Homework:

1 Prove the following proposition

PROPOSITION 22 [Real Analysis Royden Prop. 22 p. 69 3th edition].

Let f be a measurable function defined on an interval $[a,b]$, and assume that f takes values $\pm\infty$ only on a set of measure zero. Then given $\varepsilon > 0$, we can find a step function g and a continuos function h such that

$|f-g| < \varepsilon$ and $|f-h| < \varepsilon$

except on a set mesurable of mesure les than ε, i.e.

$$m\{x : |f(x)-g(x)| \geq \varepsilon\} < \varepsilon \text{ and}$$

$$m\{x : |f(x)-h(x)| \geq \varepsilon\} < \varepsilon$$

Hint:

See Wheeden Thm 4.13, Measure and Integral An introduction to real Analysis.

. . .

2) Proposition. Let A a Lebesgue measurable set. Given $\epsilon > 0$, there exist a compact set K and open set G such that
i) $K \subset A \subset G$, and
ii) $m(G \sim K) < \varepsilon$.

3) Prove property FT9 p. 296, Lebesgue Integration on Euclidean spaces, Frank Jones.

Let T be an invertible matrix and let T^{tr} denote the transpose of T. Then

$$|\det T| \widehat{f(Tx)} = \hat{f}\left((T^{tr})^{-1}\xi\right).$$

4) Enumerate and proof the Plancherel theorem; from An Introduction to Harmonic Analysis: Yitzhak Katznelson third corrected edition.

5) From the same book as above, prove the Pale - Wiener theorems (pp. 188–190).

Here m is the Lebesgue measure.