S_H + \alpha_H u_1(t) R_H \\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \delta_H + \gamma_H)I_H - u_1(t) I_H \\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H + u_1(t)(S_H + I_H) - \alpha_H u_1(t) R_H \\
\frac{dS_A}{dt} &= \Lambda + mS_H - \lambda_A S_A - \mu_A S_A - u_1(t) S_A + \alpha_R u_1(t) R_A \\
\frac{dI_A}{dt} &= \lambda_A S_A - (\mu_A + \gamma_A)I_A - u_1(t) I_A \\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \mu_A R_A + u_1(t)(S_A + I_A) - \alpha_A u_1(t) R_A
\end{aligned}
\tag{3.1}
$$

4. Existence and characterization of optimal policies

5. Numerical analysis

6. Numerical experiments

7. Concluding remarks