# ARI2204 - Reinforcement Learning

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## Project Structure

Our project is structured as follows:

### Base directory

This includes the following files:

1. main.py
    - Main file from which the project is ran
2. evaluation.py
    - File containing helper functions used throughout the evaluation process
3. plots.py
    - Contains functions used to plot data gathered during the evaluation process.
4. requirements.txt
    - Contains the packages required to run our code in a virtual environment.
5. playBlackjack:).py
    - contains code allowing us to play our blackjack game for fun...

The base directory also includes the following subdirectories:

1. agents
    - For files containing agent classes.
2. game_structure
    - For files containing classes relevant to the structure of the blackjack game itself.

## Game Structure

The blackjack game is playable through a class in the file game_structure/Blackjack.py. The class uses classes such as Hand, cCard and Deck to make the game playable. It implements methods that allow different agents to play and learn from the game.

Different methods were imperative since the *Monte Carlo algorithm* updates its *Q-Values* at a different point in the round to the *SARSA* and *Q Learning* algorithms.

## General Agent Structure

The *MonteCarlo*, *Sarsa* and *QLearning* classes all inherit from *AgentABC*. The abstract base class defines the following global constants that are used throughout the project.

### Global Variables

```
STAND = 0
HIT = 1
```

```
LOSS = -1
WIN = 1
DRAW = 0

RANDOM = 0
BEST = 1
```

## Class Variables

The *AgentABC* defines the following class variables for all the different inheriting algorithms:

```
self.state = ()
self.epsilon_method = epsilon_method
self.episodes = 0
if current_episode is None:
    current_episode = []
self.current_episode = current_episode
self.state_action_values = defaultdict(default_q_values)
```

1. *state* is a tuple of the following structure:
    - (int: agent_hand_value, str: showing_dealer_card, bool: hasUsableAce)
2. *epsilon_method* is an integer that specified which of the following ways the algorithm is to calculate epsilon when deciding whether to explore or exploit:

```
"""
0 - 1/k
1 - e^-k/1000
2 - e^-k/10000
3 - 0.1
"""
```

3. *episodes* is an integer representing the count of episodes played
    - Includes the current episode being played
4. *current_episode* is an array containing tuples of the state action pairs played in the current episode.
    - (state: state, action: HIT/STAND)
5. *state_action_values* is a dictionary that stores the *Q-values* and counts of each state action pair observed so far
    - It is keyed by tuples containing state action pairs of the same structure defined in 4.
    - Each key's value is an array with 2 elements: [Q-value, Count]

## Methods

The *AgentABC* also defines the following methods that are use throughout the project:

1. set_state(self, state)

    - Allows the blackjack game to update the state of the agent with the current state.
2. new_episode(self)
    - Allows the blackjack game to prepare the agent for a new round.
3. get_episode(self)
    - Returns the current episode count.
4. get_state_action_value(self, index)
    - Returns the state action value stored at *index* of the current episode array.
5. get_state_action_values(self, index)
    - Returns the dictionary containing all the state action values explored, their *Q* values and their counts.
6. remove_state_action_value(self, index)
    - Removes the state action value stored at *index* of the current episode array.
7. _best_action(self)
    - Returns the best action for the current state of the agent (HIT or STAND)
8. get_action(self)
    - This is the method called by the blackjack game to determine whether the agent wants to HIT or STAND. The method automatically hits when it's hand is below 12 and stands on 21, otherwise it calls the abstract method _get_action(self)
9. _get*action(self) -> _abstract method*
    - All inheriting classes must implement this method. This method should return the algorithms choice of action for the current state.

## Algorithms Discussion

One should note that throughout our project we assume γ to be 1. It is for this reason that at points it does not appear in some of the equations.

### Monte Carlo Methods

**Definition and Mechanism**: Monte Carlo methods are a family of algorithms that rely on averaging sample returns to evaluate and improve policies. They do not require knowledge of the environment's dynamics and only update estimates based on complete episodes [1-2].

**How Monte Carlo Methods Work**:

1. **Initialize Q-values**: Start with an initial estimate of Q-values for all state-action pairs.
2. **Generate Episodes**: Generate episodes using the current policy until termination.
3. **Observe Returns**: Calculate the total return for each state-action pair encountered in the episode.
4. **Update Q-Value**: Update the Q-value for each state-action pair based on the average return observed: $$ Q(s, a) \leftarrow Q(s, a) + \alpha [G - Q(s, a)] $$ where ( G ) is the return following the state-action pair.
5. **Policy Improvement**: Update the policy to be greedy with respect to the updated Q-values.

**Characteristics**:

- **Model-Free**: Does not require a model of the environment.
- **Sample Efficiency**: Utilizes complete episodes to update values.
- **Exploration**: Naturally explores all state-action pairs encountered in episodes.

- **Stability and Convergence**: Can take longer to converge as it requires complete episodes for updates.

### SARSA (State-Action-Reward-State-Action)

**Definition and Mechanism**: SARSA is an on-policy temporal difference (TD) control algorithm. It updates its Q-values based on the action actually taken by the agent. The acronym SARSA stands for State-Action-Reward-State-Action, representing the tuple used to update the Q-value [1-2].

**How SARSA Works**:

1. **Initialize Q-values**: Start with an initial estimate of Q-values for all state-action pairs.
2. **Choose Action**: Use an epsilon-greedy policy to choose an action based on current Q-values.
3. **Observe Outcome**: Execute the action, observe the reward, and the next state.
4. **Choose Next Action**: Select the next action using the same epsilon-greedy policy.
5. **Update Q-Value**: Update the Q-value for the current state-action pair using the observed reward and the estimated Q-value of the next state-action pair: $$ Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)] $$
6. **Repeat**: Continue this process for each step of the episode until termination.

**Characteristics**:

- **On-Policy**: SARSA follows the policy being learned.
- **Exploration vs. Exploitation**: Balances exploration and exploitation via the epsilon-greedy policy.
- **Stability**: Generally more stable than off-policy methods but can be slower to converge.

### Q-Learning

**Definition and Mechanism**: Q-Learning is an off-policy TD control algorithm that seeks to find the optimal policy independently of the agent's actions. It updates its Q-values using the maximum possible reward of the next state, rather than the reward of the next state-action pair chosen by the policy [1-2].

**How Q-Learning Works**:

1. **Initialize Q-values**: Start with an initial estimate of Q-values for all state-action pairs.
2. **Choose Action**: Use an epsilon-greedy policy to choose an action based on current Q-values.
3. **Observe Outcome**: Execute the action, observe the reward, and the next state.
4. **Update Q-Value**: Update the Q-value for the current state-action pair using the observed reward and the maximum Q-value of the next state: $$ Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha [r_{t+1} + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)] $$
5. **Repeat**: Continue this process for each step of the episode until termination.

**Characteristics**:

- **Off-Policy**: Learns the optimal policy independently of the actions taken by the agent.
- **Exploration vs. Exploitation**: Uses epsilon-greedy policy for action selection.
- **Efficiency**: Can converge faster to the optimal policy but may be less stable due to aggressive updates.

## Monte Carlo Methods Implementation

The *MonteCarlo* class inherits from the *AgentABC*. It defines one new class variable:

- *exploring_starts* -> This is a boolean value that determines whether the algorithm will randomly select the first action of each episode.

## Methods

This algorithm implements the following 2 methods:

1. update_q_values(self, new_values, reward)

```
def update_q_values(self, new_values, reward):
    for q in new_values:
        # update count for states visited
        self.state_action_values[q][1] += 1
        count = self.state_action_values[q][1]

        # update state-action values for states visited
        q_value = self.state_action_values[q][0]
        self.state_action_values[q][0] = q_value + np.divide(1, count) *
(reward - q_value)
```

- Given a list of state-action pairs, for each pair it updates it's count in the state_action_values dictionary defined in the ABC.
- Next it updates the pair's *Q-value8 using the following formula: $$Q(s, a)*{n} = Q(s, a)*{n-1} + \frac{1}{count_{(s,a)}}\times{(reward - Q(s, a)_{n-1})}$$

1. _get*action(self) -> _abstract method*

```
def _get_action(self):
    if self.exploring_starts and len(self.current_episode) == 0:
        action = randint(STAND, HIT)
    else:
        if self.epsilon_method == 0:
            epsilon = np.divide(1, self.episodes)
        elif self.epsilon_method == 1:
            power = np.divide(self.episodes, 1000)
            epsilon = np.exp(-power)
        else:
            power = np.divide(self.episodes, 10000)
            epsilon = np.exp(-power)

        policy = choices([RANDOM, BEST], weights=(epsilon, 1 - epsilon),
k=1)
        if policy == RANDOM:
            action = randint(STAND, HIT)
        else:
            action = self._best_action()
    self.current_episode.append((self.state, action))
    return action
```

- This method checks whether exploring starts is True, if that is the case and we are in the first state of an episode, *action* is randomly set to either *HIT* or *STAND*.
- Otherwise, the algorithm calculates ε depending on the method specified at the agents creation.
- The method then decides whether to generate a random action (*explore*), or the best action (*exploit*) using *random.choices()*.
- Finally it appends the current state action pair to the current episodes array and returns the selected action.

### Updating *Q_values*

The *Q_values* for all states action pairs of an episode are updated once the blackjack round concludes, the call to the *update_q_values()* method defined above is made in the blackjack class jsut before terminating a round.

```
if agent_value > 21:
    reward = LOSS
elif dealer_value > 21:
    reward = WIN
elif agent_value > dealer_value:
    reward = WIN
elif agent_value < dealer_value:
    reward = LOSS
else:
    reward = DRAW

self.agent.update_q_values(agent.get_episode(), reward)
return reward
```

## SARSA Implementation

The *SarsaControl* class inherits from the *AgentABC*.

### Methods

This class implements the following 2 methods:

1. update_q_value(self, 1, q_1, reward, is_final_state)

```
def update_q_values(self, q, q_1, reward, is_final_state):
    # count update
    self.state_action_values[q][1] += 1
    count = self.state_action_values[q][1]

    # q value update
    alpha = np.divide(1, (count + 1))
    q_value = self.state_action_values[q][0]
    next_q_value = self.state_action_values[q_1][0]
    if is_final_state:
        self.state_action_values[q][0] = q_value + (alpha * (reward -
q_value))

    else:
        self.state_action_values[q][0] = q_value + (alpha * (reward +
next_q_value - q_value))
```

- This method updates the *Q_values* at every state of the episode.
- It achieves this by first updating the count of the current state action pair.
- Next it calculates the new *Q_value* using the following formula: $$Q(s, a)*n = Q(s,a)*{n-1} + \frac{1}{count + 1}\times(reward + Q(s, a) - Q(s,a)_{n-1})$$

1. _get*action(self) -> _abstract_method*

```
def _get_action(self):
    if self.epsilon_method == 0:
        epsilon = np.divide(1, self.episodes)
    elif self.epsilon_method == 1:
        power = np.divide(self.episodes, 1000)
        epsilon = np.exp(-power)
    elif self.epsilon_method == 2:
        power = np.divide(self.episodes, 10000)
        epsilon = np.exp(-power)
    else:
        epsilon = 0.1

    policy = choices([RANDOM, BEST], weights=(epsilon, 1 - epsilon), k=1)
    if policy == RANDOM:
        action = randint(STAND, HIT)
    else:
        action = self._best_action()

    self.current_episode.append((self.state, action))
    return action
```

- This method implements the logic behind choosing which action to play in much the same way as Monte Carlo Control.
- First calculating ε, then choosing to explore or exploit, appending the state action pair to current episodes and finally returning the selected action.

### Updating *Q_values*

Unlike Monte Carlo methods, the *Q_values* of this algorithm are updated using the predefined method at the end of each state with the relevant rewards:

- 1 if final state and agent won

- 0 if draw or agent neither wins or looses after playing the action
- -1 if the player busts or looses

```
def play_round_SARSA_Q(self, agent):
    self.initialise_round(agent)

    # deal cards
    for _ in range(2):
        self.agent_hand.add_card(self.deck.draw_card())
        self.dealer_hand.add_card(self.deck.draw_card())

    # agent actions
    while True:
        self._set_state(self.agent_hand, self.dealer_hand)
        self.agent.set_state(self.state)
        if self.agent.get_action() == STAND:
            break

        self.agent_hand.add_card(self.deck.draw_card())
        if self._is_bust(self.agent_hand):
            agent.update_q_values(agent.get_state_action_value(-1), None,
LOSS, True)
            return LOSS
        if len(agent.get_episode()) == 2:
            agent.update_q_values(agent.get_state_action_value(-2),
                                  agent.get_state_action_value(-1), 0,
False)
            agent.remove_state_action_value(0)

    # dealer actions
    while True:
        self.dealer.set_hand(self.dealer_hand)
        if self.dealer.get_action() == STAND:
            break

        self.dealer_hand.add_card(self.deck.draw_card())
        if self._is_bust(self.dealer_hand):
            return WIN

    # calculate the winner
    agent_value = self.agent_hand.calculate_hand_value()
    dealer_value = self.dealer_hand.calculate_hand_value()

    if agent_value > dealer_value:
        # Player gets blackjack
        if len(agent.get_episode()) == 1:
            agent.update_q_values(agent.get_state_action_value(-1), None,
WIN, True)
        else:
            agent.update_q_values(agent.get_state_action_value(-2),
                                  agent.get_state_action_value(-1), WIN,
True)
```

```
        return WIN

    if agent_value < dealer_value:
        return LOSS

    return DRAW
```

## Q Learning Implementation

The *QlearningControl* class inherits from the *AgentABC*.

### Methods

It implements the following 2 methods:

1. def update_q_values(self, q, q_1, reward, is_final_state)

```
def update_q_values(self, q, q_1, reward, is_final_state):
    # count update
    self.state_action_values[q][1] += 1
    count = self.state_action_values[q][1]

    # q value update
    alpha = np.divide(1, (count + 1))
    q_value = self.state_action_values[q][0]
    if is_final_state:
        self.state_action_values[q][0] = q_value + (alpha * (reward -
q_value))
    else:
        next_state = q_1[0]
        next_q_value = max(self.state_action_values[(next_state, HIT)][0],
                           self.state_action_values[(next_state, STAND)]
[0])
        self.state_action_values[q][0] = q_value + (alpha * (reward +
next_q_value - q_value))
```

- This function updates the current *Q_value* in a very similar way to SARSA, the main difference being that instead of taking the *Q_value* of the next state action pair, it uses the max *Q_value* of the next state.

2. _getaction(self) -> _abstract method_

- This method agents next action in the exact same way as the *SarsaControl* class.

### Updating *Q_values*

This algorithm uses the same blackjack class method to play the game, thus it's values are updated at the exact same points as SARSA.

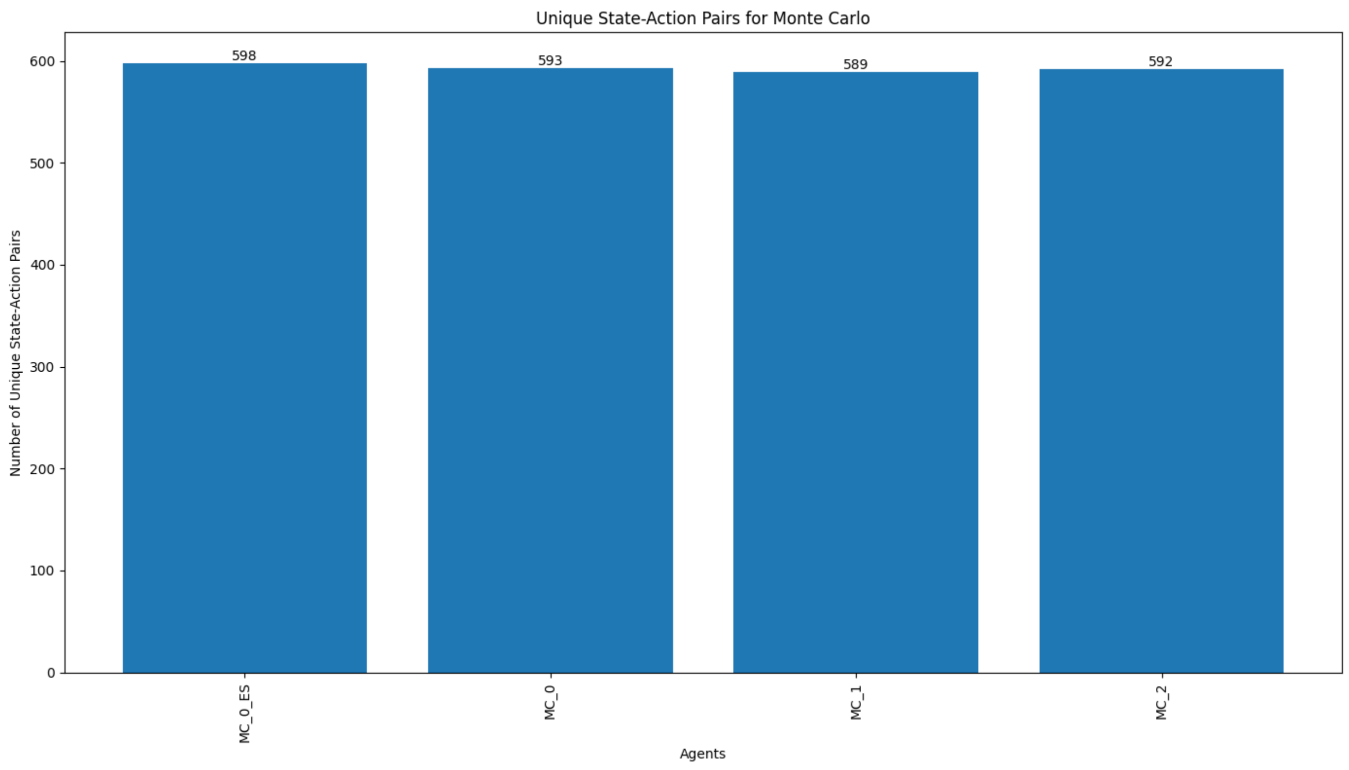
## Results and Analysis

The following functions were created to gather the relevant information for each agent:

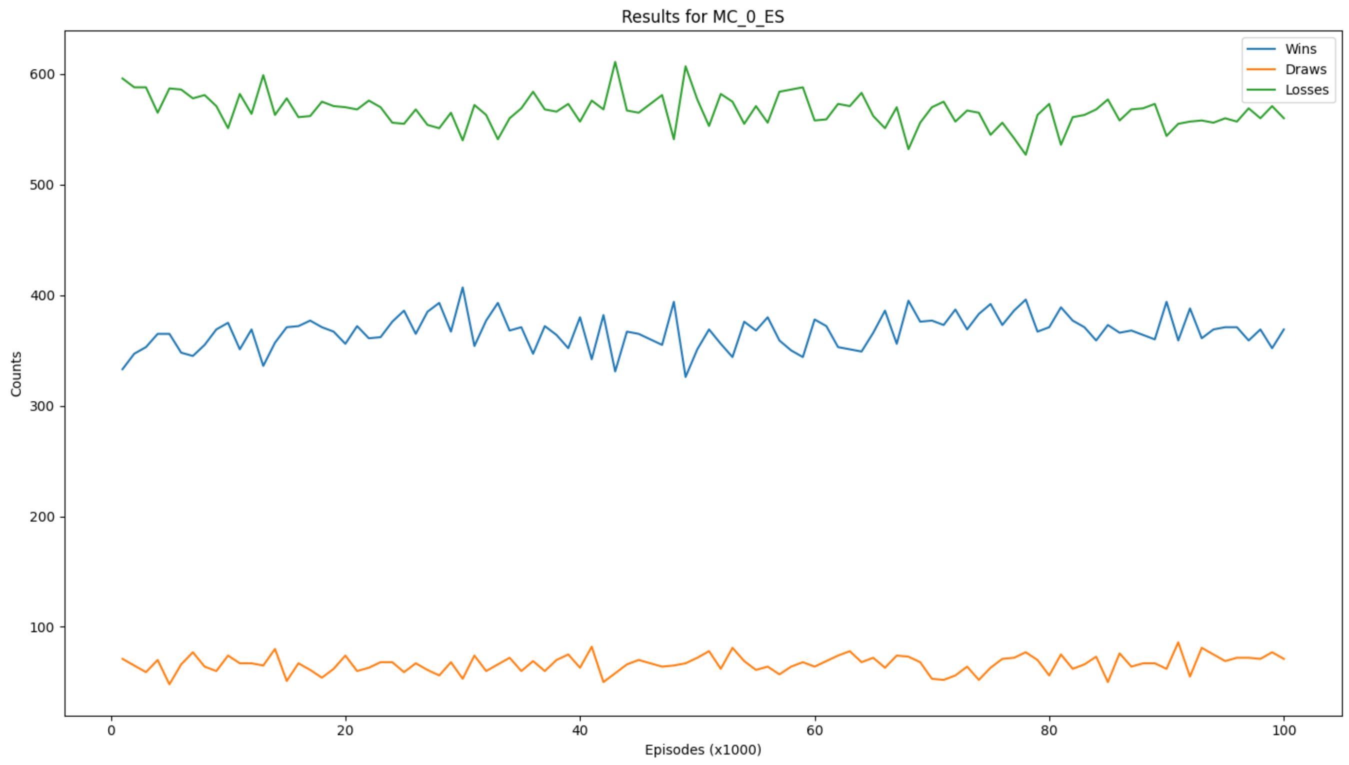
```python
def monte_carlo_round(agent, game, tally):
    reward = game.play_round_MC(agent)
    update_reward_tally(reward, tally)

def sarsa_q_round(agent, game, tally):
    reward = game.play_round_SARSA_Q(agent)
    update_reward_tally(reward, tally)

def update_reward_tally(reward, tally):
    if reward == WIN:
        tally[0] += 1
    elif reward == DRAW:
        tally[1] += 1
    else:
        tally[2] += 1

def run_evaluation(agent, game, num_episodes=100000, log_interval=1000):
    scores = []
    tally = [0, 0, 0]  # [wins, draws, losses]

    for episode in range(num_episodes):
        if isinstance(agent, MonteCarloControl):
            monte_carlo_round(agent, game, tally)
        else:
            sarsa_q_round(agent, game, tally)

        if (episode + 1) % log_interval == 0:
            scores.append(tally.copy())
            tally = [0, 0, 0]

    agent_values = agent.get_state_action_values()

    return scores, agent_values
```

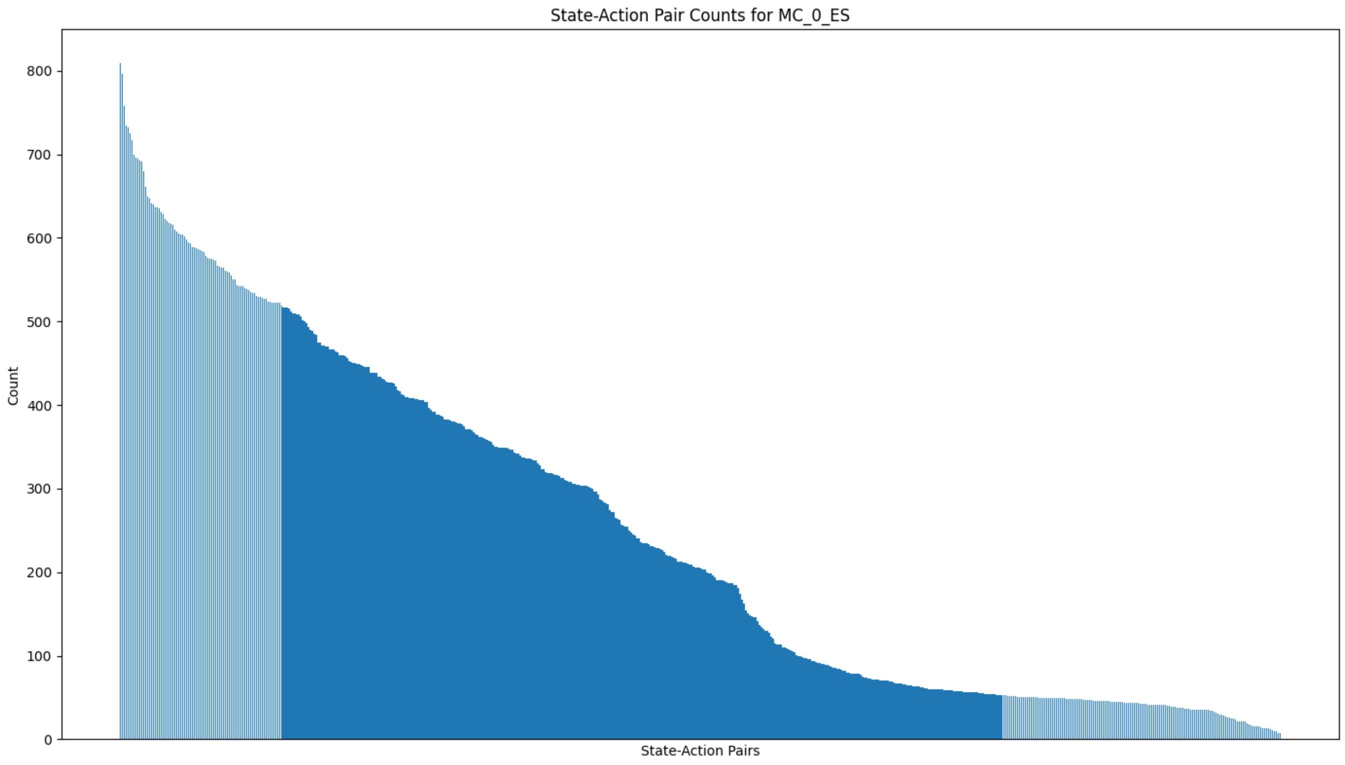
## Monte Carlo Methods

### Unique State Action Counts



## ε = 1/k and Exploring Starts





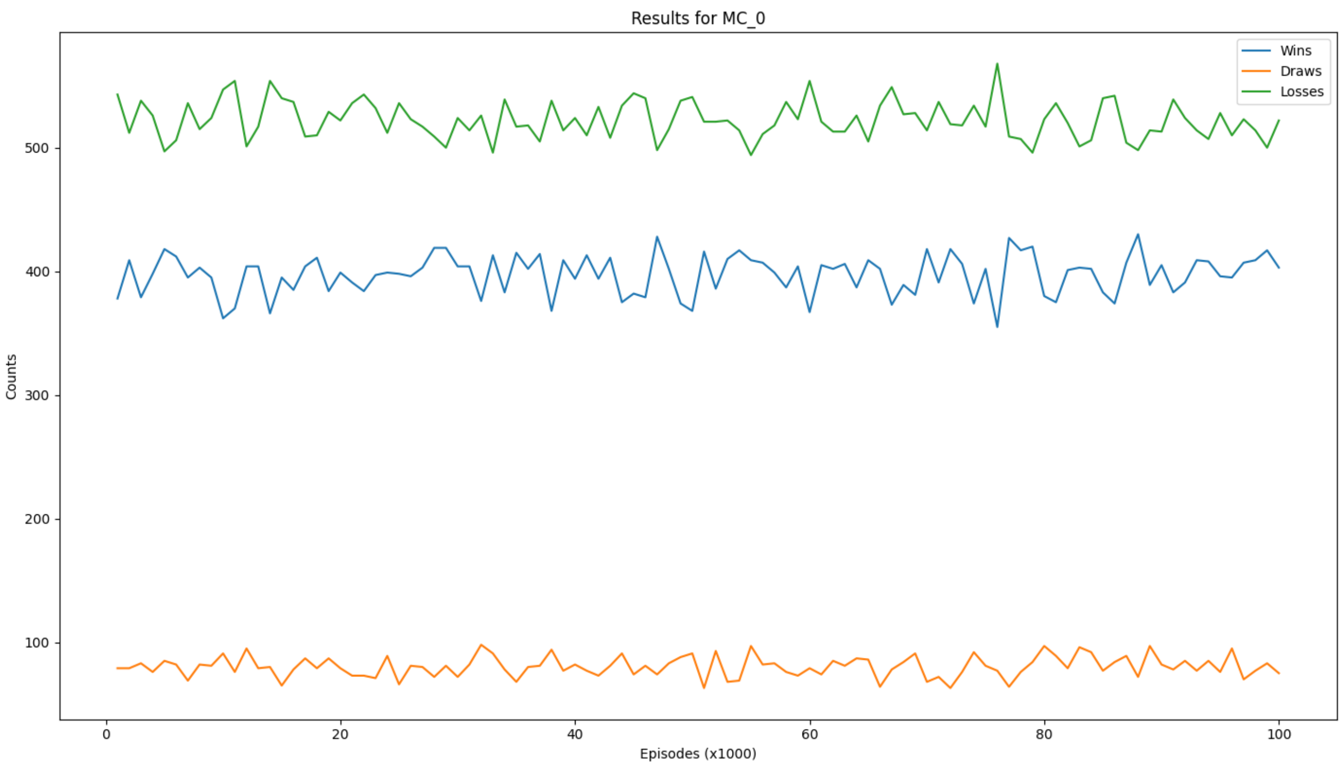
Strategy Table for MC_0_ES (with Ace)

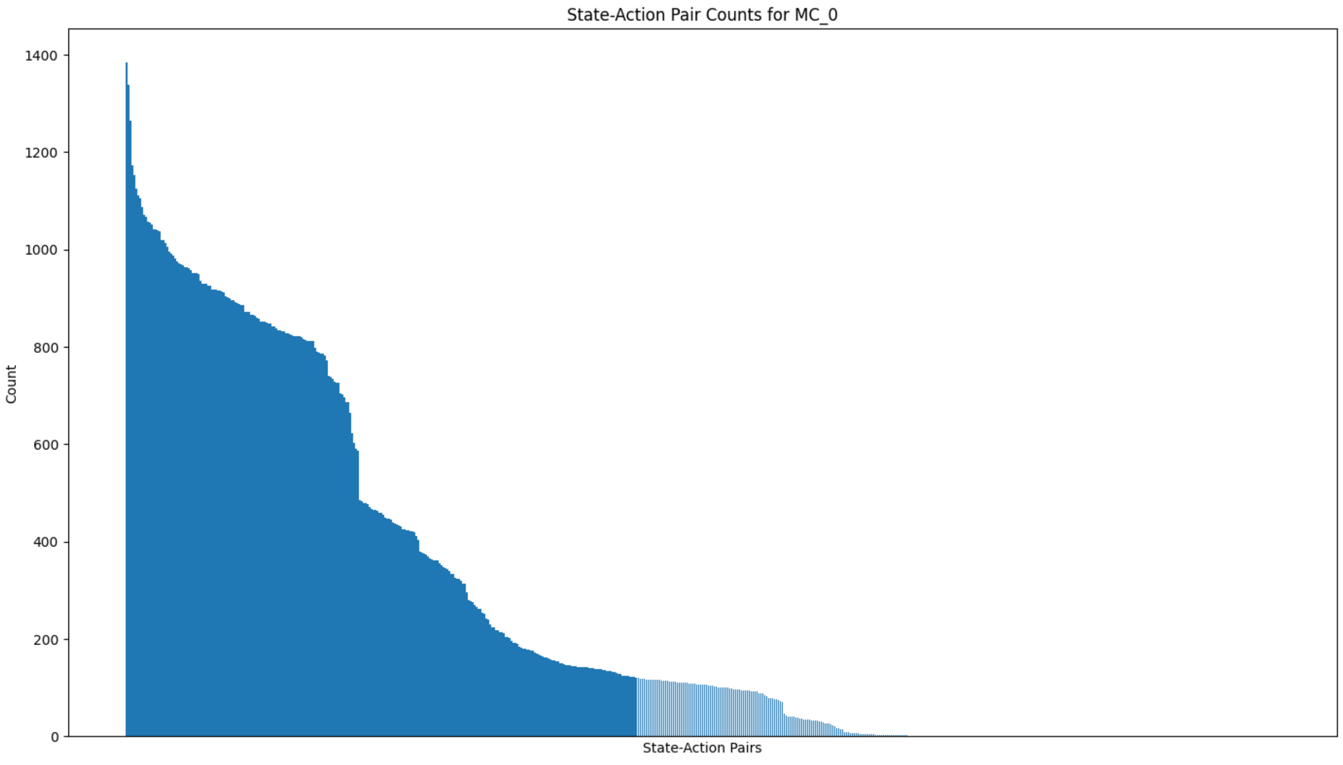
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | H | S | H | S | S |
| 18 | S | S | H | S | S | S | S | H | H | H | S | H | H |
| 17 | H | H | S | S | H | H | H | H | H | H | H | H | H |
| 16 | H | S | S | H | H | H | H | H | H | H | H | H | H |
| 15 | H | S | S | H | H | H | H | H | H | H | H | H | S |
| 14 | H | H | S | H | H | H | H | H | H | H | H | H | H |
| 13 | S | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | S | H | H | H | H | H | H | H | H | H | H |

Strategy Table for MC_0_ES (without Ace)

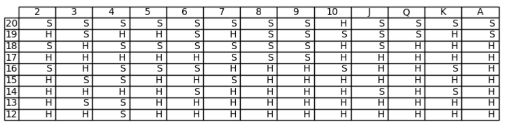
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 16 | S | S | S | S | S | H | H | S | H | H | H | S | H |
| 15 | S | S | S | S | S | H | H | H | H | H | H | H | H |
| 14 | S | S | S | S | S | H | H | H | H | H | H | H | H |
| 13 | S | S | S | S | S | H | H | H | H | H | H | H | H |
| 12 | H | H | H | S | H | H | H | H | H | H | H | H | H |

**ε = 1/k**


Results for MC_0
Wins
Draws
Losses
Counts
Episodes (x1000)
0
20
40
60
80
100
100
200
300
400
500

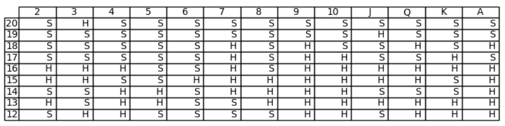

State-Action Pair Counts for MC_0
Count
State-Action Pairs
0
200
400
600
800
1000
1200
1400

Strategy Table for MC_0 (with Ace)



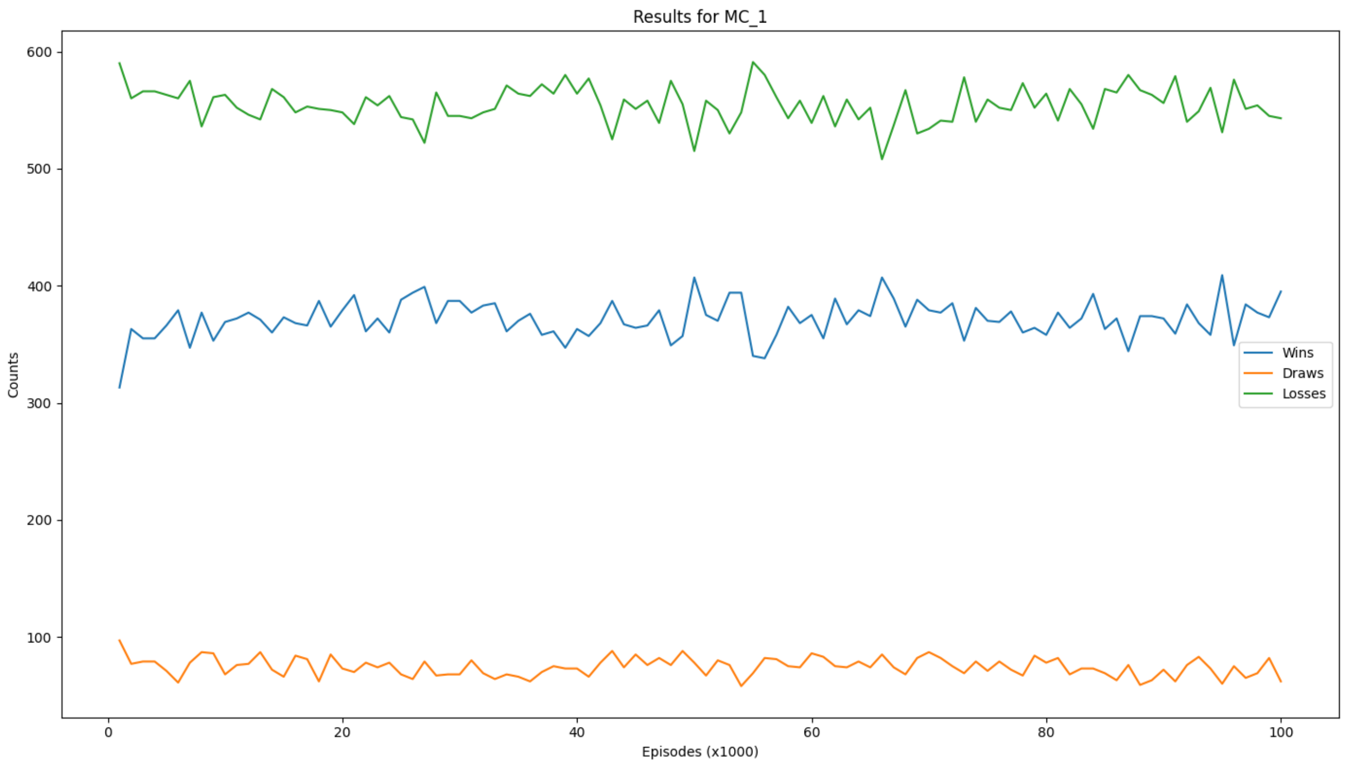
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | H | S | S | S | S |
| 19 | H | S | H | H | S | H | S | S | S | S | S | H | S |
| 18 | S | H | S | S | S | S | S | S | H | S | H | H | H |
| 17 | H | H | H | H | H | S | S | S | H | H | H | H | H |
| 16 | S | H | S | S | S | H | H | H | S | H | H | S | H |
| 15 | H | S | S | H | H | S | H | H | H | H | H | H | H |
| 14 | H | H | H | H | S | H | H | H | H | S | H | S | H |
| 13 | H | S | S | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | S | H | H | H | H | H | H | H | H | H | H |

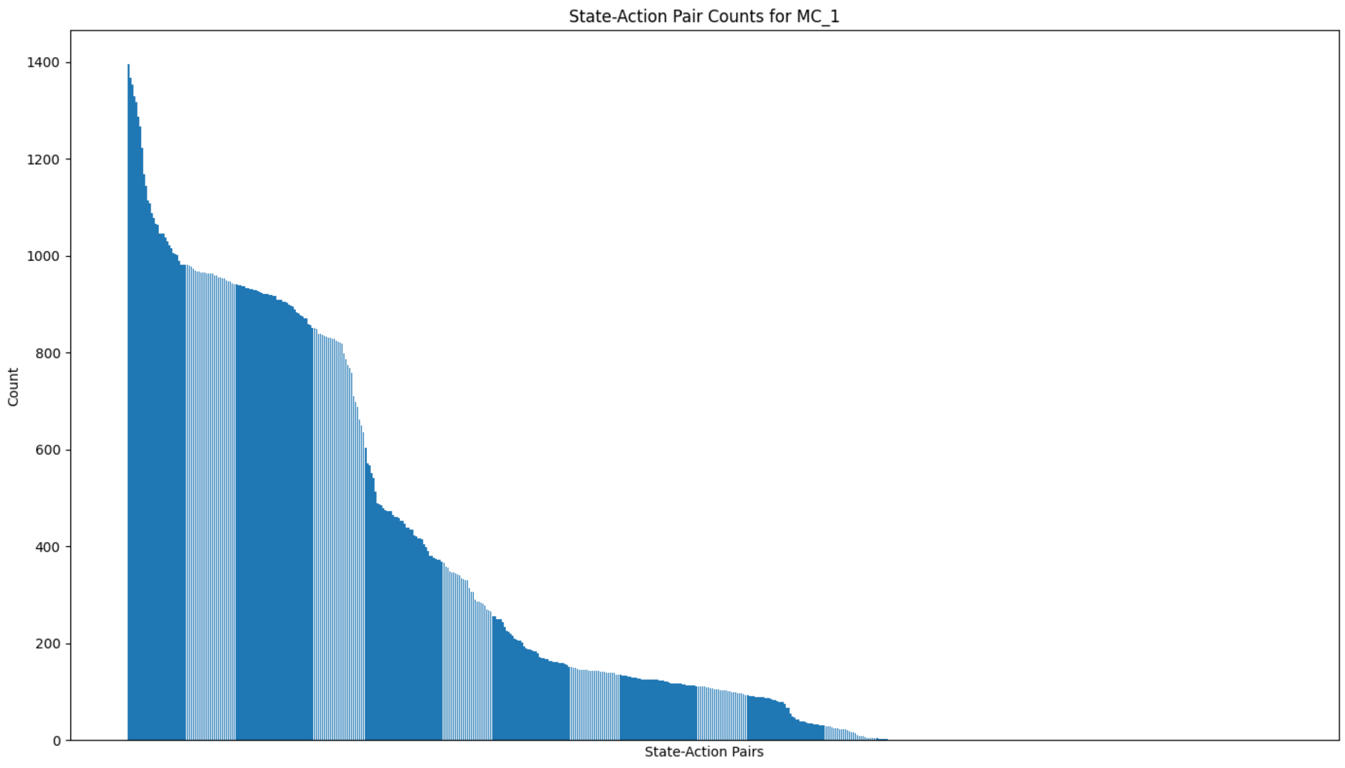
Strategy Table for MC_0 (without Ace)



| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | H | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | H | S | S | S |
| 18 | S | S | S | S | S | H | S | H | S | S | H | S | H |
| 17 | S | S | S | S | S | H | S | H | H | S | S | H | S |
| 16 | H | H | H | S | S | H | S | H | H | H | H | H | H |
| 15 | H | H | S | S | H | H | H | H | H | H | H | S | H |
| 14 | S | S | H | H | S | H | H | H | H | S | S | S | H |
| 13 | H | S | H | H | S | S | H | H | H | H | H | H | H |
| 12 | S | H | H | S | S | S | S | H | H | S | H | H | H |

## ε = e^-k/1000





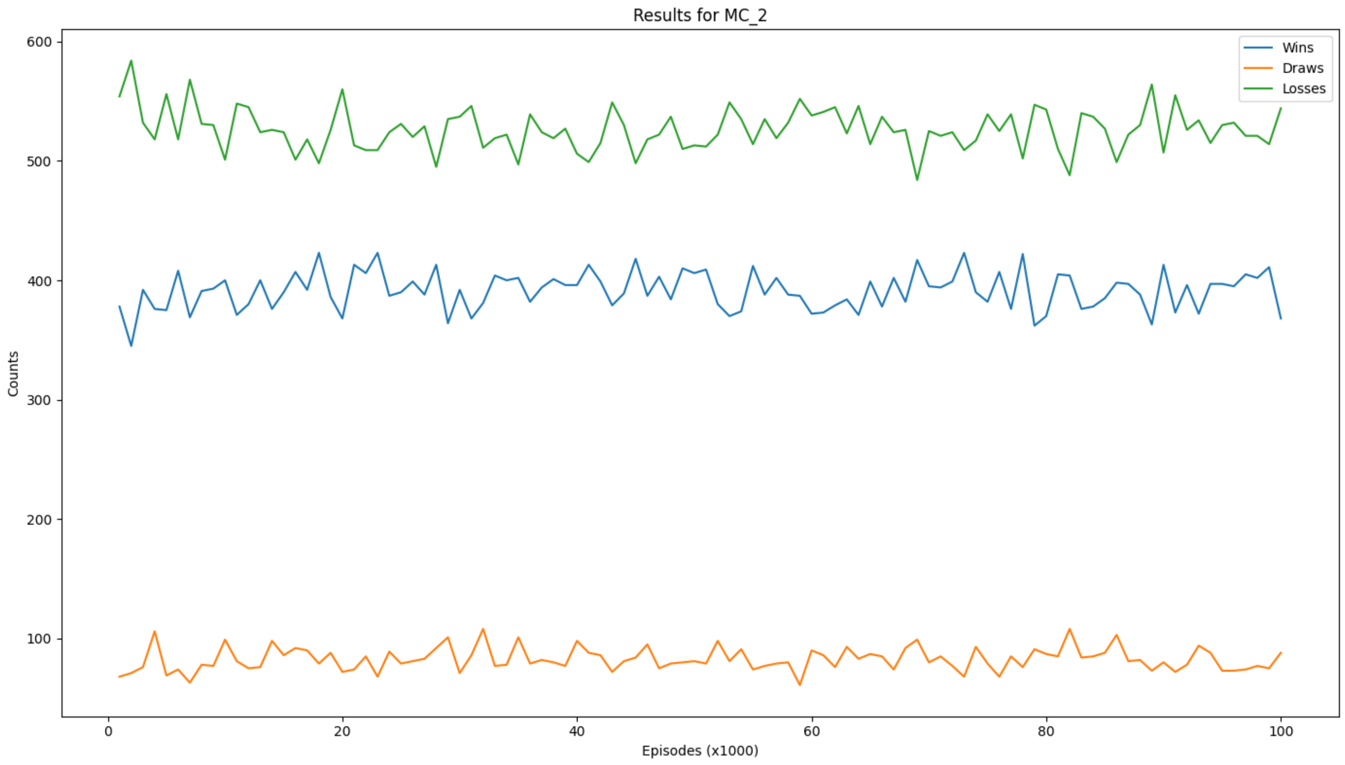
Strategy Table for MC_1 (with Ace)

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | H | H | S | S | S | S | S | S | S | S |
| 18 | S | H | H | H | H | H | S | H | H | H | H | H | H |
| 17 | S | H | H | H | H | S | S | H | H | S | H | H | H |
| 16 | H | H | H | H | H | H | H | H | S | H | H | H | H |
| 15 | S | S | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | S | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | S | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | S | H | H | H | H | H |

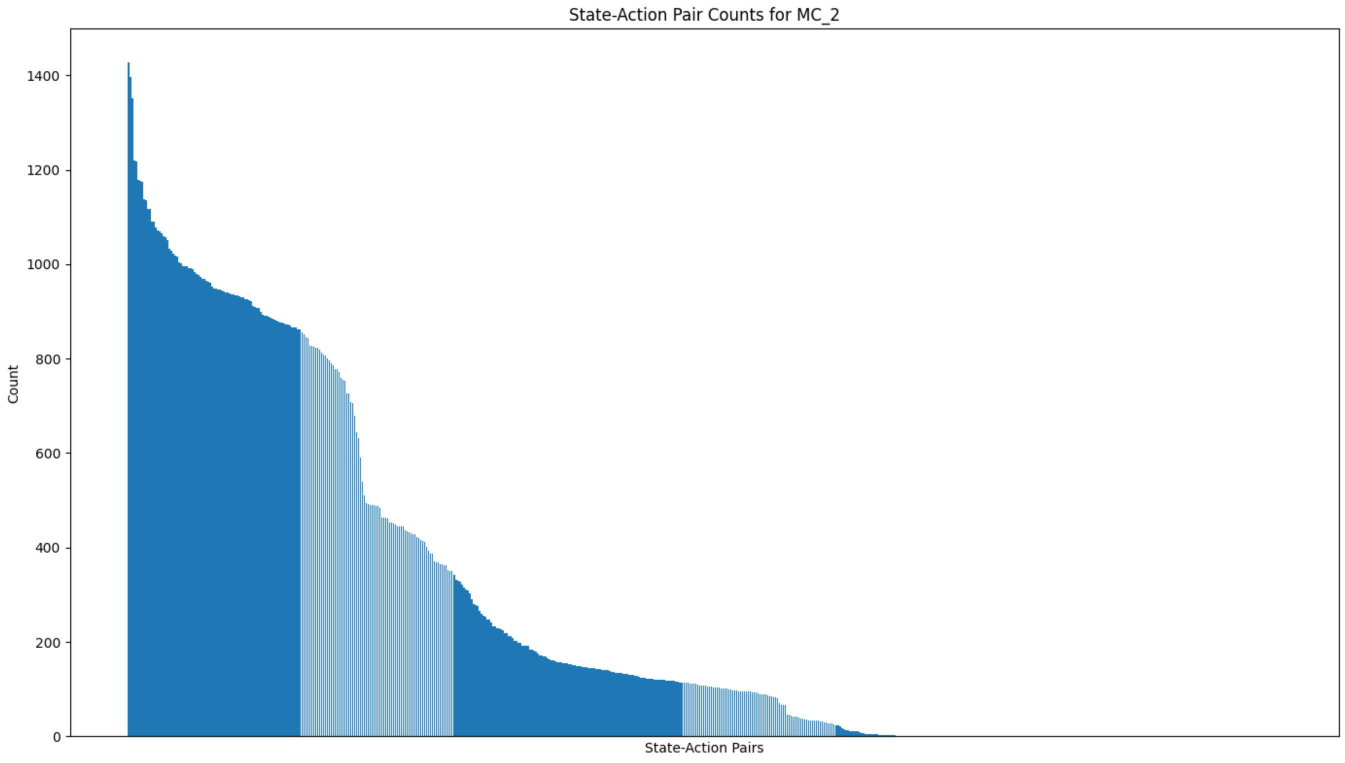
Strategy Table for MC_1 (without Ace)

| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | H | S | S | S | H | S | H | H | S | H | H | H |
| 18 | H | S | H | S | S | S | S | S | H | H | H | H | H |
| 17 | H | S | S | S | S | H | H | H | S | H | S | H | S |
| 16 | H | S | H | H | S | H | H | H | H | H | H | H | H |
| 15 | H | S | H | H | H | H | S | H | S | H | H | H | H |
| 14 | H | S | H | H | H | H | H | H | H | H | H | S | H |
| 13 | H | H | H | S | S | H | S | S | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | S | S |

**ε = e^-k/10000**



Results for MC_2
Wins
Draws
Losses
Counts
Episodes (x1000)
0
20
40
60
80
100
600
500
400
300
200
100



State-Action Pair Counts for MC_2
Count
State-Action Pairs
1400
1200
1000
800
600
400
200
0

Strategy Table for MC_2 (with Ace)

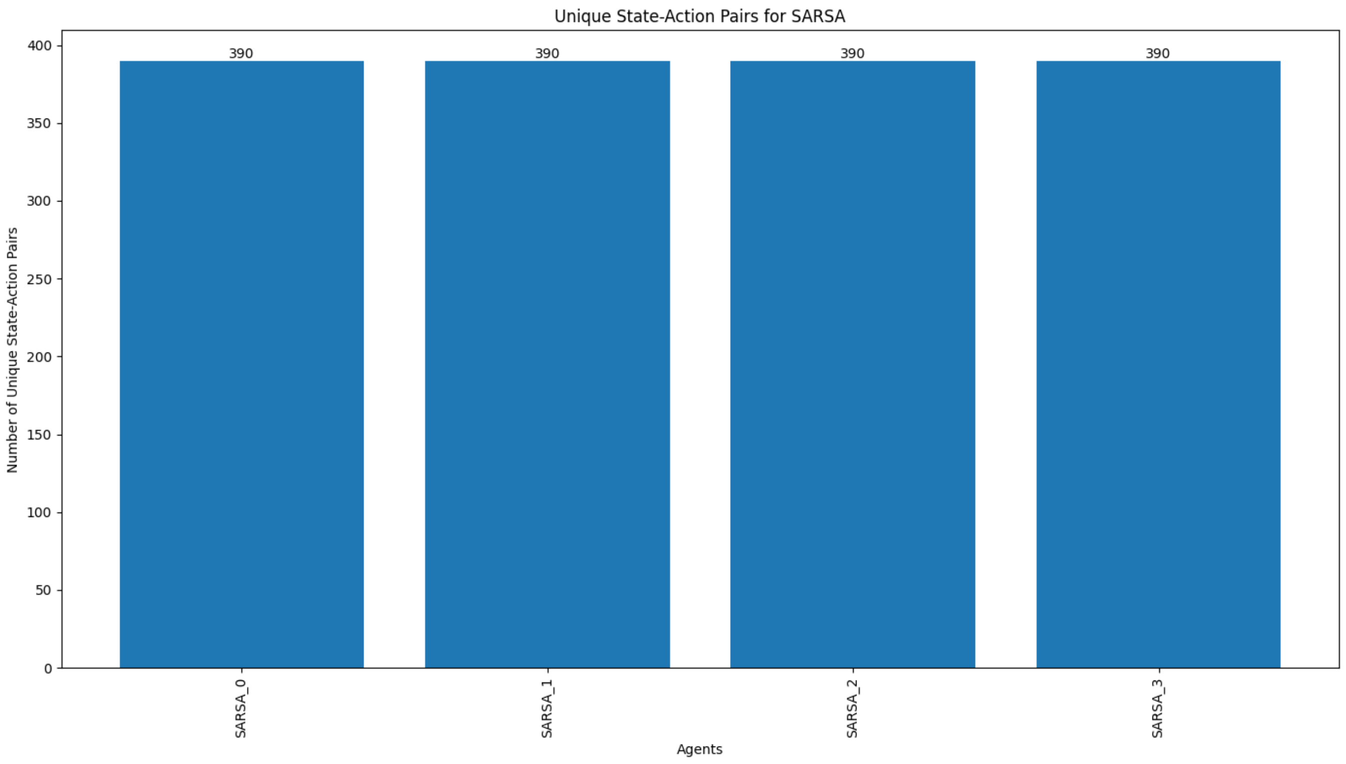
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | S | S | S | S | H | S | S | S | S | S | S |
| 19 | S | S | S | S | S | H | S | S | H | H | S | H | H |
| 18 | H | H | H | S | H | S | H | H | H | S | S | S | H |
| 17 | S | H | H | H | H | S | H | S | H | H | H | S | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | S | H | H | H | H | S | H | H | H |
| 14 | H | S | H | S | S | H | H | H | H | H | H | H | H |
| 13 | H | H | S | H | H | H | H | H | H | H | H | H | H |
| 12 | H | S | H | H | S | H | H | H | H | H | H | H | H |

Strategy Table for MC_2 (without Ace)

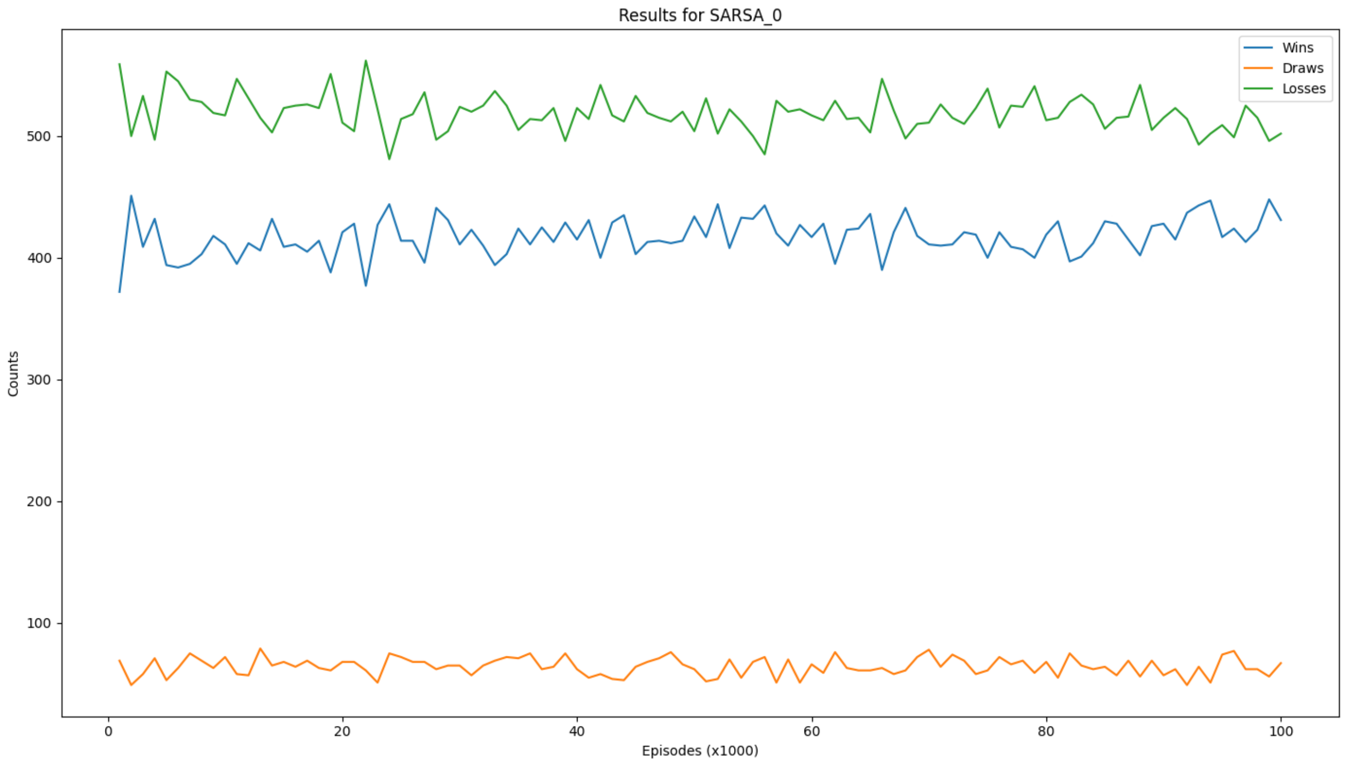
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | H |
| 19 | S | H | S | S | S | H | S | S | H | H | S | S | S |
| 18 | S | S | S | S | H | S | S | S | S | S | S | S | S |
| 17 | S | S | S | H | H | S | H | H | H | S | H | S | H |
| 16 | H | S | S | S | H | S | H | S | H | H | H | H | H |
| 15 | H | S | S | H | S | H | H | H | H | H | H | H | H |
| 14 | H | S | S | H | H | H | H | H | H | H | S | H | H |
| 13 | S | S | H | S | H | H | H | H | H | H | H | H | H |
| 12 | H | S | H | H | S | S | H | H | H | H | H | H | H |

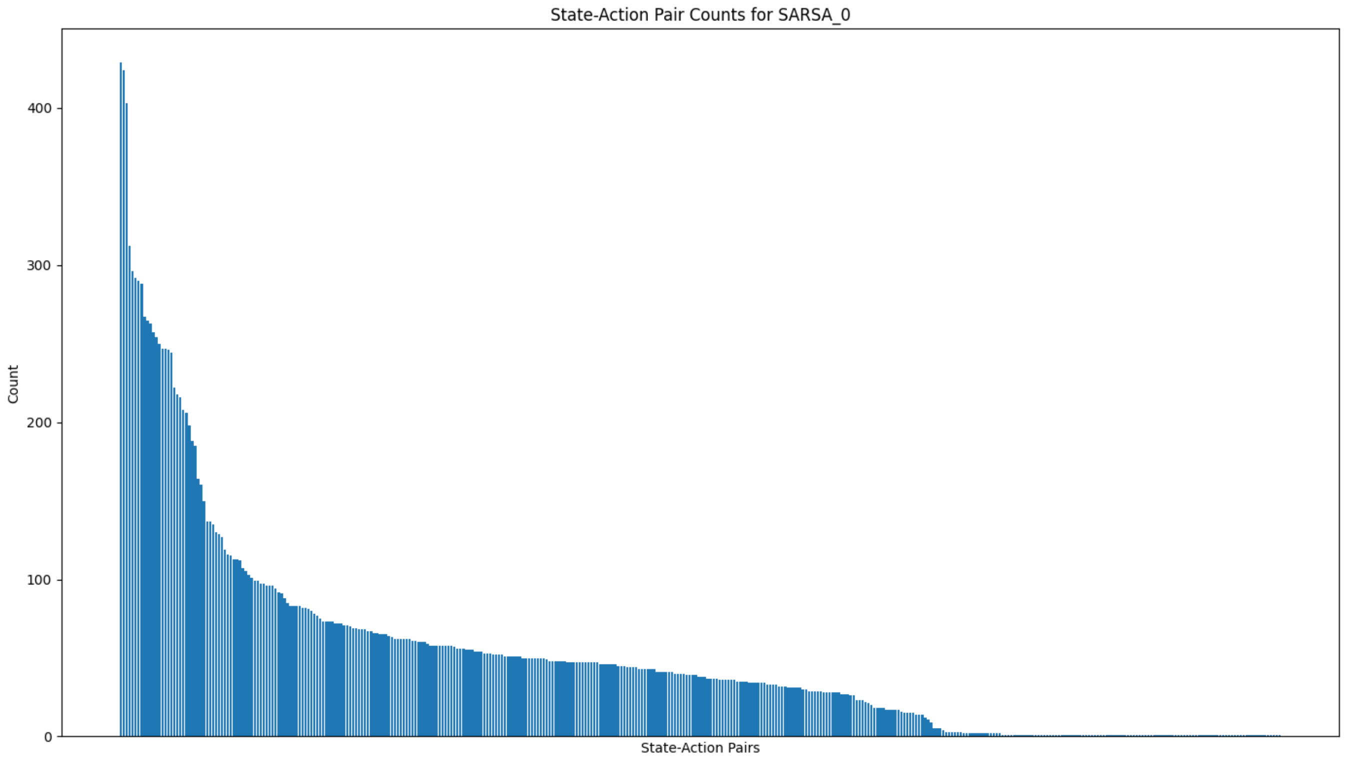
## SARSA

### Unique State Action Counts

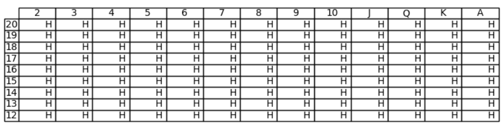


### ε = 0.1


Results for SARSA_0
Wins
Draws
Losses
Counts
Episodes (x1000)
0
20
40
60
80
100
100
200
300
400
500

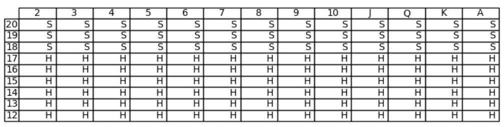

State-Action Pair Counts for SARSA_0
Count
State-Action Pairs
0
100
200
300
400

Strategy Table for SARSA_0 (with Ace)



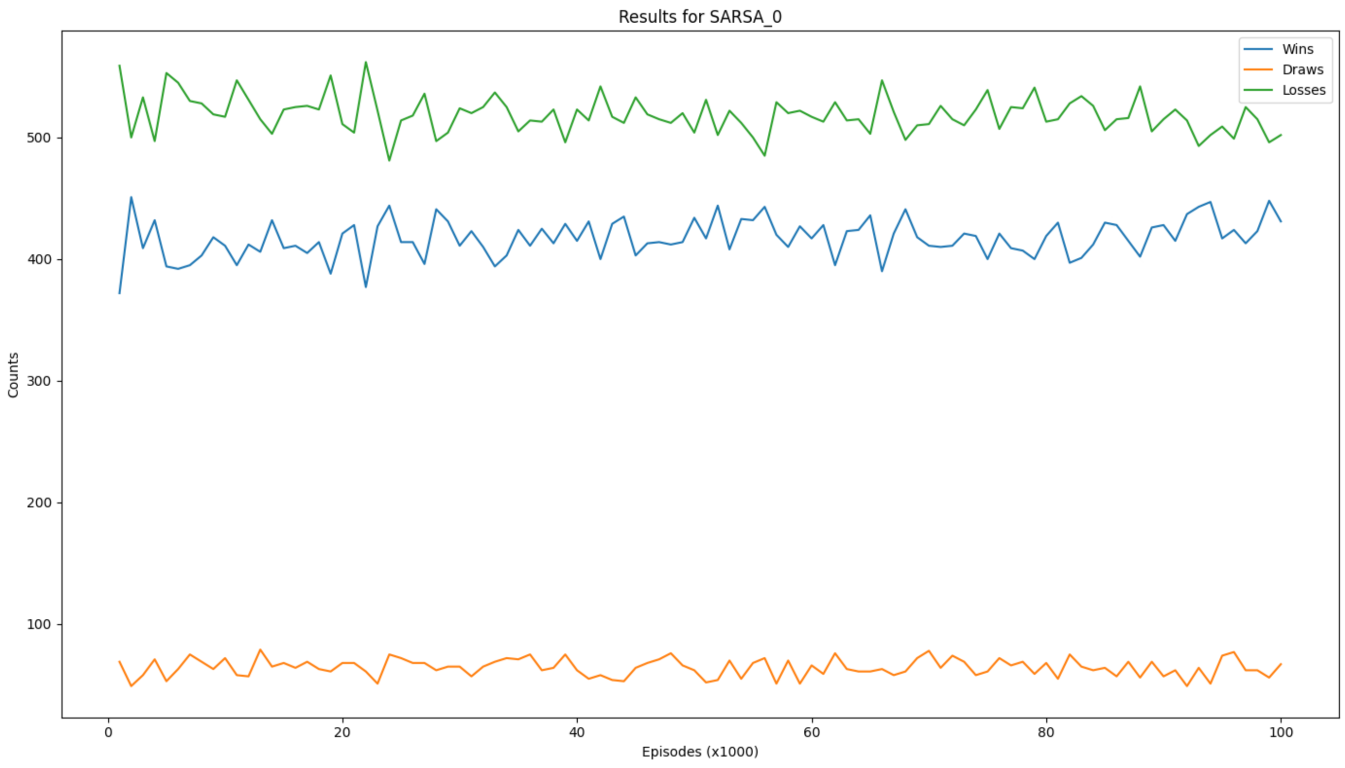
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

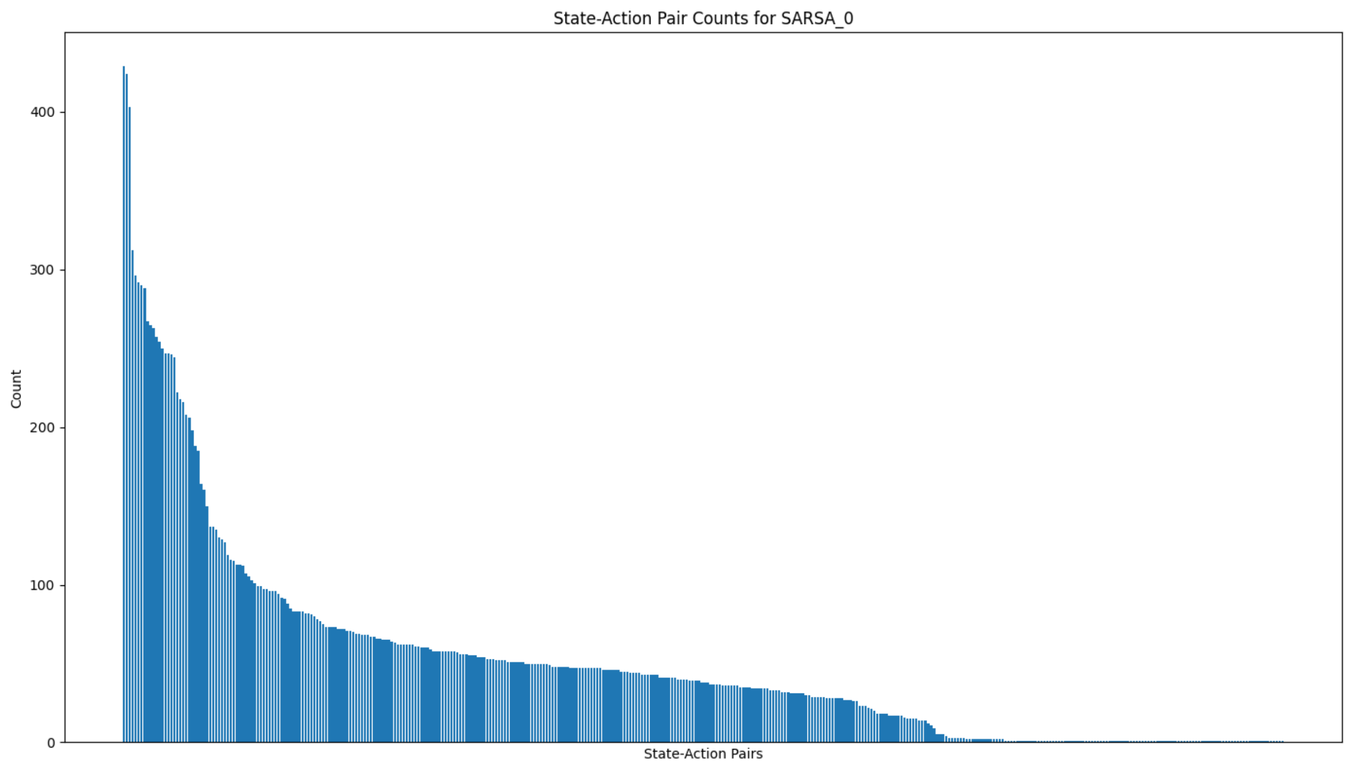
Strategy Table for SARSA_0 (without Ace)



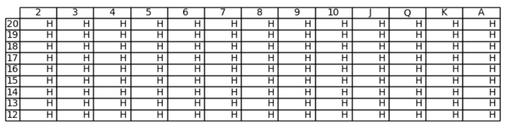
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

## ε = 1/k



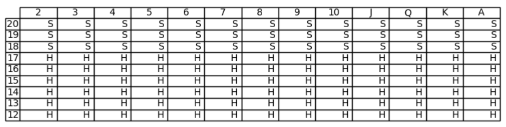


Strategy Table for SARSA_0 (with Ace)



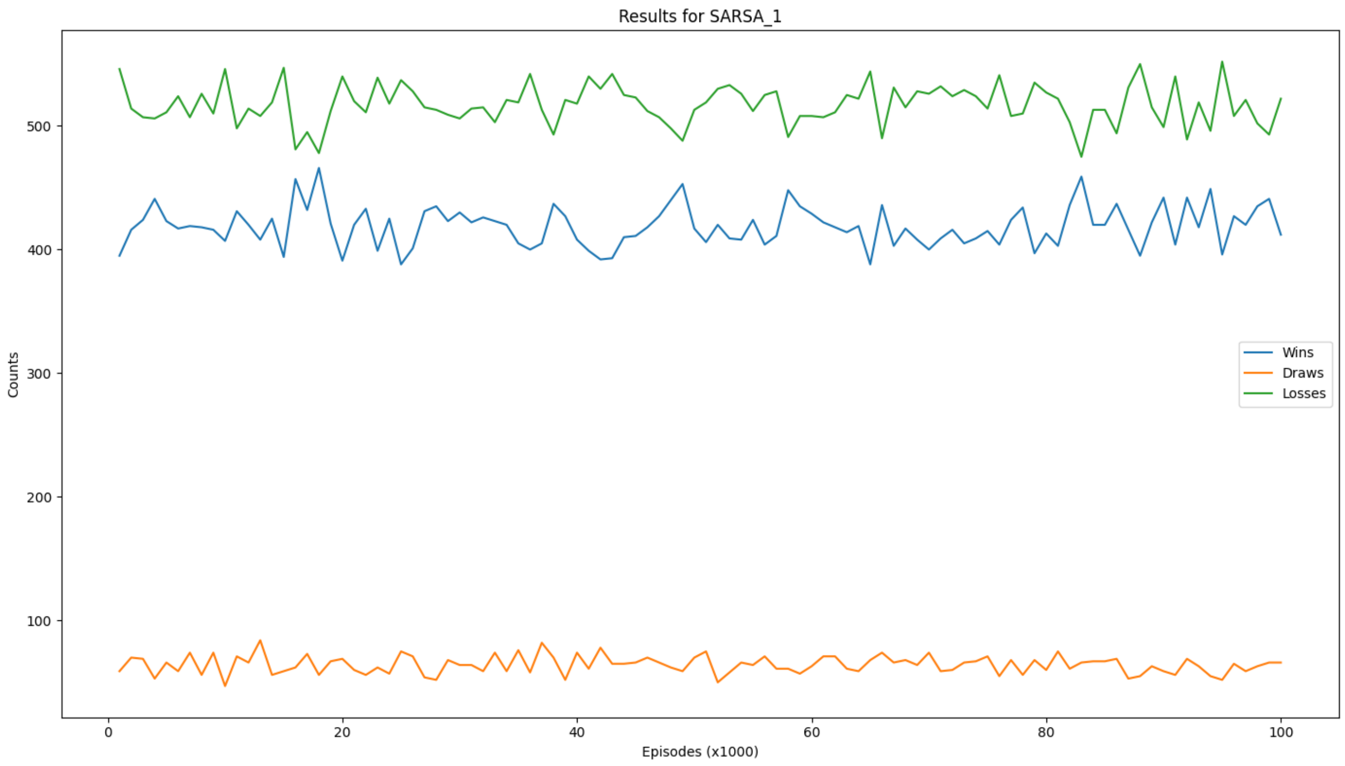
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

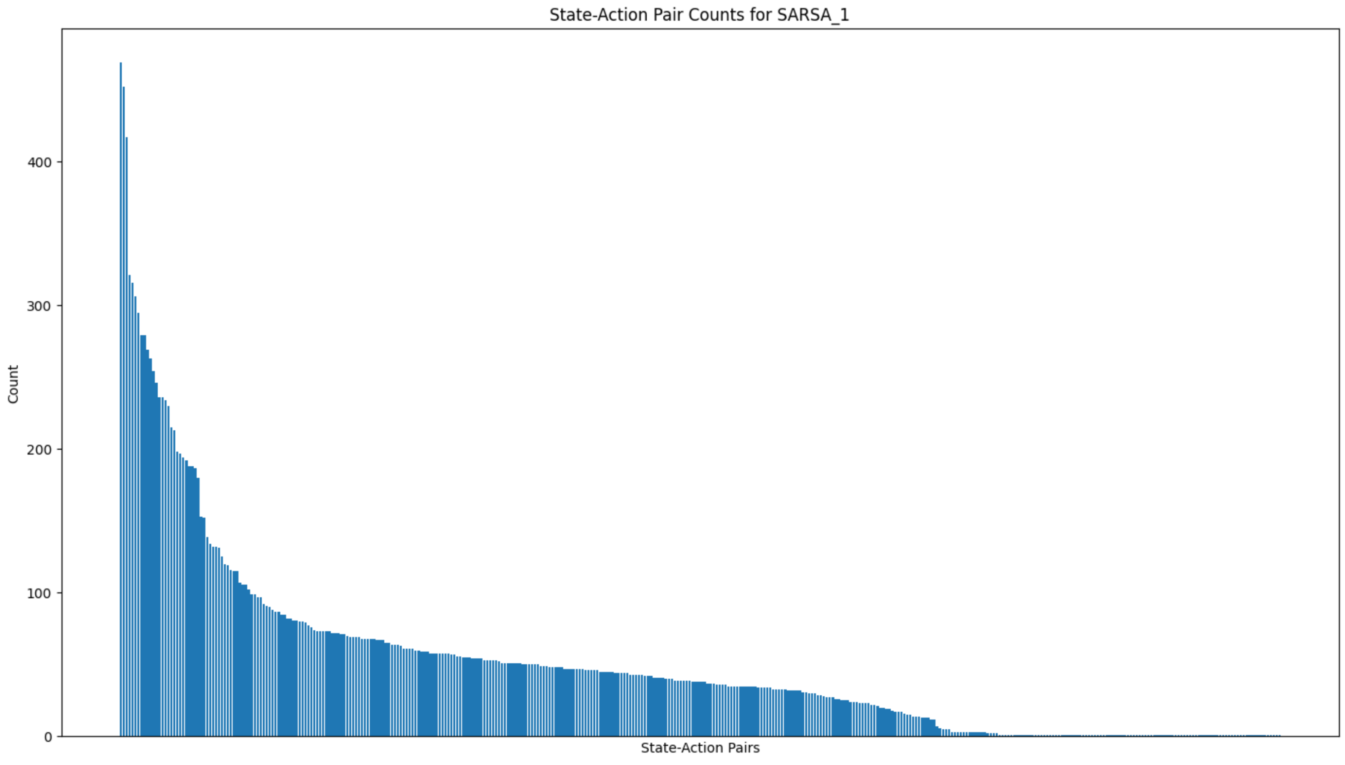
Strategy Table for SARSA_0 (without Ace)



| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

## ε = e^-k/1000


Results for SARSA_1
Counts
Episodes (x1000)
Wins
Draws
Losses
0
20
40
60
80
100
200
300
400
500


State-Action Pair Counts for SARSA_1
Count
State-Action Pairs
0
100
200
300
400

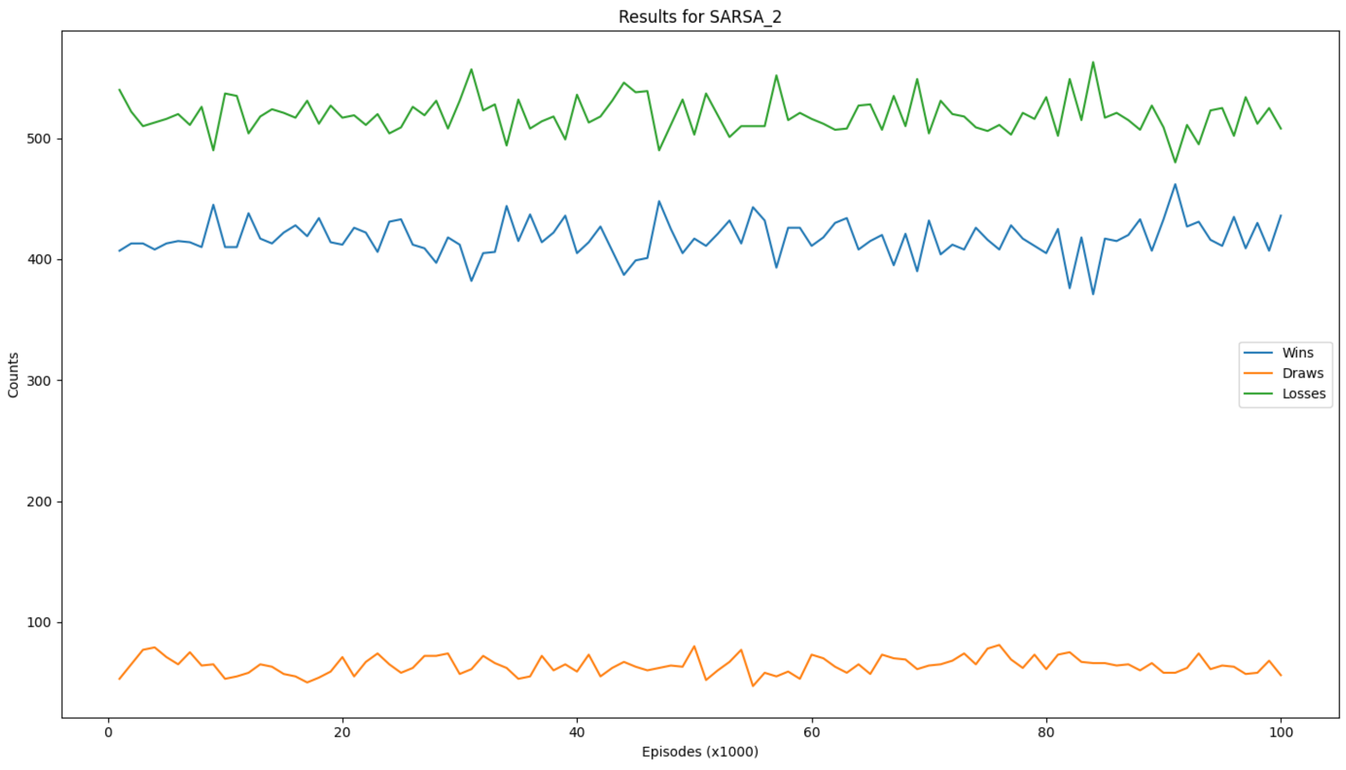
Strategy Table for SARSA_1 (with Ace)

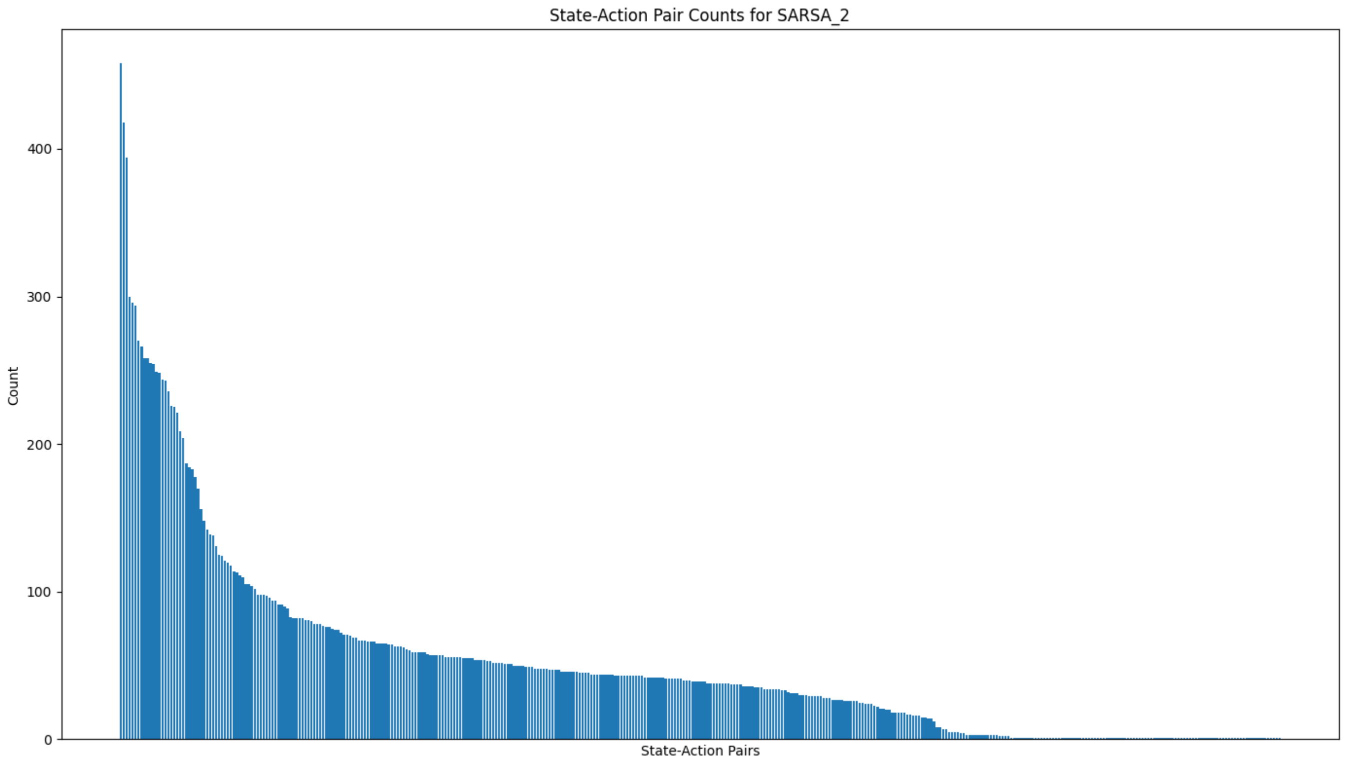
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

Strategy Table for SARSA_1 (without Ace)

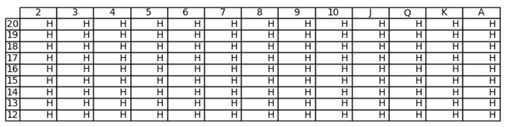
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

## ε = e^-k/10000



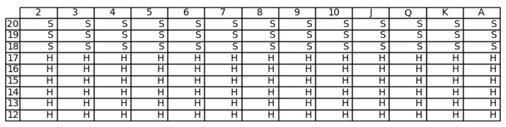


Strategy Table for SARSA_2 (with Ace)



| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

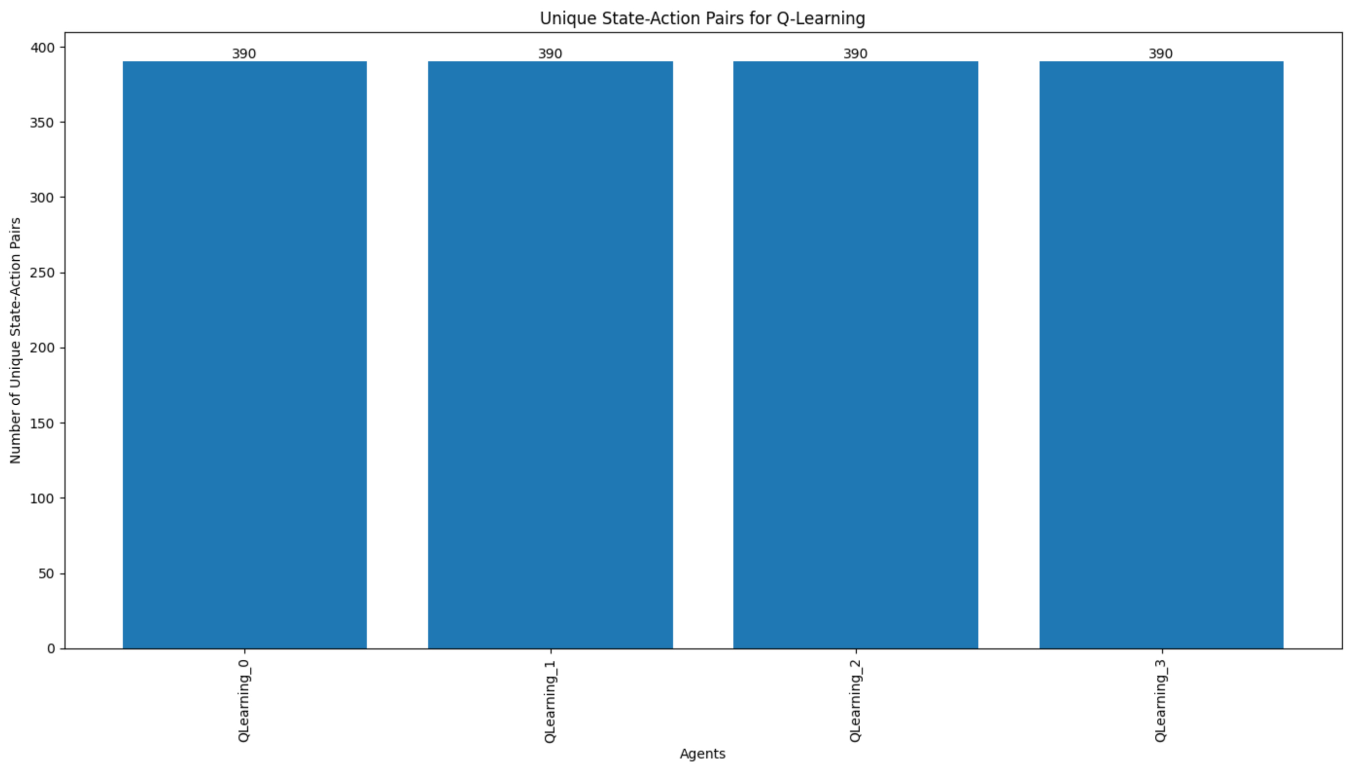
Strategy Table for SARSA_2 (without Ace)



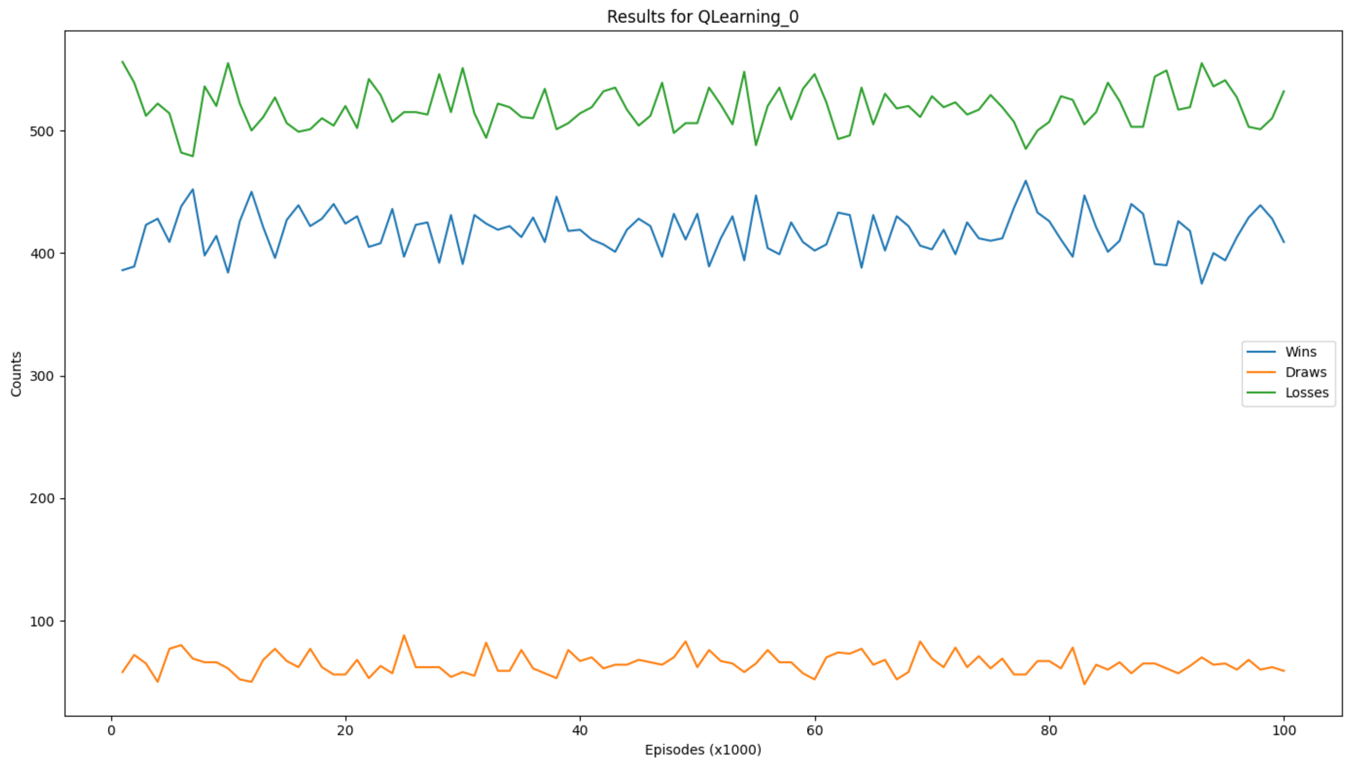
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

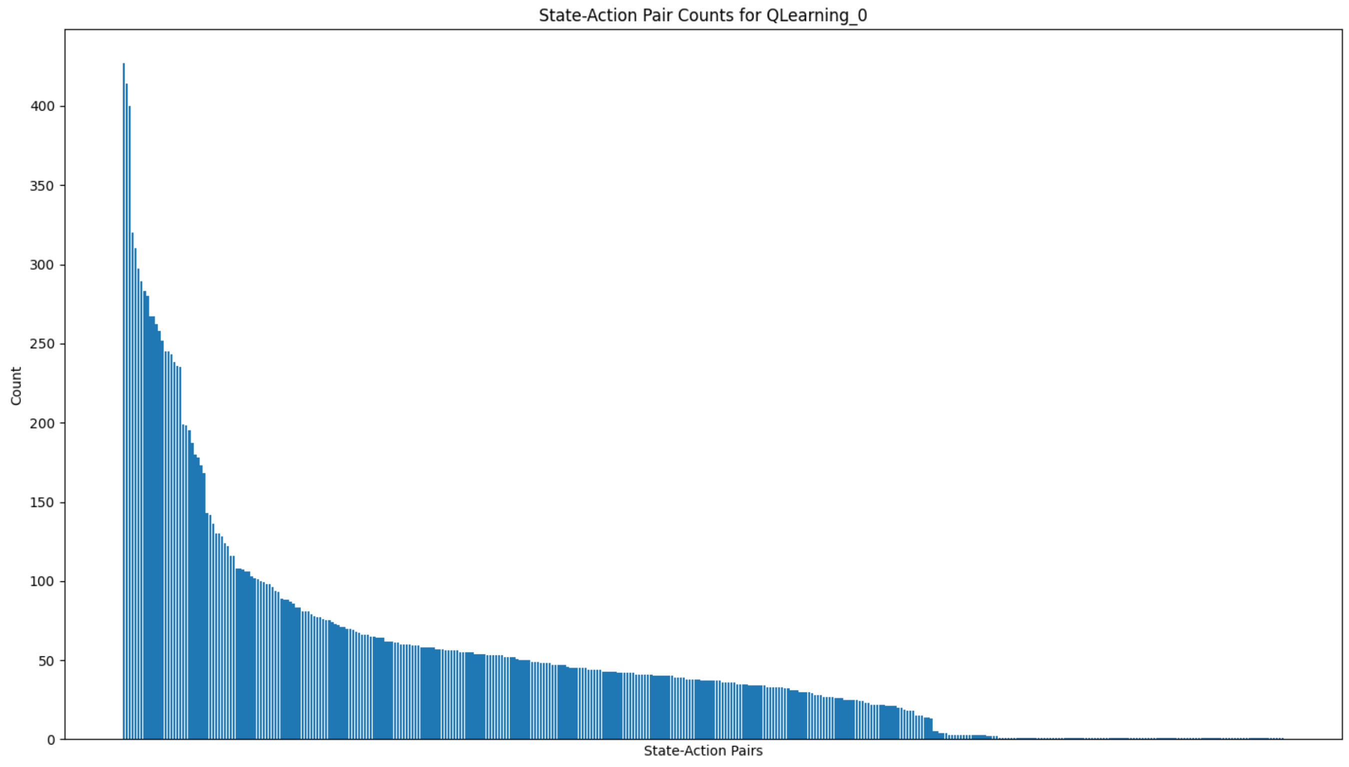
## Q Learning

### Unique State Action Counts

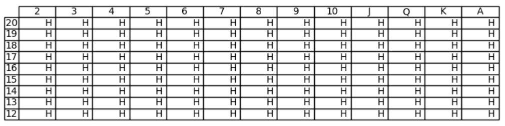

Unique State-Action Pairs for Q-Learning
Number of Unique State-Action Pairs
Agents
390
390
390
390
QLearning_0
QLearning_1
QLearning_2
QLearning_3

**ε = 0.1**


Results for QLearning_0
Counts
Episodes (x1000)
Wins
Draws
Losses

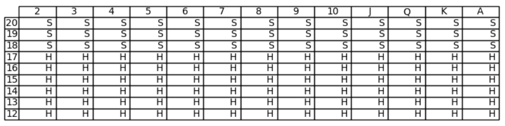


Strategy Table for QLearning_0 (with Ace)



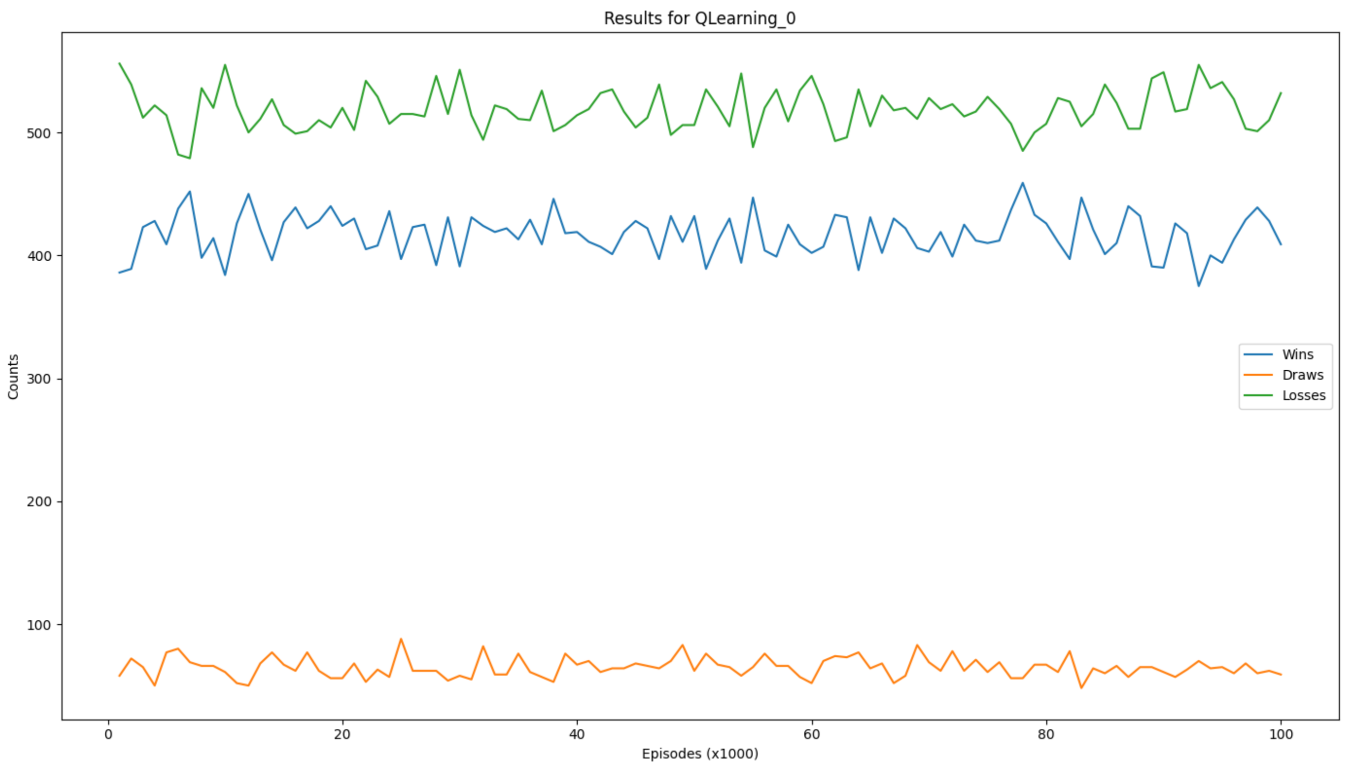
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

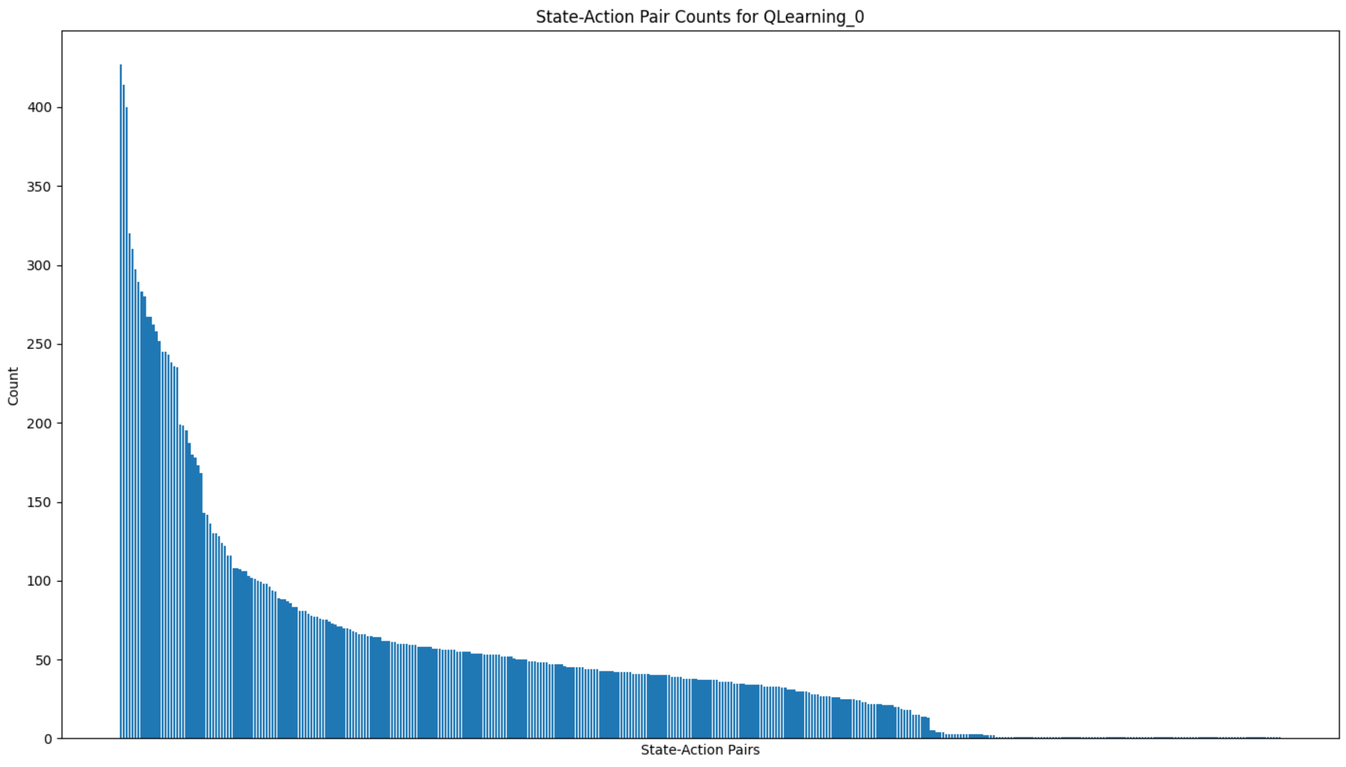
Strategy Table for QLearning_0 (without Ace)



| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

**ε = 1/k**


Results for QLearning_0
Counts
Episodes (x1000)
Wins
Draws
Losses
0
20
40
60
80
100
500
400
300
200
100


State-Action Pair Counts for QLearning_0
Count
State-Action Pairs
400
350
300
250
200
150
100
50
0

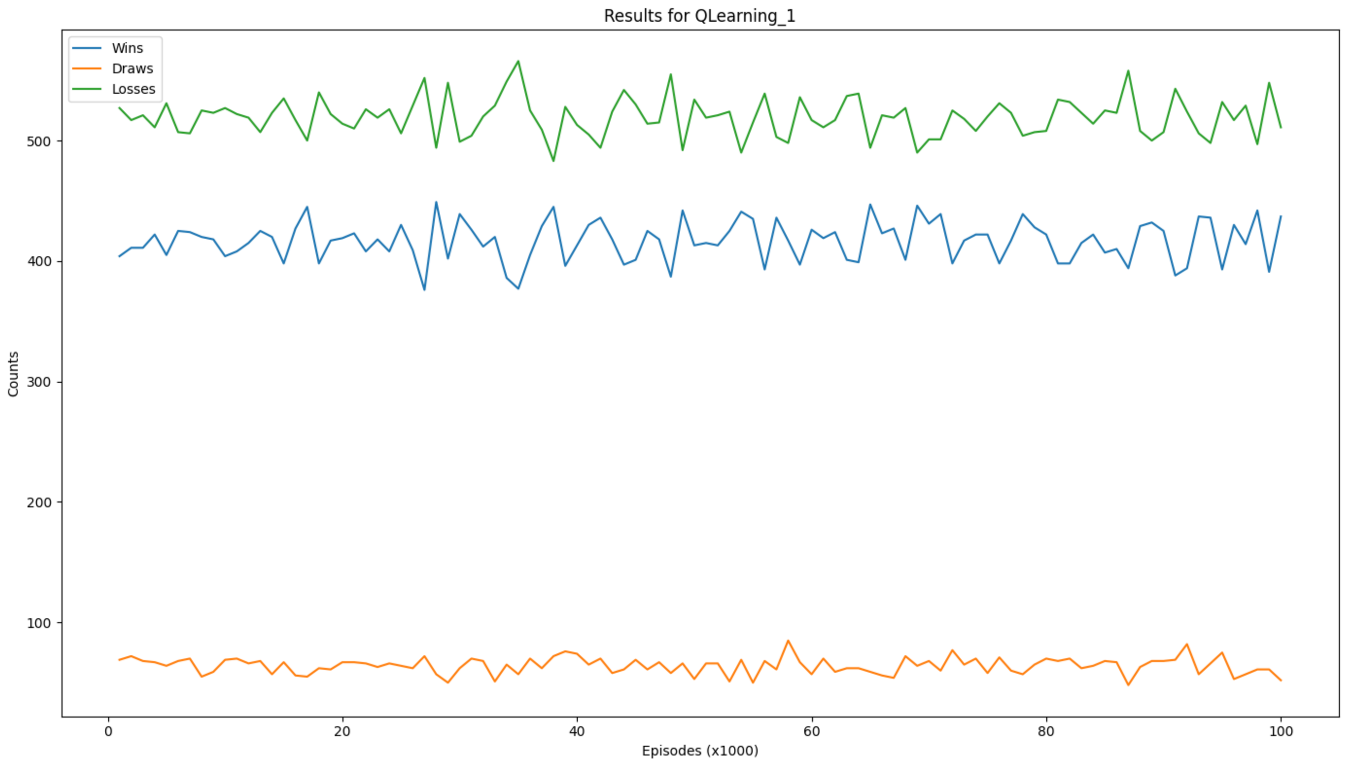
Strategy Table for QLearning_0 (with Ace)

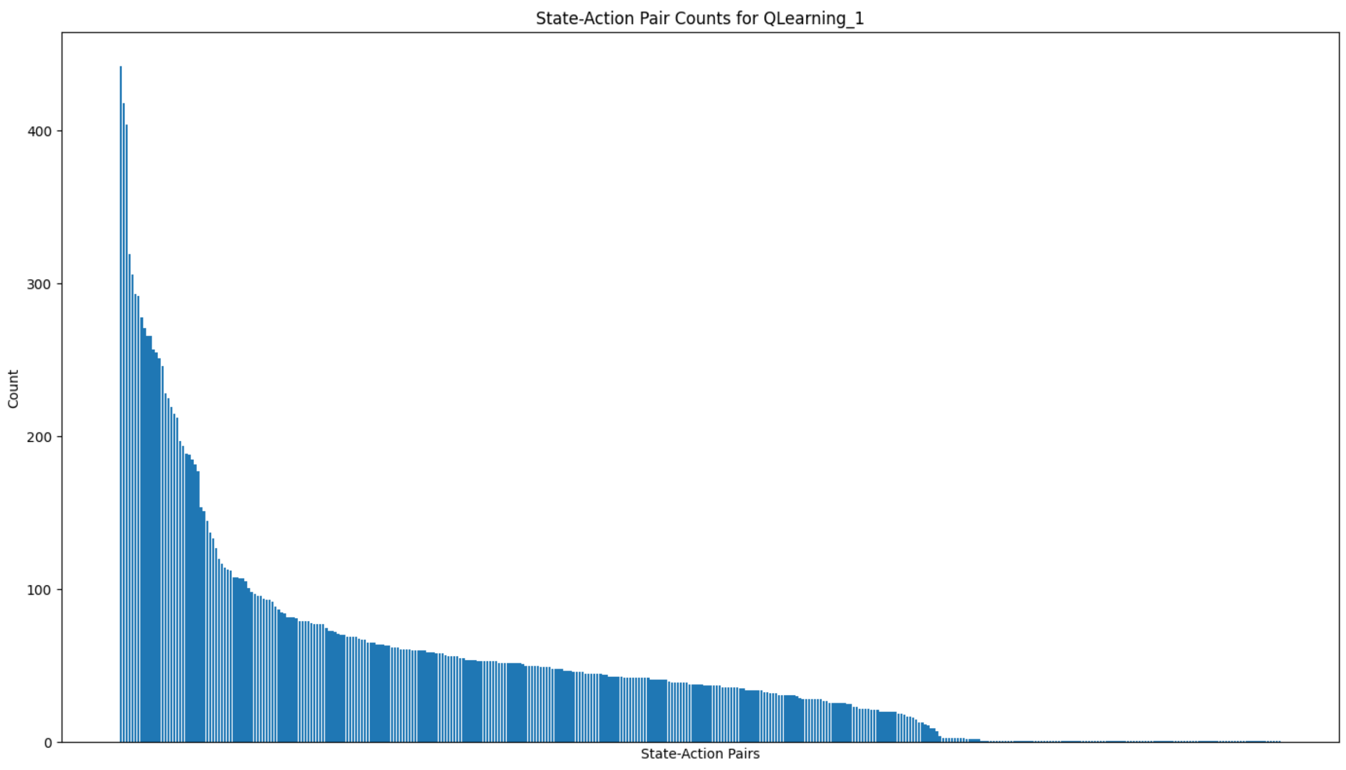
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

Strategy Table for QLearning_0 (without Ace)

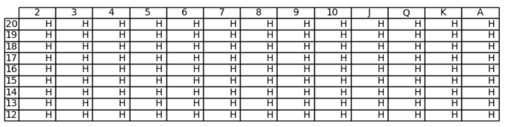
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

## ε = e^-k/1000



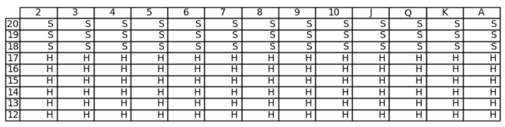


Strategy Table for QLearning_1 (with Ace)



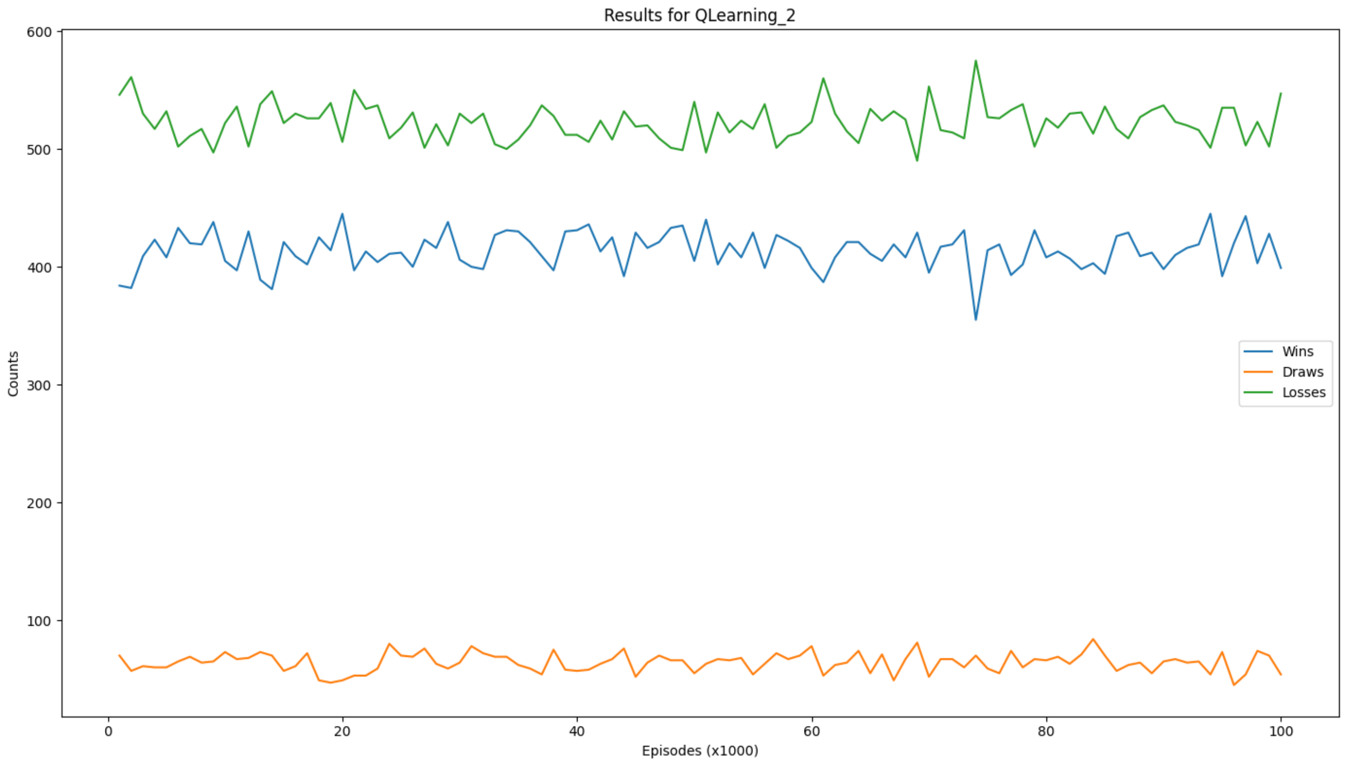
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

Strategy Table for QLearning_1 (without Ace)

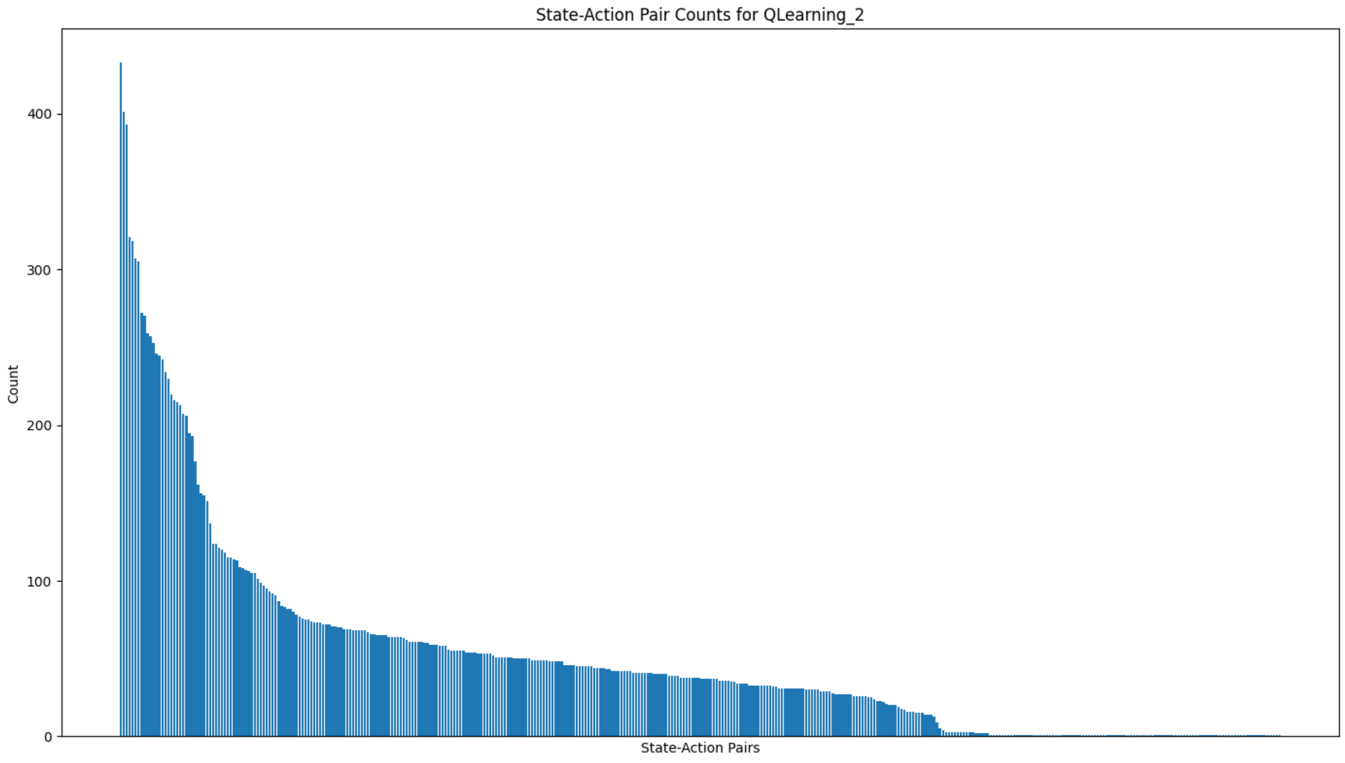


| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

**ε = e^-k/10000**

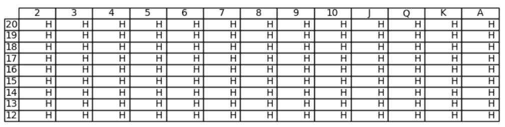


Results for QLearning_2
Counts
Episodes (x1000)
Wins
Draws
Losses
0
20
40
60
80
100
100
200
300
400
500
600



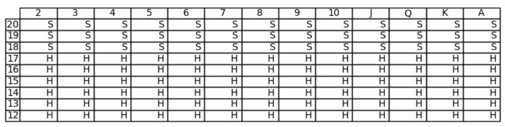
State-Action Pair Counts for QLearning_2
Count
State-Action Pairs
0
100
200
300
400

Strategy Table for QLearning_2 (with Ace)



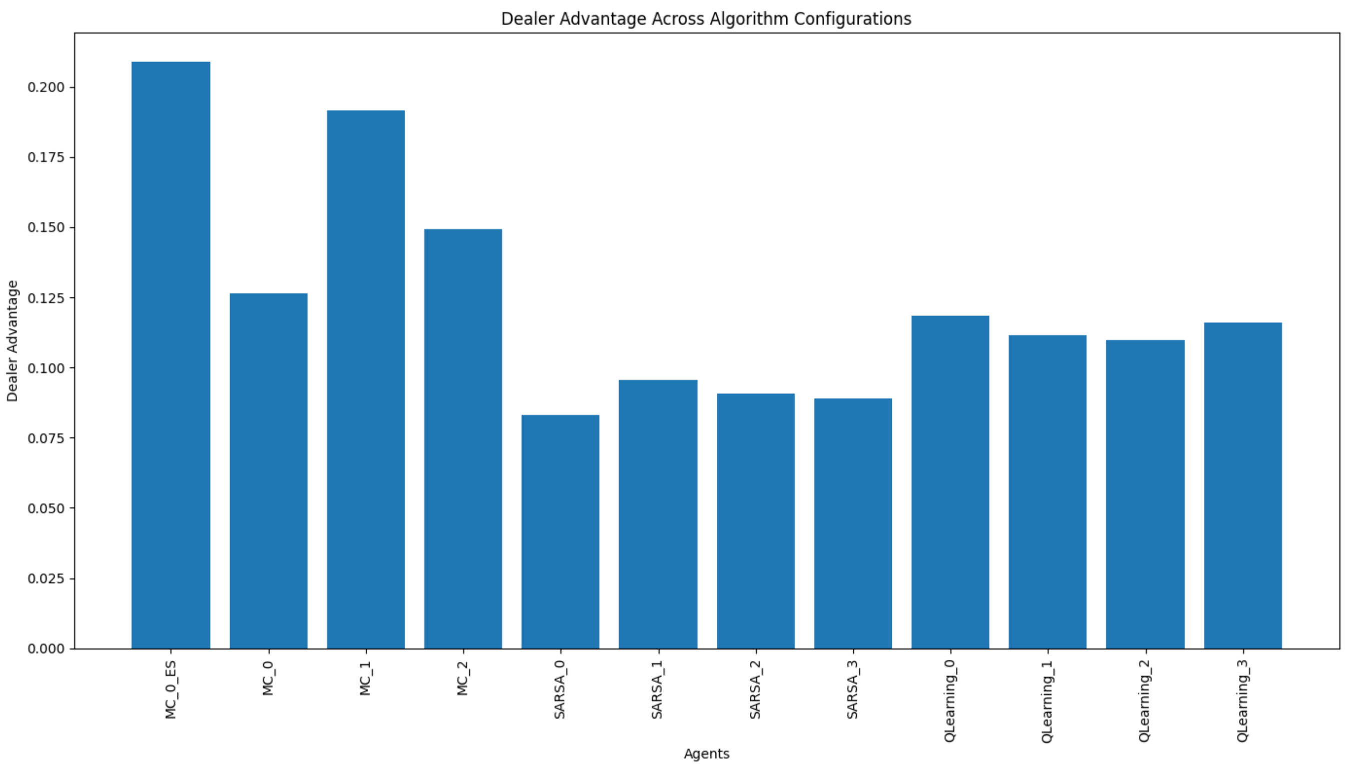
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 19 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 18 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

Strategy Table for QLearning_2 (without Ace)



| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | J | Q | K | A |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 20 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 19 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 18 | S | S | S | S | S | S | S | S | S | S | S | S | S |
| 17 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 16 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 15 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 14 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 13 | H | H | H | H | H | H | H | H | H | H | H | H | H |
| 12 | H | H | H | H | H | H | H | H | H | H | H | H | H |

### Dealer Advantage



## Analysis

### Exploration vs Exploitation

As was expected, Monte Carlo explored many more states than SARSA and QLearning, with exploring starts exploring the most of the 4 versions. However, what was not expected was that SARSA and QLearning both explored the exact same amount of states, the reason for this was not entirely clear from our analysis. As viewed in the state-action pair counts graphs, it is evident that QLearning and SARSA opted to exploit a few states at a much higher rate than others. We hypothesise that these observations would change with more simulation runs and that the policy would converge towards the optimal policy.

The effects of exploration are evident in our project since although Monte Carlo had a seemingly lower performance, it's strategy tables shows more variety than those of SARSA and QLearning. This indicates that 100000 rounds were not sufficient for any algorithm to converge towards a desired policy

**Wins and Losses**

It was observed that overall, SARSA and QLearning approaches outperformed Monte Carlo in terms of win percentage. This was observed from the results plot of each algorithm. It was also observed in the dealer advantage plots that SARSA slightly outperforms QLearning towards the end of the simulations.
In terms of ε values, a lower epsilon score seemed to correlate towards a higher win percentage overall regardless of which algorithm was being used.

**Strategies**

For Monte Carlo the strategies seem a little all over the place, this is hypothesised to be due to a higher rate of exploration that the other 2 algorithms.

SARSA and QLearning opt to hit on anything that has a usable ace, while standing when the player hand value is greater than 17. This simple strategy further supports our hypothesis that the algorithms needed more time to converge towards the optimal policy.

**Dealer Advantage**

Algorithm wise, SARSA is the algorithm with the lowest average dealer advantage, next is QLearning and finally in last place is Monte Carlo Methods with a considerable worse average score.
Of the SARSA implentations, an ε of 0.1 does the best of the 4.
Despite SARSA being the best, due to the fact that both SARSA and QLearning are following the same strategy, this result is purely down to chance and it is difficult to say which of the algorithms actually minimizes dealer advantage.