Optimal Vaccination control in Leptospiriosis

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Abstract

We survey some theoretical results about a family of optimal control problems that arise in epidemiology. We also implement the so-called forward-backward-sweep method in Python to find approximate optimal control policies via the Pontryagin maximum principle. In addition, four specific models are described and simulated.

Keywords: Multi-host, persistence, extinction, stochastic perturbation, vector transmission.

1. Introduction

2. The uncontrolled leptospirosis model

$$
\begin{aligned}
\lambda_H &:= \frac{\beta_H B}{K+B}, \qquad \lambda_A := \frac{\beta_A B}{K+B} + \beta_{AA} I_A \\
\frac{dS_H}{dt} &= b(S_A + (1-q)I_A + R_A) - \lambda_H S_H - (\mu_H + m)S_H \\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \delta_H + \gamma_H)I_H \\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H \\
\frac{dS_A}{dt} &= \Lambda_A + mS_H - \lambda_A S_A - \gamma_A d S_A \\
\frac{dI_A}{dt} &= \lambda_A S_A - (\gamma_H + \gamma_A)I_A \\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \gamma_A R_A \\
\frac{dB}{dr} &= \theta_H I_H + \theta_A I_A - kB
\end{aligned}
\tag{1}
$$

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We also consider a simplified version of the above model.

$$
\begin{aligned}
\lambda_H &:= \bar\beta_{HH} I_H + \bar\beta_{AH} I_A,\\
\lambda_A &:= \bar\beta_{HA} I_H + \bar\beta_{AA} I_A + \beta_{AA} I_A,\\
\frac{dS_H}{dt} &= b(S_A + (1-q)I_A + R_A) - \lambda_H S_H - (\mu_H + m)S_H,\\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \delta_H + \gamma_H)I_H,\\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H,\\
\frac{dS_A}{dt} &= \Lambda_A + mS_H - \lambda_A S_A - \mu_A S_A,\\
\frac{dI_A}{dt} &= \lambda_A S_A - (\mu_A + \gamma_A)I_A,\\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \mu_A R_A.
\end{aligned}
\tag{2}
$$

3. Control policies for Leptospirosis in Livestock

Notation. We denote by $u_1(t)$ the proportion of vaccinated livestock, and by $u_2(t)$ the bacterial fumigation strategies.... Our controlled model reads:

$$
\begin{aligned}
\lambda_H &:= u_2(t)(\bar\beta_{HH} I_H + \bar\beta_{AH} I_A),\\
\lambda_A &:= u_2(t)(\bar\beta_{HA} + \bar\beta_{AA}) I_A + u_1(t)\beta_{AA} I_A,\\
\frac{dS_H}{dt} &= b\left[S_A + (1-q)I_A + R_A\right] - \lambda_H S_H - (\mu_H + m)S_H - u_1(t)S_H + \alpha_H u_1(t) R_H,\\
\frac{dI_H}{dt} &= (1-\sigma)qbI_A + \lambda_H S_H - (\mu_H + \delta_H + \gamma_H)I_H - u_1(t)I_H,\\
\frac{dR_H}{dt} &= \gamma_H I_H - (\mu_H + m)R_H + u_1(t)(S_H + I_H) - \alpha_H u_1(t)R_H,\\
\frac{dS_A}{dt} &= \Lambda_A + mS_H - \lambda_A S_A - \mu_A S_A - u_1(t)S_A + \alpha_A u_1(t)R_A,\\
\frac{dI_A}{dt} &= \lambda_A S_A - (\mu_A + \gamma_A)I_A - u_1(t)I_A,\\
\frac{dR_A}{dt} &= mR_H + \gamma_A I_A - \mu_A R_A + u_1(t)(S_A + I_A) - \alpha_A u_1(t)R_A.
\end{aligned}
\tag{3}
$$

4. Existence and characterization of optimal policies

Our objective is minimize the cost function defined by

$$
J(u_1, u_2) = \int_0^{t_f} \left[a_1 I_H(t) + a_2 I_A(t) + b_1 u_1(t)^2 + b_2 u_2(t)^2\right] dt \tag{4}
$$

We employ Pontryagin's Maximum Principle to converts (3) and (4) into a problem of minimizing point-wise Hamiltonian H, with respect to (u_1, u_2). First we formulate the Hamiltonian H from the cost functional (4) and the governing dynamics (3).

$$
H = f(t) + g(t)\cdot\lambda(t), \tag{5}
$$

where $f(t)$ is the..., $g(t)$ is the right-side of system (3), and $\lambda(t) := (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6)$. It is important to mention that λ_1, λ_2, λ_3, λ_4, λ_5 and λ_6 are the adjoin or co-state variables solutions of the following system:

$$
\begin{aligned}
\frac{d\lambda_1}{dt} &= -b(S_A(t) + (1-q)I_A(t) + R_A(t)) + (m + \mu_H)S_H(t) - \alpha_H R_H(t)u_1(t) \\
&\quad + S_H(t)u_1(t) + (\bar{\beta}_{AH} I_A(t) + \bar{\beta}_{HH} I_H(t))S_H(t)u_2(t), \\
\frac{d\lambda_2}{dt} &= -bq(1-\sigma)I_A(t) - (\bar{\beta}_{AH} I_A(t) + \bar{\beta}_{HH} I_H(t))S_H(t)u_2(t) + I_H(t)u_1(t) \\
&\quad + (\gamma_H + \delta_H + \mu_H)I_H(t), \\
\frac{d\lambda_3}{dt} &= -\gamma_H I_H(t) + (m + \mu_H)R_H(t) + \alpha_H R_H(t)u_1(t) - (I_H(t) + S_H(t))u_1(t), \\
\frac{d\lambda_4}{dt} &= -\Lambda_A - u_1\alpha_A R_A(t) + \mu_A S_A(t) - mS_H(t) + S_A(t)(\beta_{AA} I_A(t)u_1(t) \\
&\quad + (\bar{\beta}_{AA} I_A(t) + \bar{\beta}_{HA} I_H(t))u_2(t)) + S_A(t)u_1(t), \\
\frac{d\lambda_5}{dt} &= (\gamma_A + \mu_A)I_A(t) + I_A(t)u_1(t) - S_A(t)(\beta_{AA} I_A(t)u_1(t) + (\bar{\beta}_{AA} I_A(t) \\
&\quad + \bar{\beta}_{HA} I_H(t))u_2(t)), \\
\frac{d\lambda_6}{dt} &= -\gamma_A I_A(t) + \mu_A R_A(t) - mR_H(t) + \alpha_A R_A(t)u_1(t) - (I_A(t) + S_A(t))u_1(t).
\end{aligned} \tag{6}
$$

The optimal conditions are given by solving the following equations by u_i.

$$
\frac{\partial H}{\partial u_i} = 0, \quad i = 1, 2.
$$

Thus,

$$
\begin{aligned}
u_1^* = \left(\frac{1}{2b_1}\right) & [\lambda_1(t)\,(S_H(t) - \alpha_H R_H(t)) + \lambda_2(t)I_H(t) + \lambda_3(t)\,(\alpha_H R_H(t) - S_H(t) - I_H(t)) \\
& + \lambda_4(t)\,(\beta_{AA} I_A(t)S_A(t) + S_A(t) - \alpha_A R_A(t)) + \lambda_5(t)\,(I_A(t) - \beta_{AA} I_A(t)S_A(t)) \\
& + \lambda_6(t)\,(\alpha_A R_A(t) - S_A(t) - I_A(t))] \\
u_2^* = \left(\frac{1}{2b_2}\right) & \left[\left(\bar{\beta}_{AH} I_A(t)S_H(t) + \bar{\beta}_{HH} I_H(t)S_H(t)\right)(\lambda_1(t) - \lambda_2(t)) \right. \\
& \left. + \left(\bar{\beta}_{AA} I_A(t)S_A(t) + \bar{\beta}_{HA} I_H(t)S_A(t)\right)(\lambda_4(t) - \lambda_5(t))\right]
\end{aligned}
$$

5. Numerical analysis

6. Numerical experiments

7. Concluding remarks

References