

Statistical Inference For Data Science

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1/16/23

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Preface

Who I am. I am Saul Diaz Infante Velasco. I just starting as assistant professor at the Data Science graduate program of Universidad de Sonora at Hermosillo Mexico. My Background is related with numerical analysis and stochastic models. I'm are a enthusiastic of this treading topic called Data-Science, but perhaps at the moment I only have just intuition about what really it is. However, I have been programming almost 20 years an moved from old programming langues as FORTRAN, Pascal, Basic, Cobol, C, C++ to the new well established treading development workflows like R, Python and Julia. This is my firs attempt in R.

More of this book is work in progress. We aim to provide material as well we review and improve our basic skills to face the study of the most popular methods in Machine learning. Thus the book try to cover fundamentals in R programming as data types, flow control structures and put particular importance in the well practice of coding functions. Then we moves to the management of data whit dplyr and other packages. To finish this part we discuss some package to visualize data. Then, we face the problem o estimation and explore techniques based on bootstrap—another sampling flavors.

This book has been started on January, 2023 as part of a course to the Master on Data Science from Universidad de Sonora.

I'm writing this book to follow a path of self learning, understanding and and joy for this matter called Data Science. I'm not try to become and expert instead I just pursuit the joy of the interaction of math, computational sciences and the generosity of this virtuous learning-teaching process.

Introduction

The focus of this course is into the programming and basic techniques for inference that are usually applied in data science. We start by reviewing and enforcing programming skills. Then we will use the database of entomological data practice and build the required bases for more structured tools like bootstrap or Jackknife cuts.

Figure 1 further explores the impact of temperature on ozone level.

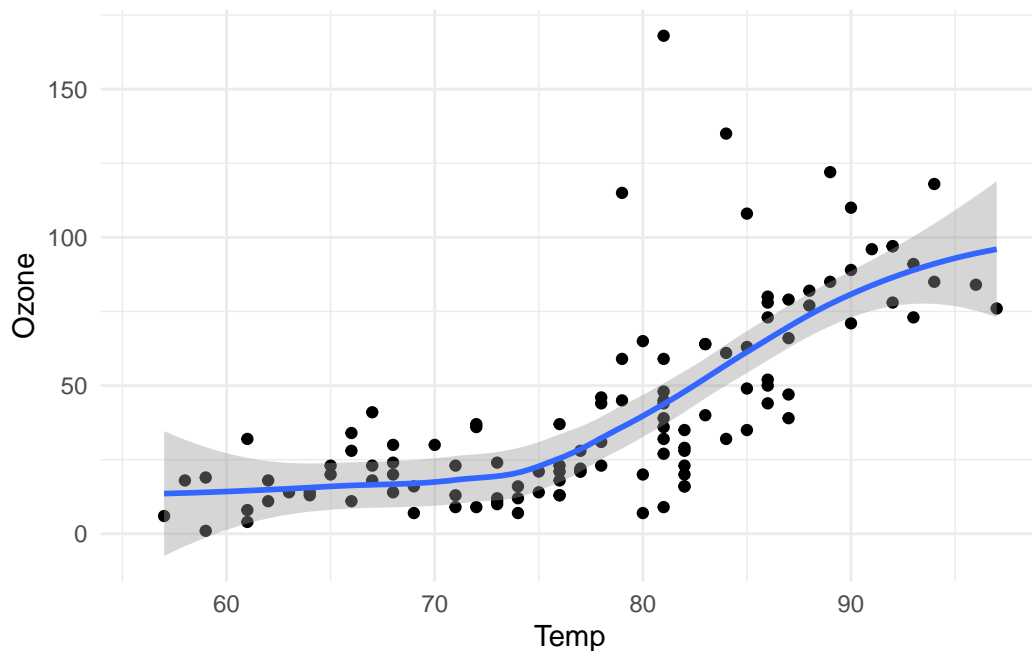


Figure 1: Temperature and ozone level.

The tidyverse

We need to install a R package. The majority of the packages that we will use are part of the so-called tidyverse package. The packages in the tidyverse share a common philosophy of data and R programming, and are designed to work together naturally.

You can install the complete tidyverse with the line of code:

then we can use it by loading in the preamble section with

```
-- Attaching packages ----- tidyverse 1.3.2 --
v tibble  3.1.8      v dplyr   1.0.10
v tidyr   1.2.0      v stringr 1.5.0
v readr   2.1.2      v forcats 0.5.2
v purrr   0.3.4
-- Conflicts ----- tidyverse_conflicts() --
x dplyr::filter() masks stats::filter()
x dplyr::lag()     masks stats::lag()
```

see <https://www.tidyverse.org/> documentation.

Part I

R fundamentals in programming, data managment and visualization

We dedicate this part to overview the basics to program in R. The aim of this part is building the basis for Machine learning, namely **data visualization**, **data manipulation** and the **good coding practices** to type script of industrial production quality.

Nuts and bolts: Data types

Entering Input: the assignment operator

The thing that we type on the R console prompt are expressions. The first expression we discuss here is the assignment operator, please watch the following video https://www.youtube.com/watch?v=vGY5i_J2c-c&t=283s

At the R console, any executable typed text that we put a side of the prompt are called expressions. We start by the `<-` symbol is the assignment operator.

```
[1] 0
```

```
[1] 0
```

```
[1] "what's up"
```

The `[1]` shown in the output indicates that `x` is a **vector** and 0 is the element at position with **index** 1.

Intro to basics

How it works

In the text editor you should type R code to solve the exercises. When you hit **ctrl + enter**, every line of code is interpreted and executed by R and you get a message whether or not your code was correct.

R makes use of the `#` sign to add comments, so that you and others can understand what the R code is about. Comments are not run as R code, so they will not influence your result. For example, Calculate $3 + 4$ in the editor on the right is a comment.

You can also execute R commands straight in the console. This is a good way to experiment with R code.

Instructions 100 XP

- In the text editor on the right there is already some sample code.
- Can you see which lines are actual R code and which are comments?
- Add a line of code that calculates the sum of 6 and 12, and hit the enter button

ex__01.R

```
# Calculate 3 + 4
3 + 4
# Calculate 6 + 12
6 + 12
```

Arithmetic with R

In its most basic form, R can be used as a simple calculator. Consider the following arithmetic operators:

- Addition: `+`

- Subtraction: -
- Multiplication: *
- Division: /
- Exponentiation: ^
- Modulo: %%

The last two might need some explaining:

- The ^ operator raises the number to its left to the power of the number to its right: for example 3^2 is 9.
- The modulo returns the remainder of the division of the number to the left by the number on its right, for example 5 modulo 3 or $5 \% 3$ is 2.

Instructions 100 XP

- Type `2^5` in the editor to calculate 2 to the power 5.
- Type `28 %% 6` to calculate 28 modulo 6.
- Run the answer in the console and have a look at the R output .
- Note how the # symbol is used to add comments on the R code.

ex__02.R

```
# An addition
5 + 5

# A subtraction
5 - 5

# A multiplication
3 * 5

# A division
(5 + 5) / 2

# Exponentiation
2 ^ 5

# Modulo
28 %% 6
```

Variable assignment

A basic concept in (statistical) programming is called a variable.

A variable allows you to store a value (e.g. 4) or an object (e.g. a function description) in R. You can then later use this variable's name to easily access the value or the object that is stored within this variable.

💡 You can assign a value 4 to a variable `my_var` with the command

```
my_var <- 4
```

Instructions 100 XP

Over to you: complete the code in the editor such that it assigns the value 42 to the variable `x` in the editor. Submit the answer. Notice that when you ask R to print `x`, the value 42 appears.

ex__03.R

```
# Assign the value 42 to x
x <- 42
# Print out the value of the variable x
print(x)
```

Variable assignment (2)

Suppose you have a fruit basket with five apples. As a data analyst in training, you want to store the number of apples in a variable with the name `my_apples`.

Instructions 100 XP

- Type the following code in the editor: `my_apples <- 5`. This will assign the value 5 to `my_apples`.
- Type: `my_apples` below the second comment. This will print out the value of `my_apples`.
- Run your answer, and look at the output: you see that the number 5 is printed. So R now links the variable `my_apples` to the value 5.

ex__04.R

```
# Assign the value 5 to the variable my_apples
my_apples <- 5
# Print out the value of the variable my_apples
```

```
print(my_apples)
```

Variable assignment (3)

Every tasty fruit basket needs oranges, so you decide to add six oranges. As a data analyst, your reflex is to immediately create the variable `my_oranges` and assign the value 6 to it. Next, you want to calculate how many pieces of fruit you have in total. Since you have given meaningful names to these values,

i you can now code this in a clear way:

```
my_apples + my_oranges
```

Instructions 100 XP

- Assign to `my_oranges` the value 6.
- Add the variables `my_apples` and `my_oranges` and have R simply print the result.
- Assign the result of adding `my_apples` and `my_oranges` to a new variable `my_fruit`.

ex_05.R

```
# Assign a value to the variables my_apples and my_oranges
my_apples <- 5
my_oranges <- 6

# Add these two variables together
my_apples + my_oranges

# Create the variable my_fruit
my_fruit <- my_apples + my_oranges
```

Apples and oranges

Common knowledge tells you not to add apples and oranges. But hey, that is what you just did, no :-)? The `my_apples` and `my_oranges` variables both contained a number in the previous exercise. The `+` operator works with numeric variables in R. If you really tried to add “apples” and “oranges”, and assigned a text value to the variable `my_oranges` (see the

editor), you would be trying to assign the addition of a numeric and a character variable to the variable `my_fruit`. This is not possible.

Instructions 100 XP

- Run the answer and read the error message. Make sure to understand why this did not work.
- Adjust the code so that R knows you have 6 oranges and thus a fruit basket with 11 pieces of fruit.

`ex_06.R`

```
# Assign a value to the variable my_apples
my_apples <- 5
# Fix the assignment of my_oranges
my_oranges <- "six"
# Create the variable my_fruit and print it out
my_fruit <- my_apples + my_oranges
my_fruit
```

Response

`ex_06.R`

```
# Assign a value to the variable my_apples
my_apples <- 5
# Fix the assignment of my_oranges
my_oranges <- 6
# Create the variable my_fruit and print it out
my_fruit <- my_apples + my_oranges
my_fruit
```

Basic data types in R

R works with numerous data types. Some of the most basic types to get started are:

- Decimal values like 4.5 are called numerics.
- Whole numbers like 4 are called integers. Integers are also numerics.
- Boolean values (TRUE or FALSE) are called logical.
- Text (or string) values are called characters.

Note how the quotation marks in the editor indicate that “some text” is a string.

Instructions 100 XP

Change the value of the:

- `my_numeric` variable to 42.
- `my_character` variable to "universe". Note that the quotation marks indicate that “universe” is a character.
- `my_logical` variable to FALSE.

i Note that R is case sensitive!

Thus despite the variables called `var`, `Var`, `vAr`, has the same fonetic characters, R understand each of these as different memory addresses.

ex_07.R

```
# Change my_numeric to be 42
my_numeric <- 42.5

# Change my_character to be "universe"
my_character <- "some text"

# Change my_logical to be FALSE
my_logical <- TRUE
```

Response

ex_07.R

```
# Change my_numeric to be 42
my_numeric <- 42

# Change my_character to be "universe"
my_character <- "universe"

# Change my_logical to be FALSE
my_logical <- FALSE
```

What's that data type?

Do you remember that when you added $5 + \text{"six"}$, you got an error due to a mismatch in data types? You can avoid such embarrassing situations by checking the data type of a variable beforehand. You can do this with the `class()` function, as the code in the editor shows.

Instructions 100 XP

Complete the code in the editor and also print out the classes of `my_character` and `my_logical`.

ex_08.R

```
# Declare variables of different types

my_numeric <- 42
my_character <- "universe"
my_logical <- FALSE
# Check class of my_numeric
class(my_numeric)

# Check class of my_character
class(my_character)

# Check class of my_logical
class(my_logical)
```

Vectors

Create a vector

Feeling lucky? You better, because this chapter takes you on a trip to the City of Sins, also known as Statisticians Paradise!

Thanks to R and your new data-analytical skills, you will learn how to uplift your performance at the tables and fire off your career as a professional gambler. This chapter will show how you can easily keep track of your betting progress and how you can do some simple analyses on past actions. Next stop, Vegas Baby... VEGAS!!

Instructions 100 XP

- Do you still remember what you have learned in the first chapter? Assign the value "Go!" to the variable vegas. Remember: R is case sensitive!

ex__08.R

```
# Define the variable vegas  
vegas <- "Go!"
```

Create a vector (2)

Let us focus first!

On your way from rags to riches, you will make extensive use of vectors. Vectors are one-dimension arrays that can hold numeric data, character data, or logical data. In other words, a vector is a simple tool to store data. For example, you can store your daily gains and losses in the casinos.

In R, you create a vector with the combine function `c()`. You place the vector elements separated by a comma between the parentheses.

i For example:

```
numeric_vector <- c(1, 2, 3)
character_vector <- c("a", "b", "c")
```

Once you have created these vectors in R, you can use them to do calculations.

Instructions 100 XP

Complete the code such that `boolean_vector` contains the three elements: `TRUE`, `FALSE` and `TRUE` (in that order).

ex__09.R

```
numeric_vector <- c(1, 10, 49)
character_vector <- c("a", "b", "c")

# Complete the code for boolean_vector
boolean_vector <- c(TRUE, FALSE, TRUE)
```

Create a vector (3)

After one week in Las Vegas and still zero Ferraris in your garage, you decide that it is time to start using your data analytical superpowers.

Before doing a first analysis, you decide to first collect all the winnings and losses for the last week:

For `poker_vector`:

- On Monday you won \$140
- Tuesday you lost \$50
- Wednesday you won \$20
- Thursday you lost \$120
- Friday you won \$240

For `roulette_vector`:

- On Monday you lost \$24
- Tuesday you lost \$50
- Wednesday you won \$100
- Thursday you lost \$350

- Friday you won \$10

You only played poker and roulette, since there was a delegation of mediums that occupied the craps tables. To be able to use this data in R, you decide to create the variables `poker_vector` and `roulette_vector`.

Instructions 100 XP

Assign the winnings/losses for roulette to the variable `roulette_vector`. You lost \$24, then lost \$50, won \$100, lost \$350, and won \$10.

ex_10.R

```
# Poker winnings from Monday to Friday
poker_vector <- c(140, -50, 20, -120, 240)

# Roulette winnings from Monday to Friday
roulette_vector <- c(-24, -50, 100, -350, 10)
```

Naming a vector

As a data analyst, it is important to have a clear view on the data that you are using. Understanding what each element refers to is therefore essential.

In the previous exercise, we created a vector with your winnings over the week. Each vector element refers to a day of the week but it is hard to tell which element belongs to which day. It would be nice if you could show that in the vector itself.

You can give a name to the elements of a vector with the `names()` function. Have a look at this example:

```
#| code-line-numbers: false
#| code-fold: false
#| code-summary: "Show the code"

some_vector <- c("John Doe", "poker player")
names(some_vector) <- c("Name", "Profession")
```

This code first creates a vector `some_vector` and then gives the two elements a name. The first element is assigned the name `Name`, while the second element is labeled `Profession`. Printing the contents to the console yields following output:

Output

Name	Profession
"John Doe"	"poker player"

Instructions 100 XP

The code in the editor names the elements in `poker_vector` with the days of the week. Add code to do the same thing for `roulette_vector`.

ex_11.R

```
# Poker winnings from Monday to Friday
poker_vector <- c(140, -50, 20, -120, 240)

# Roulette winnings from Monday to Friday
roulette_vector <- c(-24, -50, 100, -350, 10)

# Assign days as names of poker_vector
names(poker_vector) <-
  c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")

# Assign days as names of roulette_vector

names(roulette_vector) <-
  c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
```

Naming a vector (2)

If you want to become a good statistician, you have to become lazy. (If you are already lazy, chances are high you are one of those exceptional, natural-born statistical talents.)

In the previous exercises you probably experienced that it is boring and frustrating to type and retype information such as the days of the week. However, when you look at it from a higher perspective, there is a more efficient way to do this, namely, to assign the days of the week vector to a **variable**!

Just like you did with your poker and roulette returns, you can also create a variable that contains the days of the week. This way you can use and re-use it.

Instructions 100 XP

- A variable `days_vector` that contains the days of the week has already been created for you.
- Use `days_vector` to set the names of `poker_vector` and `roulette_vector`.

ex_12.R

```
# Poker winnings from Monday to Friday
poker_vector <- c(140, -50, 20, -120, 240)

# Roulette winnings from Monday to Friday
roulette_vector <- c(-24, -50, 100, -350, 10)

# The variable days_vector
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")

# Assign the names of the day to roulette_vector and poker_vector
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector
```

Calculating total winnings

Now that you have the poker and roulette winnings nicely as named vectors, you can start doing some data analytical magic.

You want to find out the following type of information:

- How much has been your overall profit or loss per day of the week?
- Have you lost money over the week in total?
- Are you winning/losing money on poker or on roulette? To get the answers, you have to do arithmetic calculations on vectors.

It is important to know that if you sum two vectors in R, it takes the element-wise sum. For example, the following three statements are completely equivalent:

You can also do the calculations with variables that represent vectors:

Instructions 100 XP

- Take the sum of the variables `A_vector` and `B_vector` and assign it to `total_vector`.
- Inspect the result by printing out `total_vector`.

`ex__13.R`

```
A_vector <- c(1, 2, 3)
B_vector <- c(4, 5, 6)

# Take the sum of A_vector and B_vector
total_vector <- A_vector + B_vector

# Print out total_vector
print(total_vector)
```

Calculating total winnings (2)

Now you understand how R does arithmetic with vectors, it is time to get those Ferraris in your garage! First, you need to understand what the overall profit or loss per day of the week was. The total daily profit is the sum of the `profit / loss` you realized on poker per day, and the `profit / loss` you realized on roulette per day.

In R, this is just the sum of `roulette_vector` and `poker_vector`.

Instructions 100 XP

Assign to the variable `total_daily` how much you won or lost on each day in total (poker and roulette combined).

`ex__14.R`

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Assign to total_daily how much you won/lost on each day
total_daily <- roulette_vector + poker_vector
```


Calculating total winnings (3)

Based on the previous analysis, it looks like you had a mix of good and bad days. This is not what your ego expected, and you wonder if there may be a very tiny chance you have lost money over the week in total?

A function that helps you to answer this question is `sum()`. It calculates the sum of all elements of a vector. For example, to calculate the total amount of money you have lost/won with poker you do:

```
total_poker <- sum(poker_vector)
```

Instructions 100 XP

- Calculate the total amount of money that you have won/lost with roulette and assign to the variable `total_roulette`.
- Now that you have the totals for roulette and poker, you can easily calculate `total_week` (which is the sum of all gains and losses of the week).
- Print out `total_week`.

ex_15.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Total winnings with poker
total_poker <- sum(poker_vector)

# Total winnings with roulette
total_roulette <- sum(roulette_vector)

# Total winnings overall
total_week <- total_poker + total_roulette

# Print out total_week
print(total_week)
```

Comparing total winnings

Oops, it seems like you are losing money. Time to rethink and adapt your strategy! This will require some deeper analysis...

After a short brainstorm in your hotel's jacuzzi, you realize that a possible explanation might be that your skills in roulette are not as well developed as your skills in poker. So maybe your total gains in poker are higher (or $>$) than in roulette.

Instructions 100 XP

- Calculate `total_poker` and `total_roulette` as in the previous exercise. Use the `sum()` function twice.
- Check if your total gains in poker are higher than for roulette by using a comparison. Simply print out the result of this comparison. What do you conclude, should you focus on roulette or on poker?

ex_16.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Calculate total gains for poker and roulette
total_poker <- sum(poker_vector)
total_roulette <- sum(roulette_vector)

# Check if you realized higher total gains in poker than in roulette

print(total_poker > total_roulette)
```

Vector selection: the good times

Your hunch seemed to be right. It appears that the poker game is more your cup of tea than roulette.

Another possible route for investigation is your performance at the beginning of the working week compared to the end of it. You did have a couple of Margarita cocktails at the end of the week...

To answer that question, you only want to focus on a selection of the `total_vector`. In other words, our goal is to select specific elements of the vector. To select elements of a vector (and later matrices, data frames, ...), you can use square brackets. Between the square brackets, you indicate what elements to select. For example, to select the first element of the vector, you type `poker_vector[1]`. To select the second element of the vector, you type `poker_vector[2]`, etc. Notice that the first element in a vector has index 1, not 0 as in many other programming languages.

Instructions 100 XP

Assign the poker results of Wednesday to the variable `poker_wednesday`.

ex_17.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Define a new variable based on a selection
poker_wednesday <- poker_vector[3]
```

Vector selection: the good times (2)

How about analyzing your midweek results?

To select multiple elements from a vector, you can add square brackets at the end of it. You can indicate between the brackets what elements should be selected. For example: suppose you want to select the first and the fifth day of the week: use the vector `c(1, 5)` between the square brackets. For example, the code below selects the first and fifth element of `poker_vector`:

```
poker_vector[c(1, 5)]
```

Instructions 100 XP

Assign the poker results of Tuesday, Wednesday and Thursday to the variable `poker_midweek`.

ex_18.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Define a new variable based on a selection
poker_midweek <- poker_vector[c(2, 3, 4)]
```

Vector selection: the good times (3)

Selecting multiple elements of `poker_vector` with `c(2, 3, 4)` is not very convenient. Many statisticians are lazy people by nature, so they created an easier way to do this: `c(2, 3, 4)` can be abbreviated to `2:4`, which generates a vector with all natural numbers from 2 up to 4.

So, another way to find the mid-week results is `poker_vector[2:4]`. Notice how the vector `2:4` is placed between the square brackets to select element 2 up to 4.

Instructions 100 XP

Assign to `roulette_selection_vector` the roulette results from Tuesday up to Friday; make use of `:` if it makes things easier for you.

ex_19.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Define a new variable based on a selection
```

```
roulette_selection_vector <- roulette_vector[2:5]
```

Vector selection: the good times (4)

Another way to tackle the previous exercise is by using the names of the vector elements (Monday, Tuesday, ...) instead of their numeric positions. For example,

```
poker_vector[c("Monday")]
```

will select the first element of `poker_vector` since “Monday” is the name of that first element.

Just like you did in the previous exercise with numerics, you can also use the element names to select multiple elements, for example:

```
poker_vector[c("Monday", "Tuesday")]
```

Instructions 100 XP

- Select the first three elements in `poker_vector` by using their names: "Monday", "Tuesday" and "Wednesday". Assign the result of the selection to `poker_start`.
- Calculate the average of the values in `poker_start` with the `mean()` function. Simply print out the result so you can inspect it.

ex_20.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Select poker results for Monday, Tuesday and Wednesday
poker_start <- poker_vector[c("Monday", "Tuesday", "Wednesday")]

# Calculate the average of the elements in poker_start
mean(poker_start)
```

Selection by comparison - Step 1

By making use of comparison operators, we can approach the previous question in a more proactive way.

The (logical) comparison operators known to R are:

- < for less than
- > for greater than
- <= for less than or equal to
- >= for greater than or equal to
- == for equal to each other
- != not equal to each other

As seen in the previous chapter, stating `6 > 5` returns `TRUE`. The nice thing about R is that you can use these comparison operators also on vectors. For example:

```
[1] FALSE FALSE TRUE
```

This command tests for every element of the vector if the condition stated by the comparison operator is `TRUE` or `FALSE`.

Instructions 100 XP

- Check which elements in `poker_vector` are positive (i.e. `> 0`) and assign this to `selection_vector`.
- Print out `selection_vector` so you can inspect it. The printout tells you whether you won (`TRUE`) or lost (`FALSE`) any money for each day.

ex_21.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Which days did you make money on poker?
selection_vector <-
  poker_vector > 0
```

```
# Print out selection_vector
print(selection_vector)
```

Selection by comparison - Step 2

Working with comparisons will make your data analytical life easier. Instead of selecting a subset of days to investigate yourself (like before), you can simply ask R to return only those days where you realized a positive return for poker.

In the previous exercises you used `selection_vector <- poker_vector > 0` to find the days on which you had a positive poker return. Now, you would like to know not only the days on which you won, but also how much you won on those days.

You can select the desired elements, by putting `selection_vector` between the square brackets that follow `poker_vector`:

```
poker_vector[selection_vector]
```

R knows what to do when you pass a logical vector in square brackets: it will only select the elements that correspond to `TRUE` in `selection_vector`.

Instructions 100 XP

Use `selection_vector` in square brackets to assign the amounts that you won on the profitable days to the variable `poker_winning_days`.

ex__22.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Which days did you make money on poker?
selection_vector <- poker_vector > 0

# Select from poker_vector these days
poker_winning_days <- poker_vector[selection_vector]
```

Advanced selection

Just like you did for poker, you also want to know those days where you realized a positive return for roulette.

Instructions 100 XP

- Create the variable `selection_vector`, this time to see if you made profit with roulette for different days.
- Assign the amounts that you made on the days that you ended positively for roulette to the variable `roulette_winning_days`. This vector thus contains the positive winnings of `roulette_vector`.

ex_23.R

```
# Poker and roulette winnings from Monday to Friday:
poker_vector <- c(140, -50, 20, -120, 240)
roulette_vector <- c(-24, -50, 100, -350, 10)
days_vector <- c("Monday", "Tuesday", "Wednesday", "Thursday", "Friday")
names(poker_vector) <- days_vector
names(roulette_vector) <- days_vector

# Which days did you make money on roulette?
selection_vector <- roulette_vector > 0

# Select from roulette_vector these days
roulette_winning_days <- roulette_vector[selection_vector]
```


Matrices

In this chapter, you will learn how to work with matrices in R. By the end of the chapter, you will be able to create matrices and understand how to do basic computations with them. You will analyze the box office numbers of the Star Wars movies and learn how to use matrices in R. May the force be with you!

What's a matrix?

In R, a matrix is a collection of elements of the same data type (numeric, character, or logical) arranged into a fixed number of rows and columns. Since you are only working with rows and columns, a matrix is called two-dimensional.

You can construct a matrix in R with the `matrix()` function. Consider the following example:

```
matrix(1:9, byrow = TRUE, nrow = 3)
```

In the `matrix()` function:

- The first argument is the collection of elements that R will arrange into the rows and columns of the matrix. Here, we use `1:9` which is a shortcut for `c(1, 2, 3, 4, 5, 6, 7, 8, 9)`.
- The argument `byrow` indicates that the matrix is filled by the rows. If we want the matrix to be filled by the columns, we just place `byrow = FALSE`.
- The third argument `nrow` indicates that the matrix should have three rows.

Instructions 100 XP

Construct a matrix with 3 rows containing the numbers 1 up to 9, filled row-wise.

ex__24.R

```
# Construct a matrix with 3 rows that contain the numbers 1 up to 9
matrix(1:9, byrow = TRUE, nrow = 3)
```

Analyze matrices, you shall

It is now time to get your hands dirty. In the following exercises you will analyze the box office numbers of the Star Wars franchise. May the force be with you!

In the editor, three vectors are defined. Each one represents the box office numbers from the first three Star Wars movies. The first element of each vector indicates the US box office revenue, the second element refers to the Non-US box office (source: Wikipedia).

In this exercise, you'll combine all these figures into a single vector. Next, you'll build a matrix from this vector.

Instructions 100 XP

- Use `c(new_hope, empire_strikes, return_jedi)` to combine the three vectors into one vector. Call this vector `box_office`.
- Construct a matrix with 3 rows, where each row represents a movie. Use the `matrix()` function to do this. The first argument is the vector `box_office`, containing all box office figures. Next, you'll have to specify `nrow = 3` and `byrow = TRUE`. Name the resulting matrix `star_wars_matrix`.

ex_25.R

```
# Box office Star Wars (in millions!)
new_hope <- c(460.998, 314.4)
empire_strikes <- c(290.475, 247.900)
return_jedi <- c(309.306, 165.8)

# Create box_office
box_office <- c(new_hope, empire_strikes, return_jedi)

# Construct star_wars_matrix
star_wars_matrix <- matrix(box_office, nrow = 3, byrow = TRUE)
```

Naming a matrix

To help you remember what is stored in `star_wars_matrix`, you would like to add the names of the movies for the rows. Not only does this help you to read the data, but it is also useful to select certain elements from the matrix.

Similar to vectors, you can add names for the rows and the columns of a matrix

```
rownames(my_matrix) <- row_names_vector
colnames(my_matrix) <- col_names_vector
```

We went ahead and prepared two vectors for you: `region`, and `titles`. You will need these vectors to name the columns and rows of `star_wars_matrix`, respectively.

Instructions 100 XP

- Use `colnames()` to name the columns of `star_wars_matrix` with the `region` vector.
- Use `rownames()` to name the rows of `star_wars_matrix` with the `titles` vector.
- Print out `star_wars_matrix` to see the result of your work.

`ex_26.R`

```
# Box office Star Wars (in millions!)
new_hope <- c(460.998, 314.4)
empire_strikes <- c(290.475, 247.900)
return_jedi <- c(309.306, 165.8)

# Construct matrix
star_wars_matrix <- matrix(c(new_hope, empire_strikes, return_jedi), nrow = 3, byrow = TRUE)

# Vectors region and titles, used for naming
region <- c("US", "non-US")
titles <- c("A New Hope", "The Empire Strikes Back", "Return of the Jedi")

# Name the columns with region
colnames(star_wars_matrix) <- region

# Name the rows with titles
rownames(star_wars_matrix) <- titles

# Print out star_wars_matrix
print(star_wars_matrix)
```

Calculating the worldwide box office

The single most important thing for a movie in order to become an instant legend in Tinseltown is its worldwide box office figures.

To calculate the total box office revenue for the three Star Wars movies, you have to take the sum of the US revenue column and the non-US revenue column.

In R, the function `rowSums()` conveniently calculates the totals for each row of a matrix. This function creates a new vector:

```
rowSums(my_matrix)
```

Instructions 100 XP

Calculate the worldwide box office figures for the three movies and put these in the vector named `worldwide_vector`.

ex__26.R

```
# Construct star_wars_matrix
box_office <- c(460.998, 314.4, 290.475, 247.900, 309.306, 165.8)
region <- c("US", "non-US")
titles <- c("A New Hope",
            "The Empire Strikes Back",
            "Return of the Jedi")

star_wars_matrix <- matrix(box_office,
                           nrow = 3, byrow = TRUE,
                           dimnames = list(titles, region))

# Calculate worldwide box office figures
worldwide_vector <- rowSums(star_wars_matrix)
```

Adding a column for the Worldwide box office

In the previous exercise you calculated the vector that contained the worldwide box office receipt for each of the three Star Wars movies. However, this vector is not yet part of `star_wars_matrix`.

You can add a column or multiple columns to a matrix with the `cbind()` function, which merges matrices and/or vectors together by column. For example:

```
big_matrix <- cbind(matrix1, matrix2, vector1 ...)
```

Instructions 100 XP

Add `worldwide_vector` as a new column to the `star_wars_matrix` and assign the result to `all_wars_matrix`. Use the `cbind()` function.

ex__27.R

```
# Construct star_wars_matrix
box_office <- c(460.998, 314.4, 290.475, 247.900, 309.306, 165.8)
region <- c("US", "non-US")
titles <- c("A New Hope",
            "The Empire Strikes Back",
            "Return of the Jedi")

star_wars_matrix <- matrix(box_office,
                           nrow = 3, byrow = TRUE,
                           dimnames = list(titles, region))

# The worldwide box office figures
worldwide_vector <- rowSums(star_wars_matrix)

# Bind the new variable worldwide_vector as a column to star_wars_matrix
all_wars_matrix <- cbind(star_wars_matrix, worldwide_vector)
```

Adding a row

Just like every action has a reaction, every `cbind()` has an `rbind()`. (We admit, we are pretty bad with metaphors.)

Your R workspace, where all variables you defined ‘live’ (check out what a workspace is), has already been initialized and contains two matrices:

- `star_wars_matrix` that we have used all along, with data on the original trilogy,
- `star_wars_matrix2`, with similar data for the prequels trilogy.

Explore these matrices in the console if you want to have a closer look. If you want to check out the contents of the workspace, you can type `ls()` in the console.

Instructions 100 XP

Use `rbind()` to paste together `star_wars_matrix` and `star_wars_matrix2`, in this order. Assign the resulting matrix to `all_wars_matrix`.

ex_28.R

```
# star_wars_matrix and star_wars_matrix2 are available in your workspace
star_wars_matrix
star_wars_matrix2

# Combine both Star Wars trilogies in one matrix
all_wars_matrix <- rbind(star_wars_matrix, star_wars_matrix2)
```

The total box office revenue for the entire saga

Just like `cbind()` has `rbind()`, `colSums()` has `rowSums()`. Your R workspace already contains the `all_wars_matrix` that you constructed in the previous exercise; type `all_wars_matrix` to have another look. Let's now calculate the total box office revenue for the entire saga.

Instructions 100 XP

- Calculate the total revenue for the US and the non-US region and assign `total_revenue_vector`. You can use the `colSums()` function.
- Print out `total_revenue_vector` to have a look at the results.

ex_29.R

```
# all_wars_matrix is available in your workspace
all_wars_matrix

# Total revenue for US and non-US
total_revenue_vector <- colSums(all_wars_matrix)

# Print out total_revenue_vector
print(total_revenue_vector)
```

Selection of matrix elements

Similar to vectors, you can use the square brackets `[]` to select one or multiple elements from a matrix. Whereas vectors have one dimension, matrices have two dimensions. You should therefore use a comma to separate the rows you want to select from the columns. For example:

- `my_matrix[1,2]` selects the element at the first row and second column.
- `my_matrix[1:3,2:4]` results in a matrix with the data on the rows 1, 2, 3 and columns 2, 3, 4.

If you want to select all elements of a row or a column, no number is needed before or after the comma, respectively:

- `my_matrix[,1]` selects all elements of the first column.
- `my_matrix[1,]` selects all elements of the first row.

Back to Star Wars with this newly acquired knowledge! As in the previous exercise, `all_wars_matrix` is already available in your workspace.

Instructions 100 XP

- Select the non-US revenue for all movies (the entire second column of `all_wars_matrix`), store the result as `non_us_all`.
- Use `mean()` on `non_us_all` to calculate the average non-US revenue for all movies. Simply print out the result.
- This time, select the non-US revenue for the first two movies in `all_wars_matrix`. Store the result as `non_us_some`.
- Use `mean()` again to print out the average of the values in `non_us_some`.

ex__30.R

```
# all_wars_matrix is available in your workspace
all_wars_matrix

# Select the non-US revenue for all movies
non_us_all <- all_wars_matrix[,2]

# Average non-US revenue
print(mean(non_us_all))

# Select the non-US revenue for first two movies
non_us_some <- all_wars_matrix[1:2, 2]

# Average non-US revenue for first two movies
print(mean(non_us_some))
```

A little arithmetic with matrices

Similar to what you have learned with vectors, the standard operators like `+`, `-`, `/`, `*`, etc. work in an element-wise way on matrices in R.

For example, `2 * my_matrix` multiplies each element of `my_matrix` by two.

As a newly-hired data analyst for Lucasfilm, it is your job to find out how many visitors went to each movie for each geographical area. You already have the total revenue figures in `all_wars_matrix`. Assume that the price of a ticket was 5 dollars. Simply dividing the box office numbers by this ticket price gives you the number of visitors.

Instructions 100 XP

- Divide `all_wars_matrix` by 5, giving you the number of visitors in millions.
- Assign the resulting matrix to `visitors`.
- Print out `visitors` so you can have a look.

ex_31.R

```
# all_wars_matrix is available in your workspace
all_wars_matrix

# Estimate the visitors
visitors <- all_wars_matrix / 5

# Print the estimate to the console
print(visitors)
```

A little arithmetic with matrices (2)

Just like `2 * my_matrix` multiplied every element of `my_matrix` by two, `my_matrix1 * my_matrix2` creates a matrix where each element is the product of the corresponding elements in `my_matrix1` and `my_matrix2`.

After looking at the result of the previous exercise, big boss Lucas points out that the ticket prices went up over time. He asks to redo the analysis based on the prices you can find in `ticket_prices_matrix` (source: imagination).

Those who are familiar with matrices should note that this is not the standard matrix multiplication for which you should use `%*%` in R.

Instructions 100 XP

- Divide `all_wars_matrix` by `ticket_prices_matrix` to get the estimated number of US and non-US visitors for the six movies. Assign the result to `visitors`.
- From the `visitors` matrix, select the entire first column, representing the number of visitors in the US. Store this selection as `us_visitors`.
- Calculate the average number of US visitors; print out the result.

ex_32.R

```
# all_wars_matrix and ticket_prices_matrix are available in your workspace
all_wars_matrix
ticket_prices_matrix

# Estimated number of visitors
visitors <- all_wars_matrix / ticket_prices_matrix

# US visitors
us_visitors <- visitors[, 1]

# Average number of US visitors
print(mean(us_visitors))
```

Factors

Data often falls into a limited number of categories. For example, human hair color can be categorized as black, brown, blond, red, grey, or white—and perhaps a few more options for people who color their hair. In R, categorical data is stored in factors. Factors are very important in data analysis, so start learning how to create, subset, and compare them now.

What’s a factor and why would you use it?

In this chapter you dive into the wonderful world of factors.

The term factor refers to a statistical data type used to store categorical variables. The difference between a categorical variable and a continuous variable is that a categorical variable can belong to a limited number of categories. A continuous variable, on the other hand, can correspond to an infinite number of values.

It is important that R knows whether it is dealing with a continuous or a categorical variable, as the statistical models you will develop in the future treat both types differently. (You will see later why this is the case.)

A good example of a categorical variable is sex. In many circumstances you can limit the sex categories to “Male” or “Female”. (Sometimes you may need different categories. For example, you may need to consider chromosomal variation, hermaphroditic animals, or different cultural norms, but you will always have a finite number of categories.)

Instructions 100 XP

Assign to variable `theory` the value “factors”.

ex_33.R

```
# Assign to the variable theory what this chapter is about!
theory <- "factors"
```

What's a factor and why would you use it? (2)

To create factors in R, you make use of the function `factor()`. First thing that you have to do is create a vector that contains all the observations that belong to a limited number of categories. For example, `sex_vector` contains the sex of 5 different individuals:

```
sex_vector <- c("Male","Female","Female","Male","Male")
```

It is clear that there are two categories, or in R-terms ‘factor levels’, at work here: “Male” and “Female”.

The function `factor()` will encode the vector as a factor:

```
factor_sex_vector <- factor(sex_vector)
```

Instructions 100 XP

- Convert the character vector `sex_vector` to a factor with `factor()` and assign the result to `factor_sex_vector`
- Print out `factor_sex_vector` and assert that R prints out the factor levels below the actual values.

ex__34.R

```
# Sex vector
sex_vector <- c("Male", "Female", "Female", "Male", "Male")

# Convert sex_vector to a factor
factor_sex_vector <- factor(sex_vector)

# Print out factor_sex_vector
print(factor_sex_vector)
```

What's a factor and why would you use it? (3)

There are two types of categorical variables: a nominal categorical variable and an ordinal categorical variable.

A nominal variable is a categorical variable without an implied order. This means that it is impossible to say that ‘one is worth more than the other’. For example, think of the categorical variable `animals_vector` with the categories “Elephant”, “Giraffe”, “Donkey” and

“Horse”. Here, it is impossible to say that one stands above or below the other. (Note that some of you might disagree ;-)).

In contrast, ordinal variables do have a natural ordering. Consider for example the categorical variable `temperature_vector` with the categories: “Low”, “Medium” and “High”. Here it is obvious that “Medium” stands above “Low”, and “High” stands above “Medium”.

Instructions 100 XP

Submit the answer to check how R constructs and prints nominal and ordinal variables. Do not worry if you do not understand all the code just yet, we will get to that.

ex_35.R

```
# Animals
animals_vector <- c("Elephant", "Giraffe", "Donkey", "Horse")
factor_animals_vector <- factor(animals_vector)
factor_animals_vector

# Temperature
temperature_vector <- c("High", "Low", "High", "Low", "Medium")
factor_temperature_vector <-
  factor(
    temperature_vector,
    order = TRUE,
    levels = c("Low", "Medium", "High")
  )
factor_temperature_vector
```

Factor levels

When you first get a dataset, you will often notice that it contains factors with specific factor levels. However, sometimes you will want to change the names of these levels for clarity or other reasons. R allows you to do this with the function `levels()`:

```
levels(factor_vector) <- c("name1", "name2", ...)
```

A good illustration is the raw data that is provided to you by a survey. A common question for every questionnaire is the sex of the respondent. Here, for simplicity, just two categories were recorded, “M” and “F”. (You usually need more categories for survey data; either way, you use a factor to store the categorical data.)

```
survey_vector <- c("M", "F", "F", "M", "M")
```

Recording the sex with the abbreviations "M" and "F" can be convenient if you are collecting data with pen and paper, but it can introduce confusion when analyzing the data. At that point, you will often want to change the factor levels to "Male" and "Female" instead of "M" and "F" for clarity.

Watch out: the order with which you assign the levels is important. If you type `levels(factor_survey_vector)`, you'll see that it outputs `[1] "F" "M"`. If you don't specify the levels of the factor when creating the vector, R will automatically assign them alphabetically. To correctly map "F" to "Female" and "M" to "Male", the levels should be set to `c("Female", "Male")`, in this order.

Instructions 100 XP

- Check out the code that builds a factor vector from `survey_vector`. You should use `factor_survey_vector` in the next instruction.
- Change the factor levels of `factor_survey_vector` to `c("Female", "Male")`. Mind the order of the vector elements here.

ex_36.R

```
# Code to build factor_survey_vector
survey_vector <- c("M", "F", "F", "M", "M")
factor_survey_vector <- factor(survey_vector)

# Specify the levels of factor_survey_vector
levels(factor_survey_vector) <- c("F", "M")

levels(factor_survey_vector) <- c("Female", "Male")
```

Summarizing a factor

Battle of the sexes

Ordered factors

Ordered factors (2)

Comparing ordered factors

Best Coding Practices for R

What we mean when say “better coding practice”

R programmers have a bad reputation writing bad code. Perhaps the main reason is that the people whose write much of the package are not programmers but scientific from other areas. Sometimes we overestimate crucial aspects from a programming standpoint. As R programmers we overcome to write the code for production. Mostly we write scripts and when we deploy it the same when we just wrap it in a function and perhaps a package. It is common to face poorly written code—**columns were referred by numbers, functions were dependent upon global environment variables, 50+ lines functions without arguments and with over-sized lines code 100 characters or more, not indentation, poor naming, conventions** etc,...,.

We strongly encourage to use a style. Yea I know, there is not a unique way to do it, but the philosophy is to follow a consistent style. With respect to this regard made yourself a favor and read this great book for R

<https://bookdown.org/content/d1e53ac9-28ce-472f-bc2c-f499f18264a3/>

Folder Structure

Code Structure

Sections

Structural Composition

Indentation

Styling

Final Comments

Tidyverse Fundamentals with R

Introduction to Tidyverse

Reshaping Data with tidyr

Project

Modeling with Data in the Tidyverse

Communication with Data in the Tidyverse

Categorical Data in the Tydiverse

Data Manipulation

Data Manipulation with dplyr

Joining Data with dplyr

Case Study: Exploratory Data Analysis in R

Data Manipulation with data.table in R

Joining Data with data.table in R

1 Data visualization with ggplot2 and friends

Part II

The whole game of statistical Inference

Our goal in this part of the book is to give you a rapid overview of the main tools of data science: **importing**, **tidying**, **transforming**, and **visualizing data**, as shown in [?@fig-ds-whole-game](#). We want to show you the “whole game” of data science giving you just enough of all the major pieces so that you can tackle real, if simple, data sets. The later parts of the book, will hit each of these topics in more depth, increasing the range of data science challenges that you can tackle.

2 Statistical Inference with resampling: Bootstrap and Jackknife.

2.1 Likelihood inference.

2.2 Variance analysis.

2.3 ROC Curves

3 Linear Regression

3.1 Linear Regression

3.2 Multiple linear regression and generalized linear regression

4 Summary

In summary, this book has no content whatsoever.

[1] 2

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